## Yury Senichev <br> Institute for Nuclear Research of Russian Academy of Sciences

Search for EDM at NICA:
Frequency Domain procedure to measure the electric dipole moment of the deuteron with an sensitivity of $10^{-29-}-10^{-30}$

The analysis done by the AD Sakharov, showed that this CPviolation is absolutely necessary to explain why on earth and in the visible universe there is a MATTER, but there is practically no ANTIMATTER

First message to search for Electric Dipole Moments(EDM) of fundamental particles: it came to understand the CP violation

Second message for Electric Dipole Moments of fundamental particles: the baryon asymmetry of the Universe that represents the fact of the prevalence of matter over antimatter

## Current results for neutron:



## LOWERING THE BAR

The standard model of particle physics predicts a small electric dipole moment (EDM) for the neutron, well below the sensitivity of previous experiments. A larger dipole, predicted by some versions of supersymmetry, should lie within the range of three current experiments.


## Current achievements in EDM measurement for fundamental particles:

The Standart Model predicts tiny non-vanishing values for EDMs of elementary particles for:
neutron EDM $d_{n} \sim 10^{-31} \div 10^{-32} \mathrm{e} \cdot \mathrm{cm}$,
electron EDM $d_{e} \sim 10^{-40} \mathrm{e} \cdot \mathrm{cm}$,
muon EDM $d_{n} \sim 10^{-38} \mathrm{e} \cdot \mathrm{cm}$.

## but

Theories beyond the Standard Model provides EDMs that are several orders of magnitude higher such as SUSY models where neutron EDM is of the order of $d_{n} \sim 10^{-26} \div 10^{-30} \mathrm{e} \cdot \mathrm{cm}$.

## at present

Despite of the efforts being made, an EDM of any elementary particle has not been found yet, and we have the limit estimation
neutron EDM $d_{n}<2.9 \cdot 10^{-26} \mathrm{e} \cdot \mathrm{cm}$ (with certainty $90 \%$ ),
electron EDM $d_{e}<10^{-29} \mathrm{e} \cdot \mathrm{cm}$, (with certainty 90\%)
muon EDM $d_{\mu}<1.8 \cdot 10^{-19} \mathrm{e} \cdot \mathrm{cm}$, (with certainty 95\%)
proton EDM $d_{p}<5.4 \cdot 10^{-24} \mathrm{e} \cdot \mathrm{cm}$ (without statistic estimation)
EDM opens the door to the "New Physics" and sheds light on the mystery of our Universe creation.

Теоре́ма Эренфе́ста: При движении частицы средние значения этих величин в квантовой механике изменяются так, как изменяются значения этих величин в классической механике. Ю.Узиков (ОИЯИ) специально обосновал этот переход для спина.
The spin is a quantum value, but in the classical physics representation the "spin" means an expectation value of a quantum mechanical spin operator:
Basic principle of EDM measurement in ring comes from "ThomasBargmann, Michel,Telegdi" equation with EDM term:

$$
\begin{aligned}
& \frac{d \vec{S}}{d t}=\vec{\Omega} \times \vec{S} \\
& \vec{\Omega}=-\frac{e}{m}\left\{G \vec{B}+\left(\frac{1}{\gamma^{2}-1}-G\right)\left(\frac{\vec{\beta} \times \vec{E}}{\mathrm{c}}\right)+\frac{\eta}{2}\left(\frac{\vec{E}}{c}+\vec{\beta} \times \vec{B}\right)\right\} \\
& \begin{array}{l}
G=\frac{g-2}{2}, G \text { is the anomaly of magnetic moment, gis the } \\
\text { gyromagnetic ratio }
\end{array} \\
& d=\eta e \hbar / 4 m c=\mathrm{EDM} \\
& \eta=\frac{4 d \cdot m c^{2}}{e \hbar c}=\frac{4 \cdot\left[10^{-31} e \cdot m\right] \cdot[938.272 M e V]}{e \cdot\left[6.582 \cdot 10^{-22} M e V \cdot \sec \right] \cdot[2.9979 m / \mathrm{sec}]} \approx 2 \cdot 10^{-15}
\end{aligned}
$$

## Frozen spin method for purely electrostatic proton ring at "magic" energy

In the FS method the beam is injected in the electrostatic ring with the spin directed along momentum $\mathbf{S} \| p$ and $S \perp E ; S=\left\{0,0, S_{z}\right\}$ and $E=\left\{E_{x}, 0,0\right\}$

$$
\begin{aligned}
& \text { at "magic" energy : } \frac{1}{\gamma_{\text {mag }}^{2}-1}-G=0 \\
& \qquad \begin{array}{l}
\frac{d \vec{S}}{d t}=\vec{\Omega} \times \vec{S} \\
\vec{\Omega}=-\frac{e}{m}\left\{G \vec{B}+\left(\frac{1}{\gamma^{2}-1}-G\right) \frac{(\vec{\beta} \times \vec{E})}{c}+\frac{\eta}{2}\left(\frac{\vec{E}}{c}+\vec{\beta} \times \vec{B}\right)\right\} \\
G=\frac{g-2}{2},
\end{array} \\
& \text { MDM spin frequency EDM spin frequency }
\end{aligned}
$$

## Frozen spin method for purely electrostatic proton ring at "magic" energy

In the method the beam is injected in the electrostatic ring with the spin directed along momentum $\mathbf{S} \| \mathrm{p}$ and $\mathrm{S} \perp \mathrm{E} ; \mathrm{S}=\left\{0,0, \mathrm{~S}_{\mathbf{z}}\right\}$ and $\mathrm{E}=\left\{\mathrm{E}_{\mathrm{x}}, \mathbf{0}, \mathbf{0}\right\}$

$$
\text { at "magic" energy : } \frac{1}{\gamma_{\text {mag }}^{2}-1}-G=00
$$

$$
\begin{aligned}
& \overrightarrow{d t}=-\frac{e}{m}\{ \\
& \vec{\Omega}= \\
& g-2
\end{aligned}+\frac{\eta}{2}\left(\frac{\vec{E}}{c}+\right.
$$



MDM spin frequency
EDM spin frequency

## EDM growth in FS concept

$$
d \vec{S}=-\frac{e \eta}{2 m}[\vec{E} \times \vec{S}] d t
$$

## Frozen Spin lattice for deuteron

The condition of the zero MDM spin precession frequency in FS lattice [1,2]

$$
\Omega_{M D M}=G \overrightarrow{B_{y}}+\left(\frac{1}{\gamma^{2}-1}-G\right)\left(\frac{\overrightarrow{\beta_{z}}}{c} \times \overrightarrow{E_{x}}\right)=0
$$

creates the relation between $E$ and $B$ fields in incorporated bending elements:

$$
E_{r} \approx G B \phi \gamma^{2}
$$



Frozen Spin lattice based on B+E elements and TWISS functions

## Quasi-Frozen Spin lattice

ARC with straight section


## QFS in COSY ring and NICA



## Main requirements to the experiment for search

for EDM in the ring:

1. Beam optics (betatron tunes, sextupoles, DA, RF, straignt sections and so on)
2. Spin coherence time maximizing up to $t_{c o h}>1000 \mathrm{sec}$ to provide the possible EDM signal observation
3. Systematic errors investigation to exclude "fake EDM signal"
4. Maximum beam polarization $\mathrm{P} \sim 80 \%$
5. Beam intensity $\sim 10^{10} \div 10^{11}$ particle per fill
6. Maximum analyzing power of polarimeter $\mathrm{A} \sim 0.6$
7. Maximum efficiency of polarimeter $f>10^{-3}$
8. Total running time of accelerator $\sim 5 \div 7$ thousand hours

## Что уже сделано на данный момент?

1. Разработана методика подавления декогеренции спина на COSY кольце за счет секступолей и экспериментально достигнуто время сохранения поляризации порядка 1000 секунд
2. Экспериментально достигнута абсолютная точность измерения прецессии спина на уровне $10^{-7}$ за один run.
3. Исследовано влияние e-cooling на длительность spin coherence time.
4. Разработан Frequency Domain Method исследования EDM малочувствительный к систематическим ошибкам.
5. Изучена 3D спин-орбитальная динамика пучка

## Cooler Synchrotron COSY in Jülich



## EDDA Polarimeter

- Left-Right asymmetry
$\Rightarrow$ vertical polarization

$$
P_{V} \propto \epsilon_{v e r}=\frac{N_{l}-N_{r}}{N_{l}+N_{r}}
$$



- Up-Down asymmetry
$\Rightarrow$ horizontal polarization
$P_{H} \propto \epsilon_{\text {hor }}=\frac{N_{u p}-N_{d n}}{N_{u p}+N_{d n}}$


$$
\epsilon\left(\varphi_{\mathrm{s}}\right)=\frac{N_{D}^{-}\left(\varphi_{\mathrm{s}}\right)-N_{U}^{-}\left(\varphi_{\mathrm{s}}\right)}{N_{D}^{+}\left(\varphi_{\mathrm{s}}\right)+N_{U}^{+}\left(\varphi_{\mathrm{s}}\right)}
$$

## Measurement of polarization asymmetry

An rf solenoid-induced spin resonance was employed to rotate the spin by $90^{\circ}$ from the initial vertical direction into the transverse horizontal direction. Subsequently, the beam was slowly extracted onto an internal carbon target using a white noise electric field applied to a stripline unit. Scattered deuterons were detected in scintillation detectors, consisting of rings and bars around the beam pipe[, and their energy deposit was measured by stopping them in the outer scintillator rings. The event arrival times, with respect to the beginning of each cycle and the frequency of the COSY if cavity, were recorded in one long-range time-to digital converter; i.e.,the same reference clock was used for all signals.


FIG.
(a) Counts $N_{U}$ and $N_{D}$ after mapping the events recorded during a turn interval of $\Delta n=10^{6}$ turns into a spin phase advance interval of $4 \pi$. (b) Count sums $N_{U, D}^{+}\left(\varphi_{s}\right)$ and differences $N_{\bar{U}, D}^{-}\left(\varphi_{\mathrm{s}}\right)$ of Eq. (6) with $\varphi_{\mathrm{s}} \in[0,2 \pi)$ using the counts $N_{U}\left(\varphi_{\mathrm{s}}\right)$ and $N_{D}\left(\varphi_{\mathrm{s}}\right)$, shown in panel (a). The vertical error bars show the statistical uncertainties, the horizontal bars indicate the bin width.

## Spin Tune Measurement

Spin vector precesses with $f_{\text {Spin }}=v f_{\text {rev }}$ in the horizontal plane

Asymmetry given by:

$$
\epsilon_{V}(t)=\frac{N_{u p}-N_{d n}}{N_{u p}+N_{d n}} \approx A P(t) \sin \left(2 \pi v f_{r e v} t+\phi\right)
$$

What do we expect? (Deuterons, $p=0.97 \mathrm{GeV} / \mathrm{c}$ )

$$
v \approx 0.16, \quad f_{\text {rev }}=750 \mathrm{kHz}
$$

Spin precession frequency: $v \cdot f_{\text {rev }} \approx 125 \mathrm{kHz}$
Detector rates: 5 kHz
Only every $25^{\text {th }}$ spin revolution is detected
$\Rightarrow$ No direct fit is possible

Figure Of Merit vs Energy: choice of energy




## Spin Coherence Time (SCT)

- Sensitivity of an EDM measurement is proportional to polarization life time
- Spin precesses with $f_{s}=\gamma G \cdot f_{\text {rev }} \approx 120 \mathrm{kHz} \longrightarrow$ decoherence
- Energy spread leads to different spin precession frequencies

- Spins decohere $->$ Loss of polarization
- Typical time scale is the Spin Coherence Time (SCT)


## Spin tune coherence:

RF field as a method for mix particles of energy
RF field $\quad \rightarrow \Delta \gamma=\Delta \gamma_{m} \cdot \cos \left(\Omega_{\text {synch }} t+\varphi\right)$
The longitudinal tune (number of longitudinal oscillations per turn) has to be one-two orders bigger of the spin tune spread without RF field:

$$
v_{z}=\frac{1}{\beta_{s}} \sqrt{\frac{e \hat{V} h \eta}{2 \pi E_{s}}} \gg v_{s}=\gamma G \cdot \frac{\Delta \gamma}{\gamma}
$$

With RF we increase SCT from $10^{-3} \mathrm{sec}$ up to $10^{2} \mathrm{sec}$

## Orbit lengthening effects and effective gamma

Momentum deviation is described by eq:

$$
\frac{d^{2} \delta}{d t^{2}}+\frac{e V_{r f} \omega_{r f}}{\beta^{2} E}\left(\alpha_{0}-\frac{1}{\gamma^{2}}\right) \cdot \delta=\frac{e V_{r f} \omega_{r f}}{\beta^{2} E} \cdot\left[-\left(\alpha_{1}-\frac{\alpha_{0}}{\gamma^{2}}+\frac{1}{\gamma^{4}}\right) \cdot \delta^{2}-\left(\frac{\Delta L}{L}\right)_{\beta}\right]
$$

Thus due to the betatron oscillation, the square term of momentum compaction factor $\alpha_{1}$
and the slip factor $\eta$ dependent on the equilibrium level energy $\delta=0$ is shifted by the value :

$$
\begin{aligned}
& \Delta \delta_{e q}= \frac{\gamma_{s}^{2}}{\gamma_{s}^{2} \alpha_{0}-1}\left[\frac{\delta_{m}^{2}}{2}\left(\alpha_{1}-\frac{\alpha_{0}}{\gamma_{s}^{2}}+\frac{1}{\gamma_{s}^{4}}\right)+\left(\frac{\Delta L}{L}\right)_{\beta}\right] \\
&\left(\frac{\Delta L}{L}\right)_{\beta}= \frac{1}{L} \oint\left(\frac{\rho+x_{\beta}}{\rho \cos \theta}-1\right) d s=\frac{1}{L} \oint\left(\frac{x_{\beta}}{\rho}+\frac{x_{\beta}^{\prime 2}+y_{\beta}^{\prime 2}}{2}\right) d s \\
& \gamma_{e f f}=\gamma_{s}+\beta_{s}^{2} \gamma_{S} \cdot \Delta \delta_{e q}
\end{aligned}
$$





## Orbit lengthening in sextupole field

Orbit lengthening due to sextupoles placed in non-zero dispersion:

$$
\begin{array}{r}
\left(\frac{\Delta L}{L}\right)_{\frac{x}{\rho}}=\frac{1}{L} \oint \frac{x}{\rho} d s=\frac{\delta}{L} \int \frac{D_{x}}{\rho} d s-\frac{\delta^{2}}{L} \sum_{i} S_{i} l_{s i} D_{x i}^{3}- \\
-\frac{\varepsilon_{x}}{2 L} \sum_{i} S_{i} l_{s i} D_{x i} \beta_{x i}+\frac{\varepsilon_{y}}{2 L} \sum_{i} S_{i} l_{s i} D_{x i} \beta_{y i} \\
\text { 1-st family 2-d family 3-d family }
\end{array}
$$

and zero-dispersion:

$$
\left(\frac{\Delta L}{L}\right)_{x^{\prime}, y^{\prime}}=\frac{1}{L} \oint \frac{x^{\prime 2}+y^{\prime 2}}{2} d s=\frac{5}{4}\left(\frac{\varepsilon_{x} \frac{1}{L} \sum_{i j} \mathcal{S} \beta_{x i}^{2}}{3}\right)^{2}+\frac{\varepsilon_{x}}{4 \beta_{x}}+\frac{\varepsilon_{y}}{4 \beta_{y}}
$$

After the sextupole correction:

$$
\begin{gathered}
\left(\frac{\Delta L}{L}\right)_{\frac{x}{\rho}}=\alpha_{0} \delta+\alpha_{1} \delta^{2}-\frac{\delta^{2}}{L} \sum_{i} S_{i} l_{s i} D_{x i}^{3} \cdots \\
\left(\frac{\Delta L}{L}\right)_{x^{\prime}, y^{\prime}}=\frac{\varepsilon_{x}}{4 \beta_{x}}-\frac{\varepsilon_{x}}{2 L} \sum_{i} S_{i} l_{s i} D_{x i} \beta_{x i}+\frac{\varepsilon_{y}}{4 \beta_{y}}+\frac{\varepsilon_{y}}{2 L} \sum_{i} S_{i} l_{s i} D_{x i} \beta_{y i}
\end{gathered}
$$

## Scan of MXG value



Best Spin Coherence Time: $\tau_{S C T} \approx 1000 s$

## Spin tune coherence: COSY ring experiment

In COSY ring in regime of "non-frozen spin " we experimentally have proved that
-SCT~1000 sec can be reached;
-spin tune measurement with relative errors $10^{-10}$ is possible, which will allow calibrating the particle energy using the clock-wise and counter clock-wise procedure.
D. Eversmann et al., (JEDI Collaboration)

New method for a continuous determination of the spin tune in storage rings and implications for precision experiments,
Phys. Rev. Lett. 115, 094801 (2015)
G. Guidoboni et al. (JEDI Collaboration) How to Reach a Thousand-Second in-Plane Polarization Lifetime with $0.97-G e V / c$ Deuterons in a Storage Ring
Phys. Rev. Lett. 117, 054801, (2016)

## E-cooling, MCF second order and spin decoherence

If the equilibrium energy $\Delta \gamma_{e q}$ depends on the particle parameters the spin tune spread for $N_{t}$ turns has incoherent spread

$$
2 \pi\left\langle\Delta v_{s}\right\rangle=2 \pi G\left\langle\Delta \gamma_{e q}\right\rangle N_{t}
$$

It reduces the spin coherence time SCT. For example, let us consider the case with the spin coherence time (SCT) limited by 1000 seconds ( $\sim 10^{9}$ turns) and $2 \pi G\left\langle\Delta \gamma_{e q}\right\rangle N_{t} \leq 1 \mathrm{rad}$ :

$$
\left\langle\frac{\Delta \gamma_{e q}}{\gamma}\right\rangle<\frac{1 \mathrm{rad}}{2 \pi \gamma G N_{t}}=\frac{1}{2 \cdot 3.14159 \cdot 1.24 \cdot 1.79 \cdot 10^{9}}=7 \cdot 10^{-11}
$$

For the momentum deviation we have:

$$
\left\langle\delta_{m}^{2}\right\rangle<\left\langle\frac{\Delta \gamma_{e q}}{\gamma}\right\rangle \cdot \frac{2}{\beta^{2}} \cdot \frac{\gamma_{s}^{2} \cdot\left(\gamma_{s}^{2} \alpha_{0}-1\right)}{\gamma_{s}^{4} \alpha_{1}-\gamma_{s}^{2} \alpha_{0}+1}
$$

At $\alpha_{0}=0.2, \gamma_{s}=1.248 \quad \alpha_{1}=2$ and $\varepsilon_{x, y}=0$ the rms momentum spread should not exceed $\left\langle\delta_{m}\right\rangle<8 \cdot 10^{-6}$ the value, and reducing the second order of MCF up to $\alpha_{1}=0.01$ we get $\quad\left\langle\delta_{m}\right\rangle<2 \cdot 10^{-5}$.

## E-cooling, MCF second order and spin decoherence

Now let us estimate the restriction for the emittance value:

$$
\varepsilon_{x y r m s}<\left\langle\frac{\Delta \gamma_{e q}}{\gamma}\right| \cdot \frac{1}{\beta^{2}} \cdot \frac{\gamma_{s}^{2} \alpha_{0}-1}{\gamma_{s}^{2}} \cdot \frac{L}{\pi v_{x, y}}
$$

At $\alpha_{0}=0.2, \quad \gamma_{s}=1.248, L=1.83 \cdot 10^{5}, v_{x, y}=3.6$ and $\delta_{m}=0$ both emittances should be $\varepsilon_{x y r m s}<1.4 \mathrm{~mm}$ mrad

From these estimations we can conclude that the contribution to the spin tune decoherence is the same for the rms values of emittance $\varepsilon_{x y r m s} \approx 1 \mathrm{~mm} \mathrm{mrad}$ and momentum spread $\quad \delta_{r m s} \approx 10^{-5}$.

## Systematic errors:

1. BNL method
2. Frequency Domain method

## Radial $B_{r}$, vertical $E_{v}$ fields

## fake EDM signal

The presence of errors in the installation of the elements (imperfections) of the ring leads to the appearance of vertical and radial components of the electric and magnetic fields, respectively.


They both change the spin components in the vertical plane, in which the EDM signal is expected, and create the systematic errors that initiate the "fake EDM" signal.

## Main ideas of BNL method:

1. 3D Frozen spin
2. CW and CCW options to minimize the systematic errors

## CW and CCW procedures

To solve this problem in the case of a proton beam, it was suggested that the procedure of simultaneously injecting two beams in the ring in two opposite directions, clockwise (CW) and counter clockwise (CCW) [BNL], be used.

$$
\Omega_{E D M}=\left(\Omega_{C W}+\Omega_{C C W}\right) / 2+\left(\Omega_{B X}^{C C W}-\Omega_{B X}^{C W}\right) / 2
$$



Adding the CW and CCW results together, the EDM can be separated from a systematic error arising due to MDM.

## BNL: what does the method measure?

In the frozen spin regime it measures the vertical projection of the spin, that is, the amplitude of the changing part of the signal $\widetilde{s}_{y}$ during a long time $\sim 1000 \mathrm{sec}$, namely
the solution at the initial

$$
\begin{aligned}
& \tilde{S}_{y}=\sqrt{\left(\frac{\Omega_{y} \Omega_{z}}{\Omega^{2}}\right)^{2}+\left(\frac{\Omega_{x}}{\Omega}\right)^{2}} \sin (\alpha+\phi) \\
& \alpha=\Omega \cdot \mathrm{t}
\end{aligned}
$$

$$
\Omega_{x}=\Omega_{e d m}+\Omega_{B_{r}} \quad \Omega=\sqrt{\left(\Omega_{e d m}+\Omega_{B_{r}}\right)^{2}+\Omega_{y}^{2}+\Omega_{z}^{2}}
$$

where $\Omega_{x}, \Omega_{y}, \Omega_{z} \quad$ are frequencies of spin precession in $\mathrm{X}, \mathrm{Y}$ and $Z$ planes.

## BNL: requirements to imperfections

First, the accuracy of the installation of magnets should be less than $10^{-9}$ meters (super unrealistic !!!);

Second, the amplitude of $\widetilde{\varsigma}_{y}$ depends on $\Omega_{x}, \Omega_{y}, \Omega_{z}$,

$$
\begin{aligned}
& \tilde{S}_{y}=\sqrt{\left(\frac{\Omega_{y} \Omega_{z}}{\Omega}\right)^{2}+\left(\frac{\Omega_{B_{r}}+\Omega_{e d m}}{\Omega}\right)^{2}} \sin (\Omega t+\phi) \\
& \alpha=\Omega \cdot \mathrm{t} \quad \text { where } \Omega^{2}=\left(\Omega_{B_{r}}+\Omega_{e d m}\right)^{2}+\Omega_{y}^{2}+\Omega_{z}^{2}
\end{aligned}
$$

At $\quad \Omega t \approx 10^{-6}$ $\square$

$$
\tilde{s}_{y}=\sqrt{\left(\Omega_{y} \Omega_{z}\right)^{2}+\left(\Omega_{B_{r}}+\Omega_{e d m}\right)^{2}} \cdot t
$$

and $\Omega_{y}, \Omega_{\mathrm{z}}, \Omega_{B_{r}}$ of the same order with $\Omega_{\mathrm{edm}}$ we do not know the value of $\tilde{S}_{y}$.

## BNL: the geometric phase

the GEOMETRIC PHASE occurs when the invariant spin axis changes direction from element to element, and when the total angle of spin rotation in each of the planes

$$
\sum_{i}^{2 n} \delta_{i}=0
$$

is zero, nevertheless after n-pairs of elements of one turn we have the non-zero MDM deviation:

$$
S_{\mathrm{y}}^{\sum}=S_{y}^{0}\left(1-n \delta^{2}\right)
$$

This is an real fact!!!

## BNL: criterion for minimizing the contribution of the geometric phase

Obviously, this contribution should be less than the EDM angle rotation per turn

$$
S_{\mathrm{y}}^{\sum}=S_{y}^{0}\left(1-n \delta^{2}\right)
$$

that is

$$
n \delta^{2}<\alpha_{e d m} \approx 10^{-6} / 10^{9}=10^{-15}
$$

Thus, at $\mathrm{n} \approx 100$ elements per one turn we must provide:

$$
n \delta^{2}<\alpha_{e d m} \approx 10^{-6} / 10^{9}=10^{-15} \text { or } \delta \approx 10^{-8} \div 10^{-9}
$$

This is again super unreal!!!

# BNL: how to restore conditions for a polarized beam at passing from CW to CCW 

no reasonable ideas

## Frequency Domain Method: what do we measure?

In Frequency Domain method we measure instead amplitude
$\tilde{S}_{y}=\sqrt{\left(\frac{\Omega_{y} \Omega_{z}}{\Omega^{2}}\right)^{2}+\left(\frac{\Omega_{x}}{\Omega}\right)^{2}} \sin [\Omega t+\phi]$,
the frequency

$$
\tilde{S}_{y}=\sqrt{\left(\frac{\Omega_{y} \Omega_{z}}{\Omega^{2}}\right)^{2}+\left(\frac{\Omega_{x}}{\Omega}\right)^{2}} \sin [\Omega t+\phi],
$$



## Frequency Domain Method: systematic errors

Let us to consider the case when in

$$
\Omega=\sqrt{\left(\Omega_{e d m}+\Omega_{B_{r}}\right)^{2}+\Omega_{\mathrm{B}_{\mathrm{v}}, E_{r}}^{2}+\Omega_{\mathrm{B}_{\mathrm{z}}}^{2}}
$$

We can not realize relation: $\Omega_{\mathrm{B}_{,}, E_{r}}^{2}, \Omega_{\mathrm{B}_{z}}^{2} \ll \Omega_{\text {edm }}^{2}$
But we can:

$$
\Omega_{B_{v}, E_{r}}^{2}, \Omega_{B_{z}}^{2} \ll\left(\Omega_{e d m}+\Omega_{B_{r}}\right)^{\prime}
$$

It is so called 2D frozen spin option:

$$
\left(\Omega_{B_{r}}\right)^{2} \gg \Omega_{\mathrm{B}_{v}, E_{r}}^{2}, \Omega_{\mathrm{B}_{z}}^{2}
$$

## Frequency Domain Method: <br> 3D frozen spin $\Longrightarrow$ 2D frozen spin

$$
\begin{array}{ll}
\Omega=\left(\Omega_{e d m}+\Omega_{B_{r}}\right)\left[1+\frac{1}{2} \frac{\Omega_{B_{v}, E_{r}}^{2}+\Omega_{B_{z}}^{2}}{\left(\Omega_{e d m}+\Omega_{B_{r}}\right)^{2}}\right] & \Rightarrow \Omega_{B_{r}}+\Omega_{e d m}+\frac{1}{2} \frac{\Omega_{B_{v}, E_{r}}^{2}+\Omega_{B_{z}}^{2}}{\Omega_{e d m}+\Omega_{B_{r}}} \\
\Omega_{e d m}>\frac{1}{2} \frac{\Omega_{B_{v}, E_{r}}^{2}+\Omega_{B_{z}}^{2}}{\Omega_{e d m}+\Omega_{B_{r}}} \quad \text { finally } \Rightarrow \quad & \Rightarrow \quad \Omega_{B_{r}}>\frac{1}{2} \frac{\Omega_{B_{v}, E_{r}}^{2}+\Omega_{B_{z}}^{2}}{\Omega_{e d m}}
\end{array}
$$

Main idea: to make the contribution from EDM frequency into the total frequency $\Omega$ bigger than from MDM additions $\Omega_{\mathrm{B}_{\mathrm{v}}, E_{r}}^{2}, \Omega_{\mathrm{B}_{\mathrm{z}}}^{2}$

## Frequency Domain Method: 2D frozen spin

Thus, we do not require 3D frozen spin when all three frequencies close to zero:

$$
\Omega_{y}, \Omega_{z}, \Omega_{B_{r}}=>0 \text { up to } \Omega_{y}, \Omega_{z}, \Omega_{B_{r}} \leq \Omega_{\mathrm{edm}}
$$

Now we realize 2D frozen spin with simple condition:

$$
\Omega_{B_{r}}\left(\sim 10^{2} \mathrm{rad} / \mathrm{sec}\right) \gg \Omega_{e d m}\left(\sim 10^{-9} \mathrm{rad} / \mathrm{sec}\right)
$$

Having the frequency $\Omega_{B_{r}} \sim 50 \div 100 \mathrm{rad} / \mathrm{sec}$ in the vertical plane and making the frequencies $\Omega_{B_{v}, E_{r}}$ and $\Omega_{B_{z}}$ in other planes much smaller $\sim 10^{-3} \mathrm{rad} / \mathrm{sec}$ we realize conditions, when the contribution of other frequencies is less than the contribution of the expected EDM frequency in the vertical frequency.

## CW\&CCW and Calibration of $\gamma_{\text {eff }}$ in horizontal plane

$$
\Omega_{E D M}=\left(\Omega_{C W}+\Omega_{C C W}\right) / 2+\left(\Omega_{B x}^{C C W}-\Omega_{B x}^{C W}\right) / 2
$$

The transverse spin rotator is switched on only for the time of calibration of the in the CW ring and for the time of its recovery in the CCW ring.


Since we are able to calibrate the frequency, that is $\gamma_{e f f}$, with the abovementioned absolute value of errors for one beam fill $\sigma_{\Omega} \approx 10^{-7}$ rad/sec and $\sigma_{\Omega} \approx 10^{-9} \mathrm{rad} / \mathrm{sec}$ with one-year statistics, and also taking into account the constant relation between the vertical and the radial components of field (11), this means that in the case of CCW we have a ring identical to the CW ring in terms of spin behaviour, and we can obtain a zero value of $\Omega_{r, m d m}^{C C W}-\Omega_{r, m d m}^{C W}$ with an accuracy of $\approx 10^{-9}$.

## EDM measurement precision

the accuracy of the frequency measurement of determines the precision of the EDM measurement.

For an absolute statistical error of measuring a frequency of the spin oscillation, we can use

$$
\sigma_{\Omega}=\delta \varepsilon_{A} \sqrt{24 / N} / T
$$

N - the total number of recorded events,
$\delta \varepsilon_{A} \approx 0.03$ - the relative error in measuring the asymmetry
$T \approx 1000 \mathrm{sec}$ is the measurement duration.
At $10^{11}$ particles per fill and a polarimeter efficiency of 0.01 an absolute error of frequency measurement is $\sigma_{\Omega}=2 \cdot 10^{-7}$.
At an average accelerator beam time of 6,000 hours per year, we can reach $\sigma_{\Omega} \approx 10^{-9} \mathrm{rad} / \mathrm{sec}$ using one-year statistics,
that is the accuracy of frequency is satisfactory and sufficient for reaching at a parameter of $d_{d} \approx 10^{-30} \mathrm{e} \cdot \mathrm{cm}$

## The fundamental features of Frequency Domain Method

The method is based on four fundamental features: the total spin precession frequency in the vertical plane due to the electric and magnetic dipole moments in an imperfect ring in a vertical plane is measured;
the position of the ring elements is unchanged from clockwise to counter-clockwise operation;
the calibration of the effective Lorentz factor using the polarization precession frequency measurement in the horizontal plane is carried out alternately in each CW and CCW operation;
the approximate relationship between the frequencies of the polarization precession in different planes is set to exclude them from mixing to the vertical frequency of the expected EDM signal at a sensitivity level approaching $10^{-29} \mathrm{e} \mathrm{cm}$

## Storage Ring EDM Project



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