



On the two-loop divergences of the 2-point hypermultiplet supergraphs for $6D$, $\mathcal{N} = (1, 1)$ SYM theory

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ABSTRACT

We consider $6D$, $\mathcal{N} = (1, 1)$ supersymmetric Yang–Mills theory formulated in $\mathcal{N} = (1, 0)$ harmonic superspace and analyze the structure of the two-loop divergences in the hypermultiplet sector. Using the $\mathcal{N} = (1, 0)$ superfield background field method we study the two-point supergraphs with the hypermultiplet legs and prove that their total contribution to the divergent part of effective action vanishes off shell.

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1. Introduction

This paper is a continuation and further development of our previous works on the structure of divergences in $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ gauge theories [1–3].

The study of supersymmetric gauge models in higher dimensions attracts much attention due to both their tight links with the superstring/brane stuff and some remarkable properties of them in the quantum domain. On the one hand, these models are non-renormalizable because of the dimensionful coupling constant, like, e.g., (super)gravity theories. On the other hand, supersymmetry may ensure canceling some divergences, so that one can expect a better ultraviolet behavior of supersymmetric gauge theories as compared to the non-supersymmetric case [4–13] (see also the review [14]).

It is known that in $4D$, $\mathcal{N} = 4$ SYM theory all divergences vanish off-shell and the theory proves to be finite just due to a large amount of supersymmetries [15–18]. At the classical level, this theory is very similar to $6D$, $\mathcal{N} = (1, 1)$ SYM theory (see, e.g., [19] for a formulation of $6D$ supersymmetry). Indeed, $4D$, $\mathcal{N} = 4$ SYM theory can be obtained from $6D$, $\mathcal{N} = (1, 1)$ theory by means

of dimensional reduction. Moreover, formulations of both theories in the harmonic superspace [20–24,12] reveal a great similarity. This resemblance suggests that $6D$, $\mathcal{N} = (1, 1)$ SYM theory could have a better ultraviolet behavior compared to other $6D$ theories. This was confirmed by the one-loop calculation, which demonstrated that one-loop divergences in this theory cancel even off-shell [2,3]. Obviously, it would be very interesting to investigate whether this remarkable quantum property persists at the two-loop level. It is known that $6D$, $\mathcal{N} = (1, 1)$ SYM theory is on-shell finite at the two-loop level [4–9,11]. In this letter we will investigate the off-shell divergences. To calculate them, we make use of the technique of the harmonic supergraphs, which allows one to perform all calculations in a manifestly $\mathcal{N} = (1, 0)$ supersymmetric way. Besides, we use the $\mathcal{N} = (1, 0)$ background superfield method which ensures preserving the classical gauge symmetry of the effective action [2,3]. The theory under consideration possesses the hidden $\mathcal{N} = (0, 1)$ supersymmetry [12]. As a consequence, it suffices to analyze the supergraphs with the hypermultiplet external lines only. All other supergraphs can be obtained from these ones via the hidden supersymmetry transformations. Therefore, if the effective action is finite in the pure hypermultiplet sector, it is finite as a total. In this letter we limit our study to the structure of the two-loop divergences of the supergraphs with the two external hypermultiplet legs only and demonstrate that all such divergences cancel off shell. The divergences of the supergraphs with four hypermultiplet legs will be a subject of the next publication.

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2. $\mathcal{N} = (1, 1)$ SYM theory in $\mathcal{N} = (1, 0)$ harmonic superspace

The 6D, $\mathcal{N} = (1, 1)$ SYM theory can be considered as a particular case of 6D, $\mathcal{N} = (1, 0)$ SYM theories. It is convenient to describe them using the formalism of the harmonic superspace, because $\mathcal{N} = (1, 0)$ supersymmetry is then a manifest symmetry of the theory at all steps of calculating quantum corrections. Besides, this theory possesses the hidden $\mathcal{N} = (0, 1)$ supersymmetry [12].

Following ref. [12] we briefly consider the harmonic superspace formulation of the theory. We introduce the harmonic variables $u^{\pm i}$, where $i = 1, 2$ and $u_i^- = (u^+)^*$, such that $u^+ u_i^- = 1$. The harmonic superspace is obtained by adding these coordinates to the set $z \equiv (x^M, \theta_i^a)$, where x^M with $M = 0, \dots, 5$, are usual 6D Minkowski coordinates and θ_i^a with $a = 1, \dots, 4$ are anticommuting left-handed spinors. The analytic coordinates are defined as $\zeta \equiv (x_A^M, \theta^{+a})$, where

$$x_A^M \equiv x^M + \frac{i}{2} \theta^- \gamma^M \theta^+, \quad \theta^{\pm a} \equiv u_i^{\pm} \theta^{ai}, \quad (1)$$

with γ^M being the six-dimensional γ -matrices. In our notation, the integration measures are written as

$$\int d^{14}z = \int d^6x d^8\theta; \quad \int d\zeta^{(-4)} \equiv \int d^6x d^4\theta^+. \quad (2)$$

In the harmonic superspace approach, 6D, $\mathcal{N} = (1, 0)$ SYM theory with the gauge group G and the hypermultiplets in the representation R is described by the action

$$S = \frac{1}{f_0^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+. \quad (3)$$

Then $\mathcal{N} = (1, 1)$ SYM theory is reproduced in the particular case, when the hypermultiplets belong to the adjoint representation, $R = \text{Adj}$. In general, the theories described by the action (3) are anomalous [25–27], but the anomalies are canceled for $\mathcal{N} = (1, 1)$ theory.

The gauge superfield $V^{++}(z, u)$ lies in the adjoint representation of the gauge group. It is real with respect to the specially defined conjugation denoted by tilde, $\widetilde{V^{++}} = V^{++}$, and satisfies the analyticity condition,

$$D_a^+ V^{++} = 0. \quad (4)$$

In the pure gauge field part of the action (3), $V^{++}(z, u) = V^{++A} t^A$, where t^A are the generators of the fundamental representation of the gauge group G , normalized by the condition

$$\text{tr}(t^A t^B) = \frac{1}{2} \delta^{AB}. \quad (5)$$

In our notation the structure constants f^{ABC} are defined by the commutation relation

$$[t^A, t^B] = i f^{ABC} t^C. \quad (6)$$

The bare coupling constant f_0 in 6D has the dimension m^{-1} .

Hypermultiplets are described by the analytic superfields $(q^+)_i$, and the covariant harmonic derivative is written as

$$\nabla^{++} = D^{++} + i V^{++} = D^{++} + i V^{++A} T^A, \quad (7)$$

where $(T^A)_i^j$ denotes generators in the representation R to which the hypermultiplet belongs.

The gauge transformations in harmonic superspace,

$$V^{++} \rightarrow e^{i\lambda} V^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}; \quad q^+ \rightarrow e^{i\lambda} q^+, \quad (8)$$

are parameterized by the Lie-algebra valued analytic superfield parameter λ .

For quantization we use the background field method [2,3]. Its basic convenience is the possibility to make the effective action manifestly invariant under the background gauge transformations by a proper choice of the gauge condition. The background-quantum splitting is linear,

$$V^{++} = \mathbf{V}^{++} + v^{++}. \quad (9)$$

Here \mathbf{V}^{++} is the background gauge superfield, and v^{++} is the quantum gauge superfield. We will use the gauge-fixing term of the form

$$S_{\text{gf}} = -\frac{1}{2f_0^2} \text{tr} \int d^{14}z du_1 du_2 \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \times D_1^{++} \left(e^{-i\mathbf{b}(z, u_1)} v^{++}(z, u_1) e^{i\mathbf{b}(z, u_1)} \right) \times D_2^{++} \left(e^{-i\mathbf{b}(z, u_2)} v^{++}(z, u_2) e^{i\mathbf{b}(z, u_2)} \right), \quad (10)$$

where $\mathbf{b}(z, u)$ is the background bridge superfield related to the background superfield \mathbf{V}^{++} by the relation

$$\mathbf{V}^{++} = -i e^{i\mathbf{b}} D^{++} e^{-i\mathbf{b}}. \quad (11)$$

The gauge-fixing term (10) is invariant under the background gauge transformations

$$\mathbf{V}^{++} \rightarrow e^{i\lambda} \mathbf{V}^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}; \quad v^{++} \rightarrow e^{i\lambda} v^{++} e^{-i\lambda}; \quad q^+ \rightarrow e^{i\lambda} q^+; \quad e^{i\mathbf{b}} \rightarrow e^{i\lambda} e^{i\mathbf{b}} e^{i\tau}, \quad (12)$$

where the gauge parameter $\tau = \tau(z)$ does not depend on the harmonic variables.

The Faddeev–Popov ghost action corresponding to the gauge fixing-term (10) reads

$$S_{\text{FP}} = \text{tr} \int d\zeta^{(-4)} du b \nabla^{++} \left(\nabla^{++} c + i[v^{++}, c] \right), \quad (13)$$

where b and c are anticommuting analytic superfields in the adjoint representation of the gauge group, and $\nabla^{++} c = D^{++} c + i[\mathbf{V}^{++}, c]$ is the background-covariant derivative. The generating functional also involves determinants corresponding to the Nielsen–Kallosh ghosts,

$$Z = \int Dv^{++} D\tilde{q}^+ Dq^+ Db Dc D\varphi \times \text{Det}^{1/2} \widehat{\square} \exp \left[i(S + S_{\text{gf}} + S_{\text{FP}} + S_{\text{NK}} + S_{\text{sources}}) \right], \quad (14)$$

where

$$S_{\text{NK}} = -\frac{1}{2} \text{tr} \int d\zeta^{(-4)} du (\nabla^{++} \varphi)^2, \quad (15)$$

and φ is a commuting analytic superfield in the adjoint representation. In our notation

$$\widehat{\square} \equiv \frac{1}{2} (D^+)^4 (\nabla^{--})^2 \quad \text{and} \quad (D^+)^4 = -\frac{1}{24} \varepsilon^{abcd} D_a^+ D_b^+ D_c^+ D_d^+. \quad (16)$$

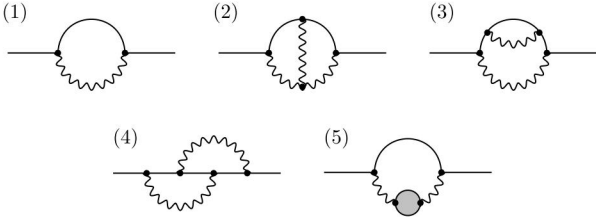


Fig. 1. One- and two-loop diagrams contributing to the two-point Green function of the hypermultiplet.

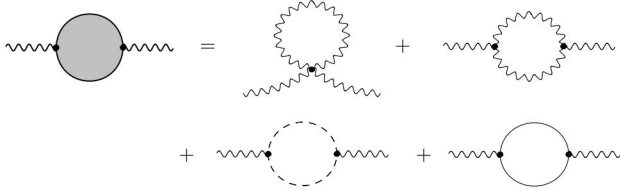


Fig. 2. One-loop subdiagrams entering the two-loop diagram (5) in Fig. 1.

The operator $\hat{\square}$ acts on a superfield σ in the adjoint representation, for which

$$\nabla^{--}\sigma \equiv D^{--}\sigma + i[\mathbf{V}^{--}, \sigma], \quad (17)$$

where $\mathbf{V}^{--} = -ie^{ib}D^{--}e^{-ib}$. The expression S_{sources} includes the relevant source terms.

The structure of divergences in the hypermultiplet sector is determined by the expression for the superficial degree of divergence in harmonic superfield formulation [2,3],

$$\omega = 2L - N_q - \frac{1}{2}N_D, \quad (18)$$

where L is a number of loops, N_q is a number of external hypermultiplet legs, and N_D is a number of spinor derivatives acting on external legs. From this equation we see that in the two-loop approximation ($L = 2$) the diagrams with two external hypermultiplet legs ($N_q = 2$) are quadratically divergent. Also we see that the diagrams with four external hypermultiplet legs ($N_q = 4$) are logarithmically divergent. In this paper we will calculate the two-point function of the hypermultiplet which corresponds to the first case only.

3. Two-loop two-point Green function of the hypermultiplet

In the one-loop approximation the two-point function of the hypermultiplet is given by the first diagram in Fig. 1. All other diagrams correspond to two loops. The last diagram (5) in Fig. 1 contains the insertion of the one-loop polarization operator, which is denoted by the gray disk. The diagrams contributing to this one-loop polarization operator are depicted in Fig. 2. Although our purpose is to calculate these diagrams for $\mathcal{N} = (1, 1)$ theory, we will consider a more general case of $\mathcal{N} = (1, 0)$ theory with hypermultiplets in the representation R . The direct calculation leads to the following contributions to the effective action coming from the diagrams drawn in Fig. 1¹:

$$(1) = 0; \quad (19)$$

$$(2) = 2C_2 f_0^4 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \times \int \frac{du_1 du_2}{(u_1^+ u_2^+)} \tilde{q}^+(p, \theta, u_1)^i (T^A T^A)_i^j q^+(-p, \theta, u_2)_j \times \int \frac{d^6 k}{(2\pi)^6} \frac{d^6 l}{(2\pi)^6} \frac{1}{k^2 l^2 (k+l)^2 (k+p)^2}; \quad (20)$$

$$(3) = 0; \quad (21)$$

$$(4) = -4f_0^4 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \times \int \frac{du_1 du_2}{(u_1^+ u_2^+)} \tilde{q}^+(p, \theta, u_1)^i (T^A T^B T^A T^B)_i^j q^+(-p, \theta, u_2)_j \times \int \frac{d^6 k}{(2\pi)^6} \frac{d^6 l}{(2\pi)^6} \frac{1}{k^2 l^2 (k+l)^2 (k+p)^2}; \quad (22)$$

$$(5) = 4(C_2 - T(R)) f_0^4 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \times \int \frac{du_1 du_2}{(u_1^+ u_2^+)} \tilde{q}^+(p, \theta, u_1)^i (T^A T^A)_i^j q^+(-p, \theta, u_2)_j \times \int \frac{d^6 k}{(2\pi)^6} \frac{d^6 l}{(2\pi)^6} \frac{1}{k^4 (k+p)^2 l^2 (k+l)^2}. \quad (23)$$

Note that all these expressions are written in the Minkowski space before the Wick rotation. The group theory coefficients entering them are defined by the relations

$$f^{ACD} f^{BCD} = C_2 \delta^{AB}; \quad T(R) \delta^{AB} = \text{tr}(T^A T^B); \quad C(R)_i^j = (T^A T^A)_i^j. \quad (24)$$

From these relations, after some algebra, we derive

$$(T^A T^B T^A T^B)_i^j = \left(C(R)^2 - \frac{1}{2} C_2 C(R) \right)_i^j. \quad (25)$$

Thus, the result for the sum of all considered diagrams can be written in the form

$$4f_0^4 \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \int \frac{du_1 du_2}{(u_1^+ u_2^+)} \times \left[\tilde{q}^+(p, \theta, u_1)^i \left(-C(R)^2 + C_2 C(R) \right)_i^j q^+(-p, \theta, u_2)_j \times \int \frac{d^6 k}{(2\pi)^6} \frac{d^6 l}{(2\pi)^6} \frac{1}{k^2 l^2 (k+l)^2 (k+p)^2} + (C_2 - T(R)) \tilde{q}^+(p, \theta, u_1)^i C(R)_i^j \times q^+(-p, \theta, u_2)_j \int \frac{d^6 k}{(2\pi)^6} \frac{d^6 l}{(2\pi)^6} \frac{1}{k^4 (k+p)^2 l^2 (k+l)^2} \right]. \quad (26)$$

We see that it is quadratically divergent, in the precise agreement with the general expression for the degree of divergence calculated in [2]. The expression (26) is written formally, because the regularization was not still introduced. It is known that quadratic divergences cannot be caught within the dimensional regularization, and it is necessary to use different regularization schemes, as, e.g., in [28]. Note, however, that for the considered theory $R = \text{Adj}$, i.e. the hypermultiplets belong to the adjoint representation of the gauge group,

$$T(\text{Adj}) = C_2; \quad C(\text{Adj})_i^j = C_2 \delta_i^j. \quad (27)$$

¹ The calculations are similar to those in [3] and here we omit the technical details.

This implies that for $\mathcal{N} = (1, 1)$ theory the expression (26) vanishes identically. In particular, the leading quadratic divergences cancel each other, so that in the case of using the dimensional regularization technique we do not miss any divergent contributions. Moreover, within the dimensional reduction scheme the result obtained from Eq. (26) by the replacement $6 \rightarrow D$ also identically vanishes. Thereby, the logarithmically divergent contributions are also absent and the considered Green function vanishes in the two-loop approximation for $\mathcal{N} = (1, 1)$ SYM theory.

It is worth to point out that the finiteness of the two-point supergraphs in the hypermultiplet sector, implied by the conditions (27), is achieved in the same way as in the one-loop case [2,3].

4. Summary

In this paper we have investigated the two-loop divergences in $6D$, $\mathcal{N} = (1, 1)$ SYM theory. This theory is a 6-dimensional analog of $4D$, $\mathcal{N} = 4$ SYM theory, for which reason one could expect a better ultraviolet behavior of this theory in comparison with other $6D$ theories. First, we calculated the two-loop divergences of the hypermultiplet two-point function in $6D$, $\mathcal{N} = (1, 0)$ vector multiplet theory coupled to the hypermultiplet in an arbitrary representation of gauge group. Then we turn to $6D$, $\mathcal{N} = (1, 1)$ SYM theory, which corresponds to the hypermultiplet in adjoint representation. We proved that the corresponding divergences identically vanish² without using the equations of motion. Moreover, the conditions of vanishing of divergences are the same as in the one-loop case. Taking into account that the Green function considered is related to other two-point Green functions of $\mathcal{N} = (1, 1)$ theory by hidden $\mathcal{N} = (0, 1)$ supersymmetry, we come to the conclusion that all two-point Green functions of the theory are finite in the two-loop approximation. However, logarithmic divergences can still appear in the four-point Green functions. To see, whether they are finite or not, it will be sufficient to calculate the four-point function of the hypermultiplet. We are going to address this problem in the forthcoming paper.

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² Here we essentially used the property that off-shell divergences in the theory under consideration are absent at one loop [2,3] and therefore there is no need to take into account the one-loop counterterms for two-loop diagrams.