Will NICA be able to test the various parametrizations of Fragmentation Functions?

Dorota Kotlorz

Opole University of Technology and BLTP JINR

Dubna, October 6, 2022

The goals

E. Christova, E. Leader and D.K. arXiv:2205.02516

- to check compatibility of the published versions of the pion and kaon fragmentation functions with the STAR data on semi-inclusive pion and kaon production in proton-proton collisions
- to try to obtain a new version of the fragmentation functions which agrees better with the STAR data than the published versions
- on the basis of these to make reliable predictions for the p_T spectra of the pions and kaons in inclusive pion and kaon production at the NICA collider

Introduction

Inclusive production of single hadrons is described in terms of

Parton Distribution Functions (PDFs),

parton-parton interaction Cross Sections calculated in the Standard Model (SM) and **Fragmentation Functions (FFs).**

While the PDFs are well known, the flavor-separated quark and gluon FFs are relatively new objects. Most directly the FFs have been extracted from one-hadron production in electron-positron collisions. However, this process in principle, cannot distinguish the quark and anti-quark FFs and information only about $D^h_{q+\bar{q}}$ is obtained.

In order to obtain separate quark and anti-quark FFs, one-hadron semi-inclusive lN and pp processes play an essential role.

Introduction

In arXiv:2205.02516, based on pQCD and taking into account all LO partonic cross sections

B. L. Combridge, J. Kripfganz, and J. Ranft, Phys. Lett. B 70, 234 (1977)
R. Cutler and D. W. Sivers, Phys. Rev. D 17, 196 (1978)
J. F. Owens, E. Reya, and M. Glück, Phys. Rev. D 18, 1501 (1978)
R. P. Feynman, R. D. Field, and G. C. Fox, Phys. Rev. D 18, 3320 (1978)
R. Baier, J. Engels, and B. Petersson, Z. Phys. C 2, 265 (1979),

we have calculated the hadronic p_T-spectra of the final pions and kaons in single hadron production in pp collisions in the high p_T region, and compared it to the STAR data:

J. Adams et al. (STAR), Phys. Lett. B 616, 8 (2005), nucl-ex/0309012
J. Adams et al. (STAR), Phys. Lett. B 637, 161 (2006), nucl-ex/0601033
B. I. Abelev et al. (STAR), Phys. Rev. C 75, 064901 (2007), nucl-ex/0607033
G. Agakishiev et al. (STAR), Phys. Rev. Lett. 108, 072302 (2012), 1110.0579

HKNS-07, AKK-08, DSEHS-14, LSS-15 and DEHSS-17 FFs:

M. Hirai, S. Kumano, T. H. Nagai, and K. Sudoh, Phys. Rev. D 75, 094009 (2007), hep-ph/0702250
S. Albino, B. A. Kniehl, and G. Kramer, Nucl. Phys. B 803, 42 (2008), 0803.2768
D. de Florian, R. Sassot, M. Epele, R. J. Hernandez-Pinto, and M. Stratmann, Phys. Rev. D 91, 014035 (2015), 1410.6027
E. Leader, A. V. Sidorov, and D. B. Stamenov, Phys. Rev. D 93, 074026 (2016), 1506.06381
D. de Florian, M. Epele, R. J. Hernandez-Pinto, R. Sassot, and M. Stratmann, Phys. Rev. D 95, 094019 (2017),

Our analysis was based mainly on the model independent approach of the difference cross sections of h^+ and h^--production,

E. Christova and E. Leader, Phys. Rev. D 79, 014019 (2009), 0809.0191.

It suggests that, both in e^+e^-, SIDIS and proton-proton collisions, if instead of the X-sections $d\sigma^h_{e^+e^-}$, $d\sigma^h_N$ and $d\sigma^h_{pp}$ for inclusive production of hadrons and their antiparticles, one deals with their differences, i.e. with $d\sigma^{h-\bar{h}}_{e^+e^-}$, $d\sigma^{h-\bar{h}}_N$ and $d\sigma^{h-\bar{h}}_{pp} \equiv d\sigma^h_{pp} - d\sigma^{\bar{h}}_{pp}$, one determines directly the non-singlet (NS) combinations of FFs $D^{h-\bar{h}}_q = D^h_{q_V}$.

This result follows solely from charge-conjugation invariance of the strong interactions without any model assumptions about the sea-quarks or favoured and unfavoured fragmentation functions.

In progress

A more accurate treatment

NLO analysis of the inclusive hadron-production at high p_T,

R. K. Ellis, M. A. Furman, H. E. Haber, and I. Hinchliffe, Nucl. Phys. B 173, 397 (1980)
R. K. Ellis and J. C. Sexton, Nucl. Phys. B 269, 445 (1986)
F. Aversa, P. Chiappetta, M. Greco, and J. P. Guillet, Nucl. Phys. B 327, 105 (1989).

New CKL fit to the STAR data on the differences of the cross sections $\sigma_{pp}^{h^+ - h^-}$ for pions and kaons.

Predictions for NICA.

Preliminary conclusions:

The NICA measurements at low $0.4 \leqslant p_T \leqslant 0.5\,\mathrm{GeV}/c$ will be able to provide new constraints on FFs both for charged pions and kaons.

$pp \to h + X$

We consider inclusive hadron production in proton-proton collisions:

$$p(P_A) + p(P_B) \to h(P^h) + X,$$

which proceeds through the hard-collision partonic subprocess:

$$a(p_a) + b(p_b) \to c(p_c) + X.$$

The basic concept underlying the theoretical analysis of most high energy interactions is Factorization.

The cross-section for large momentum-transfer reactions may be factorized into long-distance (nonperturbative) pieces that contain the desired information on the structure of the nucleon in terms of its parton densities such as PDFs and FFs, and short-distance (perturbative) parts which describe the hard interactions of the partons e.g. partonic cross section.

Fragmentation Functions

Fragmentation: hadron production from a quark, antiquark, or gluon.

$D_i^h(z, Q^2)$ - fragmentation function of hadron h from a parton i.

It is probability to find the hadron h from a parton i with the energy fraction z.

Energy conservation:

$$\sum_h \int_0^1 dz\, z D_i^h(z, Q^2) = 1$$

$$h = \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, p, \bar{p}, \ldots$$

The strength of the hard interaction is controlled by the running α_s evaluated at a large Q^2 scale.

Cross Section

In the simple parton model the expression for the cross section for $pp \to hX$ in the c.m.s. of pp has the factorized form:

$$E^h \frac{d\sigma^h_{pp}}{d^3P^h} = \frac{1}{\pi} \sum_{ab\to cd} \int_{x_{a,min}}^1 dx_a \int_{x_{b,min}}^1 dx_b \, \frac{1}{z} \times$$
$$\times \left\{ q_a(x_a) q_b(x_b) \left[\frac{d\hat{\sigma}^{cd}_{ab}}{dt} D^h_c(z) + \frac{d\hat{\sigma}^{cd}_{ab}}{du} D^h_d(z) \right] \right.$$
$$\left. + q_a(x_b) q_b(x_a) \left[\frac{d\hat{\sigma}^{cd}_{ab}}{du} D^h_c(z) + \frac{d\hat{\sigma}^{cd}_{ab}}{dt} D^h_d(z) \right] \right\},$$

where $d\hat{\sigma}^{cd}_{ab}$ are the Born cross sections of order α_s^2.
PDFs $q(x)$ and FFs D^h_i are scale independent and $z = E^h/E_c$.

In the QCD improved parton model:

$$q(x) \to q(x, Q^2), \qquad D^h_q(z) \to D^h_q(z, Q^2).$$

LO Partonic Processes

There are 8 different $2 \to 2$ partonic processes that contribute to $d\sigma^h_{pp}$:

$$\hat{\sigma}_1 : \quad q_i q_j \to q_i q_j, \quad \bar{q}_i \bar{q}_j \to \bar{q}_i \bar{q}_j, \quad q_i \bar{q}_j \to q_i \bar{q}_j, i \neq j$$

$$\hat{\sigma}_2 : \quad q_i q_i \to q_i q_i, \quad \bar{q}_i \bar{q}_i \to \bar{q}_i \bar{q}_i,$$

$$\hat{\sigma}_3 : \quad q_i \bar{q}_i \to q_j \bar{q}_j, \quad i \neq j$$

$$\hat{\sigma}_4 : \quad q_i \bar{q}_i \to q_i \bar{q}_i$$

$$\hat{\sigma}_5 : \quad q_i \bar{q}_i \to gg$$

$$\hat{\sigma}_6 : \quad gg \to q_i \bar{q}_i$$

$$\hat{\sigma}_7 : \quad q_i g \to q_i g$$

$$\hat{\sigma}_8 : \quad gg \to gg$$

$\hat{\sigma}_1$: $q_i q_j \to q_i q_j$, $\bar{q}_i \bar{q}_j \to \bar{q}_i \bar{q}_j$, $q_i \bar{q}_j \to q_i \bar{q}_j$, $\bar{q}_i q_j \to \bar{q}_i q_j$, $i \neq j$

$$\begin{aligned}\hat{\sigma}_1: \quad \sum_{i\neq j} \quad & q_i(x_a)q_j(x_b)\left[D^h_{q_i}\frac{d\hat{\sigma}_1}{dt} + D^h_{q_j}\frac{d\hat{\sigma}_1}{du}\right] \\ + \quad & \bar{q}_i(x_a)\bar{q}_j(x_b)\left[D^h_{\bar{q}_i}\frac{d\hat{\sigma}_1}{dt} + D^h_{\bar{q}_j}\frac{d\hat{\sigma}_1}{du}\right] \\ + \quad & q_i(x_a)\bar{q}_j(x_b)\left[D^h_{q_i}\frac{d\hat{\sigma}_1}{dt} + D^h_{\bar{q}_j}\frac{d\hat{\sigma}_1}{du}\right] \\ + \quad & \bar{q}_i(x_a)q_j(x_b)\left[D^h_{\bar{q}_i}\frac{d\hat{\sigma}_1}{dt} + D^h_{q_j}\frac{d\hat{\sigma}_1}{du}\right]\end{aligned}$$

$i \neq j: \ i = u, j = d, s; \ i = d, j = u, s, \quad i = s, j = u, d.$

$$\frac{d\hat{\sigma}_i(ab \to cd)}{dt} = \frac{\pi\alpha_s^2(Q^2)}{s^2}\,|M_i(s,t,u)|^2$$

NLO

A complete $O(\alpha_s^3)$ calculation of the parton-parton scattering contributing to inclusive production of a hadron at large transverse momenta was presented by

F. Aversa, P. Chiappetta, M. Greco, and J. P. Guillet,
Nucl. Phys. B 327, 105 (1989).

The result includes the radiative corrections for all parton subprocesses involving quarks and gluons.

Numerical support:

- INCNLO: P. Aurenche, T. Binoth, M. Fontannaz, J.-P. Guillet, G. Heinrich, E. Pilon and M. Werlen,
  https://lapth.cnrs.fr/PHOX_FAMILY/readme_inc.html
- Kumano et. al.'s code for calculating the FFs:
  http://research.kek.jp/people/kumanos/ffs.html

The choice $AL = 1$ is mandated by the equivalent choice made in the two loop anomalous dimensions. This ambiguity does not affect the leading terms c_i in eq. (22) and is numerically irrelevant. We have used $AL = 0$ in our numerical results.

This concludes our discussion about the basic formulas for the one hadron and jet inclusive cross sections. We present in the next section our detailed analytical results for the various processes.

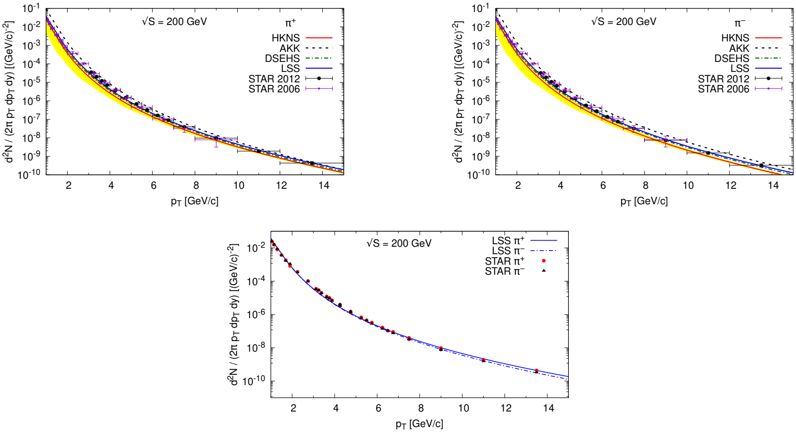
3. $O(\alpha_s^3)$ cross sections for parton–parton scattering processes

We consider the following inclusive processes:

$$\begin{aligned}
&(\mathrm{A01}) \quad q_j q_k \to q_j + X, \\
&(\mathrm{A02}) \quad \to q_k + X, \\
&(\mathrm{A03}) \quad \to g + X, && \text{(24a)} \\
&(\mathrm{A11}) \quad q_j \bar{q}_k \to q_j + X, \\
&(\mathrm{A12}) \quad \phantom{q_j \bar{q}_k} \to \bar{q}_k + X, \\
&(\mathrm{A13}) \quad \phantom{q_j \bar{q}_k} \to g + X, && \text{(24b)} \\
&(\mathrm{A21}) \quad q_j \bar{q}_j \to \bar{q}_k + X, \\
&(\mathrm{A22}) \quad \phantom{q_j \bar{q}_j} \to q_k + X, && \text{(24c)} \\
&(\mathrm{B01}) \quad q_j q_j \to q_j + X, \\
&(\mathrm{B02}) \quad \to g + X, && \text{(24d)} \\
&(\mathrm{C01}) \quad q_j g \to q_k + X, \\
&(\mathrm{C02}) \quad \to \bar{q}_k + X, \\
&(\mathrm{C03}) \quad \to \bar{q}_j + X, && \text{(24e)} \\
&(\mathrm{D01}) \quad q_j \bar{q}_j \to q_j + X, \\
&(\mathrm{D02}) \quad \phantom{q_j \bar{q}_j} \to \bar{q}_j + X, \\
&(\mathrm{D03}) \quad \phantom{q_j \bar{q}_j} \to g + X, && \text{(24f)} \\
&(\mathrm{E01}) \quad q_j g \to q_j + X, \\
&(\mathrm{E02}) \quad \to g + X, && \text{(24g)} \\
&(\mathrm{F01}) \quad gg \to g + X, \\
&(\mathrm{F02}) \quad \to q_j + X, && \text{(24h)}
\end{aligned}$$

PIONS

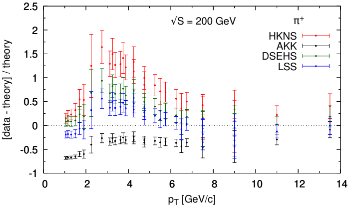
The cross sections $\sigma_{pp}^{\pi^+}$ and $\sigma_{pp}^{\pi^-}$ calculated for different sets of FFs, compared to STAR data.

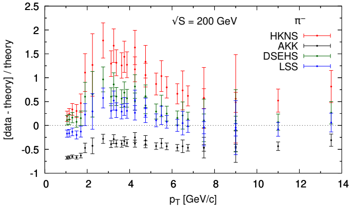


LSS: NLO QCD analysis of the HERMES data on pion multiplicities.

PIONS

The ratio (data-theory)/theory for the cross sections

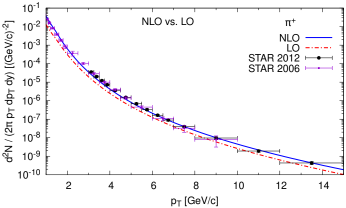


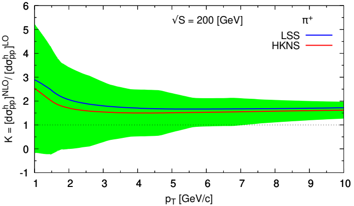


PIONS

NLO vs LO

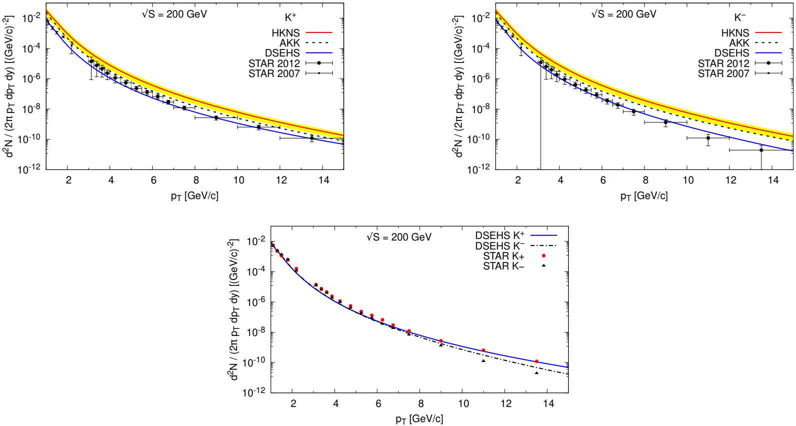
K-factor: the ratio between the NLO and the ‘Born’ cross section





KAONS

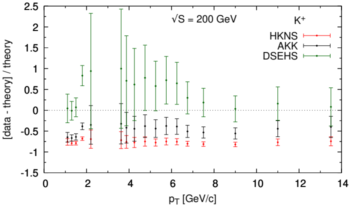
The cross sections $\sigma_{pp}^{K^+}$ and $\sigma_{pp}^{K^-}$ calculated for different sets of FFs, compared to STAR data.

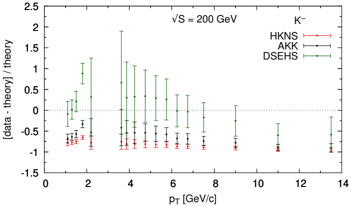


DSEHS: electron-positron annihilation, lepton-nucleon deep-inelastic scattering, and proton-proton collisions including STAR 2012.

KAONS

The ratio (data-theory)/theory for the cross sections

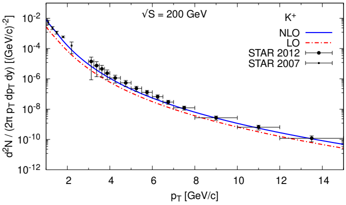


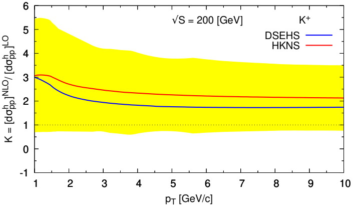


KAONS

NLO vs LO

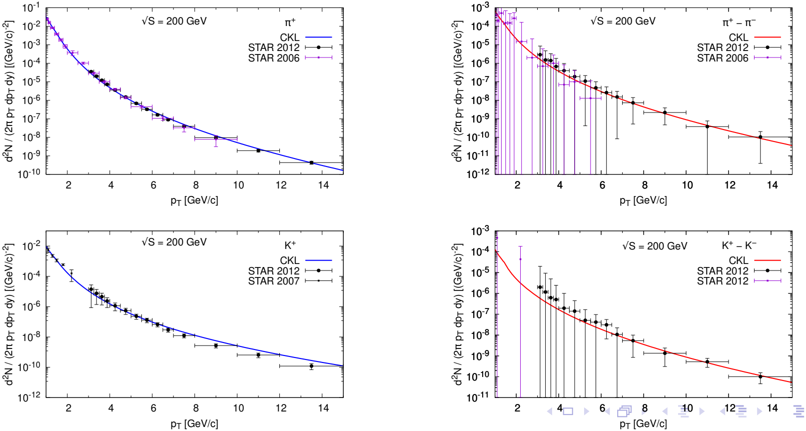
K-factor: the ratio between the NLO and the 'Born' cross section





CKL FIT to STAR data

$$D_{q_i}^{h^+}(z) = \frac{M_i\, z_i^{a}(1-z)^{b_i}}{B(a_i+2, b_i+1)}$$



The Best Fit

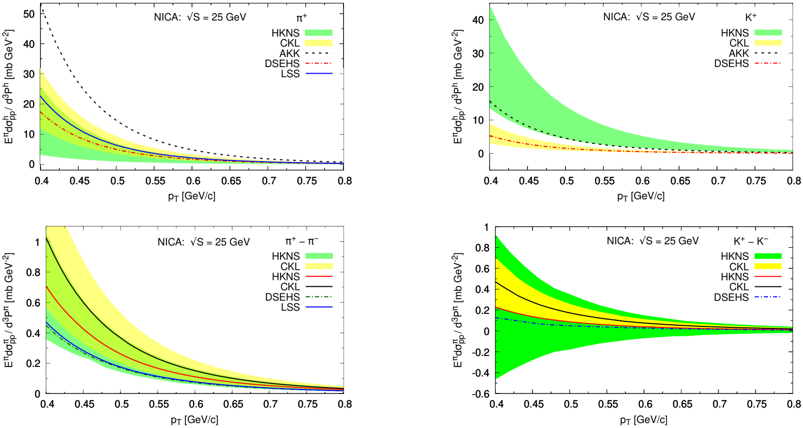
$$\chi^2 = \frac{1}{N}\sum_i^N \frac{(\sigma_i^{\text{exp}} - \sigma_i^{\text{th}})^2}{\Delta_i^2}$$

FFs	π^+	π^-	K^+	K^-	$\pi^+ - \pi^-$	$K^+ - K^-$
HKNS	10.2	11.7	124	222	0.136	0.335
AKK	77.2	83.3	16.0	34.1	-	-
DSEHS	4.55	4.39	1.18	1.31	0.148	0.329
LSS	2.83	2.46	-	-	0.143	-
CKL	1.76	1.98	2.98	9.82	0.128	0.104

We take into account the combined STAR data for $p_T > 1\ \mathrm{GeV}/c$ with $N = 30$ experimental points for pions (STAR 2006 and STAR 2012), and $N = 17$ for kaons (STAR 2007 and STAR 2012), respectively.

Predictions for NICA energies

$\sigma_{pp}^{\pi^+}$, $\sigma_{pp}^{K^+}$ **and the difference cross sections calculated for different sets of FFs including CKL**



Conclusions

- Using NLO approach, we have compared the predictions for the semi-inclusive cross-sections $pp \to \mathfrak{pions} + X$ and $pp \to \mathfrak{kaons} + X$ based on various FFs with the data from the STAR collaboration.
- LSS for pions and DSEHS for kaons are significantly better parametrizations than the others.
- A new one, CKL, provides a very good fit for the STAR data on pions and on the difference cross-sections $d\sigma_{pp}^{\pi^+ - \pi^-}$ and $d\sigma_{pp}^{K^+ - K^-}$.
- Based on the above viable FFs, we have produced estimates of various semi-inclusive cross-sections at NICA energies.
- Both for pion and kaon production, the differences between the predictions of HKNS, AKK, DSEHS, LSS and CKL become significant at the low p_T range, $0.4 \leqslant p_T \leqslant 0.5\,\mathfrak{GeV}/c$.

It is quite obviously pointless to ask after the nature of things, since such a nature, even if it should exist, always remains beyond all experience.

Anton Zeilinger