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One-loop divergences in the $6D$, $\mathcal{N}=(1,0)$ abelian gauge theory

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ABSTRACT

We consider, in the harmonic superspace approach, the six-dimensional $\mathcal{N}=(1,0)$ supersymmetric model of abelian gauge multiplet coupled to a hypermultiplet. The superficial degree of divergence is evaluated and the structure of possible one-loop divergences is analyzed. Using the superfield proper-time and background-field technique, we compute the divergent part of the one-loop effective action depending on both the gauge multiplet and the hypermultiplet. The corresponding counterterms contain the purely gauge multiplet contribution together with the mixed contributions of the gauge multiplet and hypermultiplet. We show that the theory is on-shell one-loop finite in the gauge multiplet sector in agreement with the results of [1]. The divergences in the mixed sector cannot be eliminated by any field redefinition, implying the theory to be UV divergent at one loop.

1. Introduction

The higher-dimensional supersymmetric gauge models are of interest mainly because they describe low-energy limits of the superstring/brane theory and inherit many remarkable properties of the latter. In particular, one can expect the existence of various non-renormalization theorems governing their ultraviolet (UV) behavior. In this letter we study the UV divergence structure of the six-dimensional abelian $\mathcal{N}=(1,0)$ gauge theory interacting with hypermultiplets. The analysis of this simplest model can be conducive for the further study of quantum properties of more complicated non-abelian $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ gauge theories.

An analysis of the UV divergences in the higher-dimensional supersymmetric gauge theories has been initiated by the paper [1] and continued in the subsequent papers [2–9] (and references therein). In particular, it was found that in the sector of gauge (or vector) multiplet the divergences at different loops reveal a universal structure and in many cases some counterterms can be completely eliminated by the field redefinitions. The counterterms in the hypermultiplet sector have never been calculated.

As is well known, the most efficient way to describe the quantum aspects of supersymmetric theories is to use the off-shell superfield formulations (see e.g. [10] for $4D$, $\mathcal{N}=1$ theories and [11] for $4D$, $\mathcal{N}=2$ theories). An arbitrary (n,m) representation of the six-dimensional supersymmetry is labeled by the numbers of left (n) and right (m) independent supersymmetries (see, e.g., [12]). In the case of $6D$, $\mathcal{N}=(1,0)$ supersymmetry, the models of vector multiplet and hypermultiplet can be formulated off shell in terms of superfields defined on $6D$, $\mathcal{N}=(1,0)$ harmonic superspace [13,14] (see also [5,6,8] and references therein). It allows to formulate an arbitrary six-dimensional $\mathcal{N}=(1,0)$ supersymmetric Yang–Mills theory in $6D$, $\mathcal{N}=(1,0)$ superspace as a theory of interacting unconstrained off-shell superfields describing the six-dimensional $\mathcal{N}=(1,0)$ vector multiplet and hypermultiplet. Using the appropriate set of $\mathcal{N}=(1,0)$ harmonic superfields, one can construct $\mathcal{N}=(1,1)$ supersymmetric Yang–Mills theories (see e.g. [8]), as well as the free gauge models with $\mathcal{N}=(2,0)$ supersymmetry [6]. It is worth pointing out that, in many aspects, $6D$, $\mathcal{N}=(1,0)$ SYM theory is analogous to $4D$, $\mathcal{N}=2$ SYM theory, and $6D$, $\mathcal{N}=(1,1)$ SYM theory to $4D$, $\mathcal{N}=4$ SYM theory. These $6D$ theories and their $4D$ counterparts have equal numbers of supercharges, 8 and 16, respectively. Like $4D$, $\mathcal{N}=4$ SYM theory possesses manifest off-shell $\mathcal{N}=2$ supersymmetry and an on-shell hidden $\mathcal{N}=2$ supersymmetry, $6D$, $\mathcal{N}=(1,1)$ SYM theory possesses manifest $\mathcal{N}=(1,0)$ supersymmetry and an on-shell

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hidden $\mathcal{N}=(0,1)$ supersymmetry (see [8] for details and further references).

The general analysis of the possible low-energy contributions of different conformal dimensions to the effective action of $\mathcal{N}=(1,0)$ SYM theories has been carried out in ref. [8]. It was proved that the superfield counterterm of dimension 6 in $\mathcal{N}=(1,0)$ harmonic superspace is a linear combination of three terms, where one depends only on the vector multiplet superfield, another depends only on the hypermultiplet and the third mixed term bears a dependence on both the vector multiplet and the hypermultiplet. The analysis was based on the $\mathcal{N}=(1,0)$ harmonic superspace formulation of the theory and transformation properties of the involved $\mathcal{N}=(1,0)$ harmonic superfields. Taking into account the results obtained in [8], it would be extremely interesting to demonstrate how these results can in principle be derived in the framework of quantum field theory. Namely this problem is solved in this letter for an abelian $\mathcal{N}=(1,0)$ supersymmetric gauge theory, which is an abelian $\mathcal{N}=(1,0)$ vector models coupled to $\mathcal{N}=(1,0)$ hypermultiplet.

The paper is organized as follows. In section 2 we briefly describe the formulation of abelian $6D$, $\mathcal{N}=(1,0)$ gauge theory in $\mathcal{N}=(1,0)$ harmonic superspace and fix the $6D$ notations and conventions. Section 3 presents the harmonic superspace background field method which allows one to obtain the effective action in a manifestly gauge invariant and $\mathcal{N}=(1,0)$ supersymmetric form. In section 4 we derive the superficial degree of divergence in the theory of interacting vector and hyper multiplets and discuss the structure of the one-loop divergences. In particular, we prove that the one-loop counterterms indeed match with the results of [8], except that the purely hypermultiplet divergent contribution to the effective action is absent in the one-loop approximation. Section 5 is devoted to the direct calculations of the one-loop divergences. In section 6 we summarize the results and discuss the problems for further study.

2. Abelian gauge theory in 6D, $\mathcal{N}=(1,0)$ harmonic superspace

Our consideration in this section (including notations, conventions and terminology) will closely follow ref. [8].

The basic objects of the $6D$, $\mathcal{N}=(1,0)$ superfield gauge theory are gauge covariant derivatives defined by

$$\nabla_{\mathcal{M}} = D_{\mathcal{M}} + i\mathcal{A}_{\mathcal{M}}, \tag{2.1}$$

where $D_{\mathcal{M}}=(D_M, D^i_a)$ are the flat derivatives. Here $M=0,..,5$, is the $6D$ vector index and $a=1,..4$, is the spinorial one. The superfield $\mathcal{A}_{\mathcal{M}}$ is the gauge super-connection. The covariant derivatives transform under the gauge group as

$$\nabla'_{\mathcal{M}} = e^{i\tau}\nabla_{\mathcal{M}}e^{-i\tau}, \quad \tau^\dagger = \tau\,, \tag{2.2}$$

and satisfy the algebra

$$\{\nabla^i_a, \nabla^j_b\} = -2i\varepsilon^{ij}\nabla_{\alpha\beta}, \quad [\nabla^i_c, \nabla_{ab}] = -\frac{1}{2}\varepsilon_{abcd}W^{i\,d}, \tag{2.3}$$

$$[\nabla_M, \nabla_N] = iF_{MN}\,, \tag{2.4}$$

where $W^{i\,a}$ is the superfield strength and $\nabla_{ab} = \frac{1}{2}(\gamma^M)_{ab}\nabla_M$. Further in this paper we consider only the abelian gauge theory coupled to a hypermultiplet.

The constraints (2.3) and (2.4) can be solved in the harmonic superspace framework. In the λ-frame [11], the spinor covariant derivatives ∇^+_a coincide with the flat ones D^+_a, while the harmonic covariant derivatives acquire the connections V^{++} and V^{--},

$$\nabla^{\pm\pm} = D^{\pm\pm} + iV^{\pm\pm}\,, \quad \widetilde{V}^{\pm\pm} = V^{\pm\pm}\,,$$
$$\delta V^{\pm\pm} = -\nabla^{\pm\pm}\lambda(\zeta,u)\,, \tag{2.5}$$

$$[\nabla^{--}, D^+_a] = \nabla^-_a\,, \quad [\nabla^{++}, \nabla^-_a] = D^+_a\,,$$
$$[\nabla^{++}, D^+_a] = [\nabla^{--}, \nabla^-_a] = 0\,, \tag{2.6}$$

where (ζ,u) stands for the analytic subspace coordinates. The real connection $V^{++}(\zeta,u)$ is analytic (in virtue of the third constraint in (2.6)) and it is an unconstrained potential of the theory. The component expansion of $V^{++}(\zeta,u)$ in the Wess–Zumino gauge reads

$$V^{++}_{WZ} = \theta^{+a}\theta^{+b}A_{ab}(x_{(an)}) + (\theta^+)^3_a\lambda^{ia}(x_{(an)})u^-_i$$
$$+ 3(\theta^+)^4 D^{ik}(x_{(an)})u^-_i u^-_k\,. \tag{2.7}$$

It involves the gauge field A_{ab}, the gaugino field $\lambda^{i\,a}$ and the auxiliary field $D^{(ik)}$.

The second, non-analytic harmonic connection $V^{--}(z,u)$ is uniquely determined in terms of V^{++} as a solution of the harmonic zero-curvature condition [11]. In the abelian case the latter is

$$[\nabla^{++}, \nabla^{--}] = D^0 \;\Leftrightarrow\; D^{++}V^{--} - D^{--}V^{++} = 0. \tag{2.8}$$

The equation (2.8) can be solved for V^{--} as

$$V^{--}(z,u) = \int du_1 \frac{V^{++}(z,u_1)}{(u^+u^+_1)^2}\,. \tag{2.9}$$

Using the connection V^{--}, we can construct the spinor and vector superfield connections and define the covariant spinor superfield strengths $W^{\pm a}$

$$W^{+a} = -\frac{1}{6}\varepsilon^{abcd}D^+_b D^+_c D^+_d V^{--}\,, \quad W^{-a} = D^{--}W^{+a}\,. \tag{2.10}$$

We also define the Grassmann-analytic superfield [8]

$$F^{++} = \frac{1}{4}D^+_a W^{+a} = (D^+)^4 V^{--}\,, \quad D^+_a W^{+b} = \delta^b_a F^{++}\,,$$
$$D^{++}F^{++} = 0\,, \tag{2.11}$$

which will be used for construction of the counterterms.

The superfield action of $6D$, $\mathcal{N}=(1,0)$ abelian model of gauge multiplet interacting with hypermultiplet has the form

$$S[V^{++},q^+] = \frac{1}{4f^2}\int d^{14}z\,\frac{du_1du_2}{(u^+_1u^+_2)^2}\,V^{++}(z,u_1)V^{++}(z,u_2)$$
$$-\int d\zeta^{(-4)}du\,\tilde{q}^+\nabla^{++}q^+, \tag{2.12}$$

where f is a dimensionful coupling constant ($[f]=-1$). The hypermultiplet superfield $q^+(x,\theta)$ has a short expansion $q^+(z) = f^i(x)u^+_i + \theta^{+a}\psi_a(x) + \ldots$, with a doublet of massless scalars fields $f^i(x)$ and the spinor field ψ_α as the physical fields. It also involves an infinite tail of auxiliary fields coming from the harmonic expansions. Both the superfield $q^+(\zeta,u)$ and its $\widetilde{\ }$-conjugate $\tilde{q}^+$ [11] obey the analyticity constraint, $D^+_\alpha q^+ = D^+_\alpha\tilde{q}^+ = 0$. The action (2.12) is invariant under the gauge transformation

$$V^{++\prime} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}\,, \qquad q^{+\prime} = e^{i\lambda}q^+\,, \tag{2.13}$$

where $\lambda=\lambda(\zeta,u)$ is the analytic gauge parameter introduced in (2.5). Using the zero curvature condition (2.8), one can derive a useful relation between arbitrary variations of harmonic connections [8]

$$\delta V^{--} = \frac{1}{2}(D^{--})^2\delta V^{++} - \frac{1}{2}D^{++}(D^{--}\delta V^{--})\,. \tag{2.14}$$

Classical equations of motion following from the action (2.12) read

$$\frac{\delta S}{\delta V^{++}} = 0 \Rightarrow \frac{1}{2f^2} F^{++} - i\tilde{q}^+ q^+ = 0\,,$$
$$\frac{\delta S}{\delta \tilde{q}^{++}} = 0 \Rightarrow \nabla^{++} q^+ = 0\,. \tag{2.15}$$

Note that the superfield F^{++} is real under the $\widetilde{\ }$ conjugation, $\widetilde{F}^{++} = F^{++}$. The $\widetilde{\ }$-reality of the first equation in (2.15) (as well as of the action (2.12)) is guaranteed by the conjugation rule $\widetilde{\tilde{q}^+} = -q^+$ [11].

3. Background field method

In this section we outline the background field method for the model (2.12). The construction of gauge invariant effective action in the model under consideration is very similar to that for $4D, \mathcal{N} = 2$ supersymmetric gauge theories [15,16] (see also the reviews [17]).[1]

We split the superfields V^{++}, q^+ into the sum of the "background" superfields V^{++}, Q^+ and the "quantum" ones v^{++}, q^+,

$$V^{++} \to V^{++} + f v^{++}, \qquad q^+ \to Q^+ + q^+\,, \tag{3.1}$$

and then expand the action in a power series in quantum fields. As a result, we obtain the classical action as a functional of background superfields and quantum superfields.

To construct the gauge invariant effective action, we need to impose the gauge-fixing conditions only on quantum superfields. As in the four-dimensional case [15], we introduce the gauge-fixing function in the form

$$\mathcal{F}^{(+4)} = D^{++} v^{++}. \tag{3.2}$$

We consider the abelian gauge theory, where gauge-fixing function (3.2) is background-field independent. This means that the Faddeev–Popov ghosts are also background-field independent and so make no contribution to the effective action. According to (3.2), the gauge-fixing part of the quantum field action has the form

$$S_{GF} = -\frac{1}{4}\int d^{14}z du_1 du_2 \frac{v^{++}(1) v^{++}(2)}{(u_1^+ u_2^+)^2}$$
$$+ \frac{1}{8}\int d^{14}z du\, v^{++} (D^{--})^2 v^{++}\,. \tag{3.3}$$

A formal expression of the effective action $\Gamma[V^{++},\ Q^+]$ for the theory under consideration is constructed by the Faddeev–Popov procedure (see the reviews [17] for details).

In the one-loop approximation, the first quantum correction to the classical action $\Gamma^{(1)}[V^{++},\ Q^+]$ is given by the following path integral [15,16]:

$$e^{i\Gamma^{(1)}[V^{++}, Q^+]} = \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}\tilde{q}^+ \, e^{iS_2[v^{++}, q^+;\, V^{++},\, Q^+]}\,. \tag{3.4}$$

Here, the full quadratic action S_2 is the sum of the classical action (2.12), with the background-quantum splitting accomplished, and the gauge-fixing action (3.3)

$$S_2 = \frac{1}{4}\int d\zeta^{(-4)} du\, v^{++} \Box_{(2,2)} v^{++}$$
$$- \int d\zeta^{(-4)} du \{\tilde{q}^+ \nabla^{++} q^+ + f\tilde{Q}^+ i v^{++} q^+ + f\tilde{q}^+ i v^{++} Q^+\}, \tag{3.5}$$

where the operator $\Box_{(2,2)} = \frac{1}{2}(D^+)^4 (D^{--})^2$ transforms the analytic superfields v^{++} into analytic superfields. The Green function, associated with $\Box_{(2,2)}$, $G^{(2,2)}(z_1, u_1|z_2, u_2) = i\langle v^{++}(z_1, u_1) v^{++}(z_2, u_2)\rangle$, is given by the expression similar to that in the $4D, \mathcal{N} = 2$ case [11]

$$G_\tau^{(2,2)}(1|2) = -2\frac{(D_1^+)^4}{\Box_1} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2)\,. \tag{3.6}$$

The action S_2 (3.5) is a quadratic form of quantum fields, with the coefficients depending on background fields. For further use, it is convenient to diagonalize this quadratic form, that is to decouple the quantum superfields v^{++} and q^+. To achieve this, one performs the special change of the quantum hypermultiplet variables[2] in the path integral, such that it removes the mixed terms,

$$q_1^+ = h_1^+ - f\int d\zeta_2^{(-4)} du_2\, G^{(1,1)}(1|2)\, i v_2^{++} Q_2^+\,, \tag{3.7}$$

with h^+ being the new independent quantum superfield. It is evident that the Jacobian of the variable change (3.7) is unity. Here $G^{(1,1)}(\zeta_1, u_1|\zeta_2, u_2) = i\langle \tilde{q}^+(\zeta_1, u_1) q^+(\zeta_2, u_2)\rangle$ is the superfield hypermultiplet Green function in the τ-frame ($G^{(1,1)}(1|2) = -G^{(1,1)}(2|1)$). This Green function is analytic with respect to both arguments and satisfies the equation

$$\nabla_1^{++} G_\tau^{(1,1)}(1|2) = \delta_A^{(3,1)}(1|2)$$
$$\Rightarrow G_\tau^{(1,1)}(1|2) = \frac{(\nabla_1^+)^4 (\nabla_2^+)^4}{\widehat{\Box}_1} \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}\,, \tag{3.8}$$

where $\delta_A^{(3,1)}(1|2)$ is the covariantly-analytic delta-function and $\widehat{\Box}$ is the covariantly-analytic d'Alembertian [6] which acts on analytic superfields as follows

$$\widehat{\Box} = \frac{1}{2}(D^+)^4 (\nabla^{--})^2$$
$$= \Box + iW^{+a}\nabla_a^- + iF^{++}\nabla^{--} - \frac{i}{4}(D_a^- W^{+a})\,, \tag{3.9}$$

with $\Box = \frac{1}{2}\varepsilon^{abcd}\nabla_{ab}\nabla_{cd} = \nabla^M \nabla_M$. Note that the covariant d'Alembertian transforms the analytic superfields into analytic superfields. After some algebra, the quadratic part of the action S_2 (3.5) splits into the vector-multiplet dependent part

$$S_2^{Vect}[V^{++}, Q^+] = \frac{1}{4}\int d\zeta_1^{(-4)} du_1\, v_1^{++}$$
$$\times \int d\zeta_2^{(-4)} du_2 \Big\{\Box \delta_A^{(2,2)}(1|2)$$
$$- 4f^2 \tilde{Q}_1^+ G^{(1,1)}(1|2) Q_2^+\Big\} v_2^{++}\,, \tag{3.10}$$

and the hypermultiplet part

$$S_2^{Hyp}[V^{++}] = -\int d\zeta^{(-4)} du\, \tilde{h}^+ (D^{++} + iV^{++}) h^+\,. \tag{3.11}$$

[1] There are two approaches for constructing the background field method for $4D, \mathcal{N} = 2$ SYM theories. One is formulated in the conventional $\mathcal{N} = 2$ superspace [18], while another in $4D, \mathcal{N} = 2$ harmonic superspace [15,19] (see also the reviews [17]). The first formulation faces some troubles basically related to an infinite number of FP ghosts. The second approach is free of such difficulties and provides a convenient tool for manifestly covariant loop calculations. In this paper we generalize it to $6D, \mathcal{N} = (1, 0)$ gauge theory. Though in our case the problem of ghosts is absent at all because we deal with the abelian theory, the harmonic background field method looks most preferable like in $4D$ case.

[2] A similar substitution was used in [16], [20] and [21] for computing one- and two-loop effective actions in supersymmetric theories, and in [22] for non-local change of fields in non-supersymmetric QED.

We see that the quadratic part of the action in the vector multiplet sector S_2^{Vect} is an analytic nonlocal functional of the quantum field v^{++}. It also contains an interaction between background vector multiplet and hypermultiplet through the background-dependent Green function $G^{(1,1)}(V^{++})$.

The actions (3.10) and (3.11) specify the one-loop quantum correction to the classical action (2.12):

$$\Gamma^{(1)}[V^{++},Q^+] = \frac{i}{2}\mathrm{Tr}\ln\left\{\Box - 4f^2\widetilde{Q}^+G^{(1,1)}Q^+\right\} + i\mathrm{Tr}\ln\nabla^{++}. \quad (3.12)$$

The expression (3.12) is the starting point for studying the one-loop effective action in the model (2.12). In the next sections we will calculate the divergent part of (3.12).

4. Structure of one-loop counterterms

In this section we analyze the superficial degree of divergence in the model under consideration. The formal structure of Green functions of the vector multiplet (3.6) and the hypermultiplet (3.8) in $6D$, $\mathcal{N}=(1,0)$ gauge theory is analogous to that in the four dimensional $\mathcal{N}=2$ case. Hence, we can directly make use of the similar analysis in four dimensional $\mathcal{N}=2$ theory [19]. As in the four-dimensional theory, the Green functions in the case under consideration contain enough number of Grassmann δ-function to prove the non-renormalization theorem according to which the loop contribution to the supergraphs defining the effective action can be written as a single integral over the total $6D, \mathcal{N}=(1,0)$ superspace.

Let us consider L-loop supergraph G with P propagators, V vertices, N_Q external hypermultiplet legs, and an arbitrary number of gauge superfield external legs. We denote by N_D the number of spinor covariant derivatives acting on the external legs as a result of integration by parts in the process of transforming the contributions to a single integral over $d^8\theta$. The superficial degree of divergence $\omega(G)$ of the supergraph G can be found by counting the degrees of momenta in the loop integrals.

The supergraph G involves L integrals over 6-momenta, which contribute $6L$ to the degree of divergence. Each of the hypermultiplet vertices contains one integration over $d^4\theta^+$. Propagators of the gauge superfields contribute the factors $1/k^2$, $(D^+)^4$, as well as the Grassmann δ-functions. Similarly, propagators of the hypermultiplet superfields contribute $1/k^2$, $(D^+)^4$ for each of two harmonic arguments of propagator (3.8) (eight D^+-factors on a whole), and also the Grassmann δ-functions. From each hypermultiplet propagator we take the operator $(D^+)^4$ and so complete $d^4\theta^+$ to $d^8\theta$ in all hypermultiplets lines, except for the number $\frac{1}{2}N_Q$ of them. Then we consider the corresponding vertices and we take $\frac{1}{2}N_Q$ operators $(D^+)^4$ off the propagators, which allow us to restore the integrations over $d^8\theta$. After calculating the supergraph we will end up with a single $d^8\theta$ integration. The other $V-1$ integrations, where V is a total number of vertices, are done due to the Grassmann δ-functions. The remaining $P-V+1=L$ Grassmann δ-functions survive. Each of them is killed by eight D_a^+. Therefore, the number of remaining D_a^+ is $4P-2N_Q-8L$. This implies that the superficial degree of divergence is

$$\omega(G) = (6L-2P)+(2P-N_Q-4L)-\frac{1}{2}N_D = 2L-N_Q-\frac{1}{2}N_D, \quad (4.1)$$

where N_D is the number of the spinor covariant derivatives acting on the external lines.

Equivalently, the degree of divergence can be calculated, using dimension reasonings. Each gauge propagator brings f^2, $[f^2]=m^{-2}$. The external gauge superfields are dimensionless, $[V]=m^0$, while the dimension of hypermultiplets is $[q]=m^2$. The effective action also contains a single integration over the full superspace. Taking into account that $[d^6x]=m^{-6}$ and $[d^8\theta]=m^4$ we see that

$$-\omega(G) = -2-2P_V+2N_Q+\frac{1}{2}N_D, \quad (4.2)$$

where P_V is the number of gauge propagators. For hypermultiplets $N_Q=2(-P_Q+V_Q)$, so that

$$\omega(G) = 2-2V+2P-N_Q-\frac{1}{2}N_D = 2L-N_Q-\frac{1}{2}N_D. \quad (4.3)$$

Our aim is to calculate a divergent part of the one-loop effective action, in this case the number of loops in Eq. (4.3) is $L=1$. Due to the analyticity of the hypermultiplet superfield, $D^+q^+=0$, the number N_D of spinor covariant derivatives acting on the external legs is equal to zero, $N_D=0$. Thus, in our case the superficial degree of divergence $\omega(G)$ (4.3) is reduced to

$$\omega_{1-\mathrm{loop}}(G) = 2-N_Q\,. \quad (4.4)$$

Let us apply the relation (4.4) to the analysis of the one-loop divergences. According to the general consideration of ref. [8], the possible contributions to divergent part of the effective action of abelian theory is given by the following integral over the analytic subspace of harmonic superspace:

$$\Gamma_{div} = \int d\zeta^{(-4)}du\left\{c_1(F^{++})^2 + ic_2\widetilde{Q}^+F^{++}Q^+ + c_3(\widetilde{Q}^+Q^+)^2\right\}. \quad (4.5)$$

Here, the coefficients c_1, c_2, c_3 depend on the regularization parameters.[3]

Let $N_Q=0$, then $\omega=2$. The corresponding divergent structure has to be quadratic in momenta and given by the full $\mathcal{N}=(1,0)$ superspace integral. The unique possibility is

$$\Gamma_1^{(1)} \sim \int d^{14}z\,du\,V^{--}\Box V^{++}\,, \quad (4.6)$$

where $\Box=\frac{1}{2}(D^+)^4(D^{--})^2$. Integrating in (4.6) by parts, we can transfer the factor $(D^+)^4$ from d'Alembertian on V^{--} and use the definition of superfield F^{++} (2.11). Then we take one factor D^{--} off the second multiplier and make use of the zero-curvature condition (2.8). More precisely,

$$\begin{aligned}\Gamma_1^{(1)} &\sim \int d^{14}z\,du\,F^{++}(D^{--})^2V^{++}\\ &= -\int d^{14}z\,du\,D^{--}F^{++}D^{--}V^{++}\\ &= -\int d^{14}z\,du\,D^{--}F^{++}D^{++}V^{--}\\ &= \int d^{14}z\,du\,D^{++}D^{--}F^{++}V^{--}.\end{aligned} \quad (4.7)$$

After that we commute the operators D^{++} and D^{--}, use the property $D^{++}F^{++}=0$ and obtain $D^{++}D^{--}F^{++}=D_0F^{++}=2F^{++}$. Finally, passing to the analytical subspace, we have

[3] In this paper we use the proper-time regularization (see [6,23] and references therein) preserving the supersymmetry at least at one loop and are interested in the logarithmic divergences only. One-loop logarithmic divergences are known to be not susceptible to such subtleties of quantum field theory as, e.g., presence of anomalies. We emphasize that the regularization aspects of six dimensional theories deserve a special attention, like those in four dimensional theories (see e.g., discussion in [24]). However, various choices of regularization scheme do not affect the form of one-loop logarithmic divergences which are the subject of our paper.

$$\Gamma_1^{(1)} = c_1 \int d\zeta^{(-4)} du\, (F^{++})^2 . \tag{4.8}$$

The coefficient c_1 is divergent in the limit of removing the regularization.

Let $N_Q = 2$, then $\omega = 0$. The unique candidate divergent term involving no dependence on momenta and representable as an integral over the full $\mathcal{N} = (1,0)$ superspace reads

$$\Gamma_2^{(1)} \sim \int d^{14}z\, du\, \widetilde{Q}^+ V^{--} Q^+ . \tag{4.9}$$

Passing to the analytic subspace and using (2.11), we immediately obtain

$$\Gamma_2^{(1)} = ic_2 \int d\zeta^{(-4)} du\, \widetilde{Q}^+ F^{++} Q^+ , \tag{4.10}$$

where, once again, the coefficient c_2 is divergent in the limit of removing the regularization. We see that the contributions (4.8) and (4.10) match with the general structure (4.5) of the divergent part of the effective action.

For all other values of N_Q the index ω is negative and the corresponding Feynmann integrals are UV finite. In particular, the divergent term of the form $(\widetilde{Q}^+ Q^+)^2$ is absent in the one-loop approximation. Such divergent terms could appear, starting with two loops.

5. Divergent part of the one-loop effective action

In the previous section we discussed the general structure of the one-loop contributions to the divergent part of effective action. Here we perform the direct calculation of the coefficients c_1 and c_2 in (4.5).

The $(F^{++})^2$ part of the effective action depends only on the background vector multiplet V^{++} and is defined by the second term in eq. (3.12). More precisely,

$$\Gamma^{(1)}_{F^2}[V^{++}] = i\mathrm{Tr}\ln \nabla^{++} = -i\mathrm{Tr}\ln G^{(1,1)} . \tag{5.1}$$

Here $G^{(1,1)}$ is the superfield propagator for hypermultiplet (3.8). The details of calculation for (5.1) were discussed in recent works [7,23]. We consider an arbitrary variation of the expression (5.1)

$$\begin{aligned}\delta\Gamma^{(1)}_{F^2}[V^{++}] &= -i\,\mathrm{Tr}\,\delta\, iV^{++} G^{(1,1)} \\ &= \int d\zeta_1^{(-4)} du_1\, \delta V^{++} G^{(1,1)}(1|2)\Big|_{2=1} .\end{aligned} \tag{5.2}$$

Our aim is to calculate the divergent part of the effective action (5.1). In the proper-time regularization scheme [6,23], the divergences are associated with the pole terms of the form $\frac{1}{\varepsilon}$, $\varepsilon \to 0$, where $\varepsilon = 6 - d$ with space-time dimension d. Taking into account the expression for Green function $G^{(1,1)}$ (3.8), one gets

$$\begin{aligned}\delta\Gamma^{(1)}_{F^2}[V^{++}] &= \int d\zeta_1^{(-4)} du_1 \delta V^{++} \int_0^\infty d(is)(is\mu^2)^{\frac{\varepsilon}{2}} \\ &\quad \times e^{is\widehat{\Box}_1} (D_1^+)^4 (D_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}\Big|^{2=1}_{\mathrm{div}} .\end{aligned} \tag{5.3}$$

Here s is the proper-time parameter and μ is an arbitrary regularization parameter of mass dimension. Like in the four- and five-dimensional cases, one makes use of the identity (see [25] for details)

$$\begin{aligned}&(D_1^+)^4 (D_2^+)^4 \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3} \\ &= (D_1^+)^4 \Big\{ (u_1^+ u_2^+)(\nabla_1^-)^4 - (u_1^- u_2^+)\Omega_1^{--} \\ &\quad - 4\,\widehat{\Box}_1 \frac{(u_1^- u_2^+)^2}{(u_1^+ u_2^+)} \Big\} \delta^{14}(z_1 - z_2) ,\end{aligned} \tag{5.4}$$

where we have introduced the notation

$$\Omega^{--} = i\nabla^{ab}\nabla_a^- \nabla_b^- + 4W^{-a}\nabla_a^- - (D_a^- W^{-a}) . \tag{5.5}$$

One can show [21] that only the first term in (5.4) gives contribution to the divergent part of the one-loop effective action

$$\begin{aligned}\delta\Gamma^{(1)}_{F^2}[V^{++}] &= \int d\zeta_1^{(-4)} du_1 \delta V^{++}(1) \times \\ &\quad \times \int_0^\infty d(is)(is\mu^2)^{\frac{\varepsilon}{2}} e^{is\widehat{\Box}_1} (u_1^+ u_2^+)(D_1^+)^4 (D_1^-)^4 \times \\ &\quad \times \delta^{14}(z_1 - z_2)\Big|^{2=1}_{\mathrm{div}} .\end{aligned} \tag{5.6}$$

Those terms in the right hand side of (5.6) which produce the divergent part read

$$\begin{aligned}e^{is\widehat{\Box}}(u_1^+ u_2^+)e^{-is\widehat{\Box}}\Big|^{2=1}_{\mathrm{div}} &= -i\frac{(is)^2}{2}(\Box F^{++}) \\ &\quad - i\frac{(is)^3}{6}\Big\{4(\partial^M \partial^N F^{++})\partial_M \partial_N\Big\} .\end{aligned} \tag{5.7}$$

Then we pass to momentum representation of the delta function and calculate the proper-time integral. This leads to the expression

$$\delta\Gamma^{(1)}_{F^2}[V^{++}] = -\frac{1}{3(4\pi)^3\varepsilon} \int d\zeta^{(-4)} du\, \delta V^{++} \Box F^{++} . \tag{5.8}$$

Let us compare (5.8) with (4.5). Keeping in mind the definition $F^{++} = (D^+)^4 V^{--}$, we can transform the variation (5.8) to the form

$$\delta\Gamma^{(1)}_{div} = 2c_1 \int d\zeta^{(-4)} du\, F^{++}(D^+)^4 \delta V^{--} . \tag{5.9}$$

Then we use the relation between δV^{--} and δV^{++} (2.14) and the property $D^{++}F^{++} = 0$. After that we restore the full $6D$, $\mathcal{N} = (1,0)$ superspace measure,

$$\delta\Gamma^{(1)}_{div} = c_1 \int dzdu\; F^{++}(D^{--})^2 \delta V^{++} + \int du\, D^{++}(\ldots) , \tag{5.10}$$

and integrate by parts with respect to $(D^{--})^2$. Omitting the total derivative terms and passing to the analytic subspace, we obtain

$$\delta\Gamma^{(1)}_{div} = c_1 \int dzdu\; (D^{--})^2 F^{++} \delta V^{++} \tag{5.11}$$

$$= c_1 \int d\zeta^{(-4)} du\, (D^+)^4 (D^{--})^2 F^{++} \delta V^{++} . \tag{5.12}$$

The derivatives $(D^+)^4$ act only on $(D^{--})^2 F^{++}$ because δV^{++} is an analytic superfield. Then we use the definition of analytic d'Alambertian $\Box = \frac{1}{2}(D^+)^4(D^{--})^2$ and finally find

$$\delta\Gamma^{(1)}_{div} = 2c_1 \int d\zeta^{(-4)} du\, \delta V^{++} \Box F^{++} . \tag{5.13}$$

As is expected, the variation of the divergent part of effective action (5.8) proved to have the same structure as (5.13). Hence we obtain, up to an unessential additive constant,

$$\Gamma^{(1)}_{F^2}[V^{++}] = -\frac{1}{6(4\pi)^3\varepsilon}\int d\zeta^{(-4)}du\,(F^{++})^2\,. \tag{5.14}$$

The hypermultiplet-dependent part $\widetilde{Q}^+F^{++}Q^+$ of the one-loop counterterm arises from the first term in (3.12).[4] In order to calculate this contribution one expands the logarithm in the first term (3.12) up to the first order and compute the functional trace

$$\begin{aligned}&\Gamma^{(1)}_{QFQ}[V^{++},Q^+]\\ &= \frac{i}{2}\mathrm{Tr}\,\ln\left\{\Box - 4f^2\widetilde{Q}^+G^{(1,1)}Q^+\right\}\\ &\approx -2if^2\int d\zeta^{(-4)}du\,\widetilde{Q}^+Q^+\frac{1}{\Box}G^{(1,1)}(1|2)\Big|^{2=1}_{\mathrm{div}}\,.\end{aligned} \tag{5.15}$$

We again use the identity (5.4) and consider only the first term here, because just this term is responsible for divergence:

$$\begin{aligned}\frac{1}{\Box}G^{(1,1)}(1|2)\Big|^{2=1}_{\mathrm{div}} &= \frac{1}{\Box}\frac{(D^+_1)^4(D^+_2)^4}{\widehat{\Box}_1}\frac{\delta^{14}(z_1-z_2)}{(u^+_1u^+_2)^3}\Big|^{2=1}_{\mathrm{div}}\\ &= \frac{1}{\Box}\frac{(D^+_1)^4(D^-_1)^4}{\widehat{\Box}_1}(u^+_1u^+_2)\delta^{14}(z_1-z_2)\Big|^{2=1}_{\mathrm{div}}\,.\end{aligned} \tag{5.16}$$

Now we observe that the first analytic d'Alembertian $\Box$ in the denominator comes from the pure vector multiplet part and does not contain background fields. The second covariant d'Alembertian $\widehat{\Box}$ in the denominator emerges from the Green function for hypermultiplet after non-local change of variables (3.7). This operator depends on the background vector multiplet as in (3.9).

To calculate the divergent part of the expression under consideration it suffices to take into account only two first terms in the $\widehat{\Box}$, namely

$$\widehat{\Box} = \Box + iF^{++}\nabla^{--} + \ldots\,.$$

Other two terms do not contribute to the divergent part of one-loop effective action in the point-coincidence limit. We expand the operator $\frac{1}{\widehat{\Box}}$ up to the first order in $iF^{++}\nabla^{--}$ and act by it on the harmonic distribution $(u^+_1u^+_2)$. Using properties of Grassmann delta-function, $(D^+_1)^4(D^-_1)^4\delta^8(\theta_1-\theta_2)\Big|_{2=1} = 1$, we obtain

$$\begin{aligned}&\frac{(D^+_1)^4(D^-_1)^4}{\Box(\Box + iF^{++}\nabla^{--}+..)}(u^+_1u^+_2)\delta^{14}(z_1-z_2)\Big|^{2=1}_{\mathrm{div}}\\ &= -iF^{++}\frac{(u^-_1u^+_2)}{\Box^3}\delta^6(x_1-x_2)\Big|_{2=1}\,.\end{aligned} \tag{5.17}$$

Then one uses the momentum representation of the space-time δ-function and calculates the momentum integral in the ε-regularization scheme. It leads to

$$\frac{1}{\Box^3}\delta^6(x_1-x_2)\Big|_{2=1} = \frac{i}{(4\pi)^3}\frac{1}{\varepsilon}\,,\quad \varepsilon\to 0\,. \tag{5.18}$$

As a result, one gets

$$\Gamma^{(1)}_{QFQ}[V^{++},Q^+] = \frac{2if^2}{(4\pi)^3\varepsilon}\int d\zeta^{(-4)}du\,\widetilde{Q}^+F^{++}Q^+\,. \tag{5.19}$$

[4] It is known that calculations of harmonic supergraphs with hypermultiplet propagators require a certain care related to coinciding harmonic singularities [26]. As argued in [26] (see [11,27] as well) this problem can be avoided in all cases of interest. In our case we also do not face such a problem.

Summing up the contributions (5.14) and (5.19), we finally obtain

$$\begin{aligned}\Gamma^{(1)}_{div}[V^{++},Q^+] = &-\frac{1}{6(4\pi)^3\varepsilon}\int d\zeta^{(-4)}du\\ &\times\left\{(F^{++})^2 - 12\,if^2\widetilde{Q}^+F^{++}Q^+\right\}.\end{aligned} \tag{5.20}$$

If the background hypermultiplet vanishes, the divergent part of the effective action is proportional to the classical equation of motion $F^{++}=0$. Therefore the divergence as a whole can be eliminated by a field redefinition ($\delta V^{++}\sim\frac{1}{\varepsilon}F^{++}$) in the classical action and the theory under consideration is one-loop finite on shell, in accordance with the results of ref. [1]. However, if the background hypermultiplet does not vanish, we obtain, after some field redefinition proportional to the equation of motion, the divergent part of on-shell effective action in the form $\Gamma^{(1)}_{div}\sim\frac{1}{\varepsilon}\int d\zeta^{(-4)}du(\widetilde{Q}^+Q^+)^2$. Thus, the on-shell divergence in the hypermultiplet sector cannot be eliminated and the full theory is not finite even at the one-loop level.

6. Summary and outlook

Let us briefly summarize the results obtained. We have considered the six-dimensional $\mathcal{N}=(1,0)$ supersymmetric theory of the abelian vector multiplet coupled to hypermultiplet in the $6D$, $\mathcal{N}=(1,0)$ harmonic superspace formulation. We have studied the quantum effective action involving dependence on both the vector multiplet and the hypermultiplet superfields. The corresponding background field method in harmonic superspace was formulated, such that it allows one to preserve manifest gauge invariance and supersymmetry at all stages of calculating the effective action. It is important to point out that the superfield propagators in the theory under consideration have, in the sector of anticommuting variables and harmonics, the same structure as the propagators in $4D$, $\mathcal{N}=2$ SYM theory. It leads to $6D$, $\mathcal{N}=(1,0)$ renormalization theorem, which states that the contribution of any supergraph in the theory under consideration can be written as a single integral over anticommuting variables of the full $6D$, $\mathcal{N}=(1,0)$ superspace. Using this result, we have calculated the superficial degree of divergences and analyzed the structure of one-loop counterterms in both the vector multiplet and the hypermultiplet sectors. It was shown, in particular, that one of the possible divergent counterterms in the purely hypermultiplet sector, which is allowed on the supersymmetry and dimension grounds [8], is actually prohibited at one loop.

We have developed an efficient manifestly gauge invariant and $\mathcal{N}=(1,0)$ supersymmetric technique to calculate the one-loop effective action. As an application of this technique, we found the one-loop divergences of the theory under consideration. The results completely match the analysis of the general structure of divergences based on considering superficial degree of divergences. It was shown that, if the background hypermultiplet superfield does not vanish, the one-loop divergences cannot be eliminated by any field redefinition and the theory is not one-loop finite.

Let us discuss some possible generalizations and extensions of the results obtained. As the next step, it is quite natural to study the structure of the effective action for the non-abelian $6D$, $\mathcal{N}=(1,0)$ SYM theories. All such theories admit a formulation in $6D$, $\mathcal{N}=(1,0)$ harmonic superspace. The background field method can be developed quite analogously to the abelian case and the one-loop divergences can be calculated. The basic difference from the abelian theory considered here will be a self-interaction of vector multiplet and a non-trivial ghost contribution to the effective action, which can change the relative coefficient between the $(F^{++})^2$ and the $\widetilde{Q}^+F^{++}Q^+$ terms in the one-loop divergent part. We expect that the purely hypermultiplet contribution to the divergent part of the one-loop effective action in non-abelian theory will be absent as in the abelian theory. Besides the divergent part of effective action, it would be interesting to study the finite contributions to low-energy effective action, which have never been considered before.

It would be extremely interesting to study the effective action in $6D$, $\mathcal{N}=(1,1)$ SYM theory. Such a theory can be formulated in $6D$, $\mathcal{N}=(1,0)$ harmonic superspace in terms of $\mathcal{N}=(1,0)$ analytic harmonic superfields, viz. the gauge connection V^{++} and the hypermultiplet $q^+, \tilde{q}^+$, both in the adjoint representation [8]. This theory exhibits the manifest off-shell $\mathcal{N}=(1,0)$ supersymmetry and an additional hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry, and in many aspects is analogous to $4D$, $\mathcal{N}=4$ SYM theory [11]. It was shown, based solely upon the invariance of the effective action under both manifest and hidden supersymmetries, that $\mathcal{N}=(1,1)$ SYM theory is one-loop finite. It would be tempting to analyze the divergences of $\mathcal{N}=(1,1)$ SYM theory within the quantum setting and explicitly calculate the one-loop counterterms (in parallel with constructing the full quantum $\mathcal{N}=(1,1)$ SYM effective action).

It is well known that the $6D$, $\mathcal{N}=(1,0)$ supersymmetric theories are anomalous (see discussions of chiral anomalies in higher dimensional supersymmetric theories in refs. [28]). It would be interesting to study such anomalies in the harmonic superspace formulation of $6D, \mathcal{N}=(1,0)$ SYM coupled to hypermultiplets and show, by a direct quantum field theoretical analysis, that the $\mathcal{N}=(1,1)$ SYM theory is anomaly-free. We are going to tackle all these problems in the forthcoming works.

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