Optical simulation of PMT

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Goals

- PMTs are widely used in different areas of physics, especially in neutrino experiments that use great amount of PMTs, both large and small, in order to determine the neutrino hierarchy and oscillation parameters through achieving excellent energy resolution.
- PMTs do collect the photo-signal from the detector and convert it into the electric signal. The quality of such converting depends on so-called PDE (photo-detection efficiency) that is connected with quantum efficiency (the property of the photocathode layer inside each PMT).
- The main goal is to describe optical processes (principally light absorption) inside the photocathode theoretically but not only by fitting experimental results as was done before because the PMT optical model is a very important ingredient in the program of the energy reconstruction.

Why should we take into account optics?

1. Optical coefficients - **refraction, transmission, absorption** - depend on the incident angle and outer medium. So the number of photoelectrons depends on a way the PMT is illuminated.
2. Absorption coefficients are different for **s-** and **p-waves**
3. Photo-Detection Efficiency (PDE), that we can measure and that determines the quality of light registration, describes both photocathode properties and optics of all PMT inner layers. It binds the number of detected electrons with the total number of photons. The PDE therefore depends on both the absorption of light and the collection of charges. After a photon has been absorbed and has generated a photo electron this electron still should be collected.

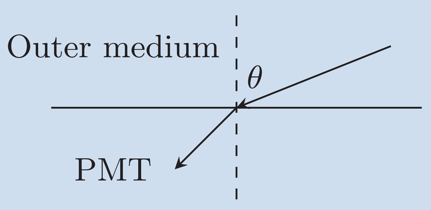
$$\langle \mathrm{PDE} \rangle = \left\langle \underbrace{\mathrm{CE}}_{a} \int_0^d dz \underbrace{\left[1 - \int_0^z dx \frac{dA}{dx}\right]}_{b} \underbrace{\sigma_{pe}}_{c} \underbrace{\frac{df}{dz}(d-z)}_{d} \right\rangle$$

(a) Collection efficiency

(b) Non-absorption (optics)

(c) Photo-effect cross-section

(d) Escape function



Differential absorption function is connected with ordinary absorption function via

$$A(\boldsymbol{\alpha}; d) = \int_0^d \frac{dA(\boldsymbol{\alpha}, z)}{dz} dz$$

Differential absorption function

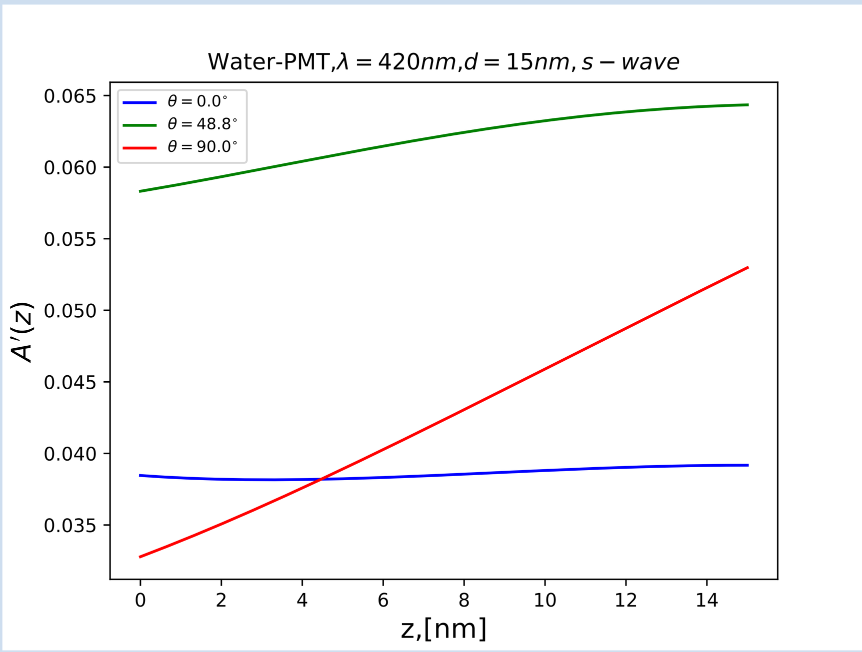
By definition the attenuation coefficient α describes the extent to which the radiant flux of a beam is reduced as it passes through a specific material. As soon as energy flux depends only on z-coordinate, we should change $\boldsymbol{r} \to z$ and then

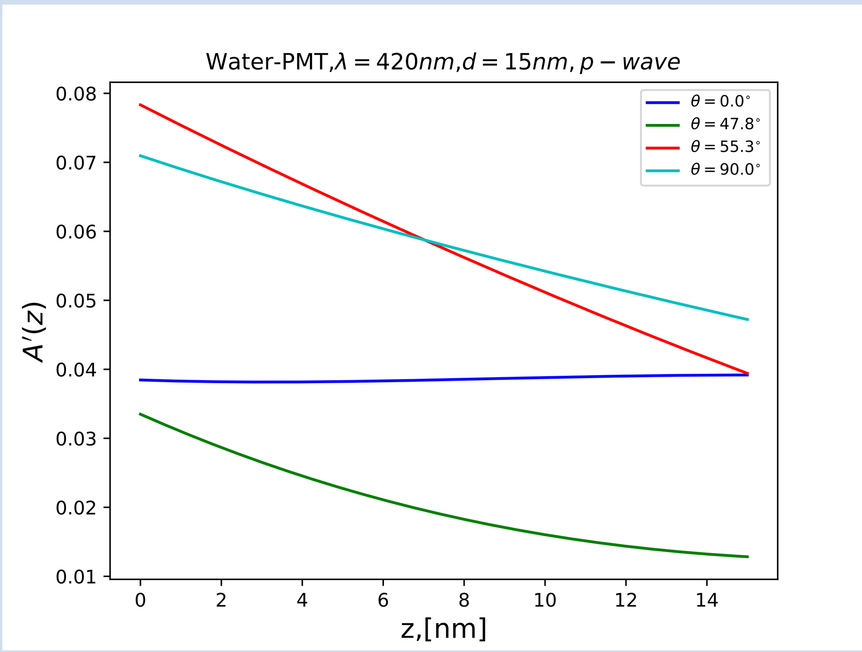
$$\alpha = -\frac{dS(z)}{dz} \Big/ S(z)$$

Differential absorption coefficient $A'(z)$ for constant incidence angle denotes what part of transmitted to the depth z normal to the boundary energy flux $S_z(z)$ is absorbed in comparison to the initial energy flux $S_z(0)$

$$\frac{dA(z)}{dz} = A'(z) = \alpha \frac{S_z(z)}{S_z(0)}$$

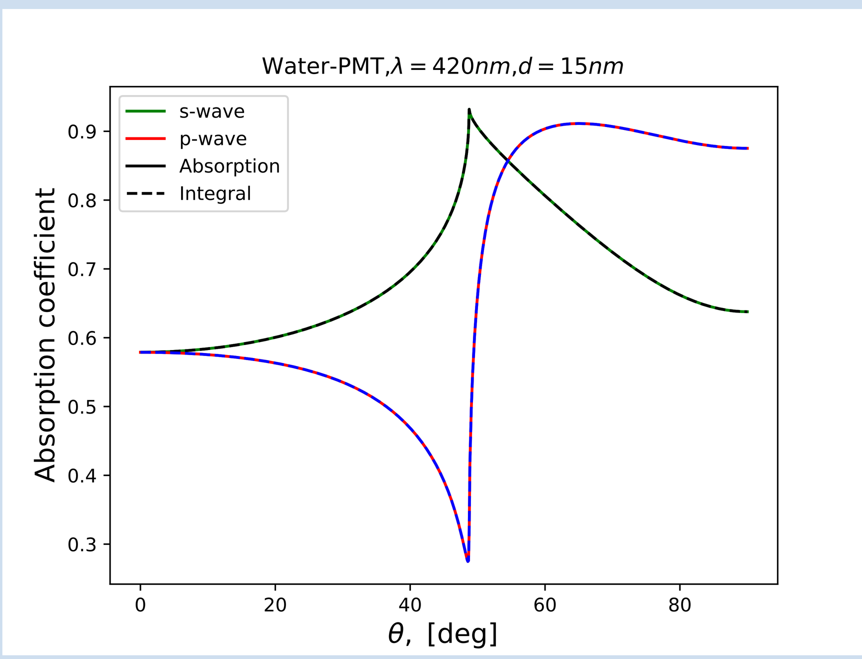
Differential absorption function also depends on the wave polarisation and outer media.





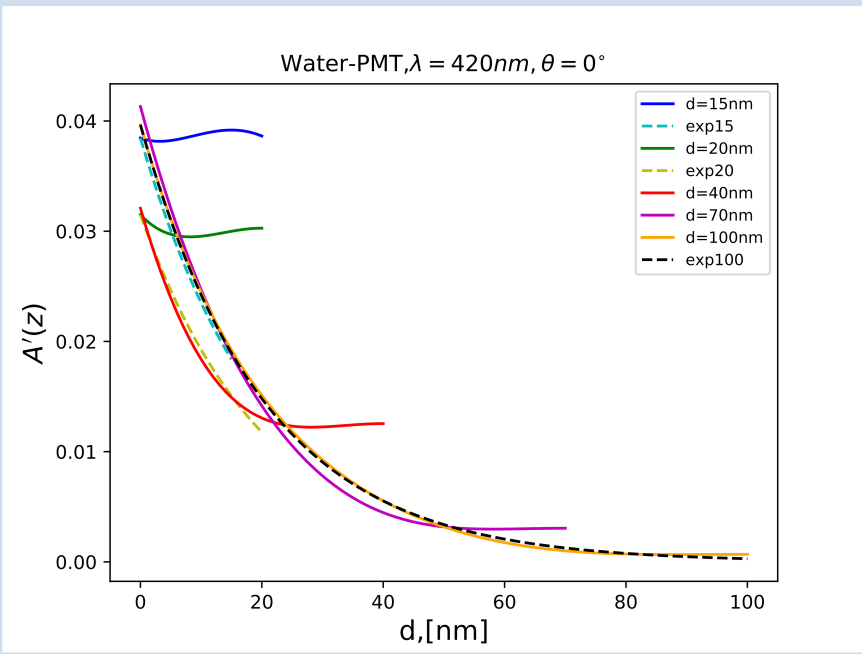
Logically the integral of a differential absorption coefficient throughout d (the thickness of the photocathode) should coincide to the integral absorption coefficient which is obtained from other optical coefficients.

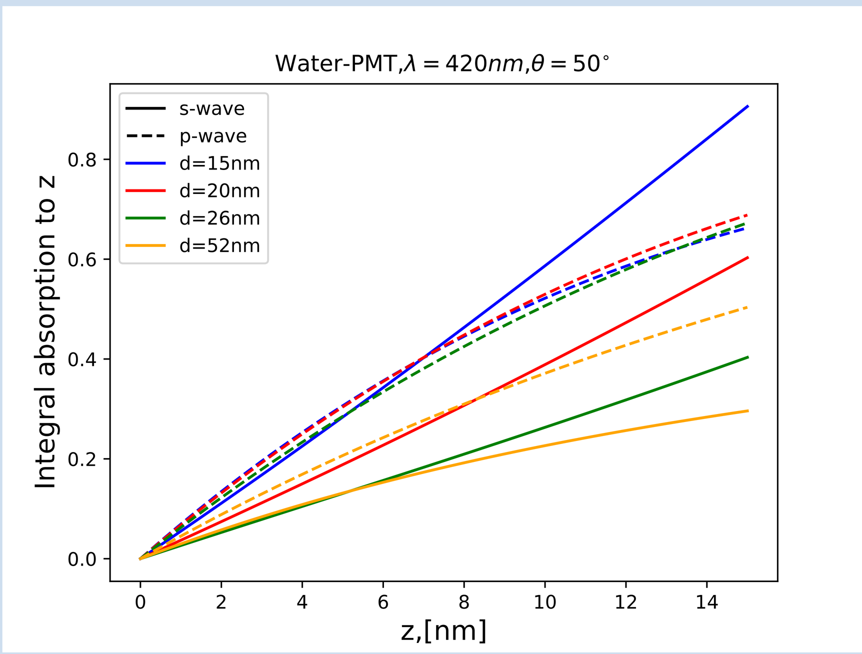
$$\int_0^d A'(z)dz = A = 1 - R - T$$



Interference effect

If there is no interference in a thin layer than absorption to a given z should look like ordinary exponent

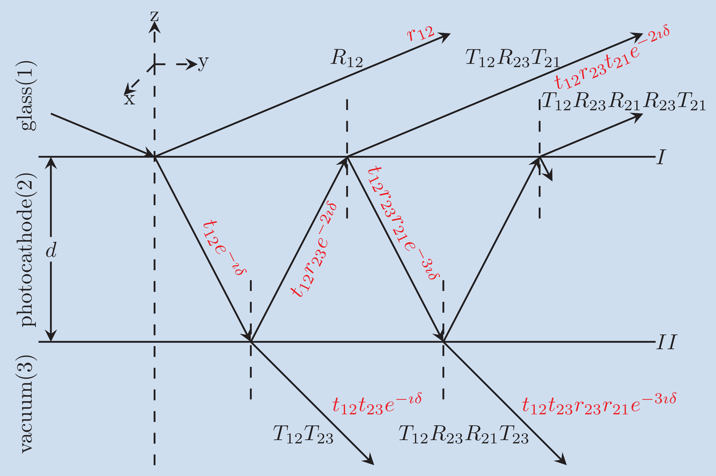


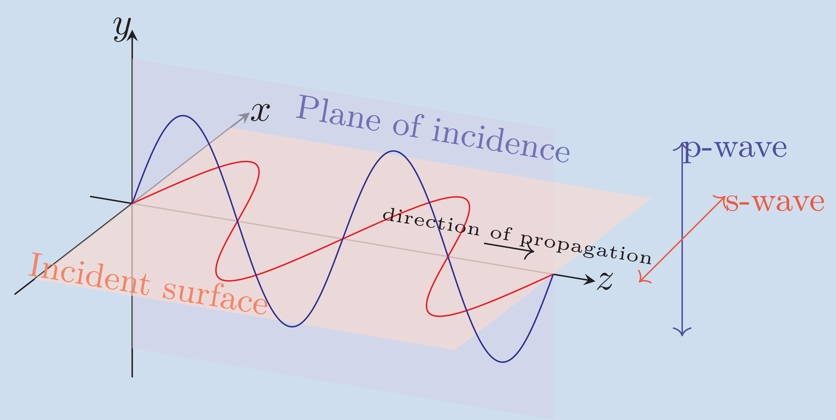


The absorption in a thin photocathode layer also depends on its width. The thicker the layer is the less absorption probability it has to the same depth z. So the less amount of electrons can be produced and can detected because the escape function depends on the residual photocathode width.

Optics overview

PMT is a complex system consisting of three media, one of which is a thin photocathode layer with thickness d_2, where the interference of light is possible. Adding layers to the system is easy enough - it is necessary to consider the already calculated system as one layer with the required coefficients, and again we will get the scheme already used above.





without interference	with interference
$R_{13} = R_{12} + T_{12}R_{23}T_{21} + T_{12}R_{23}R_{21}R_{23}T_{21} + \cdots =$	$r_{13} = r_{12} + t_{12}e^{-\imath\delta}r_{23}e^{-\imath\delta}t_{21} + \cdots =$
$= R_{12} + \dfrac{R_{23}T_{12}^2}{1 - R_{12}R_{23}},$	$= r_{12} + \dfrac{r_{23}t_{12}^2 e^{-2\imath\delta}}{1 + r_{12}r_{23}e^{-2\imath\delta}},$
$T_{13} = T_{12}T_{23} + T_{12}R_{23}R_{21}T_{23} + \cdots =$	$t_{13} = t_{12}e^{-\imath\delta}t_{23} + t_{12}e^{-\imath\delta}r_{23}e^{-\imath\delta}r_{21}e^{-\imath\delta}t_{23} + \cdots =$
$= \dfrac{T_{12}T_{23}}{1 - R_{21}R_{23}}$	$= \dfrac{t_{12}t_{23}e^{-\imath\delta}}{1 + r_{12}r_{23}e^{-2\imath\delta}}$
$A_{13} = 1 - R_{13} - T_{13}$	$R_{13} = \lvert r_{13}\rvert^2, \quad T_{13} = \lvert t_{13}\rvert^2$

All these formula work separately for different light polarisation

Wave

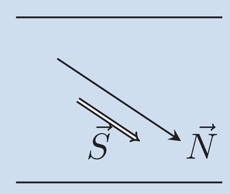
By definition Poynting vector is

$$\mathbf{S} = [\mathbf{E}\mathbf{H}^*] = (\mathbf{N} + \imath\mathbf{K})\, E_s E_s^* + \frac{\mathbf{n}^*}{\mathbf{n}} (\mathbf{N} - \imath\mathbf{K})\, E_p E_p^* + \imath\frac{2}{\mathbf{n}} [\mathbf{N}\mathbf{K}]\, E_p E_s^*$$

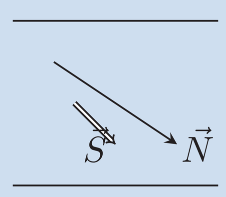
the real Poynting vector that describes the energy flow through the surface has a form of

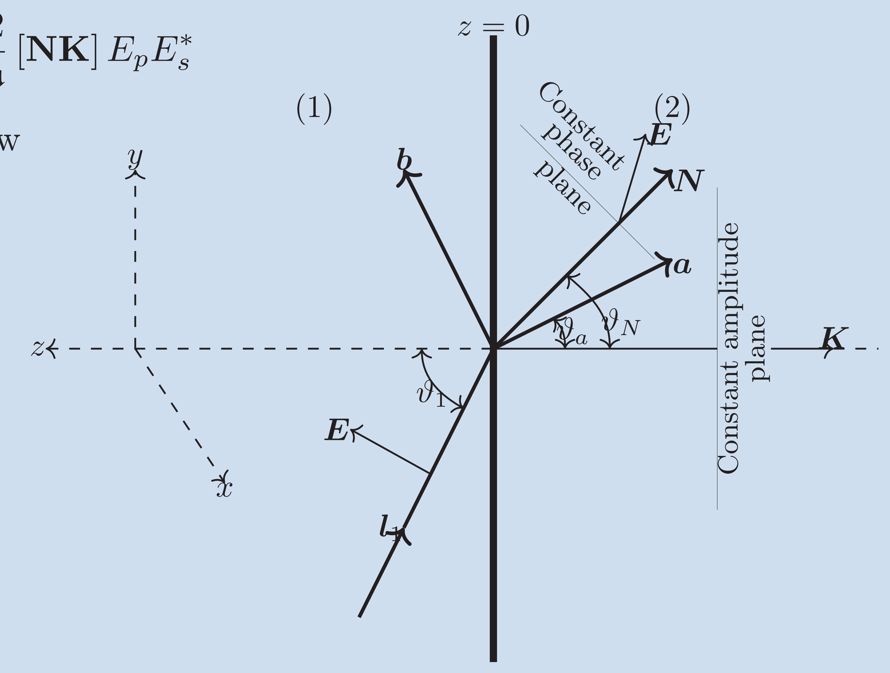
$$\mathbf{S} = \mathbf{N} E_s E_s^* + \mathbf{N}' E_p E_p^*$$

s-wave

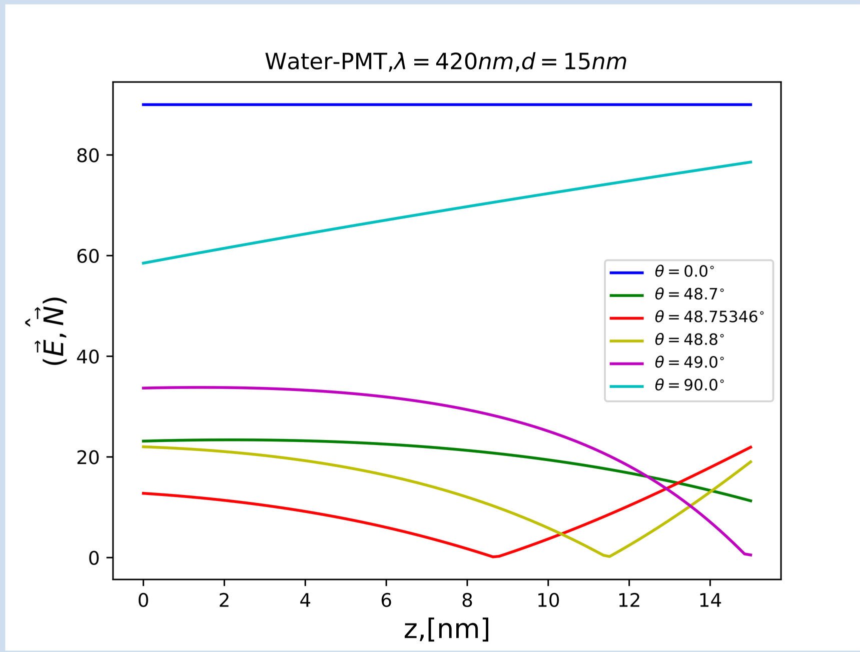


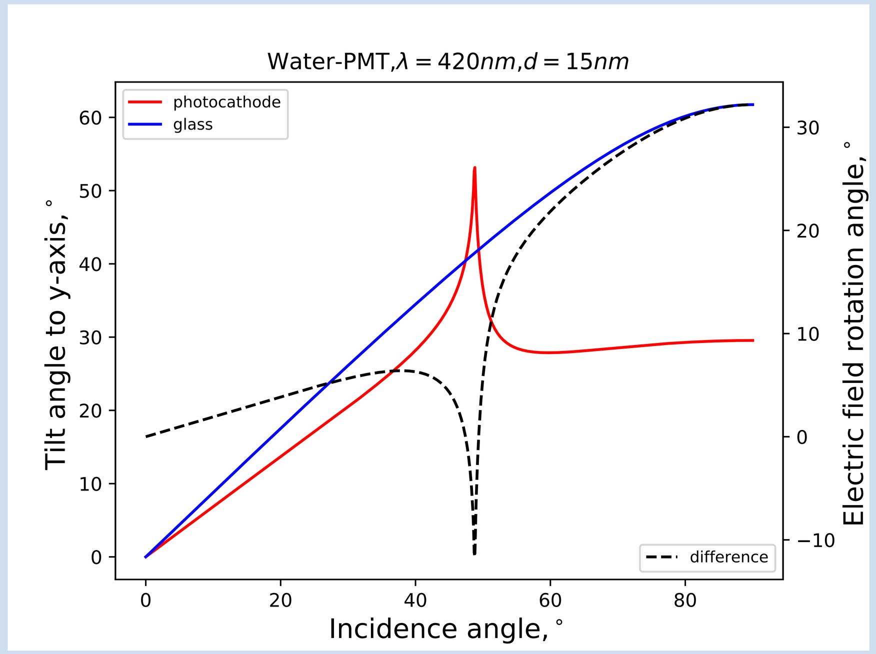
p-wave





- $\vec{N}$ is an optic ray
- $\vec{S}$ is a Poynting vector (describes the energy flow)

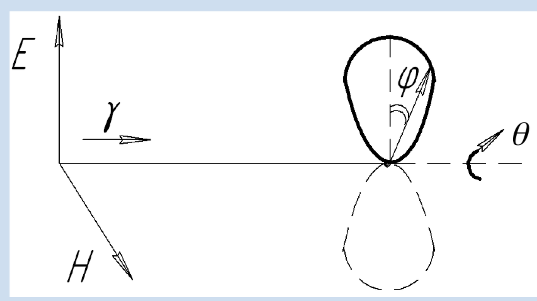




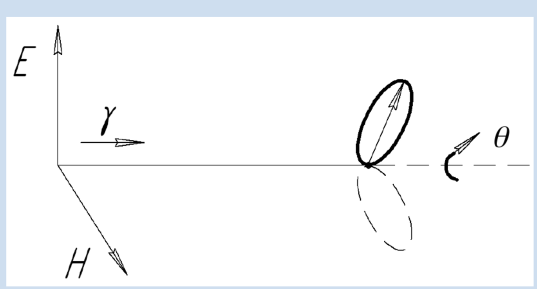
For p-wave polarisation vector $\vec{E}$ isn't normal to the optic ray $\vec{N}$

Photo electron production and the direction of its propagation.

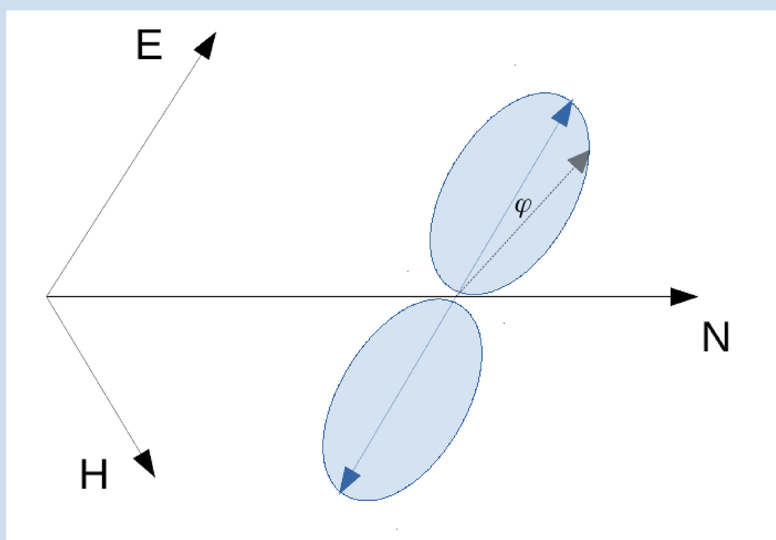
$E_\gamma \ll m_e c^2$



$E_\gamma \gg m_e c^2$



Our case is a low energy γ and some non-90° angle between $\vec{E}$ and $\vec{N}$



Photoelectric cross-section is $\sigma_{pe} \propto sin^2\theta cos^2\phi$

Future plans

- to evaluate the escape function with Geant4
- to submit a paper

Conclusion

We obtained an improved optical model that took into consideration

- interference in thin photocathode
- first principles calculation
- s and p waves

This model increases photodetection efficiency compared to naive mode.