



Supergraph analysis of the one-loop divergences in $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ gauge theories

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Abstract

We study the one-loop effective action for $6D$, $\mathcal{N} = (1, 0)$ supersymmetric Yang–Mills (SYM) theory with hypermultiplets and $6D$, $\mathcal{N} = (1, 1)$ SYM theory as a subclass of the former, using the off-shell formulation of these theories in $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace. We develop the corresponding supergraph technique and apply it to compute the one-loop divergences in the background field method ensuring the manifest gauge invariance. We calculate the two-point Green functions of the gauge superfield and the hypermultiplet, as well as the three-point gauge-hypermultiplet Green function. Using these Green functions and exploiting gauge invariance of the theory, we find the full set of the off-shell one-loop divergent contributions, including the logarithmic and power ones. Our results precisely match with those obtained earlier in [1,2] within the proper time superfield method.

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1. Introduction

Investigation of quantum corrections in higher-dimensional gauge theories is an exciting problem with a long history (see, e.g., [3–10] and references therein). On the one hand, because of

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dimensionful coupling constant, these theories are not renormalizable by formal power-counting. On the other hand, extended supersymmetry is capable to improve the ultraviolet behavior of a theory. And indeed, it was shown in the above papers that, e.g., in six-dimensional supersymmetric Yang–Mills theories the one- and two-loop amplitudes are finite. It is extremely interesting to analyze the impact of extended supersymmetry on a general structure of ultraviolet divergences in higher-dimensional gauge theories and to learn whether the supersymmetry is powerful enough for construction of the renormalizable and, perhaps, finite higher dimensional quantum field-theoretical models.

To accomplish this program, it is natural to start with such a formulation of the theory which makes manifest and off-shell as much underlying symmetries as possible. In our case these are supersymmetry and gauge invariance. In $4D$ theories, $\mathcal{N} = 1$ supersymmetry becomes manifest in the $\mathcal{N} = 1$ superfield formalism (see e.g. [11,12]). Both supersymmetries of $4D$, $\mathcal{N} = 2$ theories can also be made manifest by making use of the $\mathcal{N} = 2$ harmonic superspace approach [13–16]. The gauge invariance is manifest in the framework of background field method, which can also be formulated in superspace.

In this paper we consider $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ supersymmetric Yang–Mills (SYM) theories, which, to certain extent, are similar to $4D$, $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM theories, respectively. From the $\mathcal{N} = (1, 0)$ supersymmetry standpoint, such theories describe the interacting gauge multiplet and hypermultiplets. Both these theories can be formulated in $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace [17–22], so that $\mathcal{N} = (1, 0)$ supersymmetry remains a manifest off-shell symmetry at all steps of quantum calculations. Moreover, the gauge symmetry can be made manifest by using the background field method which has been formulated in harmonic superspace in [23–25]. Thus, the harmonic superspace approach augmented with the background field method allows one to better figure out the restrictions imposed by gauge symmetry and extended supersymmetry on the structure of the ultraviolet divergences. However, it should be noted that, in general, $\mathcal{N} = (1, 0)$ theories are plagued by anomalies [26–28] and it seems impossible to construct a regularization which would simultaneously preserve both supersymmetry and gauge symmetry. This is an essential difference from the $4D$ case, where an invariant regularization for $\mathcal{N} = 2$ supersymmetric gauge theories can be constructed [29,30] as a proper generalization of the higher-derivative regularization worked out in [31,32].

Our basic aim in this paper is to study in detail off-shell structure of the one-loop divergences of $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ SYM theories, in both the gauge multiplet and the hypermultiplet sectors.

Earlier in Refs. [1,2] we have studied the one-loop divergences using the operator proper-time method in $\mathcal{N} = (1, 0)$ harmonic superspace (for the case of non-supersymmetric theories this method was initiated in [33,34]). It has been demonstrated that the general $\mathcal{N} = (1, 0)$ theory with hypermultiplets in an arbitrary representation R of the gauge group G is divergent in the one-loop approximation. However, in the special case of $\mathcal{N} = (1, 1)$ SYM theory, which corresponds to the hypermultiplet in the adjoint representation, the divergences cancel each other and the theory proves to be one-loop finite off shell. It gave us a ground to expect a better ultraviolet behavior of this theory in higher loops as well. It is worth pointing out that the $4D$ analog of $\mathcal{N} = (1, 1)$ theory is $\mathcal{N} = 4$ SYM theory, which is finite to all loops [35–38].

In this paper we develop in detail the harmonic supergraph approach to the study of the one-loop divergences in $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ SYM theories. Such an approach for calculating the structure of divergences is more familiar, as compared to the operator proper-time method, and it provides an appropriate basis for studying the higher-loop divergences. Besides,

we will clarify and justify some subtle aspects of the calculations which have been performed in our previous papers [1,2].

The proper-time technique is very efficient for one-loop calculations. However, for calculating the higher-loop contributions to effective action this technique turns out not so convenient. Usually, for calculation of the divergent diagrams, a simpler technique is used. Just such a technique is developed in this paper, with a possibility of its further applications for higher-loop calculations. Note that in the $\mathcal{N} = (1, 0)$ harmonic superspace approach the number of divergent one-loop supergraphs is infinite, because the gauge superfield is dimensionless. Surely, it is difficult to calculate exactly a sum of infinite number of divergent supergraphs. However, it is possible to calculate divergent diagrams with small numbers of external gauge lines and then to restore the exact result by gauge symmetry arguments. In this paper we demonstrate how this method can be applied for calculating the divergent part of the one-loop effective action of $\mathcal{N} = (1, 0)$ SYM theory with the hypermultiplet in an arbitrary representation of the gauge group.

The paper is organized as follows. In Sect. 2 we formulate $\mathcal{N} = (1, 0)$ SYM theory with hypermultiplets in harmonic superspace and introduce the notation. Sect. 3 is devoted to the quantization of the theory. In particular, we construct the background field method and describe the gauge fixing procedure. Feynman rules for the theory under consideration are presented in Sect. 4. Using these rules, in Sect. 5 we calculate the divergent supergraphs with the minimal numbers of external gauge legs and then restore the full result for the divergent part of the one-loop effective action by the gauge symmetry reasonings. The results obtained are listed and discussed in Sect. 6. Technical details of the harmonic supergraph calculations are collected in Appendices A and B.

2. $\mathcal{N} = (1, 0)$ supersymmetric gauge theories in $6D$ harmonic superspace

The harmonic superspace approach [16] is convenient for describing extended supersymmetric theories, mainly because all symmetries of the theory in this approach are manifest. In our notation the harmonic variables are denoted by $u^{\pm i}$, where $u_i^- = (u^{+i})^*$. These variables are constrained by the condition $u^{+i} u_i^- = 1$. The anticommuting left-handed spinor coordinates are denoted by θ_i^a and the usual coordinates are denoted by x^M , where $M = 0, \dots, 5$. The coordinates of the ordinary $\mathcal{N} = 2$ superspace are $z \equiv (x^M, \theta_i^a)$, and $\zeta \equiv (x_A^M, \theta^{+a})$ are analytic coordinates defined as

$$x_A^M \equiv x^M + \frac{i}{2} \theta^- \gamma^M \theta^+; \quad \theta^{\pm a} \equiv u_i^{\pm} \theta^{ai}, \quad (2.1)$$

where γ^M are six-dimensional γ -matrices. This implies that the corresponding integration measures can be written as

$$\int d^{14}z = \int d^6x d^8\theta; \quad \int d\zeta^{(-4)} \equiv \int d^6x d^4\theta^+. \quad (2.2)$$

Note that

$$\int d^8\theta = \int d^4\theta^+ (D^+)^4, \quad (2.3)$$

where we have introduced the notation

$$(D^+)^4 = -\frac{1}{24} \varepsilon^{abcd} D_a^+ D_b^+ D_c^+ D_d^+ \quad (2.4)$$

with $D_a^+ = u_i^+ D_a^i$ (similarly, $D_a^- \equiv u_i^- D_a^i$).

In harmonic superspace the action of the $6D$, $\mathcal{N} = (1, 0)$ SYM theory has the form [19]

$$S_{\text{SYM}} = \frac{1}{f_0^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)}, \quad (2.5)$$

where f_0 is the bare coupling constant, which in $6D$ has the dimension m^{-1} . The gauge superfield $V^{++}(z, u)$ satisfies the analyticity condition

$$D_a^+ V^{++} = 0 \quad (2.6)$$

and is real with respect to the special “tilde” conjugation, $\widetilde{V^{++}} = V^{++}$. It can be presented as $V^{++}(z, u) = V^{++A} t^A$, where t^A are the generators of the fundamental representation of the gauge group G . In our notation they satisfy the relations

$$\text{tr}(t^A t^B) = \frac{1}{2} \delta^{AB}; \quad [t^A, t^B] = i f^{ABC} t^C, \quad (2.7)$$

where f^{ABC} are the gauge group structure constants.

The expression for the SYM action is essentially simplified in the abelian case. Namely, only terms quadratic in the gauge superfield V^{++} survive:

$$S_{U(1)} = \frac{1}{4f_0^2} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z, u_1) V^{++}(z, u_2). \quad (2.8)$$

In this paper we will consider $6D$, $\mathcal{N} = (1, 0)$ SYM theory with massless hypermultiplets residing in a certain representation R of gauge group. In the harmonic superspace formalism, the total action of such a system reads

$$S = \frac{1}{f_0^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+, \quad (2.9)$$

where the analytic superfield q^+ describes the hypermultiplet. The covariant harmonic derivative in Eq. (2.9) is defined as

$$\nabla^{++} = D^{++} + i V^{++} = D^{++} + i V^{++A} T^A. \quad (2.10)$$

The “flat” harmonic derivatives $D^{\pm\pm}$, D^0 are defined by¹

$$D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}}; \quad D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}}; \quad D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}} \quad (2.11)$$

and T^A in (2.10) are the generators of the gauge group in the representation R , such that $[T^A, T^B] = i f^{ABC} T^C$. We will consider only simple gauge groups, so that

$$\text{tr}(T^A T^B) = T(R) \delta^{AB}; \quad \text{tr}(T_{\text{Adj}}^A T_{\text{Adj}}^B) = f^{ACD} f^{BCD} = C_2 \delta^{AB}; \quad (T^A T^A)_i{}^j = C(R)_i{}^j. \quad (2.12)$$

¹ One can easily see that they form an $SU(2)$ algebra.

The representation R can in general be reducible. For an irreducible representation R , $C(R)_i{}^j$ is proportional to δ_i^j . If R is the adjoint representation, the action (2.9) describes the $\mathcal{N} = (1, 1)$ SYM theory. In this case, the action (2.9) is invariant under an extra hidden $\mathcal{N} = (0, 1)$ supersymmetry which mixes the gauge superfield and the hypermultiplet.

The $\mathcal{N} = (1, 0)$ SYM action (2.9) is invariant under the gauge transformations

$$V^{++} \rightarrow e^{i\lambda} V^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}; \quad q^+ \rightarrow e^{i\lambda} q^+, \quad (2.13)$$

where $\lambda = \lambda^A t^A$, when checking the invariance of the pure gauge-field part of the action, and $\lambda = \lambda^A T^A$ while dealing with the hypermultiplet part. The parameters λ^A are the analytic superfields which are real with respect to the tilde-conjugation, $\tilde{\lambda}^A = \lambda^A$.

One more necessary ingredient of the superfield formalism is a non-analytic superfield V^{--} introduced as a solution of the “harmonic flatness condition”

$$D^{++} V^{--} - D^{--} V^{++} + i[V^{++}, V^{--}] = 0. \quad (2.14)$$

This superfield can be solved for from (2.14) in terms of V^{++} as

$$V^{--}(z, u) \equiv \sum_{n=1}^{\infty} (-i)^{n+1} \int du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \dots (u_n^+ u^+)}. \quad (2.15)$$

Under gauge transformations (2.13) V^{--} is transformed as

$$V^{--} \rightarrow e^{i\lambda} V^{--} e^{-i\lambda} - i e^{i\lambda} D^{--} e^{-i\lambda}. \quad (2.16)$$

From the geometric point of view, this superfield is the connection covariantizing the harmonic derivative D^{--} :

$$D^{--} \Rightarrow \nabla^{--} \equiv D^{--} + i V^{--}. \quad (2.17)$$

It can be used to construct the important analytic superfield strength

$$F^{++} \equiv (D^+)^4 V^{--}, \quad (2.18)$$

which transforms homogeneously, as $F^{++} \rightarrow e^{i\lambda} F^{++} e^{-i\lambda}$.

In the abelian case, the action of the gauge theory-hypermultiplet system can be written as

$$S = \frac{1}{4f_0^2} \int d^4z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z, u_1) V^{++}(z, u_2) - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+, \quad (2.19)$$

where $\nabla^{++} = D^{++} + i V^{++}$, and it is invariant under the gauge transformations

$$V^{++} \rightarrow V^{++} - D^{++} \lambda; \quad V^{--} \rightarrow V^{--} - D^{--} \lambda; \quad q^+ \rightarrow e^{i\lambda} q^+. \quad (2.20)$$

In this case Eq. (2.14) becomes linear,

$$D^{++} V^{--} = D^{--} V^{++}, \quad (2.21)$$

and so provides the linear solution for V^{--} ,

$$V^{--}(z, u) = \int du_1 \frac{V^{++}(z, u_1)}{(u^+ u_1^+)^2}. \quad (2.22)$$

The analytic superfield F^{++} in the abelian case is gauge invariant.

3. Quantization and background field method in $\mathcal{N} = (1, 0)$ harmonic superspace

It is convenient to quantize $6D$, $\mathcal{N} = (1, 0)$ theories directly in $\mathcal{N} = (1, 0)$ harmonic superspace, thus ensuring the manifestly supersymmetric form of the quantum corrections. It is also convenient to make use of the background field method, which gives the manifestly gauge invariant effective action.

The background-quantum splitting is introduced by the substitution

$$V^{++} = \mathbf{V}^{++} + v^{++}, \quad (3.1)$$

where \mathbf{V}^{++} denotes the background gauge superfield, while v^{++} is the quantum gauge superfield. In supergraphs, external lines corresponding to the background gauge superfield will be denoted by the bold wavy lines, while the external lines corresponding to the quantum gauge superfield – by the usual wavy lines. Note that we do not make the background-quantum splitting for the hypermultiplet. This is admissible because such a splitting is linear and we will choose the gauge-fixing term to be independent of the hypermultiplet. This implies that the effective action will depend only on the sum of the quantum and background hypermultiplet superfields, so that there is no actual need to separately introduce the background hypermultiplet superfield.

After the background-quantum splitting, the gauge invariance (2.13) amounts to the background gauge invariance

$$V^{++} \rightarrow e^{i\lambda} V^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}; \quad v^{++} \rightarrow e^{i\lambda} v^{++} e^{-i\lambda}; \quad q^+ \rightarrow e^{i\lambda} q^+, \quad (3.2)$$

and the quantum gauge invariance

$$\mathbf{V}^{++} \rightarrow e^{i\lambda} \mathbf{V}^{++} e^{-i\lambda}; \quad v^{++} \rightarrow e^{i\lambda} v^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}; \quad q^+ \rightarrow e^{i\lambda} q^+. \quad (3.3)$$

To obtain the gauge invariant effective action, one should fix a gauge only with respect to the quantum superfields, without breaking the background gauge invariance (3.2). For example, it is possible to add the following gauge-fixing term:

$$\begin{aligned} S_{\text{gf}} = & -\frac{1}{2f_0^2 \xi_0} \text{tr} \int d^{14}z du_1 du_2 \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} D_1^{++} \left[e^{-ib(z, u_1)} v^{++}(z, u_1) e^{ib(z, u_1)} \right] \\ & \times D_2^{++} \left[e^{-ib(z, u_2)} v^{++}(z, u_2) e^{ib(z, u_2)} \right], \end{aligned} \quad (3.4)$$

where $b(z, u)$ is the background bridge superfield. The bridge is related to the background superfields \mathbf{V}^{++} and \mathbf{V}^{--} by the relations

$$\mathbf{V}^{++} = -i e^{ib} D^{++} e^{-ib}; \quad \mathbf{V}^{--} = -i e^{ib} D^{--} e^{-ib}. \quad (3.5)$$

Note that the hypermultiplet does not enter Eq. (3.4) and the theory is invariant under the background gauge transformations even without the background-quantum splitting for the q^+ superfields.

The expression (3.4) is an analog of the usual ξ -gauge. The terms quadratic in the quantum gauge superfield in the total action become

$$\begin{aligned} S^{(2)} + S_{\text{gf}}^{(2)} = & \frac{1}{2f_0^2} \left(1 - \frac{1}{\xi_0} \right) \text{tr} \int d^{14}z du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2} v^{++}(z, u_1) v^{++}(z, u_2) \\ & + \frac{1}{2f_0^2 \xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++}(z, u) \square v^{++}(z, u), \end{aligned} \quad (3.6)$$

where $\square \equiv \partial^2$. We observe that in the Feynman gauge $\xi_0 = 1$ this expression is essentially simplified, as in the case of the usual Yang–Mills theory. Note that, when deriving (3.6), we integrated by parts and used the relation

$$D_1^{++} \frac{1}{(u_1^+ u_2^+)^3} = \frac{1}{2} (D_1^{--})^2 \delta^{(3,-3)}(u_1, u_2). \quad (3.7)$$

Then we again integrated by parts with respect to $(D_1^{--})^2$, taking into account Eq. (2.3) and the identity

$$\frac{1}{2} (D^+)^4 (D^{--})^2 v^{++} = \square v^{++}, \quad (3.8)$$

which follows from the analyticity of the quantum gauge superfield v^{++} .

The gauge-fixing term (3.4) is invariant under the transformations (3.2) which act on the bridge as

$$e^{ib} \rightarrow e^{i\lambda} e^{ib} e^{i\tau}, \quad (3.9)$$

where $\tau = \tau(x, \theta)$ is a gauge parameter independent of the harmonic variables.

Note that in the abelian case the bridge superfield is not present in the gauge-fixing action. Also, in the abelian case it is not necessary to introduce ghosts. The latter are required by the quantization procedure only for non-abelian gauge theories. In the considered theory one is led to insert, into the generating functional, some determinants corresponding to the Faddeev–Popov and Nielsen–Kallosh ghosts.

The Faddeev–Popov ghosts b and c are anticommuting analytic superfields in the adjoint representation of the gauge group. The action for them has the form

$$S_{\text{FP}} = \text{tr} \int d\zeta^{(-4)} du b \nabla^{++} \left(\nabla^{++} c + i[v^{++}, c] \right), \quad (3.10)$$

where $\nabla^{++} c = D^{++} c + i[V^{++}, c]$ is the background covariant derivative.

Also it is necessary to insert into the generating functional the determinants corresponding to the Nielsen–Kallosh ghosts,

$$\Delta_{\text{NK}} \equiv \text{Det}^{1/2} \widehat{\square} \int D\varphi \exp(i S_{\text{NK}}), \quad (3.11)$$

where $\widehat{\square} \equiv \frac{1}{2} (D^+)^4 (\nabla^{--})^2$ and φ is a commuting analytic Nielsen–Kallosh ghost superfield in the adjoint representation of the gauge group with

$$S_{\text{NK}} = -\frac{1}{2} \text{tr} \int d\zeta^{(-4)} du (\nabla^{++} \varphi)^2. \quad (3.12)$$

Introducing anticommuting analytic superfields $\xi^{(+4)}$ and σ in the adjoint representation of the gauge group, one can write $\text{Det} \widehat{\square}$ in the form of the functional integral,

$$\text{Det} \widehat{\square} = \int D\xi^{(+4)} D\sigma \exp \left(i \text{tr} \int d\zeta^{(-4)} du \xi^{(+4)} \widehat{\square} \sigma \right). \quad (3.13)$$

Thus, the generating functional of the considered theory can be written as

$$Z = \int Dv^{++} D\tilde{q}^+ Dq^+ Db Dc D\varphi \text{Det}^{1/2} \widehat{\square} \exp \left[i(S + S_{\text{gf}} + S_{\text{FP}} + S_{\text{NK}} + S_{\text{sources}}) \right], \quad (3.14)$$



Fig. 1. Propagators of various superfields: (1), (2), (3), and (4) stand for the gauge, hypermultiplet, Faddeev–Popov and Nielsen–Kallosh ghost propagators, respectively.

where S_{sources} denotes the relevant source terms. For example, the sources for the quantum gauge and hypermultiplet superfields can be introduced as

$$\int d\zeta^{(-4)} du \left[v^{++A} J^{(+2)A} + j^{(+3)i} (q^+)_i + \tilde{j}_i^{(+3)} (\tilde{q}^+)^i \right]. \quad (3.15)$$

Likewise, it is possible to add sources for other superfields involved.

4. Propagators, vertices and supergraphs

The Feynman rules for $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theories in harmonic superspace are very similar to those in the case of $4D$, $\mathcal{N} = 2$ supersymmetric theories which have been considered in detail in [14,15].

In order to find the propagator of the quantum gauge superfield, we consider the linearized equation of motion for this superfield (setting the background gauge field equal to zero) in the presence of the source term (3.15):

$$\begin{aligned} & \frac{1}{2\xi_0 f_0^2} \square v^{++A}(z, u_1) + \frac{1}{2f_0^2} \left(1 - \frac{1}{\xi_0}\right) \\ & \times \int du_2 \frac{1}{(u_1^+ u_2^+)^2} (D_1^+)^4 v^{++A}(z, u_2) + J^{(+2)A}(z, u_1) = 0. \end{aligned} \quad (4.1)$$

Its solution can be written as

$$\begin{aligned} v^{++A}(z, u_1) = & -\frac{2\xi_0 f_0^2}{\square} J^{(+2)A}(z, u_1) + \frac{2f_0^2(\xi_0 - 1)}{\square^2} \\ & \times \int du_2 \frac{1}{(u_1^+ u_2^+)^2} (D_1^+)^4 J^{(+2)A}(z, u_2). \end{aligned} \quad (4.2)$$

This implies that the propagator of the gauge superfield in the ξ -gauge reads

$$\begin{aligned} & (G_V^{(2,2)})^{AB}(z_1, u_1; z_2, u_2) \\ & = -2f_0^2 \left(\frac{\xi_0}{\square} (D_1^+)^4 \delta^{(2,-2)}(u_2, u_1) \right. \\ & \quad \left. - \frac{\xi_0 - 1}{\square^2} (D_1^+)^4 (D_2^+)^4 \frac{1}{(u_1^+ u_2^+)^2} \right) \delta^6(x_1 - x_2) \delta^8(\theta_1 - \theta_2) \delta^{AB}. \end{aligned} \quad (4.3)$$

Below we will use the gauge $\xi_0 = 1$, in which the propagator has the simplest form, with the second term vanishing. The propagator of the gauge superfield will be graphically represented by the wavy line ending at the points 1 and 2, see Fig. 1, where it is denoted by (1).

The propagator of the hypermultiplet superfields can be defined similarly, and it is given by the expression

$$(G_q^{(1,1)})_i{}^j(z_1, u_1; z_2, u_2) = (D_1^+)^4 (D_2^+)^4 \frac{1}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \delta_i{}^j, \quad (4.4)$$



Fig. 2. Vertices coming from the hypermultiplet part of the action.

where

$$\delta^{14}(z_1 - z_2) \equiv \delta^6(x_1 - x_2) \delta^8(\theta_1 - \theta_2). \quad (4.5)$$

This propagator will be represented by the line ending at the points 1 and 2. It is denoted by the symbol (2) in Fig. 1. The external hypermultiplets will be also denoted by such lines.

The propagators of the Faddeev–Popov and Nielsen–Kallosh ghosts have the form

$$\frac{(D_1^+)^4 (D_2^+)^4}{2\Box} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \delta^{AB}. \quad (4.6)$$

They are given, respectively, by the dashed and dotted lines connecting the points 1 and 2 and denoted in Fig. 1 by the symbols (3) and (4).

Also, we will need the propagator of the superfields $\xi^{(+4)}$ and σ introduced in Eq. (3.13). It is easy to see that it is given by the expression

$$-\frac{(D_1^+)^4}{2\Box} \delta^{14}(z_1 - z_2) \delta^{(0,0)}(u_1, u_2) \delta^{AB}. \quad (4.7)$$

The only coupling of the gauge superfield with the hypermultiplet ones comes from the hypermultiplet action:

$$\begin{aligned} S_I &= -i \int d\zeta^{(-4)} du (\tilde{q}^+)^i (V^{++})_i{}^j (q^+)_j \\ &= -i \int d\zeta^{(-4)} du (\tilde{q}^+)^i (V^{++} + v^{++})_i{}^j (q^+)_j, \end{aligned} \quad (4.8)$$

where $(V^{++})_i{}^j = f_0 V^{++A} (T^A)_i{}^j$. This implies that the supergraphs can contain vertices with two hypermultiplet legs and one leg of the background or quantum gauge superfield. These vertices are presented in Fig. 2. In the abelian case, these are the only interaction vertices at all.

In the non-abelian case there are infinitely many interaction vertices, because the action (2.5) contains terms involving all powers ($n \geq 2$) of the gauge superfield V^{++} . Clearly, each line in such vertices represents the quantum superfield v^{++} or the background gauge superfield V^{++} . Expressions for the vertices with purely quantum gauge legs can be read off from Eq. (2.5). The vertices with legs of the background gauge superfield can also come from the gauge fixing action (3.4). On the external background superfield legs in such vertices there always appears the bridge superfield.

As the action (3.10) contains two background supersymmetric derivatives ∇^{++} , there are vertices with two Faddeev–Popov ghost legs and one or two legs of the gauge superfield. Note that the maximal number of the quantum gauge superfield legs equals one, because the Faddeev–Popov action contains only first degree of v^{++} .

The Nielsen–Kallosh ghosts interact only with the background gauge superfield. Like in the case of Faddeev–Popov ghosts, only three- and four-point vertices are present. The vertices

containing legs of the superfields $\xi^{(+4)}$ and σ can involve an arbitrary number of the external background superfield V^{++} legs, because the operator $\widehat{\square}$ contains the superfield V^{--} given by an infinite series (2.15).

5. Structure of one-loop divergences

5.1. General analysis

According to the general analysis performed in [39] the on-shell logarithmic divergences in the one-loop approximation can be written as

$$\Gamma_{\infty, \text{ln}}^{(1)} = \int d\xi^{(-4)} du \left[c_1 (F^{++A})^2 + i c_2 F^{++A} (\widetilde{q}^+)^i (T^A)_i{}^j (q^+)_j + c_3 \left((\widetilde{q}^+)^i (q^+)_i \right)^2 \right], \quad (5.1)$$

where c_1 , c_2 , and c_3 are numerical real coefficients. They have been found in [1] by using the proper time method.

The degree of divergence in $\mathcal{N} = (1, 0)$ gauge theory can be deduced as follows. The effective action is dimensionless. On the other hand, any contribution to this dimensionless effective action can be presented as an integral over the total superspace. The harmonic variables are dimensionless, while $[d^6x] = m^{-6}$ and $[d^8\theta] = m^4$. The gauge superfields on the external legs are dimensionless, $[V^{++}] = m^0$, while the external hypermultiplet legs contribute $[q^+] = m^2$. In our notation, each gauge propagator gives f_0^2 , where $[f_0] = m^{-1}$, and each purely gauge vertex gives f_0^{-2} . If N_D spinor derivatives act to the external lines, they also contribute $N_D/2$ to the total dimension. Therefore, taking into account that the effective action is dimensionless, we obtain that the dimension of the momentum integral ω in a supergraph with P_V gauge propagators, V_V purely gauge vertices, and N_q external hypermultiplet legs should be equal to

$$\omega = 6 - 4 + 2P_V - 2V_V - 2N_q - \frac{1}{2}N_D. \quad (5.2)$$

The quantity ω is the superficial degree of divergence. For hypermultiplets the number of external legs can be written as $N_q = 2(-P_q + V_q)$, where P_q and V_q are numbers of hypermultiplet propagators and the hypermultiplet-containing vertices, respectively. For the closed loops of the Faddeev–Popov ghosts the similar equality is $P_{\text{FP}} = V_{\text{FP}}$, where P_{FP} and V_{FP} are numbers of the Faddeev–Popov ghost propagators and vertices, respectively. Using these relations we obtain

$$\begin{aligned} \omega &= 2 + 2(P_V + P_q + P_{\text{FP}}) - 2(V_q + V_V + V_{\text{FP}}) - N_q - \frac{1}{2}N_D \\ &= 2 - 2V + 2P - N_q - \frac{1}{2}N_D, \end{aligned} \quad (5.3)$$

where

$$P = P_V + P_q + P_{\text{FP}} \quad \text{and} \quad V = V_V + V_q + V_{\text{FP}} \quad (5.4)$$

are total numbers of propagators and vertices in the considered diagram, respectively. Since the number of loops is $L = 1 - V + P$, the result for the degree of divergence can be also rewritten as

$$\omega = 2L - N_q - \frac{1}{2}N_D. \quad (5.5)$$

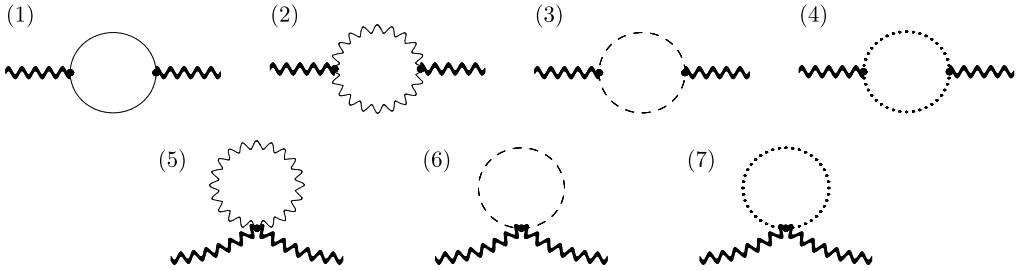


Fig. 3. One-loop contribution to the two-point Green function of the background superfield in the abelian case.

The supergraph is convergent if $\omega < 0$, otherwise it is divergent. The relation (5.5) allows one to list all possible types of divergent supergraphs and compare the corresponding counterterms with the expression (5.1). In the one-loop approximation the divergences correspond to $\omega = 2$ and 0. Further analysis depends on the choice of regularization.

Let $N_q = 0$, $N_D = 0$, so that $\omega = 2$, and use the dimensional-reduction type of regularization. Then the only admissible counterterm in the gauge multiplet sector is given by the first term in (5.1), with dimensionless divergent coefficient c_1 . Being dimensionless, this coefficient must be proportional to $1/\varepsilon$, where $\varepsilon = d - 6$ is a regularization parameter. Let still $N_q = 0$, $N_D = 0$, $\omega = 2$, but use now the cut-off regularization. In this case we have the cut-off momentum Λ and, hence, there are two admissible counterterms in the gauge multiplet sector. Like in the previous case, one of them is given by the first term in (5.1) with dimensionless divergent coefficient c_1 , which should now be proportional to $\ln \Lambda$. The second one is proportional to Λ^2 multiplied by the classical action of the pure $6D$, $\mathcal{N} = (1, 0)$ SYM theory.

Let $N_q = 2$, $N_D = 0$ (so that $\omega = 0$) and use the dimensional-reduction regularization. The admissible counterterm is given by the second term in (5.1) with the dimensionless divergent coefficient c_2 , which must be proportional to $1/\varepsilon$. In the case of the cut-off regularization we again obtain the counterterm corresponding to the second term in (5.1) with dimensionless divergent coefficient c_2 proportional to $\ln \Lambda$. Also, from Eq. (5.5) we derive that $c_3 = 0$, because the relevant structure corresponds to the convergent graphs with $N_q = 4$, $N_D = 0$ and, hence, $\omega = -2$.

In this paper we carry out the one-loop calculations both in the dimensional reduction scheme and in the cut-off regularization. We confirm the results of [1] by an independent calculation of superdiagrams in the dimensional regularization. In addition to the results of [1], we find all the one-loop counterterms in the cut-off regularization scheme and show that there is actually a counterterm proportional to S_{SYM} . In the $\mathcal{N} = (1, 1)$ SYM theory, all the one-loop divergences are canceled off shell.

5.2. Two-point Green function of the gauge superfield

We start the computation of the one-loop divergences from the two-point Green function of the background gauge superfield. In the considered approximation it is determined by the diagrams presented in Fig. 3. The bold external legs in these diagrams correspond to the background gauge superfield V^{++} . Note that in the abelian case the only non-trivial contribution comes from the diagram (1), while contributions of all other diagrams vanish. That is why we will start our analysis by calculating the coefficient c_1 for the abelian theory (2.19).

Using the standard rules of writing down the contributions of the harmonic supergraphs [16], we can present the contribution of the diagram (1) to the effective action of the abelian theory in the form

$$\begin{aligned} & \frac{i}{2} \int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 V^{++}(z_1, u_1) V^{++}(z_2, u_2) \\ & \times \frac{1}{(u_2^+ u_1^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2). \end{aligned} \quad (5.6)$$

One of the two operator factors $(D_1^+)^4 (D_2^+)^4$ in (5.6) can be used to convert both integrations over $d\zeta^{(-4)}$ into those over $d^{14}z$,

$$\begin{aligned} & -\frac{i}{2} \int d^{14}z_1 du_1 d^{14}z_2 du_2 \frac{V^{++}(z_1, u_1) V^{++}(z_2, u_2)}{(u_1^+ u_2^+)^6} \frac{1}{\square} \delta^{14}(z_1 - z_2) \\ & \times \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2). \end{aligned} \quad (5.7)$$

After this, we should take into account that

$$(D_1^+)^4 (D_2^+)^4 \delta^8(\theta_1 - \theta_2) \delta^8(\theta_1 - \theta_2) = (u_1^+ u_2^+)^4 \delta^8(\theta_1 - \theta_2) \quad (5.8)$$

and perform one of the θ -integrations with the help of the remaining Grassmann δ -function $\delta^8(\theta_1 - \theta_2)$. This gives

$$\begin{aligned} & -\frac{i}{2} \int d^6x_1 d^6x_2 d^8\theta du_1 du_2 V^{++}(x_1, \theta, u_1) V^{++}(x_2, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \frac{1}{\square} \delta^6(x_1 - x_2) \frac{1}{\square} \delta^6(x_1 - x_2). \end{aligned} \quad (5.9)$$

Next, we rewrite this expression in the momentum space:

$$\begin{aligned} & -\frac{i}{2} \int \frac{d^6p}{(2\pi)^6} \int d^8\theta du_1 du_2 V^{++}(p, \theta, u_1) V^{++}(-p, \theta, u_2) \\ & \times \frac{1}{(u_1^+ u_2^+)^2} \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2(k+p)^2}. \end{aligned} \quad (5.10)$$

Combining this expression with the tree-level result, the part of the effective action corresponding to the two-point function of the background gauge superfield can be written as

$$\begin{aligned} \Gamma_{V^{++}}^{(2)} &= \int \frac{d^6p}{(2\pi)^6} \int d^8\theta du_1 du_2 V^{++}(p, \theta, u_1) V^{++}(-p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \left[\frac{1}{4f_0^2} - \frac{i}{2} \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \right]. \end{aligned} \quad (5.11)$$

From this expression we observe that the considered Green function is quadratically divergent. To calculate the momentum integral, we resort to the standard trick of the Wick rotation to the Euclidean signature. If Λ is an UV cutoff, then, taking into account that the volume of the sphere S^5 is

$$\Omega_5 = \frac{2\pi^{(5+1)/2}}{\Gamma((5+1)/2)} = \frac{2\pi^3}{\Gamma(3)} = \pi^3, \quad (5.12)$$

the leading divergence of the considered Green function can be written in the form

$$\Gamma_{\infty, V^{++}}^{(2)} = \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V^{++}(p, \theta, u_1) V^{++}(-p, \theta, -u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{\Lambda^2}{4(4\pi)^3}. \quad (5.13)$$

This expression is evidently gauge invariant, so there is no need to add any other quadratically divergent term with higher degrees of V^{++} to the one-loop effective action. It is easy to see that in the coordinate representation it coincides, up to a numerical coefficient, with the classical action of the free gauge superfield

$$\int d^{14} z du_1 du_2 V^{++}(z, u_1) V^{++}(z, u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{\Lambda^2}{4(4\pi)^3} = \frac{f_0^2 \Lambda^2}{(4\pi)^3} S_{U(1)}. \quad (5.14)$$

This form of the quadratic divergence is in agreement with the results of Ref. [7], where the relation between leading divergences in various dimensions was analyzed. The $4D$ theory is renormalizable, so that the leading (logarithmic) divergences in the gauge-field sector are proportional to $S_{U(1)}$. They are related with the leading (quadratic) divergences in the $6D$ theory, which, thereby, should be also proportional to $S_{U(1)}$.

It is worth mentioning that it is impossible to calculate quadratic divergences using the regularization scheme by the dimensional reduction, because the dimensional reduction technique does not see these divergences. This is the reason why for computing the quadratic divergences one is led to use another regularization which does not break supersymmetries of the theory. Actually, the only regularization of this type is the Slavnov higher-derivative regularization. It was first proposed in [31,32] for non-supersymmetric theories and was generalized to the supersymmetric case in refs. [41,42]. For $4D$ $\mathcal{N} = 2$ theories it has been also worked out in the harmonic superspace approach [30]. However, to generalize this result to the $6D$ case, it is necessary to use regulators with higher degrees of covariant derivatives as compared with the $4D$ case. Now, this work is in progress. Nevertheless, any version of the higher-derivative regularization will evidently produce Eq. (5.13) for the quadratic divergences.

If the theory is regularized through the dimensional reduction, it is possible to calculate only the logarithmic divergences. In this case, using the standard Euclidean techniques, we obtain

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2(k+p)^2} = -\frac{p^2}{3\varepsilon(4\pi)^3} + \text{finite terms}, \quad (5.15)$$

where $\varepsilon \equiv 6 - D$. Taking into account that $(p^2)_E = -(p^2)_M$, within the regularization by dimensional reduction the divergent part of the effective action can be written as

$$\int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V^{++}(p, \theta, u_1) V^{++}(-p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{p^2}{6\varepsilon(4\pi)^3}. \quad (5.16)$$

It is known [39] that the only gauge invariant expression of the considered dimension (in the abelian case) is given by

$$\begin{aligned} \int d\zeta^{(-4)} du (F^{++})^2 &= \int d^{14} z du V^{--} \square V^{++} \\ &= \int d^{14} z du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(z, u_1) \square V^{++}(z, u_2), \end{aligned} \quad (5.17)$$

where $F^{++} \equiv (D^+)^4 V^{--}$ is a function of the background superfield V^{++} . Comparing this expression with Eq. (5.16) we can present the logarithmical divergences in the gauge-field sector in the form

$$\Gamma_{\infty, \ln}^{(1)} = -\frac{1}{6\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du (F^{++})^2 + \text{terms with hypermultiplets} \quad (5.18)$$

(the one-loop divergent contributions containing the hypermultiplet will be calculated below). Comparing Eq. (5.18) with Eq. (5.1),² we conclude that the coefficient c_1 in Eq. (5.1) is

$$c_1 = -\frac{1}{6\varepsilon(4\pi)^3}. \quad (5.19)$$

This result agrees with the one obtained in [1] by the proper time technique. The coincidence of the results derived by two different methods confirms the correctness of the calculations.

The calculation of the diagram (1) in the non-abelian case goes along similar lines. The only novelty is the necessity to take into account the gauge group indices and the factor

$$\text{tr}(T^A T^B) = T(R) \delta^{AB}, \quad (5.20)$$

which comes from the generators appearing in the vertices. So in the non-abelian case the diagram (1) is represented by the expression

$$\begin{aligned} & -\frac{i}{2} T(R) \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V^{++A}(p, \theta, u_1) V^{++A}(-p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2}. \end{aligned} \quad (5.21)$$

The contributions of the other diagrams depicted in Fig. 3 are calculated in Appendix A.2. There we demonstrate that the sum of all diagrams containing the loop of the quantum gauge superfield (i.e. (2) and (5)) vanishes and the net result comes solely from the ghost contributions. Adding the latter to Eq. (5.21), we obtain the total two-point function of the gauge superfield in the form

$$\begin{aligned} & \frac{i}{2} [C_2 - T(R)] \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V^{++A}(p, \theta, u_1) V^{++A}(-p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2}. \end{aligned} \quad (5.22)$$

The divergent part of this expression is calculated in the precisely same way as in the abelian case. In particular, the leading quadratic divergence can be written as

$$- [C_2 - T(R)] \int d^{14} z du_1 du_2 V^{++A}(z, u_1) V^{++A}(z, u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{\Lambda^2}{4(4\pi)^3}. \quad (5.23)$$

In the case of using the regularization by dimensional reduction the (logarithmical) divergence is parametrized by the expression

$$[C_2 - T(R)] \int d^{14} z du_1 du_2 V^{++A}(z, u_1) \square V^{++A}(z, u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{1}{6\varepsilon(4\pi)^3}. \quad (5.24)$$

² As soon as we make calculations within the background field method, it is necessary to substitute F^{++} by the similar expression \mathbf{F}^{++} constructed from the background gauge superfield.

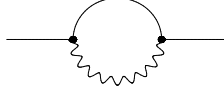


Fig. 4. The diagram contributing to the two-point Green function of the hypermultiplet.

It is worth to point out an essential difference of these results from their abelian counterparts. Actually, in the non-abelian case both V^{--} and F^{++} are non-linear functions of V^{++} . This implies that there should be also divergent contributions proportional to higher degrees of V^{++} . However, they can be easily restored by taking into account the gauge invariance of the action S_{SYM} and the expression $\text{tr}(F^{++})^2$. Comparing quadratic terms in these expressions with (5.23) and (5.24), respectively, we find that the leading quadratic divergence is

$$-\left[C_2 - T(R)\right] \frac{f_0^2 \Lambda^2}{(4\pi)^3} S_{\text{SYM}}[V^{++}], \quad (5.25)$$

while the logarithmic divergence obtained with making use of the dimensional reduction has the form

$$\frac{C_2 - T(R)}{3\varepsilon(4\pi)^3} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2, \quad (5.26)$$

that coincides with the result obtained in [2]. We see that both these expressions vanish in the case of $\mathcal{N} = (1, 1)$ SYM theory, for which $T(R) = C_2$.

Note that vanishing of quadratic divergences is quite expectable for $\mathcal{N} = (1, 1)$ theory. Actually, the quadratic divergences can appear only in the gauge-multiplet sector, while the formula for the degree of divergence (5.5) tells us that in the sector involving hypermultiplets only the logarithmically divergent terms can appear. On the other hand, the hidden $\mathcal{N} = (0, 1)$ symmetry would imply the appearance of quadratic divergences in the hypermultiplet sector, once they would be present in the gauge-multiplet sector. Since no such divergences are possible in the hypermultiplet sector, the quadratic divergences cannot appear in $\mathcal{N} = (1, 1)$ theory at all.

5.3. Two-point Green function of the hypermultiplet

Let us now calculate the two-point Green function of the matter hypermultiplet superfields. In the one-loop approximation it is determined by the diagram depicted in Fig. 4, for which $N_q = 2$. Then, according to Eq. (5.5), it is logarithmically divergent.

Note that the result for this diagram is gauge-dependent due to the presence of the gauge-superfield propagator. However, in this paper we do calculations only in the minimal gauge corresponding to the choice $\xi_0 = 1$. Then the expression constructed according to the Feynman rules has the form

$$\begin{aligned} & -2if_0^2 \int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 \tilde{q}^+(z_1, u_1)^i (T^A T^A)_i{}^j q^+(z_2, u_2)_j \frac{1}{(u_1^+ u_2^+)^3} \\ & \times \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} (D_1^+)^4 \delta^{(2, -2)}(u_2, u_1) \delta^{14}(z_1 - z_2). \end{aligned} \quad (5.27)$$

As in the case considered in the previous Subsection, the product of derivatives $(D_1^+)^4 (D_2^+)^4$ present in (5.27) makes it possible to restore two integrations over $d^{14}z$,

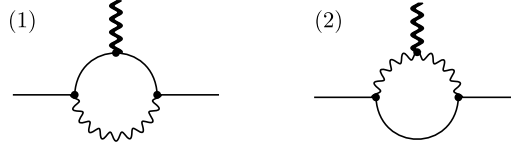


Fig. 5. The diagrams which determine the three-point gauge-hypermultiplet function. In the abelian case only the diagram (1) is non-vanishing.

$$\begin{aligned}
 & -2if_0^2 \int d^{14}z_1 du_1 d^{14}z_2 du_2 \tilde{q}^+(z_1, u_1)^i C(R)_i{}^j q^+(z_2, u_2)_j \frac{1}{(u_1^+ u_2^+)^3} \frac{1}{\square} \delta^{14}(z_1 - z_2) \\
 & \times \frac{1}{\square} (D_1^+)^4 \delta^{(2, -2)}(u_2, u_1) \delta^{14}(z_1 - z_2) = 0.
 \end{aligned} \quad (5.28)$$

The vanishing of this expression follows from the property that the product of Grassmann δ -functions does not vanish only provided there are at least eight spinor covariant derivatives acting on one of them, which is not the case for (5.28). Therefore, in the minimal gauge the hypermultiplets are not renormalized in the one-loop approximation. Obviously, this result is valid in both non-abelian and abelian cases.

5.4. Three-point gauge-hypermultiplet Green function

In order to determine the coefficient c_2 in Eq. (5.1), it suffices to consider the three-point gauge-hypermultiplet Green function, which in the one-loop approximation is represented by the diagrams depicted in Fig. 5. Evidently, in the abelian case only the diagram (1) remains. This is why we will start with studying the one-loop divergence in the abelian case. Again, we will calculate the corresponding contribution to the effective action in the minimal gauge $\xi_0 = 1$. For the abelian theory (2.19) it has the form

$$\begin{aligned}
 & -2f_0^2 \int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 d\zeta_3^{(-4)} du_3 \tilde{q}^+(z_1, u_1) q^+(z_3, u_3) \\
 & \times V^{++}(z_2, u_2) \frac{(D_1^+)^4}{\square} \delta^{(2, -2)}(u_3, u_1) \delta^{14}(z_1 - z_3) \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \\
 & \times \frac{1}{(u_2^+ u_3^+)^3} \frac{(D_2^+)^4 (D_3^+)^4}{\square} \delta^{14}(z_2 - z_3).
 \end{aligned} \quad (5.29)$$

The calculation of this diagram is described in Appendix B.1. In the momentum representation (in the Minkowski space) the result can be presented in the form

$$\begin{aligned}
 & -2f_0^2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} d^8 \theta du_1 du_2 \tilde{q}^+(q + p, \theta, u_1) q^+(-q, \theta, u_1) \\
 & \times V^{++}(-p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (q + k)^2 (q + k + p)^2}.
 \end{aligned} \quad (5.30)$$

To find a divergent part of this expression, we perform the Wick rotation in the integral over the loop momentum k and rewrite it in the Euclidean space as

$$2if_0^2 \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k + q)^2 (k + q + p)^2}. \quad (5.31)$$

Evidently, this expression is logarithmically divergent. So far, it is not well-defined, because we have not yet specified the regularization. Within the dimensional reduction method [40], it is necessary to substitute $D = 6$ by $D = 6 - \varepsilon$ with $\varepsilon \neq 0$. Then, using the standard technique, we find

$$\int \frac{d^D k}{(2\pi)^6} \frac{1}{k^2(k+q)^2(k+p)^2} = \frac{1}{\varepsilon(4\pi)^3} + \text{finite terms.} \quad (5.32)$$

Thus, in the configuration space the divergent part of the considered contribution to the effective action is given by

$$\begin{aligned} & 2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d^{14}z \int du_1 du_2 \tilde{q}^+(z, u_1) V^{++}(z, u_2) q^+(z, u_1) \frac{1}{(u_1^+ u_2^+)^2} \\ &= 2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d^{14}z \int du \tilde{q}^+(z, u) V^{--}(z, u) q^+(z, u). \end{aligned} \quad (5.33)$$

Rewriting

$$\int d^{14}z = \int d\zeta^{(-4)} (D^+)^4, \quad (5.34)$$

and taking into account the analyticity of the superfields \tilde{q}^+ and q^+ , the expression (5.33) can be written in terms of $F^{++} = (D^+)^4 V^{--}$ as

$$2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ (D^+)^4 V^{--} q^+ = 2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ F^{++} q^+. \quad (5.35)$$

Comparing this expression with Eq. (5.1), we find that

$$c_2 = \frac{2f_0^2}{\varepsilon(4\pi)^3}. \quad (5.36)$$

This result coincides with the one obtained in [1] by the proper time technique.

Next, let us consider the non-abelian case. In this case both diagrams in Fig. 5 contribute to the Green function. The diagram (1) is calculated in a close analogy with the abelian case. It can be represented as

$$\begin{aligned} & -2f_0^2 \int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 d\zeta_3^{(-4)} du_3 \tilde{q}^+(z_1, u_1)^i \\ & \times (T^A V^{++}(z_2, u_2) T^A)_i^j q^+(z_3, u_3)_j \frac{(D_1^+)^4}{\square} \delta^{(2,-2)}(u_3, u_1) \delta^{14}(z_1 - z_3) \\ & \times \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_2^+ u_3^+)^3} \frac{(D_2^+)^4 (D_3^+)^4}{\square} \delta^{14}(z_2 - z_3). \end{aligned} \quad (5.37)$$

Repeating the calculation steps described above and also taking into account that

$$\begin{aligned} & T^A V^{++} T^A = V^{++B} T^A T^B T^A = V^{++B} (T^A T^A T^B + T^A [T^B, T^A]) \\ &= V^{++B} \left[C(R) T^B - \frac{1}{2} C_2 T^B \right] = \left[C(R) - \frac{1}{2} C_2 \right] V^{++}, \end{aligned} \quad (5.38)$$

we obtain the expression for the diagram (1) in the form

$$2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du (\tilde{q}^+)^i \left[C(R)_i^k - \frac{1}{2} C_2 \delta_i^k \right] (F_{\text{linear}}^{++})_k^j (q^+)_j, \quad (5.39)$$

where

$$\mathbf{F}_{\text{linear}}^{++} \equiv (D^+)^4 \mathbf{V}_{\text{linear}}^{--} \equiv (D^+)^4 \int du_1 \frac{\mathbf{V}^{++}(z, u_1)}{(u^+ u_1^+)^2}. \quad (5.40)$$

The nonlinear terms will be restored below by the gauge-invariance reasoning.

Let us turn to calculating the second diagram in Fig. 5. The details of this calculation are described in Appendix B. The expression for the diagram (2) in Fig. 5 obtained there is as follows

$$f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta \frac{1}{k^2 (k+p)^2 (k+p+q)^2} \\ \times \int du \tilde{q}^+(q+p, \theta, u)^i \mathbf{V}_{\text{linear}}^{--}(-p, \theta, u)_i{}^j q^+(-q, \theta, u)_j. \quad (5.41)$$

The integral over the loop momentum k is calculated using the Wick rotation. In the case of the regularization by the dimensional reduction it can be found based on the result (5.32). Then, the divergent part reads

$$-\frac{i}{\varepsilon(4\pi)^3} f_0^2 C_2 \int d^{14} z du (\tilde{q}^+)^i (\mathbf{V}_{\text{linear}}^{--})_i{}^j (q^+)_j \\ = -\frac{i}{\varepsilon(4\pi)^3} f_0^2 C_2 \int d\zeta^{(-4)} du (\tilde{q}^+)^i (\mathbf{F}_{\text{linear}}^{++})_i{}^j (q^+)_j. \quad (5.42)$$

Adding this contribution to Eq. (5.39), we obtain the total result for the diagrams presented in Fig. 5,

$$2i f_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du (\tilde{q}^+)^i \left[C(R)_i{}^k - C_2 \delta_i^k \right] (\mathbf{F}_{\text{linear}}^{++})_k{}^j (q^+)_j. \quad (5.43)$$

It is linear in \mathbf{V}^{++} by construction. However, we know that the result for the one-loop divergences should be gauge invariant. The only possible gauge invariant expression yielding Eq. (5.43) in the linearization limit is as follows

$$2i f_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du (\tilde{q}^+)^i \left[C(R)_i{}^k - C_2 \delta_i^k \right] (\mathbf{F}^{++})_k{}^j (q^+)_j. \quad (5.44)$$

It agrees with the result obtained in [2] by the proper time technique. Choosing the representation R to be irreducible, we obtain the following value for the coefficient c_2 :

$$c_2 = 2 f_0^2 \frac{C(R) - C_2}{(4\pi)^3 \varepsilon}. \quad (5.45)$$

We see that the corresponding divergence vanishes for $\mathcal{N} = (1, 1)$ SYM theory, when R is adjoint representation.

5.5. Total one-loop divergences of the theory

Let us summarize the results obtained in the previous section and write down the total divergent part of the effective action for $6D$, $\mathcal{N} = (1, 0)$ SYM theory. If this theory is regularized through the dimensional reduction, then

$$(\Gamma_\infty^{(1)})_{\text{DRED}} = \frac{C_2 - T(R)}{3\varepsilon(4\pi)^3} \text{tr} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2 \\ - 2i f_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ [C_2 - C(R)] \mathbf{F}^{++} q^+. \quad (5.46)$$

However, as we have already mentioned, within the dimensional reduction technique it is impossible to catch quadratic divergencies. They can be obtained, for example, using the momentum cut-off regularization. Then the result for the one-loop divergences can be written as

$$(\Gamma_{\infty}^{(1)})_{\text{UV cut-off}} = -[C_2 - T(R)] \frac{f_0^2 \Lambda^2}{(4\pi)^3} S_{SYM}[V^{++}] + \ln \Lambda \left[\frac{C_2 - T(R)}{3(4\pi)^3} \text{tr} \int d\zeta^{(-4)} du \right. \\ \left. \times (\mathbf{F}^{++})^2 - 2if_0^2 \frac{1}{(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ [C_2 - C(R)] \mathbf{F}^{++} q^+ \right], \quad (5.47)$$

where Λ denotes the ultraviolet cut-off. The first term corresponds to the quadratic divergences, while the remaining two terms parametrize the logarithmic divergences.

In the abelian case the corresponding expressions take the form

$$(\Gamma_{\infty}^{(1)})_{\text{DRED}} = -\frac{1}{6\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2 + 2if_0^2 \frac{1}{\varepsilon(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ \mathbf{F}^{++} q^+; \\ (\Gamma_{\infty}^{(1)})_{\text{UV cut-off}} = \int d^{14}z du_1 du_2 V^{++}(z, u_1) V^{++}(z, u_2) \frac{1}{(u_1^+ u_2^+)^2} \frac{\Lambda^2}{4(4\pi)^3} \\ + \ln \Lambda \left[-\frac{1}{6(4\pi)^3} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2 + 2if_0^2 \frac{1}{(4\pi)^3} \int d\zeta^{(-4)} du \tilde{q}^+ \mathbf{F}^{++} q^+ \right]. \quad (5.48)$$

6. Summary

In this paper we have studied the quantum aspects of generic supersymmetric $6D, \mathcal{N} = (1, 0)$ gauge theory of interacting six-dimensional gauge multiplet minimally coupled to hypermultiplet. The theory is formulated in $\mathcal{N} = (1, 0)$ harmonic superspace that allows one to preserve manifest $\mathcal{N} = (1, 0)$ supersymmetry at all steps of consideration. Also we used the superfield background field method that secures, besides manifest supersymmetry, the manifest classical gauge invariance of quantum theory. The $6D, \mathcal{N} = (1, 0)$ harmonic supergraph technique was developed to study the off-shell effective action depending on the gauge and hypermultiplet superfields and it was applied for calculating the one-loop divergences. We have considered both abelian and non-abelian $\mathcal{N} = (1, 0)$ models and also $6D, \mathcal{N} = (1, 1)$ SYM theory as a particular case of the general system.

We investigated the divergent part of the one-loop effective action corresponding to the two- and three-point functions for $6D, \mathcal{N} = (1, 0)$ SYM theory interacting with hypermultiplets. Using the supergraph techniques in harmonic superspace we calculated the two-point Green functions of both the gauge superfield and the hypermultiplet. Also we found the three-point mixed gauge-hypermultiplet Green function. The results for these Green functions allowed us to restore the total gauge invariant result for the off-shell one-loop divergences in the theory under consideration. The calculations were performed for both abelian and non-abelian models.

In the non-abelian case it was demonstrated that the divergences reveal a generic structure, first found in [2] on the basis of the operator proper-time technique. Namely, all of these divergences are proportional to the difference of Casimir operators for the adjoint representation and representation R to which the hypermultiplet belongs. This leads us to conclude that the $6D, \mathcal{N} = (1, 1)$ SYM theory is completely off-shell finite in the one-loop approximation. The results for the abelian theory are also consistent with the earlier calculations in [1] where they were done by another method. It is worth pointing out that the calculations in terms of supergraphs are more transparent and simpler than those within the operator proper-time techniques as performed in

[1,2]. It should be mentioned that, besides the logarithmic divergences calculated earlier in [1,2], in the present paper we calculated the power divergences which also have an interesting structure and vanish for $\mathcal{N} = (1, 1)$ SYM theory as well.

The absence of off-shell one-loop divergences in $6D$, $\mathcal{N} = (1, 1)$ SYM theory raises a question concerning the off-shell structure of higher-loop divergences in this theory. We would like to recall that the divergences in the hypermultiplet sector have never been considered even on shell. The supergraph technique, formulated in this paper, provides a natural ground for higher-loop calculations. Note that, according to standard renormalization procedure, the calculations of the Feynman diagrams for the higher-loop divergences implicate that the divergences of all sub-diagrams have been already eliminated with the help of the lower-loop counterterms. Since $6D$, $\mathcal{N} = (1, 1)$ SYM theory is off-shell one-loop finite, the analysis of two-loop divergences is simplified because the one-loop divergences are absent and there is no need to renormalize the one-loop subgraphs. We plan to study the complete off-shell structure of the two-loop divergences of this theory in a forthcoming work.

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Appendix A. Two-point Green function of the background gauge superfield

A.1. Contribution of the Faddeev–Popov and Nielsen–Kallosh ghosts

Let us calculate the diagrams (3) and (6) in Fig. 3 that correspond to the contribution of the Faddeev–Popov ghosts. The diagram (3) is given by the expression

$$\begin{aligned}
 & i \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} du_1 du_2 f^{ABC} f^{CDA} \mathbf{V}^{++B}(z_1, u_1) \mathbf{V}^{++D}(z_2, u_2) \\
 & \times \left[D_2^{++} \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] D_1^{++} \right. \\
 & \times \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \\
 & - \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] D_1^{++} \\
 & \left. \times D_2^{++} \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \right]. \quad (\text{A.1})
 \end{aligned}$$

Using the relations (2.2), we rewrite it as

$$\begin{aligned}
 & -iC_2 \int d^{14}z_1 d^{14}z_2 du_1 du_2 \mathbf{V}^{++A}(z_1, u_1) \mathbf{V}^{++A}(z_2, u_2) \\
 & \times \left[D_2^{++} \left[\frac{1}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] D_1^{++} \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \right. \\
 & \left. - \left[\frac{1}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] D_1^{++} D_2^{++} \left[\frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \right]. \quad (\text{A.2})
 \end{aligned}$$

In order to get rid of the integrals over the anticommuting variables, we first apply the identity (5.8) and, then, calculate one of the anticommuting integral by making use of the δ -function $\delta^8(\theta_1 - \theta_2)$. This gives

$$\begin{aligned} & -iC_2 \int d^6x_1 d^6x_2 d^8\theta du_1 du_2 V^{++A}(x_1, \theta, u_1) V^{++A}(x_2, \theta, u_2) (u_1^+ u_2^+)^4 \\ & \times \left[\frac{1}{\square} \delta^6(x_1 - x_2) D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \frac{1}{\square} \delta^6(x_1 - x_2) D_1^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \right. \\ & \left. - \frac{1}{\square} \delta^6(x_1 - x_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \frac{1}{\square} \delta^6(x_1 - x_2) D_1^{++} D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \right]. \end{aligned} \quad (\text{A.3})$$

At the next step, we rewrite this expression in the momentum representation and compute the harmonic derivatives,

$$\begin{aligned} & -iC_2 \int \frac{d^6p}{(2\pi)^6} d^8\theta du_1 du_2 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_2) \\ & \times \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \left[-\frac{(u_2^+ u_1^-)(u_1^+ u_2^-)}{(u_1^+ u_2^+)^2} - \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)} \right]. \end{aligned} \quad (\text{A.4})$$

To simplify it, we use the identity

$$(u_2^+ u_1^-)(u_1^+ u_2^-) = 1 - (u_1^- u_2^-)(u_1^+ u_2^+). \quad (\text{A.5})$$

Then, for the diagram (3) we obtain

$$\begin{aligned} & iC_2 \int \frac{d^6p}{(2\pi)^6} d^8\theta du_1 du_2 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2(k+p)^2}. \end{aligned} \quad (\text{A.6})$$

The diagram (6) makes the vanishing contribution, because it contains the block

$$\begin{aligned} & \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \Big|_{z_1=z_2; u_1=u_2} \\ & = (u_1^+ u_2^+)(u_1^- u_2^-) \frac{1}{\square} \delta^6(x_1 - x_2) \Big|_{u_1=u_2} = 0. \end{aligned} \quad (\text{A.7})$$

The contribution of the Nielsen–Kallosh ghosts φ is given by the diagrams (4) and (7) in Fig. 3. Expressions for them differ from those corresponding to diagrams (3) and (6) by the factor $-1/2$.

However, the Nielsen–Kallosh ghost contribution also includes $\text{Det}^{1/2} \widehat{\square}$, which is calculated using the definition (3.13). It is easy to show that this determinant makes the vanishing contribution. Actually, the tadpole diagram (7) contains $(D^+)^4 \delta(z_1 - z_2) \Big|_{z_1=z_2} = 0$, while the diagram of the type (4) contains $(u_1^+ u_2^+) \Big|_{u_1=u_2} = 0$. Therefore, no contribution comes from the superfields $\xi^{(+4)}$ and σ at all.

Thus, the total ghost contribution to the considered part of the one-loop effective action can be written as

$$\begin{aligned} & \frac{i}{2} C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_2 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_2) \frac{1}{(u_1^+ u_2^+)^2} \\ & \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2}. \end{aligned} \quad (\text{A.8})$$

A.2. Diagrams with the loop of the gauge superfield

The contribution containing a loop of the quantum gauge superfield is given by the sum of the diagrams (2) and (5) in Fig. 3. Let us start with the diagram (2). It is convenient to split this diagram into the three parts. The first one contains two vertices coming from the classical action S_{SYM} , the second one contains one vertex from S_{SYM} and one vertex from the gauge-fixing action S_{gf} , and the last one contains two vertices, both coming from S_{gf} .

We start with a sub-diagram containing two S_{SYM} -vertices. The Feynman rules give for it the following analytical expression:

$$\begin{aligned} & -\frac{i}{4} \int d^{14} z_1 d^{14} z_2 du_1 du_2 du_3 du_4 du_5 du_6 f^{ABC} f^{ABD} V^{++C}(z_1, u_3) V^{++D}(z_2, u_6) \\ & \times \frac{1}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_1^+)(u_4^+ u_5^+)(u_5^+ u_6^+)(u_6^+ u_4^+)} \frac{(D_1^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_4) \\ & \times \frac{(D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_2, u_5). \end{aligned} \quad (\text{A.9})$$

First, we use the identity (5.8) and calculate one of the θ -integrals. Then we calculate two harmonic integrals by making use of the harmonic δ -functions. After this we obtain

$$\begin{aligned} & -\frac{i}{4} C_2 \int d^6 x_1 d^6 x_2 d^8 \theta du_1 du_2 du_3 du_6 V^{++A}(x_1, \theta, u_3) V^{++A}(x_2, \theta, u_6) \\ & \times \frac{(u_1^+ u_2^+)^2}{(u_2^+ u_3^+)(u_3^+ u_1^+)(u_2^+ u_6^+)(u_6^+ u_1^+)} \frac{1}{\square} \delta^6(x_1 - x_2) \frac{1}{\square} \delta^6(x_1 - x_2). \end{aligned} \quad (\text{A.10})$$

Then, we use the identity

$$(u_1^+ u_2^+)^2 = D_1^{++} \left[(u_1^- u_2^+) (u_1^+ u_2^+) \right] \quad (\text{A.11})$$

in the numerator of the harmonic factor and integrate by parts with respect to D_1^{++} , taking into account the relation (3.7). The resulting expression, written in the momentum representation, has the form

$$\begin{aligned} & \frac{i}{4} C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 du_6 V^{++A}(-p, \theta, u_3) V^{++A}(p, \theta, u_6) \\ & \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2} \\ & \times \frac{(u_1^- u_2^+)(u_1^+ u_2^+)}{(u_2^+ u_3^+)(u_2^+ u_6^+)} \left[\delta^{(1,-1)}(u_1, u_3) \frac{1}{(u_1^+ u_6^+)} + \delta^{(1,-1)}(u_1, u_6) \frac{1}{(u_1^+ u_3^+)} \right]. \end{aligned} \quad (\text{A.12})$$

The harmonic δ -functions allow one to do one of the harmonic integrals,

$$\begin{aligned}
& -\frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 V^{++A}(-p, \theta, u_3) V^{++A}(p, \theta, u_1) \\
& \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \frac{D_2^{++}(u_1^- u_2^-)}{(u_2^+ u_3^+)(u_1^+ u_3^+)},
\end{aligned} \tag{A.13}$$

where we used the identity $(u_1^- u_2^+) = D_2^{++}(u_1^- u_2^-)$. Integrating by parts with respect to D_2^{++} , we obtain the final expression for the considered part of the diagram (2),

$$\begin{aligned}
& \frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_3 V^{++A}(-p, \theta, u_3) V^{++A}(p, \theta, u_1) \frac{(u_1^- u_3^-)}{(u_1^+ u_3^+)} \\
& \times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2}.
\end{aligned} \tag{A.14}$$

The second sub-diagram (in the diagram (2)) contains one vertex coming from S_{SYM} and another one coming from S_{gf} . The corresponding expression reads

$$\begin{aligned}
& -\frac{i}{2}C_2 \int d^{14} z_1 d^{14} z_2 du_1 du_2 du_3 du_4 du_5 \left[\mathbf{b}^A(z_1, u_1) - \mathbf{b}^A(z_1, u_2) \right] V^{++A}(z_2, u_3) \\
& \times D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \right] \frac{1}{(u_3^+ u_4^+)(u_4^+ u_5^+)(u_5^+ u_3^+)} \frac{(D_1^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_4) \\
& \times \frac{(D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_2, u_5).
\end{aligned} \tag{A.15}$$

We start the calculation, using the identity (5.8). Then we calculate the integral over $d^8 \theta_2$ by exploiting $\delta^8(\theta_1 - \theta_2)$ and two harmonic integrals with the help of the harmonic δ -functions present in the propagators. The expression considered takes the form

$$\begin{aligned}
& -\frac{i}{2}C_2 \int d^6 x_1 d^6 x_2 d^8 \theta du_1 du_2 du_3 \left[\mathbf{b}^A(x_1, \theta, u_1) - \mathbf{b}^A(x_1, \theta, u_2) \right] V^{++A}(x_2, \theta, u_3) \\
& \times D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \right] \frac{(u_1^+ u_2^+)^3}{(u_3^+ u_1^+)(u_2^+ u_3^+)} \frac{1}{\square} \delta^6(x_1 - x_2) \frac{1}{\square} \delta^6(x_1 - x_2).
\end{aligned} \tag{A.16}$$

Calculating the action of the harmonic derivative D_1^{++} and passing to the momentum representation, we obtain

$$\begin{aligned}
& -\frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] V^{++A}(p, \theta, u_3) \\
& \times \frac{(u_1^+ u_2^+)}{(u_3^+ u_1^+)(u_2^+ u_3^+)} \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2}.
\end{aligned} \tag{A.17}$$

It is convenient to rewrite the numerator of the harmonic factor in this expression as

$$(u_1^+ u_2^+) = D_1^{++}(u_1^- u_2^+) \tag{A.18}$$

and then to integrate by parts with respect to D_1^{++} . Next, resorting to Eq. (3.7), we find

$$\begin{aligned}
& \frac{i}{2} C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} V^{++A}(p, \theta, u_3) \\
& \times \left[D_1^{++} \mathbf{b}^A(-p, \theta, u_1) \frac{(u_1^- u_2^+)}{(u_3^+ u_1^+)(u_2^+ u_3^+)} \right. \\
& \left. - \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] \frac{(u_1^- u_2^+)}{(u_2^+ u_3^+)} \delta^{(1,-1)}(u_1, u_3) \right]. \quad (\text{A.19})
\end{aligned}$$

In the first term the derivative of the bridge gives the superfield V^{++} , while in the second one it is possible to take off one of the harmonic integrals,

$$\begin{aligned}
& -\frac{i}{2} C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \\
& \times \left[\int du_1 du_2 du_3 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_3) \frac{(u_1^- u_2^+)}{(u_3^+ u_1^+)(u_2^+ u_3^+)} \right. \\
& \left. + \int du_1 du_2 \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] V^{++A}(p, \theta, u_1) \frac{(u_1^- u_2^+)}{(u_2^+ u_1^+)} \right]. \quad (\text{A.20})
\end{aligned}$$

Using the identity

$$(u_1^- u_2^+) = D_2^{++}(u_1^- u_2^-) \quad (\text{A.21})$$

and integrating by parts with respect to D_2^{++} , this expression can be rewritten as

$$\begin{aligned}
& \frac{i}{2} C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \\
& \times \left[\int du_1 du_2 du_3 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_3) \frac{(u_1^- u_2^-)}{(u_3^+ u_1^+)} \delta^{(1,-1)}(u_2, u_3) \right. \\
& \left. - \int du_1 du_2 D_2^{++} \mathbf{b}^A(-p, \theta, u_2) V^{++A}(p, \theta, u_1) \frac{(u_1^- u_2^-)}{(u_2^+ u_1^+)} \right]. \quad (\text{A.22})
\end{aligned}$$

This implies that the contribution of the sub-diagram considered is finally given by the expression

$$\begin{aligned}
& -i C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du_1 du_3 \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} V^{++A}(-p, \theta, u_1) \\
& \times V^{++A}(p, \theta, u_3) \frac{(u_1^- u_3^-)}{(u_1^+ u_3^+)}. \quad (\text{A.23})
\end{aligned}$$

The last sub-diagram contains two vertices coming from S_{gr} . It is written in the form

$$\begin{aligned}
& -\frac{i}{4} C_2 \int d^{14} z_1 d^{14} z_2 du_1 du_2 du_3 du_4 \left[\mathbf{b}^A(z_1, u_1) - \mathbf{b}^A(z_1, u_2) \right] \\
& \times \left[\mathbf{b}^A(z_2, u_3) - \mathbf{b}^A(z_2, u_4) \right] \\
& \times D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \right] D_3^{++} \left[\frac{(u_3^- u_4^+)}{(u_3^+ u_4^+)^3} \right] \frac{(D_1^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_3) \\
& \times \frac{(D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_2, u_4). \quad (\text{A.24})
\end{aligned}$$

As before, using Eq. (5.8) and δ -functions, we calculate one of the θ -integrals and two harmonic integrals. The result in the momentum representation is written as

$$-\frac{i}{4}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \int du_1 du_2 \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] \\ \times \left[\mathbf{b}^A(p, \theta, u_1) - \mathbf{b}^A(p, \theta, u_2) \right] (u_1^+ u_2^+)^4 D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \right] D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_1^+)^3} \right]. \quad (\text{A.25})$$

Calculating the harmonic derivatives in this expression, we obtain

$$-\frac{i}{4}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \int du_1 du_2 \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] \\ \times \left[\mathbf{b}^A(p, \theta, u_1) - \mathbf{b}^A(p, \theta, u_2) \right]. \quad (\text{A.26})$$

Taking into account the relation

$$D_1^{++} D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)} \right] = D_1^{++} \left[\frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)} \right] = 1 - \delta^{(1,-1)}(u_1, u_2), \quad (\text{A.27})$$

the last expression can be equivalently written in the form

$$-\frac{i}{4}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \int du_1 du_2 \left[\mathbf{b}^A(-p, \theta, u_1) - \mathbf{b}^A(-p, \theta, u_2) \right] \\ \times \left[\mathbf{b}^A(p, \theta, u_1) - \mathbf{b}^A(p, \theta, u_2) \right] D_1^{++} D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)} \right]. \quad (\text{A.28})$$

After integrating by parts with respect to the harmonic derivatives, we obtain

$$\frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \\ \times \int du_1 du_2 D_1^{++} \mathbf{b}^A(-p, \theta, u_1) D_2^{++} \mathbf{b}^A(p, \theta, u_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)}, \quad (\text{A.29})$$

that can be expressed in terms of V^{++} as

$$\frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \\ \times \int du_1 du_2 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)}. \quad (\text{A.30})$$

Summing up the contributions of the three sub-diagrams, (A.14), (A.23), and (A.30), we conclude that the diagram (2) vanishes,

$$\frac{i}{2}C_2 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2(k+p)^2} \int du_1 du_2 V^{++A}(-p, \theta, u_1) V^{++A}(p, \theta, u_2) \\ \times \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)} (1 - 2 + 1) = 0. \quad (\text{A.31})$$

Diagram (5) is a sum of two parts. The first one contains the vertex coming from the SYM action (2.5), while in the second part the vertex originates from the gauge-fixing term (3.4). It is

easy to see that the contribution of every part is vanishing separately. In particular, the first part is proportional to

$$C_2 \int d^{14}z du_1 du_2 du_3 du_4 V^{++A}(z, u_1) V^{++A}(z, u_3) \frac{1}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} \\ \times (D_2^+)^4 \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_2, u_4) \Big|_{z_1=z_2=z}. \quad (\text{A.32})$$

This expression vanishes, because only four spinor derivatives are left there to act on the Grassmann δ -function at coincident points. The second part vanishes for the same reason.

Thus, the sum of both diagrams with the loop of the quantum gauge superfield inside yields zero.

Appendix B. Gauge-hypermultiplet three-point function

B.1. Abelian case

In this section we describe details of calculating the gauge-hypermultiplet three-point function in the abelian case. For the abelian theory only the left diagram in Fig. 5 remains, while the right one is absent. In the calculations we use the minimal gauge $\xi_0 = 1$. Then the considered contribution to the effective action of the abelian theory (2.19) has the form

$$-2f_0^2 \int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 d\zeta_3^{(-4)} du_3 \tilde{q}^+(z_1, u_1) q^+(z_3, u_3) V^{++}(z_2, u_2) \\ \times \frac{(D_1^+)^4}{\square} \delta^{(2,-2)}(u_3, u_1) \delta^{14}(z_1 - z_3) \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \\ \times \delta^{14}(z_1 - z_2) \frac{1}{(u_2^+ u_3^+)^3} \frac{(D_2^+)^4 (D_3^+)^4}{\square} \delta^{14}(z_2 - z_3). \quad (\text{B.1})$$

Like in the previous cases, we start by converting the integrals over $d\zeta^{(-4)}$ into integrals over the full measure $d^{14}z$,

$$-2f_0^2 \int d^{14}z_1 du_1 d^{14}z_2 du_2 d^{14}z_3 du_3 \tilde{q}^+(z_1, u_1) q^+(z_3, u_3) V^{++}(z_2, u_2) \\ \times \frac{1}{\square} \delta^{(2,-2)}(u_3, u_1) \\ \times \delta^{14}(z_1 - z_3) \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_2^+ u_3^+)^3} \frac{1}{\square} \delta^{14}(z_2 - z_3). \quad (\text{B.2})$$

Using the δ -functions, we can calculate integrals over $d^8\theta_3$ and du_3 ,

$$2f_0^2 \int d^{14}z_1 du_1 d^{14}z_2 du_2 d^6x_3 \tilde{q}^+(z_1, u_1) q^+(x_3, \theta_1, u_1) V^{++}(z_2, u_2) \frac{1}{\square} \\ \times \delta^6(x_1 - x_3) \frac{1}{(u_1^+ u_2^+)^6} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} \delta^6(x_2 - x_3) \delta^8(\theta_1 - \theta_2). \quad (\text{B.3})$$

As the next step, using the identity (5.8), we obtain

$$2f_0^2 \int d^{14}z_1 du_1 d^{14}z_2 du_2 d^6x_3 \tilde{q}^+(z_1, u_1) q^+(x_3, \theta_1, u_1) V^{++}(z_2, u_2) \frac{1}{\square} \\ \times \delta^6(x_1 - x_3) \frac{1}{(u_1^+ u_2^+)^2} \frac{1}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} \delta^6(x_2 - x_3). \quad (\text{B.4})$$

The remaining δ -function $\delta^8(\theta_1 - \theta_2)$ can be used to perform the integration over $d^8\theta_2$. The result of these manipulations in the coordinate representation is

$$2f_0^2 \int d^6x_1 d^6x_2 d^6x_3 d^8\theta du_1 du_2 \tilde{q}^+(x_1, \theta, u_1) q^+(x_3, \theta, u_1) V^{++}(x_2, \theta, u_2) \frac{1}{\square} \\ \times \delta^6(x_1 - x_3) \frac{1}{(u_1^+ u_2^+)^2} \frac{1}{\square} \delta^6(x_1 - x_2) \frac{1}{\square} \delta^6(x_2 - x_3). \quad (\text{B.5})$$

After converting this expression to the momentum representation (in the Minkowski space) we arrive at the following final answer

$$-2f_0^2 \int \frac{d^6p}{(2\pi)^6} \frac{d^6q}{(2\pi)^6} d^8\theta du_1 du_2 \tilde{q}^+(q + p, \theta, u_1) q^+(-q, \theta, u_1) V^{++}(-p, \theta, u_2) \\ \times \frac{1}{(u_1^+ u_2^+)^2} \int \frac{d^6k}{(2\pi)^6} \frac{1}{k^2(q + k)^2(q + k + p)^2}. \quad (\text{B.6})$$

B.2. Non-abelian theory: the second diagram in Fig. 5

In this section we outline the calculation of the diagram (2) in Fig. 5. It is convenient to split it into two pieces. The first one corresponds to that part of the three-point gauge vertex which comes from the classical action S , while the second piece corresponds to that part of the vertex which comes from the gauge-fixing term S_{gf} . We calculate these two contributions separately. The expression for the first piece constructed by the Feynman rules in harmonic superspace is written as

$$-2if_0^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} d^{14}z_3 du_1 du_2 du_3 du_4 du_5 \frac{1}{(u_3^+ u_4^+)(u_4^+ u_5^+)(u_5^+ u_3^+)} \\ \times f^{ABC} V^{++C}(z_3, u_5) \\ \times \tilde{q}^+(z_1, u_1)^i (T^A T^B)_i{}^j q^+(z_2, u_2)_j \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \\ \times \frac{(D_1^+)^4}{\square} \delta^{14}(z_1 - z_3) \delta^{(2,-2)}(u_3, u_1) \frac{(D_2^+)^4}{\square} \delta^{14}(z_2 - z_3) \delta^{(2,-2)}(u_4, u_2). \quad (\text{B.7})$$

Once again, we start by converting the integrals over $d\zeta^{(-4)}$ into those over $d^{14}z$. Also we use the identity

$$2f^{ABC} T^A T^B = f^{ABC} [T^A, T^B] = iC_2 T^C. \quad (\text{B.8})$$

Then the considered part of the diagram (2) is written as

$$\begin{aligned}
& f_0^2 C_2 \int d^{14} z_1 d^{14} z_2 d^{14} z_3 du_1 du_2 du_3 du_4 du_5 \frac{1}{(u_3^+ u_4^+)(u_4^+ u_5^+)(u_5^+ u_3^+)} V^{++C}(z_3, u_5) \\
& \times \tilde{q}^+(z_1, u_1)^i (T^C)_i{}^j q^+(z_2, u_2)_j \frac{1}{(u_1^+ u_2^+)^3} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} \delta^{14}(z_1 - z_3) \\
& \times \delta^{(2,-2)}(u_3, u_1) \frac{1}{\square} \delta^{14}(z_2 - z_3) \delta^{(2,-2)}(u_4, u_2). \tag{B.9}
\end{aligned}$$

Harmonic δ -functions can be used to do two harmonic integrations,

$$\begin{aligned}
& f_0^2 C_2 \int d^{14} z_1 d^{14} z_2 d^{14} z_3 du_1 du_2 du_5 \frac{1}{(u_2^+ u_5^+)(u_5^+ u_1^+)} \tilde{q}^+(z_1, u_1)^i \\
& \times V^{++}(z_3, u_5)_i{}^j q^+(z_2, u_2)_j \\
& \times \frac{1}{(u_1^+ u_2^+)^4} \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} \delta^{14}(z_1 - z_3) \frac{1}{\square} \delta^{14}(z_2 - z_3). \tag{B.10}
\end{aligned}$$

Two θ -integrals can be calculated by using the δ -functions and the identity (5.8). This gives

$$\begin{aligned}
& f_0^2 C_2 \int d^6 x_1 d^6 x_2 d^6 x_3 d^8 \theta du_1 du_2 du_5 \tilde{q}^+(x_1, \theta, u_1)^i V^{++}(x_3, \theta, u_5)_i{}^j q^+(x_2, \theta, u_2)_j \\
& \times \frac{1}{(u_2^+ u_5^+)(u_5^+ u_1^+)} \frac{1}{\square} \delta^{14}(z_1 - z_2) \frac{1}{\square} \delta^{14}(z_1 - z_3) \frac{1}{\square} \delta^{14}(z_2 - z_3). \tag{B.11}
\end{aligned}$$

After relabeling the integration variable as $u_5 \rightarrow u_3$, this expression can be written in the momentum representation (in the Minkowski space) as

$$\begin{aligned}
& -f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \tilde{q}^+(q + p, \theta, u_1)^i \\
& \times V^{++}(-p, \theta, u_3)_i{}^j q^+(-q, \theta, u_2)_j \\
& \times \frac{1}{k^2(k+p)^2(k+q+p)^2} \frac{1}{(u_2^+ u_3^+)(u_3^+ u_1^+)}. \tag{B.12}
\end{aligned}$$

Let us now express the gauge superfield V^{++} through the bridge b in the linearized approximation,

$$V^{++} = -D^{++} b + \text{irrelevant terms}, \tag{B.13}$$

where we omitted all terms with higher degrees of the bridge superfield. This is justified, since we deal only with terms linear in the gauge superfield. Then, after integration by parts, we obtain

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \tilde{q}^+(q + p, \theta, u_1)^i \\
& \times b(-p, \theta, u_3)_i{}^j q^+(-q, \theta, u_2)_j \\
& \times \frac{1}{k^2(k+p)^2(k+q+p)^2} D_3^{++} \left[\frac{1}{(u_3^+ u_2^+)(u_3^+ u_1^+)} \right]. \tag{B.14}
\end{aligned}$$

According to the identity (3.7), the derivative D_3^{++} produces two harmonic δ -functions, so (B.14) becomes

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \tilde{q}^+(q+p, \theta, u_1)^i \\
& \times \mathbf{b}(-p, \theta, u_3)_i^j q^+(-q, \theta, u_2)_j \frac{1}{k^2(k+p)^2(k+q+p)^2} \\
& \times \left[\delta^{(1,-1)}(u_3, u_2) \frac{1}{(u_3^+ u_1^+)} + \delta^{(1,-1)}(u_3, u_1) \frac{1}{(u_3^+ u_2^+)} \right]. \quad (\text{B.15})
\end{aligned}$$

Thus, the considered part of the diagram (2) can be finally written as

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 du_3 \tilde{q}^+(q+p, \theta, u_1)^i \\
& \times [\mathbf{b}(-p, \theta, u_1) - \mathbf{b}(-p, \theta, u_2)]_i^j \\
& \times q^+(-q, \theta, u_2)_j \frac{1}{k^2(k+p)^2(k+q+p)^2} \frac{1}{(u_1^+ u_2^+)}. \quad (\text{B.16})
\end{aligned}$$

Now, let us turn to calculating that part of the diagram (2) in which the purely gauge vertex comes from the gauge-fixing term S_{gf} . This contribution is written as

$$\begin{aligned}
& 2if_0^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} d^{14} z_3 du_1 du_2 du_3 du_4 \tilde{q}^+(z_1, u_1)^i T^A T^B q^+(z_2, u_2)_j f^{ABC} \\
& \times [\mathbf{b}^C(z_3, u_4) - \mathbf{b}^C(z_3, u_3)] D_3^{++} D_4^{++} \left[\frac{(u_3^- u_4^-)}{(u_3^+ u_4^+)^3} \right] \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \\
& \times \frac{(D_1^+)^4}{\square} \delta^{14}(z_1 - z_3) \delta^{(2,-2)}(u_3, u_1) \frac{(D_2^+)^4}{\square} \delta^{14}(z_2 - z_3) \delta^{(2,-2)}(u_4, u_2). \quad (\text{B.17})
\end{aligned}$$

Following the same strategy as for the diagrams handled before, we convert integrations over $d\zeta^{(-4)}$ into integrations over $d^{14}z$ and calculate two harmonic integrals by making use of the harmonic δ -functions. This gives

$$\begin{aligned}
& -f_0^2 C_2 \int d^{14} z_1 d^{14} z_2 d^{14} z_3 du_1 du_2 \tilde{q}^+(z_1, u_1)^i [\mathbf{b}(z_3, u_2) - \mathbf{b}(z_3, u_1)]_i^j q^+(z_2, u_2)_j \\
& \times D_1^{++} D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right] \frac{(D_1^+)^4 (D_2^+)^4}{\square} \delta^{14}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \frac{1}{\square} \delta^{14}(z_1 - z_3) \\
& \times \frac{1}{\square} \delta^{14}(z_2 - z_3). \quad (\text{B.18})
\end{aligned}$$

Next, we need to calculate integrals over the anticommuting variables and to bring the result into the momentum representation. We obtain

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 \tilde{q}^+(q+p, \theta, u_1)^i \\
& \times [\mathbf{b}(-p, \theta, u_2) - \mathbf{b}(-p, \theta, u_1)]_i^j \\
& \times q^+(-q, \theta, u_2)_j \frac{1}{k^2(k+p)^2(k+p+q)^2} (u_1^+ u_2^+) D_1^{++} D_2^{++} \left[\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right]. \quad (\text{B.19})
\end{aligned}$$

Using the identity (3.7) and the equality $(u_1^- u_2^+) \delta^{(2,-2)}(u_1, u_2) = -\delta^{(2,-2)}(u_1, u_2)$, this expression can be equivalently represented as

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 \tilde{q}^+(q+p, \theta, u_1)^i \\
& \times [\mathbf{b}(-p, \theta, u_2) - \mathbf{b}(-p, \theta, u_1)]_i^j q^+(-q, \theta, u_2)_j \frac{1}{k^2(k+p)^2(k+p+q)^2} (u_1^+ u_2^+) \\
& \times \left[\frac{1}{(u_1^+ u_2^+)^2} - \frac{1}{2} (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2) \right]. \tag{B.20}
\end{aligned}$$

Integrating by parts in the last term with respect to $(D_1^{--})^2$, after some algebra we obtain

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta \frac{1}{k^2(k+p)^2(k+p+q)^2} \\
& \times \left[\int du_1 du_2 \tilde{q}^+(q+p, \theta, u_1)^i [\mathbf{b}(-p, \theta, u_2) - \mathbf{b}(-p, \theta, u_1)]_i^j q^+(-q, \theta, u_2)_j \right. \\
& \times \left. \frac{1}{(u_1^+ u_2^+)} - \int du \tilde{q}^+(q+p, \theta, u)^i D^{--} \mathbf{b}(-p, \theta, u)_i^j q^+(-q, \theta, u)_j \right]. \tag{B.21}
\end{aligned}$$

The first term in this expression cancels the previous part of the considered diagram. Therefore, taking into account that

$$V_{\text{linear}}^{--} = -D^{--} \mathbf{b} + \text{irrelevant terms}, \tag{B.22}$$

the net result for the contribution of the diagram (2) in Fig. 5 is given by the expression

$$\begin{aligned}
& f_0^2 C_2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta \frac{1}{k^2(k+p)^2(k+p+q)^2} \\
& \times \int du \tilde{q}^+(q+p, \theta, u)^i V_{\text{linear}}^{--}(-p, \theta, u)_i^j \\
& \times q^+(-q, \theta, u)_j. \tag{B.23}
\end{aligned}$$

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