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# Gauge dependence of the one-loop divergences in $6D$, $\mathcal{N} = (1, 0)$ abelian theory

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**Abstract**

We study the gauge dependence of the one-loop effective action for the abelian $6D$, $\mathcal{N} = (1, 0)$ supersymmetric gauge theory formulated in harmonic superspace. We introduce the superfield $\xi$-gauge, construct the corresponding gauge superfield propagator, and calculate the one-loop two- and three-point Green functions with two external hypermultiplet legs. We demonstrate that in the general $\xi$-gauge the two-point Green function of the hypermultiplet is divergent, as opposed to the Feynman gauge $\xi = 1$. The three-point Green function with two external hypermultiplet legs and one leg of the gauge superfield is also divergent. We verified that the Green functions considered satisfy the Ward identity formulated in $\mathcal{N} = (1, 0)$ harmonic superspace and that their gauge dependence vanishes on shell. Using the result for the two- and three-point Green functions and arguments based on the gauge invariance, we present the complete divergent part of the one-loop effective action in the general $\xi$-gauge.



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## 1. Introduction

Gauge theories with extended supersymmetries in higher dimensions attract a considerable attention for a long time [1–8]. On the one hand, such theories are non-renormalizable due to the dimensionful coupling constant (see, e.g., [9,10]). On the other hand, one can expect an improvement of the ultraviolet behavior due to the extended supersymmetry. It is very interesting to check this conjecture on the explicit examples of higher-dimensional supersymmetric theories. To be more realistic, one can expect that the full canceling of divergences is presumably possible only in the lowest loops even in the maximally extended theories (see, e.g., [11]). The problem reveals clear analogies with the most interesting case of gravity. However, the analysis in supersymmetric gauge theories is much simpler.

In order to fully display the underlying properties of theories with some symmetries it is highly desirable to be aware of the regularization and quantization schemes which do not break these symmetries. In the case of extended supersymmetries these purposes can be achieved within the harmonic superspace approach [12–17]. For $6D$ supersymmetric gauge theories (which will be the subject of the present paper) this formalism [18–23] ensures manifest $\mathcal{N}=(1,0)$ supersymmetry. With the use of the background field method in harmonic superspace [16,24], gauge symmetry can also be made manifest. For these reasons the harmonic superspace formalism seems to be most suitable for quantum calculations in $6D$ supersymmetric theories (note that $6D$, $\mathcal{N}=(1,0)$ theories are in general anomalous, see, e.g., [25–28]).

Recently, some explicit calculations based on the harmonic superspace method were done for $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ gauge theories [29–33], following the general pattern of Ref. [4]. These calculations were basically performed in the Feynman gauge $\xi=1$, which ensures the simplest form of the propagator of the gauge superfield. This considerably simplifies the calculation of quantum corrections. However, the gauge dependence of the results obtained by the harmonic superspace technique has not yet been analyzed. Meanwhile, the calculations in non-minimal gauges are frequently rather useful as compared to those in the Feynman gauge, because they are capable to make manifest divergences in the lower loops. For example, for $\mathcal{N}=1$ supersymmetric gauge theories in the one-loop approximation ghosts are not renormalized in the Feynman gauge, while divergences appear for $\xi\neq 1$ [35]. For calculations in higher orders, the knowledge of gauge dependence in the lower-order approximations is also essential, see, e.g., [36]. These are the reasons why a vast literature is devoted to calculations in non-minimal gauges. As a characteristic example, let us mention a recent paper [37].

In the present paper we consider the simplest $6D$, $\mathcal{N}=(1,0)$ supersymmetric gauge theory, namely, $\mathcal{N}=(1,0)$ supersymmetric electrodynamics, and investigate the structure of the gauge-dependent contributions to the effective action by the harmonic superspace technique. In particular, we demonstrate that (unlike the case of the Feynman gauge considered, e.g., in [29]) the two-point Green function of hypermultiplets is divergent already at the one-loop level. The gauge-dependent divergences are also present in the gauge multiplet – hypermultiplet Green functions. In this paper we explicitly calculate the one-loop three-point Green function and find its divergent part. Moreover, we derive the Ward identity in the harmonic superspace and verify that the Green functions obtained by calculating harmonic supergraphs satisfy this identity, as expected. This result is a non-trivial verification of the correctness of our calculations. One more test, which has also been done in this paper, is the demonstration of the property that the gauge dependence of the effective action vanishes on shell (this is a consequence of the general theorem, see Refs. [38–43]). Using the results for the two- and three-point Green functions, we also restore the complete result for the one-loop divergences, based on the gauge invariance of the theory under consideration.

The paper is organized as follows: In Sect. 2 we recall some basic points of the formulation of $6D, \mathcal{N}=(1,0)$ supersymmetric electrodynamics in harmonic superspace. We present the superfield action for this theory, write down the Ward identity, and formulate the harmonic superspace Feynman rules. In particular, we construct the propagator of the gauge superfield in the non-minimal gauges which are analogs of the $\xi$-gauges in the usual electrodynamics. In Sect. 3, using these Feynman rules, we investigate the gauge dependence of the one-loop two-point Green functions of the gauge superfield and the hypermultiplet. We also calculate the one-loop three-point gauge superfield – hypermultiplet Green function. Checking the Ward identities for these Green functions is the subject of Sect. 4. The vanishing of the gauge dependence on shell in the approximation we are considering is demonstrated in Sect. 5. The total divergent part of the one-loop effective action (which is an infinite series in $V^{++}$) is constructed in Sect. 6, by invoking the arguments based on the gauge invariance. Also we verify that the gauge dependence of the expression obtained vanishes on shell. Some technical details are collected in two Appendices.

## 2. Harmonic superspace formulation of $6D, \mathcal{N}=(1,0)$ electrodynamics

### 2.1. The harmonic superspace action

The harmonic superspace is very convenient for formulating $6D, \mathcal{N}=(1,0)$ supersymmetric theories, because it ensures manifest supersymmetry at all steps of quantum calculations. It is parametrized by the coordinate set $(x^M, \theta^{ai}, u_i^\pm)$ which will be referred to as the central basis. Here $x^M$ with $M=0,\ldots 5$ are the usual coordinates of the six-dimensional Minkowski space. The Grassmann anticommuting coordinates $\theta^{ai}$ with $a=1,\ldots 4$ and $i=1,2$ form a left-handed $6D$ spinor. The harmonic variables $u_i^\pm$ satisfy the condition $u^{+i}u_i^-=1$, with $u_i^-=(u^{+i})^*$. The analytic basis of the harmonic superspace is parametrized by the coordinates

$$x_A^M = x^M + \frac{i}{2}\theta^-\gamma^M\theta^+; \qquad \theta^{\pm a} = u_i^\pm\theta^{ai}; \qquad u_i^\pm, \tag{1}$$

where $\gamma^M$ are $6D$ $\gamma$-matrices. The coordinate subset $(x_A^M, \theta^{+a}, u_i^\pm)$ parametrizes the analytic harmonic subspace which is closed on its own under $6D, \mathcal{N}=(1,0)$ supersymmetry transformations.

It is convenient to define the spinor covariant derivatives

$$D_a^+ = u_i^+ D_a^i; \qquad D_a^- = u_i^- D_a^i, \tag{2}$$

such that $\{D_a^+, D_b^-\} = i(\gamma^M)_{ab}\partial_M$, and to introduce the notation

$$(D^+)^4 = -\frac{1}{24}\varepsilon^{abcd}D_a^+D_b^+D_c^+D_d^+. \tag{3}$$

Also we will need the harmonics derivatives in the central basis

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}; \qquad D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}}; \qquad D^0 = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}. \tag{4}$$

They satisfy the commutation relations of the $SU(2)$ algebra. The analytic basis form of these derivatives can be easily found and is given, e.g., in [34].

For constructing the $\mathcal{N}=(1,0)$ invariants we need the invariant superspace integration measures:

$$\int d^{14}z = \int d^6x\, d^8\theta; \qquad \int d\zeta^{(-4)} = \int d^6x\, d^4\theta^+; \tag{5}$$

$$\int d^6x\, d^8\theta = \int d^6x\, d^4\theta^+ (D^+)^4. \tag{6}$$

In this paper we consider $\mathcal{N}=(1,0)$ supersymmetric electrodynamics, which is a particular abelian case of $\mathcal{N}=(1,0)$ supersymmetric Yang–Mills theory with hypermultiplets. The harmonic superspace form of the action of $6D$, $\mathcal{N}=(1,0)$ supersymmetric Yang–Mills theory was pioneered in Ref. [20]. As opposed to the analogous $4D$, $\mathcal{N}=2$ construction, the gauge theory coupling constant $f_0$ in $6D$ has the dimension $m^{-1}$. In the harmonic superspace approach the gauge superfield $V^{++}(z,u)$ satisfies the analyticity condition

$$D^+_a V^{++} = 0 \tag{7}$$

and is real with respect to the special conjugation denoted by $\widetilde{\ }$, i.e. $\widetilde{V^{++}} = V^{++}$. The hypermultiplets are described by the analytic superfield $q^+$ and its $\widetilde{\ }$-conjugate $\widetilde{q}^+$.

Like in the non-supersymmetric case, the action of $\mathcal{N}=(1,0)$ electrodynamics is quadratic in the gauge superfield. It can be written as

$$S = \frac{1}{4f_0^2}\int d^{14}z\, \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z,u_1)V^{++}(z,u_2) - \int d\zeta^{(-4)} du\, \widetilde{q}^+ \nabla^{++} q^+, \tag{8}$$

where

$$\nabla^{++} = D^{++} + iV^{++} \tag{9}$$

and $D^{++}$ is taken in the analytic basis. The gauge transformations have the form

$$V^{++} \to V^{++} - D^{++}\lambda; \qquad q^+ \to e^{i\lambda} q^+; \qquad \widetilde{q}^+ \to e^{-i\lambda}\widetilde{q}^+, \tag{10}$$

where $\lambda$ is an analytic superfield parameter which is real with respect to the $\widetilde{\ }$-conjugation.

It is useful to introduce the non-analytic superfield

$$V^{--}(z,u) = \int du_1\, \frac{V^{++}(z,u_1)}{(u^+ u_1^+)^2}. \tag{11}$$

It satisfies the conditions $D^{++}V^{--} = D^{--}V^{++}$ and transforms as

$$V^{--} \to V^{--} - D^{--}\lambda \tag{12}$$

under the gauge transformations. Starting from this superfield, it is possible to construct the analytic superfield $F^{++} = (D^+)^4 V^{--}$, which is gauge invariant in the abelian case.

For further use, we also define the non-analytic superfield $q^-$ as a solution of the equation

$$q^+ = \nabla^{++} q^- = (D^{++} + iV^{++}) q^-. \tag{13}$$

From this definition one can derive that the gauge transformations act on $q^-$ as

$$q^- \to e^{i\lambda} q^-. \tag{14}$$

In the explicit form the solution of Eq. (13) can be expressed as the series

$$
\begin{aligned}
q^- &= \int \frac{du_1}{(u^+u_1^+)} q_1^+ - i\int \frac{du_1\,du_2}{(u^+u_1^+)(u_1^+u_2^+)} V_1^{++} q_2^+ \\
&\quad - \int \frac{du_1\,du_2\,du_3}{(u^+u_1^+)(u_1^+u_2^+)(u_2^+u_3^+)} V_1^{++} V_2^{++} q_3^+ + \dots \\
&= \sum_{n=1}^{\infty} (-i)^{n-1} \int du_1 \dots du_n \frac{V_1^{++} \dots V_{n-1}^{++}}{(u^+u_1^+)\dots(u_{n-1}^+u_n^+)} q_n^+,
\end{aligned}
\tag{15}
$$

where subscripts numerate the harmonic "points".

For quantizing the theory (8) it is necessary to fix the gauge. This can be done by adding the gauge-fixing term to the action,

$$
S_{\text{gf}} = -\frac{1}{4 f_0^2 \xi_0} \int d^{14}z\, du_1 du_2 \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} D_1^{++} V^{++}(z, u_1) D_2^{++} V^{++}(z, u_2),
\tag{16}
$$

where $\xi_0$ is the bare gauge-fixing parameter. This term corresponds to the $\xi$-gauge in the usual electrodynamics. In particular, the Feynman gauge amounts to the choice $\xi_0 = 1$. In the abelian case we are considering it is not necessary to introduce the ghosts superfields. Therefore, the generating functional for our theory can be written as

$$
Z = \int DV^{++}\, D\widetilde{q}^{+}\, Dq^+ \exp\Big\{ i(S + S_{\text{gf}} + S_{\text{sources}}) \Big\},
\tag{17}
$$

where $S_{\text{sources}}$ is a sum of the source terms,

$$
\int d\zeta^{(-4)}\, du \Big[ V^{++} J^{(+2)} + j^{(+3)} q^+ + \widetilde{j}^{(+3)} \widetilde{q}^{+} \Big].
\tag{18}
$$

Here $J^{(+2)}$ is the analytic source for the gauge superfield, while $j^{(+3)}$ and $\widetilde{j}^{(+3)}$ denote sources for the hypermultiplet superfields. The effective action is constructed from the generating functional for the connected Green functions $W = -i \ln Z$ by making the Legendre transformation,

$$
\Gamma = W - S_{\text{sources}},
\tag{19}
$$

where it is necessary to express the sources in terms of the fields with the help of the equations

$$
V^{++} = \frac{\delta W}{\delta J^{(+2)}}; \qquad q^+ = \frac{\delta W}{\delta j^{(+3)}}; \qquad \widetilde{q}^{+} = \frac{\delta W}{\delta \widetilde{j}^{(+3)}}.
\tag{20}
$$

### *2.2. Ward identity*

In the abelian gauge theory at the quantum level the gauge invariance is encoded in the Ward identity [44], which is a particular case of the Slavnov–Taylor identities [45,46]. The harmonic superspace analog of this identity can be formulated, using the standard technique. For this purpose we make the transformation (10) in the generating functional (17) which evidently remains invariant. Taking into account that the classical action is gauge invariant, in the lowest order in $\lambda$ we obtain

$$
0 = \Big\langle \int d\zeta^{(-4)}\, du \Big[ -\frac{\delta S_{\text{gf}}}{\delta V^{++}} D^{++}\lambda - J^{(+2)} D^{++}\lambda + i j^{(+3)} \lambda q^+ - i \widetilde{j}^{(+3)} \lambda \widetilde{q}^{+} \Big] \Big\rangle,
\tag{21}
$$

where we used the notation

$$\Big\langle A(V^{++},q^+,\widetilde{q}^+)\Big\rangle = \frac{1}{Z}\int DV^{++}\,D\widetilde{q}^+\,Dq^+\,A(V^{++},q^+,\widetilde{q}^+)\exp\Big\{i(S+S_{\text{gf}}+S_{\text{sources}})\Big\}. \tag{22}$$

Integrating in Eq. (21) by parts with respect to the derivatives $D^{++}$, using an arbitrariness of $\lambda$, and expressing the result in terms of superfields, we obtain

$$0 = D^{++}\frac{\delta S_{\text{gf}}}{\delta V^{++}} - D^{++}\frac{\delta\Gamma}{\delta V^{++}} - iq^+\frac{\delta\Gamma}{\delta q^+} + i\widetilde{q}^+\frac{\delta\Gamma}{\delta\widetilde{q}^+}, \tag{23}$$

where $\Gamma$ is the effective action defined by Eq. (19), and we also took into account that the gauge-fixing term is quadratic in the gauge superfield. Introducing

$$\Delta\Gamma = \Gamma - S_{\text{gf}}, \tag{24}$$

the Ward identity can be written in a more compact form,

$$D^{++}\frac{\delta\Delta\Gamma}{\delta V^{++}} = -iq^+\frac{\delta\Delta\Gamma}{\delta q^+} + i\widetilde{q}^+\frac{\delta\Delta\Gamma}{\delta\widetilde{q}^+}. \tag{25}$$

It is important that this equation is valid for arbitrary non-zero values of the involved superfields. Differentiating Eq. (25) with respect to various superfields we derive an infinite set of identities relating the longitudinal part of the $(n+1)$-point Green functions to the $n$-point Green functions. For example, differentiating with respect to $V_2^{++}$ and setting all fields equal to zero at the end, we obtain that quantum corrections to the two-point Green function of the gauge superfield are transversal,

$$D_1^{++}\frac{\delta^2\Delta\Gamma}{\delta V_1^{++}\delta V_2^{++}} = 0. \tag{26}$$

Differentiating Eq. (25) with respect to $q_2^+$ and $\widetilde{q}_3^+$ and setting the fields equal to zero at the end give an analog of the usual Ward identity relating three- and two-point Green functions:

$$\begin{aligned} D_1^{++}\frac{\delta^3\Delta\Gamma}{\delta V_1^{++}\delta q_2^+\delta\widetilde{q}_3^+} &= -i(D_1^+)^4\delta^{14}(z_1-z_2)\delta^{(-3,3)}(u_1,u_2)\frac{\delta^2\Delta\Gamma}{\delta q_1^+\delta\widetilde{q}_3^+} \\ &\quad + i(D_1^+)^4\delta^{14}(z_1-z_3)\delta^{(-3,3)}(u_1,u_3)\frac{\delta^2\Delta\Gamma}{\delta q_2^+\delta\widetilde{q}_1^+}. \end{aligned} \tag{27}$$

When deriving this equation, we have taken into account the property implied by the Grassmann analyticity

$$\frac{\delta q_1^+}{\delta q_2^+} = (D_1^+)^4\delta^{14}(z_1-z_2)\delta^{(-3,3)}(u_1,u_2), \tag{28}$$

where

$$\delta^{14}(z_1-z_2) = \delta^6(x_1-x_2)\delta^8(\theta_1-\theta_2). \tag{29}$$

It is convenient to multiply the identity (27) with the analytic superfields $\lambda_1$, $q_2^+$, and $\widetilde{q}_3^+$, and integrate the expression obtained over both analytic arguments,

$$\int d\mu\, \widetilde{q}_3^+ D^{++}\lambda_1 q_2^+ \frac{\delta^3 \Delta\Gamma}{\delta V_1^{++}\delta q_2^+ \delta\widetilde{q}_3^+} = i\int d\zeta_1^{(-4)} du_1\, d\zeta_3^{(-4)}\, du_3\, \widetilde{q}_3^+ \lambda_1 q_1^+ \frac{\delta^2 \Delta\Gamma}{\delta q_1^+ \delta\widetilde{q}_3^+}$$
$$- i\int d\zeta_1^{(-4)} du_1\, d\zeta_2^{(-4)}\, du_2\, \widetilde{q}_1^+ \lambda_1 q_2^+ \frac{\delta^2 \Delta\Gamma}{\delta q_2^+ \delta\widetilde{q}_1^+}, \tag{30}$$

where

$$\int d\mu = \int d\zeta_1^{(-4)}\, du_1\, d\zeta_2^{(-4)}\, du_2\, d\zeta_3^{(-4)}\, du_3. \tag{31}$$

This form of the Ward identity is most convenient, when checking it for one or another particular class of diagrams.

### 2.3. The Feynman rules

For the explicit calculation of quantum correction it is necessary to formulate the relevant Feynman rules. This can be accomplished quite similarly to the $4D$, $\mathcal{N}=2$ case considered in detail in Refs. [13,14]. To find the propagator of the gauge superfield in the $\xi$-gauge, we consider the sum of the gauge superfield action and the gauge-fixing term

$$S_{\text{gauge}} + S_{\text{gf}} = \frac{1}{4f_0^2}\Big(1-\frac{1}{\xi_0}\Big)\int d^{14}z\, du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(z,u_1)V^{++}(z,u_2)$$
$$+ \frac{1}{4f_0^2\xi_0}\int d\zeta^{(-4)}\, du\, V^{++}(z,u)\partial^2 V^{++}(z,u), \tag{32}$$

where we made use of the identity

$$D_1^{++}\frac{1}{(u_1^+u_2^+)^3} = \frac{1}{2}(D_1^{--})^2\delta^{(3,-3)}(u_1,u_2) \tag{33}$$

and took into account that, when acting on the analytic superfields,

$$\frac{1}{2}(D^+)^4(D^{--})^2 \Rightarrow \partial^2. \tag{34}$$

Following Ref. [31], we consider the free theory and solve the equation of motion for the superfield $V^{++}$ in the presence of the source term,

$$\frac{1}{2\xi_0 f_0^2}\partial^2 V^{++}(z,u_1) + \frac{1}{2f_0^2}\Big(1-\frac{1}{\xi_0}\Big)$$
$$\times \int du_2 \frac{1}{(u_1^+u_2^+)^2}(D_1^+)^4 V^{++}(z,u_2) + J^{(+2)}(z,u_1) = 0. \tag{35}$$

The solution can be presented as

$$V^{++}(z,u_1) = -\frac{2\xi_0 f_0^2}{\partial^2} J^{(+2)}(z,u_1)$$
$$+ \frac{2f_0^2(\xi_0-1)}{\partial^4}\int du_2 \frac{1}{(u_1^+u_2^+)^2}(D_1^+)^4 J^{(+2)}(z,u_2), \tag{36}$$

whence one extracts the $\xi$-gauge form of the propagator of the gauge superfield


Fig. 1. The propagators of the gauge superfield $V^{++}$ and of the hypermultiplets.

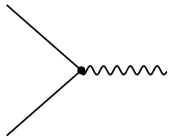

Fig. 2. The only vertex comes from the interaction of the hypermultiplet with the gauge superfield.

$$G_V^{(2,2)}(z_1,u_1;z_2,u_2) = -2f_0^2\Big(\frac{\xi_0}{\partial^2}(D_1^+)^4\delta^{(2,-2)}(u_2,u_1) - \frac{\xi_0-1}{\partial^4}(D_1^+)^4(D_2^+)^4\frac{1}{(u_1^+u_2^+)^2}\Big)\delta^{14}(z_1-z_2). \tag{37}$$

The second term vanishes in the Feynman gauge $\xi_0 = 1$. Such a choice considerably simplifies calculation of quantum corrections. However, the purpose of the present paper is to investigate the $\xi_0$-dependence of various Green functions for the generic choice of $\xi_0$.

In the left part of Fig. 1, the propagator (37) is depicted by the wavy line with the ends corresponding to the points 1 and 2.

For completeness, we also present the expression for the hypermultiplet propagator,

$$G_q^{(1,1)}(z_1,u_1;z_2,u_2) = (D_1^+)^4(D_2^+)^4\frac{1}{\partial^2}\delta^{14}(z_1-z_2)\frac{1}{(u_1^+u_2^+)^3}, \tag{38}$$

which is denoted by the solid line in the right part of Fig. 1.

The only vertex of the theory (8) is presented in Fig. 2 and stands for the interaction of the hypermultiplet with the gauge superfield

$$S_I = -i\int d\zeta^{(-4)}\,du\,\widetilde{q}^+V^{++}q^+. \tag{39}$$

The superficial degree of divergence in the theory under consideration has been calculated in Ref. [29]:

$$\omega = 2L - N_q - \frac{1}{2}N_D. \tag{40}$$

Here $L$ is a number of loops, $N_q$ is a number of external hypermultiplet legs, and $N_D$ is a number of spinor supersymmetric covariant derivatives acting on external legs. This formula implies that in the one-loop approximation only diagrams without external hypermultiplet legs or with two such legs can be divergent.

## 3. Gauge dependence of the one-loop divergences

### 3.1. Two-point function of the gauge superfield

In the one-loop approximation the two-point function of the gauge superfield $V^{++}$ is divergent. In the abelian case this divergence comes only from the diagram pictured in Fig. 3.

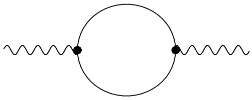

Fig. 3. The diagram representing the one-loop two-point Green function in the abelian case.

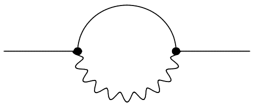

Fig. 4. The two-point Green function of the hypermultiplet in the one-loop approximation.

However, this diagram does not contain propagators of the gauge superfield and is therefore gauge-independent.

Thus, in the one-loop approximation this Green function in the $\xi$-gauge is the same as in the Feynman gauge. It is given by the expression [29]

$$\int \frac{d^6p}{(2\pi)^6} \int d^8\theta\, du_1\, du_2\, V^{++}(p,\theta,u_1) V^{++}(-p,\theta,u_2) \\ \times \frac{1}{(u_1^+ u_2^+)^2}\Big[\frac{1}{4f_0^2} - \frac{i}{2}\int \frac{d^6k}{(2\pi)^6}\frac{1}{k^2(k+p)^2}\Big]. \tag{41}$$

The corresponding divergent part of the effective action is gauge-independent and in the dimensional reduction scheme[1] can be written as

$$-\frac{1}{6\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\, du\, (F^{++})^2, \tag{42}$$

where $\varepsilon = 6 - D$.

### 3.2. Two-point hypermultiplet Green function

In the one-loop approximation the two-point Green function of the hypermultiplet is contributed to by the single logarithmically divergent diagram presented in Fig. 4.

In the Feynman gauge this superdiagram vanishes. However, it includes the propagator of the gauge superfield, for which reason we can expect that the result for it is in fact gauge-dependent. Using the Feynman rules defined above, the expression for this diagram in the generic $\xi$-gauge can be written as

$$-2if_0^2 \int d\zeta_1^{(-4)}\, du_1\, d\zeta_2^{(-4)}\, du_2\, \widetilde{q}^+(z_1,u_1) q^+(z_2,u_2) \frac{1}{(u_1^+u_2^+)^3}\frac{(D_1^+)^4(D_2^+)^4}{\partial^2} \\ \times \delta^{14}(z_1-z_2)\Big(\frac{\xi_0}{\partial^2}(D_1^+)^4\delta^{(2,-2)}(u_2,u_1) - \frac{\xi_0-1}{\partial^4}(D_1^+)^4(D_2^+)^4\frac{1}{(u_1^+u_2^+)^2}\Big)\delta^{14}(z_1-z_2). \tag{43}$$

[1] Here we use the regularization by dimensional reduction [47]. However, for calculating power divergences one should use another regularization, e.g., some modifications of the higher covariant derivative regularization [48,49]. At least for $4D$, $\mathcal{N}=2$ supersymmetric theories such a regularization can be formulated in the harmonic superspace [50].

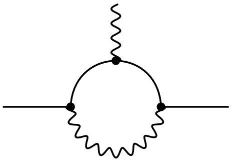

Fig. 5. The diagram representing the three-point gauge – hypermultiplet function in the one-loop approximation.

The derivatives $(D_1^+)^4(D_2^+)^4$ in the hypermultiplet propagator can be used to convert the integrations over $d\zeta^{(-4)}$ into those over $d^{14}z$,

$$-2if_0^2\int d^{14}z_1\,du_1\,d^{14}z_2\,du_2\,\widetilde{q}^+(z_1,u_1)q^+(z_2,u_2)\frac{1}{(u_1^+u_2^+)^3}\,\frac{1}{\partial^2}\delta^{14}(z_1-z_2)$$
$$\times\Big(\frac{\xi_0}{\partial^2}(D_1^+)^4\delta^{(2,-2)}(u_2,u_1)-\frac{\xi_0-1}{\partial^4}(D_1^+)^4(D_2^+)^4\frac{1}{(u_1^+u_2^+)^2}\Big)\delta^{14}(z_1-z_2). \tag{44}$$

Taking into account the identities

$$\delta^8(\theta_1-\theta_2)\,(D_1^+)^4\delta^8(\theta_1-\theta_2)=0, \tag{45}$$
$$\delta^8(\theta_1-\theta_2)\,(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)=(u_1^+u_2^+)^4\,\delta^8(\theta_1-\theta_2), \tag{46}$$

we find that the first term in this expression vanishes, reducing (44) to the form

$$2if_0^2\int d^6x_1\,d^6x_2\,d^8\theta\,du_1\,du_2\,\widetilde{q}^+(x_1,\theta,u_1)q^+(x_2,\theta,u_2)$$
$$\times\frac{(\xi_0-1)}{(u_1^+u_2^+)}\,\frac{1}{\partial^2}\delta^6(x_1-x_2)\,\frac{1}{\partial^4}\delta^6(x_1-x_2). \tag{47}$$

This expression can be rewritten in the momentum representation as

$$-2if_0^2\int\frac{d^6p}{(2\pi)^6}\,\frac{d^6k}{(2\pi)^6}\,\frac{1}{k^4(k+p)^2}\int d^8\theta\,du_1\,du_2\,\frac{(\xi_0-1)}{(u_1^+u_2^+)}\widetilde{q}^+(p,\theta,u_1)q^+(-p,\theta\,u_2). \tag{48}$$

We observe that this expression is logarithmically divergent and does not vanish, unless the Feynman gauge is chosen. If the theory is regularized by dimensional reduction, the corresponding contribution to the divergent part takes the form

$$-\frac{2f_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\,du_1\,du_2\,\frac{(\xi_0-1)}{(u_1^+u_2^+)}\widetilde{q}^+(z,u_1)q^+(z,u_2). \tag{49}$$

### 3.3. Three-point gauge-hypermultiplet Green function

According to Eq. (40), all diagrams containing two external hypermultiplet legs are logarithmically divergent, irrespective of the number of the external gauge legs. That is why in calculating the one-loop divergences it is necessary to take into account such Green functions. The simplest of them is the three-point gauge superfield – hypermultiplet Green function. In the one-loop approximation, it is contributed to by the single supergraph depicted in Fig. 5.

Calculating this diagram by Feynman rules in the general $\xi$-gauge, we obtain

$$-2f_0^2\int d\zeta_1^{(-4)}\,du_1\,d\zeta_2^{(-4)}\,du_2\,d\zeta_3^{(-4)}\,du_3\,\widetilde{q}^{+}(z_1,u_1)q^{+}(z_3,u_3)V^{++}(z_2,u_2)$$
$$\times\Big(\frac{\xi_0}{\partial^2}(D_1^+)^4\delta^{(2,-2)}(u_3,u_1)-\frac{(\xi_0-1)}{\partial^4}(D_1^+)^4(D_3^+)^4\frac{1}{(u_1^+u_3^+)^2}\Big)\delta^{14}(z_1-z_3)\,\frac{1}{(u_1^+u_2^+)^3}$$
$$\times\frac{(D_1^+)^4(D_2^+)^4}{\partial^2}\delta^{14}(z_1-z_2)\,\frac{1}{(u_2^+u_3^+)^3}\,\frac{(D_2^+)^4(D_3^+)^4}{\partial^2}\delta^{14}(z_2-z_3). \tag{50}$$

To work out this expression, we, first, convert the integrals over $d\zeta^{(-4)}$ in it into integrals over $d^{14}z$ using Eq. (6):

$$-2f_0^2\int d^{14}z_1\,d^{14}z_2\,d^{14}z_3\,du_1\,du_2\,du_3\,\widetilde{q}^{+}(z_1,u_1)q^{+}(z_3,u_3)V^{++}(z_2,u_2)\Big(\frac{\xi_0}{\partial^2}(D_1^+)^4$$
$$\times\,\delta^{(2,-2)}(u_3,u_1)-\frac{(\xi_0-1)}{\partial^4}(D_1^+)^4(D_3^+)^4\frac{1}{(u_1^+u_3^+)^2}\Big)\delta^{14}(z_1-z_3)\,\frac{1}{(u_1^+u_2^+)^3(u_2^+u_3^+)^3}$$
$$\times\frac{(D_2^+)^4}{\partial^2}\delta^{14}(z_1-z_2)\,\frac{1}{\partial^2}\delta^{14}(z_2-z_3). \tag{51}$$

Next, we integrate by parts with respect to $(D_2^+)^4$ (assuming that $D_2^+$ acts on $z_1$), taking into account that

$$\delta^8(\theta_1-\theta_2)\prod_{n=1}^{N}D^+_{i_na_n}\delta^8(\theta_1-\theta_2)=0\qquad\text{for arbitrary odd } N. \tag{52}$$

In the term containing the harmonic $\delta$-function we further integrate over $du_3$. Integrating also over $\theta_2$, we finally obtain for (50):

$$2f_0^2\int d^6x_1\,d^6x_2\,d^6x_3\,d^8\theta_1\,d^8\theta_3\,\delta^8(\theta_1-\theta_3)\Bigg\{\int du_1\,du_2\,\widetilde{q}^{+}(x_1,\theta_1,u_1)q^{+}(x_3,\theta_3,u_1)$$
$$\times\,V^{++}(x_2,\theta_1,u_2)\frac{\xi_0}{(u_1^+u_2^+)^6}\,\frac{(D_1^+)^4(D_2^+)^4}{\partial^2}\delta^{14}(z_1-z_3)\,\frac{1}{\partial^2}\delta^6(x_1-x_2)\,\frac{1}{\partial^2}\delta^6(x_2-x_3)$$
$$+\int du_1\,du_2\,du_3\,V^{++}(x_2,\theta_1,u_2)q^{+}(x_3,\theta_3,u_3)\frac{(\xi_0-1)}{(u_1^+u_3^+)^2(u_1^+u_2^+)^3(u_2^+u_3^+)^3}$$
$$\times\,\frac{1}{\partial^2}\delta^6(x_1-x_2)\frac{1}{\partial^2}\delta^6(x_2-x_3)\Bigg[(D_2^+)^4\widetilde{q}^{+}(x_1,\theta_1,u_1)\,\frac{(D_1^+)^4(D_3^+)^4}{\partial^4}\delta^{14}(z_1-z_3)$$
$$+\,\widetilde{q}^{+}(x_1,\theta_1,u_1)\frac{(D_2^+)^4(D_1^+)^4(D_3^+)^4}{\partial^4}\delta^{14}(z_1-z_3)$$
$$-\,\frac{1}{4}\varepsilon^{abcd}D^+_{2a}D^+_{2b}\,\widetilde{q}^{+}(x_1,\theta_1,u_1)\,\frac{D^+_{2c}D^+_{2d}(D_1^+)^4(D_3^+)^4}{\partial^4}\delta^{14}(z_1-z_3)\Bigg]\Bigg\}. \tag{53}$$

As the further step, we use the identities (45), (46) together with

$$\delta^8(\theta_1-\theta_2)\,D^+_{2a}D^+_{2b}(D_1^+)^4(D_3^+)^4\delta^8(\theta_1-\theta_2)$$
$$=-i(\gamma^M)_{ab}(u_2^+u_1^+)\,(u_2^+u_3^+)\,(u_1^+u_3^+)^3\,\delta^8(\theta_1-\theta_2)\partial_M; \tag{54}$$

$$\delta^8(\theta_1-\theta_2)\,(D_2^+)^4(D_1^+)^4(D_3^+)^4\delta^8(\theta_1-\theta_2)$$
$$=(u_1^+u_2^+)^2\,(u_1^+u_3^+)^2\,(u_2^+u_3^+)^2\,\delta^8(\theta_1-\theta_2)\partial^2 \tag{55}$$

in order to do the integrals over the Grassmann coordinate $\theta_2$. After renaming $\theta_1\to\theta$, the expression for the diagram in question in the momentum representation is written as

$$\begin{aligned}
&2f_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}d^8\theta\Bigg\{-\int du_1\,du_2\,\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\\
&\times q^+(-q,\theta,u_1)\frac{\xi_0}{k^2(q+k)^2(q+k+p)^2}\frac{1}{(u_1^+u_2^+)^2}\\
&+\int du_1\,du_2\,du_3\Bigg[(D_2^+)^4\,\widetilde{q}^+(q+p,\theta,u_1)\,V^{++}(-p,\theta,u_2)\\
&\times q^+(-q,\theta,u_3)\,\frac{(\xi_0-1)}{k^4(q+k)^2(q+k+p)^2}\,\frac{(u_1^+u_3^+)^2}{(u_1^+u_2^+)^3(u_2^+u_3^+)^3}\\
&-\widetilde{q}^+(q+p,\theta,u_1)\,V^{++}(-p,\theta,u_2)\,q^+(-q,\theta,u_3)\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\\
&\times\frac{1}{(u_1^+u_2^+)(u_2^+u_3^+)}-D_{2a}^+D_{2b}^+\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\,q^+(-q,\theta,u_3)\\
&\times\frac{(\xi_0-1)(\widetilde{\gamma}^M)^{ab}k_M}{2k^4(q+k)^2(q+k+p)^2}\,\frac{(u_1^+u_3^+)}{(u_1^+u_2^+)^2(u_2^+u_3^+)^2}\Bigg]\Bigg\},
\end{aligned}\tag{56}$$

where $(\widetilde{\gamma}^M)^{ab}=\varepsilon^{abcd}(\gamma^M)_{cd}/2$. The divergent part of this expression can now be found after the Wick rotation. There remains only one divergent integral

$$\int\frac{d^6k}{(2\pi)^6}\frac{1}{k^2(k+q)^2(k+q+p)^2}, \tag{57}$$

which, after regularizing it by dimensional reduction, is reduced to

$$-i\int\frac{d^DK}{(2\pi)^6}\frac{1}{K^2(K+Q)^2(K+Q+P)^2}=-\frac{i}{\varepsilon(4\pi)^3}+\text{finite terms}, \tag{58}$$

where the capital letters denote Euclidean momenta. Thus, the divergent part of the diagram in Fig. 5 can be presented as

$$\begin{aligned}
&\frac{2if_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\Bigg\{\int du_1\,du_2\,\widetilde{q}_1^+V_2^{++}q_1^+\frac{\xi_0}{(u_1^+u_2^+)^2}\\
&+\int du_1\,du_2\,du_3\,\widetilde{q}_1^+\,V_2^{++}q_3^+\frac{(\xi_0-1)}{(u_1^+u_2^+)(u_2^+u_3^+)}\Bigg\},
\end{aligned}\tag{59}$$

where the subscripts on the superfields refer to the relevant harmonic arguments.

## 4. Verification of the Ward identities

To be convinced of the correctness of the results obtained in the previous sections, let us check that the two- and three-point Green functions derived above satisfy the Ward identities.

First, for completeness, we verify the Ward identity (26). The two-point Green function of the gauge superfield is obtained by differentiating Eq. (41) with respect to $V^{++}$, using Eq. (28). This gives

$$\frac{\delta^2 \Delta\Gamma}{\delta V_1^{++} \delta V_2^{++}} = G_V(i\partial_M) \frac{1}{(u_1^+ u_2^+)^2} (D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2), \tag{60}$$

where

$$G_V(p_M) = \frac{1}{2 f_0^2} - i \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2} + \dots \tag{61}$$

Therefore,

$$\begin{aligned} D_1^{++} \frac{\delta^2 \Delta\Gamma}{\delta V_1^{++} \delta V_2^{++}} &= G_V(i\partial_M) D_1^{--} \delta^{(2,-2)}(u_1, u_2) \cdot (D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2) \\ &= G_V(i\partial_M) \Big[ D_1^{--} \Big( \delta^{(2,-2)}(u_1, u_2) (D_1^+)^4 (D_2^+)^4 \Big) \\ &- \delta^{(2,-2)}(u_1, u_2) \Big( D_1^{--} (D_1^+)^4 \Big) (D_2^+)^4 \Big] \delta^{14}(z_1 - z_2) = 0. \end{aligned} \tag{62}$$

Thus, we have verified that the Ward identity (26) is indeed satisfied.

The two-point Green function of the hypermultiplet is obtained by differentiating Eq. (48) with respect to $q^+$ and $\widetilde{q}^+$. These derivatives are calculated with the help of Eq. (28). We obtain

$$\frac{\delta^2 \Gamma}{\delta q_2^+ \delta \widetilde{q}_1^+} = G_q(i\partial_M) \frac{1}{(u_1^+ u_2^+)} (D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2), \tag{63}$$

where

$$G_q(p_M) = -2i f_0^2 \int \frac{d^6 k}{(2\pi)^6} \frac{(\xi_0 - 1)}{k^4 (k+p)^2} + \dots \tag{64}$$

The three-point gauge superfield – hypermultiplet Green function can be constructed quite similarly, starting from Eq. (56), but we prefer not to present the expression for it explicitly. Instead, we will check for it the Ward identity in the form (30). From Eq. (56) we obtain

$$\begin{aligned} &\int d\zeta_1^{(-4)} du_1 \, d\zeta_2^{(-4)} du_2 \, d\zeta_3^{(-4)} du_3 \, \widetilde{q}_3^+ D^{++} \lambda_1 q_2^+ \frac{\delta^3 \Delta\Gamma}{\delta V_1^{++} \delta q_2^+ \delta \widetilde{q}_3^+} \\ &= 2 f_0^2 \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8\theta \Bigg\{ - \int du_1 \, du_2 \, \widetilde{q}^+(q+p, \theta, u_2) D_1^{++} \lambda(-p, \theta, u_1) \\ &\times q^+(-q, \theta, u_2) \frac{\xi_0}{k^2 (q+k)^2 (q+k+p)^2} \frac{1}{(u_1^+ u_2^+)^2} \\ &+ \int du_1 \, du_2 \, du_3 \Bigg[ (D_1^+)^4 \widetilde{q}^+(q+p, \theta, u_3) \, D_1^{++} \lambda(-p, \theta, u_1) \\ &\times q^+(-q, \theta, u_2) \frac{(\xi_0 - 1)}{k^4 (q+k)^2 (q+k+p)^2} \frac{(u_3^+ u_2^+)^2}{(u_3^+ u_1^+)^3 (u_1^+ u_2^+)^3} \\ &- \widetilde{q}^+(q+p, \theta, u_3) \, D_1^{++} \lambda(-p, \theta, u_1) \end{aligned}$$

$$\times\, q^+(-q,\theta,u_2)\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\frac{1}{(u_3^+u_1^+)(u_1^+u_2^+)} - D^+_{1a}D^+_{1b}\widetilde{q}^+(q+p,\theta,u_3)$$
$$\times\, D_1^{++}\lambda(-p,\theta,u_1)\,q^+(-q,\theta,u_2)\,\frac{(\xi_0-1)(\widetilde{\gamma}^M)^{ab}k_M}{2k^4(q+k)^2(q+k+p)^2}\,\frac{(u_3^+u_2^+)}{(u_3^+u_1^+)^2(u_1^+u_2^+)^2}\Bigg]\Bigg\}. \tag{65}$$

Next, we integrate by parts with respect to the harmonic derivatives $D_1^{++}$, taking into account the identity

$$D_1^{++}\frac{1}{(u_1^+u_2^+)^n} = \frac{1}{(n-1)!}(D_1^{--})^{n-1}\delta^{(n,-n)}(u_1,u_2)$$
$$= \frac{(-1)^{n-1}}{(n-1)!}(D_2^{--})^{n-1}\delta^{(2-n,n-2)}(u_1,u_2). \tag{66}$$

After some algebra (described in Appendix A), this gives

$$\int d\mu\,\widetilde{q}_3^+D^{++}\lambda_1 q_2^+\,\frac{\delta^3\Delta\Gamma}{\delta V_1^{++}\delta q_2^+\delta\widetilde{q}_3^+}$$
$$= -2f_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{1}{k^4(k+q+p)^2}\int d^8\theta\,du_1\,du_3$$
$$\times\,\frac{(\xi_0-1)}{(u_1^+u_3^+)}\widetilde{q}^+(q+p,\theta,u_3)\lambda(-p,\theta,u_1)q^+(-q,\theta\,u_1)$$
$$-\,2f_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{1}{k^4(k+q)^2}$$
$$\times\int d^8\theta\,du_1\,du_2\,\frac{(\xi_0-1)}{(u_1^+u_2^+)}\widetilde{q}^+(q+p,\theta,u_1)\lambda(-p,\theta,u_1)q^+(-q,\theta\,u_2). \tag{67}$$

The right-hand side of this equation can be rewritten as

$$i\int d\zeta_1^{(-4)}du_1\,d\zeta_3^{(-4)}\,du_3\,\widetilde{q}_3^+\lambda_1 q_1^+\,\frac{\delta^2\Gamma}{\delta q_1^+\delta\widetilde{q}_3^+}$$
$$-\,i\int d\zeta_1^{(-4)}du_1\,d\zeta_2^{(-4)}\,du_2\,\widetilde{q}_1^+\lambda_1 q_2^+\,\frac{\delta^2\Gamma}{\delta q_2^+\delta\widetilde{q}_1^+}, \tag{68}$$

thus demonstrating that the Green functions (48) and (56) satisfy the Ward identity (30), as it should be. Obviously, they also satisfy the Ward identity in the original form (27). This completes checking the correctness of our calculation.

## 5. The vanishing of the gauge dependence on shell

According to the general theorem of Refs. [38–43], the gauge-dependent terms should disappear on shell. Let us verify that our results are in agreement with this statement.

It is convenient to represent the effective action in the form

$$\Gamma = \Gamma_{\xi_0=1} + \widetilde{\Gamma}, \tag{69}$$

where

$$\Gamma_{\xi_0=1} = S + S_{\rm gf} - \frac{i}{2}\int \frac{d^6p}{(2\pi)^6}\int d^8\theta\, du_1\, du_2\, V^{++}(p,\theta,u_1)V^{++}(-p,\theta,u_2)\frac{1}{(u_1^+u_2^+)^2}$$
$$\times \int \frac{d^6k}{(2\pi)^6}\frac{1}{k^2(k+p)^2} - \int \frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6} d^8\theta\, du_1\, du_2\, \widetilde{q}^+(q+p,\theta,u_1)$$
$$\times V^{++}(-p,\theta,u_2)q^+(-q,\theta,u_1)\frac{1}{(u_1^+u_2^+)^2}\int\frac{d^6k}{(2\pi)^6}\frac{2f_0^2}{k^2(q+k)^2(q+k+p)^2} + \dots \tag{70}$$

is the effective action in the Feynman gauge and

$$\widetilde{\Gamma} = -2if_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{1}{k^4(k+p)^2}$$
$$\times\int d^8\theta\, du_1\, du_2\, \frac{(\xi_0-1)}{(u_1^+u_2^+)}\widetilde{q}^+(p,\theta,u_1)\, q^+(-p,\theta\, u_2)$$
$$+ 2f_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}d^8\theta$$
$$\times\Bigg\{-\int du_1\, du_2\, \widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)q^+(-q,\theta,u_1)$$
$$\times\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\frac{1}{(u_1^+u_2^+)^2}$$
$$+\int du_1\, du_2\, du_3\Bigg[(D_2^+)^4\widetilde{q}^+(q+p,\theta,u_1)\, V^{++}(-p,\theta,u_2)$$
$$\times q^+(-q,\theta,u_3)\frac{(\xi_0-1)}{k^4(q+k)^2(q+k+p)^2}\frac{(u_1^+u_3^+)^2}{(u_1^+u_2^+)^3(u_2^+u_3^+)^3}$$
$$-\widetilde{q}^+(q+p,\theta,u_1)\, V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)$$
$$\times\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\frac{1}{(u_1^+u_2^+)(u_2^+u_3^+)} - D^+_{2a}D^+_{2b}\widetilde{q}^+(q+p,\theta,u_1)$$
$$\times V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)\frac{(\xi_0-1)(\widetilde{\gamma}^M)^{ab}k_M}{2k^4(q+k)^2(q+k+p)^2}\frac{(u_1^+u_3^+)}{(u_1^+u_2^+)^2(u_2^+u_3^+)^2}\Bigg]\Bigg\} + \dots \tag{71}$$

stands for the gauge-dependent remainder of the effective action.

The purpose of this section is to demonstrate, by an explicit calculation, that in the approximation considered, $\widetilde{\Gamma}$ indeed vanishes on shell. To this end, we use the equations of motion for the hypermultiplets following from the action (8),

$$0 = \nabla^{++}q^+ = D^{++}q^+ + iV^{++}q^+; \qquad 0 = \nabla^{++}\widetilde{q}^+ = D^{++}\widetilde{q}^+ - iV^{++}\widetilde{q}^+. \tag{72}$$

In Appendix B (after some lengthy calculations) we demonstrate that, with these equations taken into account, the gauge-dependent part of the one-loop effective action can be cast in the form

$$\widetilde{\Gamma} = 2f_0^2\int\frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}d^4\theta^+\, du\, \widetilde{q}^+(q+p,\theta,u)V^{++}(-p,\theta,u)q^+(-q,\theta,u)$$
$$\times\Big((q+p)^2+q^2\Big)\int\frac{d^6k}{(2\pi)^6}\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2} + O\Big((V^{++})^2\Big). \tag{73}$$

On shell, where $q^2 = 0$ and $(q + p)^2 = 0$,[2] this expression vanishes. Thereby we have proved that the gauge dependence is vanishing on shell.

Note that, while deriving this result, we ignored all terms proportional to $(V^{++})^k$ for $k \geq 2$, because in this paper we limit our attention only to the diagrams without external gauge superfield legs at all, and to those having a single gauge superfield leg. In this approximation, terms of higher orders in $V^{++}$ are irrelevant.

## 6. The total divergent part of the one-loop effective action

So far we investigated gauge dependence of the two- and three-point Green functions only. In particular, we demonstrated that the corresponding one-loop divergences are gauge-dependent. However, according to Eq. (40), the Green functions with an arbitrary number of external gauge legs (and two external hypermultiplet legs) are also divergent. Nevertheless, the total divergent part of the one-loop effective action can be found using the reasoning based on the gauge invariance. Actually, the one-loop divergences corresponding to the two- and three-point Green functions (see Eqs. (42), (49), and (59)) have the form

$$\Gamma^{(1)}_\infty = -\frac{1}{6\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\,du\,(F^{++})^2 - \frac{2f_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\,du_1\,du_2\,\frac{(\xi_0-1)}{(u_1^+u_2^+)}\widetilde{q}_1^{\,+}q_2^+$$
$$+\frac{2if_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\bigg\{\int du_1\,du_2\,\widetilde{q}_1^{\,+}V_2^{++}q_1^+\frac{\xi_0}{(u_1^+u_2^+)^2}$$
$$+\int du_1\,du_2\,du_3\,\widetilde{q}_1^{\,+}\,V_2^{++}q_3^+\frac{(\xi_0-1)}{(u_1^+u_2^+)(u_2^+u_3^+)}\bigg\} + O\Big(\widetilde{q}^{\,+}(V^{++})^2q^+\Big). \tag{74}$$

The first term in this equation is gauge invariant. The expression corresponding to the first term in the curly brackets can also be rewritten in the explicitly gauge invariant form,

$$\frac{2if_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\,du_1\,du_2\,\widetilde{q}_1^{\,+}V_2^{++}q_1^+\frac{\xi_0}{(u_1^+u_2^+)^2} = \xi_0\,\frac{2if_0^2}{\varepsilon(4\pi)^3}\int d^{14}z\,du\,\widetilde{q}^{\,+}V^{--}q^+$$
$$=\xi_0\,\frac{2if_0^2}{\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\,du\,\widetilde{q}^{\,+}F^{++}q^+. \tag{75}$$

According to Eq. (15), the remaining two terms in Eq. (74) are the lowest terms in the series expansion of the gauge invariant expression

$$-\frac{2f_0^2(\xi_0-1)}{\varepsilon(4\pi)^3}\int d^{14}z\,du\,\widetilde{q}^{\,+}q^- \tag{76}$$

in powers of $V^{++}$. Thus, the divergent part of the one-loop effective action can be written as

$$\Gamma^{(1)}_\infty = -\frac{1}{6\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\,du\,(F^{++})^2 + \frac{2if_0^2\xi_0}{\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\,du\,\widetilde{q}^{\,+}F^{++}q^+$$
$$-\frac{2f_0^2(\xi_0-1)}{\varepsilon(4\pi)^3}\int d^{14}z\,du\,\widetilde{q}^{\,+}q^-. \tag{77}$$

[2] These equations can be derived directly from the hypermultiplet free equation of motion, see Ref. [34] for details.

Note that this expression does not include $O\Big(\widetilde{q}^{+}(V^{++})^2q^{+}\Big)$, because for obtaining the gauge invariant expression such terms should contain $F^{++}$ in which the number $N_D = 4$ of spinor derivatives acts on $V^{--}$. However, according to Eq. (40) these terms are finite and do not contribute to the divergent part of the one-loop effective action. Therefore, Eq. (77) provides the exact result for the divergent part of the effective action of the theory in question.

Note that on shell the gauge dependence of Eq. (77) vanishes. Actually, on shell, as the consequence of the equation of motion $\nabla^{++}q^{+} = 0$, we have the chain of relations

$$(\nabla^{++})^2q^{-} = 0 \Rightarrow (\nabla^{++})^2\nabla^{--}q^{-} = 0 \Rightarrow \nabla^{++}\nabla^{--}q^{-} = 0 \Rightarrow \nabla^{--}q^{-} = 0. \tag{78}$$

Acting on the latter equation by $\nabla^{++}$ it is easy to find

$$q^{-} = \nabla^{--}q^{+}. \tag{79}$$

In deriving these relations, we made use of the well known properties $D^{++}\omega^{-n} = 0 \to \omega^{-n} = 0$, $D^{--}\omega^{+m} = 0 \to \omega^{+m} = 0$ for $n \geq 1, m \geq 1$.

As a consequence of (79), we obtain that on shell

$$\begin{aligned}\int d^{14}z\, du\, \widetilde{q}^{+}\, q^{-} &= \int d\zeta^{(-4)}\, du\, (D^{+})^4\Big(\widetilde{q}^{+}\, \nabla^{--}q^{+}\Big)\\ &= \int d\zeta^{(-4)}\, du\, \widetilde{q}^{+}\, (D^{+})^4\Big((D^{--} + iV^{--})q^{+}\Big) = i\int d\zeta^{(-4)}\, du\, \widetilde{q}^{+}\, F^{++}q^{+}.\end{aligned} \tag{80}$$

Thus, on shell, the one-loop divergence (77) takes the form

$$\Gamma^{(1)}_{\infty} = -\frac{1}{6\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\, du\, (F^{++})^2 + \frac{2if_0^2}{\varepsilon(4\pi)^3}\int d\zeta^{(-4)}\, du\, \widetilde{q}^{+}F^{++}q^{+}. \tag{81}$$

We see that this expression does not depend on the parameter $\xi$ and, hence, on the gauge choice.

## 7. Summary

In this paper, using the $6D, \mathcal{N} = (1,0)$ harmonic superspace formalism, we studied the gauge dependence of the one-loop effective action for the $\mathcal{N} = (1,0)$ supersymmetric quantum electrodynamics. As compared to the case of the Feynman gauge, in the general $\xi$-gauge some new divergences appear. In particular, we demonstrated that in the general case the hypermultiplet Green function is divergent already in the one-loop approximation, as opposed to the case of the Feynman gauge, in which this divergence vanishes. Moreover, we calculated the three-point gauge – hypermultiplet Green function in the general $\xi$-gauge. To check the correctness of the calculation, we have verified the relevant Ward identity. Also it was checked that the gauge dependence vanishes on shell. Taking into account the gauge invariance, we also restored the divergent part of the one-loop effective action with terms of higher orders in the gauge superfield $V^{++}$. It is given by Eq. (77) and contains a new term which is absent in the Feynman gauge. We demonstrated that the gauge dependence of this general expression also vanishes on shell.

It would be interesting to investigate the gauge dependence in the non-abelian case. In particular, from the results of this paper we can expect that in the general $\xi$-gauge the $6D, \mathcal{N} = (1,1)$ sypersymmetric Yang–Mills theory is not finite even in the one-loop approximation, while the divergent terms are vanishing on shell.

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## Appendix A. Ward identity in harmonic superspace

Let us show how to pass from Eq. (65) to its equivalent form (67). After integrating by parts with respect to the derivatives $D_1^{++}$ and using the identity (66), we obtain

$$
\begin{aligned}
&\int d\mu\, \widetilde{q}_3^+ D^{++}\lambda_1 q_2^+ \frac{\delta^3 \Delta\Gamma}{\delta V_1^{++}\delta q_2^+ \delta\widetilde{q}_3^+} \\
&= 2 f_0^2 \int \frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6} d^8\theta \Bigg\{ -\int du_1\, du_2\, \widetilde{q}^+(q+p,\theta,u_2)\lambda(-p,\theta,u_1) \\
&\times q^+(-q,\theta,u_2)\frac{\xi_0}{k^2(q+k)^2(q+k+p)^2} D_2^{--}\delta^{(0,0)}(u_1,u_2) \\
&+ \int du_1\, du_2\, du_3 \Bigg[ (D_1^+)^4\, \widetilde{q}^+(q+p,\theta,u_3)\lambda(-p,\theta,u_1) q^+(-q,\theta,u_2) \\
&\times \frac{(\xi_0-1)(u_3^+u_2^+)^2}{k^4(q+k)^2(q+k+p)^2}\left(\frac{1}{2(u_1^+u_2^+)^3}(D_3^{--})^2\delta^{(-1,1)}(u_1,u_3)\right. \\
&\left. + \frac{1}{2(u_1^+u_3^+)^3}(D_2^{--})^2\delta^{(-1,1)}(u_1,u_2)\right) - \widetilde{q}^+(q+p,\theta,u_3)\,\lambda(-p,\theta,u_1)q^+(-q,\theta,u_2) \\
&\times \frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\left(\frac{1}{(u_1^+u_3^+)}\delta^{(1,-1)}(u_1,u_2) + \delta^{(1,-1)}(u_1,u_3)\frac{1}{(u_1^+u_2^+)}\right) \\
&- D_{1a}^+ D_{1b}^+\, \widetilde{q}^+(q+p,\theta,u_3)\lambda(-p,\theta,u_1)\, q^+(-q,\theta,u_2) \\
&\times \frac{(\xi_0-1)(\widetilde{\gamma}^M)^{ab}k_M}{2k^4(q+k)^2(q+k+p)^2}\,(u_3^+u_2^+) \\
&\times \left(\frac{1}{(u_1^+u_2^+)^2} D_3^{--}\delta^{(0,0)}(u_1,u_3) + \frac{1}{(u_1^+u_3^+)^2} D_2^{--}\delta^{(0,0)}(u_1,u_2)\right)\Bigg]\Bigg\}. \qquad (82)
\end{aligned}
$$

Then we integrate by parts with respect to the derivatives $D^{--}$ and take off one harmonic integral with the help of the delta functions. Taking into account that the first term vanishes as a consequence of the analyticity of the superfields $\lambda$, $\widetilde{q}^+$, and $q^+$, the expression (82) can be further rewritten as

$$
\begin{aligned}
&2 f_0^2 \int \frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{(\xi_0-1)}{k^4(q+k)^2(q+k+p)^2}\int d^8\theta\, du_1\, \lambda(-p,\theta,u_1) \\
&\times \Bigg\{ \int du_2 \frac{1}{(u_1^+u_2^+)} q^+(-q,\theta,u_2) \Bigg[ \frac{1}{2}(D_1^+)^4\,(D_1^{--})^2 \widetilde{q}^+(q+p,\theta,u_1)
\end{aligned}
$$

$$
\begin{aligned}
&- k^2\widetilde{q}^{+}(q+p,\theta,u_1) + \frac{1}{2}(\widetilde{\gamma}^M)^{ab}k_M\, D^+_{1a}D^+_{1b}D^{--}_1\, \widetilde{q}^{+}(q+p,\theta,u_1)\Bigg] \\
&+ \int du_3\, \frac{1}{(u^+_1u^+_3)}\Bigg[\frac{1}{2}(D^+_1)^4\,\widetilde{q}^{+}(q+p,\theta,u_3)\,(D^{--}_1)^2 q^+(-q,\theta,u_1) \\
&- k^2\widetilde{q}^{+}(q+p,\theta,u_3)\,q^+(-q,\theta_1,u_1) \\
&- \frac{1}{2}(\widetilde{\gamma}^M)^{ab}k_M\, D^+_{1a}D^+_{1b}\,\widetilde{q}^{+}(q+p,\theta,u_3)\, D^{--}_1 q^+(-q,\theta,u_1)\Bigg]\Bigg\}. \qquad (83)
\end{aligned}
$$

Once again, integrating by parts and taking into account that

$$
\frac{1}{2}(D^+)^4(D^{--})^2 = \partial^2; \qquad (\widetilde{\gamma}^M)^{ab}D^+_{1a}D^+_{1b}D^{--}_1 = -4i\,\partial^M \qquad (84)
$$

on the analytic superfields, this expression can be cast in the form

$$
\begin{aligned}
&-2f_0^2\int \frac{d^6p}{(2\pi)^6}\,\frac{d^6q}{(2\pi)^6}\,\frac{d^6k}{(2\pi)^6}\,\frac{(\xi_0-1)}{k^4(k+q)^2} \\
&\times \int d^8\theta\, du_1\, du_2\, \frac{1}{(u^+_1u^+_2)}\widetilde{q}^{+}(q+p,\theta,u_1)\lambda(-p,\theta,u_1)q^+(-q,\theta\, u_2) \\
&- 2f_0^2\int \frac{d^6p}{(2\pi)^6}\,\frac{d^6q}{(2\pi)^6}\,\frac{d^6k}{(2\pi)^6}\,\frac{(\xi_0-1)}{k^4(k+q+p)^2}\int d^8\theta\, du_1\, du_3\, \frac{1}{(u^+_1u^+_3)}\widetilde{q}^{+}(q+p,\theta,u_3) \\
&\times \lambda(-p,\theta,u_1)q^+(-q,\theta\, u_1), \qquad (85)
\end{aligned}
$$

where we have also used the relations

$$
(q+p)^2 + k^2 + 2k_M(q+p)^M = (q+k+p)^2, \qquad q^2 + k^2 + 2k_M q^M = (q+k)^2. \quad (86)
$$

## Appendix B. Gauge-dependent part of the effective action and the hypermultiplet equations of motion

In this appendix we verify that the gauge-dependent part of the effective action vanishes on shell. This is an important non-trivial check of the correctness of our calculations.

First, we consider the two-point Green function of the hypermultiplet given by Eq. (48). Using the identity

$$
\begin{aligned}
\frac{1}{(u^+_1u^+_2)} &= D^{++}_1\frac{(u^-_1u^+_2)}{(u^+_1u^+_2)^2} + D^{--}_1\delta^{(1,-1)}(u_1,u_2) \\
&= D^{++}_1D^{++}_2\frac{(u^-_1u^-_2)}{(u^+_1u^+_2)^2} + D^{--}_1\delta^{(1,-1)}(u_1,u_2), \qquad (87)
\end{aligned}
$$

we rewrite it as

$$
\begin{aligned}
\widetilde{\Gamma}^{(2)} &= -2if_0^2\int \frac{d^6p}{(2\pi)^6}\,\frac{d^6k}{(2\pi)^6}\,\frac{(\xi_0-1)}{k^4(k+p)^2}\int d^8\theta\, du_1\, du_2\Big(D^{++}_1D^{++}_2\frac{(u^-_1u^-_2)}{(u^+_1u^+_2)^2} \\
&+ D^{--}_1\delta^{(1,-1)}(u_1,u_2)\Big)\widetilde{q}^{+}(p,\theta,u_1)q^+(-p,\theta\, u_2). \qquad (88)
\end{aligned}
$$

The second term in this expression vanishes due to the analyticity of the hypermultiplet superfield,

$$\int d^8\theta\, du D^{--}\widetilde{q}^{+}(p,\theta,u)q^{+}(-p,\theta,u)$$
$$= \int d^4\theta^+\, du\, (D^+)^4\Big(D^{--}\widetilde{q}^{+}(p,\theta,u)q^{+}(-p,\theta,u)\Big) = 0. \quad (89)$$

After integrating by parts with respect to the harmonic derivatives, the considered contribution to the effective action can be represented as

$$-2if_0^2 \int \frac{d^6p}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{(\xi_0-1)}{k^4(k+p)^2}$$
$$\times \int d^8\theta\, du_1\, du_2\, \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^2} D^{++}\widetilde{q}^{+}(p,\theta,u_1)\, D^{++}q^{+}(-p,\theta\, u_2). \quad (90)$$

Using the equations of motion for the hypermultiplets

$$0 = \nabla^{++}q^+ = \big(D^{++} + iV^{++}\big)q^+; \qquad 0 = \nabla^{++}\widetilde{q}^{+} = \big(D^{++} - iV^{++}\big)\widetilde{q}^{+}, \quad (91)$$

we see that on shell the expression (90) is proportional to $\widetilde{q}^{+}(V^{++})^2q^+$. However, in this paper we do not consider terms quadratic in the gauge superfield $V^{++}$. This implies that, within the accuracy of our approximation, the part of the one-loop effective action corresponding to the hypermultiplet two-point function vanishes on shell.

Next, we consider the gauge dependent part of the three-point gauge superfield – hypermultiplet Green function. It corresponds to the terms proportional to $\widetilde{q}^{+}V^{++}q^+$ in the expression (71). We will demonstrate that $\widetilde{\Gamma}^{(3)}$ vanishes on shell (in the approximation when all terms with more than one $V^{++}$ are omitted).

Using the identity

$$\frac{1}{(u_1^+u_2^+)^2} = D_2^{++}\frac{(u_2^-u_1^+)}{(u_2^+u_1^+)^3} + \frac{1}{2}(D_2^{--})^2\delta^{(2,-2)}(u_2,u_1) \quad (92)$$

and discarding terms quadratic in $V^{++}$ (coming from $D^{++}q^+$ and $D^{++}\widetilde{q}^{+}$ after using the equations of motion), we obtain

$$\int d^8\theta\, du_1\, du_2\, \widetilde{q}^{+}(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)q^+(-q,\theta,u_1)\frac{1}{(u_1^+u_2^+)^2}$$
$$\longrightarrow \frac{1}{2}\int d^8\theta\, du\, \widetilde{q}^{+}(q+p,\theta,u)(D^{--})^2V^{++}(-p,\theta,u)q^+(-q,\theta,u)$$
$$= \frac{1}{2}\int d^4\theta^+\, du\, \widetilde{q}^{+}(q+p,\theta,u)(D^+)^4(D^{--})^2V^{++}(-p,\theta,u)q^+(-q,\theta,u)$$
$$= -p^2\int d^4\theta^+\, du\, \widetilde{q}^{+}(q+p,\theta,u)V^{++}(-p,\theta,u)q^+(-q,\theta,u), \quad (93)$$

where the arrow indicates that we omitted some terms vanishing on shell, as well as $O((V^{++})^2)$ terms.

Using Eq. (87) twice, we have

$$\int d^8\theta\, du_1\, du_2\, du_3\, \widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)\frac{1}{(u_1^+u_2^+)(u_2^+u_3^+)}$$
$$\longrightarrow -\int d^8\theta\, du\, D^{--}\widetilde{q}^+(q+p,\theta,u)V^{++}(-p,\theta,u)D^{--}q^+(-q,\theta,u)$$
$$= -2q^M(q+p)_M\int d^4\theta^+\, du\, \widetilde{q}^+(q+p,\theta,u)V^{++}(-p,\theta,u)q^+(-q,\theta,u). \tag{94}$$

The remaining terms vanish. Indeed, let us consider the expression

$$\int du_1\, du_2\, du_3\, D^+_{2a}D^+_{2b}\,\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)$$
$$\times \frac{(u_1^+u_3^+)}{(u_1^+u_2^+)^2(u_2^+u_3^+)^2} \tag{95}$$

and make use of the relation $(u_1^+u_3^+) = D_1^{++}D_3^{++}(u_1^-u_3^-)$. Then, after integrating by parts with respect to the harmonic derivatives $D_1^{++}$ and $D_3^{++}$, up to the terms quadratic in $V^{++}$, we observe that on shell the resulting expression is proportional to $(u_1^-u_1^-) = 0$,

$$(92) \longrightarrow \int du_1\, du_2\, du_3\, D^+_{2a}D^+_{2b}\,\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)$$
$$\times\, q^+(-q,\theta,u_3)\,(u_1^-u_3^-)D_1^{--}\delta^{(2,-2)}(u_1,u_2)D_3^{--}\delta^{(2,-2)}(u_3,u_2) = 0. \tag{96}$$

Similarly, using the identity $(u_1^+u_3^+)^2 = D_1^{++}D_3^{++}\Big((u_1^-u_3^-)(u_1^+u_3^+)\Big)$, we obtain

$$\int du_1\, du_2\, du_3\, (D_2^+)^4\,\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)\,\frac{(u_1^+u_3^+)^2}{(u_1^+u_2^+)^3(u_2^+u_3^+)^3}$$
$$\longrightarrow \frac{1}{4}\int du_1\, du_2\, du_3\, (D_2^+)^4\,\widetilde{q}^+(q+p,\theta,u_1)V^{++}(-p,\theta,u_2)\, q^+(-q,\theta,u_3)$$
$$\times\, (u_1^-u_3^-)(u_1^+u_3^+)(D_1^{--})^2\delta^{(2,-2)}(u_1,u_2)(D_3^{--})^2\delta^{(2,-2)}(u_3,u_2) = 0. \tag{97}$$

Finally, collecting all terms, we conclude that the exploiting of the hypermultiplet equations of motion allows us to rewrite the part of $\widetilde{\Gamma}$ corresponding to the three-point gauge superfield – hypermultiplet Green function in the form

$$\widetilde{\Gamma}^{(3)} = 2f_0^2\int \frac{d^6p}{(2\pi)^6}\frac{d^6q}{(2\pi)^6}\frac{d^6k}{(2\pi)^6}\frac{(\xi_0-1)}{k^2(q+k)^2(q+k+p)^2}\Big((q+p)^2+q^2\Big)$$
$$\times\int d^4\theta^+\, du\, \widetilde{q}^+(q+p,\theta,u)V^{++}(-p,\theta,u)q^+(-q,\theta,u). \tag{98}$$

For the on-shell hypermultiplets the relations $q^2 = 0$ and $(q+p)^2 = 0$ are valid, so this expression vanishes. The conclusion is that the gauge-dependent contributions to the effective action are indeed canceled on shell in the approximation we stick to.

## References

[1] P.S. Howe, K.S. Stelle, Ultraviolet divergences in higher dimensional supersymmetric Yang–Mills theories, Phys. Lett. B 137 (1984) 175–180, https://doi.org/10.1016/0370-2693(84)90225-9.
[2] P.S. Howe, K.S. Stelle, Supersymmetry counterterms revisited, Phys. Lett. B 554 (2003) 190–196, https://doi.org/10.1016/S0370-2693(02)03271-9, arXiv:hep-th/0211279.
[3] G. Bossard, P.S. Howe, K.S. Stelle, The ultra-violet question in maximally supersymmetric field theories, Gen. Relativ. Gravit. 41 (2009) 919–981, https://doi.org/10.1007/s10714-009-0775-0, arXiv:0901.4661 [hep-th].
[4] G. Bossard, P.S. Howe, K.S. Stelle, A note on the UV behaviour of maximally supersymmetric Yang–Mills theories, Phys. Lett. B 682 (2009) 137–142, https://doi.org/10.1016/j.physletb.2009.10.084, arXiv:0908.3883 [hep-th].
[5] E.S. Fradkin, A.A. Tseytlin, Quantum properties of higher dimensional and dimensionally reduced supersymmetric theories, Nucl. Phys. B 227 (1983) 252–290, https://doi.org/10.1016/0550-3213(83)90022-6.
[6] N. Marcus, A. Sagnotti, A test of finiteness predictions for supersymmetric theories, Phys. Lett. B 135 (1984) 85, https://doi.org/10.1016/0370-2693(84)90458-1.
[7] A. Smilga, Ultraviolet divergences in non-renormalizable supersymmetric theories, Phys. Part. Nucl. Lett. 14 (2) (2017) 245, https://doi.org/10.1134/S1547477117020315, arXiv:1603.06811 [hep-th].
[8] L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev, D.E. Vlasenko, Divergences in maximal supersymmetric Yang–Mills theories in diverse dimensions, J. High Energy Phys. 1511 (2015) 059, https://doi.org/10.1007/JHEP11(2015)059, arXiv:1508.05570 [hep-th].
[9] S.J. Gates, M.T. Grisaru, M. Roček, W. Siegel, Superspace or one thousand and one lessons in supersymmetry, Front. Phys. 58 (1983) 1, arXiv:hep-th/0108200.
[10] I.L. Buchbinder, S.M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity: Or a Walk Through Superspace, IOP, Bristol, UK, 1998, 656 pp.
[11] N. Marcus, A. Sagnotti, The ultraviolet behavior of $N = 4$ Yang–Mills and the power counting of extended superspace, Nucl. Phys. B 256 (1985) 77, https://doi.org/10.1016/0550-3213(85)90386-4.
[12] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, Unconstrained $N = 2$ matter, Yang–Mills and supergravity theories in harmonic superspace, Class. Quantum Gravity 1 (1984) 469–498, https://doi.org/10.1088/0264-9381/1/5/004; Corrigendum: Class. Quantum Gravity 2 (1985) 127.
[13] A. Galperin, E.A. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic supergraphs. Green functions, Class. Quantum Gravity 2 (1985) 601–616, https://doi.org/10.1088/0264-9381/2/5/004.
[14] A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic supergraphs. Feynman rules and examples, Class. Quantum Gravity 2 (1985) 617–630, https://doi.org/10.1088/0264-9381/2/5/005.
[15] A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky, E.S. Sokatchev, Harmonic Superspace, Univ. Pr., Cambridge, UK, 2001, 306 pp.
[16] E.I. Buchbinder, B.A. Ovrut, I.L. Buchbinder, E.A. Ivanov, S.M. Kuzenko, Low-energy effective action in $\mathcal{N} = 2$ supersymmetric field theories, Phys. Part. Nucl. 32 (2001) 641–674, Fiz. Elem. Chast. Atom. Yadra 32 (2001) 1222–1264.
[17] I.L. Buchbinder, E.A. Ivanov, N.G. Pletnev, Superfield approach to the construction of effective action in quantum field theory with extended supersymmetry, Phys. Part. Nucl. 47 (3) (2016) 291–369, https://doi.org/10.1134/S1063779616030035, Fiz. Elem. Chast. Atom. Yadra 47 (3) (2016).
[18] P.S. Howe, G. Sierra, P.K. Townsend, Supersymmetry in six-dimensions, Nucl. Phys. B 221 (1983) 331–348, https://doi.org/10.1016/0550-3213(83)90582-5.
[19] P.S. Howe, K.S. Stelle, P.C. West, $N = 1$, $d = 6$ harmonic superspace, Class. Quantum Gravity 2 (1985) 815–821, https://doi.org/10.1088/0264-9381/2/6/008.
[20] B.M. Zupnik, Six-dimensional supergauge theories in the harmonic superspace, Sov. J. Nucl. Phys. 44 (1986) 512, Yad. Fiz. 44 (1986) 794–802.
[21] E.A. Ivanov, A.V. Smilga, B.M. Zupnik, Renormalizable supersymmetric gauge theory in six dimensions, Nucl. Phys. B 726 (2005) 131–148, https://doi.org/10.1016/j.nuclphysb.2005.08.014, arXiv:hep-th/0505082.
[22] E.A. Ivanov, A.V. Smilga, Conformal properties of hypermultiplet actions in six dimensions, Phys. Lett. B 637 (2006) 374–381, https://doi.org/10.1016/j.physletb.2006.05.003, arXiv:hep-th/0510273.
[23] I.L. Buchbinder, N.G. Pletnev, Construction of $6D$ supersymmetric field models in $\mathcal{N} = (1,0)$ harmonic superspace, Nucl. Phys. B 892 (2015) 21–48, https://doi.org/10.1016/j.nuclphysb.2015.01.002, arXiv:1411.1848 [hep-th].
[24] I.L. Buchbinder, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, The background field method for $N = 2$ superYang–Mills theories in harmonic superspace, Phys. Lett. B 417 (1998) 61–71, https://doi.org/10.1016/S0370-2693(97)01319-1, arXiv:hep-th/9704214.

[25] P.K. Townsend, G. Sierra, Chiral anomalies and constraints on the gauge group in higher dimensional supersymmetric Yang–Mills theories, Nucl. Phys. B 222 (1983) 493–506, https://doi.org/10.1016/0550-3213(83)90546-1.
[26] A.V. Smilga, Chiral anomalies in higher-derivative supersymmetric 6D theories, Phys. Lett. B 647 (2007) 298–304, https://doi.org/10.1016/j.physletb.2007.02.002, arXiv:hep-th/0606139.
[27] S.M. Kuzenko, J. Novak, I.B. Samsonov, The anomalous current multiplet in 6D minimal supersymmetry, J. High Energy Phys. 1602 (2016) 132, https://doi.org/10.1007/JHEP02(2016)132, arXiv:1511.06582 [hep-th].
[28] S.M. Kuzenko, J. Novak, I.B. Samsonov, Chiral anomalies in six dimensions from harmonic superspace, J. High Energy Phys. 1711 (2017) 145, https://doi.org/10.1007/JHEP11(2017)145, arXiv:1708.08238 [hep-th].
[29] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin, K.V. Stepanyantz, One-loop divergences in the $6D$, $\mathcal{N} = (1,0)$ abelian gauge theory, Phys. Lett. B 763 (2016) 375, https://doi.org/10.1016/j.physletb.2016.10.060, arXiv:1609.00975 [hep-th].
[30] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin, K.V. Stepanyantz, One-loop divergences in $6D$, $\mathcal{N} = (1,0)$ SYM theory, J. High Energy Phys. 1701 (2017) 128, https://doi.org/10.1007/JHEP01(2017)128, arXiv:1612.03190 [hep-th].
[31] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin, K.V. Stepanyantz, Supergraph analysis of the one-loop divergences in $6D$, $\mathcal{N} = (1,0)$ and $\mathcal{N} = (1,1)$ gauge theories, Nucl. Phys. B 921 (2017) 127, https://doi.org/10.1016/j.nuclphysb.2017.05.010, arXiv:1704.02530 [hep-th].
[32] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin, K.V. Stepanyantz, On the two-loop divergences of the 2-point hypermultiplet supergraphs for $6D$, $\mathcal{N} = (1,1)$ SYM theory, Phys. Lett. B 778 (2018) 252, https://doi.org/10.1016/j.physletb.2018.01.040, arXiv:1711.11514 [hep-th].
[33] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin, Leading low-energy effective action in $6D$, $\mathcal{N} = (1,1)$ SYM theory, arXiv:1711.03302 [hep-th].
[34] G. Bossard, E. Ivanov, A. Smilga, Ultraviolet behavior of $6D$ supersymmetric Yang–Mills theories and harmonic superspace, J. High Energy Phys. 1512 (2015) 085, https://doi.org/10.1007/JHEP12(2015)085, arXiv:1509.08027 [hep-th].
[35] S.S. Aleshin, A.E. Kazantsev, M.B. Skoptsov, K.V. Stepanyantz, One-loop divergences in non-abelian supersymmetric theories regularized by BRST-invariant version of the higher derivative regularization, J. High Energy Phys. 1605 (2016) 014, https://doi.org/10.1007/JHEP05(2016)014, arXiv:1603.04347 [hep-th].
[36] A.E. Kazantsev, M.D. Kuzmichev, N.P. Meshcheriakov, S.V. Novgorodtsev, I.E. Shirokov, M.B. Skoptsov, K.V. Stepanyantz, Two-loop renormalization of the Faddeev–Popov ghosts in $\mathcal{N} = 1$ supersymmetric gauge theories regularized by higher derivatives, J. High Energy Phys. 1806 (2018) 020, https://doi.org/10.1007/JHEP06(2018)020, arXiv:1805.03686 [hep-th].
[37] K.G. Chetyrkin, M.F. Zoller, Four-loop renormalization of QCD with a reducible fermion representation of the gauge group: anomalous dimensions and renormalization constants, J. High Energy Phys. 1706 (2017) 074, https://doi.org/10.1007/JHEP06(2017)074, arXiv:1704.04209 [hep-ph].
[38] B.S. DeWitt, Dynamical theory of groups and fields, Conf. Proc. C 630701 (1964) 585, Les Houches Lect. Notes 13 (1964) 585.
[39] D.G. Boulware, Gauge dependence of the effective action, Phys. Rev. D 23 (1981) 389, https://doi.org/10.1103/PhysRevD.23.389.
[40] B.L. Voronov, I.V. Tyutin, On renormalization of the Einsteinian gravity, Yad. Fiz. 33 (1981) 1710 (in Russian).
[41] B.L. Voronov, P.M. Lavrov, I.V. Tyutin, Canonical transformations and the gauge dependence in general gauge theories, Yad. Fiz. 36 (1982) 498 (in Russian).
[42] B.L. Voronov, I.V. Tyutin, Formulation of gauge theories of general form. II. Gauge invariant renormalizability and renormalization structure, Theor. Math. Phys. 52 (1982) 628, https://doi.org/10.1007/BF01027781, Teor. Mat. Fiz. 52 (1982) 14.
[43] P.M. Lavrov, I.V. Tyutin, Effective action in general gauge theories, Yad. Fiz. 41 (1985) 1658 (in Russian).
[44] J.C. Ward, An identity in quantum electrodynamics, Phys. Rev. 78 (1950) 182, https://doi.org/10.1103/PhysRev.78.182.
[45] J.C. Taylor, Ward identities and charge renormalization of the Yang–Mills field, Nucl. Phys. B 33 (1971) 436.
[46] A.A. Slavnov, Ward identities in gauge theories, Theor. Math. Phys. 10 (1972) 99, Teor. Mat. Fiz. 10 (1972) 153.
[47] W. Siegel, Supersymmetric dimensional regularization via dimensional reduction, Phys. Lett. B 84 (1979) 193–196, https://doi.org/10.1016/0370-2693(79)90282-X.
[48] A.A. Slavnov, Invariant regularization of nonlinear chiral theories, Nucl. Phys. B 31 (1971) 301.
[49] A.A. Slavnov, Invariant regularization of gauge theories, Theor. Math. Phys. 13 (1972) 1064, Teor. Mat. Fiz. 13 (1972) 174.
[50] I.L. Buchbinder, N.G. Pletnev, K.V. Stepanyantz, Manifestly $\mathcal{N} = 2$ supersymmetric regularization for $\mathcal{N} = 2$ supersymmetric field theories, Phys. Lett. B 751 (2015) 434, https://doi.org/10.1016/j.physletb.2015.10.071, arXiv:1509.08055 [hep-th].