Phase diagram of rotating QCD with $N_f = 2$ clover-improved Wilson fermions

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in collaboration with

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Phase transitions in rotating QCD

Introduction

• In non-central heavy ion collisions creation of QGP with angular momentum is expected.



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- The rotation occurs with relativistic velocities.





 $\begin{bmatrix} L. Adamczyk et al. (STAR), Nature \\ 548, 62–65 (2017), arXiv:1701.06657 \\ [nucl-ex] \end{bmatrix} \\ \langle \omega \rangle \sim 6 \text{ MeV } (\sqrt{s_{NN}}\text{-averaged})$

Introduction

- In non-central heavy ion collisions creation of QGP with angular momentum is expected.
- The rotation occurs with relativistic velocities.



• How does the rotation affect to phase transitions in QCD?

NJL (and other phenomenological models) predicts that critical temperature decreases due to the rotation.

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Our lattice results for <u>gluodynamics</u> is opposite: critical temperature <u>increases</u> with rotation.

- V. V. Braguta et al., JETP Lett. 112, 6-12 (2020)
- V. V. Braguta et al., Phys. Rev. D 103, 094515 (2021), arXiv:2102.05084 [hep-lat]
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Taking into account the contribution of rotating gluons to NJL model:

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The running effective coupling $G(\omega)$ is introduced.

 \Rightarrow Critical temperature increases due to the rotation.

A. A. Roenko (JINR, BLTP)

- QCD (at thermal equilibrium) is investigated in the reference frame which rotates with the system with angular velocity Ω .
- In this reference frame there appears an external gravitational field

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2 \Omega^2 & \Omega y & -\Omega x & 0\\ \Omega y & -1 & 0 & 0\\ -\Omega x & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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• The partition function is¹

$$Z = \int D\psi \, D\bar{\psi} \, DA \, \exp\left(-S_G[A,\Omega] - S_F[\bar{\psi},\psi,A,m,\Omega]\right). \tag{1}$$

¹A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), @rXiv:1303.6292 [hep-lat]. 7 December 2022

Rotating QCD: continuum action

The Euclidean gluon action can be written as

$$S_G = \frac{1}{4g^2} \int d^4x \sqrt{g_E} g_E^{\mu\nu} g_E^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \,. \tag{2}$$

And the quark action reads as follows²

$$S_F = \int d^4x \sqrt{g_E} \,\bar{\psi} \big(\gamma^\mu (D_\mu - \Gamma_\mu) + m \big) \psi \,, \tag{3}$$

The covariant derivative D_{μ} and the spinor affine connection Γ_{μ} is

$$D_{\mu} = \partial_{\mu} - iA_{\mu} \,, \tag{4}$$

$$\Gamma_{\mu} = -\frac{i}{4}\sigma^{ij}\omega_{\mu ij}\,,\tag{5}$$

$$\sigma^{ij} = \frac{i}{2} (\gamma^i \gamma^j - \gamma^j \gamma^i) \tag{6}$$

$$\omega_{\mu i j} = g^E_{\alpha\beta} e^{\alpha}_i \left(\partial_{\mu} e^{\beta}_j + \Gamma^{\beta}_{\nu\mu} e^{\nu}_j \right) \tag{7}$$

where e_i^{μ} is the vierbein and $\Gamma^{\alpha}_{\mu\nu}$ is the Christoffel symbol.

²A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111, 081601 (2013), arXiv: 1303.6292 [hep-lat].

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Phase transitions in rotating QCD

The Euclidean metric tensor can be obtained from $g_{\mu\nu}$ by Wick rotation $t \to i\tau$

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix} \,.$$

where imaginary angular velocity $\Omega_I = -i\Omega$ is introduced.

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The Euclidean metric tensor can be obtained from $g_{\mu\nu}$ by Wick rotation $t \to i\tau$

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix} \,.$$

where imaginary angular velocity $\Omega_I = -i\Omega$ is introduced.

Sign problem

- The Euclidean action is complex-valued function with real rotation!
- The Monte–Carlo simulations are conducted with imaginary angular velocity
- The results are analytically continued to the region of the real angular velocity

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The resulting partition function is

$$Z = \int D\psi \, D\bar{\psi} \, DU \, \exp\left(-S_G[U,\Omega_I] - S_F[\bar{\psi},\psi,m,U,\Omega_I]\right) = \int DU \, \det M[m,U,\Omega_I] \, e^{(-S_G[U,\Omega_I])}$$
(8)

- $N_f = 2$ clover-improved Wilson fermions $(c_{SW}$ from one-loop) + RG-improved (Iwasaki) gauge action are used.
- We reanalyze data for $m_{PS}a$ and m_Va at zero temperature from CP-PACS and WHOT-QCD collaborations to restore LCP's more frequently in β and set the scale.
- Simulation is performed on the lattices of size $N_t \times N_z \times N_s^2$ $(N_s = N_x = N_y)$, which rotate around z-axis. Up to now, only results with $N_t = 4$ are available, work in progress...

LCP and scale setting



To set the temperature along the given LCP we use the zero-temperature mass of vector meson $(m_V$ -input)

$$\frac{T}{m_V}(m_{PS}/m_V,\beta) = \frac{1}{N_t \times m_V a(m_{PS}/m_V,\beta)}.$$
(9)

and find

• The system should be limited in the directions, which are orthogonal to the rotation axis: $\Omega(N_s-1)a/\sqrt{2}<1$

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- The system should be limited in the directions, which are orthogonal to the rotation axis: $\Omega(N_s 1)a/\sqrt{2} < 1$
- The use of periodic/open/Dirichlet BC gives qualitatively the same results for rotating gluodynamics (boundary is screened). PBC in directions x, y are used.
- The critical temperature in gluodynamics depends mainly on the linear velocity on the boundary $v_I = \Omega_I (N_s - 1)a/2$. Thus, v_I is fixed in simulations instead of angular velocity Ω_I in physical units (e.g., MeV).

Observables

• The (spatial averaged) Polyakov loop is

$$L(\vec{x}) = \text{Tr}\left[\prod_{\tau=0}^{N_t-1} U_4(\vec{x},\tau)\right], \qquad L = \frac{1}{N_s^2 N_z} \sum_{\vec{x}} L(\vec{x}).$$
(10)

The pseudo-critical temperature T_{pc} of the confinement/deconfinement transition is determined using the Polyakov loop susceptibility

$$\chi_L = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right) , \qquad (11)$$

by means of the Gaussian fit.

• The (bare) chiral condensate is

$$\langle \bar{\psi}\psi \rangle^{bare} = -\frac{N_f T}{V} \left\langle \operatorname{Tr}(M^{-1}) \right\rangle$$
 (12)

For the chiral transition, pseudo-critical temperature T_{pc} is determined using the peak of the (disconnected) chiral susceptibility:

$$\chi_{\langle\bar{\psi}\psi\rangle}^{bare} = \frac{N_f T}{V} \left[\left\langle \operatorname{Tr}(M^{-1})^2 \right\rangle - \left\langle \operatorname{Tr}(M^{-1}) \right\rangle^2 \right] \tag{13}$$

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Rotating QCD: Periodic boundary conditions



Figure: The Polyakov loop as a function of $T/T_{pc}(\Omega = 0)$ for different values of imaginary linear velocity on the boundary v_I . Lattice $4 \times 16 \times 17^2$, LCP $m_{PS}/m_V = 0.80$.

• Pseudo-critical temperature decreases due to imaginary rotation (like in gluodynamics).

Rotating QCD: Periodic boundary conditions



Figure: The Polyakov loop susceptibility and chiral susceptibility as a function of $T/T_{pc}(\Omega = 0)$ for different values of imaginary linear velocity on the boundary v_I . Lattice $4 \times 16 \times 17^2$, LCP $m_{PS}/m_V = 0.80$.

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In order to disentangle the effect of the rotation on fermions and gluons, the separate angular velocities are introduced: $S_G(\Omega_G) + S_F(\Omega_F)$.

Rotating QCD: various rotation regimes



Figure: The Polyakov loop as a function of T/T_{pc} for various rotation regimes. Lattice $4 \times 16 \times 17^2$, $m_{PS}/m_V = 0.80$.

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Rotating QCD: various rotation regimes



Figure: The Polyakov loop susceptibility and chiral susceptibility as a function of T/T_{pc} for various rotation regimes. Lattice $4 \times 16 \times 17^2$, $m_{PS}/m_V = 0.80$.

• Rotation of fermions and gluons separately has the opposite influence on the critical temperature.

Rotating QCD: various rotation regimes



Figure: The Polyakov loop susceptibility and chiral susceptibility as a function of T/T_{pc} for various rotation regimes. Lattice $4 \times 16 \times 17^2$, $m_{PS}/m_V = 0.80$.

- Rotation of fermions and gluons separately has the opposite influence on the critical temperature.
- But only the regime with $\Omega_G = \Omega_F$ is physical.

Rotating QCD: critical temperature



LCP's with $m_{PS}/m_V = 0.65, 0.70, 0.75, 0.80, 0.85$ were considered; $v_I/c < 0.3$.

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2}$$

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Rotating QCD: critical temperature



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$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2} \implies \frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The pseudo-critical temperature increases with the angular velocity $(v \propto \Omega)$.
- The coefficient B_2 slightly grows with approaching to chiral limit.
- The chiral transition shifts to the same direction as confinement-deconfinement transition.

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Conclusions

- The separate rotation of quarks and gluons in QCD have the opposite influence on the critical temperature.
- The critical temperature in $N_f = 2$ QCD increases with angular velocity $(v \propto \Omega)$

$$\frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2} \,.$$

It's not Tolman-Ehrenfest effect!

- The coefficient B_2 slightly grows with decreasing pion mass in considered range $(m_{PS}/m_V = 0.65 \dots 0.85).$
- The (preliminary) results are similar to gluodynamics, where the critical temperature also increases with angular velocity.
- It should be noted, that NJL (and other phenomenological models) predicts that critical temperature decreases due to the rotation. But taking into account the contribution of rotating gluons leads to an increase in T_c .
- Future plans: increase statistics; simulations with smaller pion mass, on finer lattices $(N_t = 6, 8)$, with an open BC.

Thank you for your attention!

See the details in:

• V. V. Braguta et al., (2022), arXiv:2212.03224 [hep-lat]

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Rotating QCD: various rotational regimes



Figure: The pseudo-critical temperature as a function of imaginary linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions).

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2} \tag{14}$$

$$\Omega_G = \Omega_F \neq 0 \qquad \qquad \Omega_G \neq 0 \qquad \qquad \Omega_F \neq 0 B_2 > 0 \qquad \qquad B_2^{(G)} > B_2 \qquad \qquad B_2^{(F)} < 0$$

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$$\int d^4x \sqrt{g_E} \,(\dots) = \int_0^{1/T} dx_0 \sqrt{g_{44}} \int d^3x \sqrt{\gamma_E} \,(\dots) = \int_0^{1/T} dx_0 \int d^3x \sqrt{g_E} \,(\dots)$$

• Interpretation: Tolman-Ehrenfest effect. In gravitational field the temperature isn't a constant in space at thermal equilibrium:

$$T(r)\sqrt{g_{00}} = const,$$

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$$\int d^4x \sqrt{g_E} \, (\dots) = \int_0^{1/T} dx_0 \sqrt{g_{44}} \int d^3x \sqrt{\gamma_E} \, (\dots) = \int_0^{1/T} dx_0 \int d^3x \sqrt{g_E} \, (\dots)$$

• Interpretation: Tolman-Ehrenfest effect. In gravitational field the temperature isn't a constant in space at thermal equilibrium:

$$T(r)\sqrt{g_{00}} = const,$$

• For the (real) rotation one has

$$T(r)\sqrt{1-r^2\Omega^2}=const\equiv T\,,$$

• One could expect, that the rotation effectively warm up the periphery of the modeling volume

$$T(r) > T(r=0),$$

and as a result, from kinematics, the critical temperature should decreases.

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The Euclidean metric tensor can be obtained from $g_{\mu\nu}$ by Wick rotation $t \to i\tau$

$$g_{\mu\nu}^{E} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_{I} \\ 0 & 1 & 0 & -x\Omega_{I} \\ 0 & 0 & 1 & 0 \\ y\Omega_{I} & -x\Omega_{I} & 0 & 1 + r^{2}\Omega_{I}^{2} \end{pmatrix}$$

where imaginary angular velocity $\Omega_I = -i\Omega$ is introduced. Substituting the $(g_E)_{\mu\nu}$ to formula (15) one gets

$$S_{G} = \frac{1}{2g^{2}} \int d^{4}x \left[(1 + r^{2}\Omega_{I}^{2})F_{xy}^{a}F_{xy}^{a} + (1 + y^{2}\Omega_{I}^{2})F_{xz}^{a}F_{xz}^{a} + (1 + x^{2}\Omega_{I}^{2})F_{yz}^{a}F_{yz}^{a} + F_{x\tau}^{a}F_{x\tau}^{a} + F_{y\tau}^{a}F_{y\tau}^{a} + F_{z\tau}^{a}F_{z\tau}^{a} - \frac{2y\Omega_{I}(F_{xy}^{a}F_{y\tau}^{a} + F_{xz}^{a}F_{z\tau}^{a}) - 2x\Omega_{I}(F_{yx}^{a}F_{x\tau}^{a} + F_{yz}^{a}F_{z\tau}^{a}) + 2xy\Omega_{I}^{2}F_{xz}^{a}F_{zy}^{a} \right]$$

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The covariant Dirac operator depends on the choice of the vierbein. We choose the vierbein in the form^3

$$e_1^x = e_2^y = e_3^z = e_4^\tau = 1$$
, $e_4^x = -y\Omega_I$, $e_4^y = x\Omega_I$, and other $e_i^\mu = 0$

As the result, the Euclidean quark action is

$$S_F = \int d^4x \,\bar{\psi} \left(\gamma^x D_x + \gamma^y D_y + \gamma^z D_z + \gamma^\tau \left(D_\tau + i\Omega_I \frac{\sigma^{12}}{2} \right) + m \right) \psi \,, \qquad (15)$$

where the gamma matrices are given by $\gamma^{\mu}=\gamma^{i}e_{i}^{\mu}$

$$\gamma^{x} = \gamma^{1} - y\Omega_{I}\gamma^{4}, \quad \gamma^{y} = \gamma^{2} + x\Omega_{I}\gamma^{4}, \quad \gamma^{z} = \gamma^{3}, \quad \gamma^{\tau} = \gamma^{4}.$$
 (16)

The quark action contains orbit-rotation coupling term $\gamma^{\tau} \Omega_I (xD_y - yD_x)$ and spin-rotation coupling term $i\gamma^{\tau} \Omega_I \sigma^{12}/2$.

³A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013); @Xiv:**1**303.6292 [hep-lat].

Rotating QCD: gluon lattice action

We use RG-improved (Iwasaki) lattice gauge action (for non-rotating part):

$$S_{G} = \beta \sum_{x} \left((c_{0} + r^{2} \Omega_{I}^{2}) W_{xy}^{1 \times 1} + (c_{0} + y^{2} \Omega_{I}^{2}) W_{xz}^{1 \times 1} + (c_{0} + x^{2} \Omega_{I}^{2}) W_{yz}^{1 \times 1} + c_{0} \left(W_{x\tau}^{1 \times 1} + W_{y\tau}^{1 \times 1} + W_{z\tau}^{1 \times 1} \right) + y \Omega_{I} \left(W_{xy\tau}^{1 \times 1 \times 1} + W_{xz\tau}^{1 \times 1 \times 1} \right) - x \Omega_{I} \left(W_{yx\tau}^{1 \times 1 \times 1} + W_{yz\tau}^{1 \times 1 \times 1} \right) + xy \Omega_{I}^{2} W_{xzy}^{1 \times 1 \times 1} + \sum_{\mu \neq \nu} c_{1} W_{\mu\nu}^{1 \times 2} \right), \quad (17)$$

with $\beta = 6/g^2$, and $c_0 = 1 - 8c_1$, and $c_1 = -0.331$, where

$$W_{\mu\nu}^{1\times1}(x) = 1 - \frac{1}{3} \text{Re Tr } \bar{U}_{\mu\nu}(x) ,$$
 (18)

$$W_{\mu\nu}^{1\times2}(x) = 1 - \frac{1}{3} \text{Re Tr } R_{\mu\nu}(x),$$
 (19)

$$W_{\mu\nu\rho}^{1\times1\times1}(x) = -\frac{1}{3} \text{Re Tr } \bar{V}_{\mu\nu\rho}(x),$$
 (20)

 $\bar{U}_{\mu\nu}$ denotes clover-type average of 4 plaquettes,

 $R_{\mu\nu}$ is a rectangular loop,

 $\overline{V}_{\mu\nu\rho}$ is asymmetric chair-type average of 8 chairs.

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Rotating QCD: quark lattice action

The lattice quark action has the following form $(N_f = 2 \text{ clover-improved Wilson fermions are used})$

$$S_{F} = \sum_{f} \sum_{x_{1},x_{2}} \bar{\psi}^{f}(x_{1}) \left\{ \delta_{x_{1},x_{2}} - \kappa \left[(1 - \gamma^{x}) T_{x+} + (1 + \gamma^{x}) T_{x-} + (1 - \gamma^{y}) T_{y+} + (1 + \gamma^{y}) T_{y-} + (1 - \gamma^{z}) T_{z+} + (1 + \gamma^{z}) T_{z-} + (1 - \gamma^{\tau}) \exp\left(\frac{ia\Omega_{I} \sigma^{12}}{2}\right) T_{\tau+} + (1 + \gamma^{\tau}) \exp\left(-ia\Omega_{I} \sigma^{12} \frac{\sigma^{12}}{2}\right) T_{\tau-} \right] - \delta_{x_{1},x_{2}} c_{SW} \kappa \sum_{\mu < \nu} \sigma_{\mu\nu} F_{\mu\nu} \left\{ \psi^{f}(x_{2}), \quad (21) \right\}$$

where $\kappa = 1/(8+2am)$, $T_{\mu+} = U_{\mu}(x_1)\delta_{x_1+\mu,x_2}$, $T_{\mu-} = U_{\mu}^{\dagger}(x_1)\delta_{x_1-\mu,x_2}$ and $\gamma^x = \gamma^1 - y\Omega_I\gamma^4$, $\gamma^y = \gamma^2 + x\Omega_I\gamma^4$, $\gamma^z = \gamma^3$, $\gamma^\tau = \gamma^4$.

The clover coefficient is taken as $c_{SW} = (1 - W^{1 \times 1})^{-3/4} = (1 - 0.8412/\beta)^{-3/4}$ (one-loop result for the plaquette are used).

The spin-rotation coupling term is exponentiated like chemical potential.

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Figure: The local Polyakov loop $|\langle L(x,y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x, y) \rangle|$ is zero for all spatial points in the confinement phase, both with and without rotation \Rightarrow Polyakov loop still acts as the order parameter.
- In deconfinement phase the boundary is screened.



Figure: The local Polyakov loop $|\langle L(x,y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x,y)\rangle|$ is zero for all spatial points in the confinement phase, both without rotation and with nonzero angular velocity.
- The local Polyakov loop demonstrates weak dependence on the coordinate in the deconfinement phase.



Figure: The local Polyakov loop $|\langle L(x,y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x, y) \rangle|$ is equal three on the boundary in both phases.
- The boundary is screened.

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