



On the two-loop divergences in 6D, $\mathcal{N} = (1, 1)$ SYM theory

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ABSTRACT

We continue studying 6D, $\mathcal{N} = (1, 1)$ supersymmetric Yang-Mills (SYM) theory in the $\mathcal{N} = (1, 0)$ harmonic superspace formulation. Using the superfield background field method we explore the two-loop divergences of the effective action in the gauge multiplet sector. It is explicitly demonstrated that among four two-loop background-field dependent supergraphs contributing to the effective action, only one diverges off shell. It is also shown that the divergences are proportional to the superfield classical equations of motion and hence vanish on shell. Besides, we have analyzed a possible structure of the two-loop divergences on general gauge and hypermultiplet background.

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1. Introduction

Supersymmetric field theories in diverse dimensions, especially those exhibiting the maximally extended supersymmetry, display very interesting quantum properties. For example, divergences in such theories sometimes unexpectedly vanish. In some cases such miracles are caused by a hidden supersymmetry of the theory. This refers, e.g., to 4D, $\mathcal{N} = 4$ SYM theory where all possible divergent diagrams cancel each other due to the maximally extended rigid $\mathcal{N} = 4$ supersymmetry [1–4]. A consistent derivation of the 4D, $\mathcal{N} = 2$ non-renormalization theorem was given in [5] in $\mathcal{N} = 2$ harmonic superspace formulation [6–8], which is the most adequate approach to 4D $\mathcal{N} = 2$ supersymmetric gauge theories. Another very interesting example of the miraculous divergence cancellation is provided by $\mathcal{N} = 8$ supergravity, which is the maximally extended supergravity theory in four dimensions. At present, it is believed that this theory is finite up to at least seven loops, see [9] and references therein, although the possible all-loop ultraviolet finiteness is also discussed (see, e.g., [10,11]).

Similarly to 4D (super)gravity theories, the degree of divergence in higher dimensional gauge theories increases with a number of loops. One can expect that supersymmetry and, especially, the maximally extended supersymmetry, is capable to improve the ultraviolet behavior in such theories. This is the basic reason of interest in investigating UV divergences of the higher dimensional supersymmetric gauge theories. They were actually studied for a long time, see, e.g., [12–23]. In this paper we will concentrate on the 6D rigid $\mathcal{N} = (1, 1)$ SYM theory. This theory is in many aspects similar to 4D, $\mathcal{N} = 4$ SYM theory in four dimensions, and one can expect some similarity of the structure of divergences in both theories. However, they essentially differ in the UV domain. In contrast to $\mathcal{N} = 4$ SYM theory, which is finite to all loops, its 6D counterpart is non-renormalizable by power-counting. Nevertheless, the extended supersymmetry leads to the finiteness of the theory up to two loops, at least on mass shell [15–17,24]. The modern methods of computing scattering amplitudes [24] demonstrate that UV divergences in 6D $\mathcal{N} = (1, 1)$ SYM theory should start from the three-loop level (see also [12–14]).

In our previous works [26–31] we studied UV properties of 6D, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ theories in the 6D harmonic superspace formulation. In particular, it was found that 6D, $\mathcal{N} = (1, 1)$ theory is off-shell finite in the one-loop approximation in the Feynman gauge, although the divergences are still present in the non-minimal gauges [31] (they vanish on shell). The two-loop divergences in the hypermultiplet two-point Green functions were shown to also vanish off shell [28]. However, the complete two-loop calculation in the

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harmonic superspace approach has not been done so far. In the present paper we continue the study of $6D, \mathcal{N} = (1, 1)$ SYM theory at two loops. We argue that it is not finite off shell in the Feynman gauge in the two-loop approximation, although the divergences vanish on shell. Our consideration is limited to the gauge superfield sector and does not involve the background hypermultiplet. However, the result is still applicable to other sectors of the models due to the implicit $\mathcal{N} = (0, 1)$ supersymmetry. Indeed, we formulate the model in terms of interacting $\mathcal{N} = (1, 0)$ harmonic gauge multiplet and hypermultiplet in the adjoint representation of the gauge group. The action of the model is manifestly invariant under $\mathcal{N} = (1, 0)$ supersymmetry by construction. An additional $\mathcal{N} = (0, 1)$ supersymmetry is implicit and is present only if the hypermultiplet belongs to the adjoint representation of the gauge group. Note that, albeit $\mathcal{N} = (1, 0)$ theories are in general plagued by anomalies [32–35], $\mathcal{N} = (1, 1)$ SYM theory is not anomalous.

The letter is organized as follows. In section 2 we recall $6D, \mathcal{N} = (1, 1)$ SYM theory in $\mathcal{N} = (1, 0)$ harmonic superspace. Section 3 is devoted to a brief account of the effective action in the gauge multiplet sector. The effective action is formulated within the background harmonic superfield method. This allows us to perform the calculations in a manifestly gauge invariant and $\mathcal{N} = (1, 0)$ supersymmetric manner. In section 4 we analyze the structure of possible two-loop contributions to the effective action and calculate all divergent terms in this approximation. In section 5 we discuss a possible structure of the two loop divergences when the background hypermultiplet is taken into account. In the last section 6 we summarize the results.

2. $6D, \mathcal{N} = (1, 1)$ SYM in $\mathcal{N} = (1, 0)$ harmonic superspace

The six-dimensional maximally extended $\mathcal{N} = (1, 1)$ supersymmetric gauge theory can be formulated in $\mathcal{N} = (1, 0)$ harmonic superspace. In this framework it amounts to $\mathcal{N} = (1, 0)$ supersymmetric gauge theory coupled to the hypermultiplet q^+ in the adjoint representation of the gauge group. All necessary notations and conventions are collected in our previous papers, see, e.g., [26,27]. Here we recall only the basic concepts.

The coordinates of $6D, \mathcal{N} = (1, 0)$ harmonic superspace are denoted as $(z, u) = (x^M, \theta_i^a, u_i^{\pm})$, where x^M , $M = 0, \dots, 5$, are the $6D$ Minkowski space-time coordinates, θ_i^a , $a = 1, \dots, 4$, $i = 1, 2$, are Grassmann variables, and u_i^{\pm} , $u^{+i}u_i^- = 1$, are the harmonic variables [36,37]. For the analytic coordinates we use the notation $\zeta = (x_{\mathcal{A}}^M, \theta^{\pm a})$, where

$$x_{\mathcal{A}}^M \equiv x^M + \frac{i}{2} \theta^{+a} (\gamma^M)_{ab} \theta^{-b}, \quad \theta^{\pm a} = u_k^{\pm} \theta^{ak}, \quad (2.1)$$

and the antisymmetric $6D$ Weyl γ -matrices are used, $(\gamma^M)_{ab} = -(\gamma^M)_{ba}$, $(\tilde{\gamma}^M)^{ab} = \frac{1}{2} \varepsilon^{abcd} (\gamma^M)_{cd}$, with the totally antisymmetric tensor ε^{abcd} . By definition, the analytic superfields are annihilated by the spinor covariant derivative $D_a^+ = u_i^+ D_a^i$ and in the analytic basis (where D_a^+ are “short”) are defined on the analytic harmonic superspace $(\zeta, u^{\pm i})$. Also we will need the covariant derivative $D_a^- = u_i^- D_a^i$ and the harmonic derivatives

$$D^{\pm\pm} = u^{\pm i} \frac{\partial}{\partial u^{\mp i}}, \quad D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}}. \quad (2.2)$$

The spinor and harmonic derivatives satisfy the algebra

$$\{D_a^+, D_b^-\} = i(\gamma^M)_{ab} \partial_M, \quad [D^{++}, D^{--}] = D^0, \quad [D^{\pm\pm}, D_a^{\pm}] = 0, \quad [D^{\pm\pm}, D_a^{\mp}] = D_a^{\pm}. \quad (2.3)$$

The full harmonic and the analytic superspace integration measures are defined as follows

$$d^{14}z \equiv d^6x_{\mathcal{A}} (D^-)^4 (D^+)^4, \quad d\zeta^{(-4)} \equiv d^6x_{\mathcal{A}} du (D^-)^4, \quad (2.4)$$

where

$$(D^{\pm})^4 = -\frac{1}{24} \varepsilon^{abcd} D_a^{\pm} D_b^{\pm} D_c^{\pm} D_d^{\pm}. \quad (2.5)$$

In the harmonic superspace formalism the gauge field is a component of the analytic gauge superfield V^{++} . A necessary ingredient is also a non-analytic harmonic connection V^{--} obtained as a solution of the harmonic zero-curvature condition [8]

$$D^{++} V^{--} - D^{--} V^{++} + i[V^{++}, V^{--}] = 0. \quad (2.6)$$

Using these superfields one can construct the gauge covariant harmonic derivative $\nabla^{\pm\pm} = D^{\pm\pm} + iV^{\pm\pm}$. The superfield V^{--} is also used to define the spinor and vector connections in the gauge-covariant derivatives. In the λ -frame we have [38]

$$\nabla_a^+ = D_a^+, \quad \nabla_a^- = D_a^- + i\mathcal{A}_a^-, \quad \nabla_{ab} = \partial_{ab} + i\mathcal{A}_{ab}, \quad (2.7)$$

where $\nabla_{ab} = \frac{1}{2} (\gamma^M)_{ab} \nabla_M$ and $\nabla_M = \partial_M - i\mathcal{A}_M$, with the superfield connections defined as

$$\mathcal{A}_a^- = iD_a^+ V^{--}, \quad \mathcal{A}_{ab} = \frac{1}{2} D_a^+ D_b^+ V^{--}. \quad (2.8)$$

The covariant derivatives (2.7) satisfy the algebra

$$\{\nabla_a^+, \nabla_b^-\} = 2i\nabla_{ab}, \quad [\nabla_c^{\pm}, \nabla_{ab}] = \frac{i}{2} \varepsilon_{abcd} W^{\pm d}, \quad [\nabla_M, \nabla_N] = iF_{MN}. \quad (2.9)$$

The superfield $W^{a\pm}$ is the superfield strength of the gauge multiplet,

$$W^{+a} = -\frac{i}{6} \varepsilon^{abcd} D_b^+ D_c^+ D_d^+ V^{--}, \quad W^{-a} = \nabla^{--} W^{+a}. \quad (2.10)$$

Also we define the analytic superfield [22] $F^{++} \equiv (D^+)^4 V^{--}$ which satisfies the harmonic constraint $\nabla^{++} F^{++} = 0$ following from (2.6) and the analyticity of V^{++} .

The classical action of 6D, $\mathcal{N} = (1, 1)$ SYM theory in the harmonic superspace formulation is written as

$$S_0 = \frac{1}{f^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \frac{1}{2f^2} \text{tr} \int d\zeta^{(-4)} q^+{}^A \nabla^{++} q_A^+, \quad (2.11)$$

where $(u_n^+ u_1^+)^{-1}, \dots$ are the harmonic distributions defined in [8], $A = 1, 2$ is a Pauli-Gürsey group $SU(2)$ index and $q_A^+ = (q^+, -\tilde{q}^+)$, $\tilde{q}_A = \varepsilon^{AB} q_B^+$. The superfields V^{++} and q_A^+ take values in the adjoint representation of the gauge group, i.e., $V^{++} = V^{++I} t^I$, $q_A^+ = q_A^{+I} t^I$, where t^I are the gauge algebra generators subjected to the normalization condition $\text{tr}(t^I t^J) = \delta^{IJ}/2$. The action involves the negative-dimension coupling constant f , $[f] = m^{-1}$, and the covariant harmonic derivative

$$\nabla^{++} q_A^+ = D^{++} q_A^+ + i[V^{++}, q_A^+]. \quad (2.12)$$

The classical equations of motion of the theory have a form

$$F^{++} + \frac{i}{2}[q^{+A}, q_A^+] = 0, \quad \nabla^{++} q_A^+ = 0. \quad (2.13)$$

The action (2.11) is invariant under the manifest $\mathcal{N} = (1, 0)$ supersymmetry and an additional hidden $\mathcal{N} = (0, 1)$ supersymmetry. The hidden supersymmetry mixes the gauge and hypermultiplet superfields with each other [22],

$$\delta_{(0,1)} V^{++} = \epsilon^{+A} q_A^+, \quad \delta_{(0,1)} q_A^+ = -(D^+)^4 (\epsilon_A^- V^{--}), \quad \epsilon_A^\pm = \epsilon_{aA} \theta^{\pm a}. \quad (2.14)$$

As a result, the action (2.11) is invariant under 6D, $\mathcal{N} = (1, 1)$ supersymmetry. Certainly, it is also invariant under the superfield gauge transformation

$$\delta V^{++} = -\nabla^{++} \lambda, \quad \delta q_A^+ = i[\lambda, q_A^+], \quad (2.15)$$

parameterized by a real analytic superfield λ .

3. Effective action

When quantizing gauge theories, it is convenient to use the background field method allowing to construct the manifestly gauge invariant effective action. For 6D, $\mathcal{N} = (1, 0)$ SYM theory in the harmonic superspace formulation this method was worked out in [25–27]. In many aspects it is similar to that for 4D $\mathcal{N} = 2$ supersymmetric gauge theories [39,40] (see also the review [41]).

Following the background field method we split the superfield V^{++} into the sum of the “background” superfield V^{++} and the “quantum” one v^{++} ,

$$V^{++} \rightarrow V^{++} + f v^{++}. \quad (3.1)$$

Then we expand the effective action in a power series in quantum superfields and obtain a theory of the superfields v^{++} , q^+ in the background of the classical superfield V^{++} , which is treated as a functional argument of the effective action. Our aim is to study the two-loop contributions to the effective action in the gauge superfield sector. To this end, it is sufficient to assume that the hypermultiplet is purely quantum.

Using the results of refs. [25–27] the general expression for the effective action can be written in the form

$$e^{i\Gamma[V^{++}]} = \text{Det}^{1/2} \hat{\square} \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}b \mathcal{D}c \mathcal{D}\varphi \exp \left(iS_{\text{total}} - \text{tr} \int d\zeta^{(-4)} du \frac{\delta \Gamma[V^{++}]}{\delta V^{++}} v^{++} \right), \quad (3.2)$$

where the operator $\hat{\square} = \frac{1}{2}(D^+)^4 (\nabla^{--})^2$ acting on a space of analytic superfields is reduced to the covariant superfield d'Alembertian

$$\hat{\square} = \eta^{MN} \nabla_M \nabla_N + iW^{+a} \nabla_a^- + iF^{++} \nabla^{--} - \frac{i}{2} (\nabla^{--} F^{++}), \quad (3.3)$$

and η_{MN} is the 6D Minkowski metric with the mostly negative signature. The total action, $S_{\text{total}} = S_0 + S_{\text{gf}} + S_{\text{FP}} + S_{\text{NK}}$, includes the gauge-fixing term corresponding to the Feynman gauge,

$$S_{\text{gf}}[v^{++}, V^{++}] = -\frac{1}{2} \text{tr} \int d^{14}z du_1 du_2 \frac{v_\tau^{++}(1) v_\tau^{++}(2)}{(u_1^+ u_2^+)^2} + \frac{1}{4} \text{tr} \int d^{14}z du v_\tau^{++} (D^{--})^2 v_\tau^{++}, \quad (3.4)$$

the action for the fermionic Faddeev-Popov ghosts b and c , as well as the action for the bosonic real analytic Nielsen-Kallosh ghost φ ,

$$S_{\text{FP}} = -\text{tr} \int d\zeta^{(-4)} \nabla^{++} b (\nabla^{++} c + i[v^{++}, c]), \quad (3.5)$$

$$S_{\text{NK}} = -\frac{1}{2} \text{tr} \int d\zeta^{(-4)} \varphi (\nabla^{++})^2 \varphi. \quad (3.6)$$

The action (3.4) depends on the background field V^{++} through the background gauge bridge superfield, in a close analogy with 4D, $\mathcal{N} = 2$ SYM theory.

The calculation of the effective action is carried out in the framework of the loop expansion. In the one-loop approximation the quantum corrections to the classical action are determined by the quadratic part of the action S_{total} . After integration over quantum superfields this quadratic part produces the one-loop contribution $\Gamma^{(1)}$ to the effective action. The contributions coming from the Faddeev-Popov ghosts, the Nielsen-Kallosh ghost, and the quantum hypermultiplet contain divergences. However, for $\mathcal{N} = (1, 1)$ theory they cancel each other since in this case the hypermultiplet lies in the adjoint representation of the gauge group, see refs. [25–27] for details. This implies that the theory under consideration is off-shell finite in the one-loop approximation.

In this paper we will investigate the two-loop divergences. Before starting the calculations it is instructive to discuss the structure of propagators and vertices. That part of the total action S_{total} which is quadratic in quantum superfields defines the (background-superfield dependent) propagators of these superfields, which are similar to those for $4D$, $\mathcal{N} = 2$ theory [8,39,42]

$$G^{(2,2)}(\zeta_1, u_1 | \zeta_2, u_2) = i \langle v^{++}(\zeta_1, u_1) v^{++}(\zeta_2, u_2) \rangle = -2 \frac{(D_1^+)^4}{\widehat{\square}} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2), \quad (3.7)$$

$$G^{(1,1)}(\zeta_1, u_1 | \zeta_2, u_2) = i \langle q^+(\zeta_1, u_1) \tilde{q}^+(\zeta_2, u_2) \rangle = 2 \frac{(D_1^+)^4 (D_2^+)^4}{\widehat{\square}} \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}, \quad (3.8)$$

$$G^{(0,0)}(\zeta_1, u_1 | \zeta_2, u_2) = i \langle b(\zeta_1, u_1) c(\zeta_2, u_2) \rangle = -(u_1^- u_2^-) G^{(1,1)}(\zeta_1, u_1 | \zeta_2, u_2). \quad (3.9)$$

In comparison with the $4D$, $\mathcal{N} = 2$ case, the operator $\widehat{\square}$ has a different form and is given by (3.3).

For calculating the two-loop quantum corrections we will need vertices which are cubic and quartic in quantum superfields. In the theory under consideration there are several types of such vertices.

The first type includes the cubic and quartic self-interactions of the gauge superfield described by the corresponding terms in the classical action (2.11),

$$S_{\text{SYM}}^{(3)} = \frac{if}{3} \text{tr} \int d^{14}z \prod_{a=1}^3 du_a \frac{v_1^{++} v_2^{++} v_3^{++}}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_1^+)}, \quad (3.10)$$

$$S_{\text{SYM}}^{(4)} = \frac{f^2}{4} \text{tr} \int d^{14}z \prod_{a=1}^4 du_a \frac{v_1^{++} v_2^{++} v_3^{++} v_4^{++}}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)}. \quad (3.11)$$

The interaction of the gauge multiplet with hypermultiplet can be also found from classical action (2.11) and is given by the term

$$S_{\text{hyper}}^{(3)} = \frac{f}{2} \int d\zeta^{(-4)} f^{IJK} \tilde{q}_I^+ v_J^{++} q_K^+. \quad (3.12)$$

The action (3.5) describes the interaction of gauge multiplet and the Faddeev-Popov ghosts

$$S_{\text{ghost}}^{(3)} = \frac{f}{2} \int d\zeta^{(-4)} f^{IJK} (\nabla^{++} b)_I v_J^{++} c_K, \quad (3.13)$$

where f^{IJK} are the structure constants of the gauge group.

4. Off-shell two-loop divergences

Using the power counting [25] one can show that the only possible two-loop divergent contribution in the gauge superfield sector has the structure

$$\Gamma_{\text{div}}^{(2)}[V^{++}] = a \int d\zeta^{(-4)} \text{tr} (F^{++} \widehat{\square} F^{++}), \quad (4.1)$$

where a is a constant, which diverges after removing a regularization. Below we will calculate the constant a in the modified minimal subtraction scheme for the considered $\mathcal{N} = (1, 1)$ SYM theory off shell.

In the process of calculation we do not assume any restriction on the background gauge multiplet and perform the analysis in a manifestly gauge invariant form. In the two-loop approximation there are Feynman supergraphs of two different topologies, which we will call ‘ Θ ’ and ‘ ∞ ’ topologies. The graphs of the ‘ Θ ’ topology are generated by cubic interactions. In the $\mathcal{N} = (1, 1)$ theory under consideration they are presented by eqs. (3.10), (3.12), and (3.13). The graphs of ‘ ∞ ’ topology contain a vertex corresponding to the quartic interaction. It is given by eq. (3.11).

It is convenient to separately consider the diagrams containing only the gauge propagators $G^{2,2}$. They are presented in Fig. 1, where the gauge propagators are depicted by wavy lines. Also it is expedient to consider superdiagrams involving the hypermultiplet and ghosts propagators together. They are presented in Fig. 2. The hypermultiplet propagators $G^{(1,1)}$ are denoted by solid lines, and the Faddeev-Popov ghost propagators $G^{(0,0)}$ by dashed lines. In addition, we will take into account that the theory under consideration is finite at one loop. Therefore, there is no need to renormalize the one-loop subgraphs in the two-loop supergraphs.¹

¹ We emphasize that in the considered formalism all propagators are background-field dependent.

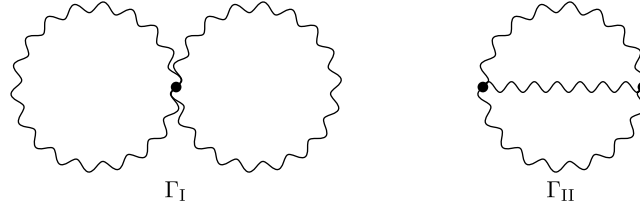


Fig. 1. Two-loop Feynman supergraphs with gauge self-interactions vertices.

The analytic expression corresponding to the diagram Γ_I (of the ‘ ∞ ’ topology) presented in Fig. 1 is written as

$$\Gamma_I = -2f^2 \text{tr}(t^I t^J t^K t^L) \int d^{14}z \int \prod_{a=1}^4 u_a \times \left\{ \frac{G_{IJ}^{(2,2)}(z, u_1; z, u_2) G_{KL}^{(2,2)}(z, u_3; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} + \frac{1}{2} \frac{G_{IJ}^{(2,2)}(z, u_1; z, u_3) G_{KL}^{(2,2)}(z, u_2; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} \right\}. \quad (4.2)$$

This expression involves two Green functions $G^{(2,2)}$ in the coincident θ limit. According to eq. (3.7), each expression for $G^{(2,2)}$ contains a harmonic δ -function. Due to these δ -functions the first term in the curly brackets will contain a singularity, since $(u_1^+ u_2^+)$ and $(u_3^+ u_4^+)$ in the denominator vanish. To avoid this problem, one should use a ‘longer form’ of the gauge superfield Green function $G^{(2,2)}$ [8,42],

$$G^{(2,2)}(\zeta_1, u_1 | \zeta_2, u_2) = -(D_1^+)^4 \frac{1}{(\square_2)^2} (D_2^+)^4 (D_2^{--})^2 \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2). \quad (4.3)$$

Next, in the first term of the expression (4.2) we should annihilate the Grassmannian delta-functions $\delta^8(\theta_1 - \theta_2)|_{\theta_2 \rightarrow \theta_1}$. This gives the factor $(u_1^+ u_2^+)^4 (u_3^+ u_4^+)^4$ in the numerator which cancels the singular terms in the denominator. The resulting expression is proportional to

$$\frac{1}{(u_1^+ u_2^+)} \frac{1}{(\square_2)^2} (u_1^+ u_2^+)^4 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2) \cdot \frac{1}{(u_3^+ u_4^+)} \frac{1}{(\square_4)^2} (u_3^+ u_4^+)^3 (D_4^{--})^2 \delta^{(-2,2)}(u_3, u_4). \quad (4.4)$$

Let us consider a part of this expression depending on u_1 and u_2 ,

$$\begin{aligned} \frac{1}{(u_1^+ u_2^+)} \frac{1}{(\square_2)^2} (u_1^+ u_2^+)^4 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2) &= \frac{1}{(u_1^+ u_2^+)} \left(\frac{1}{\square^2} - \frac{1}{\square^2} i F_2^{++} \nabla^{--} \frac{1}{\square} \right. \\ &\quad \left. - \frac{1}{\square} i F_2^{++} \nabla^{--} \frac{1}{\square^2} + \dots \right) (u_1^+ u_2^+)^4 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2), \end{aligned} \quad (4.5)$$

where

$$\square \equiv \eta^{MN} \nabla_M \nabla_N \quad (4.6)$$

and $F^{++} = F^{++A} T_{Adj}^A$ with $(T_{Adj}^A)_{IJ} = -i f^{AIJ}$. The only possible divergent contributions could appear from the terms containing D^{--} inside ∇^{--} . However,

$$\begin{aligned} \frac{1}{(u_1^+ u_2^+)} D_2^{--} \left((u_1^+ u_2^+)^4 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2) \right) &= D_2^{--} \left((u_1^+ u_2^+)^3 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2) \right) \\ + (u_1^+ u_2^-) (u_1^+ u_2^+)^2 (D_2^{--})^2 \delta^{(-2,2)}(u_1, u_2) &= \left((D_2^{--})^2 (u_1^+ u_2^+)^2 \right) \delta^{(-1,1)}(u_1, u_2) = 2\delta^{(1,-1)}(u_1, u_2). \end{aligned} \quad (4.7)$$

Therefore, the first term in eq. (4.2) diverges. Taking into account that

$$\text{tr}(t^I t^J t^K t^L) f^{AIJ} f^{BKL} = \frac{1}{4} \text{tr}([t^I, t^J][t^K, t^L]) f^{AIJ} f^{BKL} = -\frac{1}{8} (C_2)^2 \delta^{AB},$$

we see that (in the Euclidean space after the Wick rotation) its divergent part is equal to

$$4(C_2)^2 \int d^{14}z du_1 du_3 F_{1,\tau}^{++A} F_{3,\tau}^{++A} \frac{1}{(u_1^+ u_3^+)^2} \cdot \left[\left(\int \frac{d^6 k_E}{(2\pi)^6} \frac{1}{k_E^6} \right)^2 \right]_{\infty}. \quad (4.8)$$

The second term in eq. (4.2) does not contain harmonic singularities. Therefore, when using the long form of the gauge propagator, we obtain the expressions proportional to

$$(u_1^+ u_3^+)^4 (D_3^{--})^2 \delta^{(-2,2)}(u_1, u_3) \cdot (u_2^+ u_4^+)^4 (D_4^{--})^2 \delta^{(-2,2)}(u_2, u_4) = 0. \quad (4.9)$$

Therefore, the contribution of this term vanishes.

The analytic expression for the two-loop diagram Γ_{II} (of the ‘ Θ ’ topology) presented in Fig. 1 is constructed using the cubic gauge superfield vertex (3.10) and it has the form



Fig. 2. Two-loop Feynman supergraphs with hypermultiplet and ghosts vertices.

$$\Gamma_{II} = -\frac{f^2}{6} \int d^{14}z_1 d^{14}z_2 \prod_{a=1}^6 du_a f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} \frac{G_{I_1 I_2}^{(2,2)}(z_1, u_1; z_2, u_4) G_{J_1 J_2}^{(2,2)}(z_1, u_2; z_2, u_5) G_{K_1 K_2}^{(2,2)}(z_1, u_3; z_2, u_6)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_5^+)(u_5^+ u_6^+)(u_6^+ u_1^+)}. \quad (4.10)$$

As the next steps, we substitute the explicit expression for the Green function $G^{(2,2)}$ and integrate by parts with respect to one of the $(D^+)^4$ factors. Also it is possible to calculate the harmonic integrals over u_4, u_5, u_6 using the corresponding delta-functions which come out from the propagators. As a result, we obtain

$$\Gamma_{II} = -\frac{f^2}{6} \int d^{14}z_1 d^{14}z_2 \prod_{a=1}^3 du_a \frac{f^{I_1 J_1 K_1} f^{I_2 J_2 K_2}}{(u_1^+ u_2^+)^2 (u_2^+ u_3^+)^2 (u_3^+ u_1^+)^2} (\widehat{\square}^{-1})_{I_1 I_2} \delta^{14}(z_1 - z_2) \times (D_1^+)^4 (\widehat{\square}^{-1})_{J_1 J_2} (D_2^+)^4 \delta^{14}(z_1 - z_2) (\widehat{\square}^{-1})_{K_1 K_2} (D_2^+)^4 \delta^{14}(z_1 - z_2). \quad (4.11)$$

After integrating over θ_2 using the Grassmannian delta-function we are left with the coincident $\theta_2 \rightarrow \theta_1$ limit in the two remaining delta-functions. In order to annihilate these Grassmannian delta-functions in the coincident θ -point limit we need four $(D^\pm)^4$ -factors. However we have only three. The remaining $(D^-)^4$ factor should be obtained from the expansion of the inverse $\widehat{\square}$ operator. But in this case we produce an extra operator $(\partial^2)^4$, so that the overall momentum degree in the denominator will be $6 + 8 = 14$. Taking into account the presence of the integrations $d^6 k d^6 q$, we conclude that the resulting integral is convergent. Therefore, the superdiagram considered can produce only finite contributions to the effective action.

Now, let us demonstrate that in the $6D, \mathcal{N} = (1, 1)$ theory the last two contributions Γ_{III} and Γ_{IV} depicted in Fig. 2 cancel each other. The arguments are basically analogous to those used for $4D, \mathcal{N} = 4$ SYM theory in [43]. First, we note that the vertex (3.13) contains the background-dependent covariant harmonic derivative ∇^{++} , which acts on the ghost field b . After integrating by parts with respect to this derivative, the latter will act on the ghost propagator $G^{(0,0)}$ which is related to the hypermultiplet Green function by eq. (3.9). Due to this relation the analytical expression for the sum of two contributions Γ_{III} and Γ_{IV} presented in Fig. 2 takes the form

$$\Gamma_{III} + \Gamma_{IV} = f^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} \left(1 + (u_1^+ u_2^-)(u_1^- u_2^+)\right) f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} G_{I_1 I_2}^{(2,2)}(1|2) G_{J_1 J_2}^{(1,1)}(1|2) G_{K_1 K_2}^{(1,1)}(1|2). \quad (4.12)$$

As pointed out in [43], the identity $1 + (u_1^+ u_2^-)(u_1^- u_2^+) = (u_1^+ u_2^+)(u_1^- u_2^-)$ allows one to transform the contribution (4.12) to the form

$$\Gamma_{III} + \Gamma_{IV} = f^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} (u_1^+ u_2^+)(u_1^- u_2^-) f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} G_{I_1 I_2}^{(2,2)}(1|2) G_{J_1 J_2}^{(1,1)}(1|2) G_{K_1 K_2}^{(1,1)}(1|2) = 0. \quad (4.13)$$

This expression vanishes due to the useful property of the harmonic delta-function $(u_1^- u_2^-) \delta^{(2,-2)}(u_1, u_2) = 0$ [8]. Thus, these two diagrams cancel each other. Obviously, this cancellation takes place only in the case of the $\mathcal{N} = (1, 1)$ theory, when the hypermultiplet is in the adjoint representation of the gauge group. In a general $6D, \mathcal{N} = (1, 0)$ SYM theory the diagrams in Fig. 2 enter with different group factors, which prohibits the cancellation.

Thus, we see that the only divergent contribution comes from the ‘ ∞ ’ superdiagram, and the divergent part of the two-loop effective action in the gauge superfield sector is given by the expression

$$\Gamma_{\infty, \text{gauge}}^{(2)} = 8f^2 (C_2)^2 \text{tr} \int d^{14}z du_1 du_2 F_{1,\tau}^{++} F_{2,\tau}^{++} \frac{1}{(u_1^+ u_2^+)^2} \cdot \left[\left(\int \frac{d^6 k_E}{(2\pi)^6} \frac{1}{k_E^6} \right)^2 \right]_{\infty}. \quad (4.14)$$

Making use of the identity

$$F_{1,\tau}^{++} = \frac{1}{2} D_1^{++} D_1^{--} F_1^{++},$$

integrating by parts with respect to the derivative D_1^{++} and taking into account that F_{τ}^{++} is independent of the harmonic variables, we see that

$$\begin{aligned} \text{tr} \int d^{14}z du_1 du_2 F_{1,\tau}^{++} F_{2,\tau}^{++} \frac{1}{(u_1^+ u_2^+)^2} &= -\frac{1}{2} \text{tr} \int d^{14}z du_1 du_2 D_1^{--} F_{1,\tau}^{++} F_{2,\tau}^{++} D_1^{--} \delta^{(2,-2)}(u_1, u_2) \\ &= \frac{1}{2} \text{tr} \int d^{14}z du (D^{--})^2 F_{\tau}^{++} F_{\tau}^{++} = \text{tr} \int d\zeta^{(-4)} F^{++} \widehat{\square} F^{++}. \end{aligned} \quad (4.15)$$

Moreover, in the dimensional regularization scheme we have

$$\int \frac{d^D k_E}{(2\pi)^D} \frac{1}{k_E^6} \rightarrow \frac{1}{(4\pi)^{D/2}} \frac{\Gamma(3-D/2)}{\Gamma(3)} = \frac{1}{(4\pi)^3 \varepsilon} + \frac{1}{128\pi^3} (-\gamma + \ln(4\pi)) + O(\varepsilon). \quad (4.16)$$

So, in the $\overline{\text{MS}}$ -scheme,

$$\left[\left(\int \frac{d^6 k_E}{(2\pi)^6} \frac{1}{k_E^6} \right)^2 \right]_\infty = \frac{1}{(4\pi)^6 \varepsilon^2}. \quad (4.17)$$

Thus, the divergent part of the two-loop effective action can finally be written in the form

$$\Gamma_{\infty, \text{gauge}}^{(2)} = \frac{8f^2}{(4\pi)^6 \varepsilon^2} (C_2)^2 \text{tr} \int d\zeta^{(-4)} F^{++} \widehat{\square} F^{++}, \quad (4.18)$$

and the constant a appearing in Eq. (4.1) is now identified as

$$a = \frac{8f^2}{(4\pi)^6 \varepsilon^2} (C_2)^2. \quad (4.19)$$

An interesting peculiarity of the two loop divergences obtained is that they contain only leading two-loop pole $\frac{1}{\varepsilon^2}$, while the sub-leading pole $\frac{1}{\varepsilon}$ is absent. We believe that the reason for this may be hidden $\mathcal{N} = (0, 1)$ supersymmetry and the absence of the off-shell one-loop divergences in the theory under consideration. The result obtained matches with the statement of ref. [22] that the candidate two-loop counterterms in $\mathcal{N} = (1, 1)$ SYM theory vanish on mass shell, provided they are required to be $\mathcal{N} = (1, 0)$ off-shell supersymmetric and gauge invariant. More details on this point are given in the next section.

5. Hypermultiplet dependence of the two-loop divergences

In the previous section we have calculated the two-loop divergences in the gauge multiplet sector, where the background hypermultiplet q^+ is absent. Now we discuss a possible structure of the two-loop divergences in the case when the background hypermultiplet is taken into account. Of course, the hypermultiplet-dependent contribution to two-loop divergences can be obtained by the straightforward quantum computations of the two-loop effective action. However, the general form of such divergences can in principle be described without direct calculations, just starting from the expression (4.18) and assuming the invariance of the effective action under the hidden $\mathcal{N} = (0, 1)$ supersymmetry. Taking into account the result (4.18) one might expect that including the on-shell background hypermultiplet will merely lead to the replacement of F^{++} in (4.18) by the total classical equation of motion for the background gauge multiplet (2.13) coupled to hypermultiplet.

As was proved in [22,44], only the classical action (2.11) is $\mathcal{N} = (1, 1)$ supersymmetric off shell, while in any other $\mathcal{N} = (1, 1)$ invariant the hidden $\mathcal{N} = (0, 1)$ supersymmetry must be on-shell. Therefore, we will assume here that the hypermultiplet satisfies the classical equations of motion

$$\nabla^{++} q_A^+ = \nabla^{--} q_A^- = 0, \quad (5.1)$$

where $q_A^- := \nabla^{--} q_A^+$. In this case the $\mathcal{N} = (0, 1)$ supersymmetry transformation (2.14) for the non-analytic gauge potential takes the form

$$\delta_{(0,1)} V^{--} = \epsilon^{-A} q_A^-, \quad \epsilon^{-A} = \epsilon_a^A \theta^{-a}. \quad (5.2)$$

Let us now rewrite the expression (4.1) in the central basis,

$$\Gamma_{\text{div}}^{(2)}[V^{++}] = -\frac{a}{2} \int d^{14} z du \text{tr} (\nabla^{--} F^{++})^2. \quad (5.3)$$

Here we made use of the definition of covariant d'Alembertian (3.3) and integrated by parts with respect to the harmonic derivative ∇^{--} . The coefficient a is given by eq. (4.19). Our aim is to find the appropriate terms which should be added to the action (5.3) to ensure the invariance under hidden $\mathcal{N} = (0, 1)$ supersymmetry transformations. First, we rewrite the $\mathcal{N} = (0, 1)$ transformation (2.14) in the form

$$\begin{aligned} \delta_{(0,1)} F^{++} &= -i\epsilon_b^A [W^{+b}, q_A^+] - i[\epsilon^{-A} q_A^+, F^{++}], \\ \delta_{(0,1)} q^{+A} &= \epsilon_b^A W^{+b} - i[\epsilon^{-B} q_B^+, q^{+A}] - \epsilon^{-A} E^{++}, \\ \delta_{(0,1)} q^{-A} &= \epsilon_b^A W^{-b} - i[\epsilon^{-B} q_B^+, q^{-A}] - \epsilon^{-A} \nabla^{--} E^{++}, \end{aligned} \quad (5.4)$$

where $E^{++} := F^{++} + \frac{i}{2} [q^{+A}, q_A^+]$. After that one can see that the following generalization of the action (5.3),

$$\Gamma_{\text{div}}^{(2)}[V^{++}, q^+] = -\frac{a}{2} \int d^{14} z du \text{tr} \left\{ (\nabla^{--} F^{++})^2 - 2i[q^{-A}, q_A^-] F^{++} + \frac{1}{2} [q^{-A}, q_A^-] [q^{+C}, q_C^+] \right\}, \quad (5.5)$$

for the background hypermultiplet satisfying (5.1), under (5.4) is transformed as

$$\delta \Gamma_{\text{div}}^{(2)}[V^{++}, q^+] = -\frac{a}{2} \int d^{14} z du 4i \epsilon^{-A} \text{tr} q_A^- [E^{++}, \nabla^{--} E^{++}] \quad (5.6)$$

and so is invariant modulo the gauge superfield equation of motion $E^{++} = 0$. The action (5.5) can be rewritten, up to a total harmonic derivative, as

$$\Gamma_{\text{div}}^{(2)}[V^{++}, q^+] = -\frac{a}{2} \int d^{14}z du \operatorname{tr} (\nabla^{--} E^{++})^2. \quad (5.7)$$

Passing to the analytic basis, we finally obtain

$$\Gamma_{\text{div}}^{(2)}[V^{++}, q^+] = a \int d\zeta^{(-4)} \operatorname{tr} E^{++} \widehat{\square} E^{++}. \quad (5.8)$$

We see that the two-loop divergences vanish on the total mass-shell (2.13), as expected.

Finally, we note that the superficial degree of divergence in $\mathcal{N} = (1, 0)$ SYM theory was calculated in [25] in the form

$$\omega = 2L - N_q - \frac{1}{2}N_D, \quad (5.9)$$

where L is a number of loops in the supergraph with N_q external lines of hypermultiplet and N_D is a number of spinor derivatives acting on the external lines. Divergent contributions correspond to the case $\omega \geq 0$. Hence at $L = 2$, the number of the external hypermultiplet lines should be $N_q \leq 4$. Possible divergent contributions in the gauge superfield sector at two loops have the universal structure (4.1). The number of external hypermultiplet lines should be even to secure gauge invariance. Hence, the possible hypermultiplet-dependent divergent contributions have two or four external hypermultiplet lines. Taking into account these reasonings and $\mathcal{N} = (1, 0)$ supersymmetry, we obtain the following expression for the two-loop divergences

$$\Gamma_{\infty}^{(2)}[V^{++}, q^+] = a \int d\zeta^{(-4)} \operatorname{tr} \left(F^{++} \widehat{\square} F^{++} + ic_1 F^{++} \widehat{\square} [q^{+A}, q_A^+] + c_2 [q^{+A}, q_A^+] \widehat{\square} [q^{+B}, q_B^+] \right) \\ + \text{terms proportional to the hypermultiplet equations of motion}, \quad (5.10)$$

where the constant a is given by (4.19) and c_1, c_2 are the arbitrary dimensionless numerical coefficients, which can be fixed only within the quantum field theoretical computations of the effective action. Comparing (5.10) with (5.5), we observe that the role of hidden $\mathcal{N} = (0, 1)$ supersymmetry is just to relate the unknown constants c_1 and c_2 to the original constant a . Indeed, the requirement of invariance of the expression (5.10) under the $\mathcal{N} = (0, 1)$ supersymmetry yields the same expression (5.8).

6. Summary

In the present paper we have studied two-loop divergent contributions to the effective action for $6D$, $\mathcal{N} = (1, 1)$ SYM theory formulated in $\mathcal{N} = (1, 0)$ harmonic superspace. In this approach it amounts to the model (2.11) of the minimally coupled $\mathcal{N} = (1, 0)$ gauge multiplet and the hypermultiplet, both in the adjoint representation of the gauge group. The classical action of the model is invariant under an additional $\mathcal{N} = (0, 1)$ supersymmetry, so that it actually describes $\mathcal{N} = (1, 1)$ SYM theory.

In the papers [26,41] we have demonstrated by explicit calculations that, in the minimal gauge, $\mathcal{N} = (1, 1)$ SYM theory in six-dimensions is one-loop finite *off shell*. In the present paper, using the superfield background field method, we have calculated the divergent part of the two-loop effective action in the gauge multiplet sector. The corresponding background field dependent supergraphs determining the effective action are given in Fig. 1 and Fig. 2. It was shown that the divergences of the supergraphs Γ_{III} and Γ_{IV} in Fig. 2 cancel each other due to the hidden $\mathcal{N} = (0, 1)$ supersymmetry. The supergraph Γ_{II} in Fig. 1 is finite. The total divergence is only due to the supergraph Γ_I in Fig. 1. The corresponding divergent contribution to the two-loop effective action is proportional to the classical equation of motion. This means that the theory is not off-shell finite at two loops in the gauge multiplet sector even in the Feynman gauge, while the divergences vanish on shell in this sector. Nevertheless, it is worth pointing out that the two-loop divergences in the theory under consideration are ‘softer’ in some sense as compared with the general quantum field theory setting. The divergent part of the two-loop effective action (4.18) contains only the leading two-loop pole $\frac{1}{\varepsilon^2}$, the sub-leading pole $\frac{1}{\varepsilon}$ being absent. This peculiarity could be attributed to hidden $\mathcal{N} = (0, 1)$ supersymmetry.

Also, we have analyzed, on the grounds of gauge invariance, power counting, the explicit $\mathcal{N} = (1, 0)$ supersymmetry, and the hidden $\mathcal{N} = (0, 1)$ supersymmetry, a possible structure of two-loop divergences for the $\mathcal{N} = (1, 1)$ super Yang-Mills theory in an arbitrary gauge and hypermultiplet background. It was shown that such divergences vanish on the total equations of motion (2.13) and contain an arbitrary dimensionless numerical coefficient. To fix this coefficient, we must carry out the direct quantum field theoretical calculations. Thus, obviously, the most urgent problem for further study is to calculate the two-loop divergences in the general background field setting, including not only the background gauge multiplet but the background hypermultiplet as well. We hope to confirm our assertion that the total two-loop divergences involve the complete classical equation of motion.

Another interesting problem is to calculate the two-loop divergences for the general $\mathcal{N} = (1, 0)$ SYM theory without hidden $\mathcal{N} = (0, 1)$ sector. We plan to perform the detailed calculation of the two-loop divergent contributions for the general $\mathcal{N} = (1, 0)$ gauge theory in a forthcoming work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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