

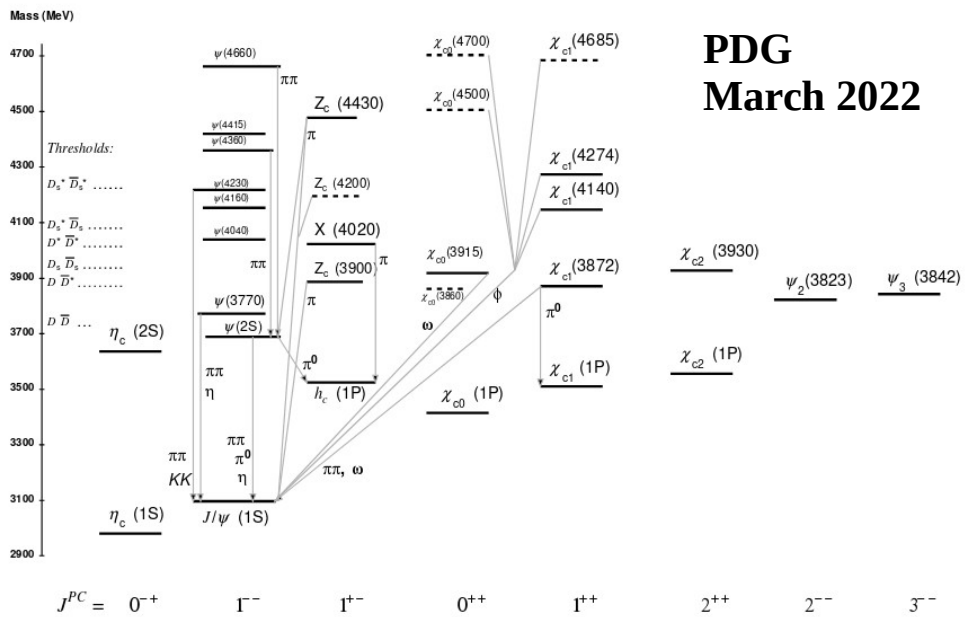
Amplitude analysis methods for experimental studies of multi-quark states

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Introduction

- The quark model of hadrons proposed by Murray Gell-Mann assumes the existence of states with structure beyond standard $q\bar{q}$ and qqq -models.
- At the moment, there are many Lattice QCD models that describe the spectrum of multi-quark states that have a large number of parameters. Experimental study of these states will help reduce the number of existing models and limit their parameters.
- One of the convenient processes for searching and studying multi-quark states is the B-hadron decays. The secondary vertex is far enough away from the primary vertex to suppress backgrounds.
- In most cases, the parameters of the model (M, Γ, J^P , decay constants) are unknown, therefore, a ready-made MC decay generator cannot be used.
- On the other hand, the use of Monte Carlo makes it possible to take into account detector effects.
- To obtain information about the spin-parity of states, in addition to Dalitz variables, it is necessary to consider the angular distributions.



[1] R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

Types of analysis:

- Model independent (can be used to confirm exotic contributions to decays, but not suitable for parameter measurements)
- [2] Phys.Rev.Lett. 117 (2016) 8, 082002
- Model dependent (precise study)

Helicity formalism (Two-body decay $A \rightarrow BC$)

The problem with using **Spin-orbit** formalism in constructing the decay amplitude is that spin and orbital angular momentum operators of particles are defined in reference frames that are not at rest with respect to one another.

Helicity operator is invariant under both rotations and boosts along momentum direction.

$$\text{Amp}_{A \rightarrow BC} = H_{\lambda_B, \lambda_C}^{A \rightarrow BC} D_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\phi_B^{[A]}, \theta_A, 0)^*$$

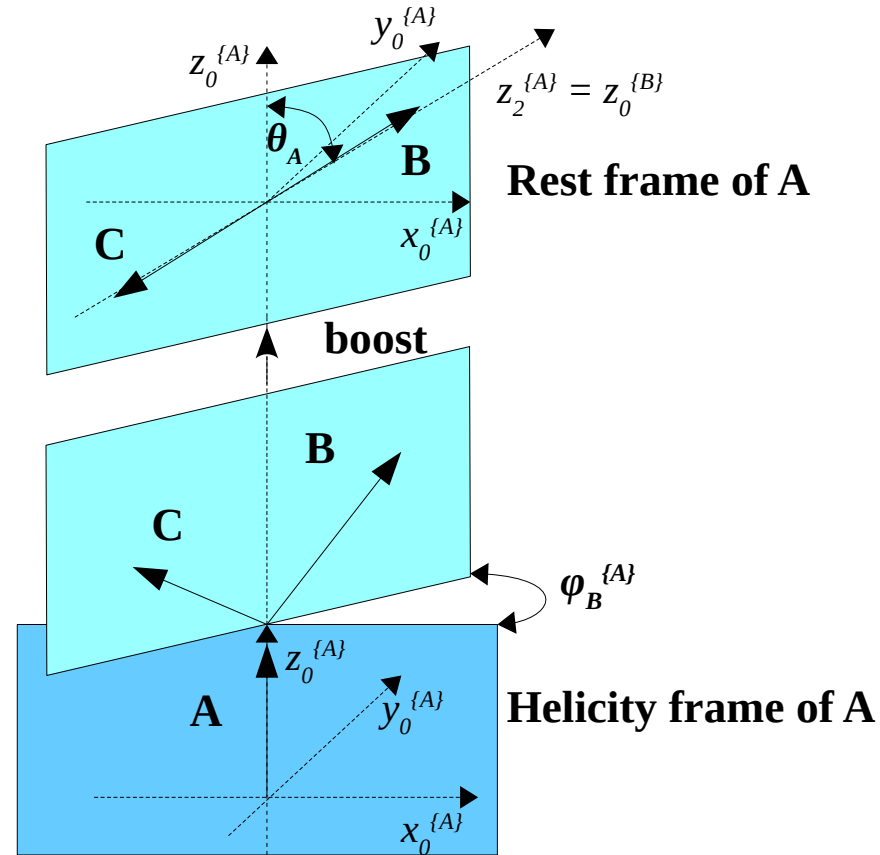
$$D_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\phi_B^{[A]}, \theta_A, 0)^* = e^{i\lambda_A \phi_B^{[A]}} d_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\theta_A)$$

The behavior of off-shell resonances must be described by the Breit-Wigner term. To describe decays under threshold, Flatté parametrization could be used.

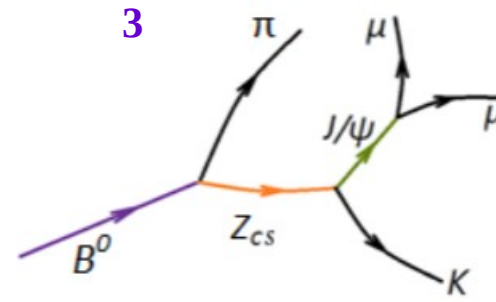
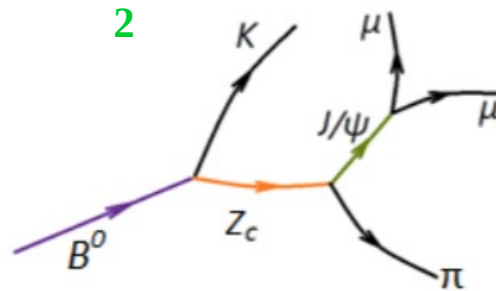
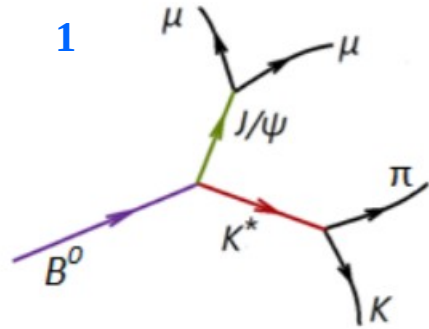
$$BW_R(M_R, m_R, \Gamma_R) = \frac{1}{M_R^2 - m_R^2 - i M_R \Gamma(m_R)}$$

$$\Gamma(m_R) = \Gamma_R \left(\frac{p_R}{p_{R0}} \right)^{2L_R+1} \left(\frac{M_R}{m_R} \right) F_{L_R}^2$$

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} = \sum_L \sum_S (-1)^{J_B - J_C + L - S + 2\lambda_B - 2\lambda_C} \sqrt{(2L+1)(2S+1)} B_{LS} \times \begin{pmatrix} J_B & J_C & S \\ \lambda_B & -\lambda_C & \lambda_C - \lambda_B \end{pmatrix} \begin{pmatrix} L & S & J_A \\ 0 & \lambda_B - \lambda_C & \lambda_C - \lambda_B \end{pmatrix}$$



Helicity formalism (example $B^0 \rightarrow J/\psi K \pi$ decays) 1



* In the next slides, the mass and width will act as free parameters of the fit

Decay chain 1 (no exotic model):

state	M, MeV	Γ , GeV	J^P
$K^*(1410)$ [1]	1414	232	1^-
$K^*_0(1430)$ [1]	1425	270	0^+
$K^*_2(1430)$ [1]	1432	109	2^+
$K^*(1680)$ [1]	1718	322	1^-
$K^*(1780)$ [1]	1779	161	1^-
$K^*(1950)$ [1]	1944	100	0^+
$K^*(2045)$ [1]	2048	199	4^+
K^* NR	1975	-	0^+

BW = 1

Decay chain 2 (model with exotic contribution):

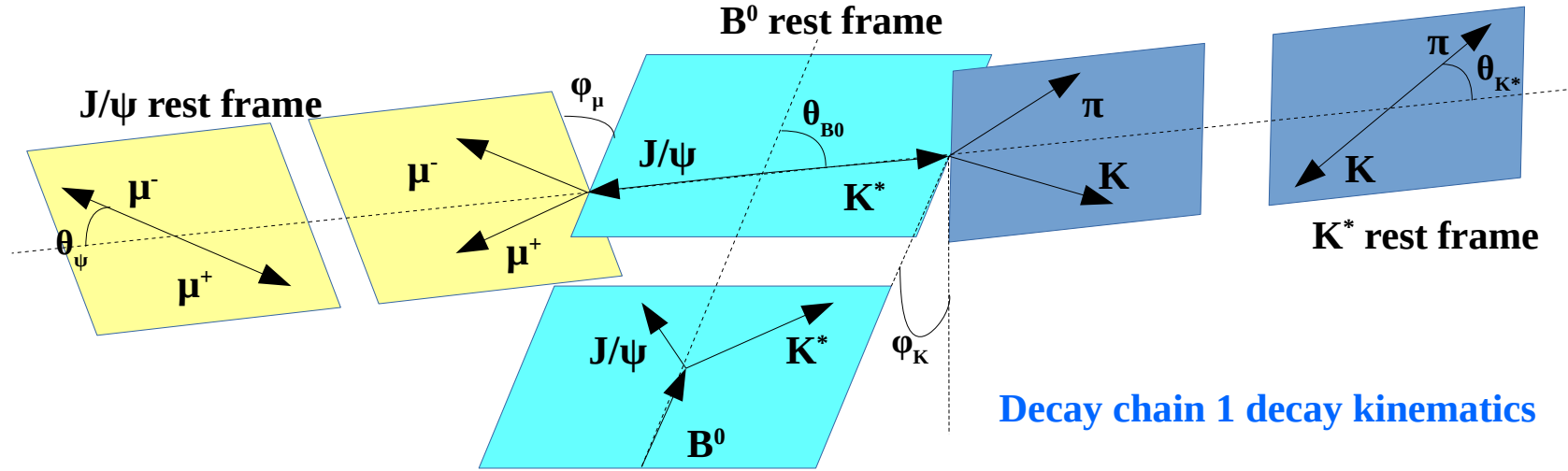
state	M, MeV	Γ , GeV	J^P
$Z_c(3900)$ [1]	3887	28	1^+
$Z_c(4200)$ [1]*	4196	370	1^+
$Z_c(4430)$ [1]	4478	181	1^+

Decay chain 3 (model with exotic contribution):

state	M, MeV	Γ , GeV	J^P
$Z_{cs}(4000)$ [2]	4003	131	1^+
$Z_{cs}(4220)$ [2]	4216	233	1^+

[3] R. Aaij *et al.* (LHCb Collaboration), *Observation of New Resonances Decaying to $J/\psi K^+$ and $J/\psi \phi$* , Phys. Rev. Lett. 127, 082001

Helicity formalism (example $B^0 \rightarrow J/\psi K \pi$ decays) 2



K* decay chain:

$B^0 \rightarrow J/\psi K^*$ (weak decay):

$$Amp_{B^0 \rightarrow J/\psi K^*} = H_{\lambda_\psi, \lambda_{K^*}}^{B^0 \rightarrow J/\psi K^*} d_{0, \lambda_{K^*} - \lambda_\psi}^0(\theta_{B^0})$$

$K^* \rightarrow K \pi$ (strong decay):

$$Amp_{K^* \rightarrow K \pi} = H_{0,0}^{K^* \rightarrow K \pi} e^{i\phi_{K^*} \lambda_{K^*}} d_{\lambda_{K^*}, 0}^{J_{K^*}}(\theta_{K^*}) * R(m(K \pi), L_{B^0}, L_{K^*})$$

$J/\psi \rightarrow \mu \mu$ (electromagnetic decay):

$$Amp_{J/\psi \rightarrow \mu \mu} = e^{i\phi_{\mu\mu} \lambda_\psi} d_{\lambda_\psi, \Delta\lambda_\mu}^0(\theta_\psi)$$

$$R(m(K \pi), L_{B^0}, L_{K^*}) = \left(\frac{p_{B^0}}{m_{B^0}}\right)^{L_{B^0}} F_{L_{B^0}} BW_{K^*}(M_{K^*}, m(K, \pi), \Gamma_{K^*}) \left(\frac{p_{K^*}}{m(K \pi)}\right)^{L_{K^*}} F_{L_{K^*}}$$

For EM or strong decays:

$$\mathcal{H}_{-\lambda_B, -\lambda_C}^{A \rightarrow BC} = P_A P_B P_C (-1)^{J_B + J_C - J_A} \mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC}$$

$$\begin{aligned} & Amp_{K^* \text{ chain}}(m(K, \pi), \text{angles 1})_{\Delta\lambda_\mu} = \\ & = \sum_{\lambda_\psi, \lambda_{K^*}} \sum_{K^* \text{ states}} Amp_{B^0 \rightarrow J/\psi K^*} Amp_{K^* \rightarrow K \pi} Amp_{J/\psi \rightarrow \mu \mu} \end{aligned}$$

Helicity formalism (example $B^0 \rightarrow J/\psi K \pi$ decays) 3

Decay chain 2/3 decay kinematics

B^0 rest frame

Z_C/Z_{CS} rest frame

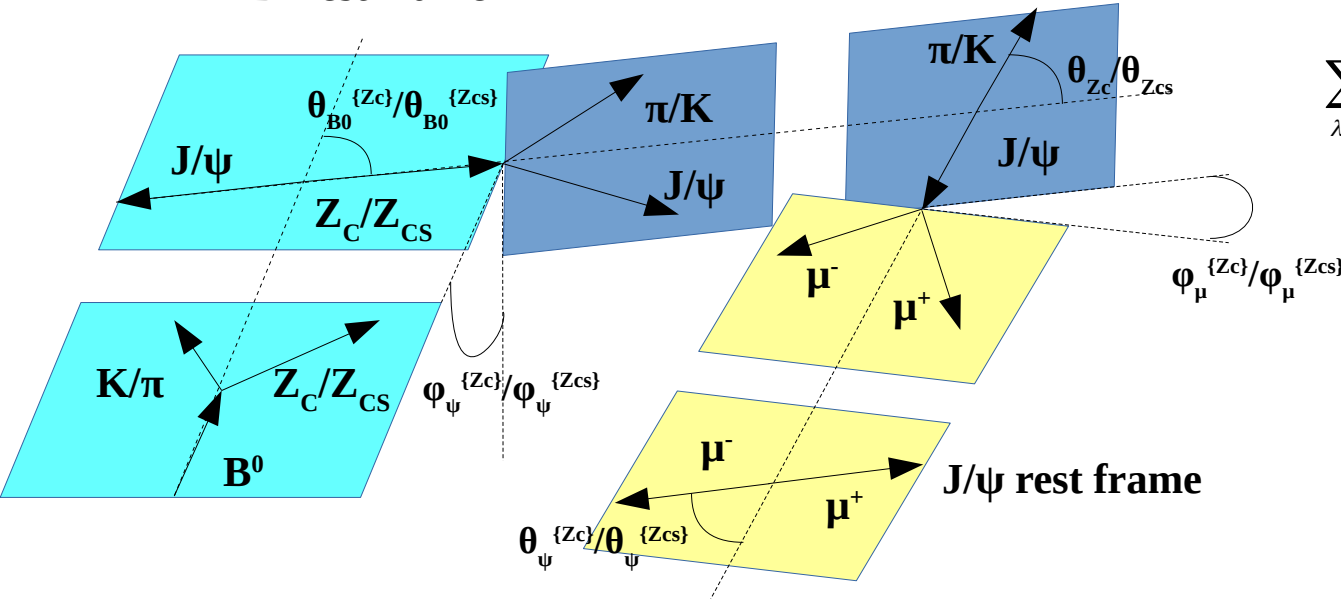
Muons are final-state particles, their helicity states in 2, 3 decay chains need to be rotated to the muon helicity states in K^* decay chain.

$$\sum_{\lambda_\mu^{Z_c}} D_{\lambda_\mu^{Z_c} \lambda_\mu}^{J_\mu}(\alpha_\mu, 0, 0)^* = \sum_{\lambda_\mu^{Z_c}} e^{i\lambda_\mu^{Z_c} \alpha_\mu} \delta_{\lambda_\mu^{Z_c} \lambda_\mu} = e^{i\lambda_\mu^{Z_c} \alpha_\mu}$$

$$\alpha_\mu^{Z_c} = \text{atan2}((\hat{p}_\mu^{[\psi]} \times \hat{x}_1) \cdot \hat{x}_2, \hat{x}_1 \cdot \hat{x}_2),$$

$$\vec{x}_2 = \hat{p}_{K^*}^{[\psi]} - (\hat{p}_{K^*}^{[\psi]} \cdot \hat{p}_\mu^{[\psi]}) \hat{p}_\mu^{[\psi]},$$

$$\vec{x}_1 = \hat{p}_\pi^{[\psi]} - (\hat{p}_\pi^{[\psi]} \cdot \hat{p}_\mu^{[\psi]}) \hat{p}_\mu^{[\psi]},$$

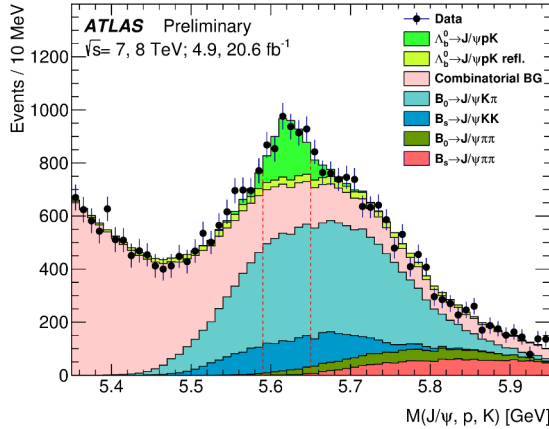


B^0 decay matrix element:

$$|M^{B^0}|^2 = \sum_{\Delta\lambda_\mu} |Amp_{K^*chain}(m(K, \pi), angles 1)_{\Delta\lambda_\mu} + e^{i\Delta\lambda_\mu \alpha_\mu^{Z_c}} Amp_{Z_Cchain}(m(J/\psi, \pi), angles 2)_{\Delta\lambda_\mu} + e^{i\Delta\lambda_\mu \alpha_\mu^{Z_{CS}}} Amp_{Z_{CS}chain}(m(J/\psi, K), angles 3)_{\Delta\lambda_\mu}|^2$$

Run1 pentaquark analysis in ATLAS

The events of pp collisions ($\sqrt{s} = 7, 8$ TeV, $L = 4.9, 20.6$ fb $^{-1}$) were selected for analysis by ATLAS. [4] ATLAS-CONF-2019-048



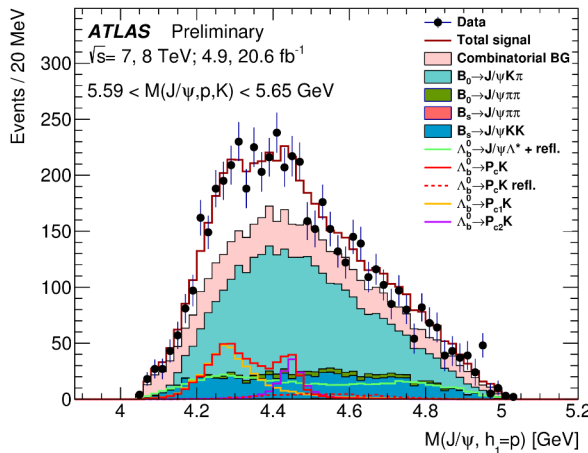
Parameter	Value	LHCb value [5]
$N(P_{c1})$	400^{+130}_{-140} (stat) $^{+110}_{-100}$ (syst)	—
$N(P_{c2})$	150^{+170}_{-100} (stat) $^{+50}_{-90}$ (syst)	—
$N(P_{c1} + P_{c2})$	540^{+80}_{-70} (stat) $^{+70}_{-80}$ (syst)	—
$\Delta\phi$	$2.8^{+1.0}_{-1.6}$ (stat) $^{+0.2}_{-0.1}$ (syst) rad	—
$m(P_{c1})$	4282^{+33}_{-26} (stat) $^{+28}_{-7}$ (syst) MeV	$4380 \pm 8 \pm 29$ MeV
$\Gamma(P_{c1})$	140^{+77}_{-50} (stat) $^{+41}_{-33}$ (syst) MeV	$205 \pm 18 \pm 86$ MeV
$m(P_{c2})$	4449^{+20}_{-29} (stat) $^{+18}_{-10}$ (syst) MeV	$4449.8 \pm 1.7 \pm 2.5$ MeV
$\Gamma(P_{c2})$	51^{+59}_{-48} (stat) $^{+14}_{-46}$ (syst) MeV	$39 \pm 5 \pm 19$ MeV

Although the data prefer model with 2 or more Pc states, the model without pentaquarks is not excluded.

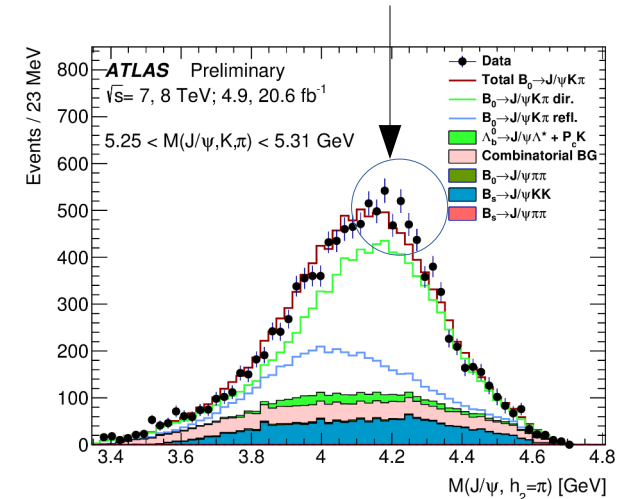
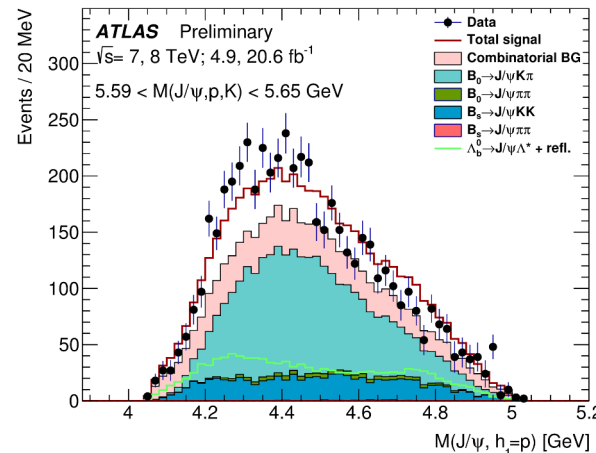
It is planned to continue the analysis on the RUN2 data.

Run1 data shows strong hint of exotic contribution to $B^0 \rightarrow J/\psi K \pi$ decays

Model with 2 pentaquarks



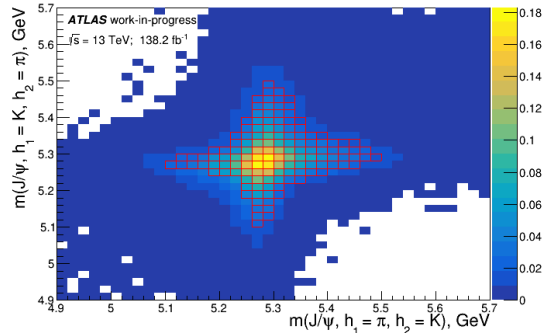
Model without pentaquarks



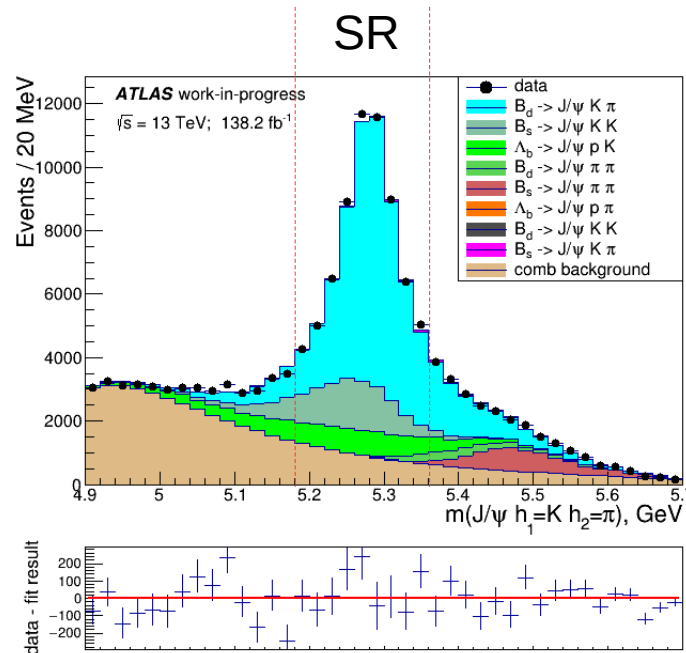
$B^0 \rightarrow J/\psi K \pi$ analysis in ATLAS experiment

The main problem of amplitude analysis of B-hadron decays in ATLAS is lack of hadron particle identification. The events of pp collisions ($\sqrt{s} = 13$ TeV, $L = 138.2$ fb $^{-1}$) were selected for analysis, consisting of 2 muon (J/ψ vertex) and 2 hadron tracks.

B^0 - \bar{B}^0 invariant mass plane
Signal area shape



For me calculation – generator events
Distributions – reconstructed events



Selected events consist of:

- $B^0 \rightarrow J/\psi K \pi$
- $B^0 \rightarrow J/\psi K K$
- $B^0 \rightarrow J/\psi \pi \pi$
- $B_s \rightarrow J/\psi K K$
- $B_s \rightarrow J/\psi K \pi$
- $B_s \rightarrow J/\psi \pi \pi$
- $\Lambda_b \rightarrow J/\psi p K$
- $\Lambda_b \rightarrow J/\psi p \pi$
- Combinatorial background

Results for full model: $K^* + Z_C(3900), Z_C(4200)$
 $J^P = 1^+, Z_C(4430), Z_{CS}(4000), Z_{CS}(4220)$

decay	N_{global}	N_{signal}
$B^0 \rightarrow J/\psi K \pi$	55500±480	41520 (69.8%)
$B^0 \rightarrow J/\psi K K$	120	8 (< 0.01%)
$B^0 \rightarrow J/\psi \pi \pi$	2470	570 (1%)
$B_s \rightarrow J/\psi K K$	13050	7670 (12.9%)
$B_s \rightarrow J/\psi K \pi$	440	200 (0.3%)
$B_s \rightarrow J/\psi \pi \pi$	5910	90 (0.2%)
$\Lambda_b \rightarrow J/\psi p K$	11920	4880 (8.2%)
$\Lambda_b \rightarrow J/\psi p \pi$	50	16 (< 0.01%)
Comb bg	48150	4510 (7.6%)

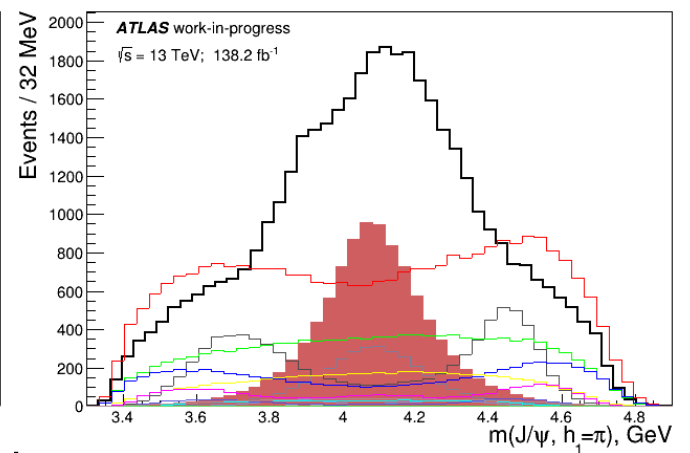
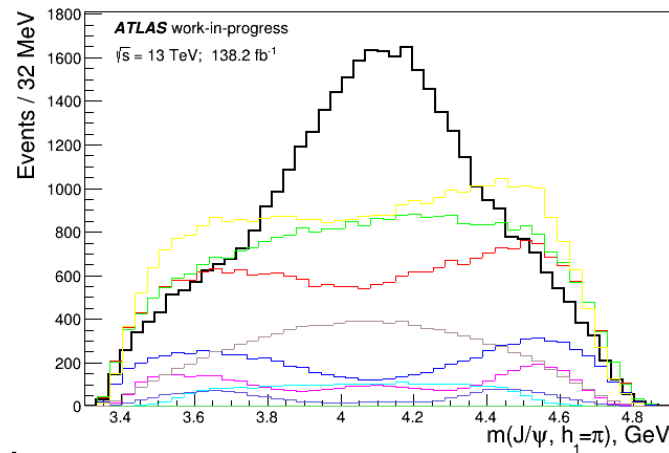
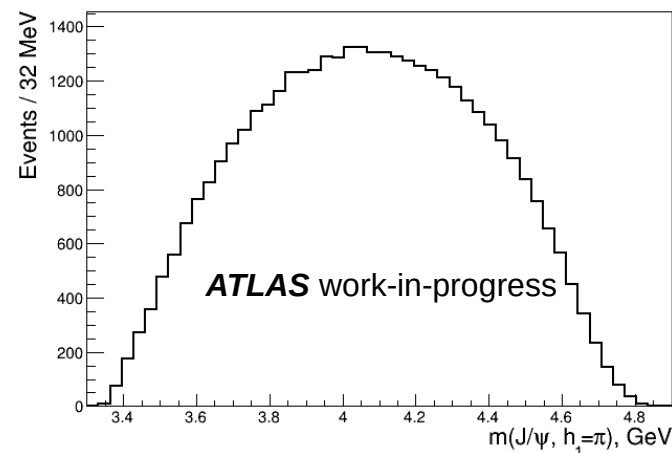
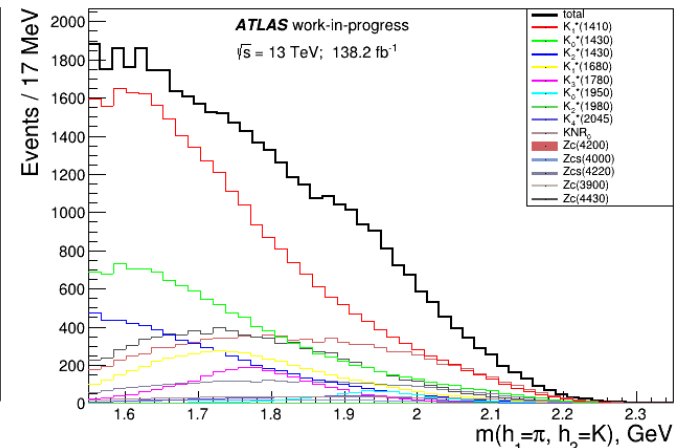
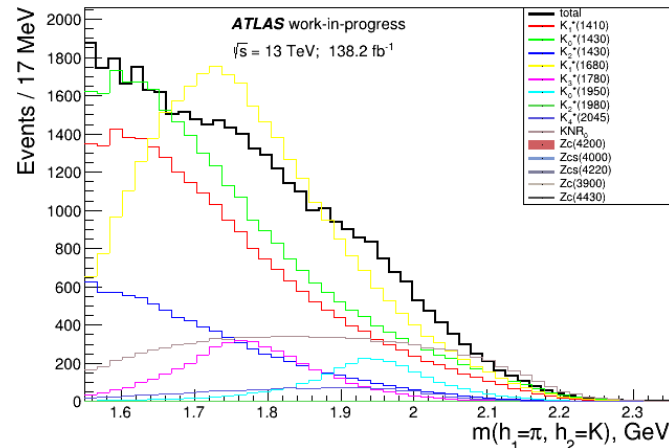
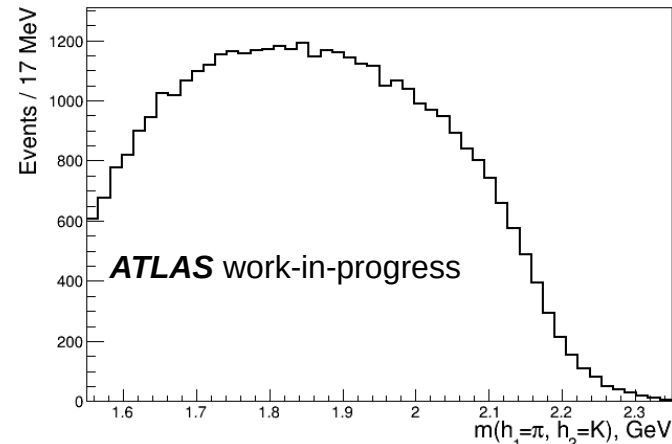
Results of decays simulation

Each generated phase-space MC event is weighted with the analytically derived decay matrix element.

Phase space MC

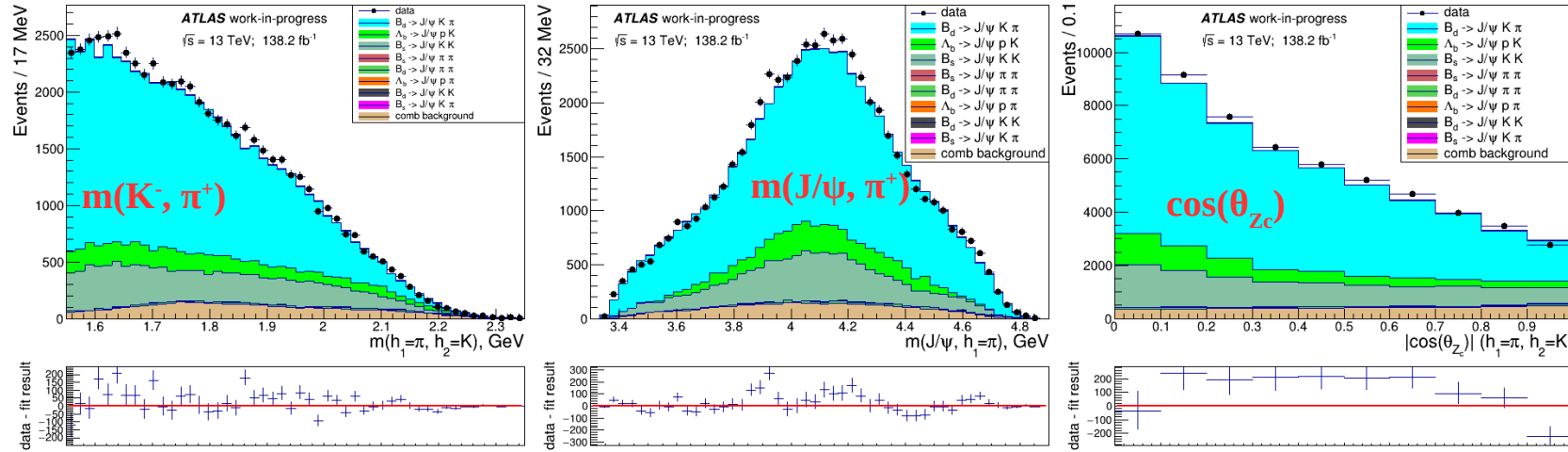
no exotic model (only K^*)

model with 3 Z_C and 2 Z_{CS} states



SR fit results

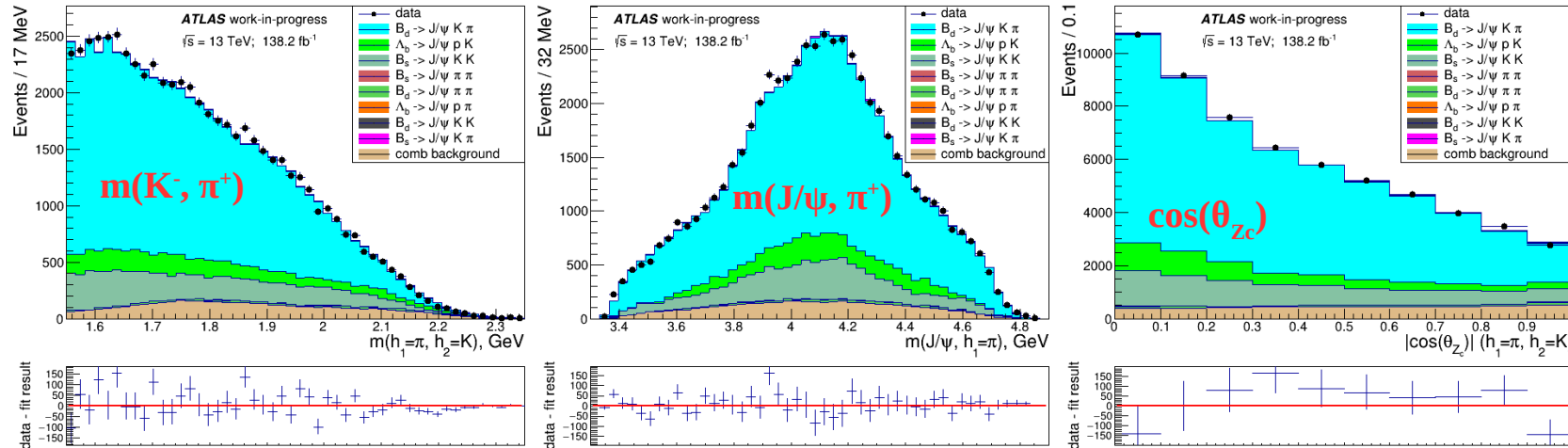
Fit results for no exotic model (only K^*)



Wilks significance of exotic contribution:
 $\Delta -2\ln(L) = 430$
 $N_{\text{parameters over no exotic}} = 30$
 $Z = 17.89\sigma$

Despite the absence of a pronounced bump in the data distribution, the formalism allows us to estimate the contribution from the states and measure the $Z_c(4200)$ parameters:

Fit results for exotic model ($K^* + Z_c(3900), Z_c(4200) J^P = 1^+, Z_c(4430), Z_{cS}(4000), Z_{cS}(4220)$)



$M = 4083 \text{ MeV}$
 $\Gamma = 224 \text{ MeV}$

Di-charmonium tetraquark study

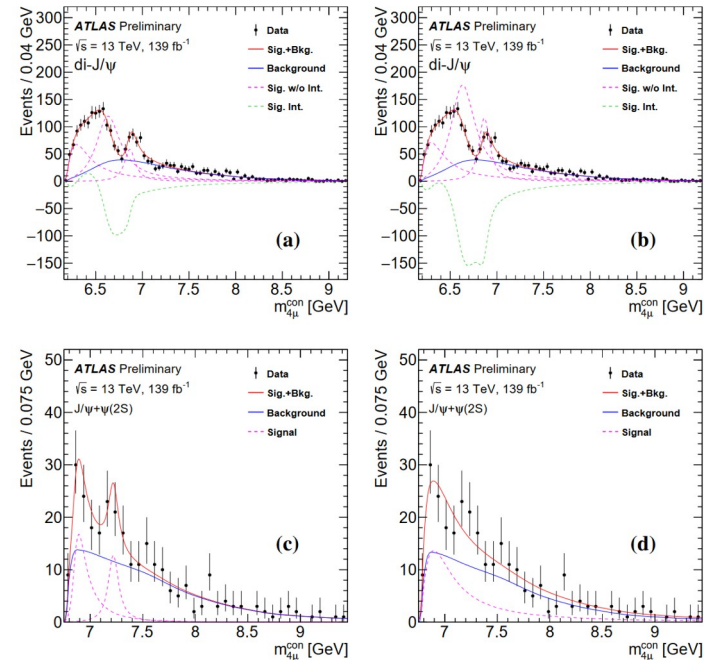
X(6900) was discovered by LHCb collaboration in 2020 in di- J/ψ mass spectrum [5].

ATLAS collaborations performed analysis of $T_{cccc}^- \rightarrow J/\psi J/\psi$ and $T_{cccc}^- \rightarrow J/\psi \psi(2S)$ in pp collisions ($s^{1/2} = 13$ TeV, $L = 139$ fb $^{-1}$).

[5] LHCb Collaboration, Observation of structure in the J/ψ -pair mass spectrum, Science Bulletin 65 (2020) 1983
 [6] ATLAS-CONF-2022-040

Background sources:

- 1) Double J/ψ background:
 - SPS (two prompt J/ψ);
 - DPS (two prompt J/ψ);
 - Two non-prompt J/ψ from $b\bar{b} \rightarrow J/\psi J/\psi X$;
- 2) Single J/ψ background:
 - prompt J/ψ with fake muons;
 - non-prompt J/ψ from B-hadron decays;
- 3) Non peaking background (no J/ψ);



(GeV)	m_0	Γ_0	m_1	Γ_1
di- J/ψ	$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$	$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$
10.0σ	m_2	Γ_2	—	—
	$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$	—	—
(GeV)	m_3	Γ_3		
$J/\psi + \psi(2S)$	model A	$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$	4.6σ
	model B	$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$	4.3σ

As a result, they obtained two models (with 3 resonances) with the same likelihood. The use of **helicity formalism** can allow more precise analysis to measure the properties of X(6900) state;

Conclusion and outlooks

- Helicity formalism is a useful tool in the problem of experimental search and study of multi-quark states. It allows you to describe the angular distributions and obtain J^P information.
- It allows for multidimensional fits, which is especially important when there is no clear bump in the data distribution.
- One example of the use of this formalism is the search for pentaquarks in the Run1 data of the ATLAS experiment. The model without pentaquarks is not excluded. It is planned to continue the analysis on the Run2 data.
- It is planned to study the $B^0 \rightarrow J/\psi K\pi$, $B_s \rightarrow J/\psi KK$ decays on the data in order to confirm the contributions of $Z_C(4200)$, $Z_{CS}(4000)$, $Z_{CS}(4220)$ and measure their parameters
- ATLAS collaborations performed analysis of $T_{cccc} \rightarrow J/\psi J/\psi$ and $T_{cccc} \rightarrow J/\psi \psi(2S)$ in pp collisions ($s^{1/2} = 13$ TeV, $L = 139$ fb $^{-1}$). As a result, they obtained two degenerate solutions with the same likelihood. The use of a formalism can allow more precise analysis to measure the properties of X(6900) state;

Thank you for your attention!

Backup (Blatt-Weisskopf factors)

$$B'_0(p, p_0, d) = 1,$$

$$B'_1(p, p_0, d) = \sqrt{\frac{1 + (p_0 d)^2}{1 + (p d)^2}},$$

$$B'_2(p, p_0, d) = \sqrt{\frac{9 + 3(p_0 d)^2 + (p_0 d)^4}{9 + 3(p d)^2 + (p d)^4}},$$

$$B'_3(p, p_0, d) = \sqrt{\frac{225 + 45(p_0 d)^2 + 6(p_0 d)^4 + (p_0 d)^6}{225 + 45(p d)^2 + 6(p d)^4 + (p d)^6}},$$

$$B'_4(p, p_0, d) = \sqrt{\frac{11025 + 1575(p_0 d)^2 + 135(p_0 d)^4 + 10(p_0 d)^6 + (p_0 d)^8}{11025 + 1575(p d)^2 + 135(p d)^4 + 10(p d)^6 + (p d)^8}},$$

$$B'_5(p, p_0, d) = \sqrt{\frac{893025 + 99225(p_0 d)^2 + 6300(p_0 d)^4 + 315(p_0 d)^6 + 15(p_0 d)^8 + (p_0 d)^{10}}{893025 + 99225(p d)^2 + 6300(p d)^4 + 315(p d)^6 + 15(p d)^8 + (p d)^{10}}}$$