

Low-energy effective action in $5D, \mathcal{N} = 2$ supersymmetric gauge theory

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Abstract

We construct $\mathcal{N} = 2$ supersymmetric low-energy effective action of $5D, \mathcal{N} = 2$ supersymmetric Yang–Mills (SYM) theory in $5D, \mathcal{N} = 1$ harmonic superspace. It is obtained as a hypermultiplet completion of the leading $W \ln W$ -term in the $\mathcal{N} = 1$ SYM low-energy effective action by invoking the second implicit on-shell $\mathcal{N} = 1$ supersymmetry. After passing to components, the $\mathcal{N} = 2$ effective action constructed displays, along with other terms, the $SO(5)$ -invariant F^4/X^3 term. Though we specialize to the case of $SU(2)$ gauge group spontaneously broken to $U(1)$, our consideration is applicable to any gauge symmetry broken to some abelian subgroup.

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1. Introduction

Extended supersymmetry in diverse dimensions imposes stringent constraints on the classical and effective quantum superfield actions of gauge theories. The most prominent example is supplied by the four-derivative term in the low-energy $4D, \mathcal{N} = 4$ SYM effective action which, in the

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sector of $\mathcal{N} = 2$ gauge multiplet, is accommodated by a non-holomorphic superfield potential [1]. An $\mathcal{N} = 4$ supersymmetric completion of this potential was constructed in $\mathcal{N} = 2$ harmonic superspace [2–5], in $\mathcal{N} = 3$ harmonic superspace [6] and in various on-shell $\mathcal{N} = 4$ harmonic superspaces [7,8] (see [9] for a review). By these works, the origin of non-renormalizability of the $\mathcal{N} = 4$ SYM low-energy effective action against higher-loop quantum corrections was revealed and links with the leading terms in the effective action of D3 brane on the $AdS_5 \times S^5$ background were established.

In 3D gauge theories, the constraints imposed by an extended supersymmetry allowed one to determine the leading quantum corrections in $\mathcal{N} = 4$ SYM theory [10,11], and to construct $\mathcal{N} = 3$ superfield ABJM action [12]. This technique proved also useful for displaying the structure of the leading terms in 2D gauge theories with extended supersymmetry [13].

A common feature of all the above-mentioned results is that the leading contributions to the low-energy effective actions in gauge theories with extended supersymmetry take a reasonably simple form upon the proper choice of the superfield description. The constraints due to the extended supersymmetry (together with the requirement of scale invariance) are strong enough to fix the form of the superfield potentials in such theories up to an overall coefficient to be found further from quantum considerations. In some exceptional cases, e.g., in 4D, $\mathcal{N} = 4$ SYM theory, the numerical value of this overall coefficient can yet be fixed on the topological grounds, without the actual need to apply to quantum computations [7].

In this paper, we study the implications of extended supersymmetry for the low-energy effective action of 5D SYM theory. This theory is of interest from several points of view. It is non-renormalizable by power-counting because of the dimensionful coupling constant g , $[g] = -1/2$. Nevertheless, it was argued that a non-perturbative quantum completion of this model describes 6D, $\mathcal{N} = (2, 0)$ superconformal field theory compactified on a circle [14–16]. An additional confirmation of this conjecture came from the exact computations of the partition function in this theory by the localization technique [17–21].

Despite the non-renormalizability of 5D, $\mathcal{N} = 1$ SYM, it is still reasonable to study one-loop quantum corrections in this theory, keeping in mind that in the odd-dimensional field theories divergences can appear (within the dimensional regularization) only at even loops. One-loop contributions to the effective action of 5D, $\mathcal{N} = 1$ SYM theory were calculated in Refs. [22,23] for the case of gauge group $SU(2)$ spontaneously broken to $U(1)$. The leading contribution is given by the 5D supersymmetric Chern–Simons term [22], while the next-to-leading one was found in [23] in the form

$$c_0 \int d^{5|8}z du W \ln \frac{W}{\Lambda}, \quad (1.1)$$

where W is the 5D, $\mathcal{N} = 1$ abelian gauge superfield strength, Λ is a scale parameter, $[\Lambda] = 1$, and the integration is performed over the full $\mathcal{N} = 1$ harmonic superspace with measure $d^{5|8}z du \equiv d^5x d^8\theta du$. It is easy to check that the action (1.1) is Λ -independent. The Chern–Simons term incorporates two-derivative quantum corrections to the effective action, while (1.1) is $\mathcal{N} = 1$ superfield extension of the four-derivative “ F^4/ϕ^3 ”-terms.

The purpose of this paper is to study leading terms in the low-energy effective action of 5D, $\mathcal{N} = 2$ SYM theory in harmonic superspace. Although such terms might be found by direct quantum computations in 5D, $\mathcal{N} = 1$ superspace, we determine them here on the symmetry grounds, just by constructing $\mathcal{N} = 2$ completion of the 5D, $\mathcal{N} = 1$ SYM effective action via addition of the proper hypermultiplet terms. The effective action constructed corresponds to the Coulomb branch of 5D, $\mathcal{N} = 2$ SYM theory, with the gauge group being broken to some abelian

subgroup (e.g., the maximal torus), and, in general, depends on the massless abelian $\mathcal{N} = 2$ gauge multiplets valued in the algebra of this subgroup. For simplicity, we concentrate on the case of the gauge group $SU(2)$ and only briefly discuss (in subsect. 2.3) the case of $SU(N)$ gauge symmetry.

To start with, we point out that the $5D$, $\mathcal{N} = 1$ Chern–Simons term does not admit an $\mathcal{N} = 2$ completion because it respects the invariance under $5D$, $\mathcal{N} = 1$ superconformal algebra $F(4)$ which is unique and has no higher \mathcal{N} extensions [24]. According to this argument, the Chern–Simons term is forbidden as a quantum correction in the low-energy effective action of the $\mathcal{N} = 2$ SYM theory. From the field-theory point of view, it is possible to show, by direct quantum computations in $5D$, $\mathcal{N} = 1$ superspace, that the two-derivative contributions (the Chern–Simons term) to the $\mathcal{N} = 2$ SYM effective action coming from the hypermultiplet and from the ghost superfields cancel each other. Indeed, the background field method for $5D$, $\mathcal{N} = 1$ SYM theory [23] mimics the one for the $4D$, $\mathcal{N} = 2$ SYM theory in harmonic superspace [25,26]. In particular, the structure of ghost superfields in these theories is the same. In Refs. [27,28] it was proved that the hypermultiplet contributions to the $4D$, $\mathcal{N} = 4$ SYM one-loop effective action are fully canceled by the contributions from the ghost superfields. Since this proof is purely formal, it holds true for the $5D$, $\mathcal{N} = 2$ SYM theory as well. Note also that this cancellation is analogous to the well-known phenomenon in $3D$ case [10,11], where Chern–Simons term cannot arise as a quantum correction to the effective action in supersymmetric gauge theories with $\mathcal{N} > 2$.

As we demonstrate in the next section, the four-derivative term (1.1), on the contrary, admits the unique hypermultiplet completion under the requirement of $\mathcal{N} = 2$ supersymmetry involving an implicit $5D$, $\mathcal{N} = 1$ on-shell supersymmetry alongside with the manifest off-shell $\mathcal{N} = 1$ one. The procedure of constructing such a hypermultiplet completion is quite analogous to the one developed in [2] for finding the $\mathcal{N} = 4$ hypermultiplet extension of the $4D$, $\mathcal{N} = 2$ SYM effective action. In the component formulation, the $5D$, $\mathcal{N} = 2$ effective action constructed displays the $F^4/|X|^3$ -term where $|X|^2$ is $SO(5)$ -invariant bilinear combination of scalar fields.

It is worth to point out that the term (1.1) (as well as its analogs for the higher-rank gauge groups) may arise in quantum theory only as a one-loop quantum correction to the effective action. Indeed, it is scale-invariant and so is independent of the dimensionful gauge coupling constant g . On the other hand, within the background field method in harmonic superspace [25, 23], all higher-loop Feynman graphs involve a gauge superfield vertex with the coupling constant g . Thus, all higher-loop quantum corrections to the effective action cannot give rise to renormalization of the coefficient c_0 in Eq. (1.1) since they violate scale invariance. However, in contrast to the $4D$ case, this coefficient is not protected against non-perturbative corrections. Such corrections will be discussed elsewhere.

Our last comment concerns the possible relation of the effective action in $5D$ gauge theory to the D-brane low-energy dynamics. The classical action of $5D$, $\mathcal{N} = 2$ SYM theory with $U(N)$ gauge group can be interpreted as an action of a stack of N D4 branes in flat space–time. The $\mathcal{N} = 2$ supersymmetric completion of the F^4/ϕ^3 -term (1.1) can presumably be identified with that of the four-derivative term in the low-energy effective action of a single D4 brane on the $AdS_6 \times S^4$ background.

2. Low-energy effective action of $\mathcal{N} = 2$ SYM theory

In this section, we construct the low-energy effective action of $\mathcal{N} = 2$ SYM theory with the gauge group $SU(2)$ as a hypermultiplet completion of the term (1.1). We start our consideration with a brief account of the $\mathcal{N} = 1$ SYM and hypermultiplet models in $5D$ harmonic superspace. We follow the notation and conventions of Refs. [29,23].

2.1. Classical action

$\mathcal{N} = 2$ gauge multiplet in $5D$, $\mathcal{N} = 1$ harmonic superspace is described by a pair of analytic superfields (V^{++}, q_a^+) , where V^{++} is the $\mathcal{N} = 1$ gauge multiplet and $q_a^+ \equiv (q^+, -\bar{q}^+)$ is the hypermultiplet. The former is described by the classical action written in the full harmonic superspace [30]

$$S_{\text{YM}} = \frac{1}{2g^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{5|8} z du_1 \dots du_n \frac{V^{++}(z, u_1) V^{++}(z, u_2) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+)(u_2^+ u_3^+) \dots (u_n^+ u_1^+)}, \quad (2.1)$$

where g is a coupling constant of mass-dimension $-1/2$. This action yields the equation of motion

$$(\mathcal{D}^+)^2 W = 0, \quad (2.2)$$

where $(\mathcal{D}^+)^2 \equiv \mathcal{D}^{+\hat{\alpha}} \mathcal{D}_{\hat{\alpha}}^+$ and W is a superfield strength of the gauge $\mathcal{N} = 1$ multiplet. It may be expressed via the non-analytic prepotential V^{--}

$$W = \frac{i}{8} (\mathcal{D}^+)^2 V^{--}, \quad (2.3)$$

which, in turn, is expressed through V^{++} by the harmonic-flatness condition

$$D^{++} V^{--} - D^{--} V^{++} + i[V^{++}, V^{--}] = 0. \quad (2.4)$$

The classical action of the hypermultiplet q^{+a} ($a = 1, 2$) in the adjoint representation of the gauge group reads [31–33]

$$S_q = \frac{1}{2g^2} \text{tr} \int d\zeta^{(-4)} q_a^+ \mathcal{D}^{++} q^{+a}, \quad (2.5)$$

where $d\zeta^{(-4)}$ is the integration measure on the analytic superspace and $\mathcal{D}^{++} = D^{++} + i[V^{++}, \cdot]$ is the gauge-covariant harmonic derivative. The corresponding equation of motion is

$$\mathcal{D}^{++} q_a^+ = 0. \quad (2.6)$$

The action of $\mathcal{N} = 2$ gauge multiplet in $\mathcal{N} = 1$ harmonic superspace is just the sum of (2.1) and (2.5),

$$S^{\mathcal{N}=2} = S_{\text{YM}} + S_q. \quad (2.7)$$

This action is invariant under the implicit $\mathcal{N} = 1$ supersymmetry

$$\delta q_a^+ = -\frac{1}{2} (D^+)^4 [\epsilon_{a\hat{\alpha}} \theta^{-\hat{\alpha}} V^{--}], \quad \delta V^{++} = \epsilon_{\hat{\alpha}}^a \theta^{+\hat{\alpha}} q_a^+, \quad (2.8)$$

where $\epsilon_{\hat{\alpha}}^a$ is the relevant anticommuting parameter. Though the equation (2.2) is modified for the total action (2.7) by the hypermultiplet source term in the right-hand-side, it is not the case for the massless Cartan-subalgebra valued abelian superfields which we will be interested in. In the abelian case, the equations of motion for the $\mathcal{N} = 1$ gauge multiplet (2.2) and hypermultiplet (2.6) are simplified to the form

$$(\mathcal{D}^+)^2 W = 0, \quad D^{++} q_a^+ = 0. \quad (2.9)$$

It is straightforward to show that on these equations the implicit supersymmetry transformations (2.8) are reduced to

$$\delta q_a^\pm = \frac{i}{2} \epsilon_a^{\hat{\alpha}} (D_{\hat{\alpha}}^\pm W), \quad \delta W = -\frac{i}{8} \epsilon_a^{\hat{\alpha}} D^{-\hat{\alpha}} q_a^+ + \frac{i}{8} \epsilon_a^{\hat{\alpha}} D^{+\hat{\alpha}} q_a^-. \quad (2.10)$$

2.2. $\mathcal{N} = 2$ effective action

The part of the superfield $\mathcal{N} = 1$ SYM effective action containing the component four-derivative term reads [23]

$$S_0 = c_0 \int d^5|8 z du W \ln \frac{W}{\Lambda}, \quad (2.11)$$

where W is the abelian gauge superfield strength, Λ is a scale parameter and c_0 is a dimensionless constant. Owing to the representation (2.3) implying $\int d^5|8 z du W = 0$, the action (2.11) is independent of the scale Λ , $dS_0/d\Lambda = 0$.

The precise value of the constant c_0 in the effective action (2.11) depends on the gauge group representation content of the hypermultiplet matter [23]. Here, we do not fix the value of this constant and construct $\mathcal{N} = 2$ supersymmetric generalization of (2.11), keeping c_0 arbitrary. This construction follows the same steps as in Ref. [2] where the similar $\mathcal{N} = 4$ completion of the leading term of the $4D$, $\mathcal{N} = 2$ SYM effective action was found.

The variation of the action (2.11) under the hidden supersymmetry transformations (2.10) may be cast in the form

$$\delta S_0 = \frac{ic_0}{4} \int d^5|8 z du \epsilon_a^{\hat{\alpha}} q_a^+ \frac{D^{-\hat{\alpha}} W}{W}. \quad (2.12)$$

In deriving this equation we employed the abelian counterparts of the relations (2.3), (2.4), the equations of motion (2.9) and integration by parts with respect to the harmonic and covariant spinor derivatives.

The expression (2.12) may be partly canceled by the variation of the action

$$S_1 = c_1 \int d^5|8 z du \frac{q^{+a} q_a^-}{W}, \quad (2.13)$$

where the coefficient c_1 will be defined below. The variation of this action under (2.10) reads

$$\delta S_1 = ic_1 \int d^5|8 z du \frac{q^{+a} \epsilon_a^{\hat{\alpha}} (D_{\hat{\alpha}}^- W)}{W} - \frac{i}{8} c_1 \int d^5|8 z du \frac{q^{+a} q_a^-}{W^2} (\epsilon_a^{\hat{\alpha}} D^{+\hat{\alpha}} q_b^- - \epsilon_a^{\hat{\alpha}} D^{-\hat{\alpha}} q_b^+). \quad (2.14)$$

The first term in the right-hand side in (2.14) cancels the variation (2.11) if

$$c_1 = -\frac{c_0}{4}, \quad (2.15)$$

while the last term in (2.14) may be cast in the form

$$\delta(S_0 + S_1) = -\frac{ic_0}{12} \int d^5|8 z du \frac{q^{+a} q_a^-}{W^3} \epsilon_a^{\hat{\alpha}} q_b^+ D^{-\hat{\alpha}} W. \quad (2.16)$$

In deriving this expression, we integrated by parts and used cyclic identities for $SU(2)$ indices. To cancel this expression we need to add the next term

$$S_2 = c_2 \int d^5|8 z du \frac{(q^{+a} q_a^-)^2}{W^3}, \quad c_2 = \frac{c_0}{24}. \quad (2.17)$$

Instead of evaluating the variation of the term (2.17) we proceed to the general case and look for the full $\mathcal{N} = 2$ effective action in the form

$$S_{\text{eff}}^{\mathcal{N}=2} = \int d^5|8 z du \left[c_0 W \ln \frac{W}{\Lambda} + \sum_{n=1}^{\infty} c_n \frac{(q^{+a} q_a^-)^n}{W^{2n-1}} \right], \quad (2.18)$$

with some coefficients c_n . Let us consider two adjacent terms in the sum in (2.18):

$$c_n \frac{(q^{+a} q_a^-)^n}{W^{2n-1}} + c_{n+1} \frac{(q^{+a} q_a^-)^{n+1}}{W^{2n+1}}. \quad (2.19)$$

It is possible to show that the variation of the denominator in the first term cancels the variation of the nominator in the second term, if the coefficients are related as

$$(n+1)c_{n+1} = -c_n \frac{n(2n-1)}{n+2}. \quad (2.20)$$

Taking into account Eq. (2.15), we find from this recurrence relation the generic coefficient

$$c_n = \frac{(-1)^n (2n-2)!}{n!(n+1)!2^n} c_0. \quad (2.21)$$

This allows us to sum up the series in (2.18) and to represent the effective action in the closed form

$$S_{\text{eff}}^{\mathcal{N}=2} = c_0 \int d^5|8 z du W \left[\ln \frac{W}{\Lambda} + \frac{1}{2} H(Z) \right], \quad (2.22)$$

where

$$Z \equiv \frac{q^{+a} q_a^-}{W^2}, \quad (2.23)$$

and

$$H(Z) = 1 + 2 \ln \frac{1 + \sqrt{1+2Z}}{2} + \frac{2}{3} \frac{1}{1 + \sqrt{1+2Z}} - \frac{4}{3} \sqrt{1+2Z}. \quad (2.24)$$

It is easy to check that $H(0) = 0$, $H'(0) = -\frac{1}{2}$, in agreement with (2.21).

The action (2.22) is $\mathcal{N} = 2$ supersymmetric extension of the effective action (2.11). It would be interesting to reproduce this result from the perturbative quantum computations in $5D$ harmonic superspace, like it has been done in the $4D$ case in [3–5].

2.3. Generalization to $SU(N)$ gauge group

In the previous subsection we found the effective action (2.22) for a single massless $\mathcal{N} = 2$ gauge multiplet. Within a field theory, this effective action is expected to come out from $\mathcal{N} = 2$ SYM theory (2.7) with the $SU(2)$ gauge group spontaneously broken to its $U(1)$ subgroup. It is straightforward to generalize this result to a higher-rank gauge group. For instance, for the $SU(N)$ gauge group spontaneously broken to the maximal torus $[U(1)]^{N-1}$ we obtain

$$S_{\text{eff}}^{\mathcal{N}=2} = c_0 \sum_{I < J}^N \int d^5|8 z du W_{IJ} \left[\ln \frac{W_{IJ}}{\Lambda} + \frac{1}{2} H(Z_{IJ}) \right], \quad (2.25)$$

where $Z_{IJ} = \frac{(q^{+a})_{IJ}(q_a^-)_{IJ}}{W_{IJ}^2}$ and $W_{IJ} = W_I - W_J$, $(q^{\pm a})_{IJ} = q_I^{\pm a} - q_J^{\pm a}$. The superfields W_I and $q_I^{\pm a}$ obey the constraints $\sum_I W_I = 0$, $\sum_I q_I^{\pm a} = 0$ and span the Cartan directions in the Lie algebra $su(N)$. The function $H(Z_{IJ})$ for each argument Z_{IJ} is given by the same expression (2.24).

2.4. Component structure

We will be interested in deriving the term F^4/X^3 from the effective action (2.18). To this end, it is enough to leave only the following component fields in the involved superfields:

$$q^{+2} \equiv q^+ = f^i(x)u_i^+, \quad q^{+1} \equiv \bar{q}^+ = -\bar{f}^i(x)u_i^+, \quad (2.26)$$

$$W = \sqrt{2}\phi(x) - 2i\theta^{+\hat{\alpha}}\theta^{-\hat{\beta}}F_{\hat{\alpha}\hat{\beta}}(x). \quad (2.27)$$

Here $\bar{\phi} = \phi$, $\overline{(f^i)} = \bar{f}_i$ are scalar fields and $F_{\hat{\alpha}\hat{\beta}} = F_{\hat{\beta}\hat{\alpha}}$ is Maxwell field strength of the $\mathcal{N} = 1$ gauge multiplet.

Substituting the superfield strength (2.27) into the first term in (2.22), we find

$$S_0 = c_0 \frac{\sqrt{2}}{3} \int d^5|8z \frac{(\theta^{+\hat{\alpha}}\theta^{-\hat{\beta}}F_{\hat{\alpha}\hat{\beta}})^4}{\phi^3} = \frac{c_0}{4\sqrt{2}} \int d^5|8z \frac{\det F}{\phi^3} (\theta^+)^2 (\theta^+)^2 (\theta^-)^2 (\theta^-)^2, \quad (2.28)$$

where $\det F = \frac{1}{4!} \varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} \varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} F_{\hat{\alpha}\hat{\mu}} F_{\hat{\beta}\hat{\nu}} F_{\hat{\gamma}\hat{\rho}} F_{\hat{\delta}\hat{\sigma}}$ and $(\theta^{\pm})^2 = \theta^{\pm\hat{\alpha}}\theta_{\hat{\alpha}}^{\pm}$. We integrate over the Grassmann variables according to the rule

$$\int d^5|8z (\theta^+)^2 (\theta^+)^2 (\theta^-)^2 (\theta^-)^2 f(x) = 4 \int d^5x f(x), \quad (2.29)$$

for some $f(x)$. Thus the action (2.28) yields the component term

$$S_0 = \frac{c_0}{\sqrt{2}} \int d^5x \frac{\det F}{\phi^3}. \quad (2.30)$$

In a similar way we can perform the integration over the Grassmann variables in the last term in (2.22),

$$\int d^5|8z WH(Z) = \sqrt{2} \int d^5x \frac{\det F}{\phi^3} [4z^4 H^{(4)}(z) + 28z^3 H'''(z) + 39z^2 H''(z) + 6zH'(z)], \quad (2.31)$$

where

$$z \equiv Z|_{\theta=0} = \frac{f^i \bar{f}_i}{2\phi^2}. \quad (2.32)$$

Substituting the function (2.24) into Eq. (2.31), we find

$$\frac{c_0}{2} \int d^5|8z WH(Z) = \frac{c_0}{\sqrt{2}} \int d^5x \frac{\det F}{(\phi^2 + f^i \bar{f}_i)^{3/2}} - \frac{c_0}{\sqrt{2}} \int d^5x \frac{\det F}{\phi^3}. \quad (2.33)$$

The last term exactly cancels (2.30). As a result, the total F^4/X^3 term in the component form of the effective action (2.22) is given by the expression

$$S_{\text{eff}}^{\mathcal{N}=2} = \frac{c_0}{\sqrt{2}} \int d^5x \frac{\det F}{|X|^3} + \dots, \quad (2.34)$$

where dots stand for the remaining terms and

$$|X| = \sqrt{\phi^2 + f^i \bar{f}_i}. \quad (2.35)$$

It is remarkable that the scalar fields appear in the denominator in (2.34) just in the $SO(5)$ invariant combination (2.35). This is a non-trivial property, since the field ϕ comes from the gauge $\mathcal{N} = 1$ multiplet, while f^i, \bar{f}_i from the hypermultiplet. In the $SU(N)$ case (2.25), $S_{\text{eff}}^{\mathcal{N}=2}$ is given by a sum of the appropriate terms (2.34).

3. Summary and outlook

In this paper, generalizing the approach of Ref. [2] to the $5D$ case, we constructed the leading term in the low-energy effective action of $5D, \mathcal{N} = 2$ SYM theory as the appropriate sum of the effective action of $5D, \mathcal{N} = 1$ SYM theory and the interactions with the hypermultiplet. This interaction is fixed, up to an overall coupling constant c_0 , by the requirement of the implicit on-shell $5D, \mathcal{N} = 1$ supersymmetry extending the manifest off-shell $\mathcal{N} = 1$ supersymmetry to an on-shell $5D, \mathcal{N} = 2$ one. We discussed in detail the case of the gauge group $SU(2)$ spontaneously broken to $U(1)$, in which case the effective action depends on a pair of single abelian $5D, \mathcal{N} = 1$ gauge multiplet and hypermultiplet, and then considered a more general situation with the $SU(N)$ gauge group broken to its maximal torus, with $N - 1$ pairs of such abelian multiplets.

The next obvious problem is to reproduce these effective actions from the appropriate set of quantum $5D, \mathcal{N} = 1$ supergraphs involving the interacting hypermultiplet and $\mathcal{N} = 1$ gauge superfields. Also, it would be interesting to establish precise links with the relevant D-brane dynamics and the $4D$ and $6D$ cousins of the $5D$ effective action constructed. Finding out the explicit form of the next-to-leading corrections to this effective action, based as well on the demand of implicit $5D, \mathcal{N} = 1$ supersymmetry, is another interesting task for the future study. Finally, we expect that results obtained can in principle be used to study the quantum aspects of the twisted $5D, \mathcal{N} = 2$ SYM theory known also as the Witten–Haydys theory [34,35] (see also [36]).

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