Supersymmetric quantum mechanics and the Riemann hypothesis

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New Physics

"One must be prepared to follow up the consequence of theory,
and feel that one just has to accept the consequences
no matter where they lead."
Paul Dirac

"... all things physical are information-
theoretic in origin ..."
John A. Wheeler

We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value.
In 1900, the British physicist Lord Kelvin is said to have pronounced: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Within three decades, quantum mechanics and Einstein's theory of relativity had revolutionized the field.

If you are only a poet,
You are not even that.
(Piet Hein)
Если ты всего лишь поэт,
Поэт ли ты, наверно нет :)
(Пит Хейн - Граф О'Мар)
I always knew that sooner or later p -
adic numbers will appear in Physics -
André Weil.
Mathematics is the queen of the sciences and
number theory is the queen of mathematics.
Carl Friedrich Gauss (1777–1855)

In the Universe, matter has manly two tipe geometric structures, homogeneous isotropic, [Weinberg, 1972] and hierarchical, Russian-Doll-Like structures, [Okun 1982]

.

The homogeneous structures are naturally described by real numbers with an infinite number of digits in the fractional part and usual archimedean metrics.
The hierarchical structures are described with p-adic numbers with an infinite number of digits in the integer part and non-archimedean metrics, [Koblitz, 1977].

New Physics

A discrete, finite, regularized, version of the homogenous structures are homogeneous lattices with constant steps and distance rising as arithmetic progression. The discrete version of the hierarchical structures is hierarchical lattice-tree with scale rising in geometric progression.

There is an opinion that present day theoretical physics needs (almost) all mathematics, and the progress of modern mathematics is stimulated by fundamental problems of theoretical physics.

The Riemann zeta function $\zeta(s)$ is defined for complex $s = \sigma + it$ and $\sigma > 1$ by the expansion

$$\begin{aligned}
\zeta(s) &= \sum_{n\geq 1} n^{-s}, \;\; Re\ s > 1, \\
&= \delta_x^{-s} \frac{x}{1-x}|_{x\to 1} = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{-\delta_x t} \frac{x}{1-x}|_{x\to 1} \\
&= \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{t\partial_\tau} \frac{1}{e^\tau - 1}|_{\tau\to 0} \\
&= \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1} dt}{e^t - 1}, \;\; x = e^{-\tau} \qquad (1)
\end{aligned}$$

All complex zeros, $s = \alpha + i\beta$, of $\zeta(\sigma + it)$ function lie in the critical stripe $0 < \sigma < 1$, symmetrically with respect to the real axe and critical line $\sigma = 1/2$. So it is enough to investigate zeros with $\alpha \leq 1/2$ and $\beta > 0$. These zeros are of three type, with small, intermediate and big ordinates.

The Riemann hypothesis states that the (non-trivial) complex zeros of $\zeta(s)$ lie on the critical line $\sigma = 1/2$.
At the beginning of the XX century Polya and Hilbert made a conjecture that the imaginary part of the Riemann zeros could be the oscillation frequencies of a physical system (ζ - (mem)brane).
After the advent of Quantum Mechanics, the Polya-Hilbert conjecture was formulated as the existence of a self-adjoint operator whose spectrum contains the imaginary part of the Riemann zeros.

The Riemann hypothesis (RH) is a central problem in Pure Mathematics due to its connection with Number theory and other branches of Mathematics and Physics.

Functional Equation for Zeta Function

The functional equation is

$$\zeta(1-s) = \frac{2\Gamma(s)}{(2\pi)^s}\cos(\frac{\pi s}{2})\zeta(s) \tag{2}$$

From this equation we see the real (trivial) zeros of zeta function:

$$\zeta(-2n) = 0, \;\; n = 1, 2, ... \tag{3}$$

Also, at s=1, zeta has pole with reside 1.

Functional Equation for Zeta Function

From Field theory and statistical physics point of view, the functional equation (2) is duality relation, with self dual (or critical) line in the complex plane, at $s = 1/2 + i\beta$,

$$\zeta(\frac{1}{2} - i\beta) = \frac{2\Gamma(s)}{(2\pi)^s} \cos(\frac{\pi s}{2}) \zeta(\frac{1}{2} + i\beta), \tag{4}$$

we see that complex zeros lie symmetrically with respect to the real axe.
On the critical line, (nontrivial) zeros of zeta corresponds to the infinite value of the free energy,

$$F = -T \ln \zeta. \tag{5}$$

At the point with $\beta = 14.134725...$ is located the first zero. In the interval $10 < \beta < 100$, zeta has 29 zeros. The first few million zeros have been computed and all lie on the critical line. It has been proved that uncountably many zeros lie on critical line.
The first relation of zeta function with prime numbers is given by the following formula,

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}, \ Re\ s > 1. \tag{6}$$

Another formula, which can be used on critical line, is

$$\zeta(s) = (1-2^{1-s})^{-1}\sum_{n\geq 1}(-1)^{n+1}n^{-s}$$
$$= \frac{1}{1-2^{1-s}}\frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}dt}{e^t+1},\;\; Re\ s > 0 \qquad (7)$$

The Riemann zeta function (RZF) can be interpreted in thermodynamic terms as a statistical sum of a system with energy spectrum: $E_n = \ln n,\ \ n = 1, 2, ...$:

$$\zeta(s) = \sum_{n\geq 1} n^{-s} = Z(\beta) = \sum_{n\geq 1} \exp(-\beta E_n),$$
$$\beta = s,\ \ E_n = \ln n,\ \ n = 1, 2, ... \tag{8}$$

Let as consider the following formula

$$\frac{1}{1-x} = (1+x)(1+x^2)(1+x^4)..., \ |x| < 1. \quad (9)$$

which can be proved as

$$p_k \equiv (1+x)(1+x^2)(1+x^4)...(1+x^{2^k}) = \frac{1-x^{2^{(k+1)}}}{1-x},$$

$$|p_k| < c(1+|x|^{2^{(k+1)}}), \ \lim_{k\to\infty} p_k = c = 1/(1-x). \quad (10)$$

The formula reminds us the boson and fermion statsums

$$Z_b = \frac{\sqrt{x}}{1-x}, \; Z_f = \frac{1+x}{\sqrt{x}}, \; x = \exp\left(-\beta\hbar\omega\right) \qquad (11)$$

and can be transformed in the following relation

$$Z_b(\omega) = Z_f(\omega) Z_f(2\omega) Z_f(4\omega)... \qquad (12)$$

Indeed, [Makhaldiani 2018]

$$Z_b(\omega) = \frac{\sqrt{x}}{1-x} = x^{a/2} Z_f(\omega) Z_f(2\omega) Z_f(4\omega)...,$$
$$a = 1 + (1 + 2 + 2^2 + ...)$$
$$= 1 + \frac{1}{1-2} = 0, \ |2|_2 = 1/2 < 1. \tag{13}$$

By the way we have an extra bonus! We see that the fermi content of the boson wears the p-adic sense. The prime $p = 2$, in this case. Also, the vacuum energy of the oscillators wear p-adic sense.

What about other primes p?
For the finite fields,

$$z_n(p) = \exp(2\pi in/p),\ \ n = 0, 1, ..., p-1,\ \sum_n z_n = 0,$$

$$Z_p(\beta) = \sum_{n=1}^{p-1} \exp\left(-\beta E_n/\hbar\right),\ \ E_n = 2\pi\hbar(n+a),$$

$$Z_p(-i/p) = 0,\ \ p = 2, 3, 5, ...13...29...137... \tag{14}$$

- Why supersymmetry is so universal?
- Supermathematics unifies discrete and continual aspects of mathematics.

Boson oscillator hamiltonian is

$$H_b = \hbar\omega(b^+b + bb^+)/2 = \hbar\omega(b^+b + a), \ a = 1/2. \quad (15)$$

corresponding energy spectrum E_{bn} and eigenfunctions $|n_b>$ are

$$H_b|n_b >= E_{bn}|n_b >, \ E_{bn} = \hbar\omega(n_b + a), \ n_b = 0, 1, 2, ..(16)$$

Fermion oscillator hamiltonian, eigenvectors and energies are

$$H_f = \hbar\omega(f^+f - ff^+)/2 = \hbar\omega(f^+f - a),$$
$$H_f|n_f >= E_{fn}|n_f >, E_{fn} = \hbar\omega(n_f - a), \ n_f = 0, 1 \quad (17)$$

For supersymmetric oscillator we have

$$H = H_b + H_f, \ H|n_b, n_f >= \hbar\omega(n_b + n_f)|n_b, n_f >,$$
$$|n_b, n_f >= |n_b > |n_f >, \ E_{n_b,n_f} = \hbar\omega(n_b + n_f) \quad (18)$$

For background-vacuum $|0, 0 >$, energy $E_{0,0} = 0$. For higher energy states $|n - 1, 1 >, \ |n, 0 >, \ E_{n-1,1} = E_{n,0}$. Supersymmetry needs not only the same frequency for boson and fermion oscillators, but also that $2a = 1$.

A minimal realization of the algebra of supersymmetry

$$\{Q, Q^+\} = H,\ \{Q, Q\} = \{Q^+, Q^+\} = 0, \quad (19)$$

is given by a point particle dynamics in one dimension, [Witten 1981]

$$\begin{aligned} Q &= f(-iP + W)/\sqrt{2}, \\ Q^+ &= f^+(iP + W)/\sqrt{2}, \\ P &= -i\partial/\partial x \end{aligned} \quad (20)$$

where the superpotential $W(x)$ is any function of x, and spinor operators f and f^+ obey the anticommuting relations

$$\{f, f^+\} = 1,\ f^2 = (f^+)^2 = 0. \quad (21)$$

There is a following representation of operators f, f^+ and σ by Pauli spin matrices

$$f = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, f^+ = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$
$$\sigma = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{22}$$

From formulae (19) and (20) then we have

$$H = (P^2 + W^2 + \sigma W_x)/2. \tag{23}$$

The simplest nontrivial case of the superpotential $W = \omega x$ corresponds to the supersimmetric oscillator with Hamiltonian

$$H = H_b + H_f, \quad H_b = (P^2 + \omega^2 x^2)/2, \quad H_f = \omega\sigma/2, \tag{24}$$

The ground state energies of the bosonic and fermionic parts are

$$E_{b0} = \omega/2, \quad E_{f0} = -\omega/2, \tag{25}$$

so the vacuum energy of the supersymmetric oscillator is

$$< 0|H|0 >= E_0 = E_{b0} + E_{f0} = 0, \\ |0 >= |n_b, n_f >= |n_b > |n_f > . \tag{26}$$

Toy - solution of the cosmological constant problem

Let us see on this toy - solution of the cosmological constant problem from the quantum statistical viewpoint. The statistical sum of the supersymmetric oscillator is

$$Z(\beta) = Z_b Z_f, \tag{27}$$

where

$$Z_b = \sum_n e^{-\beta E_{bn}} = e^{-\beta\omega/2} + e^{-\beta\omega(1+1/2)} + \ldots$$
$$= e^{-\beta\omega/2}/(1 - e^{-\beta\omega})$$
$$Z_f = \sum_n e^{-\beta E_{fn}} = e^{\beta\omega/2} + e^{-\beta\omega/2}. \tag{28}$$

Toy - solution of the cosmological constant problem

In the low temperature limit,

$$Z(\beta) = 1 + O(e^{-\beta\omega}),\ \beta = T^{-1}, \tag{29}$$

so cosmological constant

$$\lambda \sim lnZ \sim e^{-\beta\omega},\ \beta\omega \sim 10^2 \tag{30}$$

From observable values of β and the cosmological constant we estimate ω.

$$T = 3K = \frac{eV}{3868} \sim 10^{-4}eV,\ \omega \sim 10^{-2}eV \tag{31}$$

In terms of Planck units, the cosmological constant is on the order of $\Lambda L_P^2 \sim 10^{-123}$.

Negative Binomial Distribution

Negative binomial distribution (NBD) is defined as

$$P(n) = \frac{\Gamma(n+r)}{n!\Gamma(r)} p^n (1-p)^r, \; \sum_{n \geq 0} P(n) = 1, \qquad (32)$$

The Bose-Einstein distribution is a special case of NBD with $r = 1$.

NBD provides a very good parametrization for multiplicity distributions in e^+e^- annihilation; in deep inelastic lepton scattering; in proton-proton collisions; in proton-nucleus scattering. Hadronic collisions at high energies (LHC) lead to charged multiplicity distributions whose shapes are well fitted by a single NBD in fixed intervals of central (pseudo)rapidity η.

Multiplicative Properties of NBD and Corresponding Motion Equations

A Bose-Einstein, or geometrical, distribution is a thermal distribution for single state systems. An useful property of the negative binomial distribution with parameters

$$< n >, \ k$$

is that it is (also) the distribution of a sum of k independent random variables drawn from a Bose-Einstein distribution with mean $< n > /k$,

$$P_n = \frac{1}{< n > +1}(\frac{< n >}{< n > +1})^n$$
$$= (e^{\beta\hbar\omega/2} - e^{-\beta\hbar\omega/2})e^{-\beta\hbar\omega(n+1/2)}, \ \ T = \frac{\hbar\omega}{\ln \frac{<n>+1}{<n>}}$$

Multiplicative Properties of NBD and Corresponding Motion Equations

Indeed, for

$$n = n_1 + n_2 + ... + n_k, \tag{34}$$

with n_i independent of each other, the probability distribution of n is

$$P_n = \sum_{n_1,...,n_k} \delta(n - \sum n_i) p_{n_1} ... p_{n_k},$$
$$P(x) = \sum_n x^n P_n = p(x)^k \tag{35}$$

Multiplicative Properties of NBD and Corresponding Motion Equations

This has a consequence that an incoherent superposition of N emitters that have a negative binomial distribution with parameters $k, < n >$ produces a negative binomial distribution with parameters $Nk, N < n >$.
So, for the GF of NBD we have (N=2)

$$F(k, < n >)F(k, < n >) = F(2k, 2 < n >) \qquad (36)$$

And more general formula (N=m) is

$$F(k, < n >)^m = F(mk, m < n >) \qquad (37)$$

Multiplicative Properties of NBD and Corresponding Motion Equations

We can put this equation in the closed nonlocal form

$$Q_q F = F^q, \tag{38}$$

where

$$Q_q = q^D, \quad D = \frac{kd}{dk} + \frac{< n > d}{d < n >} = \frac{x_1 d}{dx_1} + \frac{x_2 d}{dx_2} \tag{39}$$

Note that temperature defined in (33) gives an estimation of the Glukvar temperature when it radiates hadrons. If we take $\hbar\omega = 100MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 1.5$ If we take $\hbar\omega = 10MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 20$. A singular behavior of $< n >$ may indicate corresponding phase transition and temperature. At that point we estimate characteristic quantum $\hbar\omega$.

We see that universality of NBD in hadron-production is similar to the universality of black body radiation.

Let us consider the negative binomial distribution (NBD) for normed topological cross sections

$$\frac{\sigma_n}{\sigma} = P(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left(\frac{k}{<n>}\right)^k\left(1+\frac{k}{<n>}\right)^{-(n+k)}$$
$$= \frac{1}{n}\frac{\Gamma(n+k)}{\Gamma(n)\Gamma(k)}\left(1+\frac{k}{<n>}\right)^{-n}\left(1+\frac{<n>}{k}\right)^{-k}$$
$$= \frac{1}{n}\frac{\Gamma(n+k)}{\Gamma(n)\Gamma(k)}\left(\frac{<n>}{<n>+k}\right)^{n}\left(\frac{k}{k+<n>}\right)^{k},$$
$$= \frac{1}{n}\frac{\Gamma(n+k)}{\Gamma(n)\Gamma(k)}\frac{\left(\frac{k}{<n>}\right)^k}{\left(1+\frac{k}{<n>}\right)^{k+n}}, \tag{40}$$

where $k > 0$.

The generating function for NBD is

$$F(h) = (1 + \frac{<n>}{k}(1-h))^{-k}$$
$$= (1 + \frac{<n>}{k})^{-k}(1-ah))^{-k}, /a = \frac{<n>}{<n>+k}. \quad (41)$$

Indeed,

$$(1-ah))^{-k} = \frac{1}{\Gamma(k)} \int_0^\infty dt t^{k-1} e^{-t(1-ah)}$$
$$= \frac{1}{\Gamma(k)} \int_0^\infty dt t^{k-1} e^{-t} \sum_0^\infty \frac{(tah)^n}{n!}$$
$$= \sum_0^\infty \frac{\Gamma(n+k)a^n}{\Gamma(k)n!} h^n, \quad (42)$$

Negative binomial distribution

$$
\begin{aligned}
P(n) &= (1+\frac{<n>}{k})^{-k}\frac{\Gamma(n+k)}{\Gamma(k)n!}(\frac{<n>}{<n>+k})^{n} \\
&= \frac{k^k\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)}(<n>+k)^{-(n+k)}<n>^{n} \\
&= \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}(\frac{k}{<n>})^{k}(1+\frac{k}{<n>})^{-(n+k)} \\
&= \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}(\frac{k}{<n>})^{k}(1+\frac{k}{<n>})^{-(n+k)} \qquad (43)
\end{aligned}
$$

Note that KNO characteristic function (159) coincides with the NBD generating function (41) when
$t =< n > (h-1), \; c = k$
For negative (integer) values of $k = -N$, we have Binomial GF

$$F_{bd} = (1 + \frac{< n >}{N}(h-1))^N = (a + bh)^N,$$
$$a = 1 - \frac{< n >}{N}, \; b = \frac{< n >}{N},$$
$$P_{bd}(n) = C_N^n(\frac{< n >}{N})^n(1 - \frac{< n >}{N})^{N-n} \qquad (44)$$

In a sense we have a quantum spectrum for the parameter k, which contains any positive real values and (finite number of states) the negative integer values, $(0 \leq n \leq N)$

Let us consider the values $q = n, n = 1, 2, 3, ...$ and take sum of the corresponding equations (38), we find

$$\zeta(-D)F = \frac{F}{1-F} \quad (45)$$

In the case of the NBD we know the solutions of this equation.

From Qlike to Zeta Equations

Now we invent a Hamiltonian H with spectrum corresponding to the set of nontrivial zeros of the zeta function, in correspondence with Riemann hypothesis,

$$-D_n = \frac{n}{2} + iH_n,\ H_n = i(\frac{n}{2} + D_n),$$
$$D_n = x_1\partial_1 + x_2\partial_2 + ... + x_n\partial_n,\ H_n^+ = H_n = \sum_{m=1}^{n} H_1(x_m),$$
$$H_1 = i(\frac{1}{2} + x\partial_x) = -\frac{1}{2}(x\hat{p} + \hat{p}x),\ \hat{p} = -i\partial_x \tag{46}$$

The Hamiltonian $H = H_n$ is hermitian, its spectrum is real. The case $n = 1$ corresponds to the Riemann hypothesis.

From Qlike to Zeta Equations

The case $n = 2$, corresponds to NBD,

$$\zeta(1 + iH_2)F = \frac{F}{1-F}, \ \zeta(1 + iH_2)|_F = \frac{1}{1-F},$$
$$F(x_1, x_2; h) = (1 + \frac{x_1}{x_2}(1-h))^{-x_2} \tag{47}$$

Let us scale $x_2 \to \lambda x_2$ and take $\lambda \to \infty$ in (47), we obtain

$$\zeta(\frac{1}{2} + iH_1(x))e^{-(1-h)x} = \frac{1}{e^{(1-h)x} - 1},$$
$$\frac{1}{\zeta(\frac{1}{2} + iH(x))} \frac{1}{e^{\varepsilon x} - 1} = e^{-\varepsilon x}, \ \varepsilon = 1 - h,$$
$$H(x) = i(\frac{1}{2} + x\partial_x) = -\frac{1}{2}(x\hat{p} + \hat{p}x), \ H^+ = H \tag{48}$$

Let us take an eigenvector $|n>$ with eigenvalue E_n of H, than

$$< n|\zeta(\frac{1}{2}+iH(x))e^{-(1-h)x} >= \zeta(\frac{1}{2}+iE_n(x)) < n|e^{-(1-h)}$$
$$=< n|\frac{1}{e^{(1-h)x}-1} >$$

For zeros of Zeta function, E_n, the eigenfunctions fulfils the following conditions

$$< n|\frac{1}{e^{(1-h)x}-1} >= 0,\ < n|e^{-(1-h)x} >\neq 0. \quad (50)$$

For eigenvalues of H, we have

$$
\begin{aligned}
&H|n>=E_n|n>,\ H=i(\frac{1}{2}+x\partial_x),\\
&|n>\sim x^{s_n},\ s_n=-\frac{1}{2}-iE_n,\\
&<n|\frac{1}{e^{(1-h)x}-1}>=\zeta(\frac{1}{2}+iE_n)/(1-h)^s,\\
&<n|e^{-(1-h)x}>=\Gamma(\frac{1}{2}+iE_n)/(1-h)^s. \qquad (51)
\end{aligned}
$$

From Qlike to Zeta Equations

$$\zeta(-x_1\partial_{x_1} - x_2\partial_{x_2})F = \frac{F}{1-F} = \frac{1}{F^{-1}-1},$$
$$F(x_1, x_2) = (1 + \frac{x_1}{x_2}(1-h))^{-x_2} \qquad (52)$$

For $x_2 \to \infty$, after $x_1 \to x/(1-h)$ we obtain

$$\zeta(-x\partial_x)e^{-x} = \frac{1}{e^x - 1}, \qquad (53)$$

After multiplication bouth sides on $x^{s-1}$and integration by parts we obtain known formula

$$\int_0^\infty dxx^{s-1}\zeta(-x\partial_x)e^{-x} = \int_0^\infty dxe^{-x}\zeta(1+x\partial_x)x^{s-1}$$
$$= \zeta(s)\Gamma(s) = \int_0^\infty \frac{dxx^{s-1}}{e^x-1} \tag{54}$$

Let us consider the following finite approximation of the Riemann zeta function

$$\zeta_N(s) = \sum_{n=1}^{N} n^{-s} = \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} \frac{e^{-t} - e^{-(N+1)t}}{1 - e^{-t}}$$
$$= \zeta(s) - \Delta_N(s), \;\; Re\ s > 1$$
$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1}}{e^t - 1},$$
$$\Delta_N(s) = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1} e^{-Nt}}{e^t - 1} \tag{55}$$

Another formula, which can be used on critical line, is

$$\zeta(s) = (1-2^{1-s})^{-1}\sum_{n\geq 1}(-1)^{n+1}n^{-s}$$
$$= \frac{1}{1-2^{1-s}}\frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}dt}{e^t+1}, \quad Re\ s > 0 \qquad (56)$$

Corresponding finite approximation of the Riemann zeta function is

$$\zeta_N(s) = (1-2^{1-s})^{-1}\sum_{n=1}^{N}(-1)^{n-1}n^{-s}$$
$$= \frac{1}{1-2^{1-s}}\frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}(1-(-e^{-t})^N)dt}{e^t+1} = \zeta(s) - \Delta_N($$
$$\Delta_N(s) = \frac{1}{\Gamma(s)}\int_0^\infty dt\frac{t^{s-1}(-e^{-t})^N)}{e^t+1} \sim \pm N^{-s} \qquad ($$

at a (nontrivial) zero of the zeta function, s_0, $\zeta_N(s_0) = -\Delta_N(s_0)$. In the integral form, dependence on N is analytic and we can consider any complex valued N.

It is interesting to see dependence (evolution) of zeros with N. For the simplest nontrivial integer $N = 2$,

$$\zeta_2(s) = (1 - 2^{1-s})^{-1}(1 - 2^{-s}) \\ = \frac{1 - 2^{-s}}{1 - 2^{1-s}} = \frac{2^s - 1}{2^s - 2} = \frac{2^{s-1/2} - 1/\sqrt{2}}{2^{s-1/2} - \sqrt{2}} \qquad (58)$$

we have zeros at $s_n = 2\pi in/\ln 2, \ n = 0, \pm 1, \pm 2, ...$
$2\pi/\ln 2 = 9.06$, so, in the interval $Ims_n \in (0, 100)$ we have 10 nontrivial zeros. The first nontrivial zero of the zeta function, by Mathematica, is: $s_1 = 1/2 + i14.1347$. The last zero in the interval $Ims_n \in (0, 100)$ is:
$s_{29} = 1/2 + i98.8312$.
Another finite approximation of the zeta function is

Now let me present a recent paper: Pushpa Kalauni, Kimball A Milton, Supersymmetric quantum mechanics and the Riemann hypothesis,
https://arxiv.org/abs/2211.04382v1

Axion-like particles (ALPs) a are very light, neutral, spin zero bosons predicted by many theories which try to extend and complete the standard model of elementary particles. ALPs interact primarily with two photons and can generate photon-ALP oscillations in the presence of an external magnetic field. Effective interaction is given by the following term

$$L_a = \int d^D x a tr G_{\mu\nu} G^{\mu\nu} = \int d^D x tr \partial_\mu a j^\mu,$$
$$j_\mu = \epsilon_{\mu\nu\rho\sigma} A_\nu (\partial_\rho A_\sigma + 2/3 A_\rho A_\sigma) \quad (60)$$

It contributs in neutron electric dipol moment and is restricted as $\theta < 10^{-9}$.

Let us solve the folloving matrix linear first order equation

$$\dot{\psi} = A\psi,$$
$$\psi(t) = U(t)\psi(0), \; U(t) = P\exp(\int_0^t d\tau A(\tau)) \quad (61)$$

Neither the Schrödinger equation nor the quantum wave function are fundamental structures. Rather, they both originate from a pre-quantum operator algebra.

Most fundamintal string theory model - bosonic string model in the most general embeding space - Minkovsky sphace needs - predicts $D = 26$ dimensional embeded - balck space. Next step toward interecting string theory, strings in external - bakground fields - string $\sigma-$models. Renormdinamic motion equatins for external fields - "coupling constants" are

The propagation of strings in nontrivial spacetime backgrounds is of interest because one expects string consistency conditions to restrict the possible backgrounds and so give information on possible compactifications, cosmologies, etc. Spacetime backgrounds in which strings may consistently propagate correspond to conformally invariant non-linear signal models and the condition of conformal invariance can be expressed as a field equation for the background spacetime.

Holography principle or AdS/CFT correspondence

According to the holography principle , quantum gravity and string theory on certain manifolds with boundary can be studied in terms of a conformal field theory on the boundary. The holography principle postulates the existence of strong ties between certain field theories on a $d+1$ dimensional manifold M^{d+1}-bulk space and d dimensional manifold N^d-boundary respectively.

Holography principle or AdS/CFT correspondence

The holography principle was originally suggested by 't Hooft in order to reconcile unitarity with gravitational collapse. In this case M is a black hole and N is the event horizon. Thus the bulk space should be imagined as (a part of) space–time. There are other models where the boundary can play the role of space–time with the bulk space involving an extra dimension (e. g. the renormalization group scale) and a Kaluza–Klein type reduction, and brane world scenarios where one models our universe as a brane in higher dimensional space–time, with gravity confined to the brane. In condenced matter physics,e.g. graphen 2-dimensional boundary and QED 3+1 dimensional correspondence

Idential particles, charge conservation, extended particles

Feynman, I know why
there are identical particles -
Wheeler

According to Wheeler, elementary particles are identical becaus there are just one particle which lives in the bulk and traverce our Univerce in different points. If so we will have charge nonconservation, as pair of particle and antiparticle may appeare not simaltinuously.

Idential particles, charge conservation, extended particles

If we have string in the bulk, then particle and antiparticle born in one point and we have electric charge conservation. If we have a membrane in the bulk, we may observe production of corresponding string and so on. E.g. ball lightning may be indication on four dimensional particle in the bulk.

The equation of state in nuclear physics relates the energy density with the pressure and is the main ingredient in the understanding of neutron stars as well as heavy ion collisions.

Only nuclear physics has provided experimental support for the usefulness of superalgebras in Nature. The theory is based on the superalgebra $SU(6/M)$ in which the 3-dimensional rotation group which assigns spins to nuclear states is embedded. This provides a classification scheme for many low lying nuclear states of several even-even (bosonic) and even-odd (fermionic) nuclei in the platinum-gold region. The classification in a single irrep. of $SU(6/M)$ predicts: an energy formula for parterns of many nuclear levels, relations among decay rates of excited states and relations between nucleon-transfer reactions among such nuclei. Superalgebras have thus provided a rather general approach for correlating and organizing nuclear

Spin–orbit coupling is a relativistic effect that connects the spin angular momentum of the charge carrier with the electrostatic potential of its environment. Thus, it can be and has been used for spin manipulation. The fact that relativistic effects are larger in heavy atoms (such as Au) than in lighter ones (such as Ag) results from the steeper potential gradients at the nucleus when the atomic number Z is large.

A standart model of predator-prey interactions, the Lotka-Volterra model (LVM) is defined by the following system of equations

$$\dot{x}_n = a_n x_n + x_n k_{nm} x_m, \; 1 \le n, m \le N \tag{62}$$

The most famous special case of Lotka-Volterra system is the KM system (also known as the Volterra system) defined by

$$\dot{x}_n = x_n(x_{n+1} - x_{n-1}), \; x_0 = 0, \; x_{N+1} = 0, \tag{63}$$

which was first solved by Kac and van-Moerbeke, using a discrete version of inverse scattering due to Flaschka.

In the paper, the following three dimensional LVM were considared

$$\dot{x}_1 = x_1(rx_2 + sx_3),$$
$$\dot{x}_2 = x_2(tx_3 - rx_1),$$
$$\dot{x}_3 = x_3(-sx_1 - tx_2), \quad (64)$$

After change of sign $s \to -s$ we put the system in the form

$$\dot{x}_1 = rx_1(x_2 - s/rx_3),$$
$$\dot{x}_2 = tx_2(x_3 - r/tx_1),$$
$$\dot{x}_3 = sx_3(x_1 - t/sx_2), \quad (65)$$

We discuss integrable dynamical systems

$$\dot{x}_n = v_n(x), \;\; n = 1, 2, ..., N \tag{66}$$

in the standard configuration space R^N. Suppose that we have managed to find a pair of matrices, L and A (the so-called Lax pair), whose elements depend on the dynamical variables x so that equations (433) are equivalent to the matrix equation

$$\dot{L} = [L, A] = LA - AL \tag{67}$$

This form of writing the equations of motion will be called a Lax representation. It follows from (67) that $L(t)$ undergoes a similarity transformation

$$L(t) = U^{-1}L(0)U(t), \;\; U(t) = e^{tA}U(0) \tag{68}$$

Therefore, the eigenvalues of $L(t)$ are time-independent and so are integrals of motion. Equivalently, the matrix $L(t)$ is isospectrally deformed with time. Instead of the eigenvalues it is often more convenient to take their symmetric functions as integrals of motion, for example,

$$H_n = tr(L^n)/n \tag{69}$$

If in such a way one can find K functionally independent integrals of motion and show that they are in involution, then the system in question is completely integrable, when $N = 2M,\ K = M$. When $N - 1 \geq K > M$, the system is superintegrable. From $K = M + 1$ to $K = 2M - 1$, we might have a fine structure of superintegrability. With only one motion integral, Hamiltonian, we have an ergodic system: trajectories cover energy shell homogeneously. With more than one integrals of motion the energy shell divides on invariant subspaces, on less ergodic structures.

Let us consider the following dynamical system

$$\dot{a}_n = e^{a_{n+1}} - e^{a_{n-1}}, \; n \in \mathbb{Z}, \; a_n \in \mathbb{C} \tag{70}$$

which can be presented as

$$\begin{aligned}
&\dot{L} = [L, A], \\
&L_{nm} = e^{a_n/2}\delta_{n,m+1} + e^{a_m/2}\delta_{n+1,m}, \\
&A_{nm} = (e^{(a_n+a_{n-1})/2}\delta_{n,m+2} - e^{(a_m+a_{m-1})/2}\delta_{n+2,m})/2, \\
&L_{nm} = L_{mn}, \; A_{nm} = -A_{mn}, \\
&H_n = trL^n/n, \; \dot{H}_n = 0, \; H_1 = 0, \; H_2 = \sum_n e^{a_n}
\end{aligned} \tag{71}$$

Indeed, let as consider one nontrivial element of the matrix equation,

$$\dot{L}_{n,n-1} = \dot{a}_n e^{a_n/2}/2 = L_{n,n+1}A_{n+1,n-1} - A_{n,n-2}L_{n-2,n-1}$$
$$= e^{a_n/2}/2(e^{a_{n+1}} - e^{a_{n-1}}) \quad (72)$$

For the following deformed dynamical system

$$\dot{a}_n = \gamma_n(e^{a_{n+1}} - e^{a_{n-1}}),\ n \in \mathbb{Z},\ a_n \in \mathbb{C} \tag{73}$$

which can be presented as

$$\begin{aligned}
&\dot{L} = [L, A],\\
&L_{nm} = e^{a_n/2}/\gamma_n^{1/2}\delta_{n,m+1} + e^{a_m/2}/\gamma_m^{1/2}\delta_{n+1,m},\\
&A_{nm} = (e^{(a_n+a_{n-1})/2}\gamma_n^{1/2}\gamma_{n-1}^{1/2}\delta_{n,m+2}\\
&-e^{(a_m+a_{m-1})/2}\gamma_m^{1/2}\gamma_{m-1}^{1/2}\delta_{n+2,m})/2,\\
&L_{nm} = L_{mn},\ A_{nm} = -A_{mn},\\
&H_n = trL^n/n,\ \dot{H}_n = 0,\ H_1 = 0,\ H_2 = \sum_n e^{a_n}/\gamma_n
\end{aligned} \tag{74}$$

After change of the variables $a_n = \gamma_n b_n$, we see that the deformed system is eqvivalent to the following system

$$\dot{b}_n = e^{\gamma_{n+1} b_{n+1}} - e^{\gamma_{n-1} b_{n-1}}, \ n \in \mathbb{Z}, \ b_n \in \mathbb{C} \tag{75}$$

We may generate finite dimensional sistems by imposing periodic ocoditions: $a_{n+N} = a_n$. In the minimal nontrivial case $N = 3$,

$$L = \begin{pmatrix} 0 & e^{a_2/2}/\gamma_2^{1/2} & 0 & 0 \\ e^{a_2/2}/\gamma_2^{1/2} & 0 & e^{a_3/2}/\gamma_3^{1/2} & 0 \\ 0 & e^{a_3/2}/\gamma_3^{1/2} & 0 & e^{a_1/2}/\gamma_1^{1/2} \\ 0 & 0 & e^{a_1/2}/\gamma_1^{1/2} & 0 \end{pmatrix} \quad (76)$$

$$L^2 = \begin{pmatrix} \frac{e^{a_2}}{\gamma_2} & 0 & \frac{e^{(a_2+a_3)/2}}{(\gamma_2\gamma_3)^{1/2}} & 0 \\ 0 & \frac{e^{a_2}}{\gamma_2} + \frac{e^{a_3}}{\gamma_3} & 0 & \frac{e^{(a_1+a_3)/2}}{(\gamma_1\gamma_3)^{1/2}} \\ \frac{e^{(a_2+a_3)/2}}{(\gamma_2\gamma_3)^{1/2}} & 0 & \frac{e^{a_3}}{\gamma_3} + \frac{e^{a_1}}{\gamma_1} & 0 \\ 0 & \frac{e^{(a_1+a_3)/2}}{(\gamma_1\gamma_3)^{1/2}} & 0 & \frac{e^{a_1}}{\gamma_1} \end{pmatrix} \quad (77)$$

$$L = \begin{pmatrix} 0 & e^{a_2/2}/\gamma_2^{1/2} & e^{a_1/2}/\gamma_1^{1/2} \\ e^{a_2/2}/\gamma_2^{1/2} & 0 & e^{a_3/2}/\gamma_3^{1/2} \\ e^{a_1/2}/\gamma_1^{1/2} & e^{a_3/2}/\gamma_3^{1/2} & 0 \end{pmatrix} \tag{78}$$

$$L^2 = \begin{pmatrix} \frac{e^{a_1}}{\gamma_1} + \frac{e^{a_2}}{\gamma_2} & \frac{e^{(a_1+a_3)/2}}{(\gamma_1\gamma_3)^{1/2}} & \frac{e^{(a_2+a_3)/2}}{(\gamma_2\gamma_3)^{1/2}} \\ \frac{e^{(a_1+a_3)/2}}{(\gamma_1\gamma_3)^{1/2}} & \frac{e^{a_2}}{\gamma_2} + \frac{e^{a_3}}{\gamma_3} & \frac{e^{(a_1+a_2)/2}}{(\gamma_1\gamma_2)^{1/2}} \\ \frac{e^{(a_2+a_3)/2}}{(\gamma_2\gamma_3)^{1/2}} & \frac{e^{(a_1+a_2)/2}}{(\gamma_1\gamma_2)^{1/2}} & \frac{e^{a_1}}{\gamma_1} + \frac{e^{a_3}}{\gamma_3} \end{pmatrix} \tag{79}$$

$$H_3 = trL^3/3 = 2\frac{e^{(a_1+a_2+a_3)/2}}{(\gamma_1\gamma_2\gamma_3)^{1/2}}, \qquad (80)$$

So, we have the following integral of motion

$$H = a_1 + a_2 + a_3 = 2\log(H_3(\gamma_1\gamma_2\gamma_3)^{1/2}/2) \qquad (81)$$

When $a_n \in \mathbb{C}$,

$$\begin{aligned} &H = a_1 + a_2 + a_3 = H_0 + 2\pi ni, \\ &a_n = a + 2\pi i/3k_n, \; H_0 = 3a, \; k_1 + k_2 + k_3 = n, \\ &H_0 = \pm 1 \Rightarrow a = \pm 1/3, \\ &a_{nm} = a_n + a_m = A + 2\pi i/3(k_n + k_m), \; A = 2a(82) \end{aligned}$$

$$(L^4)_{11} = (e^{a_1}/\gamma_1 + e^{a_2}/\gamma_2))^2 + e^{(a_1+a_3)}/(\gamma_1\gamma_3)$$
$$+e^{(a_2+a_3)}/(\gamma_2\gamma_3)$$
$$(L^4)_{22} = (e^{a_2}/\gamma_2 + e^{a_3}/\gamma_3))^2 + e^{(a_1+a_2)}/(\gamma_1\gamma_2)$$
$$+e^{(a_1+a_3)}/(\gamma_1\gamma_3)$$
$$(L^4)_{33} = (e^{a_1}/\gamma_1 + e^{a_3}/\gamma_3))^2 + e^{(a_1+a_2)}/(\gamma_1\gamma_2)$$
$$+e^{(a_2+a_3)}/(\gamma_2\gamma_3)$$
$$trL^4 = 2(\frac{e^{a_1}}{\gamma_1} + \frac{e^{a_2}}{\gamma_2} + \frac{e^{a_3}}{\gamma_3})^2 = 2(trL^2)^2 \tag{83}$$

$$A = \begin{pmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{pmatrix}$$
$$A_{12} = e^{(a_1+a_3)/2}(\gamma_1\gamma_3)^{1/2}$$
$$A_{13} = -e^{(a_2+a_3)/2}(\gamma_2\gamma_3)^{1/2}$$
$$A_{23} = e^{(a_1+a_2)/2}(\gamma_1\gamma_2)^{1/2} \tag{84}$$

e.g.

$$\dot{L}_{12} = \dot{a}_2 e^{a_2}/\gamma^{1/2} = L_{13}A_{32} - A_{13}L_{32}$$
$$= \gamma_2^{1/2} e^{a_2}(e^{a_3} - e^{a_1}) \tag{85}$$

For $N = 4$,

$$L = \begin{pmatrix} 0 & e^{a_2/2}/\gamma_2^{1/2} & 0 & e^{a_1/2}/\gamma_1^{1/2} \\ e^{a_2/2}/\gamma_2^{1/2} & 0 & e^{a_3/2}/\gamma_3^{1/2} & 0 \\ 0 & e^{a_3/2}/\gamma_3^{1/2} & 0 & e^{a_4/2}/\gamma_4^{1/2} \\ e^{a_1/2}/\gamma_1^{1/2} & 0 & e^{a_4/2}/\gamma_4^{1/2} & 0 \end{pmatrix} \quad (86)$$

$$H_2 = trL^2/2 = \sum_{n=1}^{4} e^{a_n}/\gamma_n \quad (87)$$

$$L = \begin{pmatrix} 0 & L_{12} & 0 & L_{14} \\ L_{12} & 0 & L_{23} & 0 \\ 0 & L_{23} & 0 & L_{34} \\ L_{14} & 0 & L_{34} & 0 \end{pmatrix} \tag{88}$$

$$L^2 = \begin{pmatrix} L_{11}^2 & 0 & L_{13}^2 & 0 \\ 0 & L_{22}^2 & 0 & L_{24}^2 \\ L_{13}^2 & 0 & L_{33}^2 & 0 \\ 0 & L_{24}^2 & 0 & L_{44}^2 \end{pmatrix} \tag{89}$$

$$
\begin{aligned}
L_{11}^2 &= \frac{e^{a_1}}{\gamma_1} + \frac{e^{a_2}}{\gamma_2}, \\
L_{13}^2 &= \frac{e^{(a_2+a_3)/2}}{(\gamma_2\gamma_3)^{1/2}} + \frac{e^{(a_1+a_4)/2}}{(\gamma_1\gamma_4)^{1/2}}, \\
L_{22}^2 &= \frac{e^{a_2}}{\gamma_2} + \frac{e^{a_3}}{\gamma_3}, \\
L_{24}^2 &= \frac{e^{(a_1+a_2)/2}}{(\gamma_1\gamma_2)^{1/2}} + \frac{e^{(a_3+a_4)/2}}{(\gamma_3\gamma_4)^{1/2}}, \\
L_{33}^2 &= \frac{e^{a_3}}{\gamma_3} + \frac{e^{a_4}}{\gamma_4}, \\
L_{44}^2 &= \frac{e^{a_1}}{\gamma_1} + \frac{e^{a_4}}{\gamma_4}
\end{aligned}
\tag{90}
$$

$$
\begin{aligned}
&trL^3 = 0, \\
&trL^4 = tr(L^2)^2 \\
&= (L^2_{11})^2 + (L^2_{13})^2 \\
&+(L^2_{22})^2 + (L^2_{24})^2 \\
&+(L^2_{33})^2 + (L^2_{13})^2 \\
&+(L^2_{44})^2 + (L^2_{24})^2
\end{aligned} \tag{91}
$$

For $N = 4$, from motion equations we find the following integrals of motion

$$\frac{a_1}{\gamma_1} + \frac{a_3}{\gamma_3} = H_1, \quad \frac{a_2}{\gamma_2} + \frac{a_4}{\gamma_4} = H_2 \tag{92}$$

and Nambu-Poisson representation

$$\dot{a}_n = \{a_n, H_1, H_2, H_3\} = f_{nmkl}\frac{\partial H_1}{\partial a_m}\frac{\partial H_2}{\partial a_k}\frac{\partial H_3}{\partial a_l},$$
$$f_{nmkl} = \rho\epsilon_{nmkl}, \ \rho = \gamma_1\gamma_2\gamma_3\gamma_4,$$
$$H_1 = \frac{a_1}{\gamma_1} + \frac{a_3}{\gamma_3}, \ H_2 = \frac{a_2}{\gamma_2} + \frac{a_4}{\gamma_4},$$
$$H_3 = e^{a_1}/\gamma_1 + e^{a_2}/\gamma_2 + e^{a_3}/\gamma_3 + e^{a_4}/\gamma_4 \tag{93}$$

For even values $N = 2M$, from motion equations we have the following two motion integrals

$$H_1 = a_1/\gamma_1 + a_3/\gamma_1 + \ldots + a_{2M-1}/\gamma_{2M-1},$$
$$H_2 = a_2/\gamma_2 + a_4/\gamma_4 + \ldots + a_{2M}/\gamma_{2M} \qquad (94)$$

We may put $a_0 = a_{N+1} = -\infty$ and consider corresponding open chane dynamical system

$$\dot{a}_1 = \gamma_1 e^{a_2},$$
$$\dot{a}_n = \gamma_n (e^{a_{n+1}} - e^{a_{n-1}}),\ 2 \leq n \leq N-1,\ a_n \in \mathbb{C},$$
$$\dot{a}_N = -\gamma_{N-1} e^{a_{N-1}} \tag{95}$$

For $N = 3$, we have the following motion equations and integrals

$$\begin{aligned}
\dot{a}_1 &= \gamma_1 e^{a_2}, \\
\dot{a}_2 &= \gamma_2(e^{a_3} - e^{a_1}), \\
\dot{a}_3 &= -\gamma_3 e^{a_2}, \\
H_1 &= a_1/\gamma_1 + a_3/\gamma_3, \\
H_2 &= e^{a_1}/\gamma_1 + e^{a_2}/\gamma_2 + e^{a_3}/\gamma_3
\end{aligned} \tag{96}$$

We may solve the system (98) as

$$
\begin{aligned}
a_3 &= \gamma_3(H_1 - a_1/\gamma_1),\\
a_2 &= \ln(\gamma_3(H_2 - e^{a_1}/\gamma_1 - e^{a_3}/\gamma_3),\\
\int_{a_1(t_0)}^{a_1(t)} da/f(a) &= t - t_0,\\
f(a) &= \gamma_1\gamma_2(H_2 - e^{a}/\gamma_1 - e^{\gamma_3(H_1-a/\gamma_1)}/\gamma_3)
\end{aligned}
\quad (97)
$$

For $N = 4$, we have the following motion equations and integral

$$\begin{aligned}
\dot{a}_1 &= \gamma_1 e^{a_2},\\
\dot{a}_2 &= \gamma_2(e^{a_3} - e^{a_1}),\\
\dot{a}_3 &= \gamma_3(e^{a_4} - e^{a_2}),\\
\dot{a}_4 &= -\gamma_4 e^{a_3},\\
H &= e^{a_1}/\gamma_1 + e^{a_2}/\gamma_2 + e^{a_3}/\gamma_3 + e^{a_4}/\gamma_4 = H_1 + H_2,\\
H_1 &= a_1/\gamma_1 + a_3/\gamma_3,\\
H_2 &= a_2/\gamma_2 + a_4/\gamma_4,
\end{aligned} \tag{98}$$

$$
\begin{aligned}
&e^{a_2} = \dot{a}_1/\gamma_1,\ \ e^{a_3} = -\dot{a}_4/\gamma_4,\\
&e^{a_1} = e^{a_3} - \dot{a}_2/\gamma_2 = -\dot{a}_4/\gamma_4 - \dot{a}_2/\gamma_2 = -\dot{H}_2,\\
&e^{a_4} = e^{a_2} + \dot{a}_3/\gamma_3 = \dot{a}_1/\gamma_1 + \dot{a}_3/\gamma_3 = \dot{H}_1,\\
&e^{a_4} - e^{a_1} = \dot{H} = 0\\
&a_4 = a_1 + 2n\pi i,\\
&a_3 = a_2 + \ln(\gamma_1/\gamma_4) + (2n+1)\pi i,\\
&H = (1/\gamma_1 + 1/\gamma_4)e^{a_1} + (1/\gamma_2 - \gamma_1/\gamma_3/\gamma_4)e^{a_2},\\
&e^{a_2} = H/(1/\gamma_2 - \gamma_1/\gamma_3/\gamma_4)\\
&-(1/\gamma_1 + 1/\gamma_4)/(1/\gamma_2 - \gamma_1/\gamma_3/\gamma_4)e^{a_1} = f(a_1) \quad (99)
\end{aligned}
$$

Now we have solution as

$$\int_{a_1(0)}^{a_1} \frac{da}{f(a)} = t,$$
$$a_2 = \ln f(a_1),$$
$$a_3 = a_2 + \ln(\gamma_1/\gamma_4) + (2n+1)\pi i$$
$$a_4 = a_4 = a_1 + 2n\pi i \qquad (100)$$

Feynman integrals enter the evaluation of many physical observable quantities in particle physics, gravitational physics, statistical physics, and solid-state physics.

The polylogarithm (PL) function is defined by a power series in z, which is also a Dirichlet series in s:

$$Li_s(z) = \sum_{k\geq 1} \frac{z^k}{k^s}, \;\; Li_1(z) = \ln \frac{1}{1-z} = \int_0^z \frac{dt}{1-t} \qquad (101)$$

This definition is valid for arbitrary complex order s and for all complex arguments z with $|z| < 1$; it can be extended to $|z| \geq 1$ by the process of analytic continuation. (Here the denominator n^s is understood as $\exp(s \ln(n))$. The special case $s = 1$ involves the ordinary natural logarithm, $Li_1(z) = -\ln(1-z)$, while the special cases $s = 2$ and $s = 3$ are called the dilogarithm (also referred to as Spence's function) and trilogarithm respectively.

The name of the function comes from the fact that it may also be defined as the repeated integral of itself:

$$Li_{s+1}(z) = \int_0^z \frac{dt}{t} Li_s(t),$$
$$Li_2 = \int_0^z \frac{dt}{t} Li_1(t) = \int_0^z \frac{dt}{t} \ln \frac{1}{1-t} \tag{102}$$

For nonpositive integer orders s, the polylogarithm is a rational function.

Multiple polylogarithms (MPLs)

MPLs are a class of special functions defined by

$$G(a_1, ..., a_n; x) = \int_1^x \frac{dt}{t - a_1} G(a_2, ..., a_n; t), \ \ G(; x) = 1 \tag{103}$$

In the special case where all the a_i's are zero, we have

$$G(0, ..., 0; x) = \frac{1}{n!} \ln^n x \tag{104}$$

Recently CDF collaboration has published [Aaltonen et al. (CDF) 2022] new measured value of the W-boson mass

$$m_W = 80.4335 \pm 0.0094 \, GeV \qquad (105)$$

which is in excess of the SM prediction [G.Patrignani et al. (Particle Data Group) 2016]

$$m_{SMW} = 80.375 \pm 0.006 \, GeV \qquad (106)$$

at 7σ level. A lot of explanations of this result has appeared.

New Physics

Assuming that the measurement at CDF II is correct, we discuss the possibility to explain the anomaly in the constituent Higgs (125 GeV), W(80 GeV) and Z(91 GeV) model. Nearly a decade after the discovery of the Higgs boson at LHC, the true shape of the Higgs sector is still unknown. On the other hand, the Higgs sector is often extended from the minimal form in the standard model (SM) for models beyond the SM (BSM), which can explain neutrino oscillations, dark matter and baryon asymmetry of the Universe. Therefore, unveiling the structure of the Higgs sector is quite important to narrow down BSM scenarios.

New Physics

The W boson anomaly is a signature of BSM scenarios. Given the sizable difference in the W mass, the new physics scale needs to be not too far above the TeV scale. Moreover, the new physics could be at the electroweak scale if generating this discrepancy via loops. Direct new physics searches at the LHC and other experiments will certainly reveal or rule out the new physics model candidates. The electroweak precision program and the Higgs precision program will also further extract the possible imprints of new physics.

We propose milimal supersimmetric constituent model with scalar ϕ and fermion ψ supermultiplet (ϕ, ψ) with valense mass $m \sim 40$ Gev. In this model, W is $\bar{\psi}\psi$ vector bound state and H is three ϕ bound state.
In the SM and its extensions the W-boson mass can be evaluated from

$$m_W^2(1 - m_W^2/m_Z^2) = a(1+\delta) = A, \;\; a = \frac{\pi\alpha}{\sqrt{2}G_F} \quad (107)$$

where G_F is the Fermi constant, α is the fine structure constant, and δ represents the sum of all non-QED loop diagrams to the muon-decay amplitude which itself depends on M_W as well.

We can solve the equation (107) as

$$m_W^2 = (1 \pm \sqrt{1 - 4A/m_Z^2})m_Z^2/2,$$
$$A/m_Z^2 < 1/4,\ \ m_Z^2 > 4A. \tag{108}$$

To the observed value of the m_W corrseponds

$$m_W^2 = (1 - \Delta)m_Z^2 = m_Z^2 - A + ...,$$
$$\Delta = 1 - \frac{m_W^2}{m_Z^2} = 0.223 \tag{109}$$

The second solution is

$$m_{W2}^2 = \Delta m_Z^2 = A + ...,\ \ m_{W2} = \sqrt{\Delta} m_Z = 43.0\ GeV \tag{110}$$

The neutrino ν was proposed in December 1930 by Pauli in order to explain the continuous energy-spectrum of the electrons measured in β-decays. Pauli named these particles neutrons because of their uncharged nature. However, after the discovery of the heavier and uncharged particle by Chadwick in 1932, Fermi changed this name to the neutrino.

Let us consider the following **discrete dynamics**:

$$S_{n+1} + S_{n-1} = \Phi(S_n), \tag{111}$$

which is obviously a (discrete) time (n) invertible in this implicit form. In the explicit form

$$S_{n+1} = F(S_n, S_{n-1}) = \Phi(S_n) - S_{n-1} \tag{112}$$

it is not obviously time invertible. If we take two step time lattice-make simplest discrete RD step and from one component-scalar S(n) construct two component-spinor $\Psi(n)$, we obtain explicit time invertible dynamics

$$\Psi_{n+1} = \Omega(\Psi_n), \ \Psi_{n+1} = \begin{pmatrix} S_{n+2} \\ S_{n+1} \end{pmatrix}, \ \Psi_n = \begin{pmatrix} S_n \\ S_{n-1} \end{pmatrix} \tag{113}$$

This **dynamical mechanism of origin spin** which connects time inversion symmetry and the spin was invented when was constructed the theory of quanputers [Makhaldiani, 2011.2].

This mechanism indicates that with time inversion symmetry we can have only composed scalar fields. With the discovery of the Higgs particle with mass 125 GeV, a nice number $m_W/m_H \simeq 2/3$ appear, which, at least for me, indicates for composed nature of W and H, with a same mass of about 40 GeV two and three valence constituents correspondingly. The fermion constituents ψ_n^a of W and scalar constituents φ_n^a of H compose scalar super multiplet (φ_n^a, ψ_n^a) with a flavor index n and color index a. Another notation is (h, sh)-(He, She:).

If we extrapolate the SM value of $\alpha^{-1}(m_Z)$ to electron masse scale, we find $\alpha^{-1}(m_e) = 137.0$
Coupling constant unification at $\alpha_u^{-1} = 29.0$ and scale $10^{16} GeV$ in MSSM [Makhaldani, 2014] has a relict on the SM scale: $\alpha_2^{-1}(m) = 29.0$ at $m = 41 GeV$.
The 40 GeV constituents may be good candidates in dark matter particles.
Recent (missing) discovery of the second Higgs particle with mass $M_H = 750 GeV$ indicates an interesting structures. It is curious that $M_H/m_h = 750/125 = 6$!

Super QCD is a supersymmetric gauge theory which resembles quantum chromodynamics (QCD) but contains additional particles and interactions which render it supersymmetric. The most commonly used version of super QCD is in 4 dimensions and contains one Majorana spinor supercharge.

Monopole mechanism of confinement and (multi)particle production

Let me draw the following scenario of confinement and particle production in QCD. In classical gluodynamics (in the simplest case, $A_0 = 0$ and finite energy assumption) particle-like solutions, monopoles, can not be due to scale (conformal) invariance. For nontrivial asymptotic A_0, we may have monopole states. In quantum gluodynamics we have not conformal invariance beyond the renormdynamic fixed points and monopoles can exist. In (one coupling constant) quantum gluodynamics we have the trivial ultraviolet fixed point at $g = 0$ and nontrivial infrared fixed point at some $g = g_c$.

Monopole mechanism of confinement and (multi)particle production

The Higher energy multiparticle production processes follow the following scenario: higher energy quarks and gluons (perturbatively) produce lower energy gluons and quarks until the intermediate energy-scale where running coupling constant reach the selfdual (fixed) value beyond of which monopoles start to produce. Later at the valence quark energies-scales, (at which $\alpha_s = 2$,) monopoles become unstable and decay into hadrons.

Confinement, Space Dimension and Supersymmetry

With exact SUSY we have cofinement by dimentional counting: superspace dimension is zero on the hadronic scale, hadrons are pointlike, color is confined inside hadrons. For SM QCD this picture indicates that at the hadronic scale we have effective SQCD, which contains scalar quarks.

Sonoluminescence refers to that remarkable phenomenon in which a small bubble of air injected into a container of water and suspended in a node of a strong acoustic standing wave emits light. More precisely, if it is driven with a standing wave of about 20,000 Hz at an overpressure of about 1 atm, the bubble expands and contracts in concert with the wave, from a maximum radius $R \sim 10^{-3}$ cm to a minimum radius of $r \sim 10^{-4}$ cm. Note that $R/r \sim 10$ one order of magnitude as in QCD with size for hadrons $R \sim 10^{-13}$ cm and perturbative size $r \sim 10^{-14}$ cm. Exactly at minimum radius roughly 1 million optical photons are emitted, for a total energy liberated of 10 MeV.

Acoustic cavitation-the formation and implosive collapse of bubbles-occurs when a liquid is exposed to intense sound. Cavitation can produce the emission of light, or sonoluminescence. The concentration of energy during the collapse is enormous: the energy of an emitted photon can exceed the energy density of the sound field by about twelve orders of magnitude, and it has long been predicted that the interior bubble temperature reaches thousands of degrees Kelvin during collapse.

Color confinement constitutes for quarks and gluons something like an event horizon which they can never cross. Signals transmitted to the outside world from inside such a horizon cannot contain information and must thus be of thermal nature.
We consider multihadron production in high energy collisions as the QCD counterpart of Hawking-Unruh radiation, encountered in black holes and for accelerated observers. This is shown to provide a common origin for thermal multihadron production.

Fundamental constituents of QCD, quarks and gluons, are colored, and by color confinement they are not allowed to exist as individual entities in the world we can observe, a single quark or gluon can never be observed as an isolated object, in contrast to a single proton or electron, for example. Other case where things remain in principle beyond our reach are given by black holes.

A black hole is the final stage of a neutron star after gravitational collapse [Fang, Ruffini 1983]. It has a mass M concentrated in such a small volume that the resulting gravitational field confines all matter and even photons to remain inside the 'event horizon' R of the system: no causal connection with the outside world is possible. Could it be that a hadron, containing colored constituents that cannot get out, is something like a black hole of strong interaction physics?

Strong interaction black holes

In general relativity, forces are assumed to modify the underlying space-time manifold. The space-time metric of this manifold is given by

$$ds^2 = qdt^2 - q^{-1}dr^2 - r^2 d\Omega^2 \tag{114}$$

with r and Ω specifying the spatial part, and t the time; for flat space, we have $q = 1$. The event horizon of a (spherical) black hole is determined by the point at which this metric is so deformed that space and time interchange, i.e., the point at which $q = 0$.

Strong interaction black holes

For gravitation, the Einstein equations give

$$q = 1 - \frac{2GM}{r} \tag{115}$$

which leads to the Schwarzschild radius of a black hole,

$$R = 2GM \tag{116}$$

where $G = 6.7 \times 10^{-39} GeV^{-2}$ is the gravitational constant and M the mass of the system. It is instructive to consider the Schwarzschild radius of a typical hadron, assuming a mass $m \sim 1$ GeV,

$$R_h = 1.3 \times 10^{-38} GeV^{-1} = 2.7 \times 10^{-52} cm \tag{117}$$

Strong interaction black holes

To become a gravitational black hole, the mass of the hadron would thus have to be compressed into a volume more than 10^{100} times smaller than its actual volume (with a radius of about 1 fm).

On the other hand, if instead we increase the interaction strength from gravitation to strong interaction [Satz 2012], we gain in the resulting 'strong' Schwarzschild radius R_s a factor α_s/Gm^2, where α_s is the dimensionless strong coupling and Gm^2 the corresponding dimensionless gravitational coupling for the case in question. This leads to

$$R_s = \frac{2\alpha_s}{m} \tag{118}$$

which for the limiting value of the strong coupling, $\alpha_s = 3$, gives $R_s = 1.2$ fm

In other words, the confinement radius of a hadron is about the size of its 'strong' Schwarzschild radius, so that we could picture quark confinement as the strong interaction version of the gravitational confinement in black holes.

Hawking predicted that when quantum matter effects are taken into account, a stationary black hole emits thermal radiation with the Planckian power spectrum characteristic of a perfect black-body at a fixed temperature. A radiating black hole is non-stationary as it loses energy and the horizon continuously shrinks.

Hierarchy Problem

The socalled hierarchy problem - in other words, our difficulty in answering the question of why the characteristic scale of gravity, $M_P \sim 10^{19}$ GeV, is 16 orders of magnitude larger than the Electro-Weak scale, $M_{EW} \sim 1$ TeV - could be solved by assuming the existence of extra dimensions in the Universe
[Antoniadis, Arkani-Hamed, Dimopoulos and Dvali 1998].
In this idea the traditional picture of Planck-length-sized additional spacelike dimensions ($l_P \simeq 10^{-33}$ cm) was abandoned, and the extra dimensions could have a size as large as 1 mm.

The upper bound on the size of the proposed Large Extra Dimensions actually matched the smallest length scale down to which the force of the gravitational interactions, and thus their $1/r^2$ dependence, had been measured. If extra dimensions of that size did exist, gravitational interactions would have a completely different dependence on r in scales smaller than 1 mm.

On the other hand, electromagnetic, weak and strong forces are indeed sensitive to the existence of extra dimensions. If, for example, gauge bosons were allowed to propagate in the extra-dimensional spacetime, their interactions would be modified beyond any acceptable phenomenological limits unless the size of the extra dimensions was smaller than 10^{-16} cm.

This problem was resolved under the assumption that all particles experiencing this type of interactions, in other words, all ordinary matter, is restricted to live on a (3+1)-dimensional hypersurface, a 3- brane, that has a width along the extra dimensions of, at most, the above order. The 3-brane, playing the role of our four-dimensional world, is then embedded in the higher-dimensional spacetime, usually called the bulk, in which only gravity can propagate.

This is analog of the graphen electrodynamics where electromagnetic forsies propagate in (3+1)-dimensional hypersurface but graphen electrons are confined in the graphen (2+1)-dimensional hypersurface.

Planck scale, M_P , is only an effective energy scale derived from the fundamental higher-dimensional one, M_p, through the following relation

$$M_P^2 = M_p^2 (M_1 R_1)^{n_1} ... (M_k R_k)^{n_k}, \; l_P << R_1 << ... << R_k,$$
$$n_1 + ... + n_k = n. \tag{119}$$

From the above, it becomes clear that, if the volume of the compact space, $V \sim R_1^{n_1} ... R_k^{n_k}$, is large, i.e if $R_1, ..., R_k >> l_P$, then the (4 + n)- dimensional Planck mass, M_p, will be much lower than the 4-dimensional one, M_P. If one chooses $M_p = M_{EW}$, then the above expression provides a relation between the scale of gravity and the scale of particle interactions. In the regime

Primordial black holes (PBH) is a hypothetical black holes that formed soon after the Big Bang (BB). In the early universe, high densities and heterogeneous conditions could have led sufficiently dense regions to undergo gravitational collapse, forming black holes. The existence of PBH first proposed by Zel'dovich and Novikov in 1966 and the theory behind their origins was first studied in depth by Stephen Hawking in 1971. Since primordial black holes did not form from stellar gravitational collapse, their masses can be far below stellar mass (2×10^{30} kg).

In the path-integral approach to the quantization of gravity one considers expressions of the form

$$Z(g,\phi) = \int dg d\phi e^{iA} \tag{120}$$

where dg is a measure on the space of metrics g; $d\phi$ is a measure on the space of matter fields ϕ, and $A(g,\phi)$ is the action. In this integral one must include not only metrics which can be continuously deformed into the flat-space metric but also homotopically disconnected metrics such as those of black holes.

The formation and evaporation of macroscopic black holes gives rise to effects such as baryon nonconservation and entropy production. One would therefore expect similar phenomena to occur on the elementary-particle level Quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature

$$T_{BH} \simeq 10^{-6} \frac{M_{\odot}}{M} K^{\circ} \tag{121}$$

This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance: any primordial black hole of mass less than about $10^{15} g$ would have evaporated by now.

There is a Generalized Second Law: $S + A/4$ never decreases, where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons.
This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy. We suppose that it may takes plase also the inverse process of criation of the leptons and baryons from primordial BH in Heavy Ion collitions.

Primordial black holes

Black holes are well-understood general-relativistic objects when their mass M_{BH} far exceeds the fundamental (higher-dimensional) Planck mass ($M_P >> TeV$). As M_{BH} approaches M_P, the BHs become 'stringy' and their properties complex.
When we will ignore this obstacle and estimate the properties of light B_Hs by simple semiclassical arguments, strictly valid for $M_{BH} >> M_P$, because of the unknown stringy corrections, results are approximate estimates.

Consider two partons with the center-of-mass (c.m.) energy M_{BH} moving in opposite directions. Semiclassical reasoning suggests that, if the impact parameter is less than the (higher-dimensional) Schwarzschild radius, a BH with the mass M_{BH} forms. Therefore the total cross section can be estimated from geometrical arguments.

As the collision energy increases, the resulting BH gets heavier and its decay products get colder.

The wavelength $\lambda = 2\pi / T_H$ corresponding to the Hawking temperature is larger than the size of the black hole.

Therefore, the BH acts as a point radiator and emits mostly s waves.

This indicates that it decays equally to a particle on the brane and in the bulk, since it is sensitive only to the radial coordinate and does not make use of the extra angular modes available in the bulk. Since there are many more particles on our brane than in the bulk, this has the crucial consequence that the BH decays visibly to standard model (SM) particles.

In order to find the average multiplicity of particles produced in the process of BH evaporation, we note that the BH evaporation is a blackbody radiation process, with the energy flux per unit of time given by Planck's formula

$$\frac{df}{dx} \sim \frac{x^3}{e^x + q}, \ x = \frac{E}{T_H} \tag{122}$$

where q is a constant, which depends on the quantum statistics of the decay products ($q = -1$ for bosons, 1 for fermions, and 0 for Boltzmann statistics).

The spectrum of the BH decay products in the massless particle approximation is given by

$$\frac{dN}{dE} \sim \frac{x^2}{e^x + q} \tag{123}$$

For averaging the multiplicity

$$< N >=< \frac{M_{BH}}{E} >= a\frac{M_{BH}}{T_H},$$
$$a = \frac{I(1)}{I(2)}, \ I(n) = \int_0^\infty \frac{x^n dx}{e^x + q} \tag{124}$$

The multiplicity of charged particle production is considered the key to understanding the particle production mechanism. The probability $P(n)$ of obtaining n charged particles in the final state is related to the particle production mechanism. It obeys Poisson distribution if the particles are produced in a final-state independent way. Negative binomial distribution (NBD) provides the best description of the high energy multiparticle production processes, has very clear physical interpretation and corresponds to the independently radiating primordial black holes (PBH and monopole) intermadiate states.

As a concrete model, we take a relativistic scalar field model with lagrangian (see e.g. [Makhaldiani, 1980])

$$L = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \frac{g}{n}\varphi^n, \;\; \mu = 0, 1, ..., D-1 \quad (125)$$

In the case

$$n = \frac{2D}{D-2} \quad (126)$$

the coupling constant g is dimensionless, and the model is renormalizable. We take an euklidian form of the QFT which unifies quantum and statistical physics problems.

The main objects of theory are Green functions - correlation functions - correlators,

$$G_m(x_1, x_2, ..., x_m) = < \varphi(x_1)\varphi(x_2)...\varphi(x_m) >$$
$$= Z_0^{-1} \int d\varphi(x)\varphi(x_1)\varphi(x_2)...\varphi(x_m)e^{-S(\varphi)} \quad (127)$$

where $d\varphi$ is an invariant measure

$$d(\varphi + a) = d\varphi. \quad (128)$$

For gaussian actions,

$$S = S_2 = \int dxdy\phi(x)A(x,y)\phi(y) = \varphi \cdot A \cdot \varphi \quad (129)$$

the QFT is solvable,

$$G_m(x_1, ..., x_m) = \frac{\delta^m}{\delta J(x_1)...J(x_m)} lnZ_J|_{J=0},$$

$$Z_J = \int d\varphi e^{-S_2 + J\cdot\varphi} = \exp(\frac{1}{4}\int dxdyJ(x)A^{-1}(x,y)J(y))$$

$$= \exp(\frac{1}{4}J \cdot A^{-1} \cdot J) \quad (130)$$

Non trivial problem is to calculate correlators for non gaussian QFT

In quantum perturbation calculations [Bogoliubov and Shirkov 1959], we find the following corrections to the classical lagrangian

$$\Delta L = (z-1)\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - (z_m-1)\frac{m^2}{2}\varphi^2 - (z_g-1)\frac{g}{n}\varphi^n \quad (131)$$

Corrected, effective, lagrangian becomes

$$L + \Delta L = z\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - z_m\frac{m^2}{2}\varphi^2 - z_g\frac{g}{n}\varphi^n \quad (132)$$

We can restore the (classical) form of the lagrangian, by corresponding renormalization (compensating) transformations,

$$\begin{aligned} \varphi &\Rightarrow z^{-1/2}\varphi = \bar{\varphi} \\ m^2 &\Rightarrow z_m^{-1} z m^2 = \bar{m}^2 \\ g &\Rightarrow z_g^{-1} z^{n/2} g = \bar{g} \end{aligned} \tag{133}$$

So, if we order the quantum correction in some discrete (or continual) way, we can include them step by step, which will be equivalent to the corresponding evolution equations for constants and fields [Wilson, Kogut 1974].

These equations define the evolution from classical theory to quantum one. The quantum corrections often are ill defined, singular or divergent, so we need some regularization. For some field theory models, e.g. Youkawa nuklon-mezon model, quantum corrections invent new structures, in the case, mezon selfinterection, so quantum theory has an extended structure [Bogoliubov and Shirkov 1959].

In this way, we can generate from classical Fermi like models the standard model of particle physics. If the structure elements of a (quantum)field theory model is finite, we have a renormalizable model, we may have also an infinite number of structure elements, in the case of a quantum gravity model e.g.

In the infinitezimal form we have the following renormdynamic motion equations

$$\dot{\varphi} \equiv \frac{\mu d}{d\mu}\bar{\varphi}|_{\mu=\mu_0} = (\frac{\mu\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \eta(g)\frac{m\partial}{\partial m})\varphi$$
$$\equiv D\varphi = -\frac{1}{2}\gamma(g)\varphi,$$
$$\dot{m} = \eta(g)m,\ \eta(g) = \frac{1}{2}(\gamma(g) - \gamma_m(g)),$$
$$\frac{\mu d}{d\mu}\bar{g}|_{\mu=\mu_0} \equiv \frac{d}{dt}g = \dot{g} = \beta(g),\ \beta(g) = \frac{n}{2}\gamma(g) - \gamma_g(g),$$
$$t = \ln(\frac{\mu}{\mu_0}) \tag{134}$$

For correlators, renormdinamic equations are

$$(D + \frac{n}{2}\gamma(g))G_n = 0,\ \ DM_n = \frac{n}{2}\gamma(g)M_n,$$
$$M_n = (G_2)^{-n}G_n \tag{135}$$

For renorminvariant quantities - renomintegrals of motion I,

$$\dot{I} = DI = 0, \tag{136}$$

Solution to the renormdynamic equation for coupling constant, $\bar{g}$, is given in the implicit form by the following integral

$$\int_{g}^{\bar{g}} \frac{dg}{\beta(g)} = \ln \frac{\bar{\mu}}{\mu} \equiv t \tag{137}$$

The mass parameter running is given as

$$m = \bar{m} \exp\left(-\int_{\mu}^{\bar{\mu}} \frac{d\mu}{\mu} \eta(g(\mu))\right) \tag{138}$$

The correlator (renorm)dynamics is given as

$$G_n(p; g, m, \mu) = \exp(\frac{n}{2}\int_{\mu}^{\bar{\mu}} \frac{d\mu}{\mu}\gamma(g(\mu))) \cdot G_n(p; \bar{g}, \bar{m}, \bar{\mu}) \quad (139)$$

As an example in perturbative calculations, let us consider the simplest nonlinear scalar field model φ^3, [Kazakov, Lomidze, Makhaldiani, Vladimirov 1974, Collins 1984]. In $d-$dimensional space-time, we have the following exact $\beta-$function

$$\beta(d, g) = (\frac{d}{2} - 3)g + \beta(g) \quad (140)$$

In one-loop approximation

$$\beta(g) = -\frac{3}{256\pi^3}g^3 + O(g^5) \tag{141}$$

For every value of the coupling constant, $g(\mu)$, in dimension

$$d_c = 6 - 2\beta(g)/g, \tag{142}$$

we have self-similar fractal structure.
Fore small g, in dimensions $d < 6$, we have asymptotic freedom, $g(\mu) \to 0$, when $t = \ln \mu/\mu_0 \to \infty$,

$$g(\mu) = (\frac{\mu}{\mu_0})^{(d/2-3)} g_0. \tag{143}$$

In dimension $d = 6$ and one-loop approximation,

$$g(\mu)^{-2} = g(\mu_0)^{-2} + a \ln \frac{\mu}{\mu_0}, \;\; a = \frac{3}{128\pi^3} \tag{144}$$

In the region of $d > 6$ and small g, we have UV fixed point,

$$\beta(d, g_c) = 0, \qquad g_c^2 = \frac{256\pi^3}{3}(\frac{d}{2} - 3),\ d = 6 + 2\varepsilon,$$
$$\varepsilon < 4.10^{-4},\ g_c < 1 \qquad (145)$$

So, according to the perturbation theory, a fractal lives in dimension $d = 6 + 10^{-3}$

In the case of several integrals of motion, H_n, $1 \leq n \leq N$, we can formulate Renormdynamics as Nambu - Poisson dynamics (see e.g. [Makhaldiani 2007])

$$\dot{\varphi}(x) = [\varphi(x), H_1, H_2, ..., H_N], \tag{146}$$

where φ is an observable as a function of the coupling constants x_m, $1 \leq m \leq M$.
In the case of Standard model [Weinberg 1995], we have three coupling constants, $M = 3$.

Renormdynamics of observable quantities in high energy physics

Let us consider one particle semiinclusive distribution

$$F(p,n) = \frac{d\sigma_n}{\bar{d}p}$$
$$= \frac{1}{(n-1)!}\int \prod_{i=1}^{n-1} \bar{d}p_i' \delta(p_1 + p_2 - p - \Sigma_{i=1}^{n-1} p_i')$$
$$\cdot |M_{n+2}(p_1, p_2, p, p_1', p_2', ..., p_{n-1}'; g(\mu), m(\mu)), \mu)|^2,$$
$$\bar{d}p \equiv \frac{d^3p}{E(p)},\ E(p) = \sqrt{p^2 + m^2}. \tag{147}$$

Renormdynamics of observable quantities in high energy physics

From the renormdynamic equation

$$DM_{n+2} = \frac{\gamma}{2}(n+2)M_{n+2}, \qquad (148)$$

we obtain

$$\begin{aligned}
&DF(p,n) = \gamma(n+2)F(p,n),\\
&DF(p) = \gamma(<n> +2)F(p),\\
&D = \gamma(<n^{k+1}(p)> - <n^k(p)><n(p)>),\\
&DC_k = \gamma <n(p)> (C_{k+1} - C_k(1 + k(C_2 - 1)))\\
&F(p) \equiv \frac{d\sigma}{\bar{d}p} = \sum_n \frac{d\sigma_n}{\bar{d}p},\ <n^k(p)> = \frac{\sum_n n^k d\sigma_n/\bar{d}p}{\sum_n d\sigma_n/\bar{d}p}\\
&C_k = \frac{<n^k(p)>}{\ldots}
\end{aligned} \qquad (149)$$

Universal scaling relations for multi particle cross sections

From dimensional considerations, the following combination of cross sections must be universal function [Koba, Nielsen, Olesen 1972]

$$< n > \frac{\sigma_n}{\sigma} = \Psi(\frac{n}{< n >}), \quad (150)$$

a similar relation for the inclusive cross sections is [Matveev, Sisakian, Slepchenko 1976].

$$< n(p) > \frac{d\sigma_n}{\bar{d}p} / \frac{d\sigma}{\bar{d}p} = \Psi(\frac{n}{< n(p) >}) \quad (151)$$

Indeed, let us define $n-$dimension of observables [Makhaldiani, 1980]

$$[n] = 1, \ [\sigma_n] = -1, \sigma = \Sigma\sigma_n, \ [\sigma] = 0, \ [< n >] = 1, \quad (152)$$

so, the following expression does not depend on any dimensional quantities and must have a corresponding universal form

$$P_n =< n > \frac{\sigma_n}{\sigma} = \Psi(\frac{n}{< n >}). \quad (153)$$

Universal scaling relations for multi particle cross sections

Let us find an explicit form of the universal functions from renormdynamic equations. From the definition of the moments we have

$$C_k = \int_0^\infty dx x^k \Psi(x), \tag{154}$$

so they are independent from different parameters,

$$\begin{aligned} &DC_k = 0 \Rightarrow C_{k+1} = (1 + k(C_2 - 1))C_k \Rightarrow \\ &C_k = (1 + (k-1)(C_2 - 1))...(1 + 2(C_2 - 1))C_2 \end{aligned} \tag{155}$$

Now we can invert momentum transform and find (see [Makhaldiani, 1980] and appendix) universal functions [Ernst, Schmitt 1976], [Darbaidze, Makhaldiani, Sisakian, Slepchenko 1978].

$$\Psi(z) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn z^{-n-1} C_n = \frac{c^c}{\Gamma(c)} z^{c-1} e^{-cz},$$
$$C_2 = 1 + \frac{1}{c} \tag{156}$$

The value of parameter c can be measured from the dispersion low,

$$D = \sqrt{< n^2 > - < n >^2} = \sqrt{C_2 - 1} < n >= \frac{1}{\sqrt{c}} < n > (157)$$

which is in accordance with $n-$dimension counting.

We can calculate also $1/<n>$ correction to the scaling function (see appendix)

$$
\begin{aligned}
&<n>\frac{\sigma_n}{\sigma} = \Psi = \Psi_0\left(\frac{n}{<n>}\right) + \frac{1}{<n>}\Psi_1\left(\frac{n}{<n>}\right),\\
&C_k = C_k^0 + \frac{1}{<n>}C_k^1,\\
&C_k^0 = \int_0^\infty dx x^k \Psi_0(x),\ \ C_k^1 = \int_0^\infty dx x^k \Psi_1(x),\\
&\Psi_1(z) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn z^{-n-1} C_n^1\\
&= \frac{C_2^1 c^2}{2}\left(z - 2 + \frac{c-1}{cz}\right)\Psi_0 \qquad (158)
\end{aligned}
$$

Characteristic function for KNO

The characteristic function we define as

$$\Phi(t) = \int_0^\infty dx e^{tx} \Psi(x) = (1 - t/c)^{-c}, \ t < c \tag{159}$$

For the moments of the distribution, we have

$$\Phi^{(k)}(0) = C_k = (-c)(-c-1)...(-c-k+1)(-1/c)^k$$
$$= \frac{\Gamma(c+k)}{\Gamma(c)c^k} \tag{160}$$

Note that it is an infinitely divisible characteristic function, i.e.

$$\Phi(t) = (\Phi_n(t))^n, \ \Phi_n(t) = (1 - t/c)^{-c/n} \tag{161}$$

If we calculate observable(mean) value of x, we find

$$< x >= \Phi'(0) = n\Phi(0)_n{}' = n < x >_n, \\ < x >_n = \frac{< x >}{n} \quad (162)$$

For the second moment and dispersion, we have

$$< x^2 >= \Phi^{(2)}(0) = n < x^2 >_n + n(n-1) < x >_n^2, \\ \mathrm{D}^2 = < x^2 > - < x >^2 = n(< x^2 >_n - < x >_n^2) = n\mathrm{D}_n^2 \\ \mathrm{D}_n^2 = \frac{\mathrm{D}^2}{n} = \frac{\mathrm{D}^2}{< x >} < x >_n \quad (163)$$

Characteristic function for KNO

In a sense, any Hamiltonian quantum (and classical) system can be described by infinitely divisible distributions, because in the functional integral formulation, we use the following step

$$U(t) = e^{-itH} = (e^{-i\frac{t}{N}H})^N \tag{164}$$

Let us consider a generating function of the topological crossections

$$F(h,g,m,\mu)=\Sigma_{n\geq 2}h^n\sigma_n,$$
$$\sigma_n=\frac{1}{n!}\frac{d^n}{dh^n}F|_{h=0},$$
$$\sigma=F|_{h=1},\quad <n>=\frac{d}{dh}\ln F|_{h=1},\ldots \qquad (165)$$

Closed equation of renormdynamics for the generating function of the observables

It is natural that for the generating function we have closed renormdynamic equation [Makhaldiani, 1980]

$$(D-\gamma(\frac{h\partial}{\partial h}+2))F=0,$$
$$F(h,g,m,\mu)=F(\bar{h},\bar{g},\bar{m},\bar{\mu})\exp(2\int_{\mu}^{\bar{\mu}}\frac{d\mu}{\mu}\gamma(\bar{g}(\mu))),$$
$$\bar{h}=h\exp(\int_{\mu}^{\bar{\mu}}\frac{d\mu}{\mu}\gamma(\bar{g}(\mu))),\ \ \bar{m}=m\exp(\int_{\mu}^{\bar{\mu}}\frac{d\mu}{\mu}\eta(\bar{g}(\mu)))$$
$$\int_{g}^{\bar{g}}\frac{dg}{\beta(g)}=\ln\frac{\bar{\mu}}{\mu} \qquad (166)$$

Characteristic function for KNO

Let us find generating function in the case of KNO scaling. From the definition of Generating function and using topological cross section from KNO, we find

$$F(h) = \sum_n h^n \frac{\sigma}{< n >} \Psi(\frac{n}{< n >}) = \frac{\sigma}{< n >} \sum \Psi(\frac{n}{< n >})$$
$$= \frac{\sigma}{< n >} \Psi(\frac{\delta}{< n >}) \frac{h^2}{1-h},$$
$$\delta \equiv h\frac{d}{dh}, \; q^\delta f(h) = f(qh), \quad (16$$

No we can find more concrete form of the generation function, with the explicit form of KNO function,

$$(\frac{\delta}{< n >})^{c-1} \exp(-c\frac{\delta}{< n >})\frac{h^2}{1-h} = (\frac{\delta}{< n >})^{c-1}\frac{q^2 h^2}{1-qh}$$
$$= \frac{1}{< n >^{c-1}}\frac{1}{\Gamma(1-c)}\int_0^\infty \frac{dt}{t^c}\frac{q^2 h^2 e^{-2t}}{1-qhe^{-t}}, \quad (168)$$

so

$$F(h)_{KNO} = \frac{c^c}{\Gamma(c)} \frac{\sigma}{< n >^c} \frac{1}{\Gamma(1-c)} \int_0^\infty \frac{dt}{t^c} \frac{q^2 h^2 e^{-2t}}{1 - qhe^{-t}},$$
$$q = \exp(-\frac{c}{< n >}) \quad (169)$$

Indeed, if we expend end than integrate under this formula, we hind

$$F(h) = \frac{c^c}{\Gamma(c)} \frac{\sigma}{< n >^c} \sum_{n \geq 2} h^n n^{c-1} \exp(-\frac{c}{< n >} n) \quad (170)$$

which corresponds to the considered explicit form of the KNO function.

For high energy and temperature the states of hadronic matter is distributed according to the Boltsman low

$$P(E) = \rho(E)e^{-\beta E} = e^{-\beta F},\ \beta^{-1} = T,$$
$$F = E - TS,\ S = \ln \rho(E) \tag{171}$$

For point particle systems the density of states rise with energy as E^a. For extended particle systems, with combinatorial degeneracy of the states

$$\rho(E) \sim E^{-b} e^{\beta_H E} \tag{172}$$

and we have maximal value of the temperature, $T < T_H = {\beta_H}^{-1}$—the Hagedorn temperature.

It is interesting to consider the information entropy as a model of hadron production

$$S_1 = -\Sigma_k p_k \ln p_k \tag{173}$$

and its q-deformations

$$S_q = \frac{1}{q-1}(1 - \sum_{k=1}^{N} p_k^q) = \sum_{k=1}^{N} \frac{1/N - p_k^q}{q-1} \tag{174}$$

When $q \mapsto 1$ the deformed entropy reduce to the previous expression.

Particulary interesting may be the prime number values of the deformation parameter $q = p = 2; 3; 5; \ldots 29; \ldots 137; \ldots$ It is interesting to consider also small values of the deformation parameter, $0 < q = 1/p < 1$. We can take as probability spectrum e.g.

$$p_k = \frac{1-q}{1-q^N} q^k, \; q = e^{-aE} \tag{175}$$

In the 1870's G.J. Stoney [Stoney, 1881], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M :

$$l_s = \frac{e}{c^2}\sqrt{G}, \;\; t_s = \frac{e}{c^3}\sqrt{G}, \;\; m_s = \frac{e}{\sqrt{G}} \qquad (176)$$

M. Planck introduced [Planck, 1899] as universal units of Nature for L, T, M:

$$m_p = \sqrt{\frac{hc}{G}} = \frac{m_s}{\sqrt{\alpha}}, \; l_p = \frac{h}{cm_p} = \frac{l_s}{\sqrt{\alpha}} = 11.7 l_s,$$
$$t_p = \frac{l_p}{c} = \frac{t_s}{\sqrt{\alpha}} \tag{177}$$

Let us derive [Makhaldiani 2021] the Stoney's units using Newton and Coulomb laws and Einstein's formula

$$V_n = G\frac{m_s^2}{l_s} = V_c = \frac{e^2}{l_s} \Rightarrow m_s = \frac{e}{\sqrt{G}},$$
$$m_s c^2 = \frac{e^2}{l_s} \Rightarrow l_s = \frac{e^2}{m_s c^2} = \frac{e}{c^2}\sqrt{G},$$
$$t_s = \frac{l_s}{c} = \frac{e}{c^3}\sqrt{G} \qquad (178)$$

Using the Planck's formula $E = h\nu = h/t_p$ we derive the Planck's units,

$$V_n = G\frac{m_p^2}{l_p} = \frac{h}{t_p} = \frac{hc}{l_p} \Rightarrow m_p = \sqrt{\frac{hc}{G}},$$
$$m_p c^2 = \frac{hc}{l_p} \Rightarrow l_p = \frac{h}{cm_p} = \sqrt{\frac{hG}{c^3}},$$
$$t_p = \frac{l_p}{c} = \sqrt{\frac{hG}{c^5}} \tag{179}$$

Note that

$$m_p c^2 - \frac{e^2}{l_s} = 0; \;\; G\frac{m_s^2}{l_s} - \frac{e^2}{l_s} = 0,$$
$$m_p^2 = 137 m_s^2, \;\; l_p^2 = 137 l_s^2, \;\; t_p^2 = 137 t_s^2 \qquad (180)$$

So, planbrane=137stonbrane; the Planck's constant is derivable from elementary charge and light velocity:

$$h = \frac{e^2}{c\alpha} \qquad (181)$$

Stoney's fundamental constants are more fundamental just because they are less than Planck's constants :) Due to the value of $\alpha^{-1} = 137$, we can consider relativity theory and quantum mechanics as deformations of the classical mechanics when deformation parameter $c = 137$ (in units $e = 1, \hbar = 1$) and $\hbar = 137$ (in units $e = 1, c = 1$), correspondingly. These deformations have an analytic sense of p-adic convergent series.

Base of the Babylonians Number System

The Babylonians used a base 60 number system which is still used for measuring time - 60 seconds in a minute, 60 minutes in an hour - and for measuring angle - 360 degrees in a full turn. The base 60 number system has its origin in the ration of the Sumerian mina (m) and Akkadian shekel (s), $m/s \simeq 60 = 3 \cdot 4 \cdot 5$.
We also can consider base 137 system for fundamental theories.

Base of the Fundamental Number System

For the nuclear physics strong coupling phenomena description we may take as a base $p = 13$.
For the hadronic physics, valence scale QCD, and graphen strong coupling phenomena description we may take as a base $p = 2$.
For the weak coupling physics SM m_Z scale and MSSM unification scale phenomena description we may take as a base $p = 29$.

Number of the Fundamental Constants

There are different opinions about the number of fundamental constants [Duff, Okun, Veneziano, 2001].
According to Okun, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$; the velocity of light, c; and Newton's constant, G.
According to Veneziano, there are only two: the string length L_s and c.
According to Duff, there are not fundamental constants at all.

The number 137 has a very interesting geometric sense,

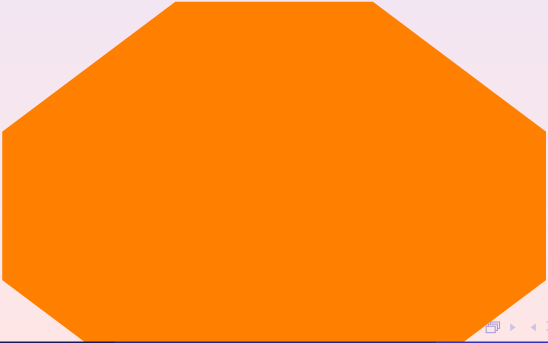
$$137 = 11^2 + 4^2, \tag{182}$$

so, $\sqrt{137}$ is the hypotenuse length of a triangle with other sides of lengths 11 and 4.

Usually $L_s = l_p$, so, the fundamental area is
$L_s^2 = 137 l_s^2 = |(4 l_s + i11 l_s|^2$.
If $z_1 = 4 + i11, \ z_2 = 11 + i4, \ |z_1 - z_2| = \sqrt{7^2 + 7^2} = \sqrt{98} = \sqrt{100 - 2} = 10(1 - 1/100 + O(10^{-4})) = 10 - 1/10 + O(10^{-3})$. The vertices $z_n = \pm 4 \pm i11$ and $\pm 11 \pm i4$ on the complex plane form an octagon with sides of length 8 and almost 10. If we cover the surface with such octagon we obtain figures of size 10 before the correction becomes of size $1 l_s$. Note that $l_p = 11.7 l_s$ This hints about microscopic origin - structure of quantum theory.
The value $s_s = l_s^2 -$ Stoney area - stonbrane, is more like on a fundamental area :)

$$R = \frac{7}{\sqrt{2}\cos(2\pi/5)} = \frac{7\sqrt{2}}{(\sqrt{5}-1)/2} = 16.0177,$$

$$\frac{100}{2\pi} = 15.9115,\ \ 5\times 8 + 10\times(10 - 1/10) = 140 - 1 = (1$$



Scale Dependent Number of Fundamental Constants

In mathematics we have two kind of structures, discrete and continuous one. If a physical quantity has discrete values, it might have no dimension. If the values are continuous - the quantity might have a dimension, a unit of measure. These structures may depend on scale, e.g. on macroscopic scale condensed state of matter (and time) is well described as continuous medium, so we use dimensional units of length (and time).

On the scale of atoms, the matter has a discrete structure, so we may count lattice sites and may not use a unit of length. If at small (e.g. at Plank) scale space (and/or time) is discrete, then we do not need a unit of length (time) for measuring, there is a fundamental length and we can just count.

Let us consider a general dynamical system described by the following system of the ordinary differential equations [Arnold, 1978]

$$\dot{x}_n = v_n(x), \ 1 \leq n \leq N, \tag{184}$$

$\dot{x}_n$ stands for the total derivative with respect to the parameter t.

When the number of the degrees of freedom is even, and

$$v_n(x) = \varepsilon_{nm}\frac{\partial H_0}{\partial x_m},\ 1 \leq n, m \leq 2M, \tag{185}$$

the system (184) is Hamiltonian one and can be put in the form

$$\dot{x}_n = \{x_n, H_0\}_0, \tag{186}$$

where the Poisson bracket is defined as

$$\{A, B\}_0 = \varepsilon_{nm}\frac{\partial A}{\partial x_n}\frac{\partial B}{\partial x_m} = A\frac{\overleftarrow{\partial}}{\partial x_n}\varepsilon_{nm}\frac{\overrightarrow{\partial}}{\partial x_m}B, \tag{187}$$

and summation rule under repeated indices has been used.

Let us consider the following Lagrangian

$$L = (\dot{x}_n - v_n(x))\psi_n \tag{188}$$

and the corresponding equations of motion

$$\dot{x}_n = v_n(x), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n}\psi_m. \tag{189}$$

The system (189) extends the general system (184) by linear equation for the variables ψ.

The extended system can be put in the Hamiltonian form [Makhaldiani, Voskresenskaya, 1997]

$$\dot{x}_n = \{x_n, H_1\}_1, \dot{\psi}_n = \{\psi_n, H_1\}_1, \tag{190}$$

where first level (order) Hamiltonian is

$$H_1 = v_n(x)\psi_n \tag{191}$$

and (first level) bracket is defined as

$$\{A, B\}_1 = A(\frac{\overleftarrow{\partial}}{\partial x_n}\frac{\overrightarrow{\partial}}{\partial \psi_n} - \frac{\overleftarrow{\partial}}{\partial \psi_n}\frac{\overrightarrow{\partial}}{\partial x_n})B. \tag{192}$$

Note that when the Grassmann grading [Berezin, 1987] of the conjugated variables x_n and ψ_n are different, the bracket (192) is known as Buttin bracket[Buttin, 1996]. In the Faddeev-Jackiw formalism [Faddeev, Jackiw, 1988] for the Hamiltonian treatment of systems defined by first-order Lagrangians, i.e. by a Lagrangian of the form

$$L = f_n(x)\dot{x}_n - H(x), \tag{193}$$

motion equations

$$f_{mn}\dot{x}_n = \frac{\partial H}{\partial x_m}, \tag{194}$$

for the regular structure function f_{mn}, can be put in the explicit Hamiltonian (Poisson; Dirac) form

$$\dot{x}_n = f^{-1}_{nm} \frac{\partial H}{\partial x_m} = \{x_n, x_m\} \frac{\partial H}{\partial x_m} = \{x_n, H\}, \tag{195}$$

where the fundamental Poisson (Dirac) bracket is

$$\{x_n, x_m\} = f^{-1}_{nm}, \;\; f_{mn} = \partial_m f_n - \partial_n f_m. \tag{196}$$

The system (189) is an important example of the first order regular hamiltonian systems. Indeed, in the new variables,

$$y_n^1 = x_n, y_n^2 = \psi_n, \tag{197}$$

lagrangian (188) takes the following first order form

$$\begin{aligned} &L = (\dot{x}_n - v_n(x))\psi_n \Rightarrow \frac{1}{2}(\dot{x}_n\psi_n - \dot{\psi}_n x_n) - v_n(x)\psi_n \\ &= \frac{1}{2}y_n^a\varepsilon^{ab}\dot{y}_n^b - H(y) \\ &= f_n^a(y)\dot{y}_n^a - H(y), f_n^a = \frac{1}{2}y_n^b\varepsilon^{ba}, H = v_n(y^1)y_n^2, \\ &f_{nm}^{ab} = \frac{\partial f_m^b}{\partial y_n^a} - \frac{\partial f_n^a}{\partial y_m^b} = \varepsilon^{ab}\delta_{nm} \end{aligned} \tag{198}$$

Corresponding motion equations and the fundamental Poisson bracket are

$$\dot{y}_n^a = \varepsilon_{ab}\delta_{nm}\frac{\partial H}{\partial y_m^b} = \{y_n^a, H\}, \{y_n^a, y_m^b\} = \varepsilon_{ab}\delta_{nm}. \quad (199)$$

To the canonical quantization of this system corresponds

$$[\hat{y}_n^a, \hat{y}_m^b] = i\hbar\varepsilon_{ab}\delta_{nm}, \ \hat{y}_n^1 = y_n^1, \ \hat{y}_n^2 = -i\hbar\frac{\partial}{\partial y_n^1} \quad (200)$$

In this quantum theory, classical part, motion equations for y_n^1, remain classical.

Nabu – Babylonian God
of Wisdom and Writing.

The Hamiltonian mechanics (HM) is in the fundamentals of mathematical description of the physical theories [Faddeev, Takhtajan, 1990]. But HM is in a sense blind; e.g., it does not make a difference between two opposites: the ergodic Hamiltonian systems (with just one integral of motion) [Sinai, 1993] and (super)integrable Hamiltonian systems (with maximal number of the integrals of motion).

Nabu mechanics (NM) [Nambu, 1973, Whittaker, 1927] is a proper generalization of the HM, which makes the difference between dynamical systems with different numbers of integrals of motion explicit (see, e.g.[Makhaldiani 2007]).

In the canonical formulation, the equations of motion of a physical system are defined via a Poisson bracket and a Hamiltonian, [Arnold, 1978]. In Nambu's formulation, the Poisson bracket is replaced by the Nambu bracket with $n+1, n \geq 1$, slots. For $n = 1$, we have the canonical formalism with one Hamiltonian. For $n \geq 2$, we have Nambu-Poisson formalism, with n Hamiltonians, [Nambu, 1973], [Whittaker, 1927].

The system of N vortexes can be described by the following system of differential equations,
[Aref, 1983, Meleshko,Konstantinov, 1993]

$$\dot{z}_n = i\sum_{m\neq n}^{N}\frac{\gamma_m}{z_n^* - z_m^*},\quad 1\leq n\leq N, \qquad (201)$$

where $z_n = x_n + iy_n$ are complex coordinate of the centre of n-th vortex.

For $N = 3$, and the quantities

$$\begin{aligned} u_1 &= ln|z_2 - z_3|^2, \\ u_2 &= ln|z_3 - z_1|^2, \\ u_3 &= ln|z_1 - z_2|^2 \end{aligned} \tag{202}$$

the system reduce to the following system

$$\begin{aligned} \dot{u}_1 &= \gamma_1(e^{u_2} - e^{u_3}), \\ \dot{u}_2 &= \gamma_2(e^{u_3} - e^{u_1}), \\ \dot{u}_3 &= \gamma_3(e^{u_1} - e^{u_2}), \end{aligned} \tag{203}$$

The system (203) has two integrals of motion

$$H_1 = \sum_{i=1}^{3} \frac{e^{u_i}}{\gamma_i}, H_2 = \sum_{i=1}^{3} \frac{u_i}{\gamma_i}$$

and can be presented in the Nambu–Poisson form, [Makhaldiani, 1997,2]

$$\dot{u}_i = \omega_{ijk} \frac{\partial H_1}{\partial u_j} \frac{\partial H_2}{\partial u_k} = \{x_i, H_1, H_2\} = \omega_{ijk} \frac{e^{u_j}}{\gamma_j} \frac{1}{\gamma_k},$$

where

$$\omega_{ijk} = \epsilon_{ijk} \rho, \rho = \gamma_1 \gamma_2 \gamma_3$$

and the Nambu–Poisson bracket of the functions A, B, C on the three-dimensional phase space is

$$\{A, B, C\} = \omega_{ijk} \frac{\partial A}{\partial u_i} \frac{\partial B}{\partial u_j} \frac{\partial C}{\partial u_k}. \tag{204}$$

This system is superintegrable: for $N = 3$ degrees of freedom, we have maximal number of the integrals of motion $N - 1 = 2$.

The reduction of the dimensionless couplings in GUTs is achieved by searching for RD integrals of motion-renormdynamic invariant (RDI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exist RDI relations among dimensional couplings that guarantee the vanishing of all beta-functions in certain GUTs even to all orders. In this case the number of the independent motion integrals N is equal to the number of the coupling constants.

Note that in superintegrable dynamical systems the number of the integrals is $\leq N - 1$, so the RD of the finite field theories is trivial, coupling constants do not run, they have fixed values, the renormdynamics is more than superintegrable, it is hyperintegrable. Developments in the soft supersymmetry breaking sector of GUTs and FUTs lead to exact RDI relations, i.e. reduction of couplings, in this dimensionful sector of the theory, too. Based on the above theoretical framework phenomenologically consistent FUTs have been constructed.

The main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the SM in terms of a few fundamental ones. In other words, to achieve reduction of couplings at a more fundamental level.

As an example of the infinite dimensional Nambu-Poisson dynamics, let me conside the following extension of Schrödinger quantum mechanics [Makhaldiani, 2000]

$$iV_t = \Delta V - \frac{V^2}{2}, \tag{205}$$

$$i\psi_t = -\Delta\psi + V\psi. \tag{206}$$

An interesting solution to the equation for the potential (205) is

$$V = \frac{4(4-d)}{r^2}, \tag{207}$$

where d is the dimension of the spase. In the case of $d = 1$, we have the potential of conformal quantum mechanics. The variational formulation of the extended quantum theory, is given by the following Lagrangian

$$L = (iV_t - \Delta V + \frac{1}{2}V^2)\psi. \tag{208}$$

The momentum variables are

$$P_v = \frac{\partial L}{\partial V_t} = i\psi, P_\psi = 0. \tag{209}$$

As Hamiltonians of the Nambu-theoretic formulation, we take the following integrals of motion

$$H_1 = \int d^d x(\Delta V - \frac{1}{2}V^2)\psi,$$
$$H_2 = \int d^d x(P_v - i\psi), \; H_3 = \int d^d x P_\psi. \tag{210}$$

We invent unifying vector notation,
$\phi = (\phi_1, \phi_2, \phi_3, \phi_4) = (\psi, P_\psi, V, P_v)$. Then it may be verified that the equations of the extended quantum theory can be put in the following Nambu-theoretic form

$$\phi_t(x) = \{\phi(x), H_1, H_2, H_3\}, \qquad (211)$$

where the bracket is defined as

$$\{A_1, A_2, A_3, A_4\} = i\varepsilon_{ijkl} \int \frac{\delta A_1}{\delta\phi_i(y)} \frac{\delta A_2}{\delta\phi_j(y)} \frac{\delta A_3}{\delta\phi_k(y)} \frac{\delta A_4}{\delta\phi_l(y)} dy$$
$$= i \int \frac{\delta(A_1, A_2, A_3, A_4)}{\delta(\phi_1(y), \phi_2(y), \phi_3(y), \phi_4(y))} dy$$
$$= idet(\frac{\delta A_k}{\delta\phi_l}). \qquad (212)$$

The basic building blocks of M theory are membranes and $M5-$branes. Membranes are fundamental objects carrying electric charges with respect to the 3-form C-field, and $M5$-branes are magnetic solitons. The Nambu-Poisson 3-algebras appear as gauge symmetries of superconformal Chern-Simons nonabelian theories in $2+1$ dimensions with the maximum allowed number of $N=8$ linear supersymmetries.

The Bagger and Lambert [Bagger, Lambert, 2007] and, Gustavsson [Gustavsson, 2007] (BLG) model is based on a 3-algebra,

$$[T^a, T^b, T^c] = f_d^{abc} T^d \tag{213}$$

where T^a, are generators and f_{abcd} is a fully anti-symmetric tensor.

Given this algebra, a maximally supersymmetric Chern-Simons lagrangian is:

$$
\begin{aligned}
&L = L_{CS} + L_{matter}, \\
&L_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f_{abcd}A^{ab}_{\mu}\partial_{\nu}A^{cd}_{\lambda} + \frac{2}{3}f_{cdag}f^{g}_{efb}A^{ab}_{\mu}A^{cd}_{\nu}A^{ef}_{\lambda}) \\
&L_{matter} = \frac{1}{2}B^{la}_{\mu}B^{\mu l}_{a} - B^{la}_{\mu}D^{\mu}X^{l}_{a} \\
&+\frac{i}{2}\bar{\psi}^{a}\Gamma^{\mu}D_{\mu}\psi_{a} + \frac{i}{4}\bar{\psi}^{b}\Gamma_{IJ}X^{I}_{c}X^{J}_{d}\psi_{a}f^{abcd} \\
&-\frac{1}{12}tr([X^{I},X^{J},X^{K}][X^{I},X^{J},X^{K}]), \\
&I = 1,2,...,8, \qquad (214)
\end{aligned}
$$

where A_μ^{ab} is gauge boson, ψ^a and $X^I = X_a^I T^a$ matter fields. If $a = 1, 2, 3, 4$, then we can obtain an $SO(4)$ gauge symmetry by choosing $f_{abcd} = f\varepsilon_{abcd}$, f being a constant. It turns out to be the only case that gives a gauge theory with manifest unitarity and $N = 8$ supersymmetry.

The action has the first order form so we can use previous formalism. The motion equations for the gauge fields

$$f^{nm}_{abcd}\dot{A}^{cd}_m(t,x) = \frac{\delta H}{\delta A^{ab}_n(t,x)}, f^{nm}_{abcd} = \varepsilon^{nm} f_{abcd} \qquad (215)$$

take canonical form

$$\dot{A}^{ab}_n = f^{abcd}_{nm}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, A^{cd}_m\}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, H\},$$
$$\{A^{ab}_n(t,x), A^{cd}_m(t,y)\} = \varepsilon_{nm} f^{abcd}\delta^{(2)}(x-y) \qquad (216)$$

Nambu-Poisson dynamics of an extended particle with spin in an accelerator

The quasi-classical description of the motion of a relativistic (nonradiating) point particle with spin in accelerators and storage rings includes the equations of orbit motion

$$
\begin{aligned}
&\dot{x}_n = f_n(x), \;\; f_n(x) = \varepsilon_{nm}\partial_m H, \;\; n, m = 1, 2, ..., 6;\\
&x_n = q_n, \;\; x_{n+3} = p_n, \;\; \varepsilon_{n,n+3} = 1, \;\; n = 1, 2, 3;\\
&H = e\Phi + c\sqrt{\wp^2 + m^2c^2}, \;\; \wp_n = p_n - \frac{e}{c}A_n \qquad (217)
\end{aligned}
$$

and Thomas-BMT equations

Nambu-Poisson dynamics of an extended particle with spin in an accelerator

[Tomas, 1927, Bargmann, Michel,Telegdi, 1959] of classical spin motion

$$\dot{s}_n = \varepsilon_{nmk}\Omega_m s_k = \{H_1, H_2, s_n\},\ \ H_1 = \Omega \cdot s,\ \ H_2 = s^2,$$
$$\{A, B, C\} = \varepsilon_{nmk}\partial_n A \partial_m B \partial_k C,$$
$$\Omega_n = \frac{-e}{m\gamma c}((1 + k\gamma)B_n - k\frac{(B\cdot\wp)\wp_n}{m^2c^2(1+\gamma)}$$
$$+\frac{1 + k(1+\gamma)}{mc(1+\gamma)}\varepsilon_{nmk}E_m\wp_k) \tag{218}$$

where, parameters e and m are the charge and the rest mass of the particle, c is the velocity of light, $k = (g-2)/2$ quantifies the anomalous spin g factor, γ is the Lorentz factor, p_n are components of the kinetic momentum vector, E_n and B_n are the electric and magnetic fields, and A_n and Φ are the vector and scalar potentials;

$$B_n = \varepsilon_{nmk}\partial_m A_k, \; E_n = -\partial_n\Phi - \frac{1}{c}\dot{A}_n,$$
$$\gamma = \frac{H - e\Phi}{mc^2} = \sqrt{1 + \frac{\wp^2}{m^2c^2}} \qquad (219)$$

The spin motion equations we put in the Nambu-Poisson form. The general method of Hamiltonization of the dynamical systems we can use also in the spinning particle case. We put this formalism in the ground of the optimal control theory of the accelerators.

Hamiltonian extension of the spinning particle dynamics

Let us invent unified configuration space
$q = (x, p, s),\ x_n = q_n,\ p_n = q_{n+3},\ s_n = q_{n+6},\ n = 1, 2, 3;$
extended phase space, (q_n, ψ_n) and hamiltonian

$$H = H(q, \psi) = v_n \psi_n,\ n = 1, 2, ...9; \tag{220}$$

motion equations

$$\dot{q}_n = v_n(q), \qquad \dot{\psi}_n = -\frac{\partial v_m}{\partial q_n}\psi_m \tag{221}$$

where the velocities v_n depends on external fields as in previous section as control parameters which can be determined according to the optimal control criterium.

EDM are one of the keys to understand the origin of our Universe [Sakharov, 1967]. Andrei Sakharov formulated three conditions for baryogenesis:

1. Early in the evolution of the universe, the baryon number conservation must be violated sufficiently strongly,
2. The C and CP invariances, and T invariance thereof, must be violated, and
3. At the moment when the baryon number is generated, the evolution of the universe must be out of thermal equilibrium.

CP violation in kaon decays is known since 1964, it has been observed in B-decays and charmed meson decays. The Standard Model (SM) accommodates CP violation via the phase in the Cabibbo-Kobayashi-Maskawa matrix.
CP and P violation entail nonvanishing P and T violating electric dipole moments (EDM) of elementary particles $\vec{d} = d\vec{s}$.
Although extremely successful in many aspects, the SM has at least two weaknesses: neutrino oscillations do require extensions of the SM and, most importantly, the SM mechanisms fail miserably in the expected baryogenesis rate.

Electric Dipole Moments (EDM) of Protons and Deuterons

Simultaneously, the SM predicts an exceedingly small electric dipole moment of nucleons $10^{-33} < d_n < 10^{-31} e \cdot cm$, way below the current upper bound for the neutron EDM, $d_n < 2.9 \times 10^{-26} e \cdot cm$. In the quest for physics beyond the SM one could follow either the high energy trail or look into new methods which offer very high precision and sensitivity. Supersymmetry is one of the most attractive extensions of the SM and S. Weinberg emphasized [Weinberg, 1993]: "Endemic in supersymmetric (SUSY) theories are CP violations that go beyond the SM. For this reason it may be that the next exciting thing to come along will be the discovery of a neutron (1932) electric dipole moment."

The SUSY predictions span typically $10^{-29} < d_n < 10^{-24} e \cdot cm$ and precisely this range is targeted in the new generation of EDM searches [Roberts, Marciano, 2010]. There is consensus among theorists that measuring the EDM of the proton, deuteron and helion is as important as that of the neutron. Furthermore, it has been argued that T-violating nuclear forces could substantially enhance nuclear EDM [Flambaum, Khriplovich, Sushkov, 1986]. At the moment, there are no significant direct upper bounds available on d_p or d_d.

Non-vanishing EDMs give rise to the precession of the spin of a particle in an electric field. In the rest frame of a particle

$$\dot{s}_n = \varepsilon_{nmk}(\Omega_m s_k + d_m E_k),\ \Omega_m = -\mu B_m, \tag{222}$$

where in terms of the lab frame fields

$$\begin{aligned} B_n &= \gamma(B_n^l - \varepsilon_{nmk}\beta_m E_k^l), \\ E_n &= \gamma(E_n^l + \varepsilon_{nmk}\beta_m B_k^l) \end{aligned} \tag{223}$$

Now we can apply the Hamiltonization and optimal control theory methods to this dynamical system.

The idea of computations on quanputers is in finding of the needed (value of the) state (wave function $\psi(t,x)$) from the initial, easy constructible, state ($\psi(0,x)$,) which is superposition of different states, including interesting one, with the same weight. During the computation the weight of the interesting state is growing till the value when we can guess the solution of the problem and then test it, which is much more easier then to find it.

Let us consider the following nonlinear evolution equation

$$iV_t = \Delta V - \frac{1}{2}V^2 + J, \quad (224)$$

extended Lagrangian and Hamiltonian

and corresponding Hamiltonian motion equations

$$iV_t = \Delta V - \frac{1}{2}V^2 + J = \{V, H\},$$
$$i\psi_t = -\Delta\psi + V\psi = \{\psi, H\},$$
$$\{V(t,x), \psi(t,y)\} = \delta^D(x-y) \qquad (226)$$

The solution of the problem is given in the form

$$|T) = U(T)|0), \ \psi(t,x) =< x|t), \ U(T) = Pexp(-i\int_0^T dtH$$

Under the programming of the quanputer we understand construction of the potential V, or the corresponding Hamiltonian. For the given potential, we calculate corresponding source J.

The discrete version of the system can be put in the form

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$
$$\Psi_m(n-1) = A_{mk}(S(n))\Psi_k(n),\ A_{mk}(S(n)) = \frac{\partial \Phi_k(S(n))}{\partial S_m(n)} \quad (228)$$

or, in the regular case, when the matrix A is regular, we obtain explicit form of the corresponding discrete dynamics

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$
$$\Psi_m(n+1) = A^{-1}_{mk}(S(n+1))\Psi_k(n), \qquad (229)$$

Now the state vector $S(n)$ and wave vector $\Psi_m(n)$ may correspond not only to the discrete values of the potential $V(n,m) = S_m(n)$, and wave function $\psi(n,m)$

GRID and Quanputing

As an example of GRID we take LHC Computing Grid. The LHC Computing Grid (LCG), is an international collaborative project that consists of a grid-based computer network infrastructure incorporating over 170 computing centers in 36 countries. It was designed by CERN to handle the prodigious volume of data produced by Large Hadron Collider (LHC) experiments. The Large Hadron Collider at CERN was designed to prove or disprove the existence of the Higgs boson, an important but elusive piece of knowledge that had been sought by particle physicists for over 40 years. A very powerful particle accelerator was needed, because Higgs bosons might not be seen in lower energy experiments, and because vast numbers of collisions

GRID and Quanputing

A design report was published in 2005. It was announced to be ready for data on 3 October 2008. It incorporates both private fiber optic cable links and existing high-speed portions of the public Internet. At the end of 2010, the Grid consisted of some 200,000 processing cores and 150 petabytes of disk space, distributed across 34 countries. The data stream from the detectors provides approximately 300 GByte/s of data, which after filtering for "interesting events", results in a data stream of about 300 MByte/s. The CERN computer center, considered "Tier 0" of the LHC Computing Grid, has a dedicated 10 Gbit/s connection to the counting room. The project was expected to generate 27 TB of raw data per day, plus 10 TB of "event

This data is sent out from CERN to eleven Tier 1 academic institutions in Europe, Asia, and North America, via dedicated 10 Gbit/s links. This is called the LHC Optical Private Network. More than 150 Tier 2 institutions are connected to the Tier 1 institutions by general-purpose national research and education networks. The data produced by the LHC on all of its distributed computing grid is expected to add up to 10-15 PB of data each year. **Today, without big efforts, we can modify (some) GRID elements in time-invertible form. After development of the quanputer technologies, we can modify (some) GRID elements in quanputer forms.**

Social profit of big collaborations

Nowadays there are several big collaborations in science, e.g. LHC. Scientific value of LHC depends on three components, the highest quality of accelerator, highest quality of detectors and distributed data processing. The first two components need good mathematical and physical modeling. Third component and the collaboration as a social structure are not under (anther) the control by scientific methods and corresponding modeling. By definition, scientific collaborations (SC) have a main scientific aim: to obtain answer on the important scientific question(s) and maybe gain extra scientific bonus: new important questions and discoveries. SC is more open information system than e.g. finance or military systems

Note that the procedure of reduction of the higher order dynamical system, e.g. second order Euler-Lagrange motion equations, to the first order dynamical systems, in the case to the Hamiltonian motion equations, can be continued using fractal calculus. E.g. first order system can be reduced to the half order one,

$$D^{1/2}q = \psi, D^{1/2}\psi = p \Leftrightarrow \dot{q} = p. \qquad (230)$$

Another representation of the halforder derivative is

$$\partial_t^{1/2} = \partial_\theta + \theta\partial_t,\ \ \partial_\theta\theta + \theta\partial_\theta = 1,\ \ \theta^2 = 0 \tag{231}$$

We may consider corresponding relativistic equation

$$\begin{aligned} &E^{1/2}\phi = H^{1/2}\phi,\ \ H^{1/2} = g_n p_n^{1/2} + bm^{1/2},\\ &E^{1/2} = (i\partial_t)^{1/2} = a(\partial_\theta + \theta\partial_t)\\ &p_n^{1/2} = (-i\partial_{x_n})^{1/2} = (\partial_{\theta_n} + \theta_n\partial_{x_n})/a, a = \exp(i\pi/4)\\ &g_n g_m + g_m g_n = \gamma_n\delta_{nm},\ \ bg_n + g_n b = 0,\\ &b^2 = 1,\ \ n = 1, 2, 3 \end{aligned} \tag{232}$$

Reduction of the higher order dynamical system and SM fermions

Dirac's equation

$$i\partial_t\psi = (\alpha_n p_n + \beta_m)\psi = H\psi \qquad (233)$$

lies in the background structure of the SM. It is known that the equation has problems with interpretation. E.g.

$$\dot{x}_k = i[H, x_k] = \alpha_k = \hat{v}_k,$$
$$v^2 = 3 > 1, \;\; v = \sqrt{< v^2 >} = \sqrt{3}c \qquad (234)$$

In (nonrelativistic, quantum) mechanics,

$$H = \frac{p^2}{2m}, \ \dot{x} = \frac{p}{m} \tag{235}$$

If we take

$$\begin{aligned} \partial_n &= \partial_{\theta_n} - \theta_n p_n^2, \\ H &= -i(\alpha_n(\partial_{\theta_n} - \theta_n p_n^2)) + \beta m, \\ \dot{x}_k &= [\alpha_n(\partial_{\theta_n} - \theta_n p_n^2)) + i\beta m, x_k] \\ &= v_k = 2i\alpha_k \theta_k p_k \end{aligned} \tag{236}$$

We may start also from nonlocal first order operator

$$E = \pm\sqrt{p^2 + m^2} \tag{237}$$

and use previous tric as

$$\sqrt{p^2 + m^2} = \partial_\theta + \theta(p^2 + m^2),$$
$$\dot{x}_k = i[H, x_k] = \pm 2i\theta p_k \tag{238}$$

Reduction of the higher order dynamical system and SM fermions

For wave function $\psi(t,x,\theta) = \psi_0 + \theta\psi_1(t,x)$,

$$\begin{aligned} &i\partial_t\psi = (\partial_\theta + \theta(p^2 + m^2))\psi, \\ &i\partial_t\psi_0 = \psi_1, \\ &i\partial_t\psi_1 = (p^2 + m^2)\psi_0, \\ &\psi(t,x,\theta) = \psi_0 + \theta\psi_1(t,x) \\ &= e^{i\theta\partial_t}\psi_0 = \psi_0(t + i\theta, x), \\ &\partial_t^2\psi_0 = (\nabla^2 - m^2)\psi_0 \end{aligned} \tag{239}$$

q-interpolation

$$D_q f(x) = \frac{f(x) - f(qx)}{(1-q)x}|_{q \to 1} \to f'(x),$$
$$D_{-1} f(x) = \frac{f(x) - f(0)}{2x},$$
$$f(\theta) = f_0 + \theta f_1,\ D_{-1} f(\theta) = f_1,\ D_{-1} = \partial_\theta \quad (240)$$

There is the Dirac-Pauli argument of the absence of a good time operator for the usual quantum dynamical systems. The Dirac's part of the argument consists in the statement that there is not a good momentum operator $\hat{p}$ if corresponding coordinate $\hat{x}$ has restricted spectrum.

Any good momentum observable permits to reach from the state with coordinate x_0 any stay with coordinate x,

$$U(x - x_0)\psi(x_0) = \psi(x),\ \hat{x}\psi(x) = x\psi(x),$$
$$U(x) = \exp(ix\hat{p}),\ [\hat{x}, \hat{p}] = i, \tag{241}$$

In the coordinate representation: $\hat{x} = x,\ \hat{p} = -i\partial_x$. In momentum representation: $\hat{p} = p,\ \hat{x} = i\partial_p$.

Time observable - operator

If we suppose that there is a good time observable $\hat{t},\ [\hat{H}, \hat{t}] = i$, than it permits from a given state corresponding to a value of energy E_0 to reach any state with corresponding energy value E. But for usual systems the spectrum is restricted from below by the ground state energy, so there can not be a good time observable. This is the Pauli's part of the argument.

Let us consider rather general example of a dynamical system with good time observable, the nonrelativistic particle of mass m, neutron (1932) e.g., in the earth gravitational field $V = -mgz$,

$$H = \frac{p^2}{2m} + V(z), \tag{242}$$

with corresponding motion equations

$$\dot{z} = \frac{\partial H}{\partial p_z} = \frac{p_z}{m},$$
$$\dot{p}_z = -\frac{\partial H}{\partial z} = mg \Rightarrow p_z(t) = mgt + p_{0z} \Downarrow \tag{243}$$

$$t = \frac{p_z(t) - p_{0z}}{mg} \Rightarrow \hat{t} = \frac{\hat{p}_z(t) - \hat{p}_{0z}}{mg},$$
$$[\hat{H}, \hat{t}] = [V, p_z(t)]/mg = i. \qquad (244)$$

So, dynamical systems with unbounded energy spectrum may play role of the quantum clock with good time observable.

The Hubbard model, with Hamiltonian

$$H = H_0 + \lambda H_1, \ H_0 = \sum_{ij} t_{ij}\psi^+_{ia}\psi_{ja},$$
$$H_1 = \sum_i (\psi^+_{ia}\psi_{ia})^2 \tag{245}$$

is the standard model of the condensed matter theory. The tight binding coefficients t_{ij} incorporate the physics of a given material, which determine the crystal structure of the ground state and the rest is supposed to be well approximated by a choice of the on site Coulomb repulsion λ.

In the simplest Hubbard models the spin index a takes on two values, representing the spin of electrons. Some materials require more complicated multi-band Hubbard models. The basic idea behind the Hubbard model is that Coulomb forces are screened, with a screening length shorter than the lattice spacing. With the exception of phonons, low energy excitations are assumed to be excitations of this low energy electron gas.

... never exist by itself, but only
as primordial part of a lager body,
from which no force can tear it loose.
Titus Lucretius Carus: De rerum natura
$\sim$55 B.C.

In the standard model of particle physics, the strong force is described by the theory of quantum chromodynamics (QCD). At ordinary temperatures or densities this force just confines the quarks into composite particles (hadrons) of size around 10^{-15} m = 1 femtometer = 1 fm (corresponding to the QCD energy scale Λ_{QCD}=200 MeV) and its effects are not noticeable at longer distances. However, when the temperature reaches the QCD energy scale (T of order 10^{12} kelvins) or the density rises to the point where the average inter-quark separation is less than 1 fm (quark chemical potential μ around 400 MeV), the hadrons are melted into their constituent quarks, and the strong interaction becomes the dominant feature of the physics.

Such phases are called quark matter or QCD matter or Gluquar. The strength of the color force makes the properties of quark matter unlike gas or plasma, instead leading to a state of matter more reminiscent of a liquid. At high densities, quark matter is a Fermi liquid, but is predicted to exhibit color superconductivity at high densities and temperatures below 10^{12} K.
QCD is the theory of the strong interactions with, as only inputs, one mass parameter for each quark species and the value of the QCD coupling constant at some energy or momentum scale in some renormalization scheme.

This last free parameter of the theory can be fixed by Λ_{QCD}, the energy scale used as the typical boundary condition for the integration of the Renormdynamic (RD) equation for the strong coupling constant. This is the parameter which expresses the scale of strong interactions, the only parameter in the limit of massless quarks.

While the evolution of the coupling with the momentum scale is determined by the quantum corrections induced by the renormalization of the bare coupling and can be computed in perturbation theory, the strength itself of the interaction, given at any scale by the value of the renormalized coupling at this scale, or equivalently by Λ_{QCD}, is one of the above mentioned parameters of the theory and has to be taken from experiment.

The RD equations play an important role in our understanding of Quantum Chromodynamics and the strong interactions. The beta function and the quarks mass anomalous dimension are among the most prominent objects for QCD RD equations. The calculation of the one-loop β-function in QCD has lead to the discovery of asymptotic freedom in this model and to the establishment of QCD as the theory of strong interactions ['t Hooft, 1972, Gross, Wilczek, 1973, Politzer, 1973].

The MS-scheme ['t Hooft, 1973] belongs to the class of massless schemes where the β-function does not depend on masses of the theory and the first two coefficients of the β-function are scheme-independent.
The Lagrangian of QCD with massive quarks in the covariant gauge is

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{q}_n(i\gamma D - m_n)q_n$$
$$-\frac{1}{2\xi}(\partial A)^2 + \partial^\mu \bar{c}^a(\partial_\mu c^a + gf^{abc}A^b_\mu c^c)$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu,$$
$$(D_\mu)_{kl} = \delta_{kl}\partial_\mu - igt^a_{kl}A^a_\mu, \qquad (246)$$

$A_\mu^a, a = 1, ..., N_c^2 - 1$ are gluon; $q_n, n = 1, ..., n_f$ are quark; c^a are ghost fields; ξ is gauge parameter; t^a are generators of fundamental representation and f^{abc} are structure constants of the Lie algebra $[t^a, t^b] = if^{abc}t^c$, we consider an arbitrary compact semi-simple Lie group G. For QCD, $G = SU(N_c), N_c = 3$.

The expression of the β-function can be obtained in the following way.

The canonical dimensions of the bare fields and constants in the d-dimensional space-time are

$$[m]=1,\ [A]=\frac{D-2}{2},\ [\Psi]=\frac{D-1}{2},$$
$$[g_b]=D-[A]-2[\Psi]=\frac{4-D}{2}=\varepsilon,$$
$$D=4-2\varepsilon,[a_b]=[g_b^2]=2\varepsilon,\ a_b=\mu^{2\varepsilon}Za,$$
$$0=da_b/dt=d(\mu^{2\varepsilon}Za)/dt=\mu^{2\varepsilon}(\varepsilon Za+\frac{\partial(Za)}{\partial a}\frac{da}{dt})$$
$$\Rightarrow\frac{da}{dt}=\beta(a,\varepsilon)=\frac{-\varepsilon Za}{\frac{\partial(Za)}{\partial a}}=-\varepsilon a+\beta(a),$$
$$\beta(a)=a^2\frac{d}{da}(Z_1) \qquad (247)$$

where

$$\beta(a,\varepsilon) = \frac{D-4}{2}a + \beta(a) \tag{248}$$

is $D-$dimensional $\beta-$function and Z_1 is the residue of the first pole in ε expansion

$$Z(a,\varepsilon) = 1 + Z_1\varepsilon^{-1} + ... + Z_n\varepsilon^{-n} + ... \tag{249}$$

Since Z does not depend explicitly on μ, the β-function is the same in all MS-like schemes, i.e. within the class of renormalization schemes which differ by the shift of the parameter μ.

The higher residue of the pole expansion can be defined from (247),

$$0=\varepsilon Za+\frac{\partial(Za)}{\partial a}\frac{da}{dt}=\varepsilon(a+\frac{aZ_1}{\varepsilon}+\frac{aZ_2}{\varepsilon^2}+...)$$
$$+(1+\frac{(aZ_1)'}{\varepsilon}+\frac{(aZ_2)'}{\varepsilon^2}+...)(-\varepsilon a+\beta(a))$$
$$=\beta-a(aZ_1)'+aZ_1+\frac{1}{\varepsilon}(aZ_2-a(aZ_2)'+\beta(aZ_1)')+...$$
$$\beta(a)=a^2\frac{dZ_1}{da};\, a^2\frac{dZ_{n+1}}{da}=\beta(a)\frac{daZ_n}{da},$$
$$n=0,1,2,... \qquad (25$$

where $Z_0 = 1$ and the last equation includes also the previous one.

$$\varepsilon Z a + (Z + a\frac{dZ}{da})(-\varepsilon a + \beta(a)) = 0 \Downarrow$$
$$a\frac{dZ}{da} = \frac{\beta}{\varepsilon a - \beta} Z \Downarrow$$
$$\ln Z/Z_0 = \int_{a_0}^{a} \frac{da}{a} \frac{\beta}{\varepsilon a - \beta} \tag{251}$$

RD equation, for the coupling constant a

$$\dot{a} = \beta_1 a + \beta_2 a^2 + \ldots \tag{252}$$

can be reparametrized,

$$\begin{aligned} a(t) &= f(A(t)) = A + f_2 A^2 + \ldots \\ &+ f_n A^n + \ldots = \sum_{n \geq 1} f_n A^n, \\ \dot{A} &= b_1 A + b_2 A^2 + \ldots = \sum_{n \geq 1} b_n A^n, \end{aligned} \tag{253}$$

$$\begin{aligned}
&\dot{a} = \dot{A}f'(A) = (b_1 A + b_2 A^2 + \ldots) \\
&\times(1 + 2f_2 A + \ldots + nf_n A^{n-1} + \ldots) \\
&= \beta_1(A + f_2 A^2 + \ldots + f_n A^n + \ldots) \\
&+\beta_2(A^2 + 2f_2 A^3 + \ldots) + \ldots \\
&+\beta_n(A^n + nf_2 A^{n+1} + \ldots) + \ldots \\
&= \beta_1 A + (\beta_2 + \beta_1 f_2)A^2 \\
&+(\beta_3 + 2\beta_2 f_2 + \beta_1 f_3)A^3 + \\
&\ldots + (\beta_n + (n-1)\beta_{n-1} f_2 \\
&+\ldots + \beta_1 f_n)A^n + \ldots
\end{aligned} \tag{254}$$

$$= \sum_{\substack{nn_1 n_2 \geq 1 \\ m_1 \dots m_k \geq 0}} A^n b_{n_1} n_2 f_{n_2} \delta_{n,n_1+n_2-1}$$

$$= \sum_{nm \geq 1} A^n \beta_m f_1^{m_1}$$

$$\dots f_k^{m_k} f(n, m, m_1, \dots, m_k),$$

$$f(n, m, m_1, \dots, m_k) = \frac{m!}{m_1! \dots m_k!}$$

$$\times \delta_{n,m_1+2m_2+\dots+km_k} \delta_{m,m_1+m_2+\dots+m_k}, \qquad (255)$$

$$b_1 = \beta_1, b_2 = \beta_2 + f_2\beta_1 - 2f_2 b_1 = \beta_2 - f_2\beta_1,$$
$$b_3 = \beta_3 + 2f_2\beta_2 + f_3\beta_1 - 2f_2 b_2 - 3f_3 b_1$$
$$= \beta_3 + 2(f_2^2 - f_3)\beta_1,$$
$$b_4 = \beta_4 + 3f_2\beta_3 + f_2^2\beta_2 + 2f_3\beta_2$$
$$-3f_4 b_1 - 3f_3 b_2 - 2f_2 b_3, \ldots$$
$$b_n = \beta_n + \ldots + \beta_1 f_n - 2f_2 b_{n-1}$$
$$\ldots - nf_n b_1, \ldots \tag{256}$$

So, by reparametrization, beyond the critical dimension ($\beta_1 \neq 0$) we can change any coefficient but β_1.

We can fix any higher coefficient with zero value, if we take

$$f_2 = \frac{\beta_2}{\beta_1},\ f_3 = \frac{\beta_3}{2\beta_1} + f_2^2,$$
$$\dots,\ f_n = \frac{\beta_n + \dots}{(n-1)\beta_1}, \dots \qquad (257)$$

In this case we have simple scale dynamics,

$$A = (\mu/\mu_0)^{-2\varepsilon} A_0 = e^{-2\varepsilon\tau} A_0,$$
$$g = f(A(\tau)). \qquad (258)$$

In the critical dimension of space-time, $\beta_1 = 0$, and we can change by reparametrization any coefficient but β_2 and β_3. From the relations (256), in the critical dimenshion ($\beta_1 = 0$), we find that, we can define the minimal form of the RD equation

$$\dot{A} = \beta_2 A^2 + \beta_3 A^3, \tag{259}$$

e.g. $b_4 = 0$ when

$$f_3 = \frac{\beta_4}{\beta_2} + \frac{\beta_3}{\beta_2} f_2 + f_2^2, \tag{260}$$

f_2 remains arbitrary and we can make choice $f_2 = 0$.

We can solve (259) as implicit function,

$$u^{\beta_3/\beta_2}e^{-u} = ce^{\beta_2 t}, \;\; u = \frac{1}{A} + \frac{\beta_3}{\beta_2} \qquad (261)$$

than, as in the noncritical case, explicit solution for a will be given by reparametrization representation (253).

If we know somehow the coefficients β_n, e.g. for first several exact and for others asymptotic values [Kazakov, Shirkov, 1980], than we can construct reparametrization function (253) and find the dynamics of the running coupling constant. This is similar to the action-angular canonical transformation of the analytic mechanics (see e.g. [Faddeev, Takhtajan, 1990]).

For any multiplicative renormalized quantity
$A, A_b = Z(\varepsilon, a)A,$

$$\dot{A} = -\gamma(\varepsilon, a)A, \gamma = \dot{Z}/Z,$$
$$A = \exp(-\int^a da \frac{\gamma(\varepsilon, a)}{-\varepsilon a + \beta(a)}), \qquad (262)$$

for $A = a, \gamma = \varepsilon - \beta(a)/a.$
In field theory models usually consider small values of ε. In statphysical models usually $D = 3, 2, 1$, so $\varepsilon = 1/2, 1, 3/2.$ Perturbative series of renormalization constants have good analytic sense when $1/\varepsilon$ is $p-$adic number.

From the relations (256), in the critical dimenshion ($\beta_1 = 0$), we find that, we can define the minimal form of the RD equation

$$\dot{A} = \beta_2 A^2 + \beta_3 A^3, \qquad (263)$$

Let us solve this fundamental equation of RD

$$\frac{dA}{\beta_2 A^3 (1/A + \beta_3/\beta_2)} = dt \Rightarrow \frac{d(1/A)1/A}{1/A + \beta_3/\beta_2} = -\beta_2 dt \Downarrow$$
$$x - a\ln(x + a) = -\beta_2 t + c,$$
$$x = 1/A, \ a = \beta_3/\beta_2 \qquad (264)$$

Nonperturbative extension means the following change

$$t = \ln \frac{p^2}{\Lambda^2} \to t_m = \ln \frac{p^2 + m^2}{\Lambda^2}, \ \frac{dt_m}{dt} = \frac{p^2}{p^2 + m^2} \quad (265)$$

In the solution (264). Let us fined corresponding RD motion equation

$$\dot{x}(1 - \frac{a}{x + a}) = -\beta_2 \frac{p^2}{p^2 + m^2} \Downarrow \quad (266)$$

$$\dot{A} = (\beta_2 A^2 + \beta_3 A^3)\frac{p^2}{p^2+m^2} = \begin{cases} \beta_{pert}, & p^2 \gg m^2, \\ 0, & p^2 \ll m^2 \end{cases},$$

$$\frac{p^2}{p^2+m^2} = 1 - \frac{m^2}{\Lambda^2} e^{(1/A-c)/\beta_2} (1/A + \beta_3/\beta_2)^{-\beta_3/\beta_2^2} \quad (267)$$

In the one loop approximation, $\beta_3 = 0$,

$$\dot{A} = \beta_2 A^2 (1 - \frac{m^2}{\Lambda^2} e^{(1/A-c)/\beta_2}) \quad (268)$$

The exact beta function of supersymmetric QCD was first found in [Novikov, Shifman, Vainshtein, Zakharov 1983]

$$\dot{a} = -a^2 \frac{\beta_0}{1 - Ca}, \ \beta_0 = 3C - TN_f, \ C = C_2/2,$$
$$T = 1/2, \ C_2 = \frac{N^2 - 1}{2N} \tag{269}$$

Note that at $a = a_c = 1/C$ we have the infrared fixed point.

PHASE TRANSITIONS IN THE EXTENDED PARTICLE SYSTEMS, HAGEDORN TEMPERATURE AND CRITICAL DENSITY

Quarks and gluons can break free from their confinement inside protons and neutrons at a temperature of around $200 MeV$ - the temperature of the universe a fraction of a second after the Big Bang. We arrived at this figure by combining the results of supercomputer calculations and heavy-ion collision experiments. It puts our knowledge of quark matter on a firmer footing. According to the Big Bang model, the very early universe was filled with quark-gluon plasma, in which quarks and gluons (the carriers of the strong nuclear force) existed as individual entities. The strong force between quarks increases rapidly

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in the description of a number of various physical systems. Two important ones are neutron stars, which probe the low temperature and intermediate baryon chemical potential domain, and heavy ion collision experiments, which explore the region of the high temperature and low baryon chemical potential. There exist low-dimensional theories, such as (1+1)-dimensional chiral Gross-Neveu (GN) type models, that possess a lot of common features with QCD (renormalizability, asymptotic freedom, dimensional transmutation, the spontaneous breaking of chiral symmetry) and can be used as a laboratory for the qualitative simulation of specific

The thermodynamics of QCD is most conveniently described by the grand canonical partition function [Le Bellac 1996]

$$Z(\alpha,\beta)=tre^{-\alpha Q-\beta H}=\int dAd\bar{q}dq\exp(-S_A-S_q),$$
$$S_A=\int_0^\beta dx_0\int_V d^3xtr(F_{\mu\nu}^2/2),$$
$$S_q=\int_0^\beta dx_0\int_V d^3x\prod_{f=1}^{N_f}\bar{q}_f(\gamma_\mu D_\mu-m_f-\alpha/\beta\gamma_0)q_f \quad (270)$$

Another form of functional representation is

$$Z(\alpha,\beta) = tre^{-\alpha Q-\beta H} = \int dAd\bar{q}dq\exp(-S_A - S_q),$$

$$S_A = \beta/\alpha\int_0^\alpha dx_0\int_V d^3x tr(F_{\mu\nu}^2/2),$$

$$S_q = \int_0^\alpha dx_0\int_V d^3x\prod_{f=1}^{N_f}\bar{q}_f(\gamma_\mu D_\mu - m_f - \gamma_0)q_f \quad (271)$$

The charge density we may interpret as a second hamiltonian: $Q = H_2,\ H_1 = H$ and consider corresponding classical Nambu's dynamics

$$\dot{A} = \{A, H_1, H_2\} \quad (272)$$

QCD at nonzero temperature and baryon chemical potential

The partition function depends on the external macroscopic parameters V, T, μ , as well as on the microscopic parameters like masses and the coupling constant. Once the partition function is known, thermodynamic properties such as free energy, pressure, average particle numbers or the thermal expectation value of an operator A readily follow,

$$F = -T \ln Z \ , \ P = -\frac{\partial F}{\partial V}, < Q >= -\frac{\partial \ln Z}{\partial \mu} \tag{273}$$

Note that the functional form is a trace from evolution operator in imaginary time. For real time the definition is formal. Similarly, it is defined for imaginary chemical potential $i\mu$ and for real μ the definition is formal. So it is natural to consider Wick rotation for both parameters

"...there will be no contradiction in our mind
if we assume that some natural forces are governed
by one special geometry, while other forces by another."
N. I. Lobachevsky

Dedicated to the memory
of V.N. Gribov

Quarkonium spectroscopy indicates that between valence quarks inside hadrons, the potential on small scales has $D = 3$ Coulomb form and at hadronic scales has $D = 1$ Coulomb one. We may add this two types of potentials and form an effective potential in which at small scales dominates $D = 3$ component and at hadronic scale –

From our point of view it is more natural to consider the dimension $D(r)$ of space of hadronic matter which is dynamically changing with r and corresponding Coulomb potential $V_D(r) \sim r^{2-D(r)}$, where effective dimension of space $D(r)$ changes from 3 at small r to 1 at hadronic scales $\sim 1fm$. In this talk we will construct such a potential and effective dimension as a functions of r, [Bureš, Makhaldiani 2019]. Heavy quarkonium is a system which can probe all scales of QCD. Hence heavy quarkonium presents an ideal laboratory for testing the interplay between perturbative and nonperturbative QCD within a controlled environment.

We have the following expression for the solution of the Poisson equation with point-like source in D-dimensional space [Makhaldiani 2019]

$$\Delta\varphi = e\delta^D(x),$$
$$\varphi(D,r) = -\frac{\Gamma(D/2)}{2(D-2)\pi^{D/2}} er^{2-D}, \ V(D,r) = e\varphi(D,r) =$$
$$\alpha(D) = \frac{e^2\Gamma(D/2)}{2(D-2)\pi^{D/2}}, \ V(3,r) = -\frac{e^2}{4\pi r}, \ V(4,r) = -\frac{}{4}$$

Indeed,

$$\int d^D x\Delta\varphi = \Omega_D r^{D-1}\frac{d}{dr}\frac{a_D}{r^{D-2}} = -(D-2)\Omega_D a_D = e, \ a_3$$
$$\int dx^D e^{-x^2} = (2\pi\int_0^\infty drre^{-r^2})^{D/2} = \pi^{D/2} = \Omega_D\int_0^\infty drr$$

As defined so far, the coupling constant has a mass dimension $d_e = (D-3)/2 = -\varepsilon$. To work with a dimensionless coupling constant e, we introduce the mass scale μ. Then, the potential energy takes the following form

$$V(D,r) = -\frac{\Gamma(D/2)}{2(D-2)\pi^{D/2}} e^2 \mu^{2\varepsilon} r^{2-D}$$
$$= -\alpha(D)(\mu r)^{2\varepsilon}/r = -\alpha(D)(x)^{2-D}\mu. \quad (277)$$

Coulomb problem in D-dimensions and Renormdynamics of QCD

Cornell potential contains QCD dynamics. We may compare it with Coulomb potential with dynamical dimension. Let us define dimension of space from the equality of (274) and (277)

$$\frac{k-x^2}{x^{3-D}} = \alpha(D) = \frac{e^2\Gamma(D/2)}{2(D-2)\pi^{D/2}} = \alpha_s \frac{2\Gamma(D/2)}{(D-2)\pi^{(D-2)/2}}, \ \alpha_s$$

For any values of x and D

$$\alpha_s(D,x) = \frac{\pi^{(D-2)/2}}{2\Gamma(D/2)}(D-2)\alpha, \ \alpha = \frac{k-x^2}{x^{3-D}} = (k-x^2)x^{D-(2}$$

At the point $D=1, \ x=x_1$

Let us consider simplest Hamiltonian dynamics

$$\dot{x}_1 = \{H, x_1\},$$
$$\dot{x}_2 = \{H, x_2\}, \qquad (281)$$

for dynamical variables (phase space) (x_1, x_2), Hamiltonian H

$$H = \frac{p^2}{2m} + V(x) = \frac{x_1^2}{2m} + V(x_2) \qquad (282)$$

and Poisson structure

$$\{A, B\} = f_{nm}\frac{\partial A}{\partial x_n}\frac{\partial B}{\partial x_m} = f_{12}\left(\frac{\partial A}{\partial x_1}\frac{\partial B}{\partial x_2} - \frac{\partial A}{\partial x_2}\frac{\partial B}{\partial x_1}\right). \qquad (283)$$

Instead of solving the system of motion equations, having one integral of motion - Hamiltonian, we may find x_1 from

The variables x, D and α are nonnegative, so it is natural to introduce, free from this restriction, variables: $t = \ln x$, $x_1 = \ln \alpha_s$ and $x_2 = \ln D$. Then from (278) we obtain the following Hamiltonian and motion equations

$$H(x_1, x_2, t) = x_1 - V(x_2, t) \Rightarrow x_1 = V(x_2, t),$$
$$\dot{x}_1 = f_{12}\frac{\partial V}{\partial x_2},$$
$$\dot{x}_2 = -f_{12}, \; V(x_2, t) = \ln(\frac{\pi^{(D-2)/2}}{2\Gamma(D/2)}(D-2)\frac{k - x^2}{x^{3-D}}). \quad (284)$$

We may also take $x_1 = \alpha$, then

$$x_1 = V(t, x_2) = (k - x^2)x^{D-3} = (k - x^2)x^{\exp(x_2)-3} = (k$$

$$\dot{x}_1 = \frac{\partial V}{\partial x_2} = (k - x^2)x^{e^{x_2}-3}\ln x e^{x_2} = (k - e^{2t})te^{t(e^{-t}-3)}e^{-}$$

$$\dot{\alpha} = \beta = te^{-t}\alpha = \beta_1\alpha,\ \ \beta_1 = \ln\frac{\alpha e^{3t}}{k - e^{2t}}$$

$$\dot{x}_2 = -1 \Rightarrow x_2 = -t,\ \ D = 1/x$$

$$\alpha_s(D, x) = \frac{\pi^{(D-2)/2}}{2\Gamma(D/2)}(D-2)\frac{k - x^2}{x^{3-D}} = \frac{\pi^{(1/x-2)/2}}{2\Gamma(1/2x)}(1/x - $$

$$= \frac{\pi^{(1/x-2)/2}}{2\Gamma(1/2x)}(1/x - 2)(\sqrt{k} - x)\frac{\sqrt{k} + x}{x^{3-1/x}},$$

Note that, $x > 0$ and $\alpha_s \geq 0$ when

$x < \min(1/2, \sqrt{k}) = 1/2$ or

Hamiltonian formulation of the space dimension dynamics

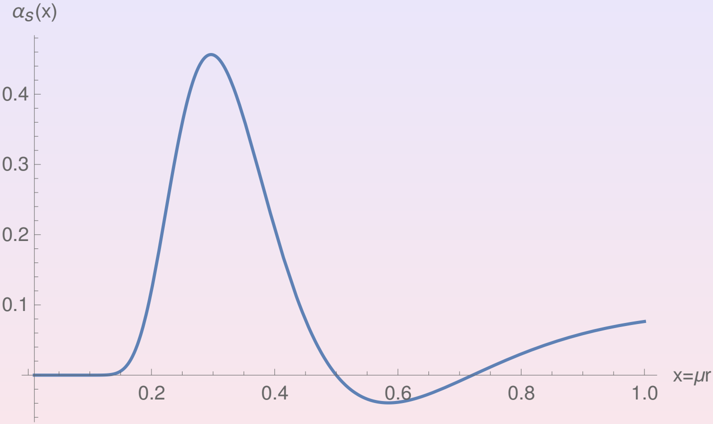


Figure: α_s as a function of $x = \mu r \in (0.01, 1.0)$

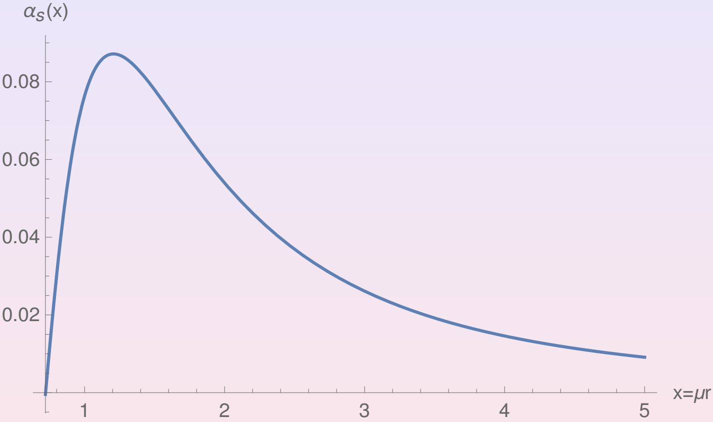


Figure: α_s as a function of $x = \mu r \in (0.72, 5)$

We may close the negative interval also taking
$\sqrt{k} = 1/2 \Rightarrow \alpha_s = 3/16 = 0.1875$

$$\alpha_s(D,x) = \frac{\pi^{(D-2)/2}}{2\Gamma(D/2)}(D-2)\frac{k-x^2}{x^{3-D}} = \frac{\pi^{(1/x-2)/2}}{2\Gamma(1/2x)}(1/x-$$

$$= \frac{\pi^{1/2x-1}}{\Gamma(1/2x)}(x-1/2)^2\frac{x+1/2}{x^{4-1/x}} \to \frac{1}{2\pi x^2},\ x \gg 1,$$

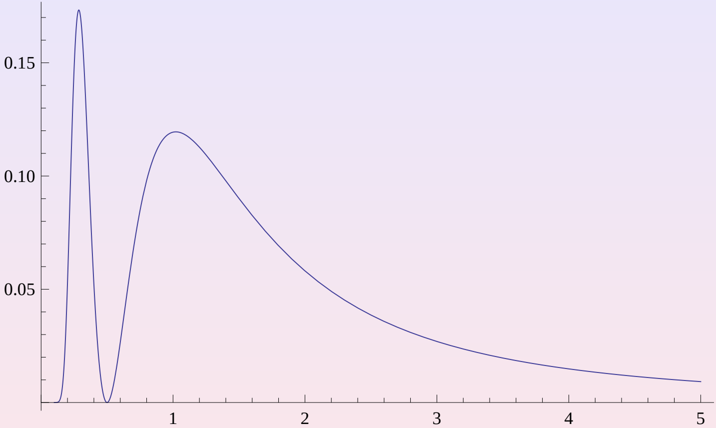


Figure: α_s as a function of $x = \mu r \in (0.1, 5.0)$

It is known that the force between two charges, e and $-e$, changes when the system is placed in a medium. In an ionized plasma, the $1/r$ potential turns into Yukawa form - Debye screening [Debye, Hückel 1923] (see also [Dixit 1989])

$$V(r) = -\frac{\alpha e^{-\mu r}}{r} = -\frac{\alpha}{r} - \sigma r + ..., \ \sigma = \alpha\mu^2/2\,. \quad (287)$$

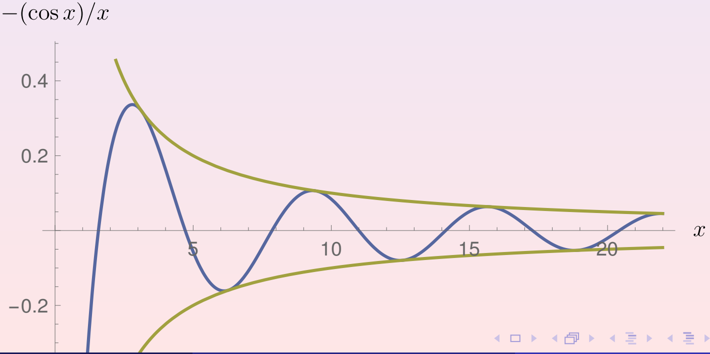
In expanded form it reminds of the "Cornell potential" (274)

$$V(r) = -\frac{k}{r} + \sigma r, \ \sigma = 1/a^2 \quad (288)$$

but the sign of the string tension σ is opposite. The positive sign corresponds to the confined phase, negative

We can take (test) as a confining potential the following one

$$V(r) = -\frac{\alpha \cos \mu r}{r} = -\frac{\mu\alpha \cos(x)}{x} = -\frac{\alpha}{r} + \sigma r + ...,$$
$$\sigma = \alpha\mu^2/2, \; x = \mu r \; . \tag{289}$$



The confining potential turns into a deconfining one when μ^2 changes sign or when exchange particle becomes tachyon.

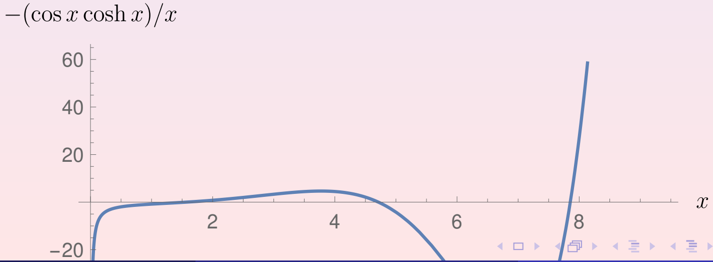
In paper [Kharzeev, Levin 2015], by proper account of the compact nature of SU(3) gauge group that gives rise to the periodic θ-vacuum of the theory, the gluon propagator was modified as

$$G(p) = (p^2 + \chi/p^2)^{-1} = \frac{p^2}{p^4 + \chi} = \frac{1}{2}(\frac{1}{p^2 + i\sqrt{\chi}} + \frac{1}{p^2 - i\sqrt{}}$$

which gives the potential (cf. Figure 6):

$$V(r) = -\frac{\alpha \cosh \mu r \cos \mu r}{r} = -\frac{\mu\alpha \cosh x \cos x}{x} = \mu\alpha(-\frac{1}{x}$$

The topological susceptibility in this formula is the only quantity which is by definition calculable in gluodynamics. Early papers of its calculation are [Di Vecchia, Fabricius, Rossi, Veneziano 1981 , Makhaldiani, Muller-Preussker 1983, Fabricius, Rossi 1983], more recent [Muller-Preussker 2015].



Potential (291) is well motivated and confining. In the minimum of the potential (291) bound states "bags" have size of the order of 11fm,

$$7/\mu = 7/0.127 \text{ GeV}^{-1} = 11 fm, \;\; \text{GeV}^{-1} \simeq 0.2 fm\,, \quad (293)$$

and can give rise to long lived states corresponding to hadronic halos or galactic (in case of gravitational) halos [Bureš, Makhaldiani 2020].
We have shown that linearly rising potential corresponds to $D = 1$. To the quadratic potential then corresponds $D = 0-$finite number of point particles - finite number of point set, and to the cubic potential - $D = -1-$ the empty space - empty set - vacuum state. We may extend Poisson

For heavy quark bound states in the framework of a non-relativistic potential model, for charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$), the Hamiltonian is given by

$$H = 2m - \frac{1}{m}\Delta + V(r, T) \qquad (294)$$

where m denotes the quark mass. For the interquark potential $V(r, T)$ we take the Cornell form

$$V(r, 0) = \sigma_0 r - \frac{\alpha}{r} \qquad (295)$$

where $\sigma_0 = 0.192 GeV^2$ and $\alpha = 0.471$,
$m_c = 1.320 GeV$, $m_b = 4.746 GeV$, as determined in a detailed in [Jacobs et al 1986];
The $1/r$ term in (295) contains both transverse string

Strongly interacting matter of sufficiently high density undergo a transition to a state of de confined quarks and gluons. Deconfinement occurs when color screening shields a given quark from the binding potential of any other quarks or antiquarks. Bound states of very heavy quarks have radii which are much smaller than those of the usual mesons and nucleons; hence they can survive in a deconfined medium until the temperature or density becomes so high that screening also prevents their tighter binding. Color screening and deconfinement for heavy quark resonances are therefore crucial for the experimental investigation of quark plasma formation.

For the study of the deconfinement of heavy quarks at

The specific screening factor for the linear part of the potential is suggested by the Schwinger model [Joos, Montvay 1983].
To apply these considerations to actual physical situations, we need to know the specific dependence of $\mu(T)$ on T. If nuclear collisions produce strongly interacting matter, then it is the temperature, not $\mu(T)$, which can be empirically determined. At $T = 0$, we have $\mu(0)=0$ only in a world without light quarks. In the presence of light quarks, the binding of any quark-antiquark system is broken when its binding energy exceeds that needed for the spontaneous creation of a $q\bar{q}$ state out of the vacuum. Hence $\mu(0) \neq 0$.
The corresponding vacuum screening length is of the order

Heavy ion collisions quickly form a droplet of quark–gluon plasma (QGP)with a remarkably small viscosity,
The physics of heavy ion collisions ranges from highly energetic quarks and gluons described by perturbative QCD to a bath of strongly interacting gluons at lower energy scales.
The running coupling possesses an infrared fixed point, $\alpha_S(0) = 8.92/N_c$ for all gauge groups $SU(N_c)$, [Busza, Rajagopal, W. van der Schee 2018]. Above one GeV the running coupling rapidly approaches its perturbative form. We will postulate that $\alpha_S(0) = 9.0/N_c = 3.0$ when $N_c = 3$.

In the multiparticle production processes of high energy particle physics there is a rule: the number of observed (charged) particles multiply by factor 1.5 (=3/2). An explanation of the rule is: if particle production is dominated by strong interactions, produced neutral particles has the same number as positive or negative (which are equal) particles. This way we estimate the number of "dark" neutral particles.

According to the contemporary observations there are extra masses in the universe interacting with the usual matter only gravitationally - dark matter. So in Newton's potential in the place of usual mass m, we should put an effective mass M,

$$V = -\frac{GM}{r} = -\frac{M}{m}\frac{Gm}{r} = -k(r)\frac{Gm}{r} \tag{298}$$

This extra factor $k(r)$ we may associate either to the Newton's constant G or to the dimension of the space in the Newton's potential in D-dimensions,

$$V_d = -k(x)\frac{Gm\mu}{x^{D-2}} = \frac{Gm\mu}{x^{d-2}},$$

$\ln k(x)$

Only motivation of DM remains just kinematic discrepancy. If there were others, we could consider them as a "proofs" :) First step: just add extra matter to the baryonic one, solves discrepancy problem but has not explanation: rise many other problems for candidates as a DM components. From a minimalist point of view it seems proper not to introduce any new DM but re parametrize the Newton potential by introducing variable dimension of space. For higher values of the DM factor $K(r)$ will corresponds lower values of the effective dimension $d(r) = D - \epsilon(r)$. From the other side, if we have some deficit as in ^7Li lack problem and/ore in ultracold neutrons in the neutron life-time experiments, we may explain that by real DM

Cosmological constant and periodic structures of the Universe

In the presence of a cosmological constant Λ, Einstein's equations is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\mu}_{\mu}) + \Lambda g_{\mu\nu} \quad (300)$$

The Schwarzschild line element for realistic (small) values of the cosmological constant is

$$ds^2 = (1 + \Lambda r^3/3 - \frac{2G_N M}{r})dt^2 - (1 + \Lambda r^3/3 - \frac{2G_N M}{r})^{-1}dr$$

we see that the Newton's potential is modified by confining potential of the same form as in the QCD case. As in QCD, we may take as a modified gravitational potential

$$G_N M \cos\mu r \cosh\mu r \qquad G_N M\mu \cos x \cosh x$$

Cosmological constant and periodic structures of the Universe

It is interesting when the cosmological constant correction become of the order of the Newton potential,

$$q = x^4/6 = \Lambda r^3/6/(G_N M/r) = \frac{\Lambda}{6G_N M} r^4 \sim 1,$$
$$x \sim (6)^{1/4} = 1.565,\ r = 1.565/\mu = 1.565/(0.32 heV)$$
$$\simeq 0.5 h^{-1} eV^{-1} = 2h^{-1} 10^{-5} cm. \qquad (303)$$

when $M = 4\pi r^3 \rho/3$,

$$q = \frac{\Lambda}{8\pi G_N \rho} r, \Lambda = \Omega_\Lambda \frac{3H_0^2}{c^2},\ \Omega_\Lambda \simeq 0.7$$
$$H_0 = 100h \times km \times s^{-1} \times Mpc^{-1},\ h \simeq 0.7,$$
$$Mpc = 10^6 pc = 3.1 \times 10^{22} m,\ pc = 3.1 \times 10^{16} m$$

On the large scale periodic structure and model independent evaluation of the number of periods of the Universe

Observations of the large scale structure of the universe suggest inhomogeneities on scales between $100h^{-1}$ and $150h^{-1}$ Mpc (where $h = 0.5 - 1$ is the Hubble constant in units of 100 $kms^{-l}Mpc^{-1}; 1pc = 3.09 \times 10^{16}m$). A deep redshift survey with a"pencil-beam"geometry of galaxies at the galactic poles indicated strong clustering,with a provocative regularity at $128h^{-1}$ Mpc [Broadhurst et al 1990].

In our potential period is $l = 2\pi/\mu$. If this period is equal to the large scale period, we define $\mu = 2\pi/l = 0.32heV$.

We estimated the number of periods in the visible Universe as 24 [Makhaldiani1992] which coincides with the effective

Extended quantum mechanics and conformal potential

In the extended quantum mechanics [Makhaldiani, 2000], usual Schroedinger equation were completed by corresponding motion equation for the potential V in the following Hamiltonian system

$$
\begin{aligned}
i\psi_t &= -\Delta\psi + V\psi, \\
iV_t &= \Delta V - \frac{1}{2}V^2
\end{aligned} \tag{305}
$$

with a partial solution for the potential

$$V = \frac{4(4-D)}{r^2}. \tag{306}$$

For dimension of space $D > 4$ we have attractive potential with possibility of quantum-mechanical bound states. We

Let us take one of the dimensions y as circle with radius R. This corresponds to the periodic structure with a point charge sources at each point

$y_n = y + 2\pi Rn, n = 0, \pm 1, \pm 2, ...$

$$\Delta\varphi = e\sum_n \delta^D(x)\delta(y_n), \varphi(D, r, y) = \sum_n \varphi(D, r, y_n),$$

$$V(D, r, y) = -\alpha(D+1)\sum_{n=-\infty}^{\infty}(r^2 + (2\pi Rn + y)^2)^{(1-D)/2} \quad (307)$$

When $D = 3$, the potential (307) can be writen in a closed form [Bureš, Siegl 2014]

$$V_3(r, y) = -\frac{\alpha(4)}{2Rr}\frac{\sinh(r/R)}{\cosh(r/R) - \cos(y/R)} = \begin{cases}\alpha(4)/(2Rr), \\ \alpha(4)/(r^2 + y^2 \end{cases}$$

Alternatively, we can rewrite (308) as

$$V_3(r,y) = -\frac{\alpha(4)}{4Rr}\left[\coth\left(\frac{r+\mathrm{i}y}{2R}\right) + \coth\left(\frac{r-\mathrm{i}y}{2R}\right)\right], \quad (309)$$

or, using

$$A^{-\alpha} = 1/\Gamma(\alpha)\int_0^\infty dt t^{\alpha-1}e^{-tA}, \quad (310)$$

by means of the Theta function as

$$V_3(r,y) = -\alpha(4)\int_0^\infty dt e^{-tr^2}\sum_{-\infty}^{\infty} e^{-t(2\pi Rn+y)^2}$$
$$= -\alpha(4)\int_0^\infty dt e^{-tr^2}\frac{\theta\left(\frac{\mathrm{i}y}{2\pi R}, e^{\frac{\mathrm{i}}{4R^2 t}}\right)}{2R\sqrt{\pi}\sqrt{t}}, \quad (311)$$

For $y = 0$, the potential takes the following simple form

$$V_3(r, y = 0) = -\frac{\alpha(4)}{2Rr} \coth \frac{r}{2R}. \tag{312}$$

For $y = \pi$, we have

$$V_3(r, y = \pi) = -\frac{\alpha(4)}{2Rr} \tanh \frac{r}{2R}. \tag{313}$$

From (308), we see that for big r, the effective dimension of space is 3 and for small r is 4. For intermediate scales, the effective dimension might change smoothly from 3 to 4. Integrating by coordinate y (or angle ϑ, see appendix B) we define mean potential depending only on the variable r, [Bureš, Siegl 2014]

Dimension dynamics with one compact dimension

As in the Cornell potential case, we define the dimension dynamics from equality between the Coulomb potentials (277) and (308)

$$\frac{\alpha(4)}{2r}\frac{\sinh(r/R)}{\cosh(r/R)-\cos(y/R)} = \alpha(D)(x)^{2-D},$$
$$\mu = 1/R,\ x = \mu r,\ r^2 = x_1^2 + ... + x_D^2. \qquad (315)$$

From this equality, the dynamical dimension of space $D(y,r)$ is defined as implicit function and needs numerical solution. Alternatively we may define y as explicit function of r and D as

$$y = R\arccos(\cosh x - \frac{\alpha_4}{2\alpha_D}x^{D-3}\sinh x)$$
$$= R\arccos(\cosh x - A(D)x^{D-3}\sinh x)$$

If we have two circlular coordinates - torus, then

$$\Delta\varphi = e\sum_{n,m}\delta^D(x)\delta(y_n)\delta(z_m),$$

$$\varphi(D,r,y,z) = \sum_{n,m}\varphi(D,r,y_n,z_m),$$

$$V(D,r,y,z) = -\alpha_{D+2}\sum_{n,m=-\infty}^{\infty}(r^2+(2\pi R_1 n+y)^2$$

$$+(2\pi R_2 m+z)^2)^{-D/2} \tag{317}$$

Dimension dynamics with more than one compact dimension

For a point quark inside hadron of size R at a temperature T we have

$$\Delta\varphi = e \sum_{k,l,n,m} \delta(\tau_k)\delta(x_l)\delta(y_n)\delta(z_m),$$

$$\varphi(0,\tau,x,y,z) = \sum_{k,l,n,m} \varphi(0,\tau_k,x_l,y_n,z_m),$$

$$V(0,\tau,x,y,z) = -\alpha_4 \sum_{k,l,n,m=-\infty}^{\infty} ((2\pi k/T+\tau)^2$$

$$+(2\pi R_1 l + x)^2 + (2\pi R_2 n + y)^2 + (2\pi R_3 m + z)^2)^{-1}$$

$$= -\alpha_4 \int_0^\infty dt t B_0(t,\tau) B_1(t,x) B_2(t,y) B_3(t,z),$$

Dimension dynamics with more than one compact dimension

For the sake of completeness, let us state the general expression for the potential in space $\mathbb{R}^D \times \mathbb{T}^d$ where $\mathbb{T}^d = S^1 \times \cdots \times S^1$ (d-times) is the d-dimensional torus. D refer to the "big" dimensions $\mathrm{x} = (x_1, \ldots x_D)$, whereas d to the "small-compactified" ones $\mathrm{y} = (y_1, \ldots y_d)$. Then

$$\Delta\varphi = e \sum_{n_1,\ldots,n_d} \delta^D(\mathrm{x})\delta(y_{1,n_1})\ldots\delta(y_{d,n_d}),$$

$$\varphi(D,d,r,y_1,\ldots,y_d) = \sum_{n_1,\ldots,n_d} \varphi(D,d,r,y_{1,n_1},\ldots,y_{d,n_d}),$$

$$V_{D,d}(r,y_1,\ldots,y_d) = \int_0^\infty dtt^{\frac{D+d-4}{2}} e^{-tr^2}\Pi_{i=1}^d e^{-ty_i^2} B_i(t,y_i),$$

∞

where we have again used (310) and written the sums in the expressions for B_i by means of the Theta function. Note that, B-factors in the integrand $\sim t^{-d/2}$ for small t, so the integral is divergent when $D \leq 2$. E.g. for total dimension $D + d = 4$ and $d = 2$, the integral is divergent. We may regularize the integral by restricting summation by some N or consider analytic continuation $D + d + \epsilon$ in the monomial factor of the integral. We may define the same conditions from direct form of the sum, before integral transform. Divergent part of the sum is estimated by integral

We may change long distance behaviour of the Coulomb potential including a variable mass term

$$\triangle V - m^2 V = -e^2\delta^D(x),\ \delta^D(x) = (2\pi)^{-D}\int d^D p e^{ipx} \Downarrow$$

$$V(r) = -\frac{e^2}{(2\pi)^D}\int d^D p(p^2 + m^2)^{-1} e^{ipx} \sim r^{2-D} e^{-mr} \sim r$$

$$m \Rightarrow m(r) = a\ln(\mu r)/r,\ d = D + a. \qquad (3$$

Note that the m^2 term plays a role of potential. Comparing this therm with the expression (306), we find corresponding dimension dynamics: $D(r) = 4 - a^2\ln^2\mu r/4$, from which we may estimate parton size r_0 , from hadron size $R \sim 1$ fm and condition $D(r) = 0$: $r_0 = R\exp(-4/a)$. The

Matrix calculus in QFT perturbation theory [Isaev, 2003], can be interpreted as operator Fractal calculus. Indeed, with the following definitions

$$[\hat{x}_n, \hat{p}_m] = i\delta_{n,m}, \ \hat{x}_n|x >= x_n|x >,$$
$$\hat{p}_m|p >= p_m|p >, \ < x|y >= \delta^D(x-y) < x|p >= \frac{1}{(2\pi)^{...}}$$
$$\int d^D p|p >< p| = \int d^D x|x >< x| = 1,$$

we have

$$G(x,y) =< x|\hat{p}^{-2\alpha}|y >= A(\alpha)(x-y)^{-2\beta},$$
$$\beta = \frac{D}{2} - \alpha, \ A(\alpha) = \frac{\Gamma(\beta)}{...}, \quad (324)$$

Fractal Calculus in Quantum Field theory
Applications

$$
\begin{aligned}
G(x,y) &= \frac{1}{\Gamma(\alpha)}\int_0^\infty dt t^{\alpha-1} < x|e^{-t\hat{p}^2}|y > \\
&= \frac{1}{\Gamma(\alpha)}\int_0^\infty dt t^{\alpha-1} \int d^D p < x|p >< p|y > \exp(- \\
&= \frac{\Gamma(\frac{D}{2}-\alpha)}{\Gamma(\alpha)2^{2\alpha}\pi^{D/2}}(x-y)^{-2(D/2-\alpha)}
\end{aligned}
$$

In coordinate representation, $\hat{p}_n = -i\partial/\partial x_n$, we have D-dimensional fractal calculus.

As an example, consider Coulomb potential, the solution of the equation for potential of point source

$$\Delta\phi = g\delta^D(x) \qquad (326)$$

Note, that, $\Delta = -\hat{p}^2$,

$$\varphi(x) = -g < 0|\frac{1}{\hat{p}^2}|x >= -g\frac{\Gamma(\frac{D}{2}-1)}{4\pi^{D/2}}\frac{1}{|x|^{D-2}} \qquad (327)$$

As another example, we take the following useful integral

$$G_3(x,y) = \int d^Dz(x-z)^{-2\alpha}(z-y)^{-2\gamma}$$

$$= A^{-1}(\beta)A^{-1}(\delta)\int d^Dz < x|\hat{p}^{-2\beta}|z >< z|\hat{p}^{-2\delta}|y >$$

Theta functions is the analytic function $\theta(z,\tau)$ in 2 variables defined by

$$\theta(z,\tau) = \sum_{n\in\mathbb{Z}} \exp[i\pi(\tau n^2 + 2nz)] = 1 + 2\sum_{n\geq 1} \exp(\mathrm{i}\pi\tau n^2) \cos$$

where $z \in \mathbb{C}$ and $\tau \in \mathbb{H}$, the upper half plane Im $\tau > 0$. The series converges absolutely and uniformly on compact sets.

Let us take the following integral

$$I(a) = \int_0^\pi \frac{d\vartheta}{a^2+1-2a\cos(\vartheta)} = \frac{\pi}{|a^2-1|} = \{ \begin{array}{ll} \pi/a^2, & a^2 \\ \pi, & a^2 \end{array}$$

Obviously, $I(1) = \infty$, but

$$I(1) = \frac{1}{2}\int_0^\pi \frac{d\vartheta}{1-\cos(\vartheta)} = \frac{1}{4}\int_0^\pi \frac{d\vartheta}{\sin^2\frac{\vartheta}{2}} = \frac{1}{2}\int_0^1 \frac{dx}{(1-x^2)}$$

$$= \frac{1}{4}\int_0^1 \frac{dy}{y^{1/2}(1-y)^{3/2}} = B(1/2,-1/2) = \frac{1}{4}\frac{\Gamma(1/2)\Gamma(-1}{\Gamma(0)}$$

$$B(\alpha,\beta) = \int_0^1 dx x^{\alpha-1}(1-x)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, Real \quad \alpha,\beta$$

In our case $a = \exp(r/R) > 1$. Let us take corresponding integral,

$$I = \frac{1}{a}\int \frac{d\theta}{b + 2\cos\theta} = \frac{1}{ia}\int \frac{dz}{z^2 + bz + 1}$$
$$= I(z, a) = \frac{1}{ia(a - 1/a)} \ln \frac{z + a}{z + 1/a},$$
$$I(a) = I(-1, a) - I(1, a) = \frac{1}{ia(a - 1/a)} \ln \frac{(-1 + a)(1 +}{(-1 + 1/a)(1}$$
$$b = a + 1/a, \; z = e^{i\theta}.$$

We may calculate the same integral by residue formula

$$\oint \frac{dz}{(z + a)(z + 1/a)} = 2\pi i \{ \begin{array}{ll} 1/(-a + 1/a), & |a| < 1 \\ 1/(-1/a + a), & |a| > 1. \end{array} \quad (333)$$

Now,

$$I = \int_0^{2\pi} \frac{d\vartheta}{a^2+1-2a\cos(\vartheta)} = I(a)+I(-a)$$
$$= \frac{2\pi}{|a^2-1|} = \frac{\pi\exp(-r/R)}{\sinh(r/R)},$$
$$\frac{1}{2\pi}\int_0^{2\pi} \frac{d\vartheta}{\cosh(r/R)-\cos(\vartheta)} = \frac{2a}{a^2-1}$$
$$= \frac{1}{\sinh(r/R)}, \ a = \exp(r/R). \qquad (335)$$

The rich structure of separated energy scales makes quarkonium an ideal probe of confinement and deconfinement. The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state. Different quarkonia will dissociate in a medium at different temperatures, providing a thermometer for the plasma.

Hagedorn temperature and decoherence problem in quanputers

Let us construct a model of a physical system with maximal (Hagedorn) temperature $T_H = \beta_H^{-1}, \;\; T < T_H$. The system consists from N identical (noninteracting) subsystems s_n each of which can be in $p > 1$ states with same energy ε. So the number of states of the system with given energy $E = N\varepsilon$ is $M = p^N$ and corresponding entropy is $S = N \ln p$. To different energies E corresponds different $N = E/\varepsilon$. The statistical sum of the system is

$$Z(\beta) = \sum_E \rho(E) e^{-\beta E} = \sum_N e^{-(\beta - \beta_H)\varepsilon N}, \;\; \beta > \beta_H = \varepsilon \ln p \quad (336$$

Note that, for QCD $T_H \simeq 150$ MeV. Quantum computers

Let us consider the integer derivatives of the monomials

$$\frac{d^n}{dx^n}x^m = m(m-1)...(m-(n-1))x^{m-n}, \quad n \leq m, \\ = \frac{\Gamma(m+1)}{\Gamma(m+1-n)}x^{m-n}. \tag{337}$$

L.Euler (1707 - 1783) invented the following definition of the fractal derivatives,

$$\frac{d^\alpha}{dx^\alpha}x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha}. \tag{338}$$

J.Liouville (1809-1882) takes exponents as a base functions,

$$\frac{d^\alpha}{dx^\alpha}e^{ax} = a^\alpha e^{ax}. \tag{339}$$

The following Cauchy formula

$$I^n_{0,x} f = \int_0^x dx_n \int_0^{x_{n-1}} dx_{n-2} \ldots \int_0^{x_2} dx_1 f(x_1) = \frac{1}{\Gamma(n)} \int_0^x dy(x$$

permits analytic extension from integer n to complex α,

$$I^\alpha_{0,x} f = \frac{1}{\Gamma(\alpha)} \int_0^x dy(x-y)^{\alpha-1} f(y) \qquad (341)$$

J.H. Holmgren invented (in 1863) the following integral transformation,

$$D_{c,x}^{-\alpha} f = \frac{1}{\Gamma(\alpha)} \int_c^x |x-t|^{\alpha-1} f(t) dt. \tag{342}$$

It is easy to show that

$$\begin{aligned} D_{c,x}^{-\alpha} x^m &= \frac{\Gamma(m+1)}{\Gamma(m+1+\alpha)} (x^{m+\alpha} - c^{m+\alpha}), \\ D_{c,x}^{-\alpha} e^{ax} &= a^{-\alpha}(e^{ax} - e^{ac}), \end{aligned} \tag{343}$$

so, $c = 0$, when $m + \alpha \geq 0$, in Holmgren's definition of the fractal calculus, corresponds to the Euler's definition, and $c = -\infty$, when $a > 0$, corresponds to the Liouville's

We considered the following modification of the $c = 0$ case [Makhaldiani, 2003],

$$D_{0,x}^{-\alpha} f = \frac{|x|^\alpha}{\Gamma(\alpha)} \int_0^1 |1-t|^{\alpha-1} f(xt)dt, = \frac{|x|^\alpha}{\Gamma(\alpha)} B(\alpha, \ \partial x) f(x)$$
$$= |x|^\alpha \frac{\Gamma(\partial x)}{\Gamma(\alpha + \partial x)} f(x), \ f(xt) = t^{x\frac{d}{dx}} f(x). \quad (345)$$

As an example, consider Euler B-function,

$$B(\alpha, \beta) = \int_0^1 dx |1-x|^{\alpha-1} |x|^{\beta-1} = \Gamma(\alpha)\Gamma(\beta) D_{01}^{-\alpha} D_{0x}^{1-\beta} 1$$
$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad (346)$$

In QFT existence of a given theory means, that we can control its behavior at some scales (short or large distances) by renormalization theory [Collins 1984].
If the theory exists, than we want to solve it, which means to determine what happens on other (large or short) scales.
This is the problem (and content) of Renormdynamics.
The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one).
We can invent scale variable λ and consider QFT on $D+1+1$ dimensional space-time-scale. For the scale variable $\lambda \in (0,1]$ it is natural to consider q-discretization

We will call RDF functions $g_n = f_n(t)$ which are solutions of the RD motion equations

$$\dot{g}_n = \beta_n(g), 1 \leq n \leq N. \tag{347}$$

In the simplest case of one coupling constant the function $g = f(t)$ is constant, $g = g_c$ when $\beta(g_c) = 0$, or is invertible (monotone). Indeed,

$$\dot{g} = f'(t) = f'(f^{-1}(g)) = \beta(g). \tag{348}$$

Each monotone interval ends by UV and IR fixed points and describes corresponding phase of the system.
Note that the simplest case of the classical dynamics, the Hamiltonian system with one degree of freedom, is already two dimensional, so we have no analog of one charge

The regular Hamiltonian systems of the classical mechanics are defined on the even dimensional phase space, so there is no analog of the three dimensional renormdynamics for the coupling constants of the SM. The fixed points of renormdynamics belong to a set of solutions of the polynomial system of equations $\beta_n(g) = 0, 1 \leq n \leq N$, in the perturbative renormdynamics. Describing the solutions is the task of contemporary algebraic and computational geometry.

The quantitative values and qualitative content of the given field theory depend on the scale (parameter, e.g. $\mu-$renormalization point, $g = g(\mu)$, $A = A(\mu)$). In QCD e.g. the effective action has the following form:

$$S(\mu) = \frac{1}{g^2(\mu)} \int d^D x \mathcal{L}(A(\mu)), \quad (349)$$

variation with respect to the change of scale gives

$$\delta S = -2\frac{\beta(g)}{g\mu}\delta\mu S + \frac{1}{g^2}\int d^D x \frac{\delta\mathcal{L}}{\delta A}\delta A \quad (350)$$

and the following two statements are equivalent:

$$\delta S = 0, \ \beta(g) = 0 \Leftrightarrow \delta S = 0, \ \frac{\delta\mathcal{L}}{\delta A} = 0. \quad (351)$$

In string theory, the connection between conformal invariance of the effective theory on the parametric world sheet and the motion equations of the fields on the embedding space is well known [Ketov, 2000]. A more recent topic in this direction is AdS/CFT Duality [Maldacena, 1999]. In this approach for QCD coupling constant the following expression was obtained [Brodsky, de Tèramond, Deur, 2010]

$$\alpha_{AdS}(Q^2) = \alpha(0)e^{-Q^2/4k^2}. \quad (352)$$

A corresponding β-function is

$$\beta(\alpha_{AdS}) = \frac{d\alpha_{AdS}}{\ln Q^2} = -\frac{Q^2}{4k^2}\alpha_{AdS}(Q^2) = \alpha_{AdS}(Q^2)\ln\frac{\alpha_{AdS}(Q^2)}{\alpha(0)}$$

For the QCD running coupling considered in [Diakonov, 2003]

$$\alpha(q^2) = \frac{4\pi}{9\ln(\frac{q^2+m_g^2}{\Lambda^2})}, \tag{354}$$

where $m_g = 0.88 GeV$, $\Lambda = 0.28 GeV$, the $\beta-$function of renormdynamics is

$$\beta(\alpha) = -\frac{\alpha^2}{k}(1 - c\exp(-\frac{k}{\alpha})),$$
$$k = \frac{4\pi}{9} = 1.40, \; c = \frac{m_g^2}{\Lambda^2} = e^{k/\alpha} = (3.143)^2 = 9.88, \tag{355}$$

for a nontrivial (IR) fixed point we have

$$\alpha_{IR} = k/\ln c = 0.61 \tag{356}$$

It is nice to have a nonperturbative $\beta-$function like (355), but it is more important to see which kind of nonperturbative corrections we need to have a phenomenological coupling constant dynamics.
It was noted [Voloshin, Ter-Martyrosian, 1984] that in valence quark parametrization $\alpha_s(m) = 2$, at a valence quark scale m.

Let us consider a potential V and corresponding force F of an elementary charge e in a scalar field φ generated in $D-$dimensional euclidian space by corresponding point charge-source g defined as a solution of the following equation

$$\Delta_D\varphi = g\delta^D(x),$$
$$\int dx^D\Delta_D\varphi = \int dS_{D-1}\nabla\varphi = \Omega_D r^{D-1}\varphi'(r) = g \Downarrow$$
$$\varphi(r) = -\frac{1}{(D-2)\Omega_D}\frac{g}{r^{D-2}},\ D\neq 2$$
$$D = 2 : 2\pi r\varphi'(r) = g \Rightarrow \varphi(r) = \frac{g}{2\pi}\ln\frac{r}{r_0},$$

For Newton potentials charges e, g play pole of masses, they are, by definition and in correspondence with observations, positive and for $D \geq 2$ we have attraction of masses. For $D < 2$ we have attraction and confinement. From Newton potential we obtain the Coulomb one if we take imaginary masses $m = ie_{\pm}$.

In polynomial approximation of a function
$f(x) \simeq P_N(x) = a_0 + a_1 x + ... + a_N x^N$,

$$
\begin{aligned}
& a_0 + a_1 x_0 + a_2 x_0^2 + ... + a_N x_0^N = f(x_0) = f_0, \\
& a_0 + a_1 x_1 + a_2 x_1^2 + ... + a_N x_1^N = f(x_1) = f_1, \\
& ..., \\
& a_0 + a_1 x_1 + a_2 x_N^2 + ... + a_N x_N^N = f(x_N) = f_N,
\end{aligned} \tag{359}
$$

the coefficients $a_n, \ n = 0, 1, ..., N$ are defined as a solutions of the linear system of equations

$$VA = F, \ A^T = (a_0, a_1, ..., a_N), \ F^T = (f_0, f_1, ..., f_N),$$

$$V = \begin{pmatrix} 1 & x_0 & x_0^2 & ... & x_0^N \\ 1 & x_1 & x_1^2 & ... & x_1^N \\ 1 & x_2 & x_2^2 & ... & x_2^N \end{pmatrix} \tag{360}$$

Determinant of the Vandermonde matrix
$\Delta_N = \prod_{N\geq m>n\geq 0}(x_m - x_n)$, ($\Delta_0 = 1$, by definition).
Indeed,

$$\Delta_1 = x_1 - x_0, \tag{361}$$

$$\Delta_2 = det\begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{pmatrix} = det\begin{pmatrix} 1 & x_0 & x_0^2 \\ 0 & x_1 - x_0 & x_1^2 - x_0^2 \\ 0 & x_2 - x_0 & x_2^2 - x_0^2 \end{pmatrix}$$
$$= (x_1 - x_0)(x_2 - x_0)det\begin{pmatrix} 1 & x_1 + x_0 \\ 1 & x_2 + x_0 \end{pmatrix}$$
$$= (x_2 - x_1)(x_2 - x_0)(x_1 - x_0),$$
$$\Delta_N = (x_N - x_{N-1})...(x_N - x_0)\Delta_{N-1} = \prod_{1\leq n\leq N} Z_n,$$

In the first, geometric progression, case

$$Z_n = (p^n - p^{n-1})(p^{n-1} - p^{n-2})...(p^n - 1)$$
$$= p^{(1+2+...+n-1)}(p-1)^n \frac{p^n - 1}{p-1}\frac{p^{n-1} - 1}{p-1}...\frac{p-1}{p-1}$$
$$= p^{n(n-1)/2}(p-1)^n[n]_p!, \ [n]_p = \frac{p^n - 1}{p-1}, \ \Delta_1 = Z_1 = (p -$$
$$\Delta_2 = (p^2 - p)(p^2 - 1)(p-1) = p(p-1)^3(p+1)$$
$$= Z_2 Z_1 = p(p-1)^2(p+1)(p-1),$$
$$\Delta_N = \prod_{1\le n\le N} Z_n = p^a(p-1)^b \prod_{1\le n\le N} [n]_p!, \qquad ($$

$$a = \frac{1}{2}\sum_0^N n(n-1) = \frac{1}{2}\Big(\sum_0^N x^n\Big)^{(2)}\Big|_{x=1} = \frac{1}{2}\frac{(1+\varepsilon)^{N+1} -}{\epsilon}$$

$$= \frac{1}{2}\Big(N+1+\frac{(N+1)N}{2}\varepsilon + \frac{(N+1)N(N-1)}{3!}\varepsilon^2 + \dots\Big)^{(2)}$$

$$= \frac{(N+1)N(N-1)}{6},$$

$$b = \sum_0^N n = N(N+1)/2,$$

$$\Delta_2 = p(p-1)^3(p+1),\ a = 1,\ b = 3.$$

For $p \gg 1$,

$$[n] \sim p^{n-1} \quad [n]! \sim p^{n(n-1)/2}$$

For $p \ll 1, \Delta_N \simeq (-1)^b p^a,\ a = N(N^2-1)/6,\ b = N(N+1)/2,\ [n]_p \simeq 1,\ \Delta_2 \simeq -p$. Having expression for Δ_N in p, it is ease to obtain corresponding expression in arithmetic progression case by putting $p = 1 + h$:
$\Delta_N(h) = h^b \prod_1^N n!,\ b = N(N+1)/2, \Delta_2 = 2h^3$. We obtain the same result by direct calculation:
$Z_n = h \times 2h \times ... \times nh = h^n n!,\ \Delta_N(h) = \prod Z_n$.

Finite Sums

We consider a novel method to generate a polynomial expression for each of the Euler sums,

$$E_k = \sum_{n=1}^{N} n^k, \ \ k \in Z^+ (k = 0, 1, 2, ...) \qquad (366)$$

One of the way of calculation of the sum

$$E_k(N) = \frac{N^{k+1}}{k+1} + P_k(N), \ \ P_k = x_k N^k + x_{k-1} N^{k-1} + ... + x_0 \qquad (367)$$

we show by explicit calculation of E_2.

For particular values $N = 1, \ 2$ and 3, we have

$$x_2 + x_1 + x_0 = 1 - 1/3 = 2/3,$$
$$4x_2 + 2x_1 + x_0 = 5 - 8/3 = 7/3,$$

$$
\begin{aligned}
3x_2 + x_1 &= 5/3, \\
5x_2 + x_1 &= 8/3
\end{aligned} \tag{369}
$$

than we have

$$
\begin{aligned}
&2x_2 = 1 \Rightarrow x_2 = 1/2 \Downarrow \\
&x_1 = 5/3 - 3x_2 = 1/6 \Rightarrow x_0 = 2/3 - x_1 - x_2 = 2/3 - 1/ \\
&E_2(N) = N^3/3 + N^2/2 + N/6 \\
&= N(N+1)(2N+1)/6 = N(N+1/2)(N+1)/3
\end{aligned}
$$

Note that, the right hand side have a sense also for $N \leq 0$ and has zeros at $N = 0, -1/2, -2$.

For general case $E_k(N)$ we have

$$x_k(N) = detV_l(N)/detW_k(N),$$

$$detW_k(N) = det\begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ . & . & . & . \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix},$$

$$detV_l(N) = det\begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ \bar{E}_l(N_1) & \bar{E}_l(N_2) & . & \bar{E}_l(N_{k+1}) \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix}, \ X =$$

$$\bar{E}_k(N_l) = E_k(N_l) - \frac{N_l^{k+1}}{\dots}, \ E = \begin{pmatrix} \bar{E}_k(N_1) \\ \bar{E}_k(N_2) \\ \dots \end{pmatrix},$$

As a numbers N_n we can take any different integers, but the simplest choice is: $N_{n+1} = N_n + 1,\ N_1 = 1$, as in considered explicit calculation for E_2. In this case, $E_k(N+1) = E_k(N) + (N+1)^k$.

Finite Sums from Generating Function

We propose the following compact form for E_k

$$E_k(N) = \frac{d^k}{dx^k} P(x,N)|_{x\Rightarrow 0} \equiv D^k P = P^{(k)}(0,N),$$
$$P(x,N) = \sum_{n=1}^{N} e^{nx} = \frac{e^{(N+1)x} - e^x}{e^x - 1} \qquad (372)$$

We take also the following slightly simpler form of $P(x,N)$, for $k = 1, 2, 3, ...$

$$P(x,N) = \sum_{n=0}^{N} e^{nx} = \frac{e^{(N+1)x} - 1}{e^x - 1} \qquad (373)$$

As an example, let us calculate $E_1(N)$,

$$E_1(N) = \frac{(N+1)e^{(N+1)x}}{e^x - 1} - \frac{(e^{(N+1)x} - 1)e^x}{(e^x - 1)^2}$$

$$= \frac{(N+1)e^{(N+1)x}(e^x - 1) - (e^{(N+1)x} - 1)e^x}{(e^x - 1)^2} \Downarrow$$

$$\frac{(N+1)(1+(N+1)x...)(x+\frac{x^2}{2}...) - ((N+1)x + \frac{(N+1)}{2}}{(x+...)^2}$$

$$= (N+1)^2 + (N+1)/2 - (N+1) - (N+1)^2/2 = N(N$$

We can present the derivative operator in the complex integral form

$$f^{(k)}(0) = \frac{k!}{2\pi i} \oint \frac{dzf(z)}{z^{k+1}} \tag{375}$$

In this form the calculation gives

$$\begin{aligned} S(1,N) &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1)z + (N+1)^2 z^2/2}{z + z^2/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1) + (N+1)^2 z/2}{1 + z/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z} \frac{(N+1)}{z}(1 - z/2) + (N+1)^2/2 \\ &= N(N+1)/2 \end{aligned} \tag{376}$$

Path integral formulation of the quantum and classical dynamics

After formulation of the mathematical framework of quantum mechanics (QM), operatorial formulation of QM, Koopman and von Neumann gave operatorial approach to classical Hamiltonian mechanics [Koopman 1931], [von Neumann 1932]. After Wiener introduction of the functional integrals, Dirac and Feynman gave formal functional integral formulation of the quantum theory [Feynman, Hibbs 1965]. Recently Gozzi invented functional integral formulation of the classical theory [Deotto, Gozzi]. The path-integral formulation of Hamiltonian classical mechanics.

For supersymmetric gauge theories stochastic quantization

Path integral formulation of the quantum and classical dynamics

Parisi-Sourlas 'dimensional reduction' of scalar field theories in external random fields [Parisi, Sourlas 1979], is closely related to both supersymmetry and stochastic quantization. This becomes apparent when one establishes the connection to the Nicolai map.

The phenomenon of dynamical 'dimensional reduction' was first noted within the context of critical phenomena associated with spin systems in random external fields. Systems very close to such a situation can in fact be created and studied in the laboratory. From renormalization group theory, the detailed long-distance behaviour of, for example, Ising spin systems can, sufficiently close to a critical point

We start in the simplest possible way by considering the Langevin equation associated with a point particle being subjected to random background noise. This corresponds to the very real physical problem of the Brownian motion of a (classical) particle in a heat bath. Surprisingly, this problem turns out to be equivalent to a supersymmetric quantum mechanical problem. Let us now see why. The Langevin equation for the particle reads

$$\frac{dx}{dt} \equiv \dot{x} = -\frac{\delta S}{\delta x} + \eta(t) \tag{378}$$

where x represents the space coordinate of the particle. Expectation values are, as usual, evaluated as the path

we now attempt to make a change of variables: $\eta \to x$.
This involves the Jacobian

$$det(\delta\eta(t)/\delta x(t')) = det((d/dt + V')\delta(t - t')) \quad (381)$$

where we have introduced $V = \delta S/\delta x$.
For partition function Z,

$$Z = \int d\eta \exp(-\frac{1}{4}\int dt\eta(t)^2)$$
$$= \int d\eta dx det(d/dt + V')\delta(\dot{x} + V - \eta(t)) \exp(-\frac{1}{4}\int dt$$
$$= \int dx det(d/dt + V') \exp(-\frac{1}{4}\int dt(\dot{x} + V)^2)$$

The theory of analytic functions of a complex variable occupies a central place in analysis. Riemann considered the unique continuation property to be the most characteristic feature of analytic functions. GPF do possess the unique continuation property, and each class of GPF has almost as much structure as the class of analytic functions. In particular, the operations of complex differentiation and complex integration have meaningful counterparts in the theory of GPF and this theory generalizes not only the Cauchy-Riemann approach to function theory but also that of Weierstrass. Such functions were considered by Picard and by Beltrami, but the first significant result was obtained by Carleman in

Analytic function $f = u + iv$ satisfy the partial differential equation $\partial_{\bar{z}} f = 0$, where complex differential operators are defined as

$$\partial_{\bar{z}} = \frac{\partial}{\partial \bar{z}} := \frac{1}{2}(\partial_x + i\partial_y), \ \partial_z = \frac{\partial}{\partial z} := \frac{1}{2}(\partial_x - i\partial_y) \quad (383)$$

Generalized analytic functions $f = u + iv$ satisfy the following generalized Cauchy-Riemann equation [Vekua 1962]

$$\partial_{\bar{z}} f = Af + B\bar{f} + J, \ A = A_0 + iA_1, \ B = B_0 + iB_1, \ J = j_1 +$$

or in terms of the real u and imaginary v components canonical form of the elliptic systems of partial differential equations of the first order

In the classical sense by a solution of the system of equations (395) we understand a pair of real continuously differentiable functions $u(x, y)$, $v(x, y)$ of the real variables x and y which satisfy this system everywhere in a domain G. Such solutions, however, exist only for a comparatively narrow class of equations.
The formal solution of the canonical equation for GPF (395) is

$$\psi = \psi_0 + RJ, \ R = (D - E)^{-1}, \ (D - E)\psi_0 = 0. \quad (387)$$

Let us introduce a length parameter $l = h^{-1}$, which is of order of the source J size, $x_n \to l x_n$. Then, for the resolvent R, we will have the longwave and shortwave expansions,

$$E^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} / \Delta_E, \; \Delta_E = ad - bc,$$

$$D^{-1} = \Delta_D^{-1} \begin{pmatrix} \partial_x & \partial_y \\ -\partial_y & \partial_x \end{pmatrix} = \Delta_D^{-1}(\partial_x + i\sigma_2 \partial_y), \; \Delta_D = \partial_x^2$$

There is a fairly complete theory of generalized analytic functions; it represents an essential extension of the classical theory preserving at the same time its principal features [Vekua 1962].

From the previous consideration it is natural to make the following four dimensional extention

$$D = \partial_x - i\sigma_2 \partial_y \Rightarrow D_4 = \partial_t - i\sigma_n \nabla_n = -i(\partial_\tau + \sigma_n \nabla_n) =$$

In matrix form

$$D_4 = \begin{pmatrix} \partial_t - i\partial_z & -i\partial_x - \partial_y \\ -i\partial_x + \partial_y & \partial_t + i\partial_z \end{pmatrix} = 2\begin{pmatrix} \partial_\zeta & -\partial_{\bar\eta} \\ \partial_\eta & \partial_{\bar\zeta} \end{pmatrix},$$
$$\zeta = t + iz,\ \eta = y + ix,$$
$$D_{13} = \begin{pmatrix} \partial_\tau + \partial_z & \partial_x - i\partial_y \\ \partial_x + i\partial_y & \partial_\tau - \partial_z \end{pmatrix} = 2\begin{pmatrix} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{pmatrix},$$
$$\Delta_4 = 4(\partial^2_{\zeta\bar\zeta} + \partial^2_{\eta\bar\eta}),\ \Delta_{13} = 4(\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma}),$$
$$\pm = \tau \pm z,\ \varsigma = x + iy, \tag{391}$$

In the Minkowski spacetime for analytic functions in matrix form $D_{13}\psi = 0$ or in components

$$\partial_- u + \partial_\varsigma v = 0,\ \partial_+ v + \partial_{\bar\varsigma} u = 0 \Rightarrow (\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma})u_n = 0,$$
$$u_1 = u,\ u_2 = v \tag{392}$$

So, u_n are harmonic or wave functions.

$$\begin{aligned} \partial_- u + \partial_\varsigma v &= au + bv + j_1, \\ \partial_+ v + \partial_{\bar\varsigma} u &= cu + dv + j_2, \end{aligned} \qquad (394)$$

or in matrix form

$$D\psi = E\psi + J,\ D = \begin{pmatrix} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{pmatrix} = \partial_\tau + \sigma_n \nabla_n,$$

$$E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ \psi = \begin{pmatrix} u \\ v \end{pmatrix},\ J = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}. \qquad (395)$$

It is curious to imagine that Hamilton knew about neutrinos equation a hundred years before Weil :) In the extended version, to the $E-$terms corresponds neutrinos mass.

Now SM is well established theory of fundamental physics with several indications, aesthetical and theoretical, on father developments, on physics beyond SM (BSM), on new physics. One well established step toward BSM is neutrino masses. In SM the neutrinos are massless. In SM we have three type of lefthanded neutrinos $\nu_n, \ n = e, \mu, \tau$ which interacts weakly with corresponding leptons, lepton number is conserved. Corresponding part of the SM lagrangian is

$$\bar{l}_n \gamma^\mu \nu'_n W_\mu + \bar{\nu}'(\gamma\partial - M)\nu', \ \bar{\nu}' = (\bar{\nu}'_e \bar{\nu}'_\mu \bar{\nu}'_\tau) \tag{396}$$

where M is a 3×3 matrix in flavor space. If the matrix is nondiagonal, we diagonalize it by an unitary transformation:

$$\bar{\nu}(\gamma\partial - M)\nu = \bar{\nu}_n(\gamma\partial - m_n)\nu_n,$$

Analytic functions and massless particles

If the Koide formula works for lepton masses, may be it works also for neutrino masses. If the lepton masses are an unique solution of the Koide formula, than neutrino masses are proportional to the lepton masses:

$m_n = qM_n, \;\; n = e, \mu, \tau.$

(Super)symmetry, stochastic dynamics and kaleidoscope effect. Time reflection invariance and dynamical origin of spin.

The meromorphic functions form a field, in fact a field extension of the complex numbers.

Weyl proposed the following 2-component equations for the zero mass spin 1/2 particles in 1929, In the mid 1950s, in experiments performed by Wu following a famous suggestion by Yang and Lee that the neutrinos did not have the parity conservation property, it was found that the neutrinos emitted during beta decay had a preferred orientation. Experimental evidence further indicated that the spin is always antiparallel to the momentum for the neutrinos so that the neutrinos are always left-handed. After Wu's experiment, Landau and Salam proposed that the Weyl equation for the left-handed neutrino-right-handed antineutrino pairs be restored as the equation satisfied by the neutrino. It is this equation that now governs massless particles, not only in Minkowski spacetime but also in curved spacetime

Complex cell-biology phenomena of life sciences can be explained by understanding of their (quantum) molecular structures.
All matter is made of atoms. The most basic phases of matter are solid, liquid and gas. By a naive classification argument, a solid holds its shape and volume, a liquid takes the shape of its container but retains a fixed volume, and a gas expands to any size, taking both the shape and the volume of the container. There are many more phases of matter which evade the standard criteria of classification. The distinction between phases of matter depends crucially on the length-scale and time-scale of consideration. A large variety of different materials, with completely different

Phases of condensed matter always spontaneously break the Poincare group simply because their equilibrium states selects a preferred reference frame, i.e. the frame in which the sample of matter is stationary. The classification of the different phases is in 1-to-1 correspondence with the classification of the possible symmetry-breaking patterns of the Poincare group. At zero temperature, the standard field theory methods are available and the formal construction has been presented in [Nicolis et al, 2015] and refined using the coset techniques in [Nicolis et al, 2014]. In this low-energy description the fundamental dynamical degrees of freedom are the Goldstone bosons corresponding to the specific symmetry breaking pattern (e.g. the phonons in a

More severe difficulty arises when trying to extend the EFT methods to describe finite-temperature systems. Standard action principles and field theory formulations do not allow for dissipation, which is an important feature of all finite temperature systems, e.g. fluids. Two tools acquired a predominant role in this direction: holography [Baggioli 2019] and Schwinger-Keldysh (SK) techniques [Kamenev 2011]. When spacetime symmetries are broken, extraneous Goldstones can be removed with so-called Inverse Higgs effect, thus allowing for fewer Goldstones than broken generators [Ivanov and Ogievetskii 1975], [Brauner and Watanabe 2014].

Living organisms are open physical systems which utilize the availability of free energy to maintain homeostasis, respond to stimuli, adapt to their environment, grow, reproduce, and to evolve [Oparin 1957]. All of these biological functions are implemented by the largescale participation and interaction of proteins. The high versatility of protein functions is achieved by linear polymerization of 20 different standard amino acids into polypeptide chains. The linear sequence of amino acids in polypeptides is a primary structure. The primary structure folds into two main types of hydrogen-bonded secondary structures, α-helices and β-sheets [Pauling 1951].

The essential biological processes that sustain life are

Mathematics is the universal language for expressing causal and functional relationships between observations. Its mainstream developments have been inspired by the needs of Physics, Chemistry and Engineering. For the twenty-first century, it is widely expected that Biology becomes a frontier for Mathematics.

The immune system is primarily about host survival of infections and for this we also need to understand the biology of the system. Understanding the cellular and molecular mechanisms that control the ability of the immune system to mount a protective response against pathogen-derived foreign antigens, but avoid a pathological response to self-antigens, is a central problem in

The challenge is to establish an interdisciplinary dialogue between mathematicians and experimentalists so that experimentation and mathematical modelling becomes an iterative process that boosts the different disciplines. The generated models that inevitably present simplifications of the underlying biological complexity must not lose touch with reality and generate testable predictions that drive, for example, perceptions of pathogen–host interactions. Such consistent models that provide a basis for quantitative analysis and predictions raises challenges for applied mathematicians related to the formulation of genuine approaches for representing the phenotypic complexity, spatial heterogeneity, hierarchical organization and control

A mathematician, working on the problems of optimizing the treatment of chronic forms of diseases, need creative contacts with immunologists, geneticist, biologists, and clinicians. The reaction of immune system to infection are the main problem in practical immunology. Understanding of regularities in immune response provides the researchers and clinicians new powerful tools for the stimulation of the immune system in order to increase its efficiency in the struggle against antigen invasion.

The standard model for the (three species) virus dynamics is defined, [Nowak et al 1996], as the following system,

$$\begin{aligned} \dot{m} &= -am + bnv, \\ \dot{n} &= c - dn - bnv, \\ \dot{v} &= km - ev, \end{aligned} \tag{399}$$

where m, n and v are, respectively, the numbers of infected cells, uninfected cells and the number of free virus particles (virons) in a fixed volume compartment; infected cells die at a rate am; the uninfected cells are produced at a constant rate c and die at a rate dn; virons infect uninfected cells at a rate proportional to the product of their numbers, bnv; infected cells produce free virus at a rate km; free virus

From the third equation of the system (399) we may define m
as a function of v,

$$m = (\dot{v} + ev)/k \tag{400}$$

Taking sum of the first and second equations, we find the following equation

$$(m+n)^{\cdot} = c - (am + dn), \tag{401}$$

with the solution when $d = a$,

$$\begin{aligned} & |c - ax(t)| = e^{-at}|c - ax_0|, \\ m + n \;=\; & x = c/a + e^{-at}|c/a - x_0|, \;\; x > c/a, \\ =\; & c/a - e^{-at}|c/a - x_0|, \;\; x < c/a \end{aligned} \tag{402}$$

We may take, e.g.

$$v(t) = v_0 \sin(\pi t/T), \ 0 \le t \le T \tag{403}$$

and find the time t_H when $m = 0$,

$$t_H = (1 - \arctan(\frac{\pi}{eT})/\pi)T, \ \ T/2 < t_H < T,$$
$$n(t_H) = x(t_H), \ v(t_H) = v_0 \sin(\pi t_H/T) = v_0 \frac{\pi e T}{\sqrt{\pi^2 + e^2 T^2}} \tag{404}$$

Parameters for a typical HIV virus infection process are:

$$a = 0.5, \ b = 2 \times 10^{-7}, \ c = 10^5, \ d = 0.1, \ k = 100, \ e = 5 \tag{405}$$

These parameters have units of inverse days: $1/d$.

Covid and traditional clothing

During covid, masks, helmets and overalls are worn in public places (doctors in the red zones). In the spring comes pollinosis, or seasonal allergic rhinoconjunctivitis, a seasonal disease caused by an allergic reaction to plant pollen. The disease is sometimes called hay fever, although hay is not a significant factor in the genesis of the disease, and fever is not characteristic of this pathology. At the beginning of the 19th century, it was believed that the cause of hay fever was freshly cut grass, which then went to make hay, hence the name of the disease. Means of protection against covid and their modifications can also protect against hay fever. Where the face and other parts of the body are traditionally hidden, masks and overalls can

This mechanism indicates that with time inversion symmetry we can have only composed scalar fields. With the discovery of the Higgs particle with mass 125 GeV, a nice number $m_W/m_H \simeq 2/3$ appear, which, at least for me, indicates for composed nature of W and H, with a same mass of about 40 GeV two and three valence constituents correspondingly. The fermion constituents ψ_n^a of W and scalar constituents φ_n^a of H compose scalar super multiplet (φ_n^a, ψ_n^a) with a flavor index n and color index a. Another notation is (h, sh)-(He, She:).

With exact SUSY we have cofinement by dimentional counting: superspace dimension is zero on the hadronic scale, hadrons are pointlike, color is confined inside hadrons. For SM QCD this picture indicates that at the hadronic scale we have effective SQCD, which contains scalar quarks.

A Solvable Model of Renormdynamics

In the Standard Model of Particle Physics (SM), the values of the coupling constants and masses of particles depends on the scale according to the Renormdynamic motion equations. One charge a, one mass m RD equations are

$$\dot{\alpha} = \beta(\alpha),$$
$$\dot{m} = \gamma(\alpha)m \qquad (406)$$

For the electron and nucleon masses, electrodynamic and pion-nucleon fine structure constants we have an empirical relation:

$$m_e/\alpha \simeq m_N/\alpha_{\pi N} \qquad (407)$$

We take the relation $m/\alpha = const$ as an integral of renormdynamic motion equations for m and α, find exact

A Solvable Model of Renormdynamics

From the integral of motion, in the minimal mass parametrization: $\gamma(\alpha) = \gamma_1 \alpha$, we obtain

$$(\ln \alpha)^{\cdot} = (\ln m)^{\cdot} \Rightarrow \beta(\alpha)/\alpha = \gamma(\alpha)$$
$$= \gamma_1 \alpha \Rightarrow \beta(\alpha) = \beta_2 \alpha^2, \ \beta_2 = \gamma_1 \tag{409}$$

so, we have the following algebraic-diofant equations for the flavor and color content of the theory

$$\beta_n = 0, \ n \geq 3,$$
$$\beta_2 = \gamma_1 \tag{410}$$

and prediction for the dimension of space-time: $D = 4$.
Solution of the motion equations are

$$\alpha(t) = \frac{\alpha_0}{1 - \alpha_0 \beta_2 t},$$

In the multidimentional renormdynamics, when we have several (N) coupling constants and masses, we assume that there are maximal number $(N-1)$ integrals of motion H_n. If the number of integrals is N, we not have dynamics, we have only statics - finite field theory,
$\alpha_n = const, \;\; n = 1, ..., N.$
The idea of reduction to the one dimensional renormdynamics is simple:

$$\frac{d\alpha_n}{dt} = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha_N) \Rightarrow \frac{d\alpha_n}{d\alpha} = B_n(\alpha_1, ..., \alpha_{(N-1)}$$
$$B_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha) = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha)/\beta_N(\alpha_1, ..., \alpha_($$
$$\alpha_n = \sum_{k>1} f_{nk}\alpha^k, \quad n = 1, 2, ..., N-1$$

Solitons are particlelike states, solutions of motion equations and they quantum extensions. Examples are solitons of SinGordon motion equation or barions-skirmions of Skyrme model. In particle theory, the skyrmion was described by Tony Skyrme in 1962 and consists of a quantum superposition of baryons and resonance states. Skyrmions as topological objects are important in solid state physics. Researchers could read and write skyrmions using scanning tunneling microscopy. The topological charge, representing the existence and non-existence of skyrmions, can represent the bit states "1" and "0".

QCD consists of quarks and gluons. Quarks possess both color (r, g, b) and flavor $(u, d, s,$etc.$)$, while gluons possess color (r, g, b) and anti-color $(\bar{r}, \bar{g}, \bar{b})$, but not flavor. An open **string** (a string with two endpoints) is ideally suited to account for such quantum numbers at its two ends. For quarks, one end represents color and the other end flavor. For gluons, one end represents color and the other anti-color. In string theory, there are **branes** (higher dimensional extended objects that are generalized membranes) to which the endpoints of an open string are confined. Applying this idea to QCD, we introduce N_c colored branes and N_f flavored branes at which open strings corresponding to quarks and gluons terminate. The energy

To describe QCD, we have to prepare Dp-branes and Dq-branes with p, q$\geq$3 for colored branes and flavored branes, respectively, and these branes should be located in the space of more than five dimensions. To evaluate the amplitude for a certain process to occur in the above picture, we have to sum up all the possible two-dimensional world sheets with the weight $\exp(iS)$, where the action S is given by S=(energy)$\times$(time)=(area of the string's world sheet)$/2\pi\alpha'$, following the Feymann path integral formulation.

Cumulative Effect: Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons is connected to the existence of **Fluctons** - droplets of dense cold nuclear matter.

Classical fields have canonical, rational for integer D, (mass)dimensions e.g. in electrodynamics

$$L = \int d^D x(\bar{\psi}(\gamma(\partial - eA) - m)\psi - \frac{1}{4}F^2),$$

$$d_\psi = [\psi] = (D-1)/2,\ d_A = (D-2)/2,\ d_e = (4-D)/2$$

Quantum corrections introduce (anomaly) corrections to the canonical dimensions, so the fields and coupling constants become **fractals.** At fixed points of RD, the fractals are self similar and their compositions present at

Qualitative picture of the (un)particle(like) objects we will illustrate with the simplest model of scalar field given by the following lagrangian

$$L = L(\Phi, M, \lambda, n) = \frac{1}{2}(\partial_\mu \Phi) - \frac{1}{2}M^2\Phi^2 - V(\Phi),\ \mu = 0, 1, 2,$$

where self interaction usually we take in the form

$$V(\Phi) = \lambda\Phi^n,\ n = -2, 1, 2, 3, 4, 6 \tag{415}$$

In renormalisible case,

$$n = \frac{2D}{D-2} = 2 + \varepsilon(D),\ \varepsilon(D) = \frac{4}{D-2},$$
$$D = \frac{2n}{n-2} = 2 + \varepsilon(n),\ \varepsilon(n) = \frac{4}{n-2}, \tag{416}$$

In the free (self non interacting) field (particle) approximation: $\lambda = 0$, but in external gravitational field we have

$$L(g, \Phi, M) = \sqrt{-g} L(\Phi, M, 0), \;\; g = det g_{\mu\nu}(x) \qquad (417)$$

Now we will see a nice composite particle mechanism :) Let us take a substitute: $\Phi = \varphi^k$, than we find

$$L(g, \Phi, M) = L((k\varphi^{k-1})^4 g, \varphi, M/k), \;\; g_{\mu\nu}(x) \Rightarrow (k\varphi^{k-1})^{4/D} g$$

Indeed

$$L(g, \Phi, M) = \sqrt{-g}(k^2\varphi^{2(k-1)}\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}M^2\varphi^{2k})$$
$$= \sqrt{-g(k\varphi^{k-1})^4}(\frac{1}{2}(\partial_\mu\varphi)^2) - \frac{1}{2}(\frac{M}{k})^2\varphi^2) \qquad (419)$$

Now, having an experience with constituent - composite particle relation, we turn attention on the self-interaction therm,

$$L = \sqrt{-g(k\varphi^{k-1})^4}(... - \frac{\lambda}{k^2}\varphi^N), \ N = kn - 2(k-1) \quad (420)$$

Most natural value of n for stable systems $(1+1 \to 1+1, \ 2 \to 2)$ is $n = 4$. In this case, $N = 2k + 2$ and only natural value of constituents for which we have a renormalizable interaction is $k = 2 \Rightarrow N = 6$ with corresponding spacetime dimension $D = 3$. The most natural value for fission-fusion interaction $(1 \leftrightarrow 2)$ is $n = 3 \Rightarrow N = k + 2$, for which we have realistic values $k = 2$ and $N = 4$, $D = 4$:) Other interesting values of

The size of particle-like states (solutions of the motion equations) are defined as $l \sim M^{-1}$, because at the boundary region, the linear part of the motion equations dominates and the Yukawa-like asymptotic $\Phi(r) \sim e^{-Mr}$ acts. In a pion-nucleon model for nucleon size we have $l_N \sim m_\pi^{-1} \simeq 1.43$ fm. The amplitude of the state (at maximum) $A \sim \lambda^{-\alpha}, \ \alpha = 1/(n-2)$. Indeed, the motion equation do not contains the coupling constant after a scaling substitution $\Phi = \lambda^{-\alpha}\phi$, so a particle-like solution ϕ dos not contains λ and corresponding solution $\Phi = \lambda^{-\alpha}\phi \sim \lambda^{-\alpha}$,

$$\triangle\Phi + M^2\Phi + \lambda n\Phi^{n-1} = \lambda^{-\alpha}(\triangle\phi + M^2\phi + \lambda^{1-(n-2)\alpha}n\phi^{n-1})$$

At not so low energies from string theory we may extract the following scalar field theory

$$L = \sqrt{-g}(\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi^3), \ \mu = 0, 1, ..., D - 1,$$

where $\varepsilon \in [0, 20]$. The one loop $\beta-$function is

$$\beta(a) = (D - 6)a - \beta_2 a^2, \ a \sim \lambda^2 \tag{423}$$

and it has stable UV fixed point at $a = (D - 6)/\beta_2$ and IR fixed point $a = 0$. Beyond this point we have an unparticle $\Phi = \phi^2$ with lagrangian

$$L = \sqrt{-g'}(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\frac{M}{2})^2\phi^2 - \frac{\lambda}{4}\phi^4), \ \mu = 0, 1, ..., D$$
$$d = 4 - \varepsilon, \ \varepsilon \in [0, 1].$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-4)\lambda + b\lambda^2 \quad (425)$$

and it has stable IR fixed point at $\lambda = (4-d)/b$. The UV fixed point is $\lambda = 0$. At this point we have reduction from higher dimentional Φ^3 to lower dimentional ϕ^4.
Another possibilities is an unparticle $\Phi = \varphi^4$ with lagrangian

$$L = \sqrt{-g''}(\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}(\frac{M}{4})^2\varphi^2 - \frac{\lambda}{16}\varphi^6),\ \mu = 0, 1, ..., d -$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-3)\lambda + c\lambda^2,\ d = 3-\varepsilon,\ \varepsilon \in [0, 2]. \quad (427)$$

Similar consideration gives reduction from higher energy ϕ^4 model to lower energy φ^6 one. Some technical questions remain. One of them concern to the substitution $\Phi = \phi^2$. It restricts Φ as $\Phi \geq 0$. OK, we already have a constraint, that the fields are real valued, we have a restriction

$$\phi^*(x) = \phi(x) = \frac{1}{(2\pi)^D} \int d^D p \exp(ipx)\hat{\phi}(p) \Rightarrow \hat{\phi}^*(p) = \hat{\phi}(-$$

To formulate positivity condition is not so easy. We will take another path, we define the interaction as $\Phi^3 = (\Phi^2)^{3/2} \geq 0$. Then the substitution $\Phi^2 = \phi^4$ will works. Bytheway by this definition we made also another improvement: the potential become bounded from below. For the reduction the substitution $\Phi^2 = \phi^4$ also works

Note that by substitution

$$(\frac{\Phi}{\Phi_0})^2 = \phi^{2k}, \ \phi^2 = \exp(\ln(\Phi^2/\Phi_0^2)/k)$$
$$= 1 + \frac{1}{k}\ln(\frac{\Phi}{\Phi_0})^2 + O(K^{-2}), \ \phi = \pm 1 + O(k^{-1}) \quad (430)$$

we reduce the field theory to a discrete theory, to a system of bits. Also, changing dimension of space D and nonlinearity n restricted by condition

$$n = \frac{2D}{D-2}, \ D = \frac{2n}{n-2}, \ \frac{1}{n} + \frac{1}{D} = \frac{1}{2} \quad (431)$$

we assume that they are functions of scale or coupling constant, due to monotonic property of the coupling constant. We have the following relation

Computers are physical devices and their behavior is determined by physical laws. The Quantum Computations [Benenti, Casati, Strini, 2004 , Nielsen, Chuang, 2000], Quantum Computing, Quanputing [Makhaldiani, 2007.2], is a new interdisciplinary field of research, which benefits from the contributions of physicists, computer scientists, mathematicians, chemists and engineers.
Contemporary digital computer and its logical elements can be considered as a spatial type of discrete dynamical systems [Makhaldiani, 2001]

$$S_n(k+1) = \Phi_n(S(k)), \qquad (433)$$

where

Definition: *We assume that the system (433) is time-reversible if we can define the reverse dynamical system*

$$S_n(k) = \Phi_n^{-1}(S(k+1)). \tag{435}$$

In this case the following matrix

$$M_{nm} = \frac{\partial \Phi_n(S(k))}{\partial S_m(k)}, \tag{436}$$

is regular, i.e. has an inverse. If the matrix is not regular, this is the case, for example, when $N(k+1) \neq N(k)$, we have an irreversible dynamical system (usual digital computers and/or corresponding irreversible gates).

Discrete dynamical systems and Quanputers

Let us consider an extension of the dynamical system (433) given by the following action function

$$A = \sum_{kn} I_n(k)(S_n(k+1) - \Phi_n(S(k))) \qquad (437)$$

and corresponding motion equations

$$S_n(k+1) = \Phi_n(S(k)) = \frac{\partial H}{\partial I_n(k)},$$

$$I_n(k-1) = I_m(k)\frac{\partial \Phi_m(S(k))}{\partial S_n(k)} = I_m(k)M_{mn}(S(k)) = \frac{\partial H}{\partial S_n(} \quad (4$$

where

$$H = \sum_{kn} I_n(k)\Phi_n(S(k)), \qquad (439)$$

From this system it is obvious that, when the initial value $I_n(k_0)$ is given, the evolution of the vector $I(k)$ is defined by evolution of the state vector $S(k)$. The equation of motion for $I_n(k)$ - Elenka is linear and has an important property that a linear superpositions of the solutions are also solutions.

Statement: *Any time-reversible dynamical system (e.g. a time-reversible computer) can be extended by corresponding linear dynamical system (quantum - like processor) which is controlled by the dynamical system and has a huge computational power,*
[Makhaldiani, 2001, Makhaldiani, 2002,
Makhaldiani, 2007.2, Makhaldiani, 2011.2]

For motion equations (438) in the continual approximation, we have

$$S_n(k+1) = x_n(t_k+\tau) = x_n(t_k) + \dot{x}_n(t_k)\tau + O(\tau^2),$$
$$\dot{x}_n(t_k) = v_n(x(t_k)) + O(\tau), \quad t_k = k\tau,$$
$$v_n(x(t_k)) = (\Phi_n(x(t_k)) - x_n(t_k))/\tau;$$
$$M_{mn}(x(t_k)) = \delta_{mn} + \tau\frac{\partial v_m(x(t_k))}{\partial x_n(t_k)}. \qquad (441)$$

(de)Coherence criterion: *the system is reversible, the linear (quantum, coherent, soul) subsystem exists, when the matrix M is regular,*

$$detM = 1 + \tau\sum_n \frac{\partial v_n}{\partial x_n} + O(\tau^2) \neq 0. \qquad (442)$$

Let me motivate an idea of construction of the reversible dynamical systems by simple example from field theory. There are renormalizable models of scalar field theory of the form (see, e.g. [Makhaldiani, 1980])

$$L = \frac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2) - g\varphi^n, \tag{444}$$

with the constraint

$$n = \frac{2d}{d-2}\ , \tag{445}$$

where d is dimension of the space-time and n is degree of nonlinearity. It is interesting that if we define d as a function of n, we find

$2n$

Generalization of the idea

Now it is natural to consider the following symmetric function

$$f(y) + f(x) = c \tag{448}$$

and define its solution

$$y = f^{-1}(c - f(x)). \tag{449}$$

This is the general method, that we will use in the following construction of the reversible dynamical systems. In the simplest case,

$$f(x) = x, \tag{450}$$

we take

$$y = S(k+1), \quad x = S(k-1), \quad c = \tilde{\Phi}(S(k)) \tag{451}$$

This dynamical system defines given state vector by previous two state vectors. We have reversible dynamical system on the time lattice with time steps of two units,

$$
\begin{aligned}
S(k+2,2) &= \Phi(S(k,2)),\\
S(k+2,2) &\equiv (S(k+2), S(k+1)),\\
S(k,2) &\equiv (S(k), S(k-1))).
\end{aligned} \tag{454}
$$

Internal, spin, degrees of freedom

Starting from a general discrete dynamical system, we obtained reversible dynamical system with internal(spin,bit) degrees of freedom

$$S_{ns}(k+2) \equiv \begin{pmatrix} S_n(k+2) \\ S_n(k+1) \end{pmatrix} = \Big(\begin{matrix} \Phi_n(\Phi(S(k)) - S(k-1 \\ \Phi_n(S(k)) - S_n(k \end{matrix}$$
$$\equiv \Phi_{ns}(S(k)), \quad s = 1,2$$

where

$$S(k) \equiv (S_{ns}(k)), \quad S_{n1}(k) \equiv S_n(k), \quad S_{n2}(k) \equiv S_n(k-1) \quad (456)$$

For the extended system we have the following action

$$A = \sum_{kns} l_{ns}(k)(S_{ns}(k+2) - \Phi_{ns}(S(k))) \qquad (457)$$

and corresponding motion equations

$$S_{ns}(k+2) = \Phi_{ns}(S(k)) = \frac{\partial H}{\partial I_{ns}(k)},$$
$$I_{ns}(k-2) = I_{mt}(k)\frac{\partial \Phi_{mt}(S(k))}{\partial S_{ns}(k)}$$
$$= I_{mt}(k)M_{mtns}(S(k)) = \frac{\partial H}{\partial S_{ns}(k)}, \qquad (458)$$

By construction, we have the following reversible dynamical system

$$S_{ns}(k+2) = \Phi_{ns}(S(k)),$$
$$I_{ns}(k+2) = I_{mt}(k)M^{-1}_{mtns}(S(k+2)), \qquad (459)$$

with classical S_{ns} and quantum I_{ns}(in the external,

p-point cluster and higher spin states reversible dynamics, or pit string dynamics

We can also consider p-point generalization of the previous structure,

$$
\begin{aligned}
& f_p(S(k+p)) + f_{p-1}(S(k+p-1)) + \ldots + f_1(S(k+1)) \\
& + f_1(S(k-1)) + \ldots + f_p(S(k-p)) = \tilde{\Phi}(S(k)), \\
& S(k+p) = \Phi(S(k), S(k+p-1), \ldots, S(k-p)) \\
& \equiv f_p^{-1}(\tilde{\Phi}(S(k)) - f_{p-1}(S(k+p-1)) - \ldots - f_p(S(k-p) \quad (46)
\end{aligned}
$$

and corresponding reversible p-oint cluster dynamical system

$$
\begin{aligned}
& S(k+p,p) \equiv \Phi(S(k,p)), \\
& S(k+p,p) \equiv (S(k+p), S(k+p-1), \ldots, S(k+1)), \\
& S(k,p) \equiv (S(k), S(k-1), \ldots, S(k-p+1)), \quad S(k,1) =
\end{aligned}
$$

p-point cluster and higher spin states reversible dynamics, or pit string dynamics

This case the quantum state function $l_{ns}, \; s = 1, 2, ...p$ will describes the state with spin $(p - 1)/2$.
Note that, in this formalism for reversible dynamics minimal value of the spin is $1/2$. There is not a place for a scalar dynamics, or the scalar dynamics is not reversible. In the Standard model (SM) of particle physics, [Beringer et al, 2012], all of the fundamental particles, leptons, quarks and gauge bosons have spin. Only scalar particles of the SM are the Higgs bosons. Perhaps the scalar particles are composed systems or quasiparticles like phonon, or Higgs dynamics is not reversible (a mechanism for 'time arrow')

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

The $NP \stackrel{?}{=} P$ problem will be solved if for some $NP-$ complete problem, e.g. TSP, a polynomial algorithm find; or show that there is not such an algorithm; or show that it is impossible to find definite answer to that question. TSP means to find minimal length path between N fixed points on a surface, which attends any point ones. We consider a system where N points with quenched positions $x_1, x_2, ..., x_N$ are independently distributed on a finite domain D with a probability density function $p(x)$. In general, the domain D is multidimensional and the points x_n are vectors in the corresponding Euclidean space. Inside the domain D we consider a polymer chain composed of N

The Hamiltonian for the system is given by

$$H = \sum_{n=1}^{N} V(|y_n - y_{n-1}|) \tag{463}$$

Here V is the interaction between neighboring monomers on the polymer chain. For convenience the chain is taken to be closed, thus we take the periodic boundary condition $x_0 = x_N$. A physical realization of this system is one where the x_n are impurities where the monomers of a polymer loop are pinned. In combinatorial optimization, if one takes $V(x)$ to be the norm, or distance, of the vector x then $H(\sigma)$ is the total distance covered by a path which visits

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

In field theory language to the TSP we correspond the calculation of the following correlator

$$G_{2N}(x_1, x_2, ..., x_N) = Z_0^{-1} \int d\varphi(x)\varphi^2(x_1)\varphi^2(x_2)...\varphi^2(x_N)e$$

$$= \frac{\delta^{2N}F(J)}{\delta J(x_1)^2...\delta J(x_N)^2},\ F(J) = \ln Z(J),$$

$$Z(J) = \int d\varphi e^{-\frac{1}{2}\varphi\cdot A\cdot\varphi + J\cdot\varphi} = e^{\frac{1}{2}J\cdot A^{-1}\cdot J},\ A^{-1}(x, y; m) = e$$

$$L_{min}(x_1, ..., x_N) = -\frac{d}{dm}\ln G_{2Ns} + O(e^{-am})$$

$$< A^{-1} >\equiv \frac{1}{\Gamma(s)}\int_0^\infty dmm^{s-1}A^{-1}(x, y; m) = \frac{1}{|x-y|^s}$$

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

If we take relativistic massive scalar field, then
$A = \Delta_d + m^2$,

$$A^{-1}(x) \sim |x|^{2-d} e^{-m|x|}, \tag{465}$$

and for $d = 2$, we also have the needed behaviour. Note that G_{2N} is symmetric with respect to its arguments and contains any paths including minimal length one.

Notes

Although many astronomical observations have confirmed the existence of dark matter, its nature has so far been unknown. Among the many dark matter models, weakly interacting massive particles (WIMPs) is the most important one Other dark matter models, such as primordial black holes (PBHs), have attracted extensive attention again [4–13]. PBHs can be formed by the collapse of large density perturbation existing in the early Universe and their masses spread a wide range (see, e.g., Refs. [5, 20]).

Equations of motion for a free field of arbitrary spin can be written as a first-order differential equation (see e.g. S. Okubo and Y. Tosa Phys. Rev. D 23, 1468 (1981))

$$(\Gamma^\mu \partial_\mu - M)\psi = 0 \tag{466}$$

where Γ^μ and M are constant matrices. The corresponding Lagrangian may be given by

$$L = \bar{\psi}(\Gamma^\mu \partial_\mu - M)\psi \tag{467}$$

where $\bar{\psi}$ is related to ψ by

$$\bar{\psi} = \psi^T C \tag{468}$$

for another constant matrix C. The superscript T denotes

First order formalism for any spin dynamics

If we consider $\bar{\psi}$ and ψ as independent complex fields, we may introduce interactions with gauge fields as usual.

Notes

All known string theory models may be obtained from the bosonic string theory. Spacetime, gravity, geometry and topology are all emergent macroscopic concepts, arising from microscopic quantum interactions.

Highly supersymmetric three dimensional conformal field theories are interesting for construction of the theory describing the worldvolume of membranes in M-theory (M2-branes) at low energies. Another motivation to study three dimensional conformal field theories is that they could describe interesting conformal fixed points in condensed matter systems. Note that

Уравнения Максвелла

Классическая электродинамика возникла в 1862 г., когда Максвелл сформулировал уравнения, связывающие электрическое и магнитное поля и с плотностями заряда и тока ρ и j:

$$
\begin{aligned}
rotB - \frac{1}{c}\partial_t E &= \frac{4\pi}{c}j, \\
rotE + \frac{1}{c}\partial_t B &= 0, \\
divE &= 4\pi\rho, \\
divB &= 0, \Downarrow, \\
i\partial_0\psi &= s \cdot p\psi + J, \\
div\psi &= \rho, \ \psi = E + iB, \ x^0 = ct
\end{aligned}
\tag{470}
$$

Уравнения Максвелла

Эти уравнения, дополненные выражением для силы Лоренца, действующей на систему электрических зарядов и токов:

$$F = \int dx^3(\rho E + \frac{1}{c} jxB) \tag{471}$$

привели к представлению о свете как об электромагнитной волне, к описанию излучения движущихся зарядов и воздействия излучения на заряженные тела.

Neytrino light theory

For a systematic study of neutrino-photon interaction it is useful to work with effective Lagrangians. The simplest example of the Effective Lagrangian is the four-fermion interactions of neutrinos ν with electrons e. For fermions with momenta much smaller than intermediate bosons mass one can integrate degrees of freedom associated with W and Z and write the Effective Lagrangian only for fermionic degrees of freedom:

$$L = g\bar{\nu}\gamma_\alpha \nu \bar{e}\Gamma^\alpha e, \;\; g = G_F/\sqrt{2},$$
$$\Gamma_\alpha = g_V\gamma_\alpha + g_A\gamma_\alpha\gamma_5, \;\; g_V = 3/2 - 2\sin^2\theta_W, \;\; g_A = 1/2 \qquad (4372)$$

Despite its success, the Standard Model (SM) still leaves us with some unanswered questions. One example is the well-stablished muon g-2 anomaly from Brookhaven E821 [1], which has been recently confirmed by E989 [2]. Another one is the latest measurement of the W boson mass at Tevatron CDFII [3], which deviates with high significance from the SM prediction [4].
On top of this, we have puzzling questions regarding Dark Matter(DM). Although we have a large amount of indirect evidence for DM, such as galaxy rotation curves and the CMB, we still don't understand its nature neither its origin.
The Weinberg-Salam model can be interpreted as a solution

R–matrix satisfying the Hecke relation,

$$R^2 + \lambda R - 1 = 0,\ \lambda = q^{-1} - q, \Downarrow$$
$$q^2 + \lambda q - 1 = 0 \Rightarrow q = \frac{-\lambda \pm \sqrt{\lambda^2 + 4}}{2} \quad (475)$$

When $\lambda = 1$, the positive root is

$$q = \frac{\sqrt{5} - 1}{2} = 0.6180 \quad (476)$$

is the golden raitio.
The Fibonacci numbers may be defined by the recurrence relation

$$F_{n+1} = F_n + F_{n-1} \Rightarrow x_{n+1} = 1 + 1/x_n,$$
$$\sqrt{5} - 1 \quad 0.6180 \quad (477)$$

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