

The renormalization structure of $6D$, $\mathcal{N} = (1, 0)$ supersymmetric higher-derivative gauge theory

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Abstract

We consider the harmonic superspace formulation of higher-derivative $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theory and its minimal coupling to a hypermultiplet. In components, the kinetic term for the gauge field in such a theory involves four space-time derivatives. The theory is quantized in the framework of the superfield background method ensuring manifest $6D$, $\mathcal{N} = (1, 0)$ supersymmetry and the classical gauge invariance of the quantum effective action. We evaluate the superficial degree of divergence and prove it to be independent of the number of loops. Using the regularization by dimensional reduction, we find possible counterterms and show that they can be removed by the coupling constant renormalization for any number of loops, while the divergences in the hypermultiplet sector are absent at all. Assuming that the deviation of the gauge-fixing term from that in the Feynman gauge is small, we explicitly calculate the divergent part of the one-loop effective action in the lowest order in this deviation. In the approximation considered, the result is independent of the gauge-fixing parameter and agrees with the earlier calculation for the theory without a hypermultiplet.

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1. Introduction

The $6D, \mathcal{N} = (1, 0)$ supersymmetric higher-derivative gauge theory was firstly constructed in [1], starting from its harmonic-superspace formulation. It describes a self-interacting non-abelian gauge multiplet with the kinetic term of the fourth order in derivatives.

The ordinary $\mathcal{N} = (1, 0)$ supersymmetric Yang-Mills (SYM) theory in $6D$ has a dimensionful coupling constant and for this reason is non-renormalizable. The UV behavior of such a theory was studied by direct quantum calculations in the component approach [2–6], by using the gauge and supersymmetry methods [7–11], by applying the background field method in superspace [12,13] and, recently, by the modern amplitude techniques (see, e.g., [14] and references therein). In contrast to the gauge theory with the standard kinetic term the higher-derivative model with four space-time derivatives [1] possesses a dimensionless coupling constant. It is renormalizable by power counting and conformally invariant [15] at the classical level, although reveals the quantum conformal anomaly [16,17]. The one-loop beta function for this theory has been calculated in [1,18] using the component formulation and, recently, by applying the supergraph analysis in harmonic superspace [19].

Although the higher-derivative models are plagued by the ghost states in the spectrum, they are widely used in classical and quantum field theory.¹ The main attractive feature of such models is seen in a possibility to improve their ultraviolet behavior as compared to the corresponding conventional theories. In particular, the inclusion of the four-derivative terms into the general relativity Lagrangian yields a renormalizable quantum gravity with matter (see, e.g., [24–26]). The higher derivatives appear naturally (and inevitably) in such field models as conformal (super)gravities and (super)conformal higher-spin theories (see, e.g., [27–32] and the references therein). In this paper we investigate the quantum aspects of the six-dimensional supersymmetric higher-derivative theory pioneered in [1].

Our aim here is to study the divergence structure of the theory under consideration in a manifestly supersymmetric and covariant way on the basis of the background superfield method in $6D, \mathcal{N} = (1, 0)$ harmonic superspace [33,34]. The general concept of the harmonic superspace has been introduced in [35] (see also the book [36]). The background field method in $6D, \mathcal{N} = (1, 0)$ harmonic superspace was formulated in our works [12,13].² Taking into account the structure of the superfield propagators and vertices, as well as the manifest supersymmetry and gauge invariance of the effective action, we calculate the superficial degree of the divergence and prove that it does not depend on the number of loops. In the case of using the regularization by dimensional reduction the power counting implies that any possible counterterm is proportional to the classical higher-derivative action. This means that the theory under consideration is multiplicatively renormalizable.

¹ In fact, there were numerous attempts to show that in the interacting higher-derivative theories (with or without supersymmetry) the ghosts can be arranged in such a way that they do not contribute to the observables (see, e.g., [16,20–23] and the references therein).

² $6D, \mathcal{N} = (1, 0)$ background superfield method is in many aspects similar to the one developed for $4D, \mathcal{N} = 2$ SYM theory [37,38].

The basic feature of the background field method is the use of special gauge conditions involving a dependence on the background fields. However, due to the presence of higher derivatives, the previously used background field gauge-fixing conditions are not convenient in the theory under consideration and the superfield gauge-fixing action should be constructed in a different way. In this paper we introduce the appropriate family of the gauge-fixing conditions depending on one real (gauge) parameter ξ_0 . The value $\xi_0 = 1$ corresponds to the minimal gauge. Nevertheless, keeping ξ_0 arbitrary provides us with an efficient tool of checking the correctness of the calculations. Indeed, the divergences of the multiplicatively renormalizable theory in the background field method should not depend on the gauge choice [39] (see also [40] and the references therein).

It is worth noting that using an arbitrary parameter ξ_0 extremely complicates the calculations of divergences, since the background-field dependent differential operator in the quadratic part of the action becomes non-minimal.³ The most powerful manifestly covariant method to work with such operators is the generalized proper-time technique [41], which can in principle be reformulated for superfield theories. However, in some cases the calculations can be further simplified, assuming that the deviation of the gauge-fixing parameter from its value in the minimal gauge is small and the calculations are performed to the lowest order in this small parameter. In this paper we follow just this strategy. We assume that the value of the gauge parameter ξ_0 is close to 1 and calculate the one-loop divergences in the lowest order in the deviation ($\xi_0 - 1$). Then we explicitly demonstrate that the logarithmic divergencies are gauge independent in the considered approximation, as is expected for a multiplicatively renormalizable theory. The numerical coefficient before the one-loop beta function precisely matches the result of refs. [1,18,19].

Also we study the UV behavior for the model in which the higher-derivative gauge multiplet is minimally coupled to the hypermultiplet with the standard kinetic term (yielding the standard second and first-order kinetic terms for the relevant physical bosonic and fermionic fields). We demonstrate that the presence of such a hypermultiplet does not destroy the renormalizability of the theory (regularized by dimensional reduction) and that no divergent contributions depending on the background hypermultiplet appear even in non-minimal gauges. At the one-loop level, the only effect of adding the hypermultiplet is the change of the absolute value of the coefficient before the beta function in the gauge superfield sector.

The paper is organized as follows. Section 2 presents the classical formulation of the theories under consideration. In Section 3 we perform the quantization of the theory with higher derivatives in the harmonic $6D$, $\mathcal{N} = (1, 0)$ superspace, based upon the background superfield method. In Section 4 we discuss the general structure of divergencies in the theory and calculate the relevant divergence degree. In Section 5 we find the divergent contribution to one-loop effective action for the higher-derivative gauge theory. The UV properties of the higher-derivative gauge theory coupled to a hypermultiplet are explored in Section 6. The concluding Section 7 contains a brief summary of our results and a list of possible directions of the future work.

2. Superfield formulation of $6D$, $\mathcal{N} = (1, 0)$ higher-derivative gauge theory

The classical action of the higher-derivative $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theory constructed in [1] is written in the harmonic superspace as

³ The higher-derivative differential operator acting on space-time fields is called non-minimal if the higher derivatives are not assembled into powers of d'Alembertian.

$$S_0 = \pm \frac{1}{2g_0^2} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2, \quad (2.1)$$

where g_0 is a dimensionless coupling constant. The integration measure over analytic subspace in harmonic superspace is denoted by $d\zeta^{(-4)} = d^6x_{(\text{an})} (D^-)^4$, see [36] for details. The covariant strength of the analytic gauge superfield V^{++} is defined by the expression

$$F^{++} = (D^+)^4 V^{--} = -\frac{1}{24} \varepsilon^{abcd} D_a^+ D_b^+ D_c^+ D_d^+ V^{--}, \quad (2.2)$$

where

$$V^{--}(z, u) = \sum_{n=1}^{\infty} (-i)^{n+1} \int du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \dots (u_n^+ u^+)} \quad (2.3)$$

is a non-analytic superfield introduced in [34]. As we noted in [19], the overall sign of the action in higher derivative theories cannot be fixed from the standard requirement of the positive energy.

The action (2.1) is invariant under the gauge transformation

$$\delta_\lambda V^{++} = -D^{++}\lambda - i[V^{++}, \lambda], \quad \delta_\lambda F^{++} = i[\lambda, F^{++}], \quad (2.4)$$

with the Hermitian analytic superfield parameter λ taking values in the Lie algebra of the gauge group. In our notation the generators are normalized by the condition $\text{tr}(t^I t^J) = \frac{1}{2} \delta^{IJ}$ and satisfy the commutation relation $[t^I, t^J] = i f^{IJK} t^K$.

The component structure of the action (2.1) was discussed in [1]. The $6D$ gauge multiplet V^{++} in the Wess-Zumino gauge involves⁴ the vector field A_M , the Weyl fermion ψ^{ai} and an $SU(2)$ triplet of scalar fields $\mathcal{D}^{(ij)}$. In the component form the action (2.1) contains four derivatives in the kinetic term for the gauge field A_M and three derivatives in the kinetic term for the gaugino ψ^{ai} . The scalar fields \mathcal{D}^{ij} are also dynamical, with standard two derivatives in the kinetic term.

The higher-derivative gauge theory (2.1) in interaction with the hypermultiplet q^+ possessing the standard kinetic term (see [36] for details) is described by the action

$$\tilde{S}_0 = S_0 - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+. \quad (2.5)$$

The hypermultiplet can be placed in an arbitrary representation R of the gauge group. Correspondingly, the harmonic covariant derivative in (2.5), $\nabla^{++} = D^{++} + iV^{++}$, is defined in this representation. The action (2.5) is invariant under the transformation (2.4) supplemented by the transformation of the hypermultiplet,

$$\delta_\lambda q^+ = i\lambda q^+. \quad (2.6)$$

The classical equations of motion for the model (2.5) read

$$\begin{aligned} \frac{\delta \tilde{S}_0}{\delta V^{++I}} &= \pm \frac{1}{2g_0^2} (\square F^{++})^I - i\tilde{q}^+ T^I q^+ = 0, \\ \frac{\delta \tilde{S}_0}{\delta \tilde{q}^+} &= -\nabla^{++} q^+ = 0, \end{aligned} \quad (2.7)$$

where T^I , $I = 1, \dots, \dim G$ are the gauge group generators in the representation R . The operator

⁴ Hereafter we denote the space-time indices as $M, N = 0, \dots, 5$, the spinor ones as $a, b = 1, \dots, 4$ and the indices of $SU(2)$ R-symmetry group as $i, j = 1, 2$.

$$\widehat{\square} = \frac{1}{2}(D^+)^4(\nabla^{--})^2 \quad (2.8)$$

acting on a space of analytic superfields is reduced to the covariant superfield d'Alembertian

$$\widehat{\square} = \eta^{MN} \nabla_M \nabla_N + i W^{+a} \nabla_a^- + i F^{++} \nabla^{--} - \frac{i}{2} (\nabla^{--} F^{++}), \quad (2.9)$$

where η_{MN} , is 6D Minkowski metric (with the signature $(+---)$) and the covariant derivatives are defined by the relations

$$\nabla^{--} = D^{--} + i V^{--}, \quad [\nabla^{--}, D_a^+] = \nabla_a^-, \quad [D_a^+, \nabla_b^-] = i(\gamma^M)_{ab} \nabla_M. \quad (2.10)$$

We have introduced in (2.9) the gauge superfield strength

$$W^{+a} = -\frac{1}{6} \varepsilon^{abcd} D_b^+ D_c^+ D_d^+ V^{--}, \quad (2.11)$$

such that $F^{++} = \frac{1}{4} D_a^+ W^{+a}$.

3. Effective action

For defining the quantum version of the theory (2.1) we use the background superfield method. Following the general procedure, the superfield V^{++} is split into the sum of the “background” superfield \mathbf{V}^{++} and the “quantum” one v^{++} ,

$$V^{++} = \mathbf{V}^{++} + v^{++}. \quad (3.1)$$

Like in [12], we use the gauge-fixing function $\mathcal{F}_\tau^{(+4)} = D^{++} v_\tau^{++}$, where the subscript τ means τ -frame which is constructed with the help of the background bridge superfield. The corresponding action for the real analytic fermionic Faddeev-Popov ghosts b and c has the same form as in 6D, $\mathcal{N} = (1, 0)$ SYM theory with the standard kinetic term [12,13]

$$S_{\text{FP}}[b, c, v^{++}, \mathbf{V}^{++}] = \text{tr} \int d\zeta^{(-4)} du b \mathbf{V}^{++} \left(\nabla^{++} c + i[v^{++}, c] \right), \quad (3.2)$$

where $\nabla^{\pm\pm} = D^{\pm\pm} + i V^{\pm\pm}$.

The effective action $\Gamma[V^{++}]$ of the theory is defined as in [12]

$$e^{i\Gamma[V^{++}]} = \int \mathcal{D}v^{++} \mathcal{D}b \mathcal{D}c \delta[\mathcal{F}^{(+4)} - f^{(+4)}] \times \exp \left(i \left\{ S_0[\mathbf{V}^{++} + v^{++}] + S_{\text{FP}}[b, c, v^{++}, \mathbf{V}^{++}] \right\} \right). \quad (3.3)$$

The superfield $f^{(+4)}(\zeta, u)$ is an external analytic superfield independent of the background superfield \mathbf{V}^{++} which takes value in the Lie algebra of the gauge group. The effective action by construction is invariant under the background gauge transformations

$$\delta \mathbf{V}^{++} = -\nabla^{++} \lambda, \quad \delta v^{++} = i[\lambda, v^{++}]. \quad (3.4)$$

Following [12], we average the delta-function $\delta(F^{(+4)} - f^{(+4)})$ with the weight

$$1 = \Delta_{\text{NK}}[V^{++}] \exp \left\{ \mp \frac{i}{2g_0^2 \xi_0} \int d^{14} z du_1 du_2 f_\tau^{(+4)}(u_1) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \widehat{\square}_2 f_\tau^{(+4)}(u_2) \right\}, \quad (3.5)$$

where $\widehat{\square} = \frac{1}{2}(D^+)^4(\nabla^{++})^2$ and ξ_0 is an arbitrary real parameter. Note that, as distinct from the theory without higher derivatives, in (3.5) there appears an extra operator $\widehat{\square}$. The factor $\Delta[V^{++}]$

yields the Nielson-Kallosh determinant. It is convenient to present it as the functional integral over the bosonic real analytic superfield φ and anticommuting analytic superfields $\chi^{(+4)}$ and σ , all in the adjoint representation of the gauge group,

$$\Delta_{\text{NK}}[V^{++}] = \int \mathcal{D}\varphi \mathcal{D}\chi^{(+4)} \mathcal{D}\sigma \exp \left\{ i S_{\text{NK}}[\varphi, V^{++}] \right\}. \quad (3.6)$$

Here,

$$S_{\text{NK}} = \text{tr} \int d\zeta^{(-4)} du \left(-\frac{1}{2} \varphi (\nabla^{++})^2 \varphi + \chi^{(+4)} \widehat{\square} \sigma \right) \quad (3.7)$$

is the Nielsen-Kallosh ghost action.

The gauge-fixing term obtained as a result of the procedure described above reads

$$S_{\text{gf}}[v^{++}, V^{++}] = \mp \frac{1}{2g_0^2 \xi_0} \text{tr} \int d^{14}z du_1 du_2 \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} (D^{++} v_{\tau}^{++})_2 (\widehat{\square}_{\tau} D^{++} v_{\tau}^{++})_1. \quad (3.8)$$

According to [19], it can be equivalently rewritten in the form

$$\begin{aligned} S_{\text{gf}} = & \pm \frac{1}{2g_0^2 \xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++} \widehat{\square}^2 v^{++} \\ & \mp \frac{1}{2g_0^2 \xi_0} \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} v_{\tau,1}^{++} \left\{ (\widehat{\square} v^{++}) + \frac{i}{2} \nabla^{--} [F^{++}, v^{++}] \right\}_{\tau,2}. \end{aligned} \quad (3.9)$$

Adding this expression to that part of the classical action which is quadratic in the quantum gauge superfield and was calculated in [19],

$$\begin{aligned} S_0^{(2)} = & \pm \frac{1}{2g_0^2} \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} \left\{ v_{\tau,1}^{++} (\widehat{\square} v^{++})_{\tau,2} + \frac{i}{2} v_{\tau,1}^{++} [(\nabla^{--} F^{++}), v^{++}]_{\tau,2} \right\} \\ & \mp \frac{i}{4g_0^2} \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} v_{\tau,1}^{++} [F^{++}, \nabla^{--} v^{++}]_{\tau,2}, \end{aligned} \quad (3.10)$$

we obtain

$$\begin{aligned} S_{\text{gauge}}^{(2)} = & \pm \frac{1}{2g_0^2 \xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++} \widehat{\square}^2 v^{++} \\ & \pm \frac{1}{2g_0^2} \left(1 - \frac{1}{\xi_0} \right) \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} \left\{ v_{\tau,1}^{++} (\widehat{\square} v^{++})_{\tau,2} \right. \\ & \left. + \frac{i}{2} v_{\tau,1}^{++} [(\nabla^{--} F^{++}), v^{++}]_{\tau,2} \right\} \\ & \mp \frac{i}{4g_0^2} \left(1 + \frac{1}{\xi_0} \right) \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} v_{\tau,1}^{++} [F^{++}, \nabla^{--} v^{++}]_{\tau,2}. \end{aligned} \quad (3.11)$$

The quantum effective action which is gauge invariant and $\mathcal{N} = (1, 0)$ supersymmetric by construction (for details see refs. [12,13,19,42]) is given by the expression

$$\begin{aligned} e^{i\Gamma[V^{++}]} = & \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}b \mathcal{D}c \mathcal{D}\varphi \mathcal{D}\chi^{(+4)} \mathcal{D}\sigma \\ & \times \exp \left(i S_{\text{total}} - \int d\zeta^{(-4)} du \frac{\delta \Gamma[V^{++}]}{\delta V^{++A}} v^{++A} \right), \end{aligned} \quad (3.12)$$

where the total action has the form

$$S_{\text{total}} = S_0[V^{++} + v^{++}] + S_{\text{gf}}[v^{++}, V^{++}] + S_{\text{gh}}, \quad (3.13)$$

and we have denoted $S_{\text{gh}} = S_{\text{FP}}[b, c, v^{++}, V^{++}] + S_{\text{NK}}[\varphi, \chi^{(+4)}, \sigma, V^{++}]$.

The effective action defined in (3.12) has the structure $\Gamma[V^{++}] = S[V^{++}] + \Delta\Gamma[V^{++}]$, where $\Delta\Gamma[V^{++}]$ accommodates all quantum corrections to the classical action.

Further we will consider the structure of the effective action in the one-loop approximation. From eq. (3.12) we see that the one-loop contribution to the effective action is given by the functional integral

$$\exp(i\Delta\Gamma^{(1)}[V^{++}]) = \int \mathcal{D}v^{++} \mathcal{D}b \mathcal{D}c \mathcal{D}\varphi \mathcal{D}\chi^{(+4)} \mathcal{D}\sigma \times \exp(iS_{\text{total}}^{(2)}[v^{++}, b, c, \varphi, \chi^{(+4)}, \sigma, V^{++}]), \quad (3.14)$$

where $S_{\text{total}}^{(2)}$ denotes a part of the total action quadratic in the quantum superfields. To present it in the most convenient form, we integrate by parts with respect to the derivative ∇_2^{--} in the last term of the expression (3.11). After this, taking into account that $e^{-ib}\nabla^{--}e^{ib} = D^{--}$, we can rewrite $S_{\text{total}}^{(2)}$ as

$$\begin{aligned} S_{\text{total}}^{(2)} = & \pm \frac{1}{2g_0^2\xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++} \widehat{\square}^2 v^{++} \\ & \pm \frac{1}{2g_0^2} \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} \left\{ \left(1 - \frac{1}{\xi_0}\right) v_{\tau,1}^{++} (\widehat{\square} v^{++})_{\tau,2} \right. \\ & \left. + i v_{\tau,1}^{++} [(\nabla^{--} \mathbf{F}^{++}), v^{++}]_{\tau,2} \right\} \\ & \mp \frac{i}{2g_0^2} \left(1 + \frac{1}{\xi_0}\right) \text{tr} \int d^{14}z du_1 du_2 \frac{(u_1^+ u_2^-)}{(u_1^+ u_2^+)^3} v_{\tau,1}^{++} [\mathbf{F}^{++}, v^{++}]_{\tau,2} \\ & + \text{tr} \int d\zeta^{(-4)} du b (\nabla^{++})^2 c \\ & + \text{tr} \int d\zeta^{(-4)} du \left(-\frac{1}{2} \varphi (\nabla^{++})^2 \varphi + \chi^{(+4)} \widehat{\square} \sigma \right). \end{aligned} \quad (3.15)$$

After integration over quantum superfields in the functional integral (3.14) we obtain the one-loop quantum correction to the effective action as a sum of three terms⁵

$$\begin{aligned} \Delta\Gamma^{(1)}[V^{++}] = & \frac{i}{2} \text{Tr}_{(2,2)} \ln \left\{ \frac{1}{\xi_0} (\widehat{\square}_1)^2 (D_1^+)^4 \delta^{(-2,2)}(u_1, u_2) \right. \\ & + \left(1 - \frac{1}{\xi_0}\right) \frac{(D_1^+)^4 \widehat{\square}_2 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \\ & + \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \left[i(\nabla^{--} \mathbf{F}^{++}) - \frac{i(u_1^+ u_2^-)}{(u_1^+ u_2^+)} \left(1 + \frac{1}{\xi_0}\right) \mathbf{F}^{++} \right]_2 \Big\}_{\text{Adj}} \\ & - i \text{Tr}_{(4,0)} \ln \widehat{\square}_{\text{Adj}} - i \text{Tr} \ln \nabla_{\text{Adj}}^{++}. \end{aligned} \quad (3.16)$$

⁵ Obviously, the one-loop quantum correction is the same for the upper and lower signs in the first three terms of (3.15).

The first term in (3.16) comes from the quantum gauge multiplet v^{++} in (3.14). The second term is produced by the Nielsen–Kallosh ghosts $\chi^{(+4)}$ and σ . As in the conventional $\mathcal{N} = (1, 0)$ SYM theory, the last term in (3.16) is the sum of the contributions coming from the Nielsen–Kallosh ghost φ and the Faddeev–Popov ghosts (the analysis of this term was carried out in [12,13]). In eq. (3.16) the functional trace over harmonic superspace is defined as

$$\text{Tr}_{(q,4-q)} \mathcal{O} = \text{tr} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} \delta_{\mathcal{A}}^{(q,4-q)}(2|1) \mathcal{O}^{(q,4-q)}(1|2), \quad (3.17)$$

where $\delta_{\mathcal{A}}^{(q,4-q)}(2|1)$ is an analytic delta-function and $\mathcal{O}^{(q,4-q)}(\zeta_1, u_1 | \zeta_2, u_2)$ is the kernel of some operator \mathcal{O} acting in the space of analytic superfields possessing the harmonic $U(1)$ charge q [36].

4. Power counting and counterterms

The general form of possible counterterms can be found using the power counting in $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace. Let us consider an arbitrary L -loop supergraph containing external and internal lines of the gauge, hypermultiplet and ghost superfields.⁶ The superfield propagators of vector multiplet, hypermultiplet and ghosts contain the Grassmann delta-functions [19]. This allows us to represent any loop supergraph as a single integral over anticommuting variables.⁷ Taking into account the locality of divergences, we conclude that each contribution to the effective action can be presented as an integral over $d^{14}z = d^6x d^8\theta$. Besides, we should take account of the fact that the quantum theory is formulated in the framework of the background field method, which implies that the quantum effective action bears invariance under the classical background gauge transformations.

Now the power counting, as usual, can be carried out, based on dimensional reasonings. A contribution to the dimensionless effective action can formally be written as

$$\int d^{14}z \prod_k du_k \left[\text{Momentum integral} \right] \left[(D)^{N_D} \right] \left[\text{Superfields} \right], \quad (4.1)$$

where the superfields correspond to the external lines, and the symbol $(D)^{N_D}$ denotes the product of N_D spinor covariant derivatives acting on these external lines (we assume that the external momenta in this expression are replaced by the relevant derivatives acting on the corresponding external lines). By definition, the degree of divergence ω coincides with the dimension of the momentum integral in units of mass,

$$\left[\text{Momentum integral} \right] = m^{\omega}. \quad (4.2)$$

Therefore, it can be found by analyzing the dimensions of various factors in (4.1).

The dimension of the anticommuting variables is $[\theta] = m^{-1/2}$, so $[d^{14}z] = m^{-6} \cdot m^4 = m^{-2}$. The harmonic variables u_k^{\pm} are dimensionless. The gauge superfield V^{++} (or v^{++}) is dimensionless, while the hypermultiplet and the Faddeev–Popov ghosts have the dimension m . Thus, the dimension of the external legs is $m^{2N_q + 2N_c}$, where N_q and N_c are the numbers of external hypermultiplet and ghost lines, respectively. If N_D spinor derivatives (each of the dimension $m^{1/2}$) act

⁶ The details of the supergraph technique for the theory under consideration are discussed in [19].

⁷ In supersymmetric theories this statement is closely related to the non-renormalization theorems, see, e.g., [43].

on external lines, then this dimension is increased by $m^{N_D/2}$. Each gauge multiplet propagator present in the supergraph contributes the factor g_0^2 .

Taking into account that the effective action is dimensionless, we obtain $1 = m^{-2} \cdot m^{N_D/2+2N_q+2N_c} \cdot m^\omega$, whence

$$\omega = 2 - 2N_q - 2N_c - N_D/2. \quad (4.3)$$

This implies that the degree of divergence does not depend on the number of loops and the number of external gauge superfield lines. Moreover, taking into account that N_q and N_c are even, we see that divergences can appear only in supergraphs with external gauge lines. Therefore, all supergraphs with the hypermultiplet or ghost external lines are finite. Since the theory is formulated in the framework of the background field method, the form of the divergences is restricted by the gauge invariance. As a result, it becomes possible to list all divergences which could present in the theory.

The only gauge invariant combination of the dimension m^{-2} corresponding to the quadratic divergences is proportional to the standard $6D$, $\mathcal{N} = (1, 0)$ SYM action [34]

$$S_{SYM} = \frac{1}{f_0^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)}. \quad (4.4)$$

In this case $N_D = 0$, so that $\omega = 2$.

The only invariant of the dimension m^0 corresponding to the logarithmic divergences reads

$$\text{tr} \int d^{14}z du V^{--} (D^+)^4 V^{--} = \text{tr} \int d\zeta^{(-4)} du (F^{++})^2. \quad (4.5)$$

It contains four spinor derivatives acting on the gauge superfields, so that $N_D = 4$ and $\omega = 0$ for it.

Therefore, the theory described by the action

$$S = S_{SYM} + \frac{1}{4g_0^2} \int d\zeta^{(-4)} du (F^{++A})^2 - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+ \quad (4.6)$$

is renormalizable: all divergences can be absorbed into the renormalization of the coupling constants g_0 and f_0 . The hypermultiplet and ghost superfields are not renormalized.⁸

When using the dimensional regularization (in the context of superfield theories it is necessary to use its modification called the dimensional reduction), only the logarithmic divergences are displayed. Therefore, the counterterms of the form (4.4) are prohibited and the only admissible counterterm is (4.5), i.e. it is proportional to the classical action (2.1) at any loop.

5. One-loop divergences

In this section, we will focus on the one-loop quantum correction $\Delta\Gamma^{(1)}[V^{++}]$ to the classical action. We use the regularization by dimensional reduction and calculate the divergent part of the one-loop effective action $\Delta\Gamma_\infty^1[V^{++}]$ in the lowest order in the parameter $(\xi_0 - 1)$ which is assumed to be small.

⁸ The Nielson-Kallosh ghosts interact only with the background gauge superfield and, therefore, no counterterms are required in this sector.

The most difficult task is to single out the divergent part of the first term in the expression (3.16)

$$\begin{aligned} & \frac{i}{2} \text{Tr}_{(2,2)} \ln \left\{ \frac{1}{\xi_0} (\widehat{\square}_1)^2 (D_1^+)^4 \delta^{(-2,2)}(u_1, u_2) + \left(1 - \frac{1}{\xi_0}\right) \frac{(D_1^+)^4 \widehat{\square}_2 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \right. \\ & + \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \left[i(\nabla^{--} \mathbf{F}^{++}) \right. \\ & \left. \left. - \frac{i(u_1^+ u_2^-)}{(u_1^+ u_2^+)} \left(1 + \frac{1}{\xi_0}\right) \mathbf{F}^{++} \right]_2 \right\} \delta^{14}(z_1 - z_2) \Big|_{Adj}, \end{aligned} \quad (5.1)$$

where $\delta^{14}(z_1 - z_2) = \delta^8(\theta_1 - \theta_2) \delta^6(x_1 - x_2)$. We present it as the sum of two logarithms,

$$\begin{aligned} & \frac{i}{2} \text{Tr}_{(2,2)} \ln \left\{ \frac{1}{\xi_0} (\widehat{\square}_1)^2 (D_1^+)^4 \delta^{(-2,2)}(u_1, u_2) \delta^{14}(z_1 - z_2) \right\} \\ & + \frac{i}{2} \text{Tr}_{(2,2)} \ln \left\{ (D_1^+)^4 \delta^{(-2,2)}(u_1, u_2) \delta^{14}(z_1 - z_2) \right. \\ & + \frac{1}{(\widehat{\square}_1)_{Adj}^2} \left\{ (\xi_0 - 1) \frac{(D_1^+)^4 \widehat{\square}_2 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \right. \\ & + \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \left[i\xi_0 (\nabla^{--} \mathbf{F}^{++}) \right. \\ & \left. \left. - (\xi_0 + 1) \frac{i(u_1^+ u_2^-)}{(u_1^+ u_2^+)} \mathbf{F}^{++} \right]_2 \right\} \delta^{14}(z_1 - z_2) \Big|_{Adj}. \end{aligned} \quad (5.2)$$

According to [13,19], the first term in this expression vanishes. To calculate the divergent part of the second term in the lowest order in $(\xi_0 - 1)$, we need to expand the logarithm up to a linear term only. Then we are to find the divergent part of the expression

$$\Delta\Gamma^{(1)}[\mathbf{V}^{++}] = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4, \quad (5.3)$$

where

$$\Gamma_1 = \frac{1}{2} \text{tr} \int d\zeta_1^{(-4)} du_1 \frac{(\xi_0 + 1)}{(\widehat{\square}_1)^2} (D_1^+)^4 (D_2^+)^4 \frac{(u_1^+ u_2^-)}{(u_1^+ u_2^+)^3} e^{ib_1} e^{-ib_2} \mathbf{F}_2^{++} \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}, \quad (5.4)$$

$$\Gamma_2 = -\frac{1}{2} \text{tr} \int d\zeta_1^{(-4)} du_1 \frac{\xi_0}{(\widehat{\square}_1)^2} \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} (\nabla^{--} \mathbf{F}^{++})_2 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}, \quad (5.5)$$

$$\Gamma_3 = \frac{i}{2} \text{tr} \int d\zeta_1^{(-4)} du_1 \frac{(\xi_0 - 1)}{(\widehat{\square}_1)^2} (D_1^+)^4 \widehat{\square}_2 (D_2^+)^4 \frac{1}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}, \quad (5.6)$$

$$\Gamma_4 = -i \text{Tr} \ln \nabla^{++}, \quad (5.7)$$

and tr stands for the usual matrix trace. The gauge and bridge (\mathbf{b}) superfields appearing in all these expressions should be expanded over the generators of the adjoint representation. Note that the term $-i\text{Tr}_{(4,0)} \ln \widehat{\square}_{Adj}$ present in Eq. (3.16) vanishes, see [13,19].

Before starting the calculation we outline our strategy. The one-loop effective action (5.2) contains the functional trace Tr defined by Eq. (3.17). The calculation of this trace includes the evaluation of the coincident-points limit for the kernel of the corresponding operator. In the expressions (5.4), (5.5) and (5.6) the operator $\widehat{\square}^{-2}$ acts on everything to the right of it, including the delta-function. Hence, before taking the coincident-points limit we should accurately calculate the action of the operator $\widehat{\square}^{-2}$ on all terms to the right and then single out the divergent contributions.

As the first step of the calculation, we consider the divergent contribution coming from (5.4),

$$\Gamma_1 = \frac{(\xi_0 + 1)}{2} \int d\zeta_1^{(-4)} du_1 ((\widehat{\square}_1)^{-2})^{IJ} (D_1^+)^4 (D_2^+)^4 \times \frac{(u_1^+ u_2^-)}{(u_1^+ u_2^+)^3} (e^{ib_1} e^{-ib_2})^{JK} (\mathbf{F}_2^{++})^{KI} \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \quad (5.8)$$

In this expression we should pull out the operator $(\widehat{\square})^{-2}$ to the right. Acting on analytic superfields, the covariant d'Alembertian (2.9) yields

$$\widehat{\square}^{IJ} = \partial^2 \delta^{IJ} + i(\mathbf{F}^{++})^{IJ} D^{--} + \dots,$$

where $(\mathbf{F}^{++})^{IJ} = -if^{KIJ} \mathbf{F}^{++K}$. The logarithmically divergent contribution in (5.8) is proportional to the third inverse power of the operator $\partial^2 = \partial^M \partial_M$ acting on the space-time delta-function $\delta^6(x_1 - x_2)$. Indeed,

$$\frac{1}{(\partial^2)^3} \delta^6(x_1 - x_2) \Big|_{2 \rightarrow 1} = \frac{i}{(4\pi)^3 \varepsilon}, \quad \varepsilon \rightarrow 0. \quad (5.9)$$

To calculate the coincident-points limit for Grassmann variables, we use the identity

$$(D_1^+)^4 (D_2^+)^4 \delta^8(\theta_1 - \theta_2) = (u_1^+ u_2^+)^4 (D_1^+)^4 (D_1^-)^4 \delta^8(\theta_1 - \theta_2). \quad (5.10)$$

Then

$$\Gamma_1 \rightarrow \frac{(\xi_0 + 1)}{2} \times \int d\zeta_1^{(-4)} du_1 ((\widehat{\square}_1)^{-2})^{IJ} (u_1^+ u_2^+) (u_1^+ u_2^-) (D_1^+)^4 (D_1^-)^4 (\mathbf{F}_2^{++})^{JI} \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \quad (5.11)$$

To evaluate this expression, we note that the derivatives D^{--} inside the operator $\widehat{\square}^{-2}$ can act on $(u_1^+ u_2^+)$. Then, taking into account that $D_1^{--} (u_1^+ u_2^+) |_{2 \rightarrow 1} = -1$ (see, e.g., [36]), we obtain that the divergent part of (5.11) is reduced to

$$\Gamma_{1,\infty} = i(\xi_0 + 1) \int d\zeta_1^{(-4)} du_1 (\mathbf{F}^{++})^{IJ} (\mathbf{F}^{++})^{JI} \frac{(D_1^+)^4 (D_1^-)^4}{(\partial^2)^3} \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \quad (5.12)$$

Finally we annihilate the Grassmann delta-function, using the identity $(D_1^+)^4 (D_1^-)^4 \delta^8(\theta_1 - \theta_2) |_{2 \rightarrow 1} = 1$ and eq. (5.9). Then we obtain

$$\Gamma_{1,\infty} = -2(\xi_0 + 1) \frac{C_2}{(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2, \quad (5.13)$$

where C_2 is the second Casimir for the adjoint representation of the gauge group.

The divergent part of the expression (5.5) vanishes. Indeed, after taking the coincident-points limit we obtain

$$\begin{aligned} \Gamma_2 \rightarrow & -\frac{(\xi_0 - 1)}{4} \\ & \times \int d\zeta_1^{(-4)} du_1 ((\widehat{\square}_1)^{-2})^{IJ} (u_1^+ u_2^+)^2 (\nabla^{--} F_2^{++})^{JI} (D_1^+)^4 (D_1^-)^4 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \end{aligned} \quad (5.14)$$

To annihilate the factor $(u_1^+ u_2^+)^2$ we have to expand the operator $(\widehat{\square}^2)^{-1}$ up to the second order, so as to gain two derivatives D^{--} . As a result, we accumulate the fourth power of inverse ∂^2 operator. Hence, this term does not contain UV divergence,

$$\Gamma_{2,\infty} = 0. \quad (5.15)$$

One more divergent contribution comes from Γ_3 given by (5.6). We again use the properties of the spinor derivatives D_a^+ and transform the corresponding expression to the form

$$\Gamma_3 \rightarrow \frac{i(\xi_0 - 1)}{2} \int d\zeta_1^{(-4)} du_1 ((\widehat{\square}_1)^{-2})^{IJ} \widehat{\square}_2^{JI} (u_1^+ u_2^+)^2 (D_1^+)^4 (D_1^-)^4 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \quad (5.16)$$

Also we reconstruct the full superspace measure in (5.8) using the property $d^{14}z = d\zeta^{(-4)}(D^+)^4$. Then the expression (5.16) can be rewritten as

$$\begin{aligned} & \frac{i(\xi_0 - 1)}{4} \int d^{14}z_1 du_1 ((\widehat{\square}_1)^{-2})^{IJ} ((\nabla_2^{--})^2)^{JI} (u_1^+ u_2^+)^2 (D_1^+)^4 (D_1^-)^4 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1} \\ & = \frac{i(\xi_0 - 1)}{4} \int d^{14}z_1 du_1 ((\nabla_1^{--})^2)^{JI} ((\widehat{\square}_1)^{-2})^{IJ} (u_1^+ u_2^+)^2 (D_1^+)^4 (D_1^-)^4 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1} \\ & = \frac{i(\xi_0 - 1)}{2} \int d\zeta_1^{(-4)} du_1 ((\widehat{\square}_1)^{-1})^{II} (u_1^+ u_2^+)^2 (D_1^+)^4 (D_1^-)^4 \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}. \end{aligned} \quad (5.17)$$

When acting by the operator $\widehat{\square}^{-1}$ on the right, we expand the inverse operator $\widehat{\square}$ up to the second order in D^{--} to remove $(u_1^+ u_2^+)^2$. After this, we calculate the coincident-points limit and extract the divergent contribution from (5.17) as

$$\Gamma_{3,\infty} = 2(\xi_0 - 1) \frac{C_2}{(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2. \quad (5.18)$$

The divergent contribution from Γ_4 in (5.7) was considered earlier in [12]. It is

$$\Delta\Gamma_{4,\infty} = \frac{C_2}{3(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2. \quad (5.19)$$

Summing up the contributions (5.13), (5.15), (5.18), and (5.19) we obtain the final result for the divergent part of the one-loop effective action,

$$\Delta\Gamma_\infty^{(1)} = \Gamma_{1,\infty} + \Gamma_{2,\infty} + \Gamma_{3,\infty} + \Gamma_{4,\infty} = -\frac{11}{3} \frac{C_2}{(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2. \quad (5.20)$$

We see that all divergent contributions depending on the gauge-fixing parameter ξ_0 in the considered approximation cancel each other. This agrees with the general statement that the renormalization of dimensionless coupling constants in multiplicatively renormalizable gauge theories does not depend on the gauge choice [39,40]. Therefore, the cancellation of terms containing the gauge-fixing parameter can be considered as a non-trivial test for the correctness of our calculations. As we have already mentioned, the wave function is not renormalized in the background (super)field method, so that all divergences are absorbed into the renormalization of the coupling constant. The coefficient agrees with the one obtained earlier in the Feynman gauge $\xi_0 = 1$, both in the component approach [1,18] and by the supergraph technique [19].

6. Adding the hypermultiplet

In this section we consider the one-loop divergences for the theory with the action (2.5), focusing on the divergences in the hypermultiplet sector. To study possible divergent contributions to the effective action we introduce the background-quantum splitting for both q^+ and V^{++} ,

$$V^{++} \rightarrow \mathbf{V}^{++} + v^{++}, \quad q^+ \rightarrow Q^+ + q^+. \quad (6.1)$$

Here we have denoted the “background” superfields by the capital letters \mathbf{V}^{++} , Q^+ and “quantum” ones by v^{++} , q^+ . The presence of the background hypermultiplets leads to the mixing of the superfields v^{++} and q^+ . All such terms can be eliminated by a special redefinition of the quantum hypermultiplet in the functional integral similarly to the case of $6D$, $\mathcal{N} = (1, 0)$ SYM theory (see [12] for details). The calculation of the one-loop divergences for the theory (2.5) is performed in a close analogy to the case of the model (2.1) described in Sec. 5. The effective action is written as

$$\begin{aligned} \Delta\Gamma^{(1)}[V^{++}, Q^+] &= \frac{i}{2} \text{Tr}_{(2,2)} \ln \left\{ \frac{1}{\xi_0} (\widehat{\square}_1)^2 (D_2^+)^4 \delta^{(2,-2)}(u_1, u_2) \right. \\ &\quad + \left(1 - \frac{1}{\xi_0}\right) \frac{(D_1^+)^4 \widehat{\square}_2 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \\ &\quad + \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \left[i(\nabla^{--} \mathbf{F}^{++}) - \frac{i(u_1^+ u_2^-)}{(u_1^+ u_2^+)^2} \left(1 + \frac{1}{\xi_0}\right) \mathbf{F}^{++} \right]_2^{IJ} \\ &\quad \left. - 2g_0 \widetilde{Q}_1^+ T^I G^{(1,1)}(1|2) T^J Q_2^+ \right\} - i \text{Tr}_{(4,0)} \ln \widehat{\square}_{Adj} - i \text{Tr} \ln \nabla_{Adj}^{++} + i \text{Tr} \ln \nabla_R^{++}, \quad (6.2) \end{aligned}$$

where

$$G^{(1,1)}(1|2) = \frac{1}{\widehat{\square}} \frac{(D_1^+)^4 (D_2^+)^4}{(u_1^+ u_2^+)^3} \delta^{14}(z_1 - z_2) \quad (6.3)$$

is the hypermultiplet Green function [36] and T^I , $I = 1, \dots, \dim G$, are generators of gauge group G in the hypermultiplet representation R .

The term $i \text{Tr} \ln \nabla_R^{++}$ in the expression (6.2) corresponds to the contribution of the quantum hypermultiplet. Taking into account that the supergraphs with Q^+ on external legs are finite, we see that the hypermultiplet can merely change the coefficient of the purely gauge contribution containing $\text{tr}(\mathbf{F}^{++})^2$. This implies that

$$\Delta\widetilde{\Gamma}_\infty^{(1)}[V^{++}] = \left(\Delta\Gamma^{(1)}[V^{++}] + i \text{Tr} \ln \nabla_R^{++} \right)_\infty, \quad (6.4)$$

where $\Delta\Gamma^{(1)}[V^{++}]$ was introduced earlier in (3.16), and $\tilde{\Gamma}$ is the effective action for the model (2.5).

The contribution of the hypermultiplet to the one-loop divergences has been calculated in [12,13]. Adding it to the expression (5.20), we obtain the total contribution in the form

$$\tilde{\Gamma}_{\infty}^{(1)}[V^{++}] = -\frac{11C_2 + T_R}{3(4\pi)^3\varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2, \quad (6.5)$$

where the constant T_R is defined by the relation $\text{tr}(T^I T^J) = T_R \delta^{IJ}$. Thus, the presence of the hypermultiplet gives rise to an increase of the absolute value of the β -function.

Now, let us explicitly verify that the part of the effective action containing the hypermultiplet is finite.⁹ To this end, we consider that term in (6.2) which depends on the background hypermultiplet Q^+ . The corresponding contribution with the maximal degree of divergence reads¹⁰

$$\begin{aligned} & \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} du_1 du_2 \left(\frac{1}{\xi_0} (\widehat{\square}_1)^2 (D_2^+)^4 \delta^{(2,-2)}(u_1, u_2) \right. \\ & + \frac{(D_1^+)^4 \widehat{\square}_2 (D_2^+)^4}{(u_1^+ u_2^+)^2} \left(1 - \frac{1}{\xi_0} \right) \Big)^{-1} \delta^{14}(z_1 - z_2) \tilde{Q}_1^+ G^{(1,1)}(1|2) Q_2^+ \\ & \sim \int d\zeta_1^{(-4)} du_1 \frac{\xi_0}{(\widehat{\square}_1)^2} \tilde{Q}_1^+ G^{(1,1)}(1|2) Q_2^+ \Big|_{2 \rightarrow 1} + \dots, \end{aligned} \quad (6.6)$$

where dots denote terms containing higher powers of $1/\partial^2$. The explicitly written term in this expression can be worked out as

$$\begin{aligned} & \int d\zeta_1^{(-4)} du_1 \frac{\xi_0}{(\widehat{\square}_1)^2} \tilde{Q}_1^+ G^{(1,1)}(1|2) Q_2^+ \Big|_{2 \rightarrow 1} \\ & = \int d\zeta_1^{(-4)} du_1 \frac{\xi_0}{(\widehat{\square}_1)^2} \tilde{Q}_1^+ Q_2^+ \frac{1}{\widehat{\square}_1} (D_1^+)^4 (D_1^-)^4 (u_1^+ u_2^+) \delta^{14}(z_1 - z_2) \Big|_{2 \rightarrow 1}, \\ & \sim \int d\zeta^{(-4)} du \tilde{Q}^+ F^{++} Q^+ \frac{1}{(\partial^2)^4} \delta^6(x - x') \Big|_{x' \rightarrow x}, \end{aligned} \quad (6.7)$$

where we have used the identity (5.10). In the expression (6.7), using the relation similar to (2.9), we expand the operators $\widehat{\square}$ up to the first order in $F^{++} \nabla^{--}$ and then act by the harmonic derivative ∇^{--} on the factor $(u_1^+ u_2^+)$ in the coincident harmonic points limit. But the hypermultiplet Green function $G^{(1,1)}$ brings the inverse power of operator $\widehat{\square}$ and the resulting power of ∂^2 in the denominator amounts to a finite contribution to the effective action. The remaining terms in (6.6) contain at least the fourth power of $\widehat{\square}$ in the denominator. Therefore, they are also finite, in agreement with the power counting arguments of section 4. This means that the one-loop divergencies in the hypermultiplet sector are actually absent.

⁹ For simplicity, we omit all gauge group indices and explicit dependence on the generators.

¹⁰ Other contributions can be analyzed in a similar manner.

7. Summary and outlook

We studied the quantum divergence structure of the higher-derivative $\mathcal{N} = (1, 0)$ supersymmetric non-abelian gauge theory in six dimensions. This theory involves four derivatives in the component gauge field sector and three derivatives in the spinor gaugino sector. The theory is characterized by a dimensionless coupling constant. Two such models were considered: the model with the gauge multiplet only and the model in which the gauge multiplet is coupled to the hypermultiplet in some representation of the gauge group, with the standard kinetic terms for the hypermultiplet physical scalars and fermions. Both models were formulated in harmonic $6D$, $\mathcal{N} = (1, 0)$ superspace ensuring manifest $\mathcal{N} = (1, 0)$ supersymmetry. The quantization was accomplished in the framework of the background superfield method with a one-parameter family of the quantum gauge-fixing conditions. The corresponding gauge invariant and manifestly supersymmetric quantum effective action was introduced and all possible divergent terms in such an action were identified.

The analysis of the divergence structure for the theories under consideration was based on the superfield power counting. It was shown that the superficial degree of divergence does not depend on a number of loops and is completely specified by the number of D -factors acting on the external lines of the gauge superfield. Then, taking into account the gauge invariance of the effective action and making use of the regularization by dimensional reduction, we conclude that the only possible counterterm in the theory is proportional to the classical action of gauge superfield. All supergraphs with the hypermultiplet external legs should be finite. This implies that the theory under consideration is multiplicatively renormalizable and the renormalization affects only the dimensionless coupling constant.

A manifestly supersymmetric and gauge invariant procedure to calculate the one-loop divergences was developed and applied for the explicit calculation of these divergencies. The result completely agrees with the one obtained earlier in [1,18,19] through direct calculations of Feynman (super)graphs, as well as with the general analysis based on the power counting. It was also shown that in the lowest order with respect to the deviation $(\xi_0 - 1)$ of the gauge-fixing parameter ξ_0 from its minimal-gauge value $\xi_0 = 1$ the divergences are independent of this parameter, which can be considered as a check of the correctness of our calculations. We also found the modification of the one-loop divergence in the gauge superfield sector by the hypermultiplet contribution. It amounts to changing the absolute value of the relevant coefficient.

There are at least four interesting directions for further generalization of the results obtained.

- It would be tempting to calculate the one-loop divergencies for the general theory (4.6) the action of which is a sum of the higher-derivative action (2.1) and the action of the standard $6D$, $\mathcal{N} = (1, 0)$ SYM theory. In this case we will deal with one dimensionless coupling constant g_0 and another dimensionful coupling constant f_0 . Such a theory is still multiplicatively renormalizable, but there can be non-trivial running coupling constant regimes.
- It is interesting to develop the superfield method for studying the superconformal anomaly in the higher-derivative theory (2.1).
- One more noteworthy prospect is to study a renormalization structure of the higher-derivative gauge superfield model coupled to the higher-derivative hypermultiplet model. The corresponding classical theory was constructed in [15]. In principle, such a consideration could allow to set up $6D$, $\mathcal{N} = (1, 1)$ supersymmetric anomaly-free higher-derivative theory. One can, e.g., conjecture that this theory is asymptotically free and even completely finite.

- An obvious generalization of our study is to find the total dependence of the divergences on the gauge-fixing parameter ξ_0 .

We hope to address all these problems in the forthcoming works.

CRediT authorship contribution statement

I.L. Buchbinder: Conceptualization, Investigation, Writing – review & editing. **E.A. Ivanov:** Conceptualization, Investigation, Writing – review & editing. **B.S. Merzlikin:** Investigation, Writing – original draft, Writing – review & editing. **K.V. Stepanyantz:** Investigation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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