Published for SISSA by Springer

Received: October 16, 2021
Accepted: November 23, 2021
Published: December 6, 2021

Unconstrained off-shell superfield formulation of $4D$, $\mathcal{N}=2$ supersymmetric higher spins

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Abstract: We present, for the first time, the complete off-shell $4D,\mathcal{N}=2$ superfield actions for any free massless integer spin $\mathbf{s}\geq 2$ fields, using the $\mathcal{N}=2$ harmonic superspace approach. The relevant gauge supermultiplet is accommodated by two real analytic bosonic superfields $h^{++}_{\alpha(s-1)\dot\alpha(s-1)}$, $h^{++}_{\alpha(s-2)\dot\alpha(s-2)}$ and two conjugated complex analytic spinor superfields $h^{+3}_{\alpha(s-1)\dot\alpha(s-2)}$, $h^{+3}_{\alpha(s-2)\dot\alpha(s-1)}$, where $\alpha(s):=(\alpha_1\ldots\alpha_s), \dot\alpha(s):=(\dot\alpha_1\ldots\dot\alpha_s)$. Like in the harmonic superspace formulations of $\mathcal{N}=2$ Maxwell and supergravity theories, an infinite number of original off-shell degrees of freedom is reduced to the finite set (in WZ-type gauge) due to an infinite number of the component gauge parameters in the analytic superfield parameters. On shell, the standard spin content $(\mathbf{s},\mathbf{s}-\mathbf{1/2},\mathbf{s}-\mathbf{1/2},\mathbf{s}-\mathbf{1})$ is restored. For $\mathbf{s}=2$ the action describes the linearized version of "minimal" $\mathcal{N}=2$ Einstein supergravity.

Keywords: Extended Supersymmetry, Higher Spin Gravity, Higher Spin Symmetry, Superspaces

ArXiv ePrint: 2109.07639

Dedicated to Emery Sokatchev on the occasion of his 70th birthday

https://doi.org/10.1007/JHEP12(2021)016

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Contents

1 Introduction

Supersymmetric higher-spin field theories attract a vast attention during a long time. There are at least two reasons for this. First, from the purely theoretical point of view, it is tempting to construct new supersymmetric models of this kind, as well as to supersymmetrize the already available higher-spin field bosonic models. The new universal methods to be developed during these studies could, in particular, shed more light on hidden relationships between fermionic and bosonic degrees of freedom for the higher-spin fields and open new possibilities for building consistent higher-spin field models due to the appearance of extended gauge (super)symmetries. Second, since the superstring theory encompasses infinite towers of bosonic and fermionic higher spin states, supersymmetric higher-spin gauge theory can serve as a bridge between superstring theory and low-energy field theory.

There is a huge literature on higher spin fields. In this introductory section (and over the whole work) we limit our discussion to the issues related *only to supersymmetric higher spin theories* and *only to the four-dimensional* versions of the latter. Respectively, our reference list mainly includes the papers of the same trend.

As is well known, there exist two different generic formulations of the supersymmetric field theories, the component (on-shell) formulation and superfield (off-shell) formulation (see, e.g., [1–3]). In the first approach, the theory is formulated in a way lacking a manifest supersymmetry, in terms of bosonic and fermionic fields forming a supermultiplet on the

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mass shell. The algebra of the supersymmetry transformations is open and becomes closed only upon using the equations of motion. To close the algebra off shell, we are led to introduce the auxiliary fields which vanish on the mass shell. In the second approach, a theory is formulated in a manifestly supersymmetric way employing superfields. The algebra of the supersymmetry transformations is automatically off-shell closed and the auxiliary fields are already built in the superfields. Clearly, due to the manifest off-shell supersymmetry, the second approach much more suits for setting up various generalizations, e.g., finding out consistent interactions proceeding from a given free theory. However, the superfield formulations of some Lagrangian field theory (both classical and quantum) are useful and efficient only providing that the relevant superfields are not subject to any algebraic constraints. For all types of supersymmetries in all dimensions, such unconstrained superfield formulations are at present generically unknown. What concerns four dimensions, there exist unconstrained superfield formulations for all $\mathcal{N} = 1$ supersymmetric theories of interest (matter, super Yang-Mills, supergravity) in conventional $4D$ superspaces, general and chiral (see, e.g., [1, 2]), and for their $\mathcal{N} = 2$ supersymmetry counterparts in terms of harmonic superspace [3–5].

Free massless bosonic and fermionic higher-spin field theories have been pioneered by Fronsdal [6, 7]. The corresponding supersymmetric generalization in the component approach can be constructed as follows. Lagrangian is written as a sum of Lagrangians for all fields of the on-shell supermultiplet. Then one should invent the appropriate supersymmetry transformations and check the invariance of the total Lagrangian. Such a description was realized for $4D$ free massless higher-spin $\mathcal{N} = 1$ supersymmetric models in works [8, 9].[1] Complete off-shell Lagrangian formulation of $4D$ free higher-spin $\mathcal{N} = 1$ models has been developed in terms of $\mathcal{N} = 1$ superfields in works [11–13] (see also section 6.9 in [2]) and further applied to study quantum effective action generated by $\mathcal{N} = 1$ superfields in AdS space in [14]. Note that in [13] the massless higher-spin $\mathcal{N} = 1$ supermultiplets in AdS_4 were constructed for the first time. The superfield approach to $\mathcal{N} = 1$ supersymmetric massless higher spin fields was further generalized in [15–23].[2] Some additional geometric aspects of this approach were explored in [25–28]. Also note an activity on $\mathcal{N} = 1$ supersymmetric massive higher spin theories (see e.g., [29–31] and the references therein), however it is out of the subject of our paper.

At present, a manifestly supersymmetric off-shell unconstrained superfield Lagrangian formulation for extended higher-spin supersymmetric theories is unknown even for the free case (modulo the superconformal theories [24] which we do not concern here). Progress in this area is associated either with the realization of extended supersymmetry in terms of $\mathcal{N} = 1$ superfields [15, 16], or in terms of light-cone $\mathcal{N}$-extended superfields [32], or in the on-shell component approach (see, e.g., [33] and the references therein). In all cases, the

[1] Later, it was shown that the free supersymmetric massless higher-spin gauge theory can be also formulated in the framework of the BRST formalism [10].

[2] It is worth noting the paper [24], where $4D, \mathcal{N} = 2$ superconformal higher-spin theory was formulated in terms of unconstrained $\mathcal{N} = 2$ superfields. The program of constructing the massless higher spin $\mathcal{N} = 2$ superfield actions was sketched but not realized there. Here we do not deal with the superconformal theories at all.

full extended supersymmetry remains non-manifest. An off-sell Lagrangian formulation of $\mathcal{N}=2$ supersymmetric higher-spin theory on AdS space in terms of unconstrained $\mathcal{N}=1$ superfields (and some Poincaré supersymmetric limits thereof) has been constructed for the first time in [16] but such a formulation does not reveal a manifest $\mathcal{N}=2$ supersymmetry. As a result we can conclude that the problem of complete off-shell description of the higher-spin extended supersymmetric theories is still open.[3]

Note that one of the actively developing directions in the theory of higher spin fields is related with the study of interactions. In particular, recently a substantial understanding of the structure of cubic interaction vertices for higher spin supersymmetric fields has been achieved in different component and $\mathcal{N}=1$ superfield approaches (see, e.g., [32, 35–40] and the references therein). Other aspects of higher-spin supersymmetric field theory are related with supersymmetric extension [41–43] of the Vasiliev theory of interacting higher spin fields (see the reviews [44–46], and the references therein). It is beyond the scope of our paper to discuss these prospective and advanced studies.

In this paper we construct the completely off-shell manifestly $\mathcal{N}=2$ supersymmetric superfield extension of arbitrary $4D$ integer-spin free massless theory. The construction is based on the use of the harmonic superspace method [3] which is at present the most adequate and convenient approach for description of $4D, \mathcal{N}=2$ supersymmetric field theories.

The paper is organized as follows. Section 2 is devoted to a brief description of the linearized $\mathcal{N}=2$ Einstein supergravity (linearized massless $\mathcal{N}=2$ spin 2 theory) in terms of unconstrained analytic harmonic superfields. In section 3 we generalize the above results and formulate the free massless $\mathcal{N}=2$ spin 3 harmonic superfield theory. Section 4 is devoted to further generalization and construction of a completely off-shell invariant action for the free massless $\mathcal{N}=2$ gauge theory with an arbitrary maximal integer spin $\mathbf{s}$ of the supermultiplet. The theory is formulated in $4D, \mathcal{N}=2$ harmonic superspace in terms of unconstrained analytic superfields. In section 5 we summarize the results and discuss possible ways of further development of the approach presented.

2 $\mathcal{N}=2$ spin 2 theory

2.1 Minimal Einstein $\mathcal{N}=2$ supergravity in the harmonic approach

We start by a sketch of the basic principles of Einstein $\mathcal{N}=2$ supergravity (SG) in harmonic superspace [3, 47]. Its linearized version provides an off-shell $\mathcal{N}=2$ supersymmetric free spin 2 action and will serve as a prototype for constructing $\mathcal{N}=2$ higher spin actions.

We will deal with $\mathcal{N}=2$ harmonic superspace (HSS) in the analytic basis as the following set of coordinates [3–5]

$$Z=(x^m,\theta^{+\mu},\bar{\theta}^{+\dot{\mu}},u^{\pm}_i,\theta^{-\mu},\bar{\theta}^{-\dot{\mu}})\equiv(\zeta,\theta^{-\mu},\bar{\theta}^{-\dot{\mu}}), \tag{2.1}$$

[3]The free equations of motion for higher-spin massless $\mathcal{N}=2$ superfields have been constructed in the conventional $\mathcal{N}=2$ AdS superspace in ref. [34]. As noted by its authors, the issue of constructing the corresponding Lagrangian formulation remained unresolved in their approach.

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where the standard notation of ref. [3] is used. In particular, $u^\pm_i$ are harmonic variables parametrizing the internal sphere S^2, $u^{+i}u^-_i = 1$, the indices $\pm$ denote the harmonic U(1) charges of various quantities and the index $i = 1, 2$ is the doublet index of the automorphism SU(2) group acting only on the harmonic variables. The set (2.1) is closed under the rigid $\mathcal{N} = 2$ supersymmetry transformations

$$\delta_\epsilon x^m = -2i(\epsilon^- \sigma^m \bar{\theta}^+ + \theta^+ \sigma^m \bar{\epsilon}^-), \; \delta_\epsilon \theta^{\pm\hat{\mu}} = \epsilon^{\pm\hat{\mu}}\,, \; \delta_\epsilon u^\pm_i = 0\,, \quad \epsilon^{\pm\hat{\mu}} = \epsilon^{\hat{\mu}i} u^\pm_i\,, \tag{2.2}$$

where we employed the condense notation, $\hat{\mu} = (\mu, \dot{\mu})$. These transformations also leave intact the harmonic analytic subspace of (2.1),

$$\zeta := (x^m, \theta^{+\mu}, \bar{\theta}^{+\dot{\mu}}, u^\pm_i). \tag{2.3}$$

The HSS formulation of $\mathcal{N} = 2$ SG is displayed in an extension of the HSS (2.1) by a fifth coordinate x^5,

$$Z \Longrightarrow (Z, x^5)\,, \tag{2.4}$$

with the following analyticity-preserving transformation law under $\mathcal{N} = 2$ supersymmetry,

$$\delta_\epsilon x^5 = 2i(\epsilon^- \theta^+ - \bar{\epsilon}^- \bar{\theta}^+). \tag{2.5}$$

This coordinate can be interpreted as associated with the central charge in $\mathcal{N} = 2$ Poincaré superalgebra.

An important ingredient of the HSS formalism is the harmonic derivatives D^{++} and D^{--} which have the following form in the analytic basis[4]

$$\begin{aligned} D^{++} &= \partial^{++} - 2i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{+\hat{\mu}}\partial^+_{\hat{\mu}} + i(\hat{\theta^+})^2\partial_5\,, \\ D^{--} &= \partial^{--} - 2i\theta^{-\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{-\hat{\mu}}\partial^-_{\hat{\mu}} + i(\hat{\theta^-})^2\partial_5\,, \end{aligned} \tag{2.6}$$

$$[D^{++}, D^{--}] = D^0\,, \quad D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+\hat{\mu}}\partial^-_{\hat{\mu}} - \theta^{-\hat{\mu}}\partial^+_{\hat{\mu}}\,. \tag{2.7}$$

The crucial difference between derivatives D^{++} and D^{--} is that D^{++} preserves analyticity, while D^{--} does not.

We will be interested in the simplest version of Einstein $\mathcal{N} = 2$ SG which is obtained from the conformal $\mathcal{N} = 2$ SG by invoking the so called nonlinear multiplet as one of the two necessary compensating multiplets. In the HSS formalism, one uses a gauge in which the analytic superfield which accommodates this compensating multiplet is gauged away to yield the fundamental group of the resulting Einstein $\mathcal{N} = 2$ SG as the following analyticity-preserving superdiffeomorphisms

$$\delta_\lambda x^m = \lambda^m(x, \theta^+, u), \qquad \delta_\lambda x^5 = \lambda^5(x, \theta^+, u)\,, \tag{2.8}$$

$$\begin{aligned} \delta_\lambda \theta^{+\mu} &= \lambda^{+\mu}(x, \theta^+, u), & \delta_\lambda \bar{\theta}^{+\dot{\mu}} &= \bar{\lambda}^{+\dot{\mu}}(x, \theta^+, u), \\ \delta_\lambda \theta^{-\mu} &= \lambda^{-\mu}(x, \theta^+, \theta^-, u), & \delta_\lambda \bar{\theta}^{-\dot{\mu}} &= \bar{\lambda}^{-\dot{\mu}}(x, \theta^+, \theta^-, u), \end{aligned} \tag{2.9}$$

$$\delta_\lambda u^\pm_i = 0. \tag{2.10}$$

[4] Hereafter, we use the notations $\hat{\mu} \equiv (\mu, \dot{\mu})$, $\partial^\pm_{\hat{\mu}} = \partial/\partial\theta^{\mp\hat{\mu}}$, $(\hat{\theta^+})^2 \equiv (\theta^+)^2 - (\bar{\theta}^+)^2$ and $\partial_{\alpha\dot{\alpha}} = \sigma^m_{\alpha\dot{\alpha}}\partial_m$. The summation rules are $\psi\chi = \psi^\alpha\chi_\alpha$, $\bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$, Minkowski metric is diag$(1, -1, -1, -1)$ and $\Box = \partial^m\partial_m = \frac{1}{2}\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$.

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Notice that neither the gauge parameters nor any of the geometrical objects used in the paper depend on the fifth coordinate x^5.

Next one defines a generalization of the flat harmonic derivatives (2.6), $\mathfrak{D}^{\pm\pm}$, such that they were covariant under (2.9)

$$\delta\mathfrak{D}^{\pm\pm} = 0 \;\Rightarrow \tag{2.11}$$

$$\mathfrak{D}^{++} = D^{++} + h^{++m}\partial_m + h^{++\hat{\mu}+}\partial^-_{\hat{\mu}} + h^{++\hat{\mu}-}\partial^+_{\hat{\mu}} + h^{++5}\partial_5\,, \tag{2.12}$$

$$\mathfrak{D}^{--} = D^{--} + h^{--m}\partial_m + h^{--\hat{\mu}+}\partial^-_{\hat{\mu}} + h^{--\hat{\mu}-}\partial^+_{\hat{\mu}} + h^{--5}\partial_5\,. \tag{2.13}$$

The components of the vielbein h^{++M} in (2.12) are analytic superfields, $h^{++M} = h^{++M}(\zeta)$, as the constraints of $\mathcal{N}=2$ SG in the HSS formulation require that [47]

$$[\partial^+_{\hat{\mu}}, \mathfrak{D}^{++}] = 0\,. \tag{2.14}$$

The negatively charged vielbeins in (2.13) are expressed in terms of those in (2.12) from the conditions implied by the harmonic constraint

$$[\mathfrak{D}^{++}, \mathfrak{D}^{--}] = D^0\,, \tag{2.15}$$

which is just a generalization of the flat superspace condition (2.7). The explicit form of the relevant relations will be given below for the linearized theory. The transformation properties of the vielbeins h^{++M} and h^{--M} are uniquely determined by (2.11), whence, in particular, $\delta_\lambda D^0 = 0$. Here we present them for h^{++M}, postponing those for h^{--M} also until the linearized case,

$$\begin{aligned}
\delta_\lambda h^{++m} &= \mathfrak{D}^{++}\lambda^m + 2i\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + 2i\theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\,,\\
\delta_\lambda h^{++5} &= \mathfrak{D}^{++}\lambda^5 - 2i\lambda^{+\hat{\mu}}\theta^+_{\hat{\mu}}\,,\\
\delta_\lambda h^{++\hat{\mu}+} &= \mathfrak{D}^{++}\lambda^{+\hat{\mu}}\,,\\
\delta_\lambda h^{++\hat{\mu}-} &= \mathfrak{D}^{++}\lambda^{-\hat{\mu}} - \lambda^{+\hat{\mu}}\,.
\end{aligned} \tag{2.16}$$

Note, that the non-analytic vielbein $h^{++\hat{\mu}-}$ and the non-analytic parameter $\lambda^{-\hat{\mu}}$ have exactly the same component contents. Therefore, this vielbein can be entirely gauged away,

$$h^{++\hat{\mu}-} = 0\,. \tag{2.17}$$

In this gauge we have the "analytic gauge" condition

$$\mathfrak{D}^{++}\lambda^{-\hat{\mu}} = \lambda^{+\hat{\mu}}\,, \tag{2.18}$$

which fully specifies $\lambda^{-\hat{\mu}}$ in terms of the components of $\lambda^{+\hat{\mu}}$.

Now, using the transformations (2.16), one can display the field content of the vielbeins in the Wess-Zumino gauge:

$$\begin{aligned}
h^{++m} &= -2i\theta^+\sigma^a\bar{\theta}^+\Phi^m_a + (\bar{\theta}^+)^2\theta^+\psi^{m\,i}u^-_i + (\theta^+)^2\bar{\theta}^+\bar{\psi}^{m\,i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2V^{m(ij)}u^-_iu^-_j\,,\\
h^{++5} &= -2i\theta^+\sigma^a\bar{\theta}^+C_a + (\bar{\theta}^+)^2\theta^+\rho^iu^-_i + (\theta^+)^2\bar{\theta}^+\bar{\rho}^iu^-_i + (\theta^+)^2(\bar{\theta}^+)^2S^{(ij)}u^-_iu^-_j\,,\\
h^{++\mu+} &= (\theta^+)^2\bar{\theta}^+_{\dot{\mu}}P^{\mu\dot{\mu}} + \left(\bar{\theta}^+\right)^2\theta^+_\nu\left[\varepsilon^{\mu\nu}M + T^{(\mu\nu)}\right]\\
&\quad + (\theta^+)^2(\bar{\theta}^+)^2\chi^{\mu i}u^-_i\,, \qquad h^{++\dot{\mu}+} = \widetilde{h^{++\mu+}}\,.
\end{aligned} \tag{2.19}$$

This is just the content of the "minimal" $\mathcal{N}=2$ Einstein supergravity multiplet [48, 49] (note that the fields $M, P^{\mu\dot{\mu}}, T^{(\mu\nu)}$ entering $h^{++\mu+}$ in (2.19) are complex). So the analytic superfields $h^{++m}, h^{++\hat{\mu}+}, h^{++5}$ are the unconstrained gauge potentials of the "minimal" $\mathcal{N}=2$ Einstein supergravity. The physical fields are $\Phi^m_a, \psi^{m\,i}_{\hat{\mu}}, C_a$, the remaining ones are auxiliary. After eliminating them from the appropriate action, we are left with the on-shell superspin 1, superisospin 0 multiplet $(\mathbf{2}, \mathbf{3/2}, \mathbf{3/2}, \mathbf{1})$. With taking into account the residual gauge freedom of the WZ gauge (2.19) (see below), the complete set of essential off-shell degrees of freedom is $\mathbf{40}+\mathbf{40}$.

2.2 Linearized theory

In what follows, we will be interested in the linearized version of the above construction.

In general, the negatively charged vielbeins in (2.13) obey rather complicated nonlinear harmonic equations following from the condition (2.15). However, at the linearized level these conditions are essentially simplified: they are reduced to the linear harmonic equations for the gauge potentials:

$$
\begin{aligned}
D^{++}h^{--\alpha\dot{\alpha}} - D^{--}h^{++\alpha\dot{\alpha}} + 4i(h^{--\alpha+}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}h^{--\dot{\alpha}+}) &= 0\,,\\
D^{++}h^{--5} - D^{--}h^{++5} - 2i(h^{--\alpha+}\theta^{+}_{\alpha} - \bar{\theta}^{+}_{\dot{\alpha}}h^{--\dot{\alpha}+}) &= 0\,, \qquad (2.20)
\end{aligned}
$$

$$
\begin{aligned}
D^{++}h^{--\alpha+} - D^{--}h^{++\alpha+} = 0\,, &\qquad D^{++}h^{--\dot{\alpha}+} - D^{--}h^{++\dot{\alpha}+} = 0\,,\\
D^{++}h^{--\alpha-} - h^{--\alpha+} = 0\,, &\qquad D^{++}h^{--\dot{\alpha}-} - h^{--\dot{\alpha}+} = 0\,. \qquad (2.21)
\end{aligned}
$$

These constraints are invariant under the following linearized form of the superfield gauge transformations (2.16) and their counterparts for the negatively charged vielbeins

$$
\begin{aligned}
\delta_\lambda h^{++m} &= D^{++}\lambda^m + 2i(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}})\,,\\
\delta_\lambda h^{++5} &= D^{++}\lambda^5 - 2i(\lambda^{+\alpha}\theta^{+}_{\alpha} - \bar{\theta}^{+}_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}),\\
\delta_\lambda h^{++\hat{\mu}+} &= D^{++}\lambda^{+\hat{\mu}}\,, \qquad (2.22)\\
\delta_\lambda h^{--m} &= D^{--}\lambda^m + 2i(\lambda^{-\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{-\dot{\alpha}} + \theta^{-\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{-\dot{\alpha}})\,,\\
\delta_\lambda h^{--5} &= D^{--}\lambda^5 - 2i(\lambda^{-\alpha}\theta^{-}_{\alpha} - \bar{\theta}^{-}_{\dot{\alpha}}\bar{\lambda}^{-\dot{\alpha}}),\\
\delta_\lambda h^{--\mu+} &= D^{--}\lambda^{+\mu} - \lambda^{-\mu}\,, \qquad \delta_\lambda h^{--\dot{\mu}+} = D^{--}\lambda^{+\dot{\mu}} - \bar{\lambda}^{-\dot{\mu}}\,, \qquad (2.23)\\
\delta_\lambda h^{--\mu-} &= D^{--}\lambda^{-\mu}\,, \qquad \delta_\lambda h^{--\dot{\mu}-} = D^{--}\lambda^{-\dot{\mu}}\,. \qquad (2.24)
\end{aligned}
$$

These transformations can still be used to choose the Wess-Zumino gauge (2.19) for analytic superfields h^{++M} in the linearized theory as well, though in this approximation h^{++M} and h^{--M} loose their geometric meaning of vielbeins. Similarly, the analytic gauge parameters $\lambda^{++m,5}$ and $\lambda^{+\hat{\mu}}$ loose their original geometric meaning of the parameters of the coordinate superdiffeomorphisms preserving the analytic subspace (2.3). The non-analytic gauge parameter $\lambda^{-\hat{\mu}}$ satisfies the linearized form of eq. (2.18),

$$
D^{++}\lambda^{-\hat{\mu}} = \lambda^{+\hat{\mu}}\,. \qquad (2.25)
$$

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As usual, fixing WZ gauge does not fully capture symmetry. The residual gauge freedom of the theory is spanned by the parameters:

$$\begin{cases} \lambda^m \ \Rightarrow \ a^m(x)\,, \\ \lambda^5 \ \Rightarrow \ b(x)\,, \\ \lambda^{\mu+} \ \Rightarrow \ \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu} l_{(\nu}{}^{\mu)}(x)\,, \\ \bar\lambda^{\dot\mu+} \ \Rightarrow \ \bar\epsilon^{\dot\mu i}(x)u^+_i + \bar\theta^{+\dot\nu} l_{(\dot\nu}{}^{\dot\mu)}(x)\,. \end{cases} \tag{2.26}$$

It is natural to make the following identification:

- $a^m(x)$ are the remnants of the diffeomorphism parameters which now form the basic gauge freedom of the free spin 2 field;
- $b(x)$ is the parameter of Abelian gauge transformations acting on the "graviphoton" A^m;
- $\epsilon^{\hat\mu i}(x)$ originate from the parameters of local supersymmetry which are now $\mathcal{N} = 2$ counterparts of the local a^m transformations;
- $l^{(\mu\nu)}$ and $l^{(\dot\mu\dot\nu)}$ are the former parameters of local Lorentz transformations which can be used to gauge away the antisymmetric part of Φ^m_a and so to leave only the symmetric part in the latter (traceless "conformal graviton" and the trace itself).

For the further consideration, it will be instructive to explicitly see (before any gauge-fixing) how the gauge freedom (2.22) allows to remove all the "superfluous" SU(2) singlet bosonic spins from the basic gauge superfields. The relevant shifting local symmetries are contained in the supergauge parameter $\lambda^{+\alpha}, \bar\lambda^{+\dot\alpha}$, while the physical dimension SU(2) singlet spins in $h^{++m,5}$. Passing, for the convenience, to the spinor notation, $h^{++}_{\alpha\dot\alpha} = (\sigma_m)_{\alpha\dot\alpha} h^{++m}$, we identify these particular components as

$$\begin{aligned} h^{++}_{\alpha\dot\alpha} \ &\Rightarrow \ (\theta^+)^2 \omega_{\alpha\dot\alpha} + (\bar\theta^+)^2 \bar\omega_{\alpha\dot\alpha} - 2i\theta^{+\beta}\bar\theta^{+\dot\beta}\Phi_{\beta\dot\beta\alpha\dot\alpha}\,, \\ h^{++5} \ &\Rightarrow \ (\theta^+)^2\omega + (\bar\theta^+)^2\bar\omega + i\theta^{+\beta}\bar\theta^{+\dot\beta} C_{\beta\dot\beta}\,, \\ \lambda^{+\alpha} \ &\Rightarrow \ \theta^{+\alpha}\, l + \theta^{+\beta} l_{(\beta}{}^{\alpha)} + \bar\theta^{+\dot\beta} l^{\alpha}_{\dot\beta}\,, \quad \bar\lambda^{+\dot\alpha} \ \Rightarrow \ \bar\theta^{+\dot\alpha}\,\bar l + \bar\theta^{+\dot\beta}\bar l_{(\dot\beta}{}^{\dot\alpha)} - \theta^{+\beta}\bar l^{\dot\alpha}_{\beta}\,. \end{aligned} \tag{2.27}$$

From the transformation laws (2.22) we find

$$\begin{aligned} \delta\omega_{\alpha\dot\alpha} &= 2i\, l_{\alpha\dot\alpha}\,, \ \delta\bar\omega_{\alpha\dot\alpha} = -2i\,\bar l_{\alpha\dot\alpha}\,, \ \delta\omega = -2il\,, \ \delta\bar\omega = 2i\bar l\,, \ \delta C_{\alpha\dot\alpha} = -2(l_{\alpha\dot\alpha} + \bar l_{\alpha\dot\alpha}), \\ \delta\Phi_{\beta\dot\beta\alpha\dot\alpha} &= -2(\varepsilon_{\alpha\beta}\bar l_{(\dot\beta\dot\alpha)} + \varepsilon_{\dot\alpha\dot\beta} l_{(\beta\alpha)})\,. \end{aligned} \tag{2.28}$$

Decomposing

$$\Phi_{\beta\dot\beta\alpha\dot\alpha} = \Phi_{(\beta\alpha)(\dot\beta\dot\alpha)} + \varepsilon_{\alpha\beta}\Phi_{(\dot\beta\dot\alpha)} + \varepsilon_{\dot\alpha\dot\beta}\Phi_{(\beta\alpha)} + \varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}\Phi\,, \tag{2.29}$$

we observe that the complex fields $\omega_{\alpha\dot\alpha}$ and ϕ are purely gauge degrees of freedom: they can be put equal to zero in accordance with the WZ gauge (2.19); using the local parameters $l_{\alpha\dot\alpha}, \bar l_{\alpha\dot\alpha}$ one can gauge away as well the spin 1 parts of the field $\Phi_{\beta\dot\beta\alpha\dot\alpha}$, to end with the off-shell spin 2 field $(\Phi_{(\beta\alpha)(\dot\beta\dot\alpha)}, \Phi)$ and the spin 1 field $C_{\alpha\dot\alpha}$ as the only surviving physical bosonic gauge fields. The standard residual gauge transformations of these fields are associated with the local parameters $a^m(x)$ and $b(x)$ coming from the gauge superfunctions $\lambda^m(\zeta), \lambda^5(\zeta)$:

$$
\begin{aligned}
\delta_\lambda \Phi_{\beta\dot\beta\alpha\dot\alpha} &= \frac{1}{2}\left(\partial_{\alpha\dot\alpha} a_{\beta\dot\beta} + \partial_{\beta\dot\beta} a_{\alpha\dot\alpha}\right), \quad \delta_\lambda \Phi = \frac{1}{4}\partial_{\alpha\dot\alpha} a^{\alpha\dot\alpha}, && (2.30)\\
\delta_\lambda C_{\alpha\dot\alpha} &= -2\partial_{\alpha\dot\alpha}\, b\,. && (2.31)
\end{aligned}
$$

It was taken into account in (2.30) that the gauge choice $\Phi_{(\beta\alpha)} = 0$ (and c.c.) expresses the parameters $l_{\alpha\dot\alpha}, \bar{l}_{\alpha\dot\alpha}$ as

$$
l_{\alpha\beta} = \frac{1}{4}\partial_{(\alpha\dot\alpha} a^{\dot\alpha}_{\beta)}, \quad l_{\dot\alpha\dot\beta} = \frac{1}{4}\partial_{\beta(\dot\alpha} a^{\beta}_{\dot\beta)}\,. \tag{2.32}
$$

In what follows, an important role is played by the realization of the rigid $\mathcal{N} = 2$ supersymmetry on the superfields $h^{\pm\pm}$. Even before passing to the linearized approximation, it is immediately seen that under the $\mathcal{N} = 2$ transformations (2.2) the covariantized harmonic derivatives (2.12), (2.13) are invariant provided the vielbeins have the following unusual transformation rules

$$
\begin{aligned}
\delta_\epsilon h^{++m} &= -2i\big(h^{++\mu+}\sigma^m_{\mu\dot\mu}\bar\epsilon^{-\dot\mu} + \epsilon^{-\mu}\sigma^m_{\mu\dot\mu} h^{++\dot\mu+}\big)\,,\\
\delta_\epsilon h^{++5} &= 2i\big(h^{++\mu+}\epsilon^-_\mu - \bar\epsilon^-_{\dot\mu} h^{++\dot\mu+}\big)\,,\\
\delta_\epsilon h^{++\hat\mu+} &= 0\,, && (2.33)\\
\delta_\epsilon h^{--m} &= -2i\big(h^{--\mu+}\sigma^m_{\mu\dot\mu}\bar\epsilon^{-\dot\mu} + \epsilon^{-\rho}\sigma^m_{\rho\dot\mu} h^{--\dot\mu+}\big)\,,\\
\delta_\epsilon h^{--5} &= 2i\big(h^{--\mu+}\epsilon^-_\mu - \bar\epsilon^-_{\dot\mu} h^{--\dot\mu+}\big)\,,\\
\delta_\epsilon h^{--\hat\mu+} &= \delta h^{--\hat\mu-} = 0\,. && (2.34)
\end{aligned}
$$

These transformation laws are valid in the linearized limit too. The difference between the nonlinear and linearized cases is that in the former case these rigid transformations form a subgroup of the gauge group (2.16) (and its counterpart for the negatively charged vielbeins), while in the latter case they constitute an independent symmetry (which form a semi-direct product with the relevant gauge transformations (2.23) and (2.24)).

Now, let us define the non-analytic objects which behave as the standard $\mathcal{N} = 2$ superfields

$$
\begin{aligned}
G^{++m} &:= h^{++m} + 2i\big(h^{++\mu+}\sigma^m_{\mu\dot\mu}\bar\theta^{-\dot\mu} + \theta^{-\mu}\sigma^m_{\mu\dot\mu} h^{++\dot\mu+}\big)\,, && (2.35)\\
G^{++5} &:= h^{++5} - 2i\big(h^{++\mu+}\theta^-_\mu - \bar\theta^-_{\dot\mu} h^{++\dot\mu+}\big)\,, && (2.36)\\
G^{--m} &:= h^{--m} + 2i\big(h^{--\mu+}\sigma^m_{\mu\dot\mu}\bar\theta^{-\dot\mu} + \theta^{-\mu}\sigma^m_{\mu\dot\mu} h^{++\dot\mu+}\big)\,, && (2.37)\\
G^{--5} &:= h^{--5} - 2i\big(h^{--\mu+}\theta^-_\mu - \bar\theta^-_{\dot\mu} h^{--\dot\mu+}\big)\,. && (2.38)
\end{aligned}
$$

It is easy to check that[5]

$$
\delta_\epsilon G^{++m} = \delta_\epsilon G^{++5} = \delta_\epsilon G^{--m} = \delta_\epsilon G^{--5} = 0\,. \tag{2.39}
$$

[5] We denote by δ_ϵ the so called passive transformations differing from the more accustomed "active" transformations δ^*_ϵ by the "transport term", $\delta^*_\epsilon = \delta_\epsilon - \delta_\epsilon Z^M \partial_M$.

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The newly introduced objects also possess simple transformation properties under the gauge transformations (2.22)–(2.24)

$$\delta_\lambda G^{\pm\pm m} = D^{\pm\pm}\Lambda^m\,, \qquad\qquad \delta_\lambda G^{\pm\pm 5} = D^{\pm\pm}\Lambda^5\,, \tag{2.40}$$

$$\Lambda^m = \lambda^m + 2i(\lambda^+\sigma^m\bar\theta^- + \theta^-\sigma^m\bar\lambda^+)\,, \qquad \Lambda^5 = \lambda^5 - 2i(\lambda^+\theta^- - \bar\theta^-\bar\lambda^+), \tag{2.41}$$

and satisfy the flatness conditions

$$D^{++}G^{--m} = D^{--}G^{++m}\,, \quad D^{++}G^{--5} = D^{--}G^{++5} \tag{2.42}$$

as a direct consequence of the harmonic equations (2.20)–(2.21). The invariant linearized action of $\mathcal{N}=2$ SG can be constructed just from these objects.

Let us pass to the spinor notation,

$$\begin{aligned} G^{\pm\pm\alpha\dot\alpha} &= (\tilde\sigma_m)^{\alpha\dot\alpha}G^{\pm\pm m}\,, \delta_\lambda G^{\pm\pm\alpha\dot\alpha} = D^{\pm\pm}\Lambda^{\alpha\dot\alpha}\,,\\ \Lambda^{\alpha\dot\alpha} &= \lambda^{\alpha\dot\alpha} + 4i(\lambda^{+\alpha}\bar\theta^{-\dot\alpha} + \theta^{-\alpha}\bar\lambda^{+\dot\alpha})\,,\\ G^{\pm\pm\alpha\dot\alpha} &= h^{\pm\pm\alpha\dot\alpha} + 4i(h^{\pm\pm\alpha+}\bar\theta^{-\dot\alpha} + \theta^{-\alpha}\bar h^{\pm\pm\dot\alpha+})\,, \end{aligned} \tag{2.43}$$

and consider the manifestly $\mathcal{N}=2$ supersymmetric action

$$S_1 = \int d^4x d^8\theta du\, G^{++\alpha\dot\alpha}G^{--}_{\alpha\dot\alpha}\,. \tag{2.44}$$

Its gauge variation, with taking into account the relation (2.42), can be reduced to the expression

$$\delta_\lambda S_1 = 2\int d^4x d^8\theta du\, D^{--}\Lambda^{\alpha\dot\alpha}G^{++}_{\alpha\dot\alpha}\,. \tag{2.45}$$

Next, we pass to the integral over the analytic subspace using

$$\int d^4x d^8\theta du = \int d\zeta^{-4}du (D^+)^4\,, \quad (D^+)^4 = \frac{1}{16}(\bar D^+)^2(D^+)^2\,. \tag{2.46}$$

After some algebra, using the relation

$$\{D^+_\alpha, \bar D^-_{\dot\alpha}\} = -2i\partial_{\alpha\dot\alpha}\,, \quad D^-_\alpha = [D^{--}, D^+_\alpha] \text{ (and c.c.)}\,,$$

as well as the property that both $\Lambda^{\alpha\dot\alpha}$ and $G^{++}_{\alpha\dot\alpha}$ are linear in $\theta^-_\alpha, \bar\theta^-_{\dot\beta}$ with analytic coefficients, we can represent this variation as

$$\delta_\lambda S_1 = 8i\int d\zeta^{-4}du(\partial_{\beta\dot\beta}\lambda^{+\beta}h^{++\dot\beta+} - \partial_{\beta\dot\beta}\bar\lambda^{+\dot\beta}h^{++\beta+}). \tag{2.47}$$

As the second step, we define

$$S_2 = \int d^4x d^8\theta du\, G^{++5}G^{--5} \tag{2.48}$$

and, applying similar manipulations, find

$$\delta_\lambda S_2 = -2i\int d\zeta^{-4}du(\partial_{\beta\dot\beta}\lambda^{+\beta}h^{++\dot\beta+} - \partial_{\beta\dot\beta}\bar\lambda^{+\dot\beta}h^{++\beta+}). \tag{2.49}$$

So we come to the conclusion that the sum

$$S_{(s=2)} \sim S_1 + 4S_2 = -\frac{1}{4}\int d^4x d^8\theta du \left(G^{++\alpha\dot{\alpha}}G^{--}_{\alpha\dot{\alpha}} + 4G^{++5}G^{--5}\right) \tag{2.50}$$

is invariant under both rigid $\mathcal{N}=2$ supersymmetry and linearized gauge transformations. So it is the invariant action of the linearized $\mathcal{N}=2$ SG and the true $\mathcal{N}=2$ extension of the free spin 2 action. It was firstly given in [50].[6] The choice of the normalization constant in this action will become clear after considering its component bosonic sector in the WZ gauge (2.19). Note that, while proving gauge invariance of S_1+4S_2, we did not make use of the precise structure of $G^{--}_{\alpha\dot{\alpha}}$ and G^{--5}, only the flatness conditions (2.42) were employed.

2.3 Passing to components

When calculating the component action, the most annoying problem is to restore the negatively charged non-analytic gauge superfields by the basic analytic ones $h^{++}_{\alpha\dot{\beta}}, h^{++5}$ and h^{+3}_{α} by using eqs. (2.20)–(2.21) (or (2.42) for $G^{\pm\pm}_{\alpha\dot{\beta}}$ and $G^{\pm\pm5}$). We shall present the full $\mathcal{N}=2$ component actions for any spin elsewhere; here we limit ourselves to their bosonic sectors. Moreover, we will be basically interested in the actions for the physical gauge fields; in all cases, the auxiliary bosonic fields produce some bilinear terms and so vanish on shell.

For the considered spin 2 case we should firstly substitute the bosonic reduction of the WZ gauge (2.19), with the additional gauge choice $\Phi_{\alpha\dot{\alpha}\beta\dot{\beta}} = \Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi$, to eqs. (2.20)–(2.21). Since the latter are linear, one can solve them separately for each term in the analytic gauge superfield. The relevant solution for the appropriate set of negatively charged gauge potentials will contain this fixed component field together with its x-derivatives. As an important example, consider the following analytic monomial

$$h^{++A}_{(\Phi)} = G^{++A}_{(\Phi)} := i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}\Phi^{A}_{\beta\dot{\beta}}\,, \tag{2.51}$$

where the precise value of the external index A is of no interest for us for the moment. For the corresponding part of the negatively charged gauge potential we obtain

$$\begin{aligned} G^{--A}_{(\Phi)} = {}& i\theta^{-\beta}\bar{\theta}^{-\dot{\beta}}\Phi^{A}_{\beta\dot{\beta}} - (\theta^-)^2\bar{\theta}^{-(\dot{\rho}}\bar{\theta}^{+\dot{\beta})}\partial^{\beta}_{\dot{\rho}}\Phi^{A}_{\beta\dot{\beta}} + (\bar{\theta}^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot{\beta}}_{\rho}\Phi^{A}_{\beta\dot{\beta}} \\ & - i(\theta^-)^2(\bar{\theta}^-)^2\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\left[\Box\Phi^{A}_{\rho\dot{\rho}} - \frac{1}{2}\partial_{\rho\dot{\rho}}\partial^{\beta\dot{\beta}}\Phi^{A}_{\beta\dot{\beta}}\right]. \end{aligned} \tag{2.52}$$

To find the component $(C,\ \Phi)$ action, one should also take into account that the field $P^{\alpha\dot{\beta}}$ in $h^{++\alpha+}$ in (2.19) (and its conjugate $\bar{P}^{\dot{\alpha}\beta}$ in $h^{++\dot{\alpha}+}$) is transformed under the gauge spin 2 transformations,

$$\delta_\lambda P^{\alpha\dot{\beta}} = i\partial^{\dot{\beta}}_{\beta}l^{\alpha\beta} = -\frac{i}{4}\left(\Box a^{\alpha\dot{\beta}} - \frac{1}{2}\partial^{\alpha\dot{\beta}}\,\partial^{\gamma\dot{\gamma}}a_{\gamma\dot{\gamma}}\right),$$

and so one needs to pass to the inert field $\tilde{P}_{\alpha\dot{\alpha}}$ through the redefinition

$$P^{\mu\dot{\mu}} = \tilde{P}^{\mu\dot{\mu}} + iB^{\mu\dot{\mu}}, \qquad \bar{P}^{\mu\dot{\mu}} = \tilde{\bar{P}}^{\mu\dot{\mu}} - iB^{\mu\dot{\mu}}\,, \tag{2.53}$$

[6] The linearized $\mathcal{N}=2$ SG in the ordinary $\mathcal{N}=2$ superspace was considered in [51].

where

$$B_{\beta\dot\beta} = \frac{1}{4}\left\{3\partial_{\beta\dot\beta}\Phi - \partial^{\alpha\dot\alpha}\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)}\right\}, \quad \delta_\lambda B_{\alpha\dot\beta} = -\frac{1}{4}\Big(\Box a_{\alpha\dot\beta} - \frac{1}{2}\partial_{\alpha\dot\beta}\,\partial^{\gamma\dot\gamma}a_{\gamma\dot\gamma}\Big). \tag{2.54}$$

All other auxiliary bosonic fields entering (2.19) are inert under gauge transformations and all, besides the tensorial one $T^{(\mu\nu)}$, produce bilinear component actions and so disappear on shell. The tensorial field plays an interesting role and should be retained.

Firstly we consider the part $G^{++5}G^{--5}$ in (2.50). The C-gauge field sector is determined by the analytic gauge potential

$$G^{++5}_{(C)} = i\theta^{+\rho}\bar\theta^{+\dot\rho}C_{\rho\dot\rho}\,. \tag{2.55}$$

In accord with the general formula (2.52):

$$\begin{aligned} G^{--5}_{(C)} &= i\theta^{-\beta}\bar\theta^{-\dot\beta}C_{\beta\dot\beta} - (\theta^-)^2\bar\theta^{-(\dot\rho}\bar\theta^{+\dot\beta)}\partial^\beta_{\dot\rho}C_{\beta\dot\beta} + (\bar\theta^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot\beta}_\rho C_{\beta\dot\beta} \\ &\quad - i(\theta^-)^2(\bar\theta^-)^2\theta^{+\rho}\bar\theta^{+\dot\rho}\left[\Box C_{\rho\dot\rho} - \partial_{\rho\dot\rho}\partial^m C_m\right]. \end{aligned} \tag{2.56}$$

As for the tensorial auxiliary field, it gives contributions to both $G^{++\alpha\dot\alpha}$ and G^{++5}, so we are led to compute both $G^{--\alpha\dot\alpha}$ and G^{--5}. However, it can be shown that $G^{++\alpha\dot\alpha}_{(T)}G^{--}_{\alpha\dot\alpha(T)}$ does not contribute to the component Lagrangian, only $G^{++5}_{(T)}$ and $G^{--5}_{(T)}$ do. For them we have the following expressions

$$G^{++5}_{(T)} = -2i(\bar\theta^+)^2\theta^+_\nu\theta^-_\mu T^{(\mu\nu)} - 2i(\theta^+)^2\bar\theta^+_{\dot\nu}\bar\theta^-_{\dot\mu}\bar T^{(\dot\mu\dot\nu)}\,, \tag{2.57}$$

$$\begin{aligned} G^{--5}_{(T)} &= -2i(\bar\theta^-)^2\theta^+_\nu\theta^-_\mu T^{(\mu\nu)} - 2i(\theta^-)^2\bar\theta^+_{\dot\nu}\bar\theta^-_{\dot\mu}\bar T^{(\dot\mu\dot\nu)} \\ &\quad + 2(\bar\theta^-)^2(\theta^-)^2\bar\theta^{+\dot\rho}\theta^+_\mu\partial_{\rho\dot\rho}T^{(\mu\rho)} + 2(\theta^-)^2(\bar\theta^-)^2\theta^{+\rho}\bar\theta^+_{\dot\mu}\partial_{\rho\dot\nu}\bar T^{(\dot\mu\dot\nu)}. \end{aligned} \tag{2.58}$$

The total contribution of

$$G^{++5}_{(C)}G^{--5}_{(C)} + G^{++5}_{(T)}G^{--5}_{(T)} + G^{++5}_{(C)}G^{--5}_{(T)} + G^{++5}_{(T)}G^{--5}_{(C)}$$

to the component Lagrangian in (2.50) reads

$$\begin{aligned} \mathcal{L}_{(C,T)} &= \frac{1}{4}F^{mn}F_{mn} - [T^{(\dot\alpha\dot\gamma)}T_{(\dot\alpha\dot\gamma)} + T^{(\alpha\gamma)}T_{(\alpha\gamma)}] \\ &\quad + i[T^{(\dot\alpha\dot\gamma)}\partial^\beta_{(\dot\alpha}C_{\beta\dot\gamma)} - T^{(\beta\rho)}\partial^{\dot\beta}_{(\beta}C_{\rho)\dot\beta}], \end{aligned} \tag{2.59}$$

where

$$\partial^{\dot\beta}_{(\beta}C_{\rho)\dot\beta} = \frac{i}{2}(\sigma^{mn})_{\beta\rho}F_{mn}\,,\ \partial^\beta_{(\dot\alpha}C_{\beta\dot\gamma)} = -\frac{i}{2}(\tilde\sigma^{mn})_{\dot\alpha\dot\gamma}F_{mn}\,,\ F_{mn} = \partial_m C_n - \partial_n C_m\,.$$

We observe the mixing between $T^{(\alpha\beta)}, T^{(\dot\alpha\dot\beta)}$ and the gauge field strength F^{mn}. After removing this mixing by redefining the tensorial fields as

$$T_{(\alpha\beta)} = \tilde T_{(\alpha\beta)} + \frac{i}{2}\partial^{\dot\beta}_{(\alpha}C_{\beta)\dot\beta}, \quad T_{(\dot\alpha\dot\beta)} = \tilde T_{(\dot\alpha\dot\beta)} - \frac{i}{2}\partial^\beta_{(\dot\alpha}C_{\beta\dot\beta)} \tag{2.60}$$

we obtain

$$\mathcal{L}_{(C,T)} = -\frac{1}{4}F^{mn}F_{mn} - [\tilde{T}^{(\dot\alpha\dot\gamma)}\tilde{T}_{(\dot\alpha\dot\gamma)} + \tilde{T}^{(\alpha\gamma)}\tilde{T}_{(\alpha\gamma)}]. \tag{2.61}$$

We see that the sign of kinetic term of the gauge field has changed after this procedure and this explain the choice of the normalization factor before the action (2.50).[7]

The pure spin 2 part of the action (2.50) is obtained from the following expressions for the pure gravitation parts of $G^{++}_{\alpha\dot\alpha}$ and G^{++5} given below.

$$G^{++\alpha\dot\alpha}_{(\Phi)} = -2i\theta^{+\beta}\bar\theta^{+\dot\beta}\Phi^{\alpha\dot\alpha}_{\beta\dot\beta} + 4(\theta^+)^2\bar\theta^{+\dot\beta}\bar\theta^{-\dot\alpha}B^{\alpha}_{\dot\beta} - 4(\bar\theta^+)^2\theta^{+\beta}\theta^{-\alpha}B^{\dot\alpha}_{\beta}\,, \tag{2.62}$$

$$G^{++5}_{(\Phi)} = -2(\theta^+)^2\bar\theta^{+\dot\rho}\theta^-_{\mu}B^{\mu}_{\dot\rho} - 2(\bar\theta^+)^2\theta^{+\beta}\bar\theta^-_{\dot\rho}B^{\dot\rho}_{\beta}\,. \tag{2.63}$$

The expressions for the relevant negatively charged potentials $G^{--\alpha\dot\alpha}_{(\Phi)}$ and $G^{--5}_{(\Phi)}$, are rather bulky and are given in appendix (eqs. (A.1) and (A.2)).

After some simple though time-consuming computation we find the contribution of $G^{++\alpha\dot\alpha}_{(\Phi)}G^{--}_{(\Phi)\alpha\dot\alpha} + 4G^{++5}_{(\Phi)}G^{--5}_{(\Phi)}$ to the component spin 2 Lagrangian

$$\begin{aligned}\mathcal{L}_{(\Phi)} = -\frac{1}{4}\Big[&\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\Box\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - \Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\alpha\dot\alpha}\partial^{\rho\dot\rho}\Phi_{(\rho\beta)(\dot\rho\dot\beta)} \\ &+2\,\Phi\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - 6\Phi\Box\Phi\Big].\end{aligned} \tag{2.64}$$

It is easy to check that this Lagrangian is invariant, up to a total derivative, under the gauge transformations (2.30). It has a correct sign agreed with that of the spin 1 Lagrangian (2.61).

So in the gauge bosonic sector we are left with the spin 2 fields $(\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}, \Phi)$ and the spin 1 field $C_{\alpha\dot\alpha}$ with the correct Lagrangians and gauge transformations. This directly extends to the $\mathcal{N} = 2$, $\mathbf{s} > \mathbf{2}$ cases.

3 Generalization to $\mathcal{N} = 2$ spin 3 theory

3.1 Superfield contents and gauge symmetries

In the $\mathcal{N} = 2$ supersymmetric theory of the free spin 2 described above, the basic analytic superfield objects have a nice geometric meaning, being linearized versions of the $\mathcal{N} = 2$ supergravity analytic supervielbein covariantizing the analyticity-preserving harmonic derivative $\mathfrak{D}^{++}$ with respect to the superdiffeomorphism group (2.8)–(2.9). For spins $s > 2$ we are not aware of such a nice geometric picture. Nevertheless, it turns out that the problem of constructing the relevant off-shell formalism can be solved just by properly generalizing the formalism of the linearized $\mathcal{N} = 2$ supergravity described in section 2.2.

We start with $\mathbf{s} = \mathbf{3}$. We introduce the real $\mathcal{N} = 2$ bosonic superfields $h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)$, $h^{++\alpha\dot\alpha}(\zeta)$ (of scaling dimension -1) and the conjugated fermionic superfields $h^{++(\alpha\beta)\dot\alpha+}(\zeta)$,

[7]This sign is inherited from the total $\mathcal{N} = 2$ SG action [3], where the Maxwell superfield h^{++5} plays the role of compensator for the underlying gauge $\mathcal{N} = 2$ superconformal group and, as is common for compensators, its action has a wrong sign as compared to any other Maxwell multiplet. E.I. thanks Bernard de Wit for useful correspondence on this issue.

$h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)$ (of dimension $-1/2$), all being unconstrained analytic. We ascribe to them the following gauge transformation rules as a direct generalization of (2.22):

$$\begin{aligned}
\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],\\
\delta h^{++\alpha\dot\alpha} &= D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
\delta h^{++(\alpha\beta)\dot\alpha+} &= D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\\
\delta h^{++(\dot\alpha\dot\beta)\alpha+} &= D^{++}\bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\,.
\end{aligned} \tag{3.2}$$

Like in the $\mathbf{s}=\mathbf{2}$ case, let us first to see which kind of unremovable bosonic SU(2) singlet ("white") gauge fields is retained in the newly defined gauge potentials. As in the case of spin 2, a simple analysis shows that all shifting SU(2) singlet local symmetries are concentrated in the gauge parameters $\lambda^{+(\alpha\beta)\dot\alpha}, \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}$, while all bosonic gauge fields in the potentials $h^{++(\alpha\beta)(\dot\alpha\dot\beta)}$ and $h^{++\alpha\dot\alpha}$. Singling out in both sets of the objects the relevant SU(2) singlet components, we find

$$\begin{aligned}
h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &\Rightarrow (\theta^+)^2\omega^{(\alpha\beta)(\dot\alpha\dot\beta)} + (\bar\theta^+)^2\bar\omega^{(\alpha\beta)(\dot\alpha\dot\beta)} - 2i\theta^+_\gamma\bar\theta^+_{\dot\gamma}\Phi^{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)}\,,\\
h^{++\alpha\dot\alpha} &\Rightarrow (\theta^+)^2\omega^{\alpha\dot\alpha} + (\bar\theta^+)^2\bar\omega^{\alpha\dot\alpha} - 2i\theta^+_\gamma\bar\theta^+_{\dot\gamma}C^{\gamma\dot\gamma\alpha\dot\alpha}\,,\\
\lambda^{+(\alpha\beta)\dot\alpha} &\Rightarrow l^{(\alpha\beta)\dot\alpha\gamma}\theta^+_\gamma + l^{(\alpha\beta)\dot\alpha\dot\gamma}\bar\theta^+_{\dot\gamma}\,,\\
\bar\lambda^{+(\dot\alpha\dot\beta)\alpha} &\Rightarrow \bar l^{(\dot\alpha\dot\beta)\alpha\dot\gamma}\bar\theta^+_{\dot\gamma} - \bar l^{(\dot\alpha\dot\beta)\alpha\gamma}\theta^+_\gamma\,.
\end{aligned} \tag{3.3}$$

The transformation laws (3.1) imply the following gauge transformations for "white" component fields:

$$\begin{aligned}
\delta\omega^{\alpha\dot\alpha} &= il^{(\alpha\beta)\dot\alpha}{}_{\beta}\,, \qquad\qquad \delta\bar\omega^{\alpha\dot\alpha} = -i\bar l^{(\dot\alpha\dot\beta)\alpha}{}_{\dot\beta}\,,\\
\delta C^{\gamma\dot\gamma\alpha\dot\alpha} &= \frac{1}{2}\big[\bar l^{(\dot\alpha\dot\gamma)\alpha\gamma} - l^{(\alpha\gamma)\dot\alpha\dot\gamma}\big]\,,\\
\delta\omega^{(\alpha\beta)(\dot\alpha\dot\beta)} &= 2il^{(\alpha\beta)(\dot\alpha\dot\beta)}\,, \quad \delta\bar\omega^{(\alpha\beta)(\dot\alpha\dot\beta)} = -2i\bar l^{(\dot\alpha\dot\beta)(\alpha\beta)}\,,\\
\delta\Phi^{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)} &= -2\big[l^{(\alpha\beta)(\dot\alpha\gamma}\varepsilon^{\dot\beta)\dot\gamma} + \bar l^{(\dot\alpha\dot\beta)(\alpha\dot\gamma}\varepsilon^{\beta)\gamma}\big].
\end{aligned}$$

We have verified that the gauge freedom associated with the complex parameters $l^{(\alpha\beta)\dot\alpha\gamma}$ and $l^{(\alpha\beta)\dot\alpha\dot\gamma}$ (and c.c.) is powerful enough to gauge away fields $\omega^{\alpha\dot\alpha}, \bar\omega^{\alpha\dot\alpha}, \omega^{(\alpha\beta)(\dot\alpha\dot\beta)}, \bar\omega^{(\alpha\beta)(\dot\alpha\dot\beta)}$. Also one can use it to gauge away all the components in $C^{\gamma\dot\gamma\alpha\dot\alpha}$ apart from

$$C^{(\gamma\alpha)(\dot\gamma\dot\alpha)} + \varepsilon^{\gamma\alpha}\varepsilon^{\dot\gamma\dot\alpha}C \tag{3.4}$$

and all the components in $\Phi^{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)}$ apart from

$$\Phi^{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)} + \varepsilon^{\dot\gamma(\dot\alpha}\varepsilon^{\gamma(\beta}\Phi^{\alpha)\dot\beta)}\,. \tag{3.5}$$

So there survive only the pairs of fields $C^{(\gamma\alpha)(\dot\gamma\dot\alpha)}, C$ and $\Phi^{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}, \Phi^{\alpha\dot\beta}$ needed for the consistent description of massless spins 2 and 3, respectively [6].

Now we perform a more detailed analysis of the gauge freedom, prior to imposing any gauge on the gauge fields C and Φ. This analysis leads to the following Wess-Zumino type gauge for the considered case

$$
\begin{aligned}
h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + (\bar\theta^+)^2\theta^+\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}u^-_i \\
&\quad + (\theta^+)^2\bar\theta^+\bar\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}u^-_i + (\theta^+)^2(\bar\theta^+)^2 V^{(\alpha\beta)(\dot\alpha\dot\beta)(ij)}u^-_i u^-_j\,, \\
h^{++\alpha\dot\alpha} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + (\bar\theta^+)^2\theta^+\rho^{\alpha\dot\alpha i}u^-_i + (\theta^+)^2\bar\theta^+\bar\rho^{\alpha\dot\alpha i}u^-_i + (\theta^+)^2(\bar\theta^+)^2 S^{\alpha\dot\alpha(ij)}u^-_i u^-_j\,, \\
h^{++(\alpha\mu)\dot\alpha+} &= (\theta^+)^2\bar\theta^+_{\dot\mu}P^{(\alpha\mu)\dot\alpha\dot\mu} + \left(\bar\theta^+\right)^2\theta^+_\nu\left[\varepsilon^{\nu(\alpha}M^{\mu)\dot\alpha} + T^{\dot\alpha(\alpha\mu\nu)}\right] + (\theta^+)^2(\bar\theta^+)^2\chi^{(\alpha\mu)\dot\alpha i}u^-_i\,, \\
h^{++\alpha(\dot\alpha\dot\mu)+} &= \widetilde{(h^{++(\alpha\mu)\dot\alpha+})}\,.
\end{aligned}
\tag{3.6}
$$

The relevant residual gauge freedom is spanned by the following set of parameters

$$
\begin{cases}
\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} \;\Rightarrow\; a^{(\alpha\beta)(\dot\alpha\dot\beta)}(x)\,, \\
\lambda^{\alpha\dot\alpha} \;\Rightarrow\; b^{\alpha\dot\alpha}(x)\,, \\
\lambda^{(\mu\alpha)\dot\alpha+} \;\Rightarrow\; \epsilon^{(\mu\alpha)\dot\alpha i}(x)u^+_i + \bar\theta^{+\dot\alpha}n^{(\mu\alpha)} + \theta^{+\nu}l_{(\nu}{}^{\mu\alpha)\dot\alpha}(x)\,, \\
\bar\lambda^{\alpha(\dot\alpha\dot\mu)+} \;\Rightarrow\; \bar\epsilon^{\alpha(\dot\alpha\dot\mu)i}(x)u^+_i + \theta^{+\alpha}n^{(\dot\alpha\dot\mu)} + \bar\theta^{+\dot\nu}l_{(\dot\nu}{}^{\alpha\dot\alpha\dot\mu)}(x)\,.
\end{cases}
\tag{3.7}
$$

These parameters are identified as:

- $a^{(\alpha\beta)(\dot\alpha\dot\beta)}(x)$ are local parameters of the spin 3 gauge transformations;
- $b^{\alpha\dot\alpha}(x)$ are local parameters of the spin 2 gauge transformations;
- $\epsilon^{(\mu\alpha)\dot\alpha i}(x)$ and $\bar\epsilon^{\alpha(\dot\alpha\dot\mu)i}(x)$ are parameters of local spin 3 fermionic symmetry (an analog of the fermionic local symmetry for spin 2 in (2.26));
- $n^{(\mu\alpha)}$ and $n^{(\dot\alpha\dot\mu)}$ are parameters of local "Lorentz rotations" (they were present in (2.26) as well);
- $l^{(\nu\mu\alpha)\dot\alpha}(x)$ and $l^{\alpha(\dot\nu\dot\alpha\dot\mu)}(x)$ are new spin 3 analogs of the local "Lorentz rotations".

Note that the latter two types of parameters have been already used when coming to the irreducible contents of the bosonic gauge fields (3.4) and (3.5) before attaining the complete WZ gauge. The bosonic fields $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}, C^{\alpha\dot\alpha}_{\rho\dot\rho}$, and the fermionic ones $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_\rho$ (and c.c.) are physical, the remaining fields are auxiliary. Keeping in mind the residual gauge freedom, we are left with the full set of $\mathbf{104+104}$ off-shell degrees of freedom. On shell, the multiplet $(\mathbf{3}, \mathbf{5/2}, \mathbf{5/2}, \mathbf{2})$ is retained.

The transformation laws (3.2) imply the following residual bosonic transformation laws:

- Spin 3 sector

$$
\delta\Phi^{(\alpha\gamma)(\dot\alpha\dot\gamma)}_{\beta\dot\beta} = \partial_{\beta\dot\beta}a^{(\alpha\gamma)(\dot\alpha\dot\gamma)} - 2l_{(\beta}{}^{\alpha\gamma)(\dot\alpha}\delta^{\dot\gamma)}_{\dot\beta} - 2l_{(\dot\beta}{}^{\dot\alpha\dot\gamma)(\alpha}\delta^{\gamma)}_{\beta}\,.
\tag{3.8}
$$

We decompose the spin 3 field into the irreducible parts as:

$$
\begin{aligned}
\Phi_{(\alpha\gamma)\beta(\dot\alpha\dot\gamma)\dot\beta} &= \Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} + \Phi_{(\alpha\gamma\beta)(\dot\alpha}\varepsilon_{\dot\gamma)\dot\beta} \\
&\quad + \Phi_{(\alpha(\dot\alpha\dot\gamma\dot\beta)}\varepsilon_{\gamma)\beta} + \Phi_{(\alpha(\dot\alpha}\varepsilon_{\dot\gamma)\dot\beta}\varepsilon_{\gamma)\beta}\,.
\end{aligned}
\tag{3.9}
$$

Using the spin 3 Lorentz transformation one can gauge away $\Phi_{(\alpha\gamma\beta)\dot\alpha}$ and $\Phi_{\alpha(\dot\alpha\dot\gamma\dot\beta)}$, thus recovering the irreducible field content (3.5):

$$\Phi_{(\alpha\gamma)\beta(\dot\alpha\dot\gamma)\dot\beta} = \Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} + \Phi_{(\alpha(\dot\alpha}\varepsilon_{\dot\gamma)\dot\beta}\varepsilon_{\gamma)\beta}\,, \tag{3.10}$$

$$\Phi_{\alpha\dot\alpha} = \frac{1}{9}\varepsilon^{\gamma\beta}\varepsilon^{\dot\gamma\dot\beta}\Phi_{(\alpha\gamma)\beta(\dot\alpha\dot\gamma)\dot\beta}\,. \tag{3.11}$$

Residual spin 3 "Lorentz" transformations are determined from preserving the gauge $\Phi_{(\alpha\gamma\beta)\dot\alpha} = \Phi_{\alpha(\dot\alpha\dot\gamma\dot\beta)} = 0$:

$$\delta\Phi_{(\alpha\gamma\beta)\dot\alpha} = \frac{2}{3}\partial_{(\alpha\dot\beta}a^{(\dot\beta}_{\gamma\beta)\dot\alpha)} - 2l_{(\alpha\gamma\beta)\dot\alpha} = 0, \;\Rightarrow\; l_{(\alpha\gamma\beta)\dot\alpha} = \frac{1}{3}\partial_{(\alpha\dot\beta}a^{(\dot\beta}_{\gamma\beta)\dot\alpha)}\,, \tag{3.12}$$

$$\delta\Phi_{\alpha(\dot\alpha\dot\gamma\dot\beta)} = \frac{2}{3}\partial_{\beta(\dot\alpha}a^{(\beta}_{\dot\gamma\dot\beta)\alpha)} - 2l_{\alpha(\dot\alpha\dot\gamma\dot\beta)} = 0, \;\Rightarrow\; l_{\alpha(\dot\alpha\dot\gamma\dot\beta)} = \frac{1}{3}\partial_{\beta(\dot\alpha}a^{(\beta}_{\dot\gamma\dot\beta)\alpha)}\,. \tag{3.13}$$

For the irreducible pieces in (3.10) we obtain the following transformations

$$\delta\Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} = \partial_{(\beta(\dot\beta}a_{\alpha\gamma)\dot\alpha\dot\gamma)}\,, \tag{3.14}$$

$$\delta\Phi_{\alpha\dot\beta} = \frac{4}{9}\partial^{\gamma\dot\gamma}a_{(\alpha\gamma)(\dot\beta\dot\gamma)} = \frac{8}{9}\partial^m a_{m\alpha\dot\beta}\,. \tag{3.15}$$

These are the correct gauge transformation laws for the spin 3 fields.

- Spin 2 sector

  The transformation law of the spin 2 field $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ entering the analytic potential $h^{++\alpha\dot\alpha}$ reads:

$$\delta C^{\alpha\dot\alpha}_{\rho\dot\rho} = \partial_{\rho\dot\rho}b^{\alpha\dot\alpha} - n^{\dot\alpha}_{\dot\rho}\delta^{\alpha}_{\rho} + n^{\alpha}_{\rho}\delta^{\dot\alpha}_{\dot\rho}\,. \tag{3.16}$$

  After decomposing this field into the irreducible parts,

$$C_{\alpha\beta\dot\alpha\dot\beta} = C_{(\alpha\beta)(\dot\alpha\dot\beta)} + C_{(\alpha\beta)}\varepsilon_{\dot\alpha\dot\beta} + C_{(\dot\alpha\dot\beta)}\varepsilon_{\alpha\beta} + C\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}\,, \tag{3.17}$$

  one can gauge away $C_{(\alpha\beta)}$ and $C_{(\dot\alpha\dot\beta)}$, using local "Lorentz" shifts. The residual transformations are found from preserving this "physical" gauge:

$$2\delta C_{(\alpha\beta)} = \partial_{(\alpha\dot\alpha}b^{\dot\alpha}_{\beta)} + 2n_{\alpha\beta} = 0 \quad\Rightarrow\quad n_{\alpha\beta} = -\frac{1}{2}\partial_{(\alpha\dot\alpha}b^{\dot\alpha}_{\beta)}\,, \tag{3.18}$$

$$2\delta C_{(\dot\alpha\dot\beta)} = \partial_{\beta(\dot\alpha}b^{\beta}_{\dot\beta)} - 2n_{\dot\alpha\dot\beta} = 0 \quad\Rightarrow\quad n_{\dot\alpha\dot\beta} = \frac{1}{2}\partial_{\beta(\dot\alpha}b^{\beta}_{\dot\beta)}\,. \tag{3.19}$$

  Finally, the spin 2 field is represented as:

$$C_{\alpha\beta\dot\alpha\dot\beta} = C_{(\alpha\beta)(\dot\alpha\dot\beta)} + C\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}, \qquad C = \frac{1}{4}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}C_{\alpha\beta\dot\alpha\dot\beta}\,, \tag{3.20}$$

  with the following transformation laws for the constituent fields:

$$\delta C_{\alpha\beta\dot\alpha\dot\beta} = \frac{1}{2}\left(\partial_{\alpha\dot\alpha}b_{\beta\dot\beta} + \partial_{\beta\dot\beta}b_{\alpha\dot\alpha}\right), \tag{3.21}$$

$$\delta C_{(\alpha\beta)(\dot\alpha\dot\beta)} = \partial_{(\beta(\dot\beta}b_{\alpha)\dot\alpha)}, \qquad \delta C = \frac{1}{4}\partial_{\alpha\dot\alpha}b^{\alpha\dot\alpha}\,. \tag{3.22}$$

  Thus the spin 2 fields have the correct transformation properties under the gauge $b_{\alpha\dot\alpha}$ symmetry.

- Auxiliary fields

  Like in the previous section, the bosonic auxiliary field $P^{(\alpha\mu)dot\alpha\dot{\mu}}$ in (3.6) is not inert under the new spin 3 Lorentz-like transformations

$$\delta P^{(\alpha\mu)\dot{\alpha}\dot{\mu}} = i\partial^{\dot{\mu}}_{\rho} l^{(\rho\mu\alpha)\dot{\alpha}}, \qquad \delta \bar{P}^{\alpha\mu(\dot{\alpha}\dot{\mu})} = -i\partial^{\mu}_{\dot{\rho}} l^{\alpha(\dot{\rho}\dot{\alpha}\dot{\mu})}\,. \tag{3.23}$$

  So we are led to redefine these fields to make them inert through adding proper terms depending on the spin 3 fields. The expressions with the necessary transformation laws are as follows:

$$\begin{aligned}
B_{(\alpha\beta)\dot{\alpha}\dot{\beta}} &= -\frac{1}{2}\left\{\partial^{\gamma\dot{\gamma}}\Phi_{(\alpha\beta\gamma)(\dot{\alpha}\dot{\gamma}\dot{\beta})} - \partial_{(\alpha(\dot{\alpha}}\Phi_{\beta)\dot{\beta})} - \partial_{(\alpha\dot{\alpha}}\Phi_{\beta)\dot{\beta}}\right\},\\
\bar{B}_{\alpha\beta(\dot{\alpha}\dot{\beta})} &= -\frac{1}{2}\left\{\partial^{\gamma\dot{\gamma}}\Phi_{(\alpha\beta\gamma)(\dot{\alpha}\dot{\gamma}\dot{\beta})} - \partial_{(\alpha(\dot{\alpha}}\Phi_{\beta)\dot{\beta})} - \partial_{\alpha(\dot{\alpha}}\Phi_{\beta\dot{\beta})}\right\},\\
\delta B_{(\alpha\beta)\dot{\alpha}\dot{\beta}} &= \partial_{\rho\dot{\alpha}} l^{(\rho}{}_{\alpha\beta)\dot{\beta}}\,, \quad \delta\bar{B}_{\alpha\beta(\dot{\alpha}\dot{\beta})} = \partial_{\alpha\dot{\rho}}\bar{l}^{(\dot{\rho}}{}_{\dot{\alpha}\dot{\beta})\beta}\,.
\end{aligned} \tag{3.24}$$

  The sought redefinitions are:

$$\begin{aligned}
P^{(\alpha\mu)\dot{\alpha}\dot{\mu}} &= \tilde{P}^{(\alpha\mu)\dot{\alpha}\dot{\mu}} + iB^{(\alpha\mu)\dot{\alpha}\dot{\mu}}, \quad \bar{P}^{\alpha\mu(\dot{\alpha}\dot{\mu})} = \tilde{\bar{P}}^{\alpha\mu(\dot{\alpha}\dot{\mu})} - i\bar{B}^{\alpha\mu(\dot{\alpha}\dot{\mu})}\,,\\
\delta\tilde{P}^{(\alpha\mu)\dot{\alpha}\dot{\mu}} &= \delta\tilde{\bar{P}}^{\alpha\mu(\dot{\alpha}\dot{\mu})} = 0\,.
\end{aligned} \tag{3.25}$$

  The component fields $M^{\alpha\dot{\alpha}}$ and $T^{\dot{\alpha}(\alpha\mu\nu)}$ in (3.6) have non-trivial transformation laws under the spin 2 gauge group

$$\delta M^{\alpha\dot{\alpha}} = -\frac{2}{3} i\partial^{\dot{\alpha}}_{\gamma} n^{(\alpha\gamma)}\,, \quad \delta T^{\dot{\alpha}(\alpha\mu\nu)} = -i\partial^{(\alpha\dot{\alpha}} n^{\mu\nu)}\,, \tag{3.26}$$

  where the induced "Lorentz" parameters $n^{(\alpha\gamma)}$ are defined in (3.18). So these fields should also be redefined to make them inert. The redefinition required is as follows

$$\begin{aligned}
T^{\dot{\alpha}(\alpha\mu\nu)} &= \tilde{T}^{\dot{\alpha}(\alpha\mu\nu)} + iH^{\dot{\alpha}(\alpha\mu\nu)}\,, \qquad & M^{\alpha\dot{\alpha}} &= \tilde{M}^{\alpha\dot{\alpha}} + iH^{\alpha\dot{\alpha}}\,,\\
H^{\dot{\alpha}(\alpha\mu\nu)} &= \partial^{(\alpha}_{\dot{\beta}} C^{\mu\nu)(\dot{\alpha}\dot{\beta})}\,, & H^{\alpha\dot{\alpha}} &= \partial^{\alpha\dot{\alpha}} C - \frac{1}{3}\partial_{\beta\dot{\beta}} C^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}\,.
\end{aligned} \tag{3.27}$$

3.2 Invariant action

To construct the invariant action for $\mathcal{N} = 2$ spin 3 theory we need to define the negative charge non-analytic superfields analogous to those appearing in the spin 2 case. These additional gauge potentials are

$$h^{--(\alpha\beta)(\dot{\alpha}\dot{\beta})}, \quad h^{--\alpha\dot{\alpha}}, \quad h^{--(\alpha\beta)\dot{\alpha}+},\ h^{--(\dot{\alpha}\dot{\beta})\alpha+}, \quad h^{--(\alpha\beta)\dot{\alpha}-}, \quad h^{--(\dot{\alpha}\dot{\beta})\alpha-}\,, \tag{3.28}$$

and they satisfy the following harmonic equations

$$\begin{aligned}
D^{++}h^{--(\alpha\beta)(\dot{\alpha}\dot{\beta})} - D^{--}h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 4i\big[h^{--(\alpha\beta)(\dot{\alpha}+}\bar{\theta}^{+\dot{\beta})} - h^{--(\dot{\alpha}\dot{\beta})(\alpha+}\theta^{+\beta)}\big] &= 0\,,\\
D^{++}h^{--\alpha\dot{\beta}} - D^{--}h^{++\alpha\dot{\beta}} - 2i\big[h^{--(\alpha\beta)\dot{\beta}+}\theta^{+}_{\beta} - \bar{\theta}^{+}_{\dot{\alpha}}h^{--(\dot{\beta}\dot{\alpha})\alpha+}\big] &= 0\,,\\
D^{++}h^{--(\dot{\alpha}\dot{\beta})\alpha+} - D^{--}h^{++(\dot{\alpha}\dot{\beta})\alpha+} = 0\,,\ D^{++}h^{--(\alpha\beta)\dot{\alpha}+} - D^{--}h^{++(\alpha\beta)\dot{\alpha}+} &= 0\,,\\
D^{++}h^{--(\dot{\alpha}\dot{\beta})\alpha-} - h^{--(\dot{\alpha}\dot{\beta})\alpha+} = 0\,, \qquad D^{++}h^{--(\alpha\beta)\dot{\alpha}-} - h^{--(\alpha\beta)\dot{\alpha}+} &= 0\,.
\end{aligned} \tag{3.29}$$

These equations are covariant under the gauge transformations (3.2), provided that the negatively charged potentials are transformed as

$$\begin{aligned}
\delta_\lambda h^{--(\alpha\beta)(\dot\alpha\dot\beta)} &= D^{--}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\lambda^{-(\alpha\beta)(\dot\alpha}\bar\theta^{-\dot\beta)} - \bar\lambda^{-(\dot\alpha\dot\beta)(\beta}\theta^{-\alpha)}\big]\,,\\
\delta_\lambda h^{--\alpha\dot\beta} &= D^{--}\lambda^{\alpha\dot\beta} - 2i\big[\lambda^{-(\alpha\beta)\dot\beta}\theta^-_\beta + \bar\lambda^{-(\dot\alpha\dot\beta)\alpha}\bar\theta^-_{\dot\alpha}\big],\\
\delta_\lambda h^{--(\alpha\beta)\dot\alpha+} &= D^{--}\lambda^{+(\alpha\beta)\dot\alpha} - \lambda^{-(\alpha\beta)\dot\alpha}\,,\\
\delta_\lambda h^{--(\dot\alpha\dot\beta)\alpha+} &= D^{--}\bar\lambda^{+(\dot\alpha\dot\beta)\alpha} - \bar\lambda^{-(\dot\alpha\dot\beta)\alpha}\,,\\
\delta_\lambda h^{--(\alpha\beta)\dot\alpha-} &= D^{--}\lambda^{-(\alpha\beta)\dot\alpha}\,, \quad \delta_\lambda h^{--(\dot\alpha\dot\beta)\alpha-} = D^{--}\bar\lambda^{-(\dot\alpha\dot\beta)\alpha}\,,
\end{aligned} \tag{3.30}$$

with

$$D^{++}\lambda^{-(\alpha\beta)\dot\alpha} = \lambda^{+(\alpha\beta)\dot\alpha}\,, \quad D^{++}\bar\lambda^{-(\dot\alpha\dot\beta)\alpha} = \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\,. \tag{3.31}$$

It is now rather straightforward to check that these harmonic equations are also covariant under the following modified rigid $\mathcal{N}=2$ supersymmetry

$$\begin{aligned}
\delta_\epsilon h^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} &= -4i\big[h^{\pm\pm(\alpha\beta)(\dot\alpha+}\bar\epsilon^{-\dot\beta)} - h^{\pm\pm(\dot\alpha\dot\beta)(\alpha+}\epsilon^{-\beta)}\big]\,,\\
\delta_\epsilon h^{\pm\pm\alpha\dot\beta} &= 2i\big[h^{\pm\pm(\alpha\beta)\dot\beta+}\epsilon^-_\beta - \bar\epsilon^-_{\dot\alpha}h^{\pm\pm(\dot\alpha\dot\beta)\alpha+}\big].
\end{aligned} \tag{3.32}$$

The passive supersymmetry variations of all other gauge potentials are vanishing, like in the spin 2 case.

The next step is to define the corresponding non-analytic objects transforming as scalar superfields under $\mathcal{N}=2$ supersymmetry

$$\begin{aligned}
G^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} &= h^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[h^{\pm\pm(\alpha\beta)(\dot\alpha+}\bar\theta^{-\dot\beta)} - h^{\pm\pm(\dot\alpha\dot\beta)(\alpha+}\theta^{-\beta)}\big]\,,\\
G^{\pm\pm\alpha\dot\beta} &= h^{\pm\pm\alpha\dot\beta} - 2i\big[h^{\pm\pm(\alpha\beta)\dot\beta+}\theta^-_\beta - \bar\theta^-_{\dot\alpha}h^{\pm\pm(\dot\alpha\dot\beta)\alpha+}\big],
\end{aligned} \tag{3.33}$$

$$\delta_\epsilon G^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} = \delta_\epsilon G^{\pm\pm\alpha\dot\beta} = 0\,, \tag{3.34}$$

$$D^{++}G^{--(\alpha\beta)(\dot\alpha\dot\beta)} - D^{--}G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,, \tag{3.35}$$

$$D^{++}G^{--\alpha\dot\beta} - D^{--}G^{++\alpha\dot\beta} = 0. \tag{3.36}$$

These superfields possess simple gauge transformation laws

$$\delta_\lambda G^{\pm\pm(\alpha\beta)(\dot\alpha\dot\beta)} = D^{\pm\pm}\Lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}\,, \quad \delta_\lambda G^{\pm\pm\alpha\dot\beta} = D^{\pm\pm}\Lambda^{\alpha\dot\beta}\,, \tag{3.37}$$

$$\Lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 4i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{-\dot\beta)} - \bar\lambda^{+(\dot\alpha\dot\beta)(\alpha}\theta^{-\beta)}\big]\,, \tag{3.38}$$

$$\Lambda^{\alpha\dot\beta} = \lambda^{\alpha\dot\beta} - 2i\big[\lambda^{+(\alpha\beta)\dot\beta}\theta^-_\beta - \bar\theta^-_{\dot\alpha}\bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\big]\,. \tag{3.39}$$

Passing through the same technical steps as in section 2.3, it is a matter of direct calculation to check that the manifestly $\mathcal{N}=2$ supersymmetric action

$$S_{(s=3)} = \int d^4x d^8\theta du \left\{G^{++(\alpha\beta)(\dot\alpha\dot\beta)}G^{--}_{(\alpha\beta)(\dot\alpha\dot\beta)} + 4G^{++\alpha\dot\beta}G^{--}_{\alpha\dot\beta}\right\} \tag{3.40}$$

is invariant as well under all gauge transformations and so solves the problem of finding an invariant superfield action for $\mathcal{N}=2$ supersymmetric spin 3 theory. The coefficient before this invariant and its sign can be fixed by those of the spin 3 field component action.

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The relevant pieces of the component action come out from the following parts of $G^{++(\alpha\beta)(\dot\alpha\dot\beta)}$ and $G^{++\alpha\dot\alpha}$

$$
\begin{aligned}
G^{++(\alpha\beta)(\dot\alpha\dot\beta)}_{(\Phi,B)} &= -2i\theta^{+\gamma}\bar\theta^{+\dot\gamma}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\gamma\dot\gamma} + 4(\theta^+)^2\bar\theta^{+\dot\gamma}\bar\theta^{-(\dot\alpha}B^{\dot\beta)(\alpha\beta)}_{\dot\gamma} \\
&\quad - 4(\bar\theta^+)^2\theta^{+\gamma}\theta^{-(\alpha}\bar B^{\beta)(\dot\alpha\dot\beta)}_{\gamma}\,, \qquad (3.41)\\
G^{++\alpha\dot\alpha}_{(\Phi,B)} &= -2(\theta^+)^2\bar\theta^{+\dot\rho}\theta^-_\mu B^{(\mu\alpha)\dot\alpha}_{\dot\rho} - 2(\bar\theta^+)^2\theta^{+\beta}\bar\theta^-_{\dot\rho}\bar B^{\alpha(\dot\rho\dot\alpha)}_{\beta}\,. \qquad (3.42)
\end{aligned}
$$

Second and third terms in (3.41) and both terms in (3.42) follow from the redefinition (3.25).

The corresponding parts of $G^{--(\alpha\beta)(\dot\alpha\dot\beta)}_{(\Phi,B)}$ and $G^{--\alpha\dot\beta}_{(\Phi,B)}$ are given in appendix (eqs. (A.4) and (A.5)). Substituting all this in the superfield action (3.40), we obtain the following component action for the spin 3 fields

$$
\begin{aligned}
S_{(s=3)} = \int d^4x \Big\{ &\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)}\Box\Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \\
&- \frac{3}{2}\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)}\partial_{\alpha_1\dot\alpha_1}\partial^{\rho\dot\rho}\Phi_{(\rho\alpha_2\alpha_3)(\dot\rho\dot\alpha_2\dot\alpha_3)} \\
&+ 3\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)}\partial_{\alpha_1\dot\alpha_1}\partial_{\alpha_2\dot\alpha_2}\Phi_{\alpha_3\dot\alpha_3} - \frac{15}{4}\Phi^{\alpha\dot\alpha}\Box\Phi_{\alpha\dot\alpha} \\
&+ \frac{3}{8}\partial_{\alpha_1\dot\alpha_1}\Phi^{\alpha_1\dot\alpha_1}\partial_{\alpha_2\dot\alpha_2}\Phi^{\alpha_2\dot\alpha_2}\Big\}\,. \qquad (3.43)
\end{aligned}
$$

It is straightforward to check that (3.43) is invariant under the spin 3 gauge group (3.14), (3.15). The action (3.43) involves fields $\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)}$ and $\Phi^{\alpha\dot\alpha}$ needed for the consistent description of spin 3 and coincides with the relevant Fronsdal action. For spin 2 (which is now a superpartner of the spin 3 and is described by the fields $C^{(\alpha\beta)(\dot\alpha\dot\beta)}, C$) also a correct Fronsdal-type action can be derived, details are given in appendix B.

More detailed analysis of the component $\mathcal{N} = 2$ supersymmetric spin 2 and spin 3 actions (including the fermionic contributions) will be presented elsewhere.

4 General case: $\mathcal{N} = 2$ integer spin s theory

The construction described above for spins 2 and 3 can rather directly be extended to an arbitrary integer spin **s**. Here we sketch its basic steps, without details.

The set of analytic potentials is formed by the following analytic $\mathcal{N} = 2$ superfields

$$
h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta),\; h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta),\; h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta),\; h^{++\dot\alpha(s-1)\alpha(s-2)+}(\zeta), \qquad (4.1)
$$

where symbols $\alpha(s)$ and $\dot\alpha(s)$ denote totally symmetric combinations of s spinor indices, $\alpha(s) := (\alpha_1 \dots \alpha_s)$, $\dot\alpha(s) := (\dot\alpha_1 \dots \dot\alpha_s)$. The first two potentials are bosonic, the last two are conjugated fermionic. The corresponding gauge group is spanned by the transformations

$$
\begin{aligned}
\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-1)} &= D^{++}\lambda^{\alpha(s-1)\dot\alpha(s-1)} + 4i\big[\lambda^{+\alpha(s-1)(\dot\alpha(s-2)}\bar\theta^{+\dot\alpha_{s-1})} \\
&\quad + \theta^{+(\alpha_{s-1}}\bar\lambda^{+\alpha(s-2))\dot\alpha(s-1)}\big], \\
\delta_\lambda h^{++\alpha(s-2)\dot\alpha(s-2)} &= D^{++}\lambda^{\alpha(s-2)\dot\alpha(s-2)} - 2i\,\big[\lambda^{+(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)}\theta^+_{\alpha_{s-1}} \\
&\quad + \bar\lambda^{+(\dot\alpha(s-2)\dot\alpha_{s-1})\alpha(s-2)}\bar\theta^+_{\dot\alpha_{s-1}}\big], \\
\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-2)+} &= D^{++}\lambda^{+\alpha(s-1)\dot\alpha(s-2)}\,, \\
\delta_\lambda h^{++\dot\alpha(s-1)\alpha(s-2)+} &= D^{++}\bar\lambda^{+\dot\alpha(s-1)\alpha(s-2)}\,. \qquad (4.2)
\end{aligned}
$$

These transformations can be used to choose the appropriate WZ gauge, like in the $s = 2$ and $s = 3$ cases, and then to show that the physical multiplet involves spins $(\mathbf{s}, \mathbf{s-1/2}, \mathbf{s-1/2}, \mathbf{s-1})$.

The next step is to define the appropriate negatively charged potentials

$$h^{--\alpha(s-1)\dot\alpha(s-1)}(Z),\ h^{--\alpha(s-2)\dot\alpha(s-2)}(Z)\,,\ h^{--\alpha(s-1)\dot\alpha(s-2)+}(Z),\ h^{--\dot\alpha(s-1)\alpha(s-2)+}(Z) \tag{4.3}$$

(the potentials with the charges -3 are not essential, being fully specified by $h^{--\alpha(s-1)\dot\alpha(s-2)+}$ and c.c.). These potentials are related to (4.1) by the corresponding harmonic flatness conditions. Then one finds that these conditions require a non-standard realization of $\mathcal{N} = 2$ supersymmetry on the sets of potentials introduced. Namely,

$$\begin{aligned}
\delta_\epsilon h^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} &= -4i\big[h^{\pm\pm\alpha(s-1)(\dot\alpha(s-2)+}\bar\epsilon^{-\dot\alpha_{s-1})} - h^{\pm\pm\dot\alpha(s-1)(\alpha(s-2)+}\,\epsilon^{-\alpha_{s-1})}\big]\,,\\
\delta_\epsilon h^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} &= 2i\big[h^{\pm\pm(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)+}\epsilon^-_{\alpha_{s-1}} + h^{\pm\pm\alpha(s-2)(\dot\alpha(s-2)\dot\alpha_{s-1})+}\,\bar\epsilon^-_{\dot\alpha_{s-1}}\big]
\end{aligned}$$

(all other potentials have the standard $\mathcal{N} = 2$ superfield "passive" transformation rules, e.g., $\delta_\epsilon h^{\pm\pm\alpha(s-1)\dot\alpha(s-2)+} = 0$).

Next, one constructs $\mathcal{N} = 2$ singlet superfields

$$\begin{aligned}
G^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} &= h^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} + 4i\big[h^{\pm\pm\alpha(s-1)(\dot\alpha(s-2)+}\bar\theta^{-\dot\alpha_{s-1})}\\
&\quad -h^{\pm\pm\dot\alpha(s-1)(\alpha(s-2)+}\,\theta^{-\alpha_{s-1})}\big],\\
G^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} &= h^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} - 2i\big[h^{\pm\pm(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)+}\theta^-_{\alpha_{s-1}}\\
&\quad +h^{\pm\pm\alpha(s-2)(\dot\alpha(s-2)\dot\alpha_{s-1})+}\,\bar\theta^-_{\dot\alpha_{s-1}}\big],
\end{aligned} \tag{4.4}$$

which are transformed by the gauge group as

$$\delta_\lambda G^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} = D^{\pm\pm}\Lambda^{\alpha(s-1)\dot\alpha(s-1)}\,, \quad \delta_\lambda G^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} = D^{\pm\pm}\Lambda^{\alpha(s-2)\dot\alpha(s-2)}\,,$$

where

$$\begin{aligned}
\Lambda^{\alpha(s-1)\dot\alpha(s-1)} &= \lambda^{\alpha(s-1)\dot\alpha(s-1)} + 4i\big[\lambda^{+\alpha(s-1)(\dot\alpha(s-2)}\bar\theta^{-\dot\alpha_{s-1})} - \bar\lambda^{+\dot\alpha(s-1)(\alpha(s-2)}\theta^{-\alpha_{s-1})}\big],\\
\Lambda^{\alpha(s-2)\dot\alpha(s-2)} &= \lambda^{\alpha(s-2)\dot\alpha(s-2)} - 2i\big[\lambda^{+(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)}\theta^-_{\alpha_{s-1}}\\
&\quad -\bar\theta^-_{\dot\alpha_{s-1}}\bar\lambda^{+(\dot\alpha(s-2)\dot\alpha_{s-1})\alpha(s-2)}\big]\,.
\end{aligned} \tag{4.5}$$

They satisfy the harmonic flatness conditions

$$\begin{aligned}
D^{++}G^{--\alpha(s-1)\dot\alpha(s-1)} &= D^{--}G^{++\alpha(s-1)\dot\alpha(s-1)}\,,\\
D^{++}G^{--\alpha(s-2)\dot\alpha(s-2)} &= D^{--}G^{++\alpha(s-2)\dot\alpha(s-2)}\,.
\end{aligned}$$

The invariant action, up to a normalization factor, is written uniformly for any s:

$$\begin{aligned}
S_{(s)} &= (-1)^{s+1}\int d^4x d^8\theta du\,\Big\{G^{++\alpha(s-1)\dot\alpha(s-1)}G^{--}_{\alpha(s-1)\dot\alpha(s-1)}\\
&\quad + 4G^{++\alpha(s-2)\dot\alpha(s-2)}G^{--}_{\alpha(s-2)\dot\alpha(s-2)}\Big\}\,.
\end{aligned} \tag{4.6}$$

Its $\mathcal{N}=2$ supersymmetry is manifest, while gauge invariance is checked by bringing the gauge variation to the form

$$\begin{aligned}\delta_\lambda S_{(s)} &= 2(-1)^{s+1}\int d^4x d^8\theta du\Big\{D^{--}\Lambda^{\alpha(s-1)\dot\alpha(s-1)}G^{++}_{\alpha(s-1)\dot\alpha(s-1)}\\ &\quad + 4D^{--}\Lambda^{\alpha(s-2)\dot\alpha(s-2)}G^{++}_{\alpha(s-2)\dot\alpha(s-2)}\Big\}\end{aligned} \tag{4.7}$$

and further proceeding as in the check of invariance of the actions (2.50) and (3.40). Finally, one gets $\delta_\lambda S_{(s)}=0$.

The component actions can be deduced from (4.6) by means of the same tools as those used when deriving the component actions for the spin 3 case. In the WZ gauge the basic bosonic gauge fields are contained in the analytic potentials $h^{++\alpha(s-1)\dot\alpha(s-1)}$ and $h^{++\alpha(s-2)\dot\alpha(s-2)}$,

$$\begin{aligned}h^{++\alpha(s-1)\dot\alpha(s-1)} &= -2i\theta^{+\alpha_s}\bar\theta^{+\dot\alpha_s}\Phi^{\alpha(s-1)\dot\alpha(s-1)}_{\alpha_s\dot\alpha_s}+\dots,\\ h^{++\alpha(s-2)\dot\alpha(s-2)} &= -2i\theta^{+\alpha_{s-1}}\bar\theta^{+\dot\alpha_{s-1}}C^{\alpha(s-2)\dot\alpha(s-2)}_{\alpha_{s-1}\dot\alpha_{s-1}}+\dots.\end{aligned} \tag{4.8}$$

The residual gauge freedom in the WZ gauge proves to be so powerful that it allows one to remove from the gauge fields $\Phi^{\alpha(s-1)\dot\alpha(s-1)}_{\alpha_s\dot\alpha_s}$ and $C^{\alpha(s-2)\dot\alpha(s-2)}_{\alpha_{s-1}\dot\alpha_{s-1}}$ all the irreducible components except for

$$\big\{\Phi^{\alpha(s)\dot\alpha(s)},\quad \Phi^{\alpha(s-2)\dot\alpha(s-2)}\big\},\quad \big\{C^{\alpha(s-1)\dot\alpha(s-1)},\quad C^{\alpha(s-3)\dot\alpha(s-3)}\big\}, \tag{4.9}$$

which are just pairs of tensor fields needed for the consistent off-shell description of the massless spins $\mathbf{s}$ and $\mathbf{s-1}$ in the Fronsdal approach.[8] Their correct gauge transformation laws can easily be derived from the superfield ones on the pattern of the previously considered $\mathcal{N}=2$ supersymmetric spin $\mathbf{s=2}$ and spin $\mathbf{s=3}$ models.

The gauge freedom allowing to gauge away all the "white" (SU(2) singlet) bosonic components from the basic gauge super potentials beyond those in (4.9) is contained in the following pieces of the spinor gauge superfunctions $\lambda^{+\alpha(s-1)\dot\alpha(s-2)}$, $\bar\lambda^{+\alpha(s-2)\dot\alpha(s-1)}$:

$$\lambda^{+\alpha(s-1)\dot\alpha(s-2)} \;\Rightarrow\; \omega^{\alpha(s-1)\beta\dot\alpha(s-2)}\theta^+_\beta + \omega^{\alpha(s-1)\dot\alpha(s-2)\dot\beta}\bar\theta^+_{\dot\beta}\,,\quad \text{(and c.c.)}. \tag{4.10}$$

5 Summary and outlook

In this paper we presented an off-shell $\mathcal{N}=2$ supersymmetric extension of the Fronsdal theory [6] for integer spins in terms of unconstrained $\mathcal{N}=2$ superfields. For any spin $\mathbf{s}\geq 2$ the relevant off-shell multiplet is described by a triad of unconstrained harmonic analytic superfields $h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta)$, $h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta)$ and $h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta)$ (and c.c.), which are subjected to gauge transformations with the analytic superfield parameters. The on-shell content of the spin $\mathbf{s}$ multiplet is $(\mathbf{s},\mathbf{s-1/2},\mathbf{s-1/2},\mathbf{s-1})$.[9] For these

[8] Since the on-shell $\mathcal{N}=2$ supermultiplet contains two integer spins, $\mathbf{s}$ and $\mathbf{s-1}$, after reductions to components we naturally obtain a sum of Fronsdal actions for spins $\mathbf{s}$ and $\mathbf{s-1}$.

[9] One can include the spin $\mathbf{s=1}$ into this hierarchy as well: it is described by a single analytic superfield h^{++5} and encompasses the Abelian gauge $\mathcal{N}=2$ multiplet (spins $(\mathbf{1},\mathbf{1/2},\mathbf{1/2},\mathbf{0})$ on shell). Note that the off-shell contents of $\mathcal{N}=2$ multiplets with $\mathbf{s}=\mathbf{1},\mathbf{2},\mathbf{3}$ as the higher spins amount to $n_{\mathbf{s}}=2\times 8\big[\mathbf{s}^2+(\mathbf{s}-\mathbf{1})^2\big]$ essential degrees of freedom. It would be interesting to derive this universal formula for any spin $\mathbf{s}$ from a purely group-theoretical consideration.

superfields we found the $\mathcal{N}=2$ supersymmetric and gauge invariant superfield actions which surprisingly have the universal form (4.6). For $\mathbf{s}=\mathbf{2}$ and $\mathbf{s}=\mathbf{3}$ we have explicitly shown that this off-shell superfield action yields the correct gauge invariant actions for the spin $\mathbf{s}$ and $\mathbf{s}-\mathbf{1}$ components of the relevant multiplets.

These findings raise a lot of problems we are going to address in the nearest future.

- A natural next step would be construction of an analogous $\mathcal{N}=2$ supersymmetric extensions of theories with the half-integer highest spin [7];
- We would like also to learn how the harmonic superspace construction could be extended to AdS (and more general conformally-flat) space-time backgrounds;
- It is of interest to explore possible relationships with the $\mathcal{N}=2$ superconformal higher spins which recently received some attention [18, 22–24]. $\mathcal{N}=2$ conformal supergravity also admits a geometric formulation in HSS [3, 52], so it is natural to expect that there exist some higher-spin HSS models generalizing the linearized version of such a formulation;
- There exist a few non-equivalent off-shell versions of Einstein $\mathcal{N}=2$ SG related to different choices of the superconformal compensator for $\mathcal{N}=2$ Weyl multiplet. Our formulation of $\mathcal{N}=2$ higher spins is built on a generalization of the minimal version. It would be interesting to construct analogous off-shell formulations (if exist), proceeding from the linearizations of other versions of Einstein $\mathcal{N}=2$ SG;
- As usual, the most difficult problem would be constructing a self-consistent interacting theory with the free actions presented here as a point of departure, and finding out appropriate deformations of the higher-spin superfield gauge symmetries. As a first step towards this goal one could attempt to couple the theory to full $\mathcal{N}=2$ Einstein supergravity by replacing the flat harmonic derivatives $D^{\pm\pm}$ altogether by the covariantized ones $\mathfrak{D}^{\pm\pm}$, though for the time being it is unclear how to generalize the action (4.6). Anyway, the interactions will involve the same off-shell analytic harmonic superfields as the free theory discussed here. It is highly likely that the interaction case will require considering at once an infinite sequence of such actions (with all spins), in accord with the well-known Fradkin-Vasiliev arguments [53, 54];
- A related problem is to couple the higher $\mathcal{N}=2$ spins to the hypermultiplet matter to which all other matter $\mathcal{N}=2$ multiplets are related by the proper superfield duality transformations [3];
- It is known that the $4D,\mathcal{N}=4$ super Yang-Mills theory can be formulated in terms of $4D,\mathcal{N}=2$ harmonic superfields as a theory of coupled $\mathcal{N}=2$ vector multiplet and hypermultiplet [3]. Based on this analogy, one can hope that it will be possible to construct $\mathcal{N}=4$ supersymmetric higher-spin theory in terms of proper $\mathcal{N}=2$ harmonic superfields;
- At last, it is interesting to work out an analogous harmonic superspace setting for higher spins with extended supersymmetry in other dimensions (e.g., in $6D$ case).

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The above formulation suggests the following geometric conjecture. As was already pointed out, the basic analytic potentials of the $\mathbf{s}=\mathbf{2}$ case originate from the analytic vielbein of $\mathcal{N}=2$ supergravity in HSS which covariantizes the analyticity-preserving harmonic derivative $\mathfrak{D}^{++}$ and their index structure matches with that of the derivatives $\frac{\partial}{\partial x^{\mu\dot{\mu}}}$ and $\frac{\partial}{\partial\theta^{\mu,\dot{\mu}}}$ inside $\mathfrak{D}^{++}$. Then it is natural to assume that the higher-spin analogs of these potentials could be associated with some non-trivial extensions of the standard superspace by new tensorial and spinorial coordinates of the type $x^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}, \theta^{+(\alpha\beta)\dot{\alpha}}, \bar{\theta}^{+(\dot{\alpha}\dot{\beta})\alpha}$ (and their multi-index analogs). In the complete hypothetical supergravity-type theory, the gauge functions like $\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}, \lambda^{(\alpha\beta)\dot{\alpha}}$ could geometrically appear as local shifts of these new coordinates. Also, the plenty of spinor indices $\alpha,\beta,\dot{\alpha},\dot{\beta}\dots$ which characterize the basic objects of the theories considered could seemingly be hidden by introducing the commuting twistor-like spinorial variables $\tau_\alpha,\bar{\tau}_{\dot{\alpha}}$ and contracting the spinor indices with them. Adding such extra variables could essentially facilitate dealing with various objects of the $\mathcal{N}=2$ higher-spin theories constructed and their various generalizations, even though the geometric meaning of such variables within the present context is as yet unclear.

Acknowledgments

We thank M. Vasiliev for useful discussion and comments and S. Kuzenko for bringing to our attention ref. [34]. Work of I. B. and E. I. was supported in part by the Ministry of Education of Russian Federation, project FEWF-2020-0003.

A Some technical issues

Spin 2 sector. Here we present the negatively charged potentials for $G^{++\alpha\dot{\alpha}}_{(\Phi)}$ and $G^{++5}_{(\Phi)}$ defined in eqs. (2.62) and (2.63). The expressions for them are as follows

$$\begin{aligned} G^{--\alpha\dot{\alpha}}_{(\Phi)} &= -2i\theta^{-\beta}\bar{\theta}^{-\dot{\beta}}\Phi^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} + 2(\theta^-)^2\bar{\theta}^{-(\dot{\rho}}\bar{\theta}^{+\dot{\beta})}\partial^{\beta}_{\dot{\rho}}\Phi^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} - 2(\bar{\theta}^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot{\beta}}_{\rho}\Phi^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} \\ &\quad + 4(\theta^-)^2\bar{\theta}^{+\dot{\alpha}}\bar{\theta}^{-\dot{\beta}}B^{\alpha}_{\dot{\beta}} - 4(\bar{\theta}^-)^2\theta^{+\beta}\theta^{-\alpha}B^{\dot{\alpha}}_{\beta} - 4i(\theta^-)^2(\bar{\theta}^-)^2\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\mathcal{G}^{\alpha\dot{\alpha}}_{\rho\dot{\rho}}\,, \quad \text{(A.1)} \\ G^{--5}_{(\Phi)} &= 2(\theta^-)^2\bar{\theta}^{-\dot{\rho}}\theta^+_{\mu}B^{\mu}_{\dot{\rho}} + i(\theta^+)^2(\bar{\theta}^-)^2(\theta^-)^2\partial_{\rho\dot{\rho}}B^{\rho\dot{\rho}} \\ &\quad + 2\,(\bar{\theta}^-)^2\theta^{-\beta}\bar{\theta}^+_{\dot{\rho}}B^{\dot{\rho}}_{\beta} - i(\bar{\theta}^+)^2(\bar{\theta}^-)^2(\theta^-)^2\partial_{\rho\dot{\rho}}B^{\rho\dot{\rho}}\,. \quad \text{(A.2)} \end{aligned}$$

In eq. (A.1) the following notation was used

$$\mathcal{G}_{\alpha\beta\dot{\alpha}\dot{\beta}} = \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\mathcal{R} \quad \text{(A.3)}$$

and

$$\begin{aligned} \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= \frac{1}{2}\partial_{(\alpha(\dot{\alpha}}\partial^{\sigma\dot{\sigma}}\Phi_{\beta)\sigma\dot{\beta})\dot{\sigma}} - \frac{1}{2}\Box\Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \frac{1}{2}\partial_{(\alpha(\dot{\alpha}}\partial_{\beta)\dot{\beta})}\Phi\,, \\ \mathcal{R} &= \frac{1}{8}\partial^{\alpha\dot{\alpha}}\partial^{\beta\dot{\beta}}\Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \frac{3}{4}\Box\Phi\,. \end{aligned}$$

The object $\mathcal{G}_{\alpha\beta\dot\alpha\dot\beta}$ is just the linearized form of Einstein tensor, $\mathcal{R}_{(\alpha\beta)(\dot\alpha\dot\beta)}$ and $\mathcal{R}$ are related to the linearized scalar curvature R and Ricci tensor R_{mn} in the tensor notation as:

$$\mathcal{R}_{(\alpha\beta)(\dot\alpha\dot\beta)} + \mathcal{R}\,\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} = (\sigma^m)_{\alpha\dot\alpha}(\sigma^n)_{\beta\dot\beta}R_{(mn)}\,, \quad \mathcal{R} = \frac{1}{2}R\,,$$
$$R = \partial^m\partial^n h_{mn} - \Box h\,, \quad R_{mn} = \frac{1}{2}(\partial^k\partial_m h_{nk} + \partial^k\partial_n h_{mk} - \Box h_{mn} - \partial_m\partial_n h),$$

where

$$h_{mn} = \frac{1}{4}(\tilde\sigma_m)^{\dot\alpha\alpha}(\tilde\sigma_n)^{\dot\beta\beta}[\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} + \Phi\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}].$$

It is straightforward to check that the expressions (A.1) and (A.2), together with (2.62) and (2.63), solve the harmonic flatness conditions (2.42).

Spin 3 sector. Here we present the negatively charged potentials for $G^{++(\alpha\beta)(\dot\alpha\dot\beta)}_{(\Phi,B)}$ and $G^{++\alpha\dot\alpha}_{(\Phi,B)}$ defined in eqs. (3.41) and (3.42). The expressions for them are as follows

$$\begin{aligned}
G^{--(\alpha\beta)(\dot\alpha\dot\beta)}_{(\Phi,B)} &= -2i\theta^{-\beta}\bar\theta^{-\dot\beta}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\beta\dot\beta} \\
&\quad + 2(\theta^-)^2\bar\theta^{-(\dot\rho}\bar\theta^{+\dot\beta)}\partial^{\beta}_{\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\beta\dot\beta} - 2(\bar\theta^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot\beta}_{\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\beta\dot\beta} \\
&\quad + 4(\theta^-)^2\bar\theta^{+(\dot\alpha}\bar\theta^{-\dot\rho}B^{\dot\beta)(\alpha\beta)}_{\dot\rho} - 4(\bar\theta^-)^2\theta^{+\rho}\theta^{-(\alpha}\bar B^{\beta)(\dot\alpha\dot\beta)}_{\rho} \\
&\quad - 3i(\theta^-)^2(\bar\theta^-)^2\theta^{+\rho}\bar\theta^{+\dot\rho}\mathcal{G}^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}\,,
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
G^{--\alpha\dot\alpha}_{(\Phi,B)} &= 2(\theta^-)^2\bar\theta^{-\dot\rho}\theta^+_\mu B^{(\mu\alpha)\dot\alpha}_{\dot\rho} + i(\theta^+)^2(\bar\theta^-)^2(\theta^-)^2\partial_{\rho\dot\rho}B^{(\rho\alpha)\dot\rho\dot\alpha} \\
&\quad + 2\,(\bar\theta^-)^2\theta^{-\beta}\bar\theta^+_{\dot\rho}\bar B^{\alpha(\dot\alpha\dot\rho)}_{\beta} - i(\bar\theta^+)^2(\bar\theta^-)^2(\theta^-)^2\partial_{\rho\dot\rho}\bar B^{\rho\alpha(\dot\rho\dot\alpha)}\,.
\end{aligned} \tag{A.5}$$

In eq. (A.4) the following notation was used:

$$\mathcal{G}_{(\alpha_1\alpha_2)\alpha_3(\dot\alpha_1\dot\alpha_2)\dot\alpha_3} = \mathcal{R}_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} - \frac{4}{9}\mathcal{R}_{(\alpha_1(\dot\alpha_1}\epsilon_{\dot\alpha_2)\dot\beta}\epsilon_{\alpha_2)\beta} \tag{A.6}$$

and

$$\mathcal{R}_{\alpha_1\dot\alpha_1} = \partial^{\alpha_2\dot\alpha_2}\partial^{\alpha_3\dot\alpha_3}\Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} - \frac{1}{4}\partial_{\alpha_1\dot\alpha_1}\partial^{\alpha_2\dot\alpha_2}\Phi_{\alpha_2\dot\alpha_2} - \frac{5}{2}\Box\Phi_{\alpha_1\dot\alpha_1}\,, \tag{A.7}$$

$$\begin{aligned}
\mathcal{R}_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} &= \partial_{(\alpha_1(\dot\alpha_1}\partial^{\rho\dot\rho}\Phi_{\alpha_2\alpha_3)\rho\dot\alpha_2\dot\alpha_3)\dot\rho} - \frac{2}{3}\Box\Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \\
&\quad - \partial_{(\alpha_1(\dot\alpha_1}\partial_{\alpha_2\dot\alpha_2}\Phi_{\alpha_3)\dot\alpha_3)}\,.
\end{aligned} \tag{A.8}$$

B Spin 2 sector of $\mathcal{N} = 2$ spin 3 theory

Here we present the relevant pieces of the analytic gauge potential in the spin 2 sector of $\mathcal{N} = 2$ spin 3 theory and the corresponding parts of the negatively charged potentials. Using them, we derive the component form of the superfield action (3.40) in the spin 2 sector.

As a consequence of the redefinition of (3.27) and relations (3.33) we have in the spin 2 sector:

$$G^{++(\alpha\beta)(\dot\alpha\dot\beta)}_{(s=2)} = -4(\bar\theta^+)^2\theta^+_\rho\bar\theta^{-(\dot\alpha}H^{\dot\beta)\rho(\alpha\beta)} + 4(\theta^+)^2\bar\theta^+_{\dot\rho}\theta^{-(\alpha}\bar H^{\beta)\dot\rho(\dot\alpha\dot\beta)}\,, \tag{B.1}$$

$$G^{++\alpha\dot\alpha}_{(s=2)} = -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + 2(\theta^+)^2\bar\theta^+_{\dot\rho}\bar\theta^-_{\dot\mu}\bar H^{\alpha\dot\rho(\dot\mu\dot\alpha)} + 2(\bar\theta^+)^2\theta^+_\rho\theta^-_\mu H^{\dot\alpha\rho(\alpha\mu)}\,. \tag{B.2}$$

The negatively charged potentials can be obtained as a solution of eqs. (3.35) and (3.36):

$$G^{--(\alpha\beta)(\dot\alpha\dot\beta)}_{(s=2)} = -4(\bar\theta^-)^2\theta^-_\rho\bar\theta^{+(\dot\alpha}H^{\dot\beta)\rho(\alpha\beta)} + 4(\theta^-)^2\bar\theta^-_{\dot\rho}\theta^{+(\alpha}\bar H^{\beta)\dot\rho(\dot\alpha\dot\beta)} + \ldots\,, \tag{B.3}$$

$$G^{--\alpha\dot\alpha}_{(s=2)} = -2i\theta^{-\rho}\bar\theta^{-\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + 2(\theta^-)^2\bar\theta^{-(\dot\rho}\bar\theta^{+\dot\beta)}\partial^\beta_{\dot\rho}C^{\alpha\dot\alpha}_{\beta\dot\beta} - 2(\bar\theta^-)^2\theta^{-(\rho}\theta^{+\beta)}\partial^{\dot\beta}_\rho C^{\alpha\dot\alpha}_{\beta\dot\beta} \tag{B.4}$$
$$+ 2(\theta^-)^2\bar\theta^+_{\dot\rho}\bar\theta^-_{\dot\mu}\bar H^{\alpha\dot\rho(\dot\mu\dot\alpha)} + 2(\bar\theta^-)^2\theta^+_\rho\theta^-_\mu H^{\dot\alpha\rho(\mu\alpha)} + 4i(\theta^-)^2(\bar\theta^-)^2\theta^+_\rho\bar\theta^+_{\dot\rho}\mathcal{G}^{\alpha\rho\dot\alpha\dot\rho}\,.$$

Here, $\mathcal{G}^{\alpha\rho\dot\alpha\dot\rho}$ is the linearized form of Einstein tensor (A.3). We also used the notations:

$$\bar H^{\alpha\dot\rho(\dot\alpha\dot\mu)} := \bar H^{\alpha(\dot\rho\dot\alpha\dot\mu)} + \epsilon^{\dot\rho(\dot\alpha}\bar H^{\alpha\dot\mu)}\,, \qquad H^{\dot\alpha\rho(\mu\alpha)} := H^{\dot\alpha(\rho\mu\alpha)} + \epsilon^{\rho(\alpha}H^{\dot\alpha\mu)}\,,$$
$$\bar H^{\alpha(\dot\rho\dot\alpha\dot\mu)} = -\,\partial^{(\dot\rho}_\beta C^{\dot\alpha\dot\mu)(\alpha\beta)}\,, \qquad H^{\dot\alpha(\rho\mu\alpha)} = \partial^{(\alpha}_{\dot\beta}C^{\mu\rho)(\dot\alpha\dot\beta)}\,,$$
$$H^{\mu\dot\mu} = \partial^{\mu\dot\mu}C - \frac{1}{3}\partial_{\rho\dot\rho}C^{(\mu\rho)(\dot\mu\dot\rho)} = -\bar H^{\mu\dot\mu}\,.$$

Substituting all this in the superfield action (3.40), we obtain the spin 2 action of the $\mathcal{N}=2$ spin 3 theory

$$S_{(s=2)} = 16\int d^4x\, C^{\alpha\beta\dot\alpha\dot\beta}\mathcal{G}_{\alpha\beta\dot\alpha\dot\beta} = -8\int d^4x\,\Big[C^{(\alpha\beta)(\dot\alpha\dot\beta)}\Box C_{(\alpha\beta)(\dot\alpha\dot\beta)}$$
$$- C^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\alpha\dot\alpha}\partial^{\rho\dot\rho}C_{(\rho\beta)(\dot\rho\dot\beta)} + 2\,C\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}C_{(\alpha\beta)(\dot\alpha\dot\beta)} - 6C\Box C\Big]. \tag{B.5}$$

This action is the linearized Einstein action and, up to a normalization factor, coincides with the action corresponding to the Lagrangian (2.64).

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