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# Leading low-energy effective action in $6D$ , $\mathcal{N} = (1, 1)$ SYM theory

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**ABSTRACT:** We elaborate on the low-energy effective action of  $6D$ ,  $\mathcal{N} = (1, 1)$  supersymmetric Yang-Mills (SYM) theory in the  $\mathcal{N} = (1, 0)$  harmonic superspace formulation. The theory is described in terms of analytic  $\mathcal{N} = (1, 0)$  gauge superfield  $V^{++}$  and analytic  $\omega$ -hypermultiplet, both in the adjoint representation of gauge group. The effective action is defined in the framework of the background superfield method ensuring the manifest gauge invariance along with manifest  $\mathcal{N} = (1, 0)$  supersymmetry. We calculate leading contribution to the one-loop effective action using the on-shell background superfields corresponding to the option when gauge group  $SU(N)$  is broken to  $SU(N-1) \times U(1) \subset SU(N)$ . In the bosonic sector the effective action involves the structure  $\sim \frac{F^4}{X^2}$ , where  $F^4$  is a monomial of the fourth degree in an abelian field strength  $F_{MN}$  and  $X$  stands for the scalar fields from the  $\omega$ -hypermultiplet. It is manifestly demonstrated that the expectation values of the hypermultiplet scalar fields play the role of a natural infrared cutoff.

**KEYWORDS:** Extended Supersymmetry, Superspaces, Supersymmetric Gauge Theory

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## 1 Introduction

The low-energy effective action plays an important role in supersymmetric gauge theories, providing a link between superstring/brane theory and quantum field theory. On the one hand, such an effective action can be calculated in the quantum field theory setting and, on the other, it can be derived within the brane stuff. As a result, the low-energy effective action allows one, in principle, to describe the low-energy string effects by methods of quantum field theory and vice versa (see reviews [1–3]).

It is known that D3-branes are related to  $4D, \mathcal{N} = 4$  SYM theory (see, e.g., [4, 5]). Interaction of D3-branes is described in abelian bosonic sector by the Born-Infeld action, with the leading low-energy correction of the form  $\sim \frac{F^4}{X^4}$ , where  $F^4$  is a structure of fourth degree in an abelian field strength  $F_{mn}$  and  $X$  stands for the scalar fields of  $4D, \mathcal{N} = 4$  gauge (vector) multiplet. The one-loop calculation of such an effective action in the Coulomb branch of  $\mathcal{N} = 4$  SYM theory, both in the component approach and in terms of  $\mathcal{N} = 1, 2$  superfields, has been performed in refs. [6–15]. The complete  $\mathcal{N} = 4$  structure of the one-loop low-energy effective action has been established in [16, 17]. The two-loop contributions to the low-energy effective actions of  $\mathcal{N} = 4$  SYM theory have been studied in [18, 19]. The structure of the low-energy effective action in the mixed Coulomb - Higgs branch was a subject of ref. [20]. A review of the results related to the calculations of low-energy effective actions in four-dimensional extended supersymmetric gauge theories can be found, e.g., in [1, 2].

Another interesting class of the extended objects in superstring/brane theory is presented by D5-branes (see e.g., [4, 5]). These objects are related to  $6D, \mathcal{N} = (1, 1)$  SYM theory likewise D3-branes are related to  $4D, \mathcal{N} = 4$  SYM theory. Similarly to the D3-brane case, the interaction of D5-branes is described by the  $6D$  Born-Infeld action [21] (see [22–27] for aspects of the Born-Infeld action in diverse dimensions). Since D5-brane is related to  $6D, \mathcal{N} = (1, 1)$  SYM theory, it is natural to expect that the D5-brane interaction in the low-energy domain can be calculated on the basis of the low-energy quantum effective action of this theory.

It is also worth mentioning that maximally supersymmetric  $6D, \mathcal{N} = (1, 1)$  Yang-Mills theory is closely related to the so called “little string theories” (see [48] for a review). In

refs. [49, 50], there were computed the massless gluon amplitudes in  $6D$ ,  $\mathcal{N} = (1, 1)$  Yang-Mills theory with the gauge group  $SU(N)$  broken to  $\Upsilon(1)^{N-1}$ . Up to four loops, there was found the precise relation between these amplitudes and tree-level amplitudes of the corresponding massless string modes in the double-scaled little string theory.

In the present paper we study the quantum aspects of  $6D$ ,  $\mathcal{N} = (1, 1)$  SYM theory. It is the maximally extended supersymmetric gauge model in six dimensions, and it involves eight left-handed and eight right-handed supercharges. An equal number of spinors with mutually-opposite chiralities guarantees the absence of chiral anomaly in the theory. From the point of view of  $6D$ ,  $\mathcal{N} = (1, 0)$  supersymmetry, the model is built on a gauge (vector) multiplet and a hypermultiplet. Respectively, the bosonic sector of the model includes a real vector gauge field and two complex (or four real) scalar fields.

Although  $6D$ ,  $\mathcal{N} = (1, 1)$  non-abelian SYM theory is non-renormalizable by power counting, it was proved that it is on-shell finite at one and two loops [28–35]. Moreover, it was recently shown that this theory is one-loop finite even off-shell [36–38] and that the two-loop diagrams with hypermultiplet legs are also off-shell finite [39].

Here we develop a method to determine the one-loop effective action in general  $6D$ ,  $\mathcal{N} = (1, 1)$  SYM theory and to calculate the leading low-energy contributions to it. To preserve as many manifest supersymmetries as possible we use the harmonic superspace approach [40, 41]. The theory under consideration is formulated in terms of  $\mathcal{N} = (1, 0)$  harmonic superfields describing the gauge multiplet and the hypermultiplet. Therefore it possesses the manifest  $\mathcal{N} = (1, 0)$  supersymmetry and, in addition, a non-manifest (hidden) on-shell  $\mathcal{N} = (0, 1)$  supersymmetry mixing  $\mathcal{N} = (1, 0)$  gauge multiplet and hypermultiplet. These supersymmetries close on the total on-shell  $\mathcal{N} = (1, 1)$  supersymmetry. Such a formulation of  $\mathcal{N} = (1, 1)$  SYM theory was described in detail in the paper [42] (see also refs. [43, 44]).<sup>1</sup> An essential difference of our consideration here is the use of the so called “ $\omega$ -form” of the hypermultiplet (see below).

The theory under consideration is quantized in the framework of  $\mathcal{N} = (1, 0)$  supersymmetric background field method [37, 38]. In this method, the effective action depends on the background superfields of  $6D$ ,  $\mathcal{N} = (1, 0)$  gauge multiplet and hypermultiplet. By construction, it exhibits manifest gauge invariance under the classical gauge transformations and  $\mathcal{N} = (1, 0)$  supersymmetry. To calculate the one-loop effective action we make use of the superfield proper-time technique [47], which ensures the manifest gauge invariance and  $\mathcal{N} = (1, 0)$  supersymmetry at all steps of calculation. The low-energy effective action is obtained, when we impose the restriction that both the background superfield strength and the background hypermultiplet are space-time-independent. The leading low-energy approximation amounts to keeping those terms in the effective action which are of the lowest order in the superfield strength. We also assume that the background superfields satisfy the classical equations of motion, that guarantees the gauge independence of the effective action.

We consider the case when gauge symmetry  $SU(N)$  is broken to  $SU(N-1) \times \Upsilon(1) \subset SU(N)$ . Technically, this means that background superfields align through the fixed gen-

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<sup>1</sup>There is also another superfield formulation for maximally supersymmetric Yang-Mills theories based on the pure spinor superfield formalism [45, 46]. However, the scheme of quantum calculations within this approach has not been worked out so far.

erator of Cartan subalgebra of  $SU(N)$ , which corresponds to an abelian subgroup  $\Upsilon(1)$ . In this case the effective action of the theory depends only on the abelian vector multiplet and hypermultiplet. In the bosonic sector we find out the effective action for the single real  $\Upsilon(1)$  gauge field and four real scalar fields.

The paper is organized as follows. In section 2 we formulate an arbitrary  $6D, \mathcal{N} = (1, 1)$  SYM theory in terms of  $\mathcal{N} = (1, 0)$  harmonic superfields representing the gauge multiplet and the hypermultiplet. Unlike the majority of the previous papers on effective action in  $4D$  and  $6D$  harmonic superspaces, we prefer to work with  $\omega$ -form of the hypermultiplet. Such a formulation has certain merits over the more accustomed formulation in terms of  $q$ -hypermultiplet. Although the  $\omega$ - and  $q$ - descriptions of the hypermultiplet are classically equivalent [41], and this equivalency apparently extends to the exact quantum theory, the approximate schemes for calculating the quantum effective action in terms of these superfields can be different. Besides, the  $\omega$ -hypermultiplet possesses an advantage of being real, *i.e.* carrying no external  $\Upsilon(1)$  charges. This property essentially simplifies the construction of the super-invariants in the  $\omega$ -representation. In section 3, besides giving details of the  $6D, \mathcal{N} = (1, 1)$  SYM action in terms of the harmonic superfields  $V^{++}$  and  $\omega$ , we derive the transformations of the hidden on-shell  $\mathcal{N} = (0, 1)$  supersymmetry which mixes the superfields  $V^{++}$  and  $\omega$  and leaves the action invariant. In section 3 we develop a procedure of constructing the one-loop effective action which depends on both the gauge and the hypermultiplet background superfields. Also we demonstrate advantages of the  $\omega$ -hypermultiplet formulation and construct some on-shell invariants depending on  $V^{++}$  and  $\omega$  superfields. Analogous invariants have never been constructed in terms of  $q$ -hypermultiplet. Section 4 describes the calculation of leading low-energy contributions to the one-loop effective action. To this end, we fix the background superfields by requiring them to be space-time independent and to satisfy the classical equations of motion. We consider the case of background superfields breaking  $SU(N)$  gauge group of the original Lagrangian to  $SU(N-1) \times \Upsilon(1)$ . In this case the effective action depends on an abelian  $\Upsilon(1)$  gauge superfield. It is explicitly demonstrated that the  $\omega$ -hypermultiplet acts as an infrared regulator securing the absence of the infrared singularities in the low-energy effective action. The last section contains a brief summary of the results obtained and a list of some problems for the future study.

## 2 The model and conventions

We consider the formulation of  $6D, \mathcal{N} = (1, 1)$  SYM theory in terms of  $6D, \mathcal{N} = (1, 0)$  harmonic analytic superfields  $V^{++}$  and  $\omega$ , which represent the gauge multiplet and the hypermultiplet.<sup>2</sup> The action of  $\mathcal{N} = (1, 1)$  SYM theory is written as

$$S_0[V^{++}, q^+] = \frac{1}{f^2} \left\{ \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \frac{1}{2} \text{tr} \int d\zeta^{(-4)} \nabla^{++} \omega \nabla^{++} \omega \right\}, \quad (2.1)$$

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<sup>2</sup>These superfields satisfy the Grassmann harmonic analyticity conditions  $D_a^+ V^{++} = 0$  and  $D_a^+ \omega = 0$ , where  $D_a^+ = \frac{\partial}{\partial \theta^a}$ .

where  $f$  is a dimensionful coupling constant ( $[f] = -1$ ) and the measure of integration over the analytic subspace  $d\zeta^{(-4)}$  includes the integration over harmonics,  $d\zeta^{(-4)} = d^6x_{(\text{an})} du (D^-)^4$ . Both  $V^{++}$  and  $\omega$  superfields take values in the adjoint representation of the gauge group. The covariant harmonic derivative  $\nabla^{++}$  acts on the hypermultiplet  $\omega$  as

$$\nabla^{++}\omega = D^{++}\omega + i[V^{++}, \omega]. \quad (2.2)$$

The action (2.1) is invariant under the infinitesimal gauge transformations

$$\delta V^{++} = -\nabla^{++}\Lambda, \quad \delta\omega = i[\Lambda, \omega], \quad (2.3)$$

where  $\Lambda(\zeta, u) = \tilde{\Lambda}(\zeta, u)$  is a real analytic gauge parameter.

Besides the analytic gauge connection  $V^{++}$  we introduce a non-analytic one  $V^{--}$  as a solution of the zero curvature condition [41]

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0. \quad (2.4)$$

Using  $V^{--}$  we can define one more covariant harmonic derivative  $\nabla^{--} = D^{--} + iV^{--}$  and the  $\mathcal{N} = (1, 0)$  gauge superfield strength

$$W^{+a} = -\frac{i}{6}\varepsilon^{abcd}D_b^+D_c^+D_d^+V^{--}, \quad (2.5)$$

possessing the useful off-shell properties

$$\nabla^{++}W^{+a} = \nabla^{--}W^{-a} = 0, \quad W^{-a} = \nabla^{--}W^{+a}. \quad (2.6)$$

Introducing an analytic superfield  $F^{++}$ ,

$$F^{++} = \frac{1}{4}D_a^+W^{+a} = i(D^+)^4V^{--}, \quad D_a^+F^{++} = \nabla^{++}F^{++} = 0, \quad (2.7)$$

we can write the classical equations of motion corresponding to the action (2.1) as

$$F^{++} + [\omega, \nabla^{++}\omega] = 0, \quad (\nabla^{++})^2\omega = 0. \quad (2.8)$$

The  $\mathcal{N} = (1, 0)$  superfield action (2.1) enjoys the additional  $\mathcal{N} = (0, 1)$  supersymmetry

$$\delta V^{++} = (\epsilon^{+A}u_A^+)\omega - (\epsilon^{+A}u_A^-)\nabla^{++}\omega = 2(\epsilon^{+A}u_A^+)\omega - \nabla^{++}((\epsilon^{+A}u_A^-)\omega), \quad (2.9)$$

$$\delta\omega = -(D^+)^4((\epsilon^{-A}u_A^-)V^{--}) = i(\epsilon^{-A}u_A^-)F^{++} - i(\epsilon_a^A u_A^-)W^{+a}, \quad (2.10)$$

where  $A = 1, 2$  is the Pauli-Gürsey SU(2) index. To check this, one first derives, using (2.9) and (2.10), the  $\mathcal{N} = (0, 1)$  transformation law of  $\nabla^{++}\omega$

$$\delta(\nabla^{++}\omega) = i((\epsilon^{-A}u_A^+) + (\epsilon^{+A}u_A^-))F^{++} - i(\epsilon_a^A u_A^+)W^{+a} + i(\epsilon^{+A}u_A^-)[\omega, \nabla^{++}\omega]. \quad (2.11)$$

Then one varies the classical action (2.1) with respect to (2.9) and (2.11)

$$\delta S = \frac{1}{f^2} \left\{ \text{tr} \int d^{14}z du V^{--} \delta V^{++} - \text{tr} \int d\zeta^{(-4)} \nabla^{++}\omega \delta(\nabla^{++}\omega) \right\}. \quad (2.12)$$

In the first integral, we pass to the integration over the analytic subspace and use the explicit form of the variations (2.9) and (2.11)

$$\delta S = -\frac{i}{f^2} \text{tr} \int d\zeta^{(-4)} \left\{ 2F^{++}(\epsilon^{+A}u_A^+)\omega + \nabla^{++}\omega((\epsilon^{-A}u_A^+) + (\epsilon^{+A}u_A^-))F^{++} \right. \\ \left. - F^{++}\nabla^{++}((\epsilon^{+A}u_A^-)\omega) - \epsilon_a^A u_A^+ \nabla^{++}\omega W^{+a} \right\} = 0. \quad (2.13)$$

The last two terms in (2.13) are the total harmonic derivative  $\nabla^{++}$  due to the properties of  $F^{++}$  and  $W^{+a}$  and so they vanish under  $d\zeta^{(-4)}$ . The first two terms cancel each other after integration by parts with respect to the harmonic derivative  $\nabla^{++}$  and using the properties  $\nabla^{++}\epsilon^{-A} = \epsilon^{-A}$  and  $\nabla^{++}u_A^- = u_A^+$ . Finally, the term  $\text{tr}(\nabla^{++}\omega[\omega, \nabla^{++}\omega])$  vanishes due to the cyclic property of trace.

The zero curvature condition (2.4) allows one to express the transformation of the non-analytic gauge connection  $\delta V^{--}$  through  $\delta V^{++}$

$$\nabla^{++}\delta V^{--} - \nabla^{--}\delta V^{++} = 0, \quad (2.14)$$

and to define the transformation law of the gauge superfield strength  $W^{+a}$  under the hidden supersymmetry

$$\delta W^{+a} = \varepsilon^{adbc} \epsilon_d^A \nabla_{bc}(u_A^+\omega - u_A^-\nabla^{++}\omega) + i\epsilon^{-A}[W^{+a}, u_A^+\omega - u_A^-\nabla^{++}\omega], \quad (2.15)$$

where

$$\nabla_{bc} = \partial_{bc} - \frac{1}{2}D_b^+ D_c^+ V^{--}. \quad (2.16)$$

Note that, while deriving (2.15), we essentially used the  $\omega$ -hypermultiplet equation of motion  $(\nabla^{++})^2\omega = 0$  and some its consequences.

### 3 One-loop effective action in the background field method

The background field method for  $4D, \mathcal{N} = 2$  gauge theories in the harmonic superspace was worked out in [9]. It was generalized to  $6D$  theories in our recent works [37, 38]. Following these techniques, we represent the original superfields  $V^{++}$  and  $\omega$  as a sum of the “background” superfields  $\mathbf{V}^{++}, \mathbf{\Omega}$  and the “quantum” ones  $v^{++}, \omega$ ,

$$V^{++} \rightarrow \mathbf{V}^{++} + f v^{++}, \quad \omega \rightarrow \mathbf{\Omega} + f \omega, \quad (3.1)$$

and then expand the action in a power series with respect to the quantum fields. The one-loop contribution to the effective action  $\Gamma^{(1)}$  for the model (2.1) is given by

$$e^{i\Gamma^{(1)}[\mathbf{V}^{++}, \mathbf{\Omega}]} = \text{Det}^{1/2} \widehat{\square} \int \mathcal{D}v^{++} \mathcal{D}\omega \mathcal{D}\mathbf{b} \mathcal{D}\mathbf{c} \mathcal{D}\varphi \, e^{iS_2[v^{++}, \omega, \mathbf{b}, \mathbf{c}, \varphi, \mathbf{V}^{++}, \mathbf{\Omega}]}, \quad (3.2)$$

where

$$S_2 = S_{gh} + \frac{1}{2} \text{tr} \int d\zeta^{(-4)} v^{++} \widehat{\square} v^{++} - \frac{1}{2} \text{tr} \int d\zeta^{(-4)} (\nabla^{++}\omega)^2 \\ - i \text{tr} \int d\zeta^{(-4)} \left\{ \nabla^{++}\omega[v^{++}, \mathbf{\Omega}] + \nabla^{++}\mathbf{\Omega}[v^{++}, \omega] + \frac{i}{2}[v^{++}, \mathbf{\Omega}]^2 \right\}, \quad (3.3)$$

$$S_{gh} = \text{tr} \int d\zeta^{(-4)} \mathbf{b}(\nabla^{++})^2 \mathbf{c} + \frac{1}{2} \text{tr} \int d\zeta^{(-4)} \varphi(\nabla^{++})^2 \varphi. \quad (3.4)$$

The ghost action  $S_{gh}$  (3.4) involves the Faddeev-Popov ghosts  $\mathbf{b}$  and  $\mathbf{c}$  and also Nielsen-Kallosh ghost  $\phi$ . The covariantly-analytic d'Alembertian  $\widehat{\square}$  is defined as  $\widehat{\square} = \frac{1}{2}(D^+)^4(\nabla^{--})^2$ , where the harmonic covariant derivative  $\nabla^{--} = D^{--} + i\mathbf{V}^{--}$  contains the background superfield  $\mathbf{V}^{--}$ . While acting on an analytic superfield, the operator  $\widehat{\square}$  is given by

$$\widehat{\square} = \eta^{MN} \nabla_M \nabla_N + \mathbf{W}^{+a} \nabla_a^- + \mathbf{F}^{++} \nabla^{--} - \frac{1}{2}(\nabla^{--} \mathbf{F}^{++}), \quad (3.5)$$

where  $\eta^{MN} = \text{diag}(1, -1, -1, -1, -1, -1)$  is the six-dimensional Minkowski metric,  $M, N = 0, \dots, 5$ , and  $\nabla_M = \partial_M + i\mathbf{A}_M$  is the background-dependent vector supercovariant derivative (see [42] for details).

In the action (3.2) the background superfields  $\mathbf{V}^{++}$  and  $\mathbf{\Omega}$  are analytic but unconstrained otherwise. The gauge group of the theory (2.1) is assumed to be  $\text{SU}(N)$ . For the further consideration, we will also assume that the background fields  $\mathbf{V}^{++}$  and  $\mathbf{\Omega}$  align in a fixed direction in the Cartan subalgebra of  $su(N)$

$$\mathbf{V}^{++} = V^{++}(\zeta, u)H, \quad \mathbf{\Omega} = \Omega(\zeta, u)H, \quad (3.6)$$

where  $H$  is a fixed generator in the Cartan subalgebra generating some abelian subgroup  $\Upsilon(1)$ .<sup>3</sup> Our choice of the background corresponds to the spontaneous symmetry breaking  $\text{SU}(N) \rightarrow \text{SU}(N-1) \times \Upsilon(1)$ . We have to note that the pair of the background superfields  $(V^{++}, \Omega)$  forms an abelian vector  $\mathcal{N} = (1, 1)$  multiplet which, in the bosonic sector, contains a single real gauge vector field  $A_M(x)$  and four real scalars  $\phi(x)$  and  $\phi^{(ij)}(x)$ ,  $i, j = 1, 2$ , where  $\phi$  and  $\phi^{(ij)}$  are the scalar components of  $\Omega$  hypermultiplet [41]. The abelian vector field and four scalars in six-dimensional space-time describe just the bosonic world-volume degrees of freedom of a single D5-brane [4, 5].

The classical equations of motions (2.8) for the background superfields  $V^{++}$  and  $\Omega$  are free

$$F^{++} = 0, \quad (D^{++})^2 \Omega = 0. \quad (3.7)$$

In what follows we assume that the background superfields solve the classical equation of motion (3.7). We will also consider the background slowly varying in space-time, i.e. assume that

$$\partial_M W^{+a} = 0, \quad \partial_M \Omega = 0. \quad (3.8)$$

Finally we are left with an abelian background analytic superfields  $V^{++}$  and  $\Omega$ , which satisfy the classical equation of motion (3.7) and the conditions (3.8). Under these assertions the gauge superfield strength  $W^{+a}$  is analytic,<sup>4</sup>  $D_a^+ W^{+b} = \delta_a^b F^{++} = 0$ . For further analysis it is convenient to use the  $\mathcal{N} = (0, 1)$  transformation for gauge superfield strength  $W^{+a}$  (2.15). In the case of the slowly varying abelian on-shell background superfields

<sup>3</sup>We denote the  $H$  component of  $\mathbf{V}^{++}$  by the same letter  $V^{++}$  as the original non-abelian harmonic connection, with the hope that this will not create a misunderstanding. The same concerns the abelian superfield strength  $W^{+a}$ .

<sup>4</sup>In general this is not true and  $F^{++} \neq 0$ .



the hidden  $\mathcal{N} = (0, 1)$  supersymmetry transformations (2.10) and (2.15) have the very simple form

$$\delta\Omega = -i(\epsilon_a^A u_A^-) W^{+a} \quad \delta W^{+a} = 0. \quad (3.9)$$

These transformation rules follow from the abelian version of the transformations (2.10), (2.15) in which one should take into account the conditions (3.8) and (3.7). It is worth to point out that these conditions on their own are covariant under  $\mathcal{N} = (0, 1)$  supersymmetry.

In conclusion of this section, let us consider the simplest  $\mathcal{N} = (1, 1)$  invariants which can be constructed out of the abelian analytic superfields  $W^{+a}$  and  $\Omega$  under the assumptions (3.7) and (3.8). It is evident that the following gauge-invariant action

$$I = f^2 \int d\zeta^{(-4)} (W^+)^4 \mathcal{F}(f\Omega), \quad (3.10)$$

where  $(W^+)^4 = -\frac{1}{24}\varepsilon_{abcd}W^{+a}W^{+b}W^{+c}W^{+d}$  and  $\mathcal{F}(f\Omega)$  is an arbitrary function of  $\Omega$ , is invariant under the transformation (3.9) due to the nilpotency condition  $(W^+)^5 \equiv 0$ . For our further consideration, of the main interest is the choice

$$I_1 = c \int d\zeta^{(-4)} \frac{(W^+)^4}{\Omega^2}, \quad (3.11)$$

which corresponds to  $\mathcal{F} = \frac{1}{f^2\Omega^2}$  in (3.10). The coefficient  $c$  in (3.11) cannot be fixed only on the symmetry grounds and should be calculated in the framework of the quantum field theory. In the next section we will find it from the calculation of the leading low-energy contribution to the effective action of the theory (2.1).

## 4 Leading low-energy contributions to one-loop effective action

We choose the Cartan-Weyl basis for the  $SU(N)$  gauge group generators, so that the quantum superfield  $v^{++}$  has the decomposition

$$v^{++} = v_i^{++} H_i + v_\alpha^{++} E_\alpha, \quad i = 1, \dots, N-1, \quad \alpha = 1, \dots, N(N-1), \quad (4.1)$$

where  $E_\alpha$  is the generator corresponding to the root  $\alpha$  normalized as  $\text{tr}(E_\alpha E_{-\beta}) = \delta_{\alpha\beta}$  and  $H_i$  are the Cartan subalgebra generators,  $[H_i, E_\alpha] = \alpha_{H_i} E_\alpha$ . In this case the background covariant d'Alembertian (3.5) under the conditions (3.7) acts on the quantum superfield  $v^{++}$  as

$$\begin{aligned} \widehat{\square} v^{++} &= \frac{1}{2} (D^+)^4 \left\{ (D^{--})^2 v^{++} + i\alpha_H D^{--} V^{--} v_\alpha^{++} E_\alpha \right. \\ &\quad \left. + i\alpha_H V^{--} D^{--} v_\alpha^{++} E_\alpha - \alpha_H^2 (V^{--})^2 v_\alpha^{++} E_\alpha \right\} \end{aligned} \quad (4.2)$$

$$= \widehat{\square}_H v_\alpha^{++} E_\alpha + \partial_M \partial^M v_i^{++} H_i, \quad (4.3)$$

where we have introduced the operator

$$\widehat{\square}_H := \square + \alpha_H W^{+a} D_a^-. \quad (4.4)$$



The one-loop effective action (3.2) for the background superfields  $V^{++}$  and  $\Omega$  subjected to the conditions (3.7) and (3.8) thus reads

$$\begin{aligned} \Gamma^{(1)} = & \frac{i}{2} \text{Tr}_{(2,2)} \ln \left( \widehat{\square}_H - \alpha_H^2 \Omega^2 \right) + \frac{i}{2} \text{Tr} \ln \left[ (\nabla_H^{++})^2 + A_{(+)} \frac{\alpha_H^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} A_{(-)} \right] \\ & - \frac{i}{2} \text{Tr}_{(4,0)} \ln \widehat{\square}_H - i \text{Tr} \ln (\nabla_H^{++})^2 + \frac{i}{2} \text{Tr} \ln (\nabla_H^{++})^2, \end{aligned} \quad (4.5)$$

where we have defined  $\nabla_H^{++} = D^{++} + \alpha_H V^{++}$  and  $A_{(\pm)}(\Omega) = \Omega \nabla_H^{++} \pm \frac{3}{2} (D^{++} \Omega)$ .

The first term in the first line of the expression (4.5) is the contribution from the gauge multiplet, while the second one is the total contribution from the hypermultiplet. The first term in the second line comes from  $\text{Det}^{1/2} \widehat{\square}$  in (3.2), while the second and the third ones are contributions from the ghost action (3.4). We use the standard definition for the functional trace over harmonic superspace in (4.5)

$$\text{Tr}_{(q,4-q)} \mathcal{O} = \text{tr} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} \delta_{\mathcal{A}}^{(q,4-q)}(1|2) \mathcal{O}^{(q,4-q)}(1|2).$$

Here  $\delta_{\mathcal{A}}^{(q,4-q)}(1|2)$  is an analytic delta-function [41] and  $\mathcal{O}^{(q,4-q)}(\zeta_1, u_1 | \zeta_2, u_2)$  is the kernel of an operator acting in the space of analytic superfields with the harmonic U(1) charge  $q$ .

As the next step we rewrite the contribution from  $\text{Det}^{1/2} \widehat{\square}$  as

$$\frac{i}{2} \text{Tr}_{(4,0)} \ln \widehat{\square}_H = \frac{i}{2} \text{Tr}_{(4,0)} \ln \left( \widehat{\square}_H - \alpha_H^2 \Omega^2 \right) + \frac{i}{2} \text{Tr}_{(4,0)} \left( 1 + \frac{\alpha_H^2 \Omega^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} \right). \quad (4.6)$$

Hence the one-loop contribution to effective action (4.5) is divided as

$$\Gamma^{(1)} = \Gamma_{\text{lead}}^{(1)} + \Gamma_{\text{high}}^{(1)}, \quad (4.7)$$

where

$$\Gamma_{\text{lead}}^{(1)} = \frac{i}{2} \text{Tr}_{(2,2)} \ln \left( \widehat{\square}_H - \alpha_H^2 \Omega^2 \right) - \frac{i}{2} \text{Tr}_{(4,0)} \ln \left( \widehat{\square}_H - \alpha_H^2 \Omega^2 \right), \quad (4.8)$$

and

$$\begin{aligned} \Gamma_{\text{high}}^{(1)} = & \frac{i}{2} \text{Tr} \ln \left[ (\nabla_H^{++})^2 + A_{(+)} \frac{\alpha_H^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} A_{(-)} \right] \\ & - \frac{i}{2} \text{Tr}_{(4,0)} \left( 1 + \frac{\alpha_H^2 \Omega^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} \right) - \frac{i}{2} \text{Tr} \ln (\nabla_H^{++})^2. \end{aligned} \quad (4.9)$$

Then we consider the contribution from the quantum hypermultiplet in (4.5). For on-shell superfields the covariant harmonic derivative  $\nabla_H^{++}$  commutes with  $\widehat{\square}_H$ , but it is not true for the operator  $\widehat{\square}_H - \alpha_H^2 \Omega^2$ . Moreover, the operators  $A_{(\pm)}$  also contain background hypermultiplet and as a consequence do not commute with  $\widehat{\square}_H - \alpha_H^2 \Omega^2$  even for the constant on-shell background superfields

$$\frac{i}{2} \text{Tr} \ln \left[ (\nabla_H^{++})^2 + A_{(+)} \frac{\alpha_H^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} A_{(-)} \right] = \frac{i}{2} \text{Tr} \ln \left[ (\nabla_H^{++})^2 + (\nabla_H^{++})^2 \frac{\alpha_H^2 \Omega^2}{\widehat{\square}_H - \alpha_H^2 \Omega^2} + \dots \right], \quad (4.10)$$

where dots stand for terms involving the harmonic derivative of the hypermultiplet,  $D^{++}\Omega$ . Our aim is to demonstrate that the  $\mathcal{N} = (1,1)$  invariant action (3.11) can be evaluated as the leading contribution to the one-loop effective action  $\Gamma_{\text{lead}}^{(1)}$  (4.8). The action (3.11) contains only the gauge superfield strength  $W^{+a}$  and  $\Omega$  without terms  $D^{++}\Omega$ ,  $D_a^-\Omega$  and  $D_a^-W^{+b}$ . Hence we will systematically neglect such terms in our computations. In this case the contributions from ghosts in (4.5) and the second term in (4.6) are canceled by the corresponding terms in (4.10), and so  $\Gamma_{\text{high}}^{(1)}$  collects terms with  $D^{++}\Omega$  and spinorial derivatives of the background superfields only. Thus in what follows the contribution  $\Gamma_{\text{high}}^{(1)}$  will be ignored.

Computation of the expression (4.8) repeats the analogous one in the four-dimensional case [14]. Both terms in (4.8) contain harmonic singularities in the coincident points limit. According to the analysis of [14], the well-defined expression for the contribution  $\Gamma_{\text{lead}}^{(1)}$  to the one-loop effective action reads<sup>5</sup>

$$\Gamma_{\text{lead}}^{(1)} = -\frac{i}{2} \text{Tr} \int_0^\infty \frac{d(is)}{(is)} e^{is(\widehat{\square}_H - \alpha_H^2 \Omega^2)} \Pi_{\text{T}}^{(2,2)}, \quad (4.11)$$

where  $\Pi_{\text{T}}^{(2,2)}(\zeta_1, u_1; \zeta_2, u_2)$  is the projector on the space of covariantly analytic transverse superfields

$$\Pi_{\text{T}}^{(2,2)}(1|2) = \delta_A^{(2,2)}(1|2) - \nabla_1^{++} \nabla_2^{++} G^{(0,0)}(1|2). \quad (4.12)$$

The Green function  $G^{(0,0)}(\zeta_1, u_1; \zeta_2, u_2)$  satisfies the equation

$$(\nabla_1^{++})^2 G^{(0,0)}(1|2) = -\delta_A^{(4,0)}(1|2), \quad (4.13)$$

and it can be given explicitly [41] as

$$G^{(0,0)}(\zeta_1, u_1; \zeta_2, u_2) = \frac{(D_1^+)^4 (D_2^+)^4}{\widehat{\square}_1} \delta^{14}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3}. \quad (4.14)$$

By explicit calculation one can show that in our case of the on-shell background the projector (4.12) acquires the simple form

$$\Pi_{\text{T}}^{(2,2)} = -\frac{(D_1^+)^4}{\widehat{\square}_1} \left\{ (\nabla_1^-)^4 (u_1^+ u_2^+)^2 - \Omega_1^{--} (u_1^- u_2^+) (u_1^+ u_2^+) + \widehat{\square}_1 (u_1^- u_2^+)^2 \right\} \delta^{14}(z_1 - z_2). \quad (4.15)$$

We substitute the expression (4.15) for  $\Pi_{\text{T}}^{(2,2)}$  in the one-loop contribution  $\Gamma_{\text{lead}}^{(1)}$  (4.11) and take the coincident-harmonic points limit  $u_2 \rightarrow u_1$ . We see that only the third term in (4.15) survives in this limit. Thus we have

$$\Gamma_{\text{lead}}^{(1)} = -\frac{i}{2} \text{tr} \int d\zeta_1^{(-4)} \int_0^\infty \frac{d(is)}{(is)} e^{is(\widehat{\square}_H - \alpha_H^2 \Omega^2)} (D_1^+)^4 \delta^{14}(z_1 - z_2) \Big|_{2=1}. \quad (4.16)$$

---

<sup>5</sup>We have to note that the harmonic derivative commutes with the covariant d'Alembertian on shell. But it is not the case for the operator  $\widehat{\square}_H - \alpha_H^2 \Omega^2$ . Indeed,  $[\widehat{\square}_H - \alpha_H^2 \Omega^2, \nabla_H^{++}] \sim D^{++}\Omega$ . However, as was mentioned above, we omit all such terms since they provide next-to-leading order corrections to the low-energy approximation.

The trace over matrix indices in (4.16) is reduced to a sum over non-zero roots  $\alpha_H$ , taking  $H = \frac{1}{\sqrt{N(N-1)}} \text{diag}(1, \dots, 1, 1 - N)$ . In order to get rid of the Grassmann delta function by using the identity  $(D^+)^4(D^-)^4\delta^8(\theta_1 - \theta_2)|_{2=1} = 1$ , we collect the fourth power of derivative  $D_a^-$  from the exponent in (4.16). Then we pass to the momentum representation and calculate the integral over proper-time  $s$ . Finally we obtain

$$\Gamma_{\text{lead}}^{(1)} = \frac{N-1}{(4\pi)^3} \int d\zeta^{(-4)} \frac{(W^+)^4}{\Omega^2}. \quad (4.17)$$

As expected, the leading low-energy contribution (4.17) to the effective action in the model (2.1) is just the  $\mathcal{N} = (1, 1)$  invariant  $I_1$  (3.11). The coefficient  $c$  now takes the precise value

$$c = \frac{N-1}{(4\pi)^3}. \quad (4.18)$$

The expression for  $c$  is similar to that in the four-dimensional  $\mathcal{N} = 4$  SYM theory (see, e.g., [19] and references therein). In the bosonic sector the effective action (4.17) has the structure

$$\Gamma_{\text{bos}}^{(1)} \sim \int d^6x \frac{F^4}{\phi^2} \left( 1 + \frac{\phi^{(ij)}\phi_{(ij)}}{\phi^2} + \dots \right), \quad (4.19)$$

where  $F^4 = F_{MN}F^{MN}F_{PQ}F^{PQ} - 4F^{NM}F_{MR}F^{RS}F_{SN}$  and  $F_{MN}$  is the abelian gauge field strength.

## 5 Conclusions

In this paper we have studied the quantum aspects of the six-dimensional  $\mathcal{N} = (1, 1)$  super-Yang-Mills theory. We formulated the model in  $6D$ ,  $\mathcal{N} = (1, 0)$  harmonic superspace in terms of  $\mathcal{N} = (1, 0)$  gauge multiplet and  $\omega$ -hypermultiplet, all being in the adjoint representation of gauge group  $SU(N)$ . By construction, the theory possesses the manifest  $\mathcal{N} = (1, 0)$  supersymmetry and an additional non-manifest  $\mathcal{N} = (0, 1)$  one.

We studied the effective action of  $6D$ ,  $\mathcal{N} = (1, 0)$  SYM theory in the framework of the background field method. There was considered the special case of the slowly varying background superfields which break the initial gauge symmetry  $SU(N)$  down to  $SU(N-1) \times \Upsilon(1) \subset SU(N)$  and are subject to the free classical equations of motion. We provided a general analysis of possible  $\mathcal{N} = (1, 1)$  invariants which can be constructed out of the background gauge superfield strength and hypermultiplet. We argued that one of these invariants can be treated as the leading low-energy contribution to the one-loop effective action of  $\mathcal{N} = (1, 1)$  SYM theory. Our main result is the expression (4.17). It is worth pointing out that it is given in the superfield form and reveals the manifest  $\mathcal{N} = (1, 0)$  supersymmetry in parallel with the hidden  $\mathcal{N} = (0, 1)$  supersymmetry.

It is instructive to compare our results on the one-loop low-energy effective action in  $6D$ ,  $\mathcal{N} = (1, 1)$  SYM theory with the recent activity on calculating the one-loop on-shell amplitudes in the same theory [49–53]. The four-point amplitude nicely matches with the

$F^4$  component term in the superfield effective action (4.19) and, as a function of the inverse square of v.e.v. of scalar field, in the leading order is in full agreement with [49]. However, we emphasize that unlike the amplitudes, the result for the effective action in our paper was derived in a manifest superfield form. The four-point amplitude in [51–53] does not allow to directly restore this effective action. The point is that the effective action (4.17) contains the hypermultiplet superfield whose scalar component serves as a natural infrared regulator. In the above papers the amplitudes were calculated in the gauge multiplet sector only, that is not sufficient for deriving the true full effective action. In this respect, it is worth to mention once again the paper [49], where the four-point amplitude in the gauge multiplet sector was calculated under the assumption that the gauge symmetry is spontaneously broken and the gauge field acquires a mass by Higgs mechanism. Such a mass can be treated as a partial case of the constant scalar component of the hypermultiplet in our general superfield effective action (4.17). Then the structure of the first term in the bosonic component effective action (4.19) is completely analogous to the four-point amplitude of ref. [49].

We have to note that even in the one-loop approximation there might exist more complicated contributions to the effective action, which can be expected on the basis of a general analysis. One can consider, e.g., a  $6D$  analog of supersymmetric Heisenberg-Euler type effective action in  $\mathcal{N} = (1, 1)$  SYM theory. Also, it should be noticed that we studied only those contributions to the effective action which contain no harmonic derivatives of the hypermultiplet superfield. The contributions involving such derivatives were beyond the scope of our consideration. It would be interesting to analyze such contributions and to consider a more general class of the background superfields. Namely, it is tempting to find a way to consider the complete effective action with the whole dependence on the harmonic derivatives of the background hypermultiplet included. In this way we expect to obtain the complete  $\mathcal{N} = (1, 1)$  supersymmetric quantum effective action possessing both explicit  $\mathcal{N} = (1, 0)$  and hidden  $\mathcal{N} = (0, 1)$  supersymmetries. We hope to address these issues in the forthcoming works.

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