



On gauge dependence of the one-loop divergences in $6D$, $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ SYM theories

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ABSTRACT

We study the gauge dependence of one-loop divergences in a general matter-coupled $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theory in the harmonic superspace formulation. Our analysis is based on the effective action constructed by the background superfield method, with the gauge-fixing term involving one real parameter ξ_0 . A manifestly gauge invariant and $\mathcal{N} = (1, 0)$ supersymmetric procedure for calculating the one-loop effective action is developed. It yields the one-loop divergences in an explicit form and allows one to investigate their gauge dependence. As compared to the minimal gauge, $\xi_0 = 1$, the divergent part of the general-gauge effective action contains a new term depending on ξ_0 . This term vanishes for the background superfields satisfying the classical equations of motion, so that the S -matrix divergences are gauge-independent. In the case of $6D$, $\mathcal{N} = (1, 1)$ SYM theory we demonstrate that some divergent contributions in the non-minimal gauges do not vanish off shell, as opposed to the minimal gauge.

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1. Introduction

The study of quantum aspects of the higher-dimensional supersymmetric gauge field theories attracts a wide interest for a long time, mainly because of their use for the low-energy description of diverse sectors of superstring theory (see, e.g., [1,2]). From the field-theoretical point of view, such theories possess a rather unusual UV behavior. Although they are non-renormalizable by power counting, the relevant amplitudes can be still finite for some low numbers of loops. In particular, in the six-dimensional maximally extended $\mathcal{N} = (1, 1)$ super Yang-Mills theory the one- and two-loop amplitudes are finite [3–11].

Recently, using techniques of the six-dimensional ($6D$) $\mathcal{N} = (1, 0)$ harmonic superspace [12–14] (which is a direct generalization of $4D$, $\mathcal{N} = 2$ harmonic superspace [15–17]), we studied the quantum structure of $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ supersymmetric $6D$ gauge theories [18–22] (see also the review [23]), paying a

special attention to off-shell divergences. We considered $6D$, $\mathcal{N} = (1, 0)$ non-abelian Yang-Mills theory coupled to the hypermultiplet in an arbitrary representation of the gauge group and calculated the one-loop divergences of the superfield effective action. It was shown that, in the particular case of $\mathcal{N} = (1, 1)$ theory, i.e. with the hypermultiplet in the adjoint representation, all one-loop divergences vanish off shell. The calculations were performed in the minimal gauge, in which the gauge superfield propagator has the simplest form. The natural question to be posed was as to whether the vanishing of the off-shell one-loop divergences depends on the choice of the gauge-fixing condition.

We started studying the gauge dependence of the one-loop divergences in $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theories in our previous work [24]. As a simplest example of a supersymmetric theory, where the problem of the gauge dependence occurs, we considered the abelian $\mathcal{N} = (1, 0)$ gauge theory and investigated the structure of the gauge-dependent divergent contributions to the one-loop effective action. It was shown that the one-loop divergences are actually gauge-dependent. The present paper generalizes this study to the generic non-abelian $\mathcal{N} = (1, 0)$ SYM theory interacting with a set of hypermultiplets in an arbitrary representation of the gauge group. We consider the gauge conditions involving one arbitrary gauge parameter ξ_0 (analogs of usual

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ξ -gauges in the non-supersymmetric case) and analyze the dependence of the one-loop divergences on this parameter. We wish to emphasize that the gauge dependence of the off-shell divergences for non-abelian extended SYM theories has never been studied earlier even in $4D, \mathcal{N} = 2, \mathcal{N} = 4$ SYM theories and this aspect deserves a separate study.

The analysis of the effective action in refs. [18–22] was carried out in the framework of the harmonic superfield background field method, which ensures both the classical gauge invariance and $\mathcal{N} = (1, 0)$ supersymmetry as manifest symmetries. This method was earlier developed for the minimal gauge only.¹ Now we formulate the background superfield method for the one-parametric family of the gauge conditions. The detailed analysis of the structure of divergences in the non-minimal gauges can hopefully be useful for a better understanding of the UV behavior of the theory under consideration.

The paper is organized as follows. Sect. 2 provides some basic knowledge about $6D, \mathcal{N} = (1, 0)$ SYM theory interacting with hypermultiplets in the harmonic superspace formulation. We briefly discuss the superfield content and the action of the model. In Sect. 3 we develop the background superfield method with the gauge-fixing term containing an arbitrary real parameter and derive the formal expression for the corresponding one-loop effective action. In Sect. 4 we calculate the one-loop divergences as functions of the gauge-fixing parameter. We explicitly demonstrate that the divergences are gauge-dependent for both $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ theories.² We show that the gauge dependence of the divergences vanishes for the background superfields satisfying the classical equations of motion. This implies that the divergences of S -matrix are gauge-independent, in agreement with the general theorems (see, e.g., [27]).³ We also discuss the one-loop divergences for the case of $\mathcal{N} = (1, 1)$ SYM theory, that is $\mathcal{N} = (1, 0)$ SYM theory minimally coupled to a hypermultiplet in the adjoint representation of the gauge group. Sect. 5 contains a summary of the results obtained and a proposal for further work.

2. Basic notions

The harmonic $6D, \mathcal{N} = (1, 0)$ superspace in the central basis is parametrized by the coordinates $(z, u) \equiv (x^M, \theta_i^a, u^{\pm i})$. Here x^M , $M = 0, \dots, 5$, are $6D$ Minkowski space-time coordinates, θ_i^a , $a = 1, \dots, 4$, $i = 1, 2$, are Grassmann variables, and the additional harmonic variables u_i^{\pm} , $u^{+i}u_i^- = 1$, represent the coset $SU(2)/U(1)$, with $SU(2)$ being R -symmetry group of $6D, \mathcal{N} = (1, 0)$ Poincaré superalgebra [12,13].

The harmonic superspace in the analytic basis is parametrized, along with the harmonic variables, by the analytic coordinate $z_A \equiv (x_A^M, \theta^{\pm a})$, where $x_A^M \equiv x^M + \frac{i}{2}\theta^{+a}(\gamma^M)_{ab}\theta^{-b}$ and $\theta^{\pm a} = u_k^{\pm}\theta^{ak}$. We use the antisymmetric representation for $6D$ γ -matrices

$$(\gamma^M)_{ab} = -(\gamma^M)_{ba}, \quad (\tilde{\gamma}^M)^{ab} = \frac{1}{2}\varepsilon^{abcd}(\gamma^M)_{cd}, \quad (2.1)$$

where ε^{abcd} is the totally antisymmetric tensor. By definition, analytic superfields depend only on the coordinates (ζ, u) , with $\zeta \equiv (x_A^M, \theta^{\pm a})$.

¹ $6D, \mathcal{N} = (1, 0)$ background field method is a generalization of $4D, \mathcal{N} = 2$ background field method worked out in [25]. For a review of its various applications see ref. [26].

² It is worth pointing out that $6D, \mathcal{N} = (1, 0)$ gauge theories are in general anomalous [28–31]. However, while investigating the one-loop divergences, the anomalies do not matter.

³ The gauge dependence of the one-loop divergences in $\mathcal{N} = (1, 0)$ SYM theory was also discussed in ref. [10] in the framework of the component formulation.

We define the spinor and vector $6D$ derivatives in the analytic basis as

$$D_a^+ = \partial_{-a}, \quad D_a^- = -\partial_{+a} - 2i\theta^{-b}\partial_{ab}, \quad \partial_{ab} = \frac{1}{2}(\gamma^M)_{ab}\partial_M. \quad (2.2)$$

Also we will need the harmonic derivatives

$$D^{\pm\pm} = \partial^{\pm\pm} + i\theta^{\pm a}\theta^{\pm b}\partial_{ab} + \theta^{\pm a}\partial_{\mp a}, \quad D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+a}\partial_{+a} - \theta^{-a}\partial_{-a}, \quad (2.3)$$

where $\partial_a^{\pm}\theta^{\pm b} = \delta_a^b$ and the partial harmonic derivatives are defined as $\partial^{\pm\pm} = u^{\pm i}\frac{\partial}{\partial u^{\mp i}}$ (in the central basis the latter coincide with the full harmonic derivatives). The spinor and harmonic derivatives satisfy the algebra

$$\{D_a^+, D_b^-\} = 2i\partial_{ab}, \quad [D^{++}, D^{--}] = D^0, \quad [D^{\pm\pm}, D_a^{\pm}] = 0, \quad [D^{\pm\pm}, D_a^{\mp}] = D_a^{\pm}. \quad (2.4)$$

Finally, the full and analytic superspace integration measures are defined as

$$d^{14}z \equiv d^6x_A (D^-)^4 (D^+)^4, \quad d\zeta^{(-4)} \equiv d^6x_A (D^-)^4, \quad (D^{\pm})^4 = -\frac{1}{24}\varepsilon^{abcd}D_a^{\pm}D_b^{\pm}D_c^{\pm}D_d^{\pm}V^{--}. \quad (2.5)$$

Now we briefly recall, basically following ref. [14,19–23], some details of the harmonic superspace formulation of $6D, \mathcal{N} = (1, 0)$ SYM interacting with a hypermultiplet. The classical action of the theory has the form

$$S_0[V^{++}, q^+] = \frac{1}{f_0^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \times \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+, \quad (2.6)$$

where f_0 is a dimensionful coupling constant ($[f_0] = m^{-1}$). Here V^{++} is the hermitian analytic gauge connection taking values in the Lie algebra of the gauge group G ,

$$V^{++} = (V^{++})^A T^A, \quad [T^A, T^B] = if^{ABC} T^C, \quad A, B, C = 1, \dots, d_G. \quad (2.7)$$

Here f^{ABC} are the totally antisymmetric structure constants and d_G is the dimension of the gauge group.

We assume that the hypermultiplet belongs to an irreducible representation R of the gauge group G . Then the covariant harmonic derivative ∇^{++} in eq. (2.6) acts on the hypermultiplet as

$$(\nabla^{++})_m^n q_n^+ = D^{++} q_m^+ + i(V^{++})^A (T^A)_m^n q_n^+, \quad (2.8)$$

where the generators of the gauge group T^A satisfy the conditions

$$\text{tr}(T^A T^B) = T(R)\delta^{AB}, \quad (T^A)_m^l (T^A)_l^n = C(R)\delta_m^n. \quad (2.9)$$

Here $C(R)$ is the second-order Casimir for the representation R and $T(R) = C(R)d_R/d_G$, with d_R being the dimension of the irreducible representation R . For the adjoint representation the generators are written as $(T_{\text{Adj}}^C)_A^B = if^{ACB}$ and

$$T(\text{Adj}) = C(\text{Adj}) \equiv C_2. \quad (2.10)$$

The generators of the fundamental representation $T_F^A = t^A$ are normalized in the standard way, $\text{tr}(t^A t^B) = \frac{1}{2} \delta^{AB}$. Hereafter we omit the representation indices of the hypermultiplet.

The action (2.6) is invariant under the gauge transformation

$$\begin{aligned} (V^{++})' &= e^{i\lambda^A T^A} V^{++} e^{-i\lambda^A T^A} - i e^{i\lambda^A T^A} D^{++} e^{-i\lambda^A T^A}, \\ (q^+)' &= e^{i\lambda^A T^A} q^+, \end{aligned} \quad (2.11)$$

where $\lambda^A(\zeta, u)$ is a real (with respect to the “tilde” conjugation) gauge group parameter.

We also introduce the non-analytic harmonic connection $V^{--} = (V^{++})^{AT^A}$ [17] and construct the second covariant harmonic derivative ∇^{--}

$$\nabla^{--} = D^{--} + iV^{--}. \quad (2.12)$$

The superfield V^{--} is a solution of the harmonic zero-curvature condition

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0, \quad (2.13)$$

and its explicit expression in terms of V^{++} is given by

$$\begin{aligned} V^{--}(z, u) &= \sum_{n=1}^{\infty} (-i)^{n+1} \\ &\times \int du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \dots (u_n^+ u^+)}. \end{aligned} \quad (2.14)$$

Using the zero-curvature condition (2.13), one can derive a useful relation between variations of the gauge harmonic connections

$$\delta V^{--} = \frac{1}{2} (\nabla^{--})^2 \delta V^{++} - \frac{1}{2} \nabla^{++} (\nabla^{--} \delta V^{--}). \quad (2.15)$$

The superfield V^{--} can be used to construct the spinor and vector gauge-covariant derivatives. In the λ -frame they read

$$\nabla_a^+ = D_a^+, \quad \nabla_a^- = D_a^- + iA_a^-, \quad \nabla_{ab} = \partial_{ab} + iA_{ab}, \quad (2.16)$$

where $\nabla_{ab} = \frac{1}{2}(\gamma^M)_{ab} \nabla_M$ and $\nabla_M = \partial_M - iA_M$. The superfield connections in eq. (2.16) are defined as

$$A_a^- = iD_a^+ V^{--}, \quad A_{ab} = \frac{1}{2} D_a^+ D_b^+ V^{--}. \quad (2.17)$$

The covariant derivatives (2.16) satisfy the algebra

$$\begin{aligned} \{\nabla_a^+, \nabla_b^-\} &= 2i\nabla_{ab}, \quad [\nabla_c^\pm, \nabla_{ab}] = \frac{i}{2} \varepsilon_{abcd} W^{\pm d}, \\ [\nabla_M, \nabla_N] &= iF_{MN}, \end{aligned} \quad (2.18)$$

where $W^{a\pm}$ is the superfield strength of the gauge multiplet,

$$W^{+a} \equiv -\frac{i}{6} \varepsilon^{abcd} D_b^+ D_c^+ D_d^+ V^{--}, \quad W^{-a} = \nabla^{--} W^{+a}. \quad (2.19)$$

Also we define the Grassmann analytic superfield F^{++} [14],

$$F^{++} \equiv (D^+)^4 V^{--}, \quad \nabla^{++} F^{++} = 0. \quad (2.20)$$

Using the relation between the variations of the gauge connections V^{++} and V^{--} , we can derive the classical equations of motion for the model (2.6),

$$\frac{\delta S}{\delta (V^{++})^A} = 0 \Rightarrow (F^{++})^A - 2if_0 \tilde{q}^+ T^A q^+ = 0, \quad (2.21)$$

$$\frac{\delta S}{\delta \tilde{q}^+} = 0 \Rightarrow \nabla^{++} q^+ = 0. \quad (2.22)$$

Finally note that for the hypermultiplet belonging to the adjoint representation of the gauge group the action (2.6) possesses an additional $\mathcal{N} = (0, 1)$ supersymmetry [14]. In this case the action (2.6) describes 6D, $\mathcal{N} = (1, 1)$ SYM theory.

3. The one-loop effective action

The background superfield method for the model (2.6) was developed in refs. [19–23]. In many aspects it is similar to that for 4D, $\mathcal{N} = 2$ supersymmetric gauge theories [25] (see also the review [26]). Following this method, we split the superfields V^{++}, q^+ into the sum of the *background* superfields \mathbf{V}^{++}, Q^+ and the *quantum* ones v^{++}, q^+ ,

$$V^{++} \rightarrow \mathbf{V}^{++} + f_0 v^{++}, \quad q^+ \rightarrow Q^+ + q^+. \quad (3.1)$$

The effective action is invariant under the *background* gauge transformations:

$$\delta \mathbf{V}^{++} = -\nabla^{++} \lambda, \quad \delta v^{++} = -i[v^{++}, \lambda], \quad (3.2)$$

where

$$\nabla^{\pm\pm} \equiv D^{\pm\pm} + i\mathbf{V}^{\pm\pm} \quad (3.3)$$

are the background harmonic covariant derivatives.⁴

We use the gauge-fixing function similar to that in the 4D case [25,26],

$$\mathcal{F}_\tau^{(+4)} = D^{++} v_\tau^{++} = e^{-i\mathbf{b}} (\nabla^{++} v_\tau^{++}) e^{i\mathbf{b}} = e^{-i\mathbf{b}} \mathcal{F}^{(+4)} e^{i\mathbf{b}}, \quad (3.4)$$

where $\mathbf{b}(z, u)$ is the background bridge superfield (see, e.g., [17]). In this paper we will use the more general gauge-fixing term

$$\begin{aligned} S_{\text{gf}}[v^{++}, V^{++}] &= -\frac{1}{2\xi_0} \text{tr} \int d^{14}z du_1 du_2 \frac{v_\tau^{++}(1) v_\tau^{++}(2)}{(u_1^+ u_2^+)^2} \\ &+ \frac{1}{4\xi_0} \text{tr} \int d^{14}z du v_\tau^{++} (D^{--})^2 v_\tau^{++}. \end{aligned} \quad (3.5)$$

The action (3.5) includes an arbitrary real parameter ξ_0 and depends on the background field \mathbf{V}^{++} through the background gauge bridge \mathbf{b} , $v_\tau^{++} = e^{-i\mathbf{b}} v_\tau^{++} e^{i\mathbf{b}}$.

The one-loop quantum correction $\Gamma^{(1)}[\mathbf{V}^{++}, Q^+]$ to the classical action (2.6) is given by the following path integral [19,25]:

$$\begin{aligned} &\exp\left(i\Gamma^{(1)}[\mathbf{V}^{++}, Q^+]\right) \\ &= \text{Det}^{1/2} \hat{\square} \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}b \mathcal{D}c \mathcal{D}\varphi \\ &\times \exp\left(iS_2[v^{++}, q^+, b, c, \varphi, \mathbf{V}^{++}, Q^+]\right), \end{aligned} \quad (3.6)$$

where $\hat{\square} = \frac{1}{2}(D^+)^4 (\nabla^{--})^2$ is the covariant d'Alembertian. When acting on the analytic superfields, it is reduced to

$$\hat{\square} = \eta^{MN} \nabla_M \nabla_N + \mathbf{W}^{+a} \nabla_a^- + \mathbf{F}^{++} \nabla^{--} - \frac{1}{2} (\nabla^{--} \mathbf{F}^{++}). \quad (3.7)$$

In the expression (3.6) S_2 denotes that part of the total action which is quadratic in the quantum superfields. It includes the classical action (2.6) in which the background-quantum splitting is performed, the gauge-fixing action (3.5), and the actions for the ghost superfields,

$$\begin{aligned} S_2 &= S_{\text{gh}} + \frac{1}{2\xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++} \hat{\square} v^{++} \\ &+ \frac{1}{2} \left(1 - \frac{1}{\xi_0}\right) \text{tr} \int d^{14}z du_1 du_2 \frac{v_\tau^{++}(1) v_\tau^{++}(2)}{(u_1^+ u_2^+)^2} \end{aligned}$$

⁴ Herewith, the bold letters denote the objects constructed out of the background gauge superfield \mathbf{V}^{++} .

$$\begin{aligned}
& - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+ \\
& - if_0 \int d\zeta^{(-4)} du \left(\tilde{Q}^+ v^{++} q^+ + \tilde{q}^+ v^{++} Q^+ \right). \quad (3.8)
\end{aligned}$$

The ghost actions are written as

$$S_{\text{gh}} = \frac{1}{2} \text{tr} \int d\zeta^{(-4)} du \varphi (\nabla^{++})^2 \varphi + \text{tr} \int d\zeta^{(-4)} du b (\nabla^{++})^2 c. \quad (3.9)$$

The superfields b, c are the Faddeev-Popov ghosts and φ stands for the Nielsen-Kallosh ghost.

The action S_2 (3.8) contains mixed terms in which both quantum superfields v^{++} and q^+ are present. Following ref. [19], it is convenient to diagonalize S_2 by means of the special change of the quantum hypermultiplet variables in the path integral,⁵

$$q^+(1) = h^+(1) - if_0 \int d\zeta_2^{(-4)} du_2 G^{(1,1)}(1|2) v^{++}(2) Q^+(2). \quad (3.10)$$

Here, h^+ is a set of new independent quantum hypermultiplet superfields and $G^{(1,1)}(1|2)$ is the hypermultiplet Green function,

$$G^{(1,1)}(\zeta_1, u_1 | \zeta_2, u_2) = i \langle q^+(\zeta_1, u_1) \tilde{q}^+(\zeta_2, u_2) \rangle = \frac{(D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2)}{\widehat{\square}_1 (u_1^+ u_2^+)^3}. \quad (3.11)$$

This Green function is analytic with respect to its both super arguments and satisfies the equation

$$\nabla_1^{++} G^{(1,1)}(1|2) = \delta_A^{(3,1)}(1|2), \quad (3.12)$$

where $\delta_A^{(3,1)}(1|2)$ is the covariantly-analytic delta-function [17].

After performing the shift (3.10), the action S_2 (3.8) takes the diagonal form,

$$\begin{aligned}
S_2 = S_{\text{gh}} & + \frac{1}{2\xi_0} \text{tr} \int d\zeta^{(-4)} du v^{++} \widehat{\square} v^{++} \\
& + \frac{1}{2} \left(1 - \frac{1}{\xi_0}\right) \text{tr} \int d^{14} z du_1 du_2 \frac{v^{++}(1) v^{++}(2)}{(u_1^+ u_2^+)^2} \\
& - f_0^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} du_1 du_2 v_1^{++} \tilde{Q}_1^+ G^{(1,1)}(1|2) Q_2^+ v_2^{++} \\
& - \int d\zeta^{(-4)} du \tilde{q}^+ \nabla^{++} q^+. \quad (3.13)
\end{aligned}$$

The action (3.13) includes a new term which is quadratic in the quantum vector superfield v^{++} and contains a non-local contribution involving the Green function $G^{(1,1)}$.

Integrating over the quantum superfields in the path integral (3.6) with the action (3.13), we find the one-loop contribution $\Gamma^{(1)}$ to the effective action,

$$\begin{aligned}
\Gamma[\mathbf{V}^{++}, Q; \xi_0] & = \frac{i}{2} \text{Tr} \ln \left\{ \frac{1}{\xi_0} \widehat{\square}^{AB} + \left(1 - \frac{1}{\xi_0}\right) \delta^{AB} \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right. \\
& \quad \left. - 4f_0^2 \tilde{Q}_1^+ (T^A G^{(1,1)} T^B)(1|2) Q_2^+ \right\}
\end{aligned}$$

⁵ A similar shift of variables in the path integral of non-supersymmetric QED was used in ref. [32]. The supersymmetric generalization of this procedure was applied in refs. [33–35], while calculating the one- and two-loop contributions to effective actions of supersymmetric gauge theories.

$$- \frac{i}{2} \text{Tr} \ln \widehat{\square} - \frac{i}{2} \text{Tr} \ln (\nabla_{\text{Adj}}^{++})^2 + i \text{Tr} \ln \nabla_R^{++}. \quad (3.14)$$

The subscripts Adj and R in (3.14) mean that the corresponding operators act in the adjoint and R representations of the gauge group. The functional trace Tr in eq. (3.14) is defined as

$$\text{Tr} \mathcal{O} = \text{tr} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} du_1 du_2 \delta_{\mathcal{A}}^{(q,4-q)}(1|2) \mathcal{O}^{(q,4-q)}(1|2). \quad (3.15)$$

In this expression $\mathcal{O}^{(q,4-q)}(\zeta_1, u_1 | \zeta_2, u_2)$ is the kernel of an operator acting in the space of the covariantly analytic superfields with the harmonic $U(1)$ charge q , and $\delta_{\mathcal{A}}^{(q,4-q)}(1|2)$ is the corresponding analytic delta-function [17],

$$\begin{aligned}
\delta_{\mathcal{A}}^{(q,4-q)} & = (D_2^+)^4 \delta^{14}(z_1 - z_2) \delta^{(q,-q)}(u_1, u_2), \\
\delta^{14}(z_1 - z_2) & = (\theta_1^+ - \theta_2^+)^4 (\theta_1^- - \theta_2^-)^4 \delta^6(x_1 - x_2). \quad (3.16)
\end{aligned}$$

The expression (3.14) depends on the background superfields \mathbf{V}^{++} and Q^+ . Also, it contains the parameter ξ_0 of the gauge-fixing term. Our further purpose will be to study the divergent part of eq. (3.14) as a function of ξ_0 , without assuming *a priori* any restriction on the background superfields.

4. Gauge dependence of the one-loop divergences

First, we consider the part of the effective action which is specified by the last two terms in (3.14),

$$\Delta_1 \Gamma^{(1)}[\mathbf{V}^{++}] = - \frac{i}{2} \text{Tr} \ln (\nabla_{\text{Adj}}^{++})^2 + i \text{Tr} \ln \nabla_R^{++}. \quad (4.1)$$

It does not depend on the gauge-fixing parameter ξ_0 and has already been analyzed earlier in ref. [19]. Omitting details of the calculation, we present the final expression for the divergent part of (4.1)

$$\Delta_1 \Gamma_{\infty}^{(1)} = \frac{C_2 - T(R)}{3(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2. \quad (4.2)$$

Here, $\varepsilon \equiv 6 - D \rightarrow 0$ and $\mathbf{F}^{++} = (\mathbf{F}^{++})^A t^A$, with t^A being the generators of the fundamental representation.

The remaining gauge-dependent part of the one-loop counterterms comes out from the first two terms in eq. (3.14)

$$\begin{aligned}
\Delta_2 \Gamma^{(1)}[\mathbf{V}^{++}, Q^+; \xi_0] & = \frac{i}{2} \text{Tr} \ln \left\{ \frac{1}{\xi_0} \widehat{\square}^{AB} + \left(1 - \frac{1}{\xi_0}\right) \delta^{AB} \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right. \\
& \quad \left. - 4f_0^2 \tilde{Q}_1^+ (T^A G^{(1,1)} T^B)(1|2) Q_2^+ \right\} - \frac{i}{2} \text{Tr} \ln \widehat{\square}. \quad (4.3)
\end{aligned}$$

We start with the calculation of the gauge-dependent divergences in the pure gauge superfield sector. So we switch off the background hypermultiplet, $Q^+ = 0$, and expand the logarithm up to the first order in the inverse $\widehat{\square}$ operator,

$$\begin{aligned}
\Delta_{2, F^2} \Gamma^{(1)}[\mathbf{V}^{++}; \xi_0] & = \frac{i}{2} \text{Tr} \ln \left\{ \frac{1}{\xi_0} \widehat{\square} + \left(1 - \frac{1}{\xi_0}\right) \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right\} - \frac{i}{2} \text{Tr} \ln \widehat{\square} \\
& \rightarrow \frac{i}{2} (\xi_0 - 1) \int d\zeta^{(-4)} du (\widehat{\square}_1^{-1})^{AA} \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \delta_{\mathcal{A}}^{(2,2)}(1|2) \Big|_{2=1}. \quad (4.4)
\end{aligned}$$

Then we expand the operator $\widehat{\square}^{-1}$ up to the second order in the harmonic derivative D^{--} ,

$$(\widehat{\square}^{-1})^{AA} \rightarrow \frac{1}{(\partial^2)^3} (f^{ACB}(\mathbf{F}^{++})^C)(f^{BDA}(\mathbf{F}^{++})^D)(D^{--})^2 + \dots, \quad (4.5)$$

and use the explicit expression (3.16) for the analytic delta function $\delta_A^{(2,2)}(1|2)$. On the next step we use the identity

$$(D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2) \Big|_{\theta_2 = \theta_1} = (u_1^+ u_2^+)^4 \delta^6(x_1 - x_2), \quad (4.6)$$

and pass to the momentum representation, taking the coincident-point limit,

$$\frac{1}{(\partial^2)^3} \delta^6(x_1 - x_2) \Big|_{2=1} = \frac{i}{(4\pi)^3 \varepsilon} + \text{finite terms}. \quad (4.7)$$

As a result, we derive the following expression for the divergent part of $\Delta_{2, F^2} \Gamma^{(1)}$:

$$\Delta_{2, F^2} \Gamma_\infty^{(1)}[\mathbf{V}^{++}; \xi_0] = \frac{2(\xi_0 - 1)C_2}{(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2. \quad (4.8)$$

Now we consider the divergent part of the one-loop effective action which depends on the hypermultiplet Q^+ ,

$$\begin{aligned} \Delta_{2, Q} \Gamma^{(1)}[\mathbf{V}^{++}, Q^+; \xi_0] &= \frac{i}{2} \int d\zeta_1^{(-4)} du_1 \left(\frac{1}{\xi_0} \widehat{\square} + \left(1 - \frac{1}{\xi_0}\right) \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right)^{-1 AB} \\ &\quad \times \left(-4f_0^2 \widetilde{Q}_1^+ (T^B G^{(1,1)} T^A)(1|2) Q_2^+ \right) \Big|_{2=1} \\ &= -2if_0^2 \int d\zeta_1^{(-4)} du_1 \left(\frac{\xi_0}{\widehat{\square}} - (\xi_0 - 1) \frac{1}{\widehat{\square}^2} \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right)^{AB} \\ &\quad \times \left(\widetilde{Q}_1^+ (T^B G^{(1,1)} T^A)(1|2) Q_2^+ \right) \Big|_{2=1}. \end{aligned} \quad (4.9)$$

The divergent contribution to the one-loop effective action, which contains $\widetilde{Q}^+ F^{++} Q^+$, can be found similarly to the case of the minimal gauge. It comes out from the term

$$-2i\xi_0 f_0^2 \int d\zeta_1^{(-4)} du_1 (\widehat{\square}^{-1})^{AB} \left(\widetilde{Q}_1^+ (T^B G^{(1,1)} T^A)(1|2) Q_2^+ \right) \Big|_{2=1}. \quad (4.10)$$

The divergent part of this expression was calculated in ref. [19]. Here we omit details of the calculation and present the result,

$$\Delta_{2, QFQ} \Gamma_\infty^{(1)} = -\frac{2i\xi_0 f_0^2 (C_2 - C(R))}{(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} du \widetilde{Q}^+ \mathbf{F}^{++} Q^+. \quad (4.11)$$

In the case of non-minimal gauges there appears an additional contribution to the divergent part of the one-loop effective action. It comes from the second term in the (4.9),

$$\begin{aligned} 2if_0^2 (\xi_0 - 1) \int d\zeta_1^{(-4)} du_1 \left(\frac{1}{\widehat{\square}^2} \frac{(D_1^+)^4}{(u_1^+ u_2^+)^2} \right)^{AB} \\ \times \left(\widetilde{Q}_1^+ (T^B G^{(1,1)} T^A)(1|2) Q_2^+ \right) \Big|_{2=1}. \end{aligned} \quad (4.12)$$

To cast it in the explicit form, we first use the antisymmetry of the hypermultiplet propagator (3.11),

$$\widetilde{Q}_1^+ Q_2^+ G^{(1,1)}(1|2) = \frac{1}{2} (\widetilde{Q}_1^+ Q_2^+ - \widetilde{Q}_2^+ Q_1^+) G^{(1,1)}(1|2). \quad (4.13)$$

Then we rewrite the Green function $G^{(1,1)}$ as [36–40]

$$\begin{aligned} G^{(1,1)}(1|2) &= \frac{(D_1^+)^4}{\widehat{\square}} \left\{ (D_1^-)^4 (u_1^+ u_2^+) - \mathbf{\Omega}_1^{--} (u_1^- u_2^+) \right. \\ &\quad \left. - 4 \widehat{\square} \frac{(u_1^- u_2^+)^2}{(u_1^+ u_2^+)} \right\}, \end{aligned} \quad (4.14)$$

where

$$\mathbf{\Omega}^{--} \equiv i \nabla^{ab} \nabla_a^- \nabla_b^- - \mathbf{W}^{-a} \nabla_a^- + \frac{1}{4} (\nabla_a^- \mathbf{W}^{-a}). \quad (4.15)$$

The divergent contribution comes from the first term in the propagator (4.14). Plugging it in eq. (4.12), we obtain the expression

$$\begin{aligned} if_0^2 (\xi_0 - 1) \int d\zeta_1^{(-4)} du_1 \frac{(D_1^+)^4}{\widehat{\square}_{\text{Adj}}^2} \frac{1}{(u_1^+ u_2^+)^2} \\ \times \left\{ (\widetilde{Q}_1^+ Q_2^+ - \widetilde{Q}_2^+ Q_1^+) \frac{(D_1^+)^4 (D_1^-)^4}{\widehat{\square}_R} (u_1^+ u_2^+) \right\} \delta^{14}(z_1 - z_2) \Big|_{2=1}. \end{aligned} \quad (4.16)$$

We reconstruct the full superspace measure in this expression by taking away the factor $(D^+)^4$ and then use the identities

$$Q_2^+ = (u_1^+ u_2^+) Q_1^- - (u_1^- u_2^+) Q_1^+, \quad (4.17)$$

$$(\widetilde{Q}_1^+ Q_2^+ - \widetilde{Q}_2^+ Q_1^+) = (u_1^+ u_2^+) (\widetilde{Q}^+ Q^- - \widetilde{Q}^- Q^+). \quad (4.18)$$

As the last step, we get rid of the Grassmann delta-function in the limit of coincident points by making use of the operators $(D^+)^4$ and $(D^-)^4$ and collect the third power of the $(\partial^2)^{-1}$ operator acting on the space-time delta-function (4.7) to extract the divergent contribution. Finally, we obtain

$$\Delta_{2, QQ} \Gamma_\infty^{(1)} = -\frac{f_0^2 (\xi_0 - 1) C(R)}{(4\pi)^3 \varepsilon} \int d^{14}z du (\widetilde{Q}^+ Q^- - \widetilde{Q}^- Q^+). \quad (4.19)$$

Summing up the contributions (4.2) (4.8), (4.11) and (4.19), we can present the total divergent part of the one-loop effective action in the form

$$\begin{aligned} \Gamma_\infty^{(1)}[\mathbf{V}^{++}, Q^+; \xi_0] &= \Delta_1 \Gamma_\infty^{(1)} + \Delta_{2, F^2} \Gamma_\infty^{(1)} + \Delta_{2, QFQ} \Gamma_\infty^{(1)} + \Delta_{2, QQ} \Gamma_\infty^{(1)} \\ &= \frac{1}{(4\pi)^3 \varepsilon} \left(\frac{1}{3} (C_2 - T(R)) + 2(\xi_0 - 1) C_2 \right) \text{tr} \int d\zeta^{(-4)} du (\mathbf{F}^{++})^2 \\ &\quad - \frac{2i\xi_0 f_0^2 (C_2 - C(R))}{(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} du \widetilde{Q}^+ \mathbf{F}^{++} Q^+ \\ &\quad - \frac{f_0^2 (\xi_0 - 1) C(R)}{(4\pi)^3 \varepsilon} \int d^{14}z du (\widetilde{Q}^+ Q^- - \widetilde{Q}^- Q^+). \end{aligned} \quad (4.20)$$

Note that for the minimal gauge choice, $\xi_0 = 1$, this expression reproduces the result of refs. [19,20].

According to the general theorem (see, e.g., [27]), the gauge dependence should vanish on shell. This condition allows one to check the correctness of eq. (4.20). Let us suppose that the background superfields satisfy the classical equations of motion (2.22).

Then it is easy to show that all terms containing the gauge-fixing parameter ξ_0 are mutually canceled. To this end, we note that the superfields Q^+ and Q^- are not independent on shell, $Q^- = \nabla^{--} Q^+$. Therefore, taking into account that $F^{++} = (D^+)^4 V^{--}$, we obtain

$$\int d^{14}z du (\tilde{Q}^+ Q^- - \tilde{Q}^- Q^+) = 2 \int d^{14}z du \tilde{Q}^+ F^{++} Q^+. \quad (4.21)$$

Consequently, the divergent part of the one-loop effective action takes on shell the form

$$\begin{aligned} \Gamma_{\infty}^{(1)} \Big|_{\text{on shell}} &= \frac{1}{(4\pi)^3 \varepsilon} \left(\frac{1}{3} (C_2 - T(R)) + 2(\xi_0 - 1) C_2 \right) \\ &\times \text{tr} \int d\zeta^{(-4)} du (F^{++})^2 - \frac{2if_0^2 (\xi_0 C_2 - C(R))}{(4\pi)^3 \varepsilon} \\ &\times \int d\zeta^{(-4)} du \tilde{Q}^+ F^{++} Q^+. \end{aligned} \quad (4.22)$$

Next, using the equation of motion for the background gauge superfield, $(F^{++})^A = 2if_0^2 \tilde{Q}^+ T^A Q^+$, we obtain that on shell the whole gauge dependence disappears,

$$\begin{aligned} \Gamma_{\infty}^{(1)} \Big|_{\text{on shell}} &= \frac{(C_2 - T(R))}{3(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2 \\ &- \frac{2if_0^2 (C_2 - C(R))}{(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} du \tilde{Q}^+ F^{++} Q^+. \end{aligned} \quad (4.23)$$

Finally, we note that for the hypermultiplet in the adjoint representation the on-shell result (4.23) evidently vanishes, while the off-shell result (4.20) does not,

$$\begin{aligned} \Gamma_{\infty}^{(1)} \Big|_{\mathcal{N}=(1,1) \text{ SYM}} &= \frac{2(\xi_0 - 1)C_2}{(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2 \\ &- \frac{f_0^2 (\xi_0 - 1)C_2}{(4\pi)^3 \varepsilon} \int d^{14}z du (\tilde{Q}^+ Q^- - \tilde{Q}^- Q^+). \end{aligned} \quad (4.24)$$

Eqs. (4.20) and (4.24) are the main results of the paper. These expressions are obtained in the general ξ -gauge and generalize the results of refs. [19,20] which are valid only for the particular case $\xi_0 = 1$. We see that the divergent part of the one-loop effective action nontrivially depends on the gauge parameter ξ_0 . This dependence significantly changes the result for $\mathcal{N} = (1, 1)$ SYM theory, because for $\xi_0 \neq 1$ the divergences are present off shell and vanish only on shell. Note that in the case $\xi_0 = 1$ considered in refs. [19, 20] the one-loop divergences for this theory are absent even off shell.

5. Summary

We considered the general six-dimensional $\mathcal{N} = (1, 0)$ supersymmetric gauge theory in the harmonic superspace formulation and studied the dependence of the one-loop divergences on the gauge-fixing parameter. The theory describes $\mathcal{N} = (1, 0)$ vector gauge multiplet coupled to the hypermultiplet in an arbitrary representation of the gauge group. The effective action was constructed by means of the background superfield method, with making use of the one-parameter family of gauges analogous to the ξ -gauges of the non-supersymmetric case. The divergent part of the one-loop effective action was calculated by the superfield proper-time technique, and the gauge dependence of the divergences was found in the explicit form.

It was shown that the divergent part of the effective action in the generic gauge contains an additional contribution as compared to the case of the minimal gauge. However, the whole gauge

dependence of the divergences vanishes for the background superfields satisfying the classical equations of motion. This confirms the correctness of the calculations performed and ensures that the divergent part of the S -matrix is gauge-independent.

When the hypermultiplet sits in the adjoint representation, the results of this paper yield the one-loop divergences of $\mathcal{N} = (1, 1)$ SYM theory. While using the minimal gauge, such a theory is off-shell finite at one-loop [19,20,23]. In this paper we demonstrated that the one-loop effective action in the non-minimal gauge is divergent off shell, and this divergence vanishes only on shell.

One interesting prospect for the future study is related to a recent activity on constructing the higher-derivative $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theory and studying the quantum corrections in this theory [41–43]. It was noted that such a theory is renormalizable (modulo anomalies to be manifested in higher loops) and one-loop counterterms were calculated. Note that these divergences were analyzed in the component formulation, in which supersymmetry is not manifest. So, it would be extremely interesting to fulfill the superfield quantum consideration of this higher-derivative theory and to explore the corresponding effective action in the harmonic superspace formalism. One can expect that such a manifestly $6D$, $\mathcal{N} = (1, 0)$ supersymmetric analysis will help to reveal more profound aspects of the structure of quantum corrections in this theory.

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