

Clusters of cold dense nuclear matter and particle production in the cumulative region of central rapidities and large transverse momenta at NICA MPD

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Cumulative Particle Production

Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons.

Cumulative Pion Production

1970 - Dubna – beams of relativistic deuterons ($p_0=5 \text{ GeV}/c/\text{nucleon}$)

Stavinskiy V.S. =>

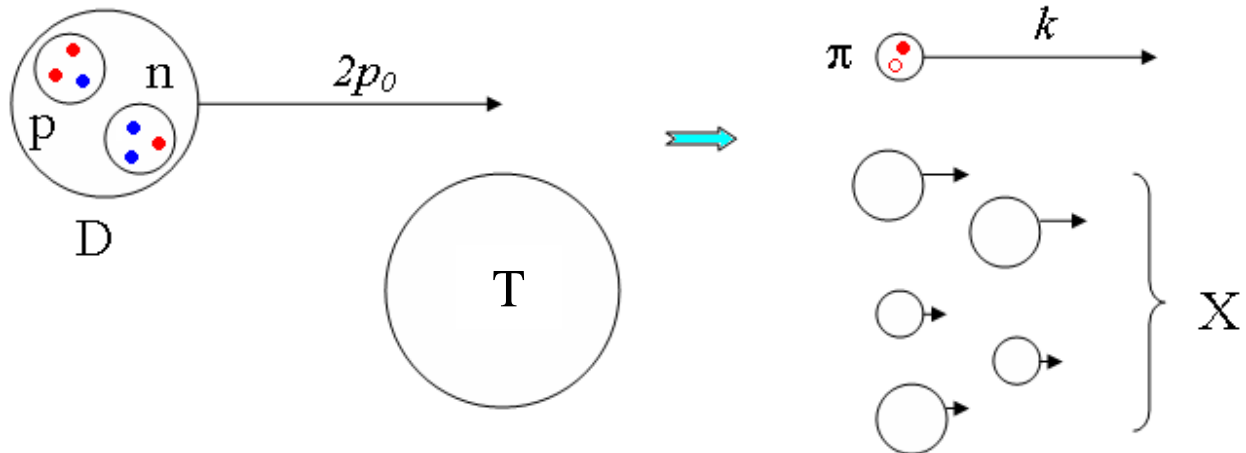
Fragmentation of projectile deuterons, D, on some target, T.

Baldin A.M. et al., Yad.Fiz.18 (1973) 79

$$D + T \Rightarrow \pi + X$$

$p_0 \gg m_N$: $p_0 < k < 2p_0$ - cumulative pions

Later – superconducting Nuclotron. Now – NICA.



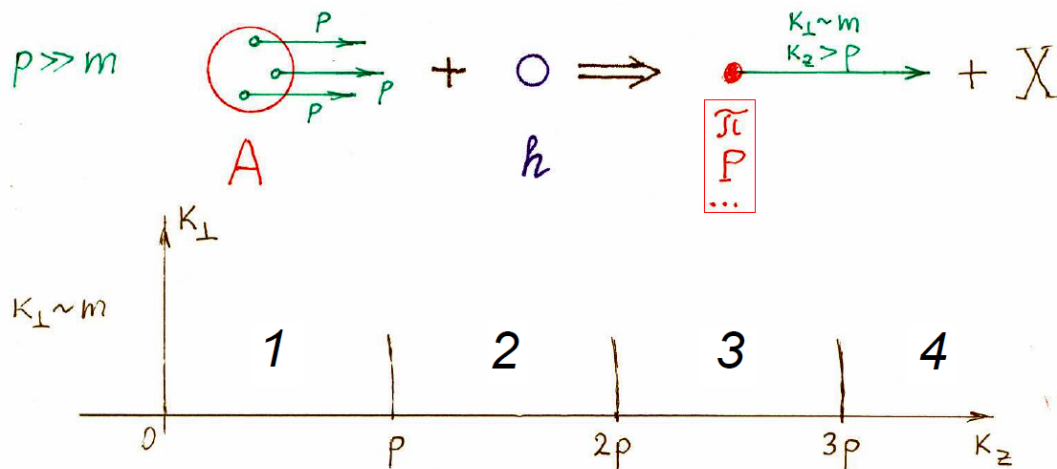
Kinematics of cumulative production

Fragmentation of **projectile** nucleus

$$x \equiv \frac{k_+}{p_+} = \frac{k_0 + k_z}{p_0 + p_z} \approx \frac{k_z}{p}$$

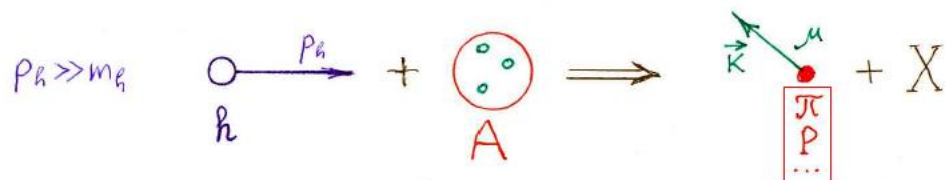
$k_z, p \gg m$, m – nucleon mass

$$x = 1, 2, 3, \dots, A$$



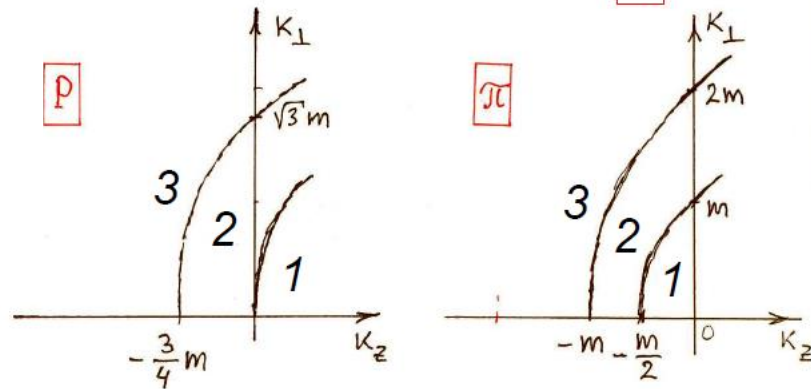
The borders increase with p

Fragmentation of **target** nucleus



$$x \equiv \frac{k_-}{p_-} = \frac{\tilde{k}_0 - \tilde{k}_z}{m} = \frac{\sqrt{\tilde{k}_z^2 + k_\perp^2 + \mu^2} - \tilde{k}_z}{m}$$

$$\tilde{k}_z = -\frac{xm}{2} + \frac{k_\perp^2 + \mu^2}{2xm}$$



The borders are fixed at $p \gg m$

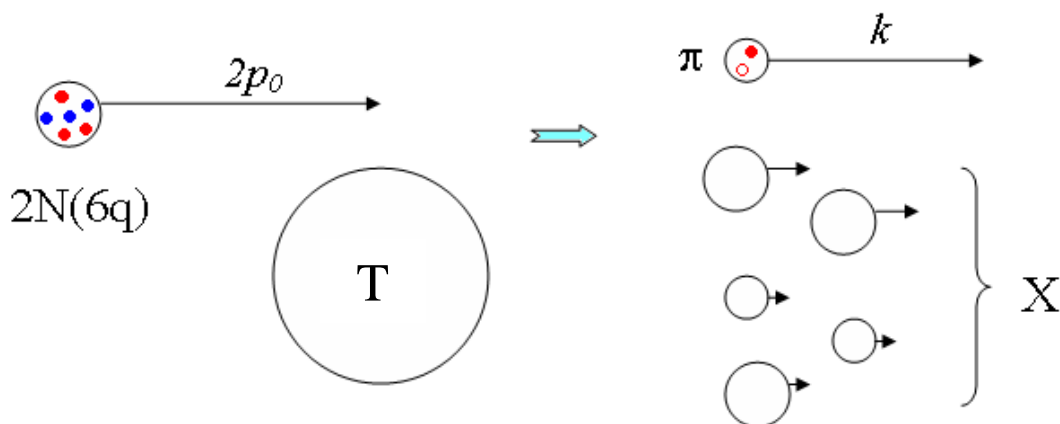
Flucton – intrinsic droplet of dense cold nuclear matter in a nucleus

Blokhintsev D.I., JETP 33 (1957) 1295

($2N$ flucton – 6 quark state)

$\sim 5\% 2N(6q)$ – flucton
admixture in D

Common $6q$ -bag



Fragmentation of **projectile** nucleus \Leftrightarrow Fragmentation of **target** nucleus
(the same phenomenon in different frames of reference)

Cumulative fragmentation of **target** nucleus:

The 1st experimental observations of the **backward** particle production
in $p+A$ collisions on a **fixed target** nucleus:

G.A. Leksin et al., ZhETF 32, 445 (1957)

L.S. Azhgirej et al., ZhETF 33, 1185 (1957)

Yu.D. Bayukov et al., Izv. AN SSSR 30, 521 (1966)

The Reserford-like experiments indicating the presence of **droplets of dense nuclear matter in a target nucleus** (fluctons).

Different cumulative variables

$x = \frac{k_+}{p_+}$ - light cone variable

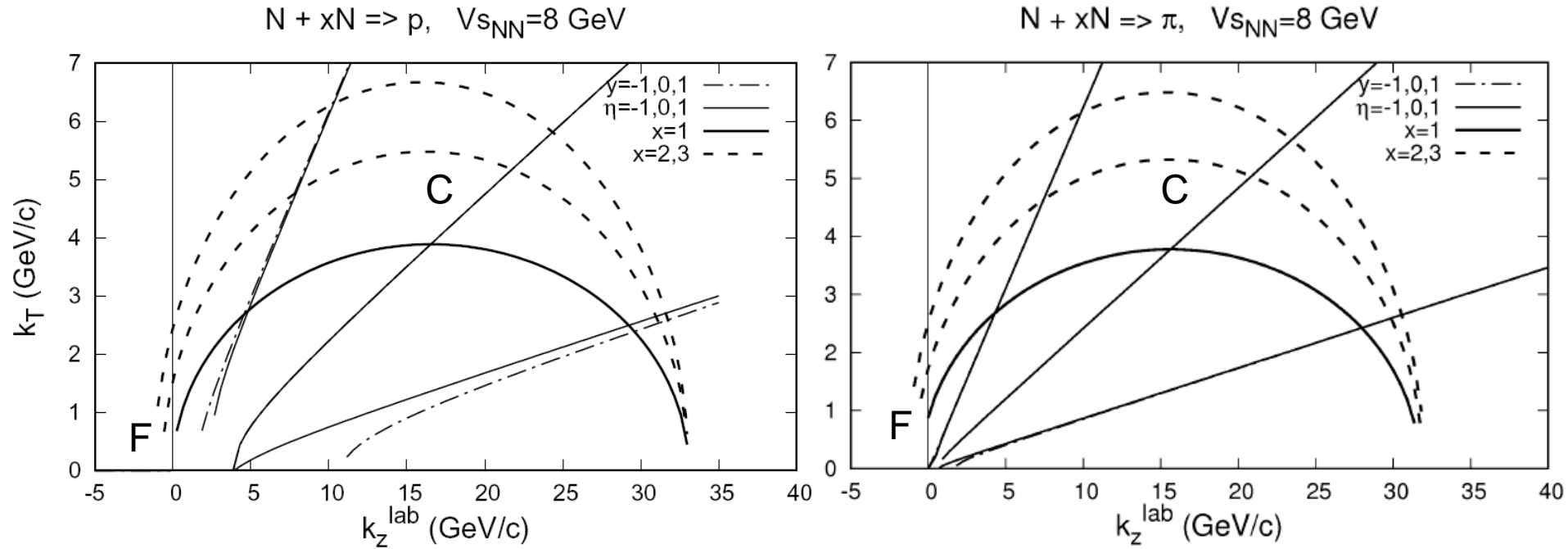
$x_F = \frac{k_z}{k_z^{max}}$ - Feynman variable

$M_f^{min} = X m_N$ - cumulative number

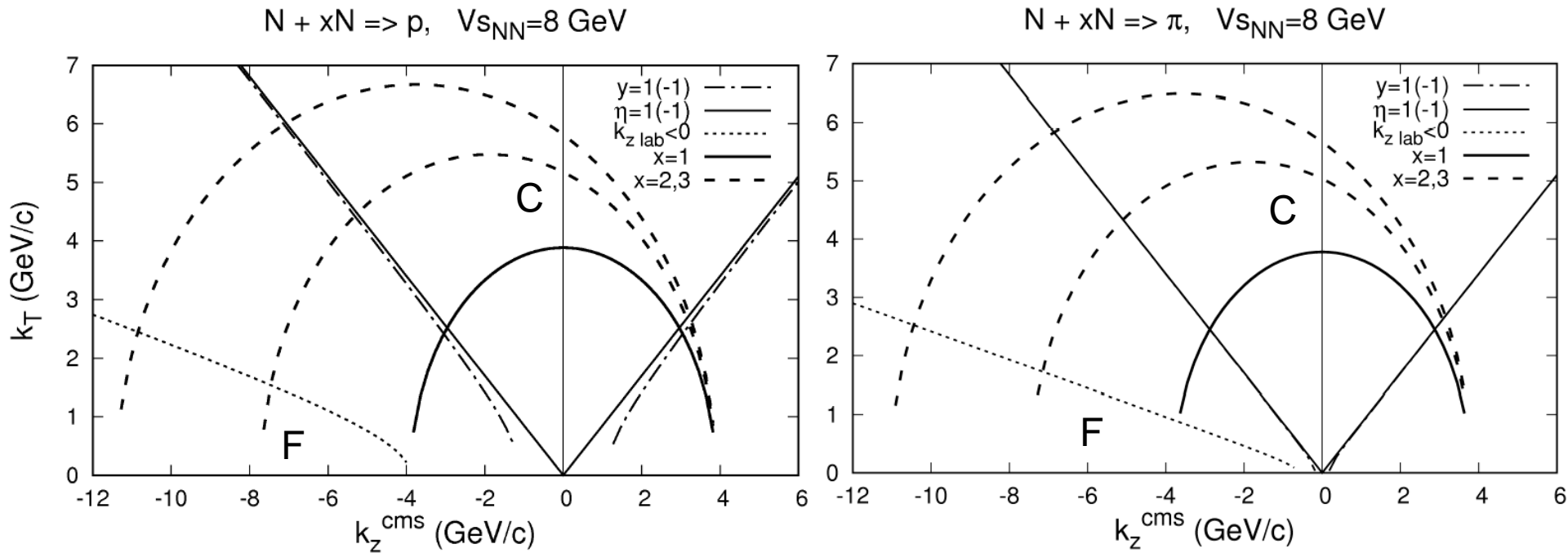
$x \approx x_F \approx X$ at $s \rightarrow \infty$

$$\frac{m_N^2}{E^{*2}} = \frac{4m_N^2}{s}$$

New cumulative region -
– central rapidities and large transverse momenta
(in the fragmenting nucleus rest system)



New cumulative region -
 – central rapidities and large transverse momenta
 (in the c.m. system)



Experimental data on cumulative production in pA collisions in nucleus fragmentation region

S.V. Boyarinov et al., Sov.J.Nucl.Phys. 46, 871 (1987)

S.V. Boyarinov et al., Physics of Atomic Nuclei 57, 1379 (1994)

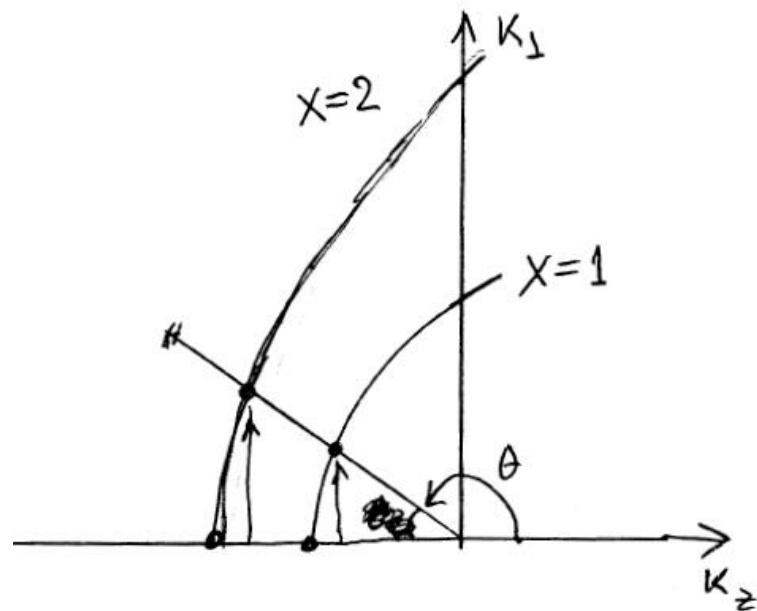
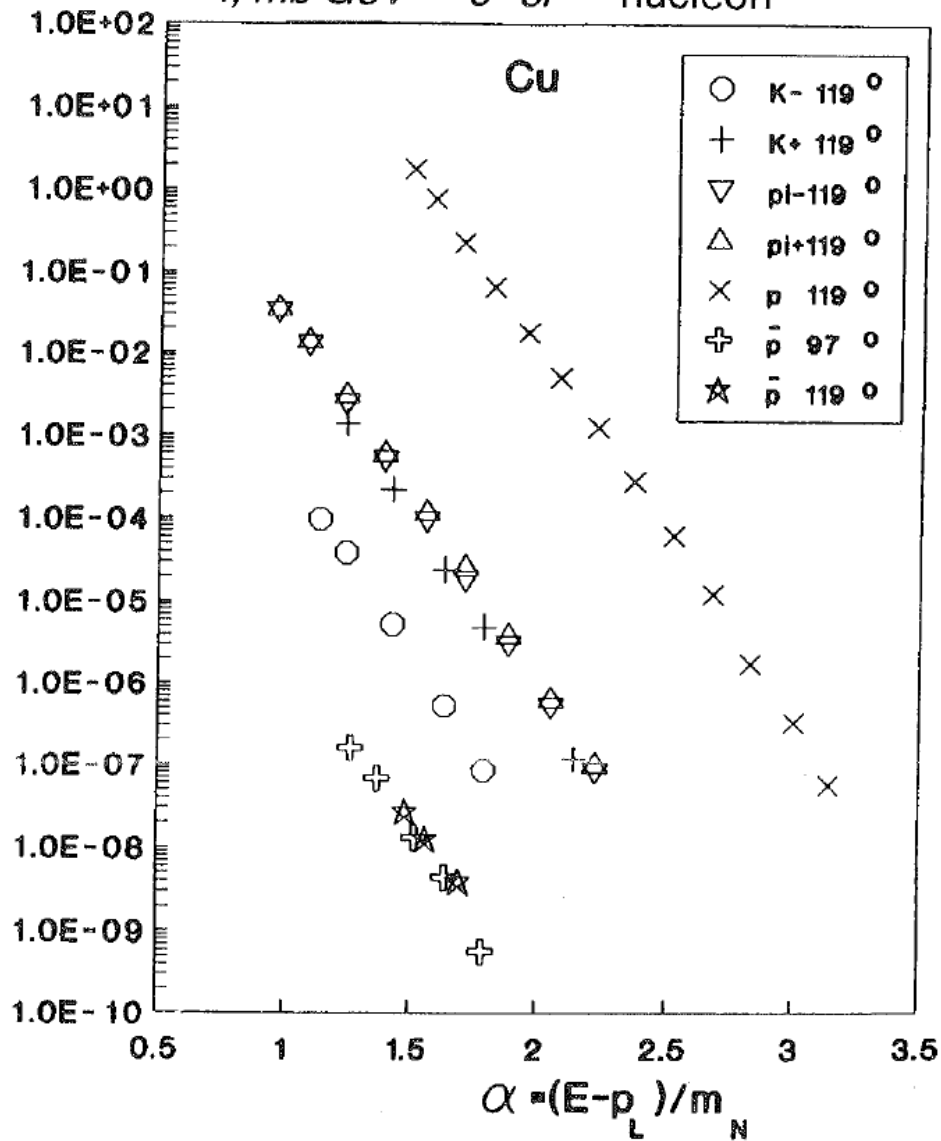
S.V. Boyarinov et al., Sov.J.Nucl.Phys. 55, 917 (1992)

To estimate the yield of cumulative particles with large transverse momenta at central rapidities we use the parametrization of the experimental data on the production of cumulative pions and protons in p+A reactions in the region of fragmentation of the nucleus at the incident proton energy of 10.14 GeV in the laboratory frame ($\sqrt{s_{NN}} = 4.56$ GeV).

$f(X, k_T) = (E/A)(d\sigma/d^3\mathbf{k})$ - the relativistically invariant quantity

$f(X) = C(\theta)\exp(-X/X_0)$ X_0 is 0.139 for pions and 0.135 for protons.

$f, mb GeV^{-2} c^3 sr^{-1} nucleon^{-1}$



$$f_1(k_T) = \exp(-k_T^2 / \langle k_T^2 \rangle)$$

$$f_2(k_T) = \exp(-2k_T / \langle k_T \rangle),$$

The values of $\langle k_T^2 \rangle$ and $\langle k_T \rangle$ depend on X !!!

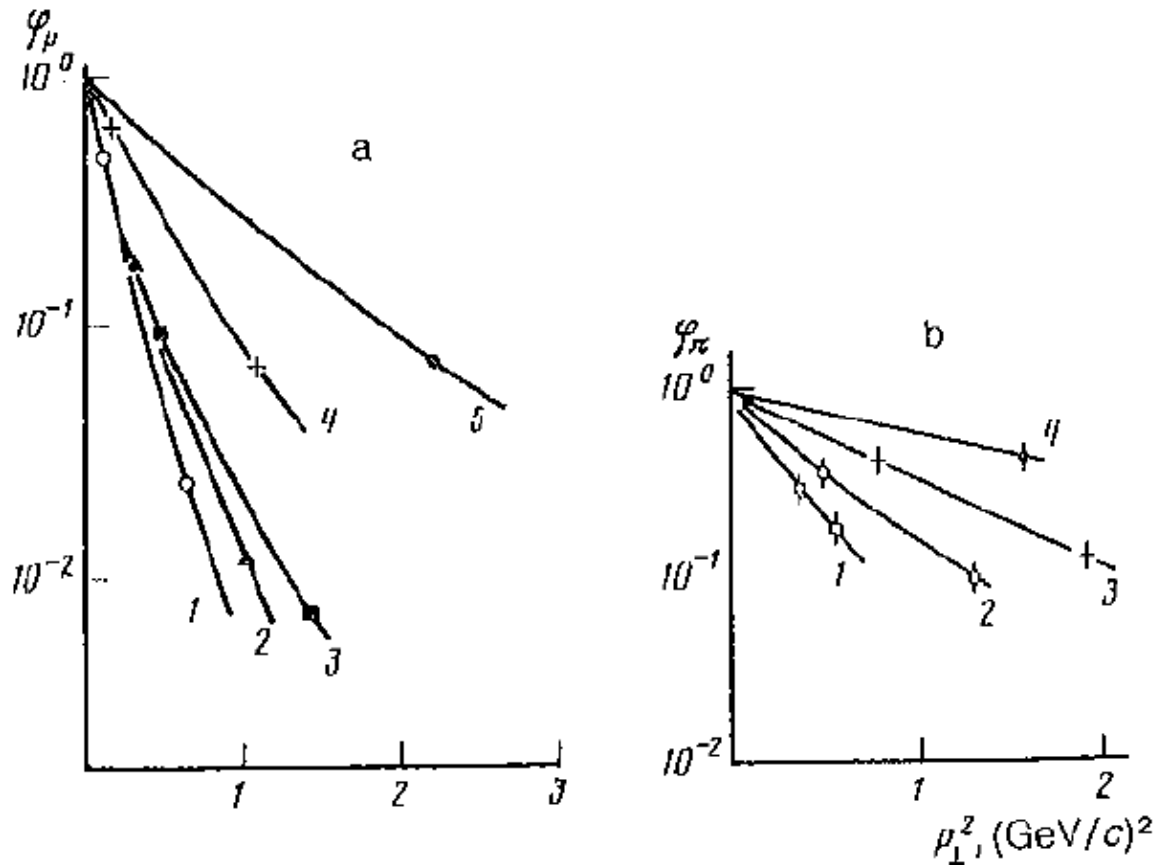


FIG. 5. Graphs of $\varphi(p_T^2) = f(x, p_T^2) / f(x, 0)$ for constant x : (a)—protons, 1— $x = 1.4$, 2— $x = 1.6$, 3— $x = 1.8$, 4— $x = 2.5$, 5— $x = 3.5$; (b)—pions, 1— $x = 0.8$, 2— $x = 1.6$, 3— $x = 2.0$, 4— $x = 3.0$.

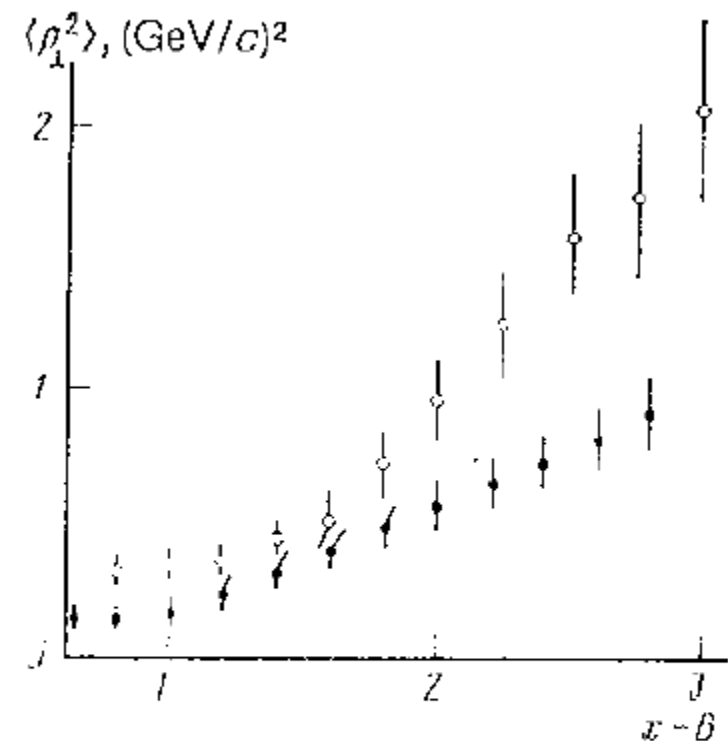


FIG. 6. Mean square transverse momentum as a function of x : ●—protons, $B = 1$; ○—pions, $B = 0$.

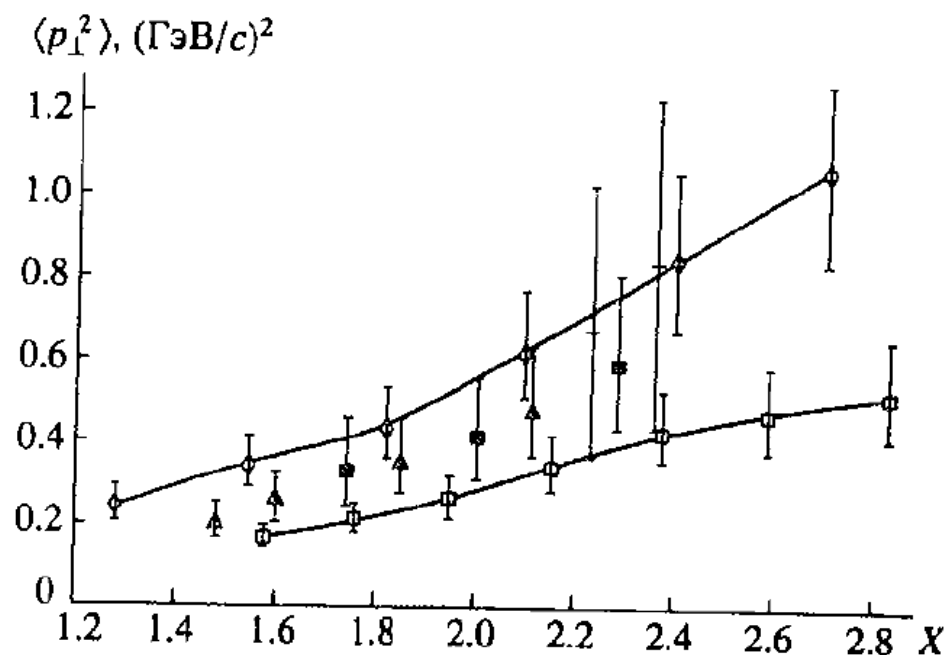


Рис. 3. Зависимость среднего квадрата поперечного импульса $\langle p_1^2 \rangle$ от кумулятивного числа X для p , π^\pm , K^+ , K^- и \bar{p} . Точки: □ — p , ◇ — π^\pm , △ — K^+ , ■ — K^- , + — \bar{p} . Соответствующие точки для p и π^\pm соединены для наглядности ломаными линиями.

Description of the hadron asymptotics at $x \rightarrow 1$

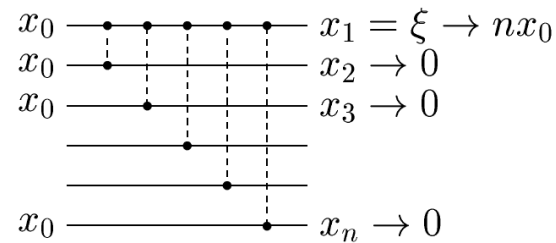
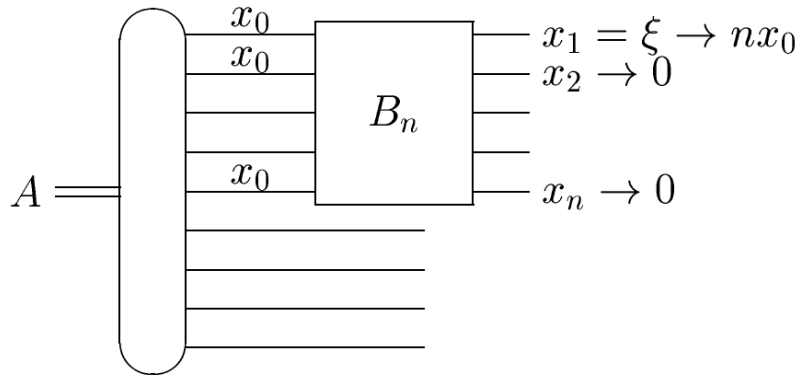
by the intrinsic diagrams of QCD in light-cone gauge
with low- x spectator quarks interact with the target

Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl.Phys. B369 (1992) 519

Description of the flucton asymptotic at $x \rightarrow f$,

f - the number of nucleons in flucton, n - the number of quarks in flucton, $x_0 = f/n (=1/3)$.

M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614. (DIS in cumulative region)

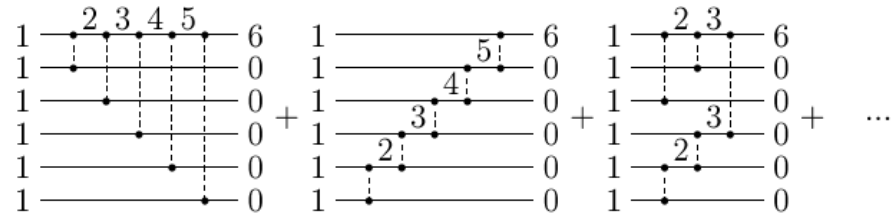


$$\sim \Delta^{2p-1}$$

$$p=n-1$$

$$\Delta = f - \mathbf{x} = nx_0 - \xi$$

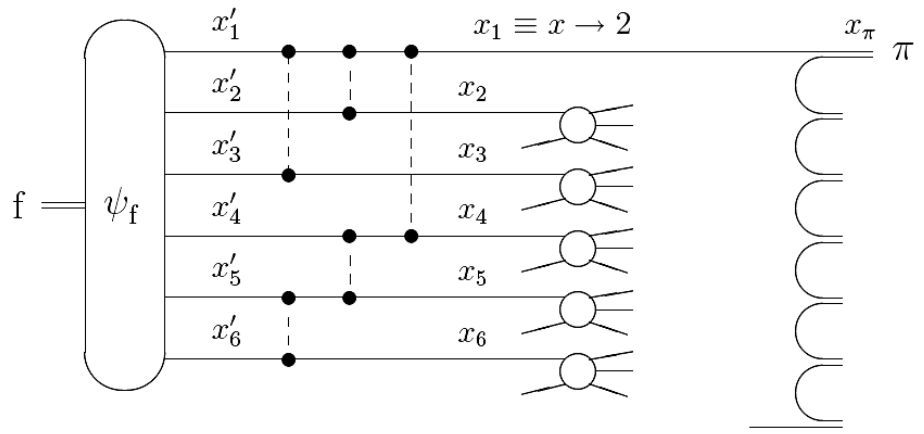
At $x_1 \rightarrow f \Rightarrow$ all $x_2, \dots, x_n \rightarrow 0 \Rightarrow$ all $|q_i| \gg m \Rightarrow$
pQCD works \Rightarrow min.number of hard exchanges.
Simple instantaneous Coulomb part dominates
in light-cone gauge.



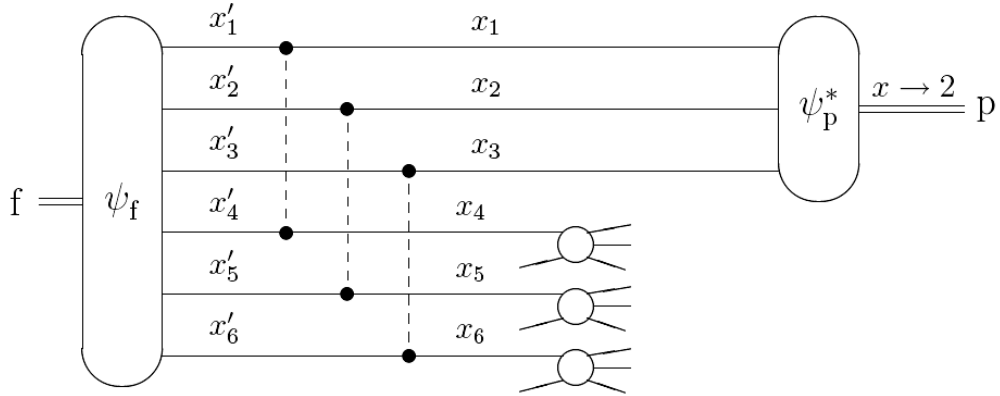
$$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{|c|} \hline X_n \\ \hline \end{array} \begin{array}{c} n \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} = \sum_{k=1}^{n-1} C_{n-2}^{k-1} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{|c|} \hline X_k \\ \hline \end{array} \begin{array}{c} k \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{c} n-k \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

- the recurrence relation

Coherent Quark Coalescence and Production of Cumulative Pions and Protons



- the cumulative pion production by hadronization of one fast quark
M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427, 614 (1994); Phys.Atom.Nucl. 60, 432 (1997); 63, 1831 (2000)

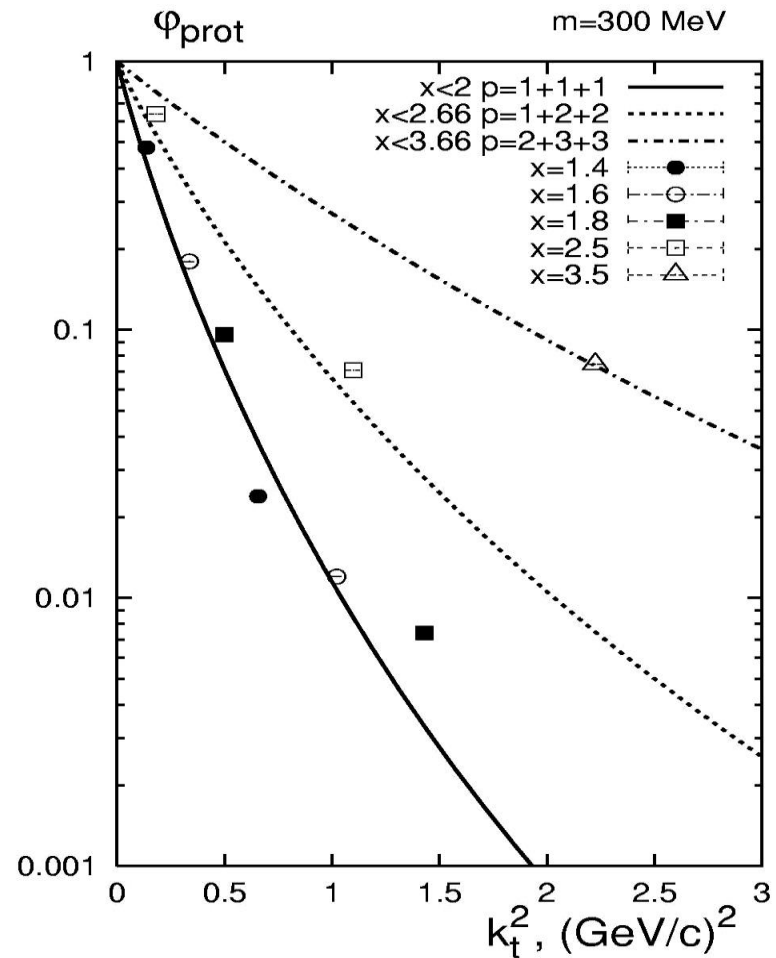
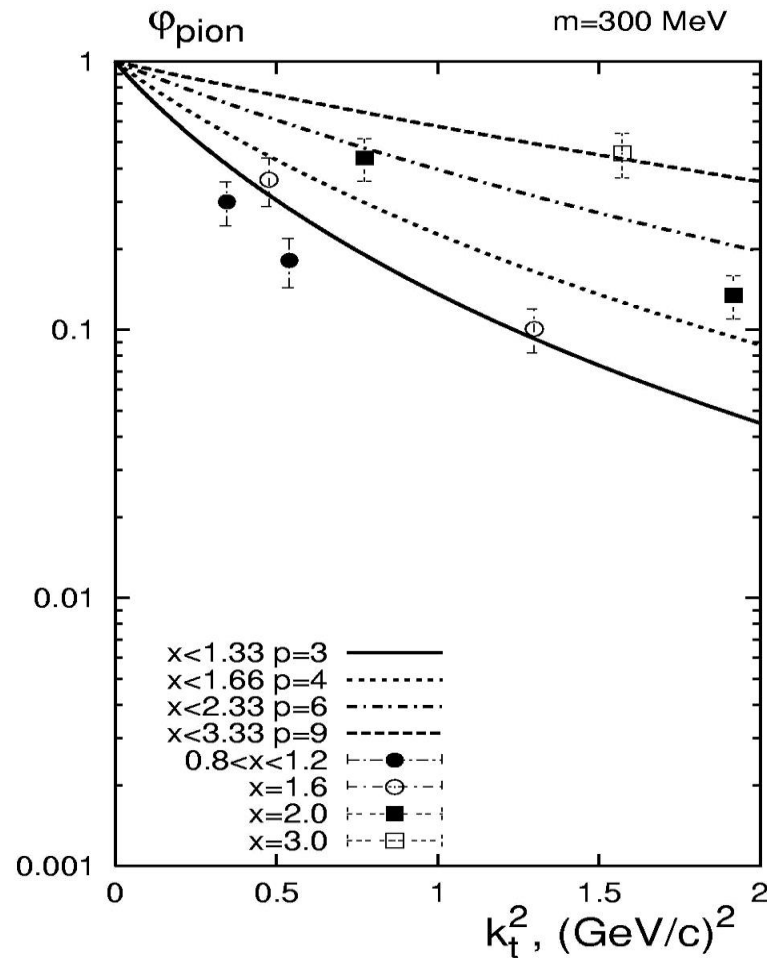


- the cumulative proton production by **coherent** quark coalescence mechanism:
M.A. Braun, V.V. Vechernin, Nucl.Phys.B 92, 156 (2001); Theor.Math.Phys 139, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.

The last diagram **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).

But instead of using the relativistic generalization of non-relativistic NN wave function **in our approach we use the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge.**

No free parameters (!) only m – the constituent quark mass: $m = 300 \text{ MeV}$.



V. Vechernin, AIP
Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **46**, 871 (1987)
S.V. Boyarinov et al., *Physics of Atomic Nuclei* **57**, 1379 (1994)
S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **55**, 917 (1992)

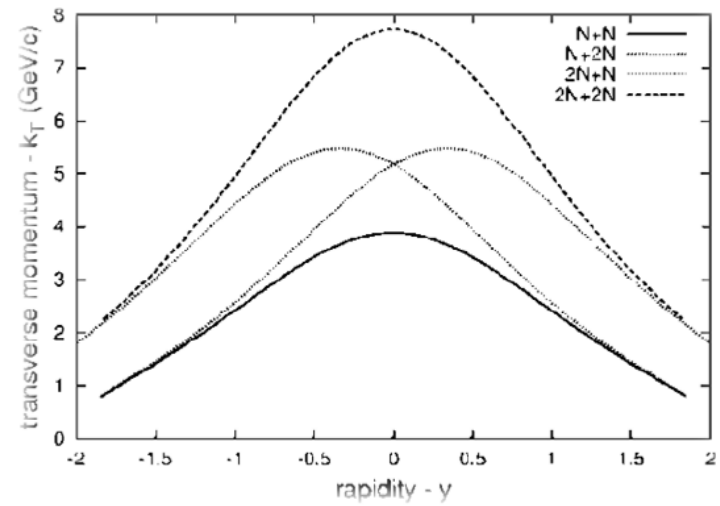
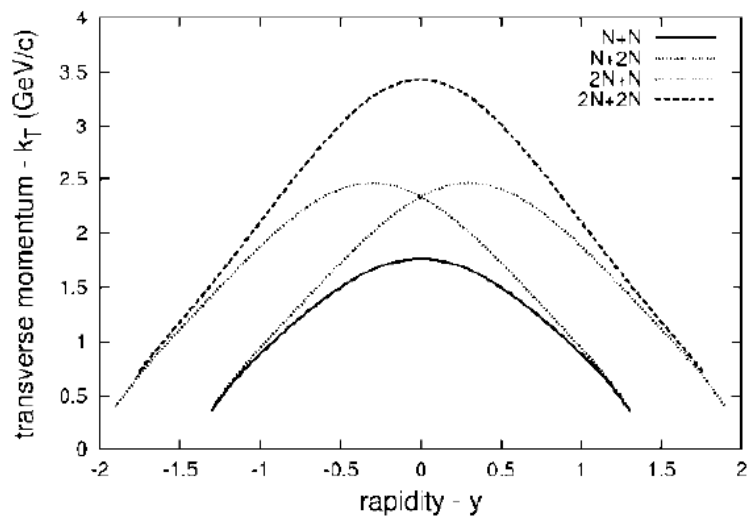


Figure 5. Kinematic boundaries of the cumulative region at central rapidities for the yield of protons at $\sqrt{s_{NN}} = 4\text{GeV}$ (left panel) π 8 GeV (right panel).

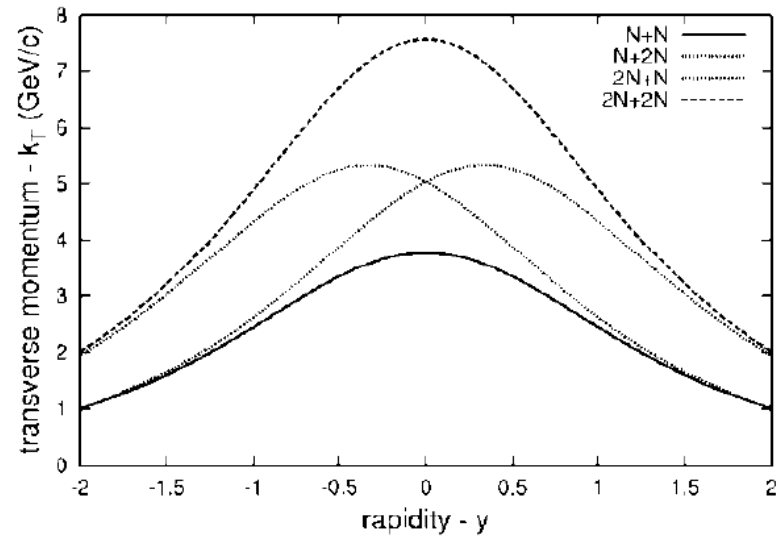
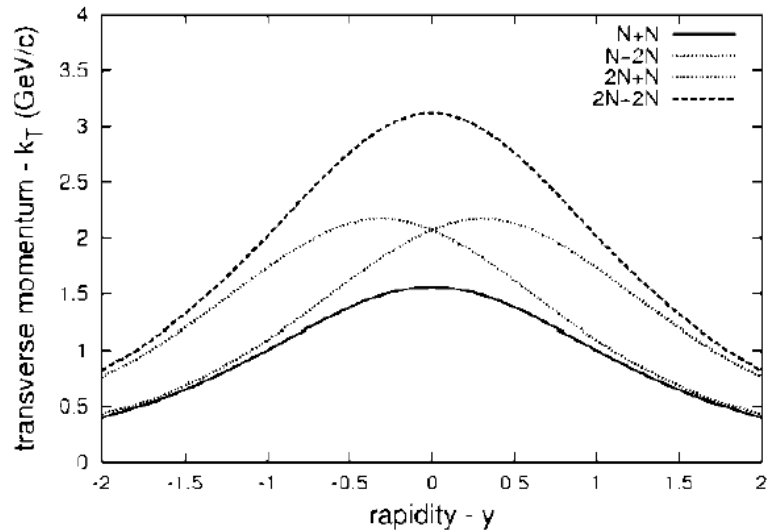
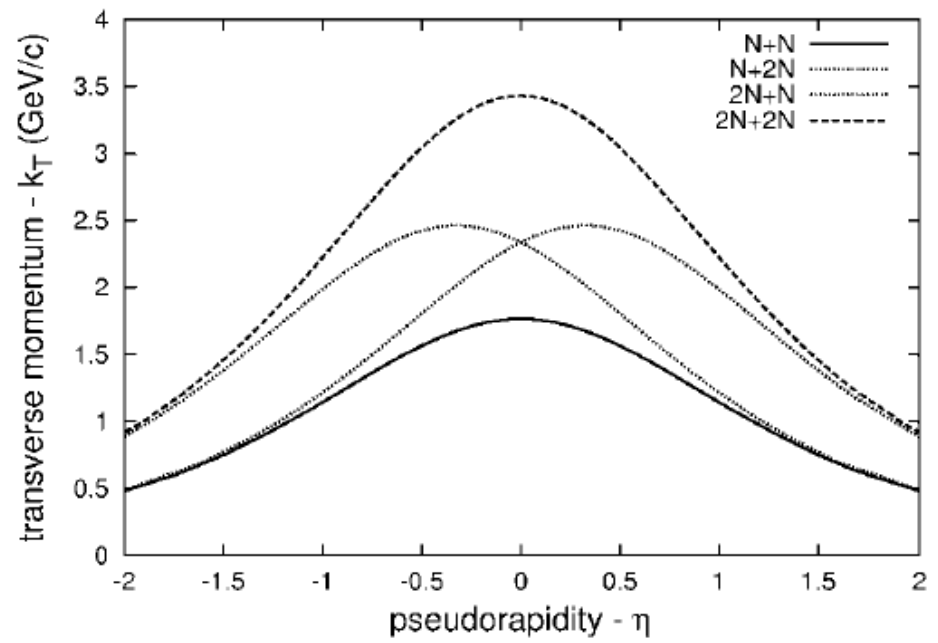
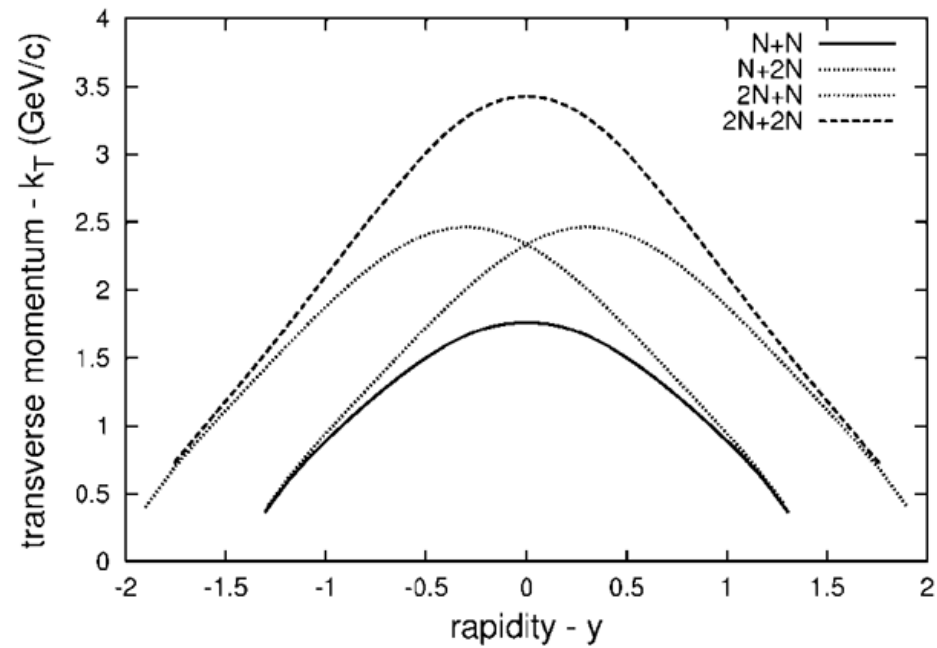


Figure 6. The same as in Fig. 5, but for the yield of pions.



Kinematic boundaries for the yield of cumulative protons at an initial energy $\sqrt{s_{NN}} = 4 \text{ TeV}$, as a function of rapidity and pseudorapidity

The multiplicity of cumulative particles with $X > 1.6$ in the rapidity acceptance of $-1 < y < 1$ in pAu collisions

$$f(X, k_T) = C_0 \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right) \exp\left(-\frac{X}{X_0}\right). \quad - \text{ Gauss}$$

$$f(X, k_T) = C_0 \exp\left(-\frac{2k_T}{\langle k_T \rangle}\right) \exp\left(-\frac{X}{X_0}\right) \quad - \text{ Exponent}$$

$$f(y, k_T) \equiv \frac{E}{A} \frac{d\sigma}{d^3\mathbf{k}} = \frac{1}{A \pi} \frac{d\sigma}{dy dk_T^2} \quad - \text{ the relativistically invariant quantity}$$

$$\langle n \rangle_{pAu} \sigma_{pAu}^{tot} = \pi \int_{-1}^1 dy \int_{X_{min}}^{\infty} dX \frac{dk_T^2}{dX} f(y, k_T) \quad \sigma_{pAu}^{tot} = 2 \text{ bn}$$

V.V. Vechernin, “Yield of particles in the cumulative region at central rapidities and large transverse momenta at the NICA collider”,
Physics of Particles and Nuclei 53 (2) 433-440 (2022).

The multiplicity of cumulative particles with $X > 1.6$ in the rapidity acceptance of $-1 < y < 1$ in pAu collisions

Table 5. Estimation of the multiplicity of cumulative pions and protons with $X > 1.6$ at mid rapidities in p+Au collisions at $\sqrt{s_{NN}} = 4$ and 8 GeV.

$\sqrt{s_{NN}}$	4 GeV		8 GeV	
k_T -fit	Gaussian	Exponent	Gaussian	Exponent
$\langle n_\pi \rangle_{pAu}$	$5 \cdot 10^{-5}$	$2 \cdot 10^{-4}$	$2 \cdot 10^{-11}$	$1.3 \cdot 10^{-5}$
$\langle n_p \rangle_{pAu}$	$7 \cdot 10^{-5}$	$3 \cdot 10^{-3}$	$9 \cdot 10^{-15}$	$4 \cdot 10^{-7}$

p+Au => Au+Au

Based on the estimates made for the multiplicity of cumulative particles with large transverse momenta at mid rapidities in p+Au collisions, we can obtain estimates for their multiplicity in this region in the Au+Au reaction. When replacing an incident proton with a nucleus, the number of projectile nucleons interacting with the flucton in another nucleus increases, that can be taken into account by introducing the corresponding factor γ . The value of this factor was estimated through the ratio of the number of nucleon-nucleon collisions in p+Au and Au+Au reactions: $\gamma_{coll} = \langle N_{coll} \rangle_{AuAu} / \langle N_{coll} \rangle_{pAu}$, which was chosen equal to 20. The obtained estimates for the multiplicity of cumulative particles with $X > 1.6$ at mid rapidities in Au+Au collisions at the NICA collider energies are given in Table 6. In performing these estimates, we, of course, took into account the symmetric contribution when the flucton in the first nucleus interacts with the nucleon in the second nucleus. Note also that for now, we leave without consideration a new physically interesting contribution arising from flucton-flucton scattering as an object for future research.

Cumulative yields at NICA

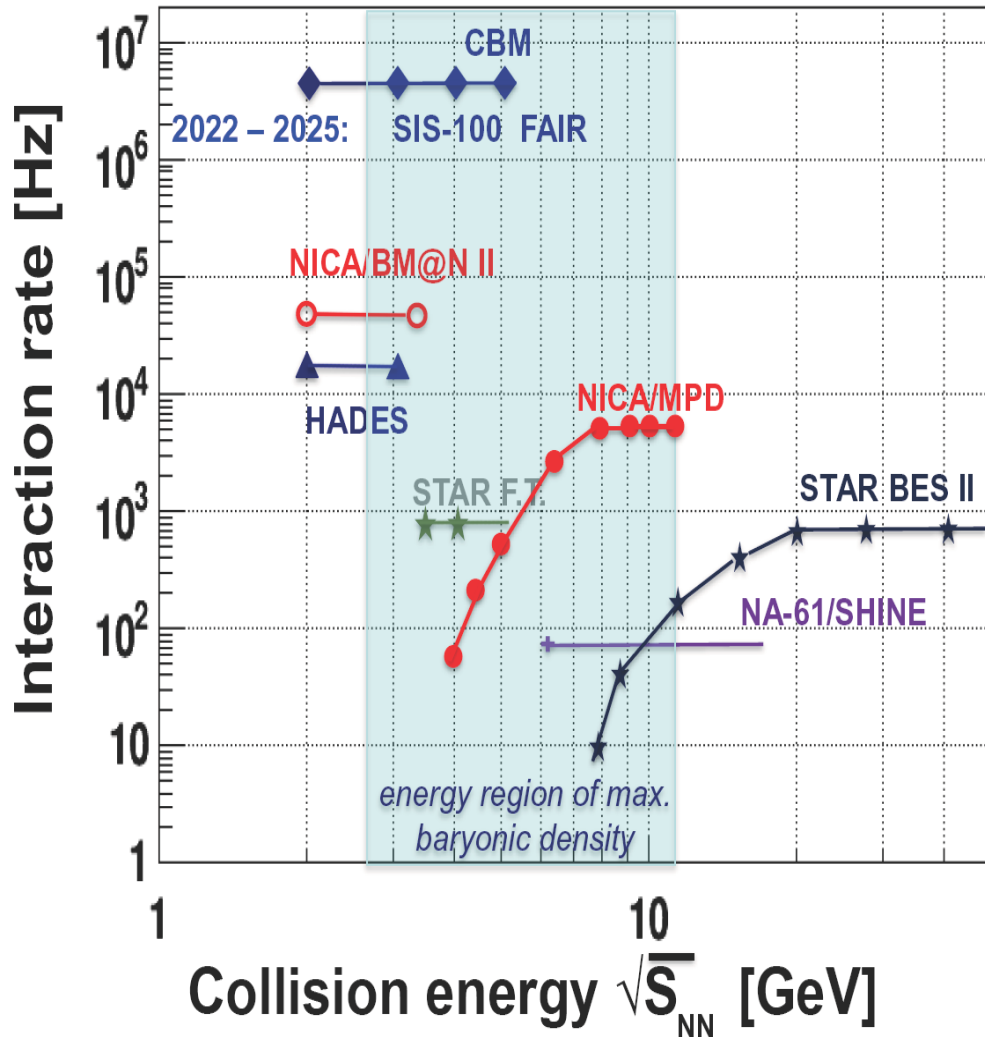
Table 6. Estimation of the multiplicity and yield of cumulative pions and protons with $X > 1.6$ at mid rapidities in Au+Au collisions at $\sqrt{s_{NN}} = 4$ and 8 GeV due to nucleon-flucton scattering.

$\sqrt{s_{NN}}$	4 GeV		8 GeV	
k_T -fit	Gaussian	Exponent	Gaussian	Exponent
$\langle n_\pi \rangle_{AuAu}$	$2 \cdot 10^{-3}$	$8 \cdot 10^{-3}$	$9 \cdot 10^{-10}$	$5 \cdot 10^{-4}$
$\langle n_p \rangle_{AuAu}$	$3 \cdot 10^{-3}$	$1.1 \cdot 10^{-1}$	$4 \cdot 10^{-13}$	$1.6 \cdot 10^{-5}$
$\langle Y_\pi \rangle_{AuAu}$	50	200	$2 \cdot 10^{-3}$	1300
$\langle Y_p \rangle_{AuAu}$	70	2700	$9 \cdot 10^{-7}$	40

Taking into account the fact that the luminosity of the NICA collider at the energy $\sqrt{s} = 4$ GeV will be 100 times lower than at the energy $\sqrt{s} = 8$ GeV, we obtain estimates of the yields of cumulative particles, Y , for 1 hour of collider operation, which are given in table.6.

Here we took also into account the reduction of the final statistics by the factor of 10 in the process of an event selection by various triggers and an interaction vertex position.

Present and future HI experiments



V. Kekelidze, A. Kovalenko,
 R. Lednicky, V. Matveev,
 I. Meshkov, A. Sorin, G. Trubnikov,
 Feasibility study of heavy-ion
 collision physics at NICA,
 Nuclear Physics A 967 (2017)
 884–887.

A completely different relationship between the yield of protons and pions in a new cumulative region of mid rapidities and large transverse momenta

(compared to the previously studied nucleus fragmentation region, where the p/π ratio is of the order of 10^4 and the yield of cumulative protons is dominant)

In the new region of mid rapidities and large transverse momenta:

- at initial energy $\sqrt{s_{NN}} = 4$ GeV their yields become approximately the same order of magnitude.

- at energy $\sqrt{s_{NN}} = 8$ GeV the yield of cumulative pions is already almost two orders of magnitude higher than the yield of protons **with the same cumulative number X**.

Theoretically, it can be explained by different mechanisms of the formation of these cumulative particles - **the coherent coalescence (recombination) of three flucton quarks for a proton** and **the fragmentation of one flucton quark for a pion**.

Observation of this impressive effect in the production of cumulative particles with large transverse momenta at mid rapidities with the MPD setup using a vertex detector would allow us to verify these theoretical ideas about the mechanisms of particle formation in the cumulative region.

Note in this connection that

the last results obtained by SPIN collaboration

Artem Semak report “High p_t anti-proton and meson production in cumulative pA reaction at 50 GeV/c” at LXX International Conference “NUCLEUS – 2020”, St Petersburg, October 11-17, 2020.

https://indico.cern.ch/event/839985/contributions/3983657/attachments/2124000/3576745/NUCLEUS_2020_Semak.pdf

on the production of particles with **large transverse momenta** at the angle of 40 degree

on fixed nuclear targets by 50 GeV/c protons, which corresponds to the **cumulative number**

X up to 1.2, show that in this region the **p/π ratio at same value of the X** is of the order of **10^2** ,

what is considerably **smaller** than **10^4** in the traditional cumulative region

of nucleus fragmentation with **small transverse momenta**.

A role of the MPD vertex detector in the investigation of the cumulative production

*V.I. Zherebchevsky, V.P. Kondratiev, V.V. Vechernin, S.N. Igolkin,
NIM A 985 (2021) 164668*

*The concept of the MPD vertex detector for the detection of rare events
in Au+Au collisions at the NICA collider.*

We see that in the framework of the assumption of the Gaussian dependence of the yields of cumulative particles on the transverse momentum, the study of a particle production in the cumulative region at the projected luminosity **will be possible only at the energy of $\sqrt{s} = 4 \text{ GeV}$** whereas the observation of cumulative production at **higher energy requires an increase in the luminosity** by at least 3 orders of magnitude to $L=10^{30} \text{ cm}^{-2} \text{ s}^{-1}$ for heavy ion collisions, i.e. to the level expected to be reached in 2022-25 at the SIS-100 in FAIR.

At high collider luminosity (at future upgrades of the NICA collider) it should be also taken into account the influence of the pile-up effects on the obtained results. This is particularly important **for the registration of the particles formed from fragmentation of the flucton residue.**

O.Denisovskaya, K.Mikhailov, et. al., Dense cold nuclear matter study with cumulative trigger, arXiv:0911.1658 (2009).

A. Stavinskiy, Dense cold matter with cumulative trigger, Physics of Particles and Nuclei Letters 8 (2011) 912.

A role of the MPD vertex detector in the investigation of the cumulative production

The MPD vertex detector with high spatial resolution is needed to remove of all particle tracks coming from the other collision vertices which distort the particle spectrum from fragmentation of the flucton residue.

For example, at the mentioned increased luminosity, $L=10^{30} \text{ cm}^{-2} \text{ s}^{-1}$ for heavy ion collisions, the average number of inelastic Au-Au interaction, occurring during the collisions of two bunches for the NICA setup, will be close to 1, instead of $0.5 \cdot 10^{-3}$ as is currently planned. Supposing their event-by-event Poissonian distribution we can estimate the probability that one more Au-Au interaction vertex will occur closer than at a given distance l along the z axis to the cumulative particle production vertex.

As a result we found that the probability to have another Au-Au interaction vertex at a distance closer than 0.5 and 0.1 mm is equal to 0.18% and 0.037% respectively for the NICA setup.

This indicates the need to use the future internal vertex tracker at MPD for the suppression of the contribution from the event pile-up when registering the particles formed from fragmentation of the flucton residue.

Conclusions

1. The prediction of a completely different relationship between the yield of protons and pions in a new cumulative region of mid rapidities and large transverse momenta.

- at initial energy $\sqrt{s_{NN}} = 4$ GeV their yields become approximately the same order of magnitude.

- at energy $\sqrt{s_{NN}} = 8$ GeV the yield of cumulative pions is already almost two orders of magnitude higher than the yield of protons **with the same cumulative number X**

(whereas in the previously studied nucleus fragmentation region the p/π ratio is of the order of 10^4 and the yield of cumulative protons is dominant).

=> The confirmation of the different mechanisms of the formation of these cumulative particles from flucton.

2. The using the future internal vertex tracker at MPD for the suppression of the contribution from the event pile-up when registering the particles formed from fragmentation of the flucton residue.

=> The confirmation of the flucton mechanism of the cumulative particle production.

Acknowledgments

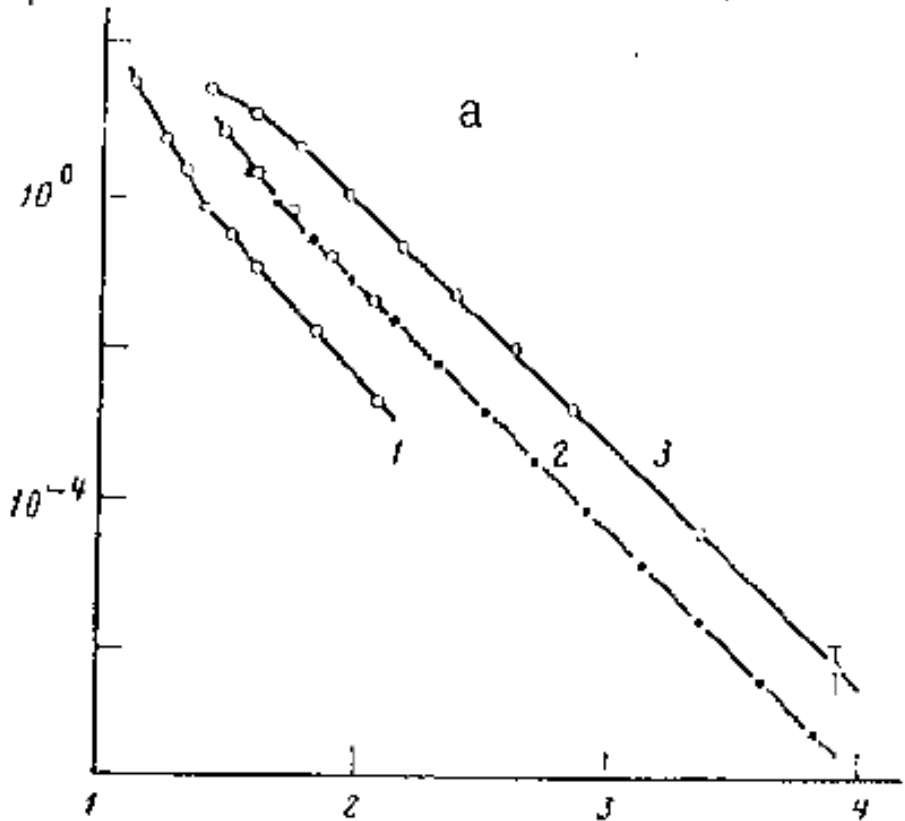
This research was funded by the Russian Foundation for Basic Research (project number 18-02-40075_mega) and by St. Petersburg State University (project № 93025435).

References:

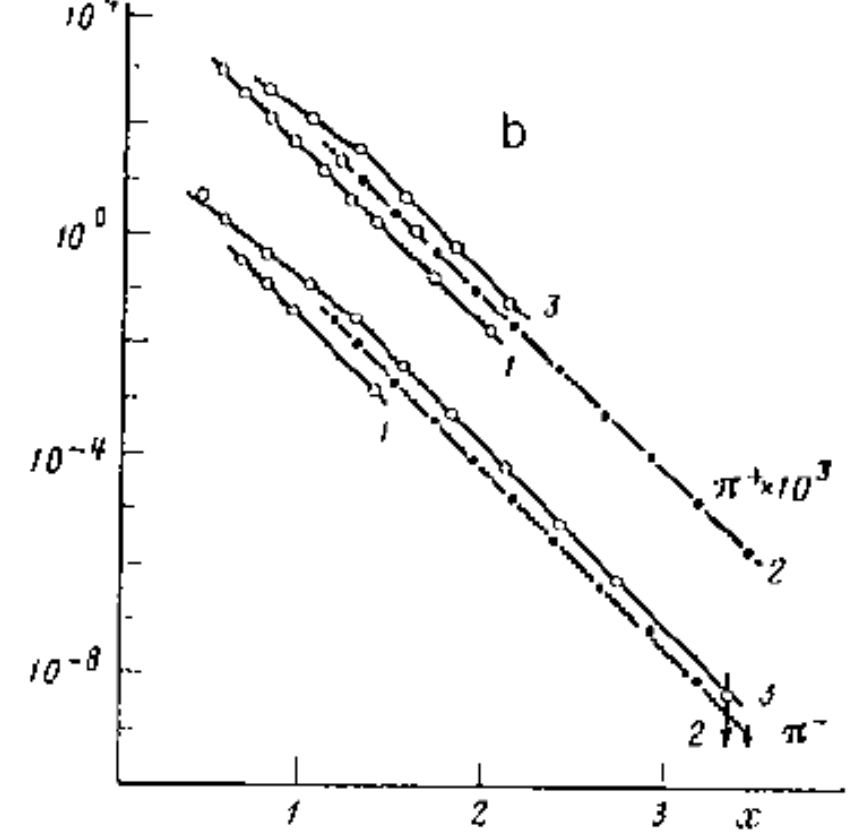
1. **V.V. Vechernin**, “Yield of particles in the cumulative region at central rapidities and large transverse momenta at the NICA collider”,
Physics of Particles and Nuclei **53 (2)** 433-440 (2022).
DOI: 10.1134/S1063779622020861
<https://doi.org/10.1134/S1063779622020861>
2. **V.V. Vechernin** “Clusters of cold dense nuclear matter and their registration with the MPD vertex detector”,
Physics of Particles and Nuclei **52 (4)** 604-608 (2021).
DOI: 10.1134/S1063779621040596
<https://link.springer.com/article/10.1134/S1063779621040596>
3. **V.I. Zhrebchevsky, V.P. Kondratiev, V.V. Vechernin, S.N. Igoikin**,
“The concept of the MPD vertex detector for the detection of rare events in Au+Au collisions at the NICA collider”,
Nuclear Inst. and Methods in Physics Research, A **985 (2021)** 164668
<https://doi.org/10.1016/j.nima.2020.164668>

Backup slides

$f, \text{mb} \cdot \text{GeV}^{-2} \cdot \text{c}^3 \cdot \text{sterad}^{-1} \cdot \text{nucleon}^{-1}$



$f, \text{mb} \cdot \text{GeV}^{-2} \cdot \text{c}^3 \cdot \text{sterad}^{-1} \cdot \text{nucleon}^{-1}$



874 Sov. J. Nucl. Phys. 46 (5), November 1987

FIG. 4. Graph of f as a function of x for different angles of emission of the particle from Pb and Ta: (a)—protons, 1— $\theta = 90^\circ$, 2— $\theta = 120^\circ$, 3— $\theta = 180^\circ$; (b)—pions, ●—our data, ○—data from Refs. 4 and 13.

$$W_j: j=1,2,3.$$

$$n=n_1+n_2+n_3$$

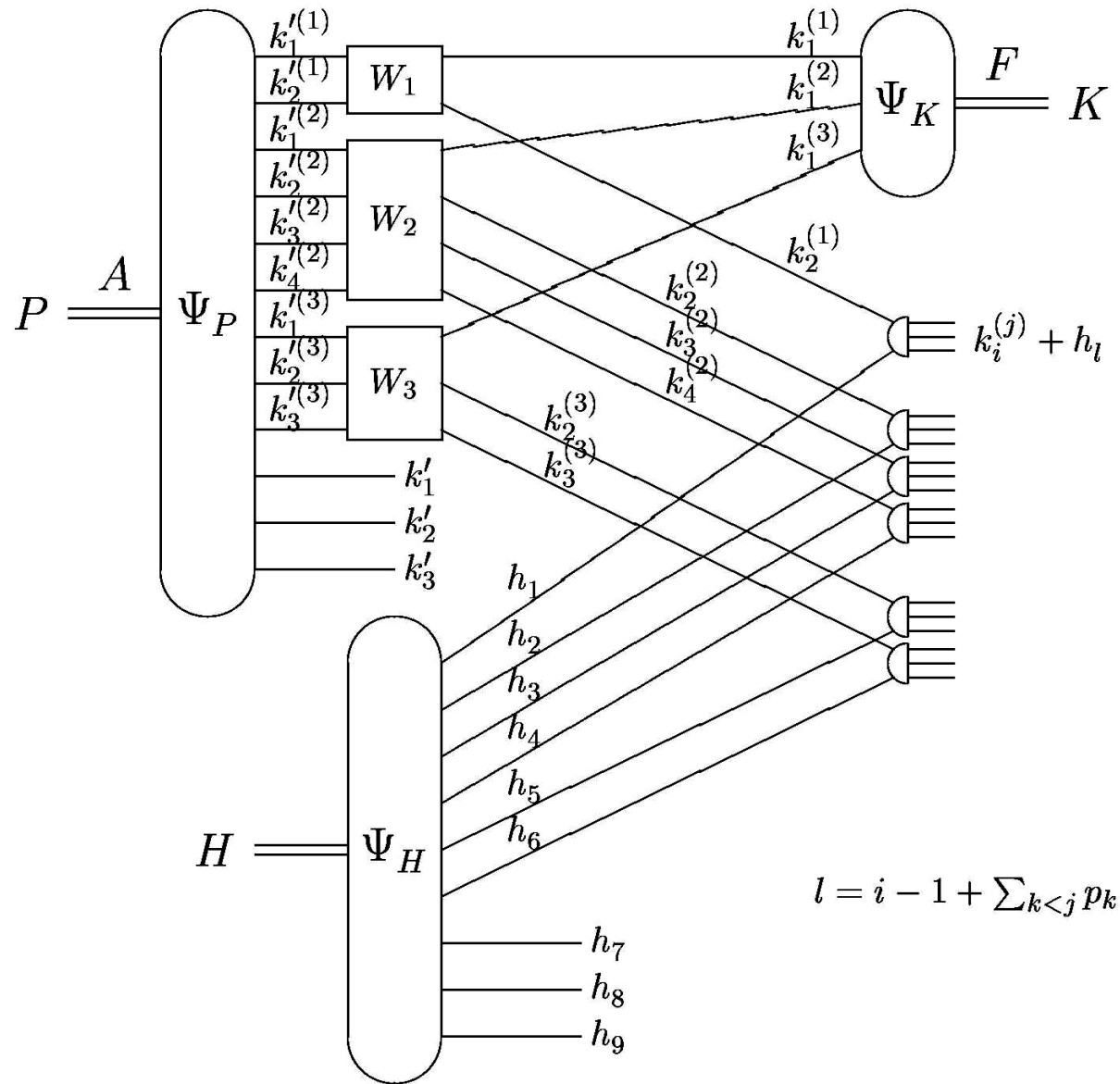
$$p_1=n_1-1$$

$$p_2=n_2-1$$

$$p_3=n_3-1$$

$$p=p_1+p_2+p_3=1+3+2=6$$

$$n=p+3=9$$



$$\sigma_{pion}(x, k_{\perp}; p) = C(p) (x_{frag} - x)^{2p-1} f_p \left(\frac{k_{\perp}}{m} \right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

$p=n-1$

M.A. Braun, V.V. Vechernin, Phys.Atom.Nucl. 63, 1831 (2000)

$$\sigma_{prot}(x, k_{\perp}; p_1, p_2, p_3) = C(p_1, p_2, p_3) (x_{coal} - x)^{2p-1} f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)$$

$$x < x_{coal}(p) = 1 + p/3, \quad p = p_1 + p_2 + p_3$$

$p=n-3$

M.A. Braun, V.V. Vechernin, Theor.Math.Phys. 139, 766 (2004)

$$f_p(t) = 2\pi \int_0^{\infty} dz z J_0(tz) [z K_1(z)]^p$$

$J_0(z)$ - the Bessel function, $K_1(z)$ - the modified Bessel function.

$$(2\pi)^{-2} \int f_p(|\mathbf{b}|) d^2\mathbf{b} = (2\pi)^{-1} \int_0^{\infty} f_p(t) t dt = 1$$

$$e^{-b_s x} = 10^2,$$

$$b_s \approx 7, \quad x = -2/3$$

Note that for $p=1$ it can be simplified to $f_1(t) = 4\pi/(t^2 + 1)^2$

$$\varphi_{pion}(k_{\perp}, p) \equiv \sigma_{pion}(x, k_{\perp}; p) / \sigma_{pion}(x, 0; p) = f_p \left(\frac{k_{\perp}}{m} \right) / f_p(0)$$

$$\varphi_{prot}(k_{\perp}, p) \equiv \sigma_{prot}(x, k_{\perp}; p) / \sigma_{prot}(x, 0; p)$$

$$\varphi_{prot}(k_{\perp}, p) = \frac{\sum_{p_1, p_2, p_3} \delta_{p, p_1+p_2+p_3} C(p_1, p_2, p_3) f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)}{\sum_{p_1, p_2, p_3} \delta_{p, p_1+p_2+p_3} C(p_1, p_2, p_3) f_{p_1}(0) f_{p_2}(0) f_{p_3}(0) \dots}$$

$$\varphi_{prot}(k_{\perp}, p_1, p_2, p_3) \equiv \frac{\sigma_{prot}(x, k_{\perp}; p_1, p_2, p_3)}{\sigma_{prot}(x, 0; p_1, p_2, p_3)} = \frac{f_{p_1} \left(\frac{k_{\perp}}{3m} \right) f_{p_2} \left(\frac{k_{\perp}}{3m} \right) f_{p_3} \left(\frac{k_{\perp}}{3m} \right)}{f_{p_1}(0) f_{p_2}(0) f_{p_3}(0)}$$

No free parameters (!) only m – the constituent quark mass: $m = 300$ MeV.

V. Vechernin, AIP Conference Proceedings 1701 (2016) 060020.