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Quantum calculation of the low-energy effective action in $5D$, $\mathcal{N}=2$ SYM theory

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ABSTRACT

We consider $5D$, $\mathcal{N}=2$ supersymmetric Yang-Mills (SYM) theory in $5D$, $\mathcal{N}=1$ harmonic superspace as a theory of the interacting adjoint $5D$, $\mathcal{N}=1$ gauge multiplet and hypermultiplet. Using the background superfield method, we compute the leading low-energy contribution to the one-loop effective action. The result of quantum calculations precisely matches the effective action derived earlier in `arXiv:1812.07206` on the pure symmetry grounds.

1. Introduction

The study of maximally extended supersymmetric gauge theories in dimensions larger than four is basically motivated by the relationships of such theories to the low-energy string/brane dynamics (see e.g. [1,2]). In the present paper we consider the quantum field aspects of $\mathcal{N}=2$ SYM theory in five dimensions. This theory bears an obvious interest because of its various connections with $6D$, $\mathcal{N}=(2,0)$ superconformal field theory compactified on a circle [3–5] and also as a nice example of applications of the localization technique [6–10]. Quantum effective action can be thought of as a universal tool of analyzing connections between the low-energy effects in string theory and in quantum field theory.

The leading term of the low-energy effective action of $5D$, $\mathcal{N}=2$ SYM theory depending on all fields of $5D$, $\mathcal{N}=2$ vector gauge multiplet was constructed in ref. [11]. This was accomplished by the method similar to that employed in [12] for a similar calculation in $4D$, $\mathcal{N}=4$ SYM theory. The latter was formulated in $\mathcal{N}=2$ harmonic superspace as a theory of $\mathcal{N}=2$ vector gauge multiplet coupled to the hypermultiplet in adjoint representation. Such a theory, being manifestly $\mathcal{N}=2$ supersymmetric, possesses an additional hidden on-shell $\mathcal{N}=2$ supersymmetry. As a result, it proves to enjoy the total $\mathcal{N}=4$ supersymmetry. It was shown that the effective action depending on both the gauge multiplet and the hypermultiplet can be found in a closed form, starting from the known effective action in the $\mathcal{N}=2$ gauge multiplet sector and invoking the invariance under the hidden $\mathcal{N}=2$ supersymmetry. Such a purely symmetry-based analysis allowed to determine the effective action up to a numerical coefficient. To specify the coefficient, one should carry out the explicit quantum calculation. The latter was performed in [13], where the result of [12] was entirely confirmed and the unknown overall coefficient was fixed.

In ref. [11], $5D$, $\mathcal{N}=2$ SYM theory was formulated in $5D$, $\mathcal{N}=1$ harmonic superspace as a theory of interacting $\mathcal{N}=1$ gauge multiplet and hypermultiplet in the adjoint representation. The theory is manifestly $\mathcal{N}=1$ supersymmetric and, in addition, possesses an implicit on-shell $\mathcal{N}=1$ supersymmetry. Its effective action in the $\mathcal{N}=1$ gauge multiplet sector was calculated some time ago in [14]. Like in the $4D, \mathcal{N}=4$ case, the total $\mathcal{N}=2$ supersymmetric effective action of this theory was restored in [11] through the completion of the $\mathcal{N}=1$ gauge multiplet action by the proper hypermultiplet-dependent terms, such that the full expression for the effective action respect the additional implicit $\mathcal{N}=1$ supersymmetry. The resulting effective action can be written as an integral over the full $5D$, $\mathcal{N}=1$ superspace [11],

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$$S_{\rm eff}^{\mathcal{N}=2} = c_0 \int d^{13}z W \left[\ln W + \frac{1}{2} H(Z) \right], \tag{1.1}$$

where

$$H(Z) = 1 + 2\ln\frac{1+\sqrt{1+2Z}}{2} + \frac{2}{3}\frac{1}{1+\sqrt{1+2Z}} - \frac{4}{3}\sqrt{1+2Z}\,, \qquad Z = \frac{Q^{+a}Q^{-}_{a}}{W^2}\,. \tag{1.2}$$

Here c_0 is an arbitrary real numerical coefficient, W is the $\mathcal{N}=1$ gauge superfield strength and Q^{+a}, Q^{-}_{a} are the hypermultiplet superfields in the harmonic superspace formulation.[1] Let us point out once more that the result (1.1), (1.2) was obtained, based on the purely symmetry consideration.

The aim of the present paper is to evaluate the leading low-energy effective action of $5D$, $\mathcal{N}=2$ SYM by the explicit calculation of the one-loop effective action in the quantum superfield perturbation theory. We perform the quantum superfield derivation of the action (1.1), (1.2) and specify the one-loop value of the coefficient c_0. To preserve the classical symmetries in the quantum case, we make use of the background superfield method in $5D, \mathcal{N}=1$ harmonic superspace. It is a $5D$ version of the method developed earlier in [14,15] (see also [16]). Following the approach of [11], we formulate $5D$, $\mathcal{N}=2$ gauge multiplet as a collection of $\mathcal{N}=1$ gauge multiplet and the hypermultiplet, both being in the adjoint representation of gauge group. In the process of calculation we assume that the background superfields align in the Cartan subalgebra of $su(2)$ algebra and obey the classical equations of motion. Also we restrict our consideration to the background superfields slowly varying in space-time, as this approximation is sufficient for finding the low-energy effective action. The expression for the effective action is obtained as an integral over the analytic harmonic subspace. After passing to the full superspace, this expression reproduces the effective action of ref. [11], with $c_0 = \frac{1}{48\pi^2}$.

The paper is organized as follows. Section 2 sketches the formulation of $5D$, $\mathcal{N}=2$ SYM theory in $\mathcal{N}=1$ harmonic superspace. In section 3 we describe the manifestly gauge covariant and $\mathcal{N}=1$ supersymmetry-preserving procedure for calculating the one-loop effective action. Section 4 is devoted to the evaluation of the leading low-energy contribution to the one-loop effective action. In the last section we give a brief summary of the results obtained and indicate possible future directions of the study.

2. The model

Throughout the paper we use the notations and conventions of [11] and [14]. We formulate $\mathcal{N}=2$ SYM theory in $5D, \mathcal{N}=1$ harmonic superspace in terms of the gauge superfield V^{++} and the hypermultiplet one $q^{+}_{a} \equiv (q^{+}, -\bar{q}^{+})$, $a=1,2$, both being analytic. The classical action of the theory is written as

$$\begin{aligned} S = \frac{1}{2g^2}\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\,\mathrm{tr}\int d^{13}z du_1\ldots du_n \frac{V^{++}(z,u_1)V^{++}(z,u_2)\ldots V^{++}(z,u_n)}{(u^+_1u^+_2)(u^+_2u^+_3)\ldots(u^+_nu^+_1)} \\ -\frac{1}{2g^2}\,\mathrm{tr}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^{+}_{a}\,, \end{aligned} \tag{2.1}$$

where g is a coupling constant of mass-dimension $-1/2$. We denote the full superspace integration measure as $d^{13}z = d^5x(\mathcal{D}^-)^4(D^+)^4$ and the analytic subspace measure as $d\zeta^{(-4)} = d^5x(\mathcal{D}^-)^4 du$, where du stands for the integration over harmonics. The powers of the covariant derivatives are defined as $(\mathcal{D}^\pm)^4 = -\frac{1}{32}(\mathcal{D}^\pm)^2(\mathcal{D}^\pm)^2$, where $(\mathcal{D}^\pm)^2 = \mathcal{D}^{\pm\hat\alpha}\mathcal{D}^{\pm}_{\hat\alpha}$ $\hat\alpha, \hat\beta = 1,2$. The covariant harmonic derivative $\mathcal{D}^{++}$ acts on the hypermultiplet according to the rule [17], $\mathcal{D}^{++}q^+_a = D^{++}q^+_a + i[V^{++}, q^+_a]$. The action (2.1) is invariant under the gauge transformation

$$\delta V^{++} = -\mathcal{D}^{++}\Lambda\,, \qquad \delta q^+_a = -[q^+_a, \Lambda]\,, \tag{2.2}$$

with an analytic superfield gauge parameter $\Lambda = \Lambda(\zeta, u)$.

The classical equation of motion associated with the action (2.1) read

$$(\mathcal{D}^+)^2 W + i[q^{+a}, q_{+a}] = 0\,, \qquad \mathcal{D}^{++}q^+_a = 0\,, \tag{2.3}$$

where $W = \frac{i}{8}(\mathcal{D}^+)^2V^{--}$ is the superfield strength of the gauge multiplet. Here we introduced the non-analytic superfield V^{--} as a solution of the harmonic zero-curvature condition [17]

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0\,. \tag{2.4}$$

The action (2.1) is formulated in $5D, \mathcal{N}=1$ harmonic superspace and hence respects the manifest off-shell $\mathcal{N}=1$ supersymmetry. Since the hypermultiplet is in the adjoint representation of gauge group, like V^{++}, the action (2.1) also exhibits invariance under an additional implicit $\mathcal{N}=1$ supersymmetry. One can check that the transformations

$$\delta q^+_a = -\frac{1}{2}(D^+)^4[\epsilon_{a\hat\alpha}\theta^{-\hat\alpha}V^{--}]\,, \qquad \delta V^{++} = \epsilon^{a}_{\hat\alpha}\theta^{+\hat\alpha}q^+_a\,, \tag{2.5}$$

where $\epsilon^a_{\hat\alpha}$ is the relevant anticommuting parameter, leave the action (2.1) invariant.

[1] Actually, the superfield Lagrangian in (1.1) does not depend on the harmonic variables on shell.

3. One-loop effective action

We construct the one-loop effective action for $\mathcal{N}=2$ SYM theory with the "microscopic" action (2.1) within the background superfield field formulation. The background superfield method in $5D, \mathcal{N}=1$ harmonic superspace [14] is a direct generalization of the $4D, \mathcal{N}=2$ one [18–21] and it is based on the background-quantum splitting of the initial superfields into the 'background' $\mathbf{V}^{++}, \mathbf{Q}^+_a$ and the 'quantum' v^+, q^+_a parts:

$$V^{++} \to \mathbf{V}^{++} + g v^{++}, \qquad q^+_a \to \mathbf{Q}^+_a + g q^+_a . \tag{3.1}$$

While quantizing the gauge theory with the action (2.1) by the background superfield technique, we as usual impose the gauge-fixing conditions on the quantum gauge superfield v^{++} only. Then we introduce the gauge-fixing action and the corresponding ghost action. One of the main features of the background superfield method is that the original infinitesimal gauge symmetry (2.2) is separated into the 'background' and 'quantum' transformations:

$$\begin{aligned} &\delta \mathbf{V}^{++} = -D^{++}\Lambda - i[\mathbf{V}^{++}, \Lambda], \qquad &&\delta v^{++} = i[\lambda, v^{++}], \\ &\delta \mathbf{Q}^+_a = -[Q^+_a, \Lambda], &&\delta q^+_a = 0 . \end{aligned} \tag{3.2}$$

By construction, the effective action calculated loop by loop depends only on the background superfields and hence is invariant under the background gauge transformations.

As was said, in the framework of the background (super)field method, we should fix the gauge with respect to the quantum gauge transformations. We choose the gauge-fixing function as in $4D$ case [18,20]

$$F^{(+4)} = D^{++} v^{++}. \tag{3.3}$$

Under the quantum gauge group it transforms as follows

$$\delta F^{(+4)} = (\mathcal{D}^{++}(\mathcal{D}^{++}\lambda + i[v^{++}, \lambda])). \tag{3.4}$$

Then the action of the corresponding Faddeev-Popov ghosts $\mathbf{b}, \mathbf{c}$ is written as [19]

$$S_{FP} = \mathrm{tr} \int d\zeta^{(-4)}\, \mathbf{b} (\mathcal{D}^{++})^2 \mathbf{c}. \tag{3.5}$$

The harmonic superfield effective action for $5D$ gauge theories is constructed in the same way as in $4D, \mathcal{N}=2$ [20] and $6D, \mathcal{N}=(1,0)$ [22] cases. For $5D$ supersymmetric gauge theories the background superfield method was developed in refs. [15] and [14]. The one-loop quantum correction to the effective action $\Gamma^{(1)}[\mathbf{V}^{++}, \mathbf{Q}^+]$ is defined by the functional integral over quantum fields v^{++}, q^+_a and ghosts fields as

$$e^{i\Gamma^{(1)}} = \mathrm{Det}^{1/2}_{(4,0)} \widehat{\Box} \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}\mathbf{b} \mathcal{D}\mathbf{c} \mathcal{D}\varphi \; e^{iS^{(2)}_{\rm quant}[v^{++}, q^+_a, \mathbf{b}, \mathbf{c}, \varphi, \mathbf{V}^{++}, \mathbf{Q}^+]}, \tag{3.6}$$

where the bilinear in quantum superfields part of the quantum action is

$$S^{(2)}_{\rm quant} = S^{(2)}_0 + S_{\rm gf} + S_{FP} + S_{NK}, \tag{3.7}$$

and we introduced the background-dependent operator $\widehat{\Box} = \frac{1}{2}(D^+)^4 (\boldsymbol{\mathcal{D}}^{--})^2$. The definition of the functional determinant $\mathrm{Det}_{(4,0)}\widehat{\Box}$ is given in ref. [20]. On a space of analytical superfields the operator $\widehat{\Box}$ is reduced to [15]

$$\widehat{\Box} = \boldsymbol{\mathcal{D}}^{\hat a}\boldsymbol{\mathcal{D}}_{\hat a} + (D^{+\hat\alpha}\mathbf{W})\boldsymbol{\mathcal{D}}^-_{\hat\alpha} - \frac{1}{4}(D^{+\hat\alpha}D^+_{\hat\alpha}\mathbf{W})\boldsymbol{\mathcal{D}}^{--} + \frac{1}{4}(D^{+\hat\alpha}\boldsymbol{\mathcal{D}}^-_{\hat\alpha}\mathbf{W}) - \mathbf{W}^2 . \tag{3.8}$$

Here, all 'bold' symbols involve only the background gauge multiplet. For instance, the covariant space-time derivative is written through the background gauge connection as $\boldsymbol{\mathcal{D}}_{\hat a} = \partial_{\hat a} - i\mathbf{A}_{\hat a},\ \hat a = 0, .., 4$.

The quadratic action (3.7) includes the Faddeev-Popov ghost action (3.5), in which the harmonic covariant derivative depends on the background superfield $\mathbf{V}^{++}$, and the action for Nielsen-Kallosh ghost φ

$$S_{NK} = \frac{1}{2}\mathrm{tr} \int d\zeta^{(-4)} \varphi (\boldsymbol{\mathcal{D}}^{++})^2 \varphi . \tag{3.9}$$

The action (3.7) also contains the sum of the quadratic part of the classical action S_0 and the gauge-fixing action $S_{\rm gf}$

$$\begin{aligned} S^{(2)}_0 + S_{\rm gf} = &-\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}\, v^{++}\widehat{\Box} v^{++} - \frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}\, q^{+a}\boldsymbol{\mathcal{D}}^{++} q^+_a \\ &-\frac{i}{2}\mathrm{tr}\int d\zeta^{(-4)} \Big\{ \mathbf{Q}^{+a}[v^{++}, q^+_a] + q^{+a}[v^{++}, \mathbf{Q}^+_a] \Big\} . \end{aligned} \tag{3.10}$$

The action (3.10) involves terms which mix the quantum gauge multiplet v^{++} and the quantum hypermultiplet q^+_a. These terms can be eliminated in R_ξ gauge (see, e.g., [23] for an example of application of the R_ξ gauge in $6D, \mathcal{N}=(1,1)$ SYM theory). In this case the action for the Faddeev-Popov ghosts would depend on both the background gauge multiplet and hypermultiplet and involve inverse powers of the operator $\widehat{\square}$. Instead of imposing R_ξ gauge, we use a special change of quantum hypermultiplet [22] in the functional integral (3.6)

$$q^+_a(1) = h^+_a(1) - i\int d\zeta_2^{(-4)}\, G^{(1,1)}(1|2)_a{}^b[v^{++}(2), \mathbf{Q}^+_b(2)]\,, \tag{3.11}$$

with h^+_a being a set of new independent quantum superfields. The change (3.11) leads to the cancellation of mixed terms in the action (3.10). The Jacobian of the change (3.11) equals one and so it does not affect the integration measure in (3.6). After changing the variables as in (3.11), the action (3.10) acquires the form

$$\begin{aligned} S_0^{(2)} + S_{\rm gf} = &\frac{1}{2}{\rm tr}\int d\zeta_1^{(-4)}\, d\zeta_2^{(-4)}\, v_1^{++}\Big\{\widehat{\square}\delta_A^{(3,1)}(1|2) - 2\,\mathbf{Q}^{+a}(1)G^{(1,1)}(1|2)\mathbf{Q}^+_a(2)\Big\}v_2^{++} \\ &-\frac{1}{2}{\rm tr}\int d\zeta^{(-4)}\, h^{+a}\boldsymbol{\mathcal{D}}^{++}h^+_a\,. \end{aligned} \tag{3.12}$$

The Green function appearing in (3.11) and (3.12), $G^{(1,1)}(\zeta_1, u_1|\zeta_2, u_2)_a{}^b = i\langle 0|{\rm T}q^+_a(\zeta_1, u_1)q^{+b}(\zeta_2, u_2)|0\rangle$, is the background-dependent superfield hypermultiplet Green function in the τ-frame. It is analytic with respect to its both arguments and satisfies the equation

$$\boldsymbol{\mathcal{D}}_1^{++}G^{(1,1)}(1|2)_a{}^b = \delta_a{}^b\delta_{\mathcal{A}}^{(3,1)}(1|2)\,. \tag{3.13}$$

In the τ-frame the Green function can be written as $G^{(1,1)}(1|2)_a{}^b = \delta_a{}^b G^{(1,1)}(1|2)$, where

$$G^{(1,1)}(1|2) = \frac{(D_1^+)^4(D_2^+)^4}{\widehat{\square}_1}\frac{\delta^{14}(z_1 - z_2)}{(u_1^+u_2^+)^3}\,, \tag{3.14}$$

and $\delta_A^{(3,1)}(1|2)$ is a covariantly-analytic delta-function [17].

In the effective action (3.6) the background superfields $\mathbf{V}^{++}$ and $\mathbf{Q}^+_a$ are analytic but unconstrained otherwise. The gauge group of the theory (2.1) is assumed to be $SU(2)$. For further consideration, we will also assume that the background fields $\mathbf{V}^{++}$ and $\mathbf{Q}^+_a$ align in the Cartan subalgebra of $su(2)$

$$\mathbf{V}^{++} = V^{++}(\zeta, u)H\,, \qquad \mathbf{Q}^+_a = Q_a{}^+(\zeta, u)\,H\,, \tag{3.15}$$

where $H = \frac{1}{2}\sigma_3$ and σ_3 is Pauli matrix. The components of the background superfields associated with the $E_\pm$ generators ($[E_+, E_-] = 2H$ and $[H, E_\pm] = \pm E_\pm$) are assumed to vanish. Our choice of the background corresponds to the spontaneous symmetry breaking $SU(2) \to U(1)$. We denote the non-zero components of background superfield $\mathbf{V}^{++}$ in (3.15) by the same letter V^{++} as in the classical action (2.1), with the hope that this will not result in a confusion. The same remark refers to the abelian superfield strength W constructed out of V^{++}.

We assume that the background superfields $\mathbf{V}^{++}$ and $\mathbf{Q}^+_a$ satisfy the classical equations of motion (2.3). The conditions (3.15) then imply free equations of motion for the superfields V^{++} and Q_a^+,

$$D^{+\hat\alpha}D^+_{\hat\alpha}W = 0 \qquad D^{++}Q_a^+ = 0\,. \tag{3.16}$$

We also consider the case of the slowly varying background gauge superfield strength and hypermultiplet

$$\partial_{\hat a}W \simeq 0 \qquad \partial_{\hat a}Q_a^+ \simeq 0\,. \tag{3.17}$$

With our choice of the background superfields as described above, it is convenient to rewrite the implicit supersymmetry transformations (2.5) in terms of the gauge superfield strength [11]. They are

$$\delta Q_a^+ = \frac{i}{2}\epsilon_a^{\hat\alpha}(D^+_{\hat\alpha}W)\,, \qquad \delta W = -\frac{i}{4}\epsilon^a_{\hat\alpha}D^{-\hat\alpha}Q_a^+\,. \tag{3.18}$$

The further strategy is as follows. We substitute (3.15) in the action (3.12) and in the actions for the ghosts superfields S_{FP} and S_{NK}. As the next step, we integrate over quantum superfields v^{++} and h^+_a in the functional integral (3.6). As in 4D [20] and 6D [22] cases the contributions of the ghost superfields exactly cancel the contribution of the quantum hypermultiplet. Thus we are left with the difference between the contribution from the quantum gauge multiplet v^{++} and the contribution from the additional determinant ${\rm Det}^{1/2}_{(4,0)}\widehat{\square}$ in (3.6). The presence of this determinant is necessary for eliminating the contributions from the longitudinal component of the superfield v^{++}, in full analogy with 4D and 6D cases (see [13], [21] and [24]). Finally, for the one-loop contribution $\Gamma^{(1)}$ to the effective action we obtain the expression

$$\Gamma^{(1)} = i{\rm Tr}_{\rm T}\ln\Big(\mathcal{D}^{\hat a}\mathcal{D}_{\hat a} + (D^{+\hat\alpha}W)D^-_{\hat\alpha} - W^2 - 2\,Q^{+a}G^{(1,1)}Q_a^+\Big), \tag{3.19}$$

where we executed the trace over matrix indices. The functional trace in (3.19) is defined as a trace over the transversal component of the superfield v^{++}

$$\Gamma^{(1)} = i\int d\zeta^{(-4)}\ln\Big(\mathcal{D}^{\hat a}\mathcal{D}_{\hat a} + (D^{+\hat\alpha}W)D^-_{\hat\alpha} - W^2 - 2\,Q^{+a}G^{(1,1)}Q_a^+\Big)\Pi_{\rm T}^{(2,2)}(1|2)\Big|_{2\to 1}\,, \tag{3.20}$$

where the projector $\Pi_{\rm T}^{(2,2)}(1|2)$ is analytic in both arguments and is defined as [21]

$$\Pi^{(2,2)}_{\rm T}(1|2) = \delta^{(2,2)}_{\mathcal{A}}(1|2) - \mathcal{D}^{++}_1\mathcal{D}^{++}_2 \frac{(D^+_1)^4(D^+_2)^4}{\widehat{\Box}_1}\delta^{13}(z_1-z_2)\frac{(u^-_1u^-_2)}{(u^+_1u^+_2)^3}, \tag{3.21}$$

with $\delta^{(2,2)}_{\mathcal{A}}(1|2)$ being an analytic delta-function [17]. For our calculation, we do not need to know the explicit form of the projector $\Pi^{(2,2)}_{\rm T}$, which for the $5D$ case was found in [14]. All what we need is the expression in the limit of coincident harmonic arguments, $u_2 \to u_1$, in the case of slowly varying on-shell background superfields. The latter condition implies the simplest form for the projector $\Pi^{(2,2)}_{\rm T}$,

$$\Pi^{(2,2)}_{\rm T}(1|2)\big|_{u_2\to u_1} = (D^+_1)^4\delta^{13}(z_1-z_2). \tag{3.22}$$

Thus we finally arrive at the following expression for the one-loop contribution $\Gamma^{(1)}$ (3.19):

$$\Gamma^{(1)} = i\int d\zeta^{(-4)} \ln\Big(\mathcal{D}^{\hat a}\mathcal{D}_{\hat a} + (D^{+\hat\alpha}W)D^-_{\hat\alpha} - W^2 - 2\,Q^{+a}G^{(1,1)}Q^+_a\Big)(D^+_1)^4\delta^{13}(z_1-z_2)\Big|_{2\to1}. \tag{3.23}$$

It is the starting point for the evaluation of the leading low-energy contribution to the effective action in the model under consideration.

4. Leading low-energy contribution

Here we demonstrate how the exact expression for the leading low-energy contribution to effective action derived in [11] can be recovered from the one-loop effective action (3.23).

First of all we have to note that the effective action (3.23) contains the non-local contribution $Q^{+a}(1)G^{(1,1)}(1|2)Q^+_a(2)$ and so one should extract the local part from it. One can use the following identity [14]

$$(D^+_1)^4(D^+_2)^4\frac{1}{(u^+_1u^+_2)^3} = (D^+_1)^4\Big\{(\mathcal{D}^-_1)^4(u^+_1u^+_2) - \frac{1}{4}(u^-_1u^+_2)\Delta^{--}_1 - \frac{(u^-_1u^+_2)^2}{(u^+_1u^+_2)}\widehat{\Box}_1\Big\}, \tag{4.1}$$

where $\Delta^{--} = i\mathcal{D}^{\hat\alpha\hat\beta}\mathcal{D}^-_{\hat\alpha}\mathcal{D}^-_{\hat\beta} + W(\mathcal{D}^-)^2 + 4(\mathcal{D}^{-\hat\alpha}W)\mathcal{D}^-_{\hat\alpha}$. An analog of this identity for $4D, \mathcal{N}=2$ supersymmetric gauge theory was originally derived in [21]. Then we use the decomposition $Q^+_a(2) = (u^+_1u^+_2)Q^-_a(1) - (u^-_1u^+_2)Q^+_a(1)$ and eq. (4.1) to properly transform the Green function $G^{(1,1)}(1|2)$ (3.14).[2] We have

$$Q^{+a}(1)G^{(1,1)}(1|2)Q^+_a(2) = Q^{+a}Q^-_a(u^-_1u^+_2)^2\delta^{13}(z_1-z_2) + \ldots, \tag{4.2}$$

where dots stand for terms proportional to $(u^+_1u^+_2)$ and so vanishing in the $u_2 \to u_1$ limit.

Thus we obtain for the one-loop contribution (3.23)

$$\Gamma^{(1)} = i\int d\zeta^{(-4)} \ln\Big(\mathcal{D}^{\hat a}\mathcal{D}_{\hat a} + (D^{+\hat\alpha}W)D^-_{\hat\alpha} - W^2 - 2Q^{+a}Q^-_a\Big)(D^+_1)^4\delta^{13}(z_1-z_2)\Big|_{2\to1}. \tag{4.3}$$

In order to evaluate the leading low-energy contribution to the effective action we need to calculate the functional trace in (4.3) in the coincident-point limit. First we calculate the $\theta^\pm_2 \to \theta^\pm_1$ limit using the presence of Grassmann delta-functions in (4.3). In the full superspace delta-function

$$\delta^{13}(z_1-z_2) = \delta^5(x_1-x_2)\delta^4(\theta^+_1-\theta^+_2)\delta^4(\theta^-_1-\theta^-_2) \tag{4.4}$$

the operator $(D^+)^4$ annihilate one of the Grassmann delta-functions according to the rule

$$(D^+)^4\delta^4(\theta^-_1-\theta^-_1) = -2\,. \tag{4.5}$$

In order to remove the remaining delta-function $\delta^4(\theta^-_1-\theta^-_2)$, we need to collect the fourth power of the derivative $D^-_{\hat\alpha}$. To this end, we expand the logarithm in (4.3) in the power series, up to the fourth power of $(D^{+\hat\alpha}W)D^-_{\hat\alpha}$:

$$\Gamma^{(1)} = \frac{i}{2}\int d\zeta^{(-4)} \frac{(D^{+\hat\alpha}WD^+_{\hat\alpha}W)^2}{(\partial^{\hat a}\partial_{\hat a} - W^2 - 2Q^{+a}Q^-_a)^4}\delta^5(x_1-x_2) + \ldots, \tag{4.6}$$

where dots mean all contribution with the derivatives of hypermultiplet, $D^-_{\hat\alpha}Q^{+a}$, which can in principle be evaluated explicitly. In what follows we omit all such contributions, assuming that they can be reconstructed by using the analyticity condition for the integrand in (4.6) and the implicit $\mathcal{N}=1$ supersymmetry (3.18).

Then we pass to the momentum representation for the space-time delta-function and calculate the momentum integral

$$\int \frac{d^5p}{(2\pi)^5}\frac{1}{(p^2+M^2)^4} = \frac{i}{6(8\pi)^2}\frac{1}{M^3}\,. \tag{4.7}$$

After that we obtain for the $\Gamma^{(1)}$ (4.6) the following expression

[2] See the detailed analysis of the similar contribution in $6D, \mathcal{N}=(1,1)$ SYM theory in ref. [24].

$$\Gamma^{(1)} = -\frac{1}{12(8\pi)^2}\int d\zeta^{(-4)}\frac{(D^{+\hat\alpha}W D^+_{\hat\alpha}W)^2}{(W^2+2Q^{+a}Q^-_a)^{3/2}} + \dots, \tag{4.8}$$

where dots as in (4.6) mean terms with derivatives of the hypermultiplet. The expression (4.8) is the leading low-energy contribution to the effective action of $5D, \mathcal{N}=5$ SYM theory. It is written as an integral over the analytic subspace. In the paper [11] the effective action of $\mathcal{N}=2$ SYM theory was obtained as a hypermultiplet completion of the leading $W\ln W$-term in the $\mathcal{N}=1$ SYM low-energy effective action and it was written as an integral over the whole superspace. This effective action was evaluated in the form (1.1), up to an overall constant c_0.

Let us demonstrate that, passing to the full superspace in (4.8), one can reproduce the expression (1.1). To this end, we first expand the function $H(Z)$ in the expression (1.1) in the power series

$$S^{\mathcal{N}=2}_{\text{eff}} = c_0\int d^{13}z\left[W\ln W + \sum_{n=1}^{\infty}\frac{(-1)^n(2n-2)!}{n!(n+1)!2^n}\frac{(Q^{+a}Q^-_a)^n}{W^{2n-1}}\right]. \tag{4.9}$$

Then we decompose the factor $(W^2+2Q^{+a}Q^-_a)^{-3/2}$ in (4.8) as

$$\Gamma^{(1)} = -\frac{1}{12(8\pi)^2}\int d\zeta^{(-4)}(D^{+\hat\alpha}W D^+_{\hat\alpha}W)^2\Big(\frac{1}{W^3} + \sum_{n=1}^{\infty}\frac{(-1)^n 2^n\Gamma[n+\frac{3}{2}]}{\Gamma[\frac{3}{2}]\Gamma[n+1]}\frac{(Q^{+a}Q^-_a)^n}{W^{2n+3}}\Big). \tag{4.10}$$

After this we pass to the full superspace by restoring $(D^+)^4$ in all terms of the series by the rules

$$\begin{aligned}
&(D^+)^4 W\ln W = -\frac{1}{16}\frac{(D^{+\hat\alpha}W D^+_{\hat\alpha}W)^2}{W^3},\\
&(D^+)^4\frac{1}{W^{2n-1}} = -\frac{1}{8}n(n+1)(2n+1)(2n-1)\frac{(D^{+\hat\alpha}W D^+_{\hat\alpha}W)^2}{W^{2n+3}},
\end{aligned} \tag{4.11}$$

keeping in mind the on-shell condition for the background gauge field strength W and omitting all terms with derivatives of the hypermultiplet. One can show that, after using (4.11) in (4.10) and employing the property,

$$\Gamma[n+\tfrac{1}{2}] = \frac{2n!\sqrt{\pi}}{4^n n!},$$

the second term in (4.10) immediately takes the same form as in (4.9). Indeed, it is straightforward to check that

$$(D^+)^4\Big(W\ln W + \tfrac{1}{2}WH\big(\tfrac{Q^{+a}Q^-_a}{W^2}\big)\Big) = -\frac{1}{16}\frac{(D^{+\hat\alpha}W D^+_{\hat\alpha}W)^2}{(W^2+2Q^{+a}Q^-_a)^{3/2}} + \dots, \tag{4.12}$$

where dots denote terms with spinor derivatives of the hypermultiplet. Thus for the leading term in the low-energy effective action we obtain the expression

$$\Gamma^{(1)} = \frac{1}{48\pi^2}\int d^{13}zW\left[\ln W + \frac{1}{2}H\Big(\frac{Q^{+a}Q^-_a}{W^2}\Big)\right], \tag{4.13}$$

where the function $H(Z)$ was defined in (1.2).

We observe the complete agreement of the method based on the symmetry considerations with the direct quantum computations. The latter also yield the precise value for the coefficient c_0.

5. Summary

We have studied the problem of computing the leading contribution to the one-loop low-energy effective action of $5D$, $\mathcal{N}=2$ SYM theory in the $5D$, $\mathcal{N}=1$ harmonic superspace formulation. The effective action was constructed in the framework of the background field method which allow to preserve the manifest gauge invariance and $5D$, $\mathcal{N}=1$ supersymmetry at all stages of calculations. The effective action derived in this way depends on all fields of $5D$, $\mathcal{N}=2$ gauge multiplet and is completely $5D$, $\mathcal{N}=2$ supersymmetric. We have shown that the superfield quantum considerations yield the same leading contribution to one-loop low-energy effective action as the analysis carried out in ref. [11] on the purely symmetric grounds.

The results obtained here can be further generalized at least in two directions. First, it would be interesting to find out the explicit forms of the next-to-leading corrections to the effective action (1.1). Second, it is tempting to study the quantum aspects of the twisted $5D$, $\mathcal{N}=2$ SYM theory [25–27], using similar techniques.

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