Freeze-out and Statistical Model
in
High Energy Collisions

Lecture 3

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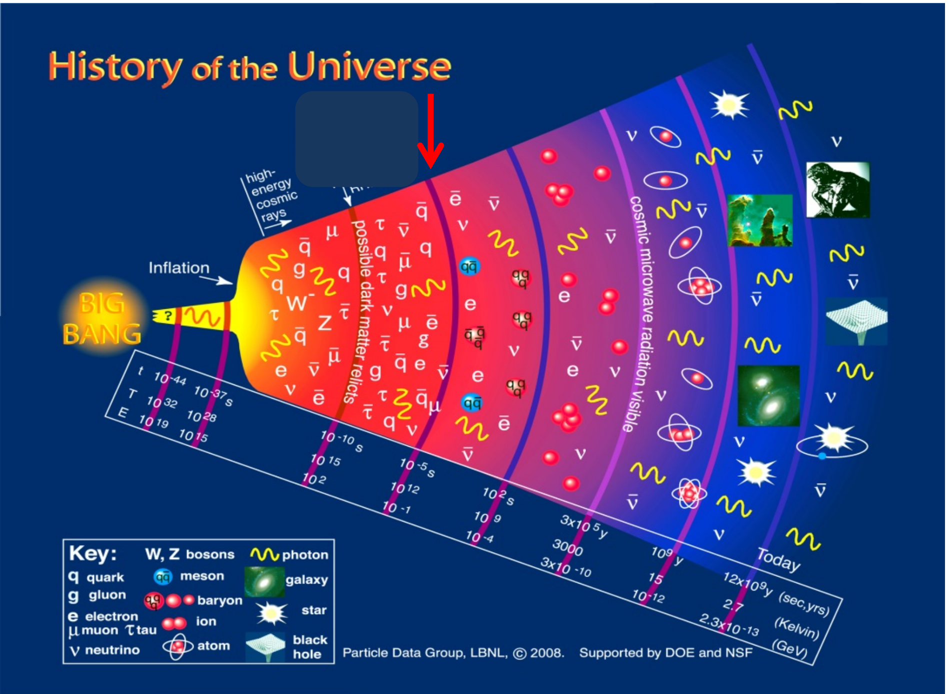
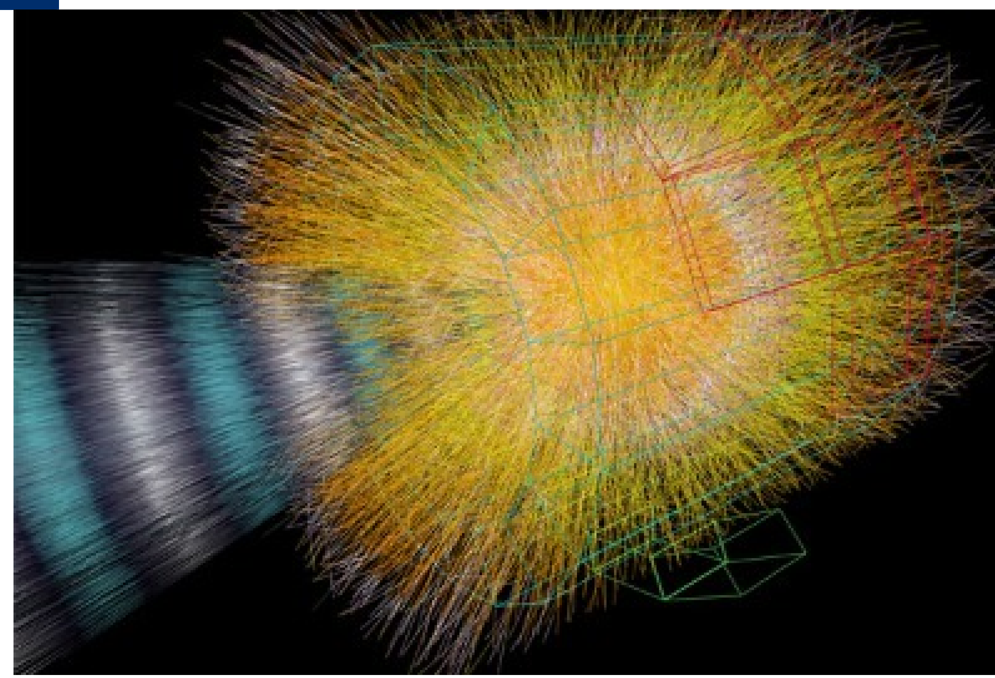
Dense Matter and SQM

DUBNA

28 June 2015
11 July 2015

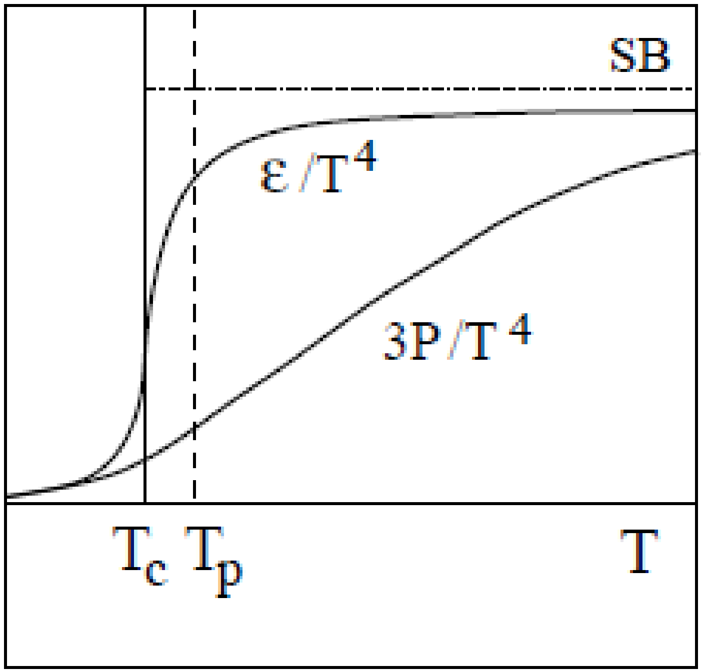
SQM DUBNA 2015

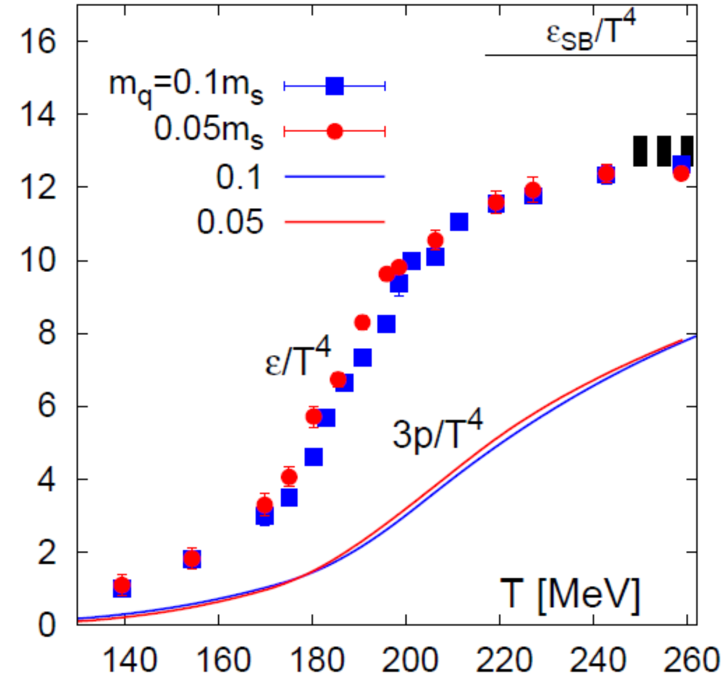


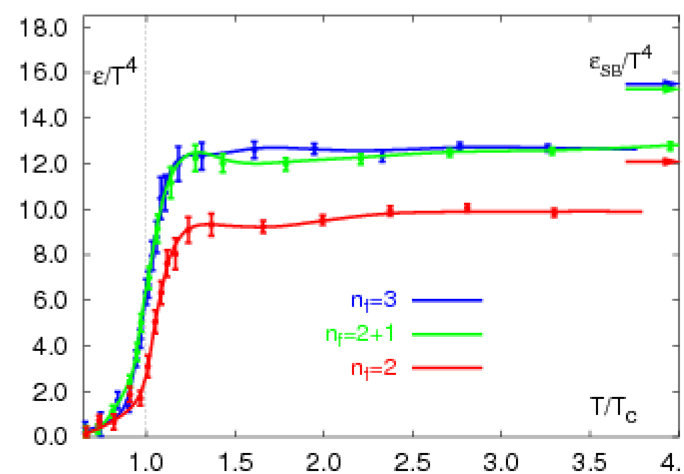


Statistical pattern $e^{-E/T}$

QCD – energy density and pressure at finite temperature





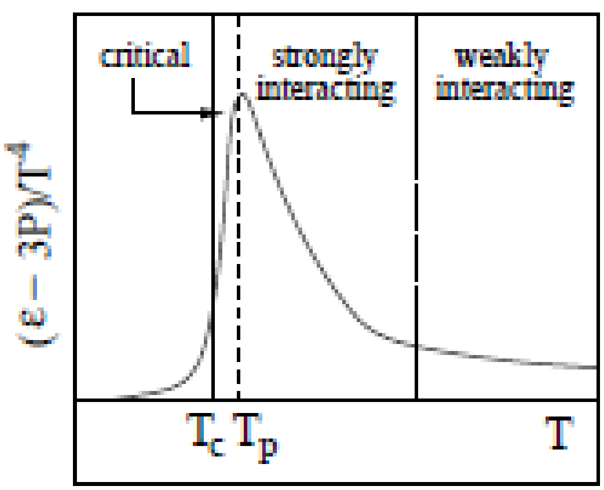


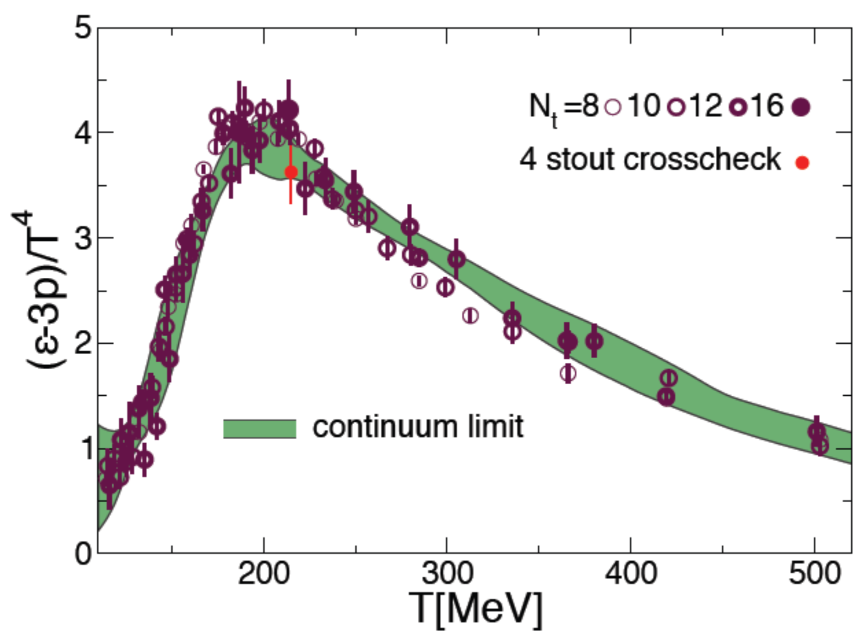
$$\Theta^{\mu}_{\mu} = \varepsilon - 3p$$

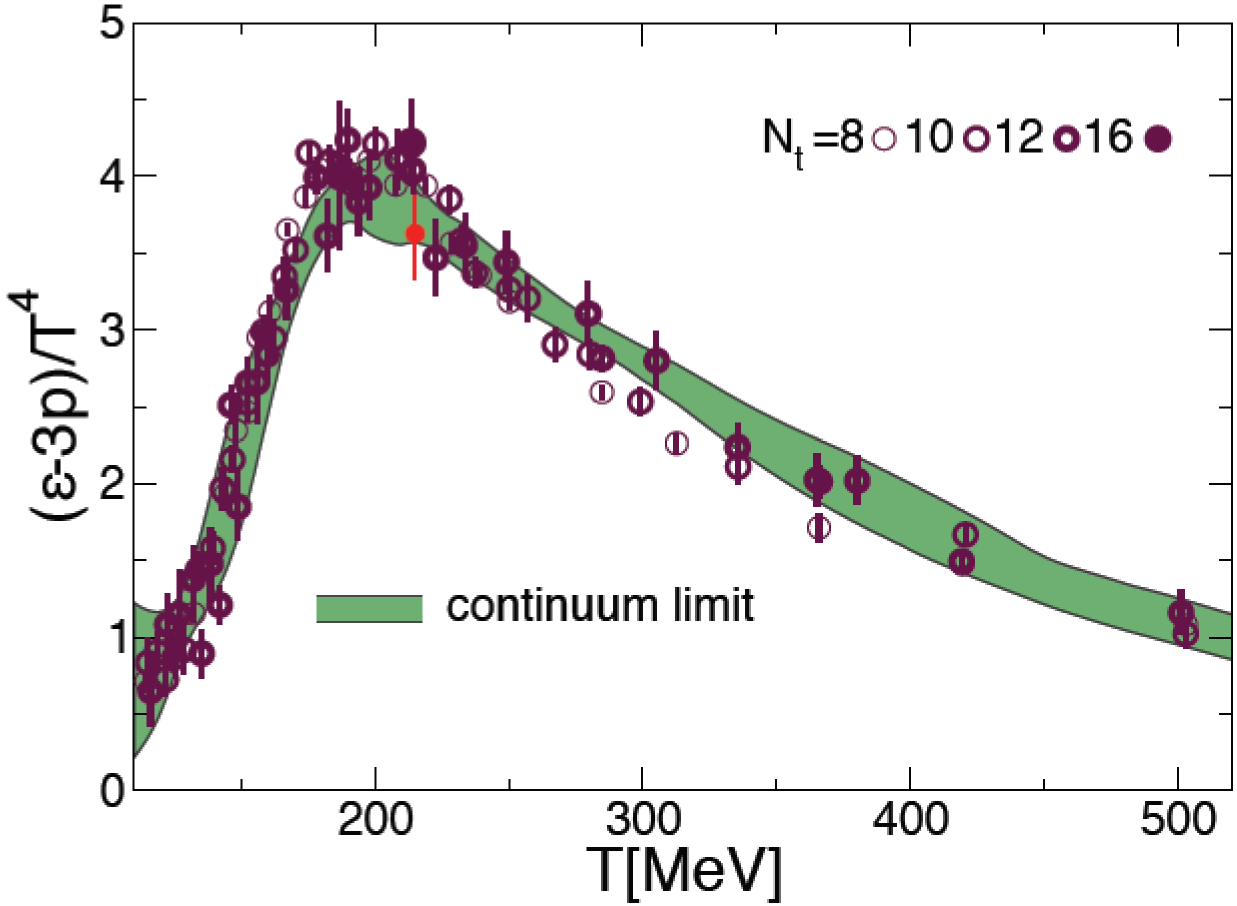
Energy density　　　pressure

$$\Delta = \Theta^{\mu}_{\mu} / T^4 = (\varepsilon - 3p) / T^4$$

Interaction measure since $\Delta = 0$ in a free , massless theory

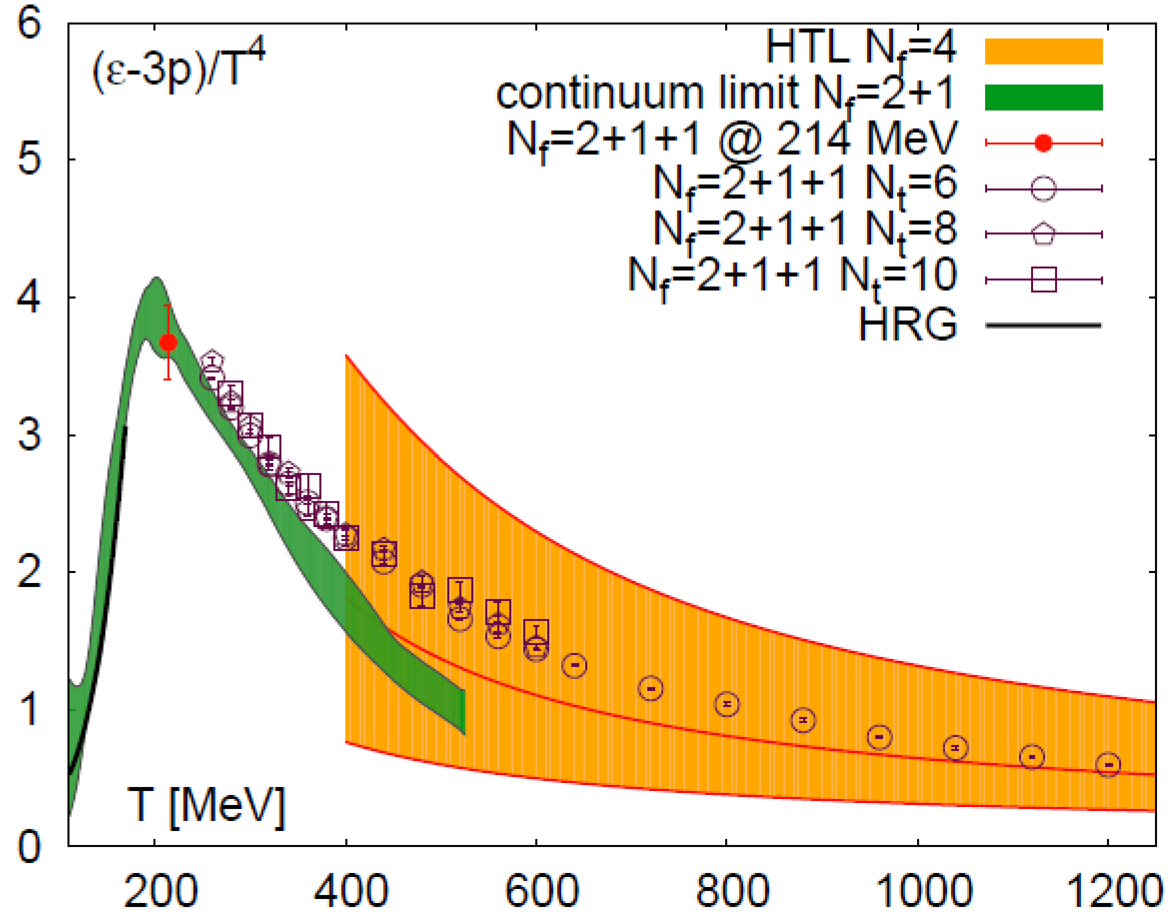






S.Borsanyi et al. Phys. Lett. B 370 (2014) 99
A.Bazavov (HotQCD collaboration) PRD90(2014)094503

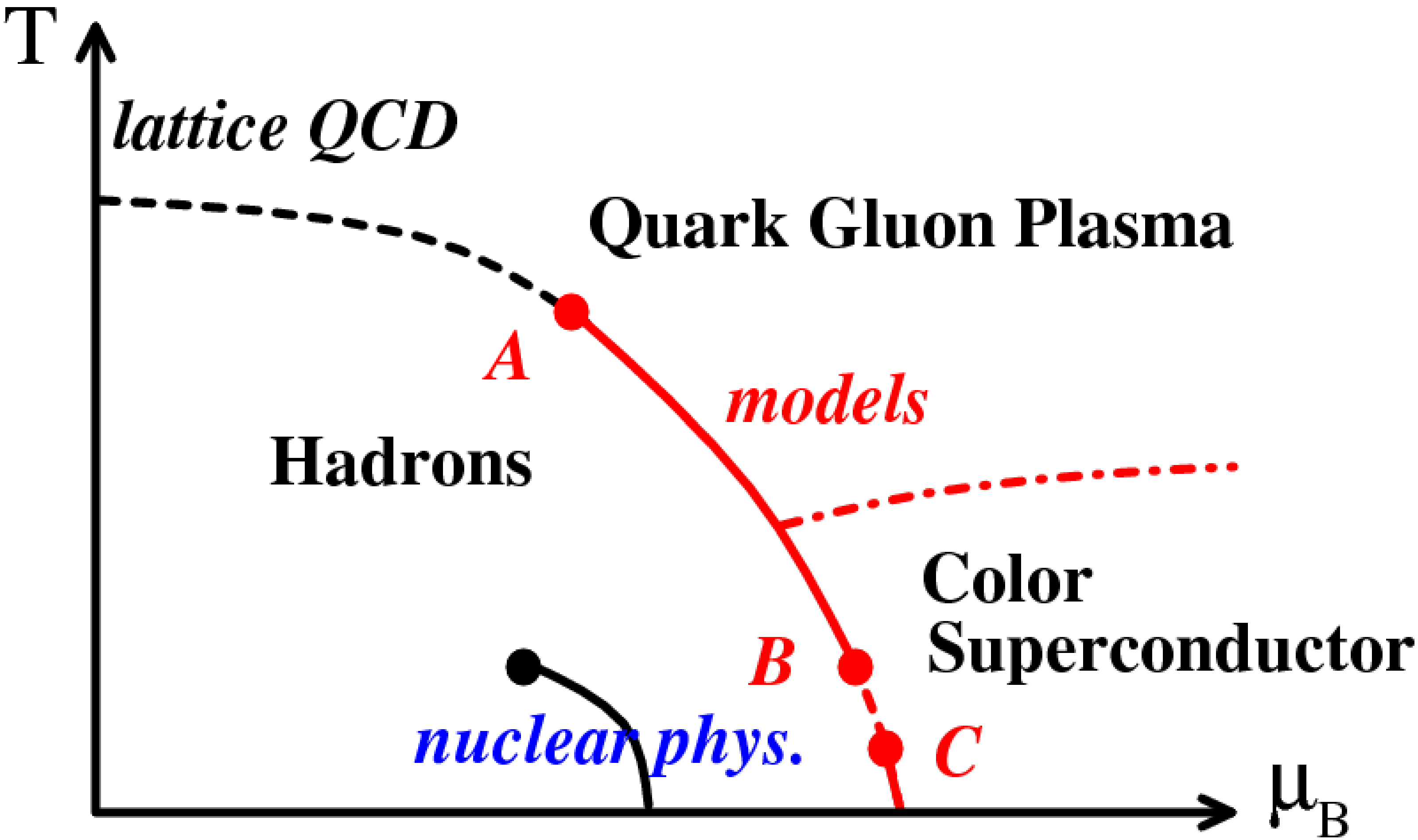
Thermodynamics of strong-interaction matter from Lattice QCD
Heng-Tong Ding, Frithjof Karsch, Swagato Mukherjee **arXiv:1504.05274**



Non perturbative origin

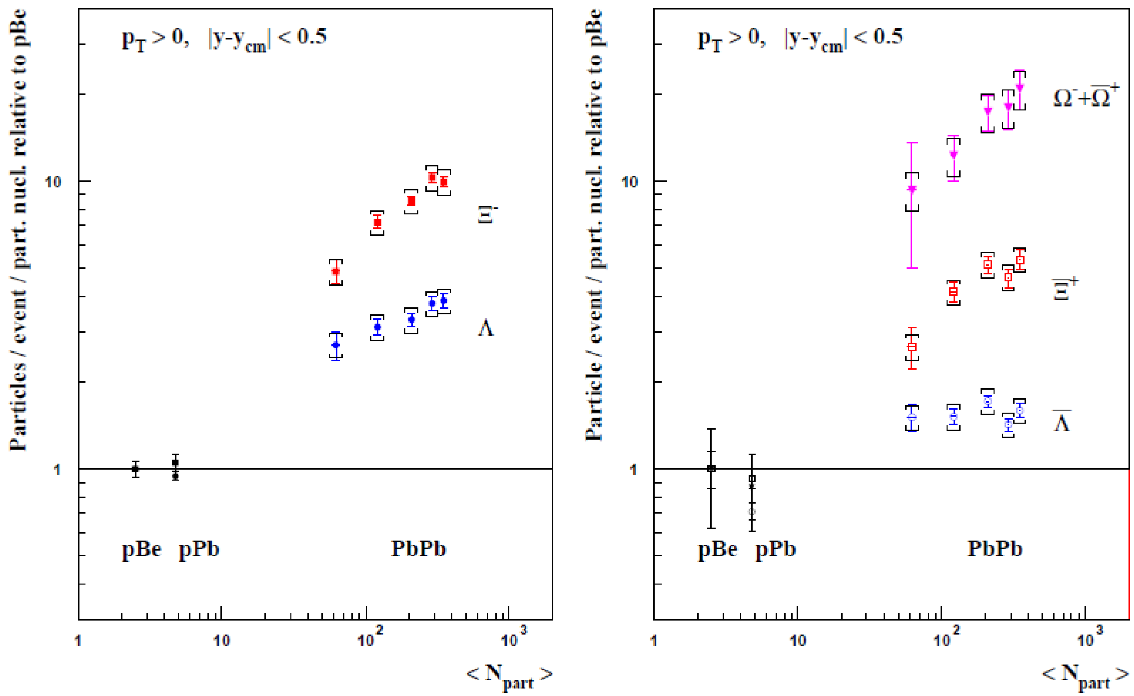
for T in the range 1-3 Tc

O.Kaczmarek lectures 1+2


T
lattice QCD
Quark Gluon Plasma
A
models
Hadrons
Color
Superconductor
B
nuclear phys.
C
μ_B

Review SPS to RICH C. Blume and C. Markert arxiv:11052798

$$E_S = \left(\frac{1}{\langle N_{\mathrm{part}} \rangle} \frac{\mathrm{d}N(\mathrm{Pb+Pb})}{\mathrm{d}y} \bigg|_{y=0} \right) \Bigg/ \left(\frac{1}{2} \frac{\mathrm{d}N(\mathrm{p+p(Be)})}{\mathrm{d}y} \bigg|_{y=0} \right)$$

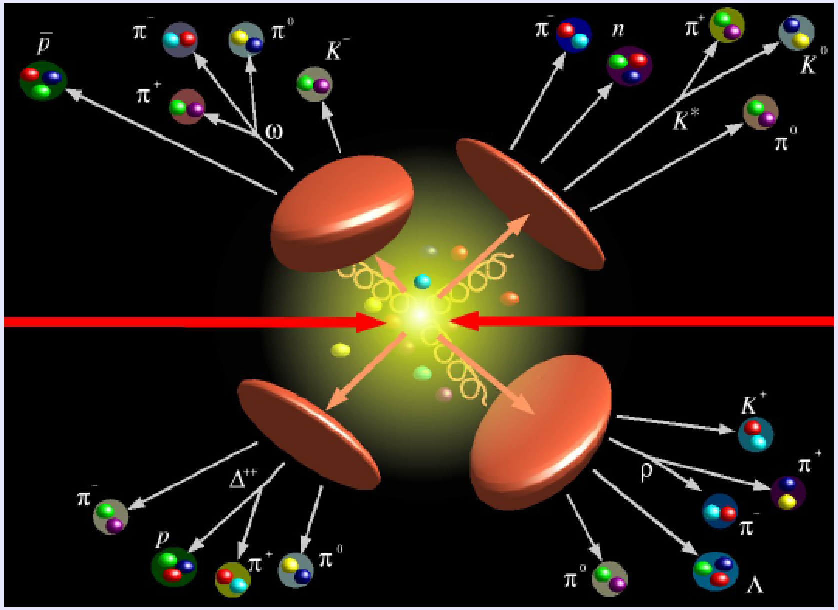


The enhancement of strange particle production with respect to p+Be collisions as a function of the number of participating nucleons, measured by the NA57 collaboration for Pb+Pb collisions at 158*A* GeV

$$\phi(s\bar{s}),\ \mathrm{K}(q\bar{s}),\ \overline{\mathrm{K}}(\bar{q}s),\ \Lambda(qqs),\ \overline{\Lambda}(\bar{q}\bar{q}\bar{s}),\ \Xi(qss),\ \overline{\Xi}(\bar{q}\bar{s}\bar{s}),\ \Omega(sss),\ \overline{\Omega}(\bar{s}\bar{s}\bar{s})$$

Statistical Model

In modern view, the statistical model is a model of hadronization, describing
the process of hadron formation at the scale where QCD is no longer perturbative



J.Cleymans lectures 1+2

basic observation in all high energy multihadron production

thermal production pattern

- species abundances $\sim$ ideal resonance gas at T_H
- universal $T_H \simeq 165 \pm 15 Mev$ for all (large) $\sqrt{s}$

caveats

- strangeness suppression in elementary collisions
- strangeness suppression weakened/removed in nuclear collisions

F. Becattini, Z. Phys. C69 (1996) 485.

F. Becattini, *Universality of thermal hadron production in pp, $p\bar{p}$ and e^+e^- collisions*, in *Universality features in multihadron production and the leading effect*, Erice 1966, World Scientific, Singapore (1998) 74-104; arXiv:hep-ph/9701275.

F. Becattini and G. Passaleva, Eur. Phys. J. C23 (2002) 551.

F. Becattini and U. Heinz, Z. Phys. C76 (1997) 268.

J. Cleymans et al., Phys. Lett. B 242 (1990) 111.

J. Cleymans and H. Satz, Z. Phys. C57 (1993) 135.

K. Redlich et al., Nucl. Phys. A 566 (1994) 391.

P. Braun-Munzinger et al., Phys. Lett. B344 (1995) 43.

F. Becattini, M. Gazdzicki and J. Sollfrank, Eur. Phys. J. C5 (1998) 143.

F. Becattini et al., Phys. Rev. C64 (2001) 024901.

P. Braun-Munzinger, K. Redlich and J. Stachel, in *Quark-Gluon Plasma 3*, Hwa and X.-N Wang (Eds.), World Scientific, Singapore 2003.

1. Thermal Hadron Production

what is "thermal"? J.Cleymans lectures 1+2

- equal *a priori* probabilities for all states in accord with given overall average energy $\Rightarrow$ temperature T;

- partition function of ideal resonance gas

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T)$$

Boltzmann factor $\phi(m_i, T) = 4\pi m_i^2 T K_2(m_i/T) \sim \epsilon^{-m_i/T}$;

- relative abundances $\frac{N_i}{N_j} = \frac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)} \sim \epsilon^{-(m_i - m_j)/T}$

predicted in terms of temperature T

In the grand-canonical formulation of the statistical model, the mean hadron multiplicities are defined as

$$\langle N_i \rangle = (2J_i + 1)\frac{V}{(2\pi)^3}\int d^3p \, \frac{1}{\gamma_s^{-s_i}\exp[(E_i - \mu \cdot q_i)/T_{ch}] \pm 1}$$

Ch**emi**cal Freeze –out Temperature = hadronic abundances get frozen

Fireball volume

Strangeness suppression

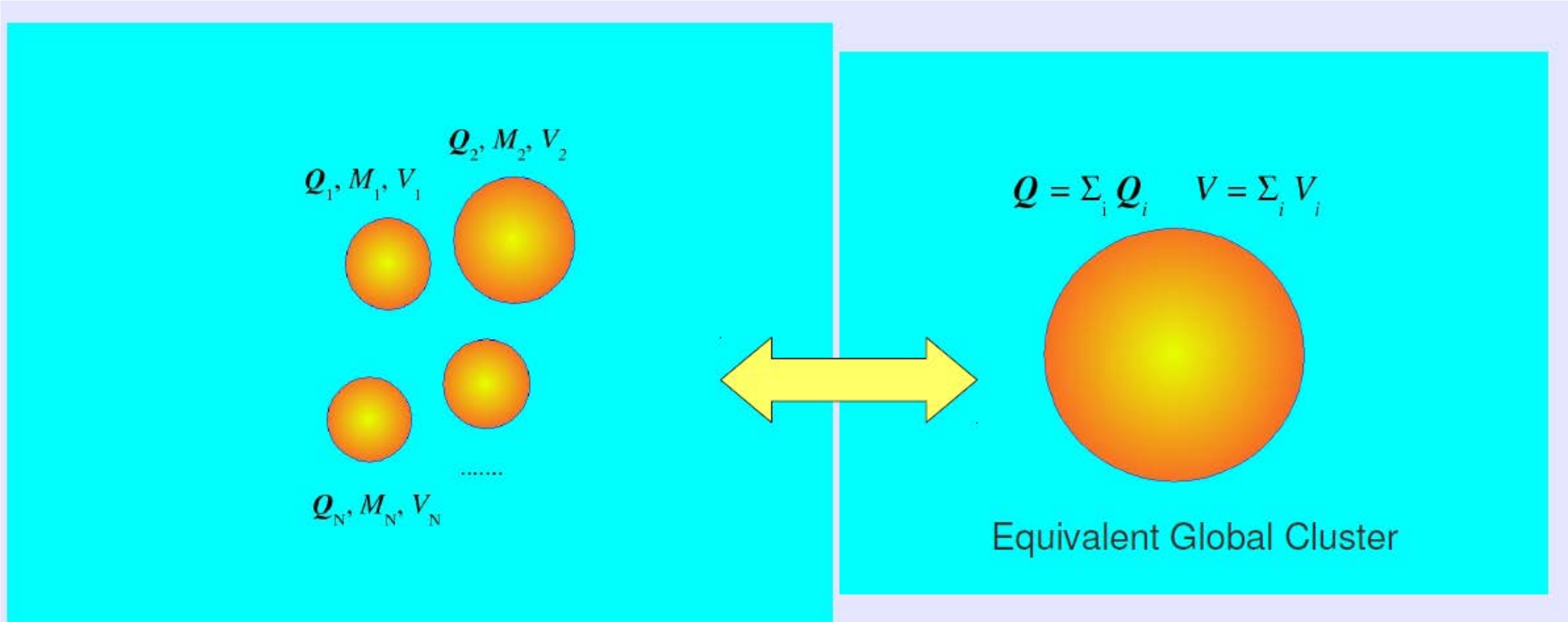
Number of s or anti-s

chemical potentials

baryon number B_i, third component of the isospin I_{3i}, strangeness S_i, and charmness C_i, the chemical potential is $\mu_i = \mu_b B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$. The chemical potentials related to baryon number (μ_b), isospin (μ_{I_3}), strangeness (μ_S) and charm (μ_C) ensure the conservation (on average) of the respective quantum numbers: i) baryon number: $V\sum_i n_i B_i = N_B$; ii) isospin: $V\sum_i n_i I_{3i} = I_3^{tot}$; iii) strangeness: $V\sum_i n_i S_i = 0$; iv) charm: $V\sum_i n_i C_i = 0$.

- massive colorless clusters distributed over rapidities, each decays statistically
- mass and charge distributions of clusters again statistically

  $\Rightarrow$ equivalent global cluster
- $V = \Sigma V_i$, $Q = \Sigma Q_i$; large enough for thermodynamics



First, a primary hadron yield $\langle n_j \rangle^{\mathrm{primary}}$ is calculated using previous equations.

As a second step, all resonances in the gas which are unstable against strong decays are allowed to decay into lighter stable hadrons, using appropriate branching ratios (B) for the decay k → j published by the PDG. The abundances in the final state are thus determined by

$$\langle n_j \rangle = \langle n_j \rangle^{\mathrm{primary}} + \sum \langle n_k \rangle BR(k \to j).$$

$$T, V, \gamma_s, \mu_b$$

Particle	Measured dN/dy (E)	Relative error	Model dN/dy (M)	Residual	(M - E)/E (%)
pp collisions at $\sqrt{s} = 200$ GeV					
π^+	1.44 ± 0.11	0.076	1.403	-0.34	-2.62
π^-	1.42 ± 0.11	0.077	1.384	-0.33	-2.59
K^+	0.150 ± 0.013	0.087	0.1522	0.17	1.48
K^-	0.145 ± 0.013	0.090	0.1460	0.076	0.68
p	0.138 ± 0.012	0.087	0.1491	0.92	7.42
$\bar{p}$	0.113 ± 0.010	0.088	0.1120	0.66	5.56
ϕ	0.0180 ± 0.0029	0.16	0.01130	-2.31	-59.3
Λ	0.0436 ± 0.0041	0.094	0.04348	-0.030	-0.28
$\bar{\Lambda}$	0.0398 ± 0.0038	0.095	0.03686	-0.77	-7.96
Ξ^-	0.0026 ± 0.00092	0.35	0.003070	0.51	15.3
$\bar{\Xi}^+$	0.0029 ± 0.00104	0.36	0.002728	-0.17	-6.29
$\Omega + \bar{\Omega}$	0.00034 ± 0.00019	0.56	0.0005712	1.22	40.5
K_S^0	0.134 ± 0.011	0.082	0.1467	1.15	8.64
ρ^0	0.259 ± 0.039	0.15	0.1861	-1.87	-39.2
$(K^{*0} + \bar{K}^{*0})/2$	0.0508 ± 0.0063	0.12	0.05151	0.11	1.38
$\Sigma^{*+} + \Sigma^{*-}$	0.0107 ± 0.00146	0.14	0.01028	-0.29	-4.12
$\bar{\Sigma}^{*+} + \bar{\Sigma}^{*-}$	0.0089 ± 0.00126	0.14	0.008650	-0.20	-2.89
$\Lambda(1520) + \bar{\Lambda}(1520)$	0.0069 ± 0.0011	0.16	0.005606	-1.18	-23.1

	pp $\sqrt{s} = 200$ GeV
Overall fit	
T(MeV)	170.1± 3.5
Normalization	0.027± 0.011
VT^3	135 ± 60
γ_S	0.569± 0.031
μ_B/T	
χ^2/dof	15.6/14
Fit with standard sample	
T(MeV)	169.8± 4.2
Normalization	0.028± 0.012
VT^3	131 ± 60
γ_S	0.600± 0.033
μ_B/T	
χ^2/dof	15.0/8

Abundances

e^+e^-, LEP Data [Becattini 1996]

Fit relative abundances to ideal resonance gas of all hadronic resonances, with $M \leq 1.7$ GeV, two parameters T and γ_s

$T = 169.9 \pm 2.6$ MeV

$\gamma_s = 0.691 \pm 0.053$

estimate systematic error by varying resonance gas scheme, contributing resonances

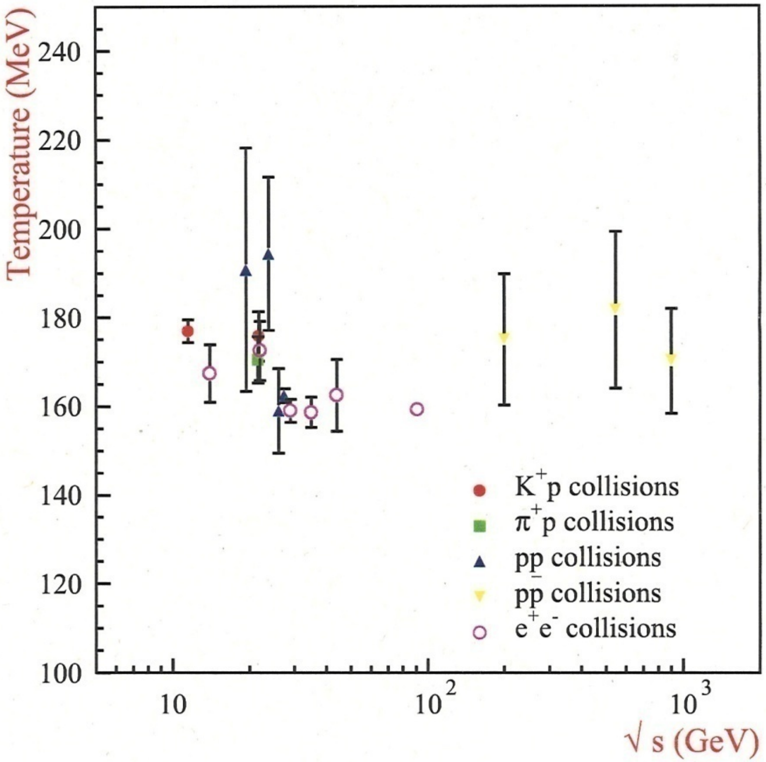
different implementation schemes

e^+e^- $\sqrt{s} = 91.2$ GeV				
species	measured			fit
π^+	8.53	$\pm$	0.40	8.72
π^0	9.18	$\pm$	0.82	9.83
K^+	1.18	$\pm$	0.052	1.06
K^0	1.015	$\pm$	0.022	1.01
η	0.934	$\pm$	0.13	0.908
ρ^0	1.21	$\pm$	0.22	1.16
K^{*+}	0.357	$\pm$	0.027	0.349
K^{*0}	0.372	$\pm$	0.027	0.343
η'	0.13	$\pm$	0.05	0.1070
p	0.488	$\pm$	0.059	0.484
ϕ	0.10	$\pm$	0.0090	0.167
Λ	0.185	$\pm$	0.0085	0.152
Ξ^-	0.0122	$\pm$	0.00079	0.011
Ξ^{*0}	0.0033	$\pm$	0.00047	0.00391
Ω	0.0014	$\pm$	0.00046	0.000782

$$T = 170 \pm 10 \text{ MeV}, \; \gamma_s \simeq 0.7 \pm 0.1$$

corresponding analyses for hadronic collisions

- pp at $\sqrt{s} = 19.4,\ 23.8,\ 26.0,\ 27.4$ GeV
- $p\bar{p}$ at $\sqrt{s} = 200,\ 500,\ 900$ GeV
- $\pi^+ p$ at $\sqrt{s} = 21.7$ GeV
- $K^+ p$ at $\sqrt{s} = 11.5,\ 21.7$ GeV



Result:

$$T \simeq 170 \pm 20 \text{ MeV}$$

$$\gamma_s = 0.7 \pm 0.1$$

independent of
- collision energy
- collision configuration

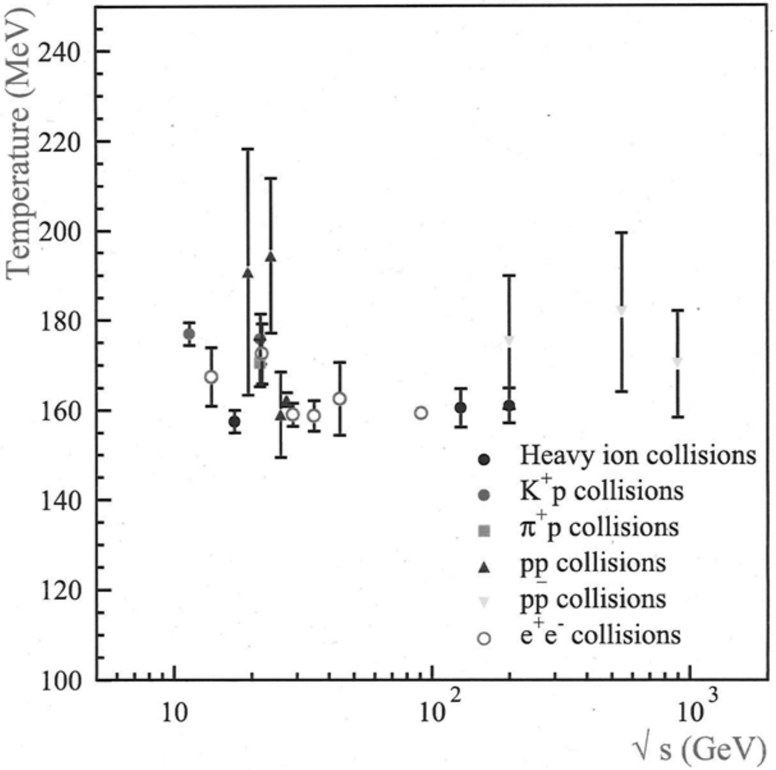
Heavy ion collisions ⇒ baryon density

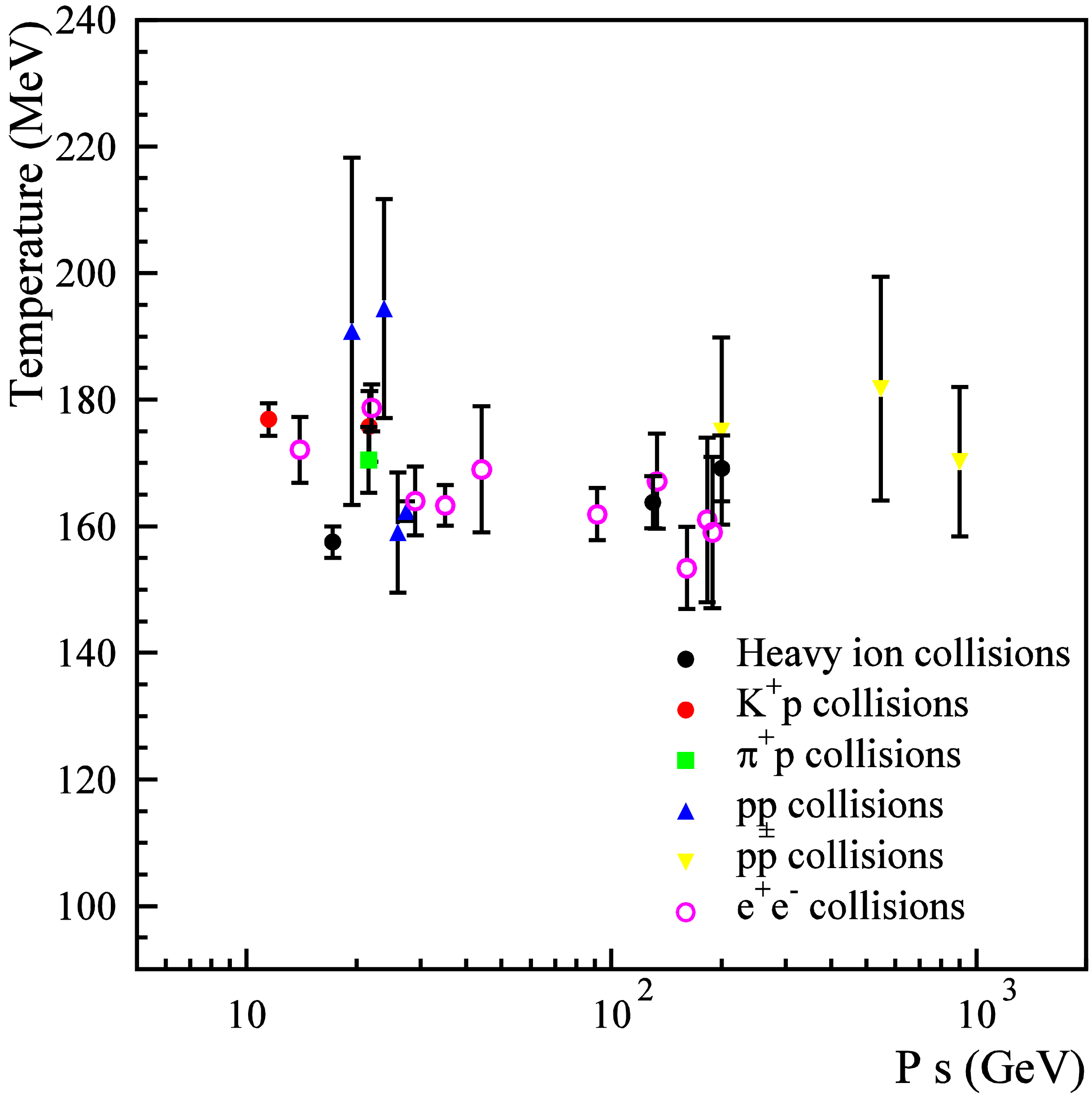
- resonance gas at T, μ_B; $\mu_B \Downarrow$ for $\sqrt{s} \Uparrow$
- consider species abundances in high energy heavy ion collisions

Result:

same hadronization temperature
for high energy heavy ion
and elementary collisions,
collision energy independent

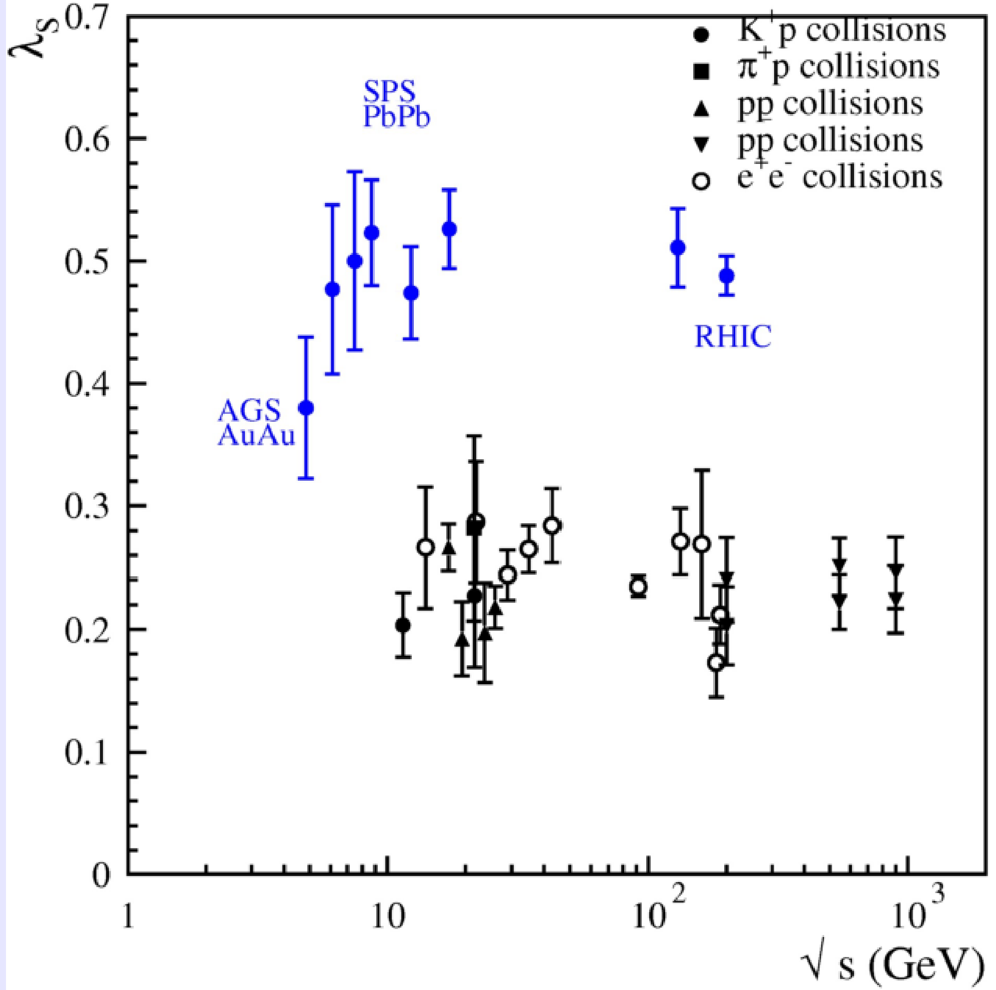
increased strangeness
$\gamma_s \longrightarrow 0.8 - 1.0$ for high
energy heavy ion collisions



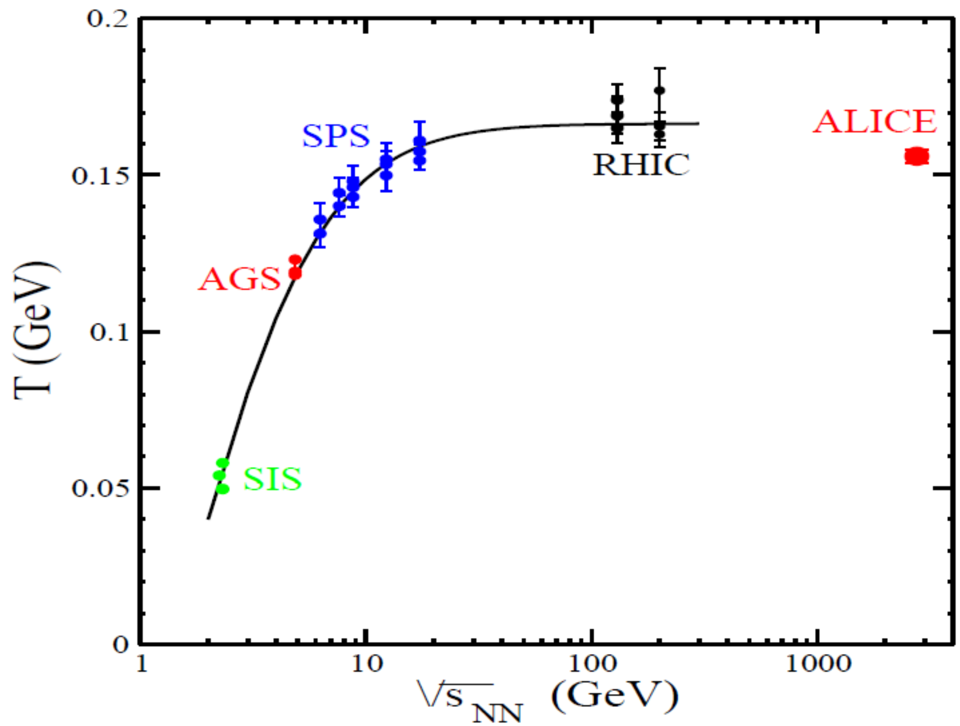


Temperature (MeV)
240
220
200
180
160
140
120
100
10
10^2
10^3
P s (GeV)
Heavy ion collisions
K+p collisions
π+p collisions
pp collisions
pp± collisions
e+e- collisions

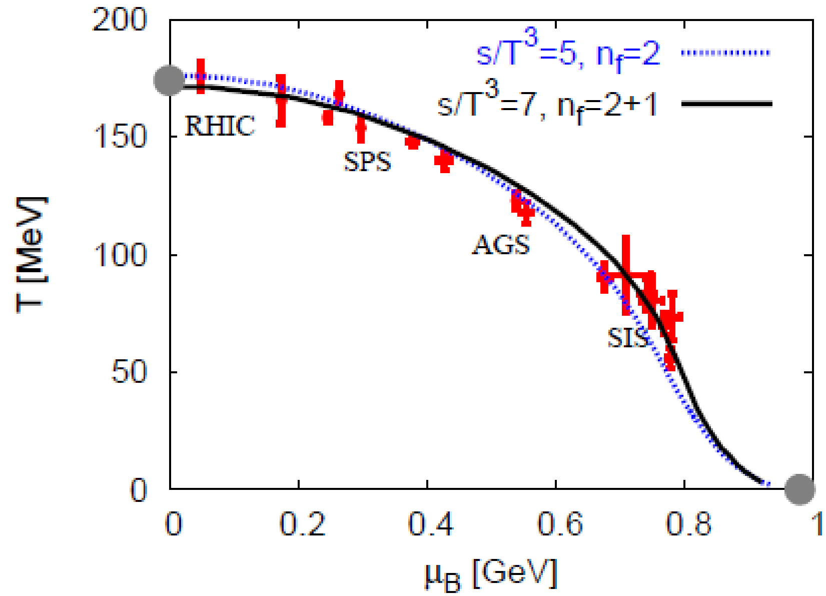
Wroblewski ratio $\lambda_S = \frac{2\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}$

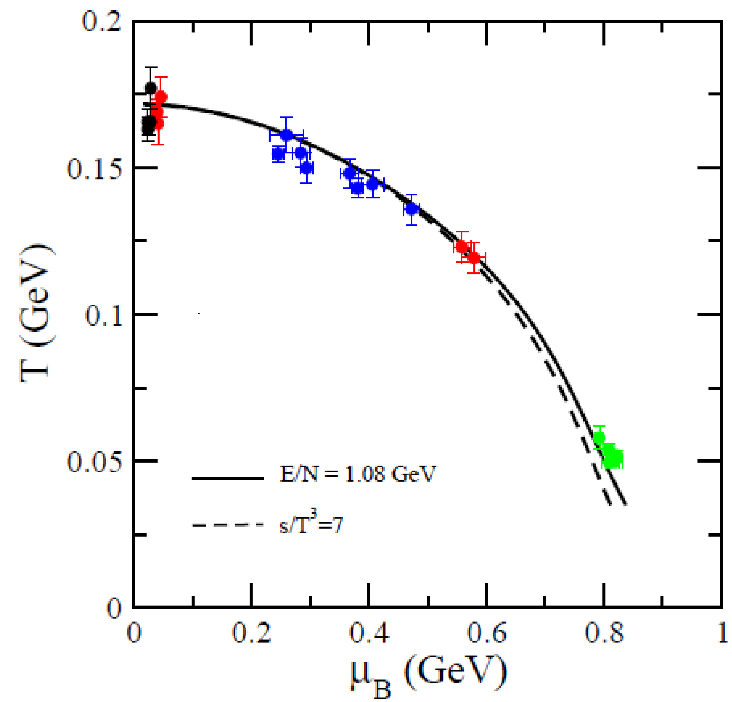


M. Floris: Nucl. Phys. A931, (2014) 103-112



Freeze-out
s/T^3 = 7





J. Cleymans and K. Redlich, Phys. Rev. Lett. 81 (1998) 5284.

J. Cleymans and K. Redlich, Phys. Rev. C 61 (1999) 054908.

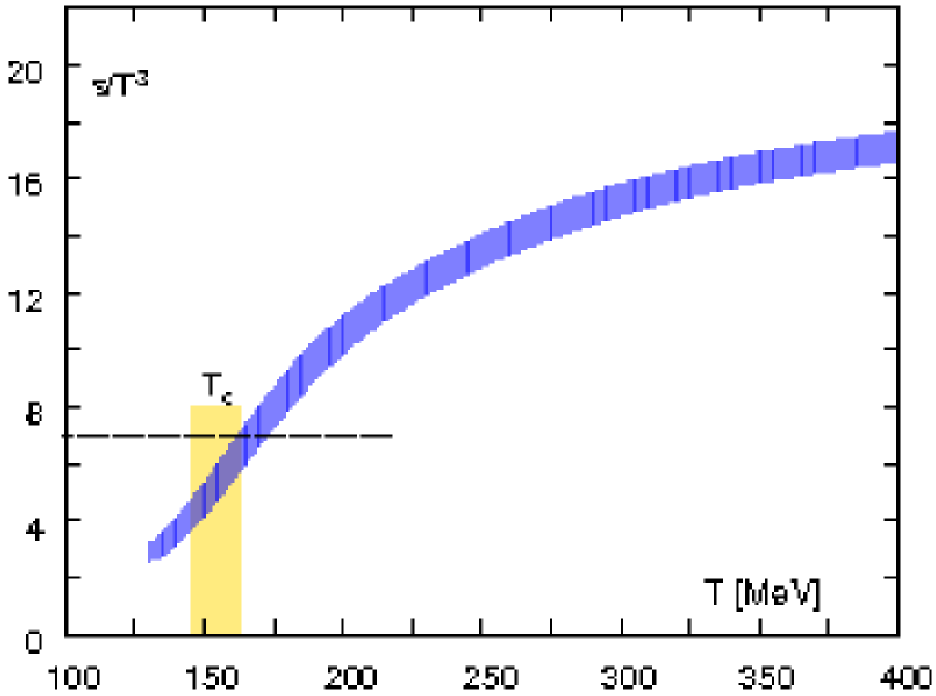
J. Cleymans et al., arXiv:hep-ph/0511094

P. Braun-Munzinger and J. Stachel, J. Phys. G 28 (2002) 1971.

V. Magas and H. Satz, Eur. Phys. J. C32 (2003) 115.

J. Cleymans et al., Phys. Lett. B 615 (2005) 50.

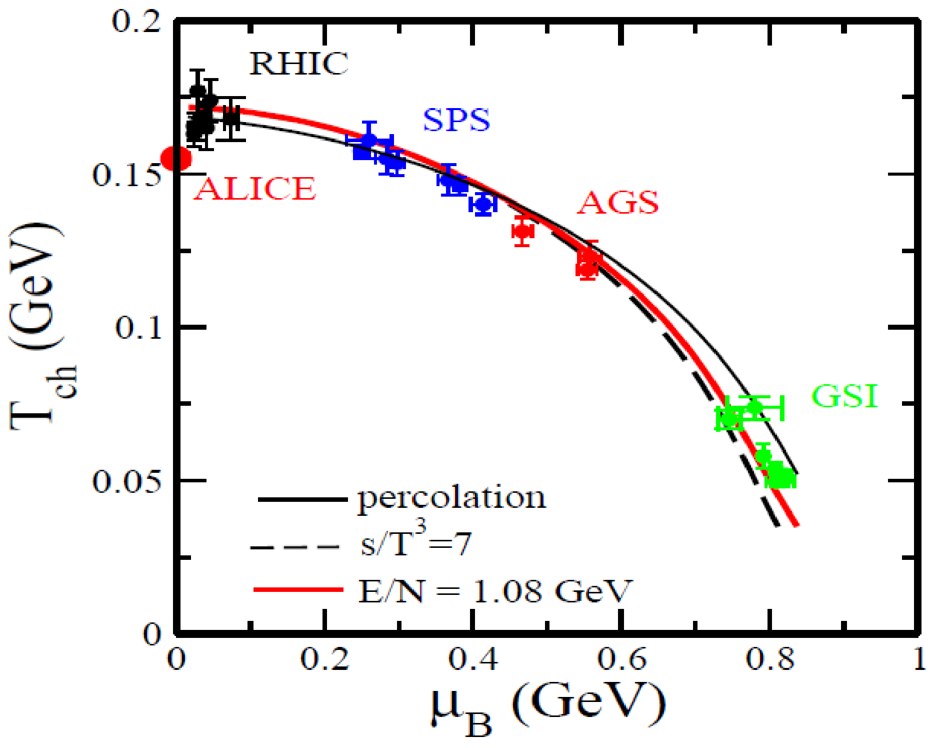
A. Tawfik, J. Phys. G 31 (2005) S1105; hep-ph/0507252 and hep-ph/050824.



Lattice results for s/T^3 as function of the temperature

A. Bazazov et al. (HotQCD Collaboration), arXiv:1407.6387

M. Floris: Nucl. Phys. A931, (2014) 103-112



Conclude:

Hadron abundances in all high energy collisions (e^+e^- annihilation, hadron-hadron interactions and heavy ion collisions) are those of an ideal resonance gas at a universal temperature

$$T_H \simeq 170 \pm 20 \text{ MeV}.$$

Strangeness production in elementary collisions is uniformly suppressed by $\gamma_s \simeq 0.6 - 0.7$

suppression weakened/removed in heavy ion collisions

1) ***Why do elementary high energy collisions*** **show a statistical behavior?**

2***) Why is strangeness production universally suppressed*** **in elementary collisions?**

3) ***Why (almost) no strangeness suppression in nuclear collisions?***

4) ***Why hadron freeze-out for s/T^3 = 7 or E/N=1.08 Gev***

5) ***Why thermalization in so short time (0.5- 1 fm/c)***

Canonical suppression

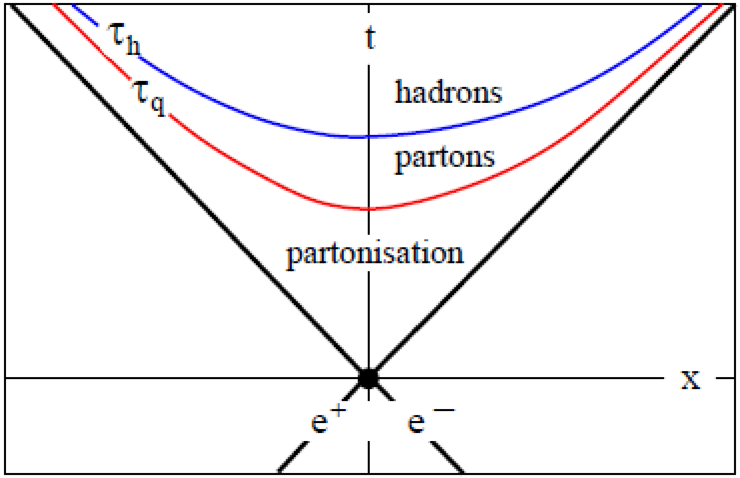
Statistical Thermodynamics in Relativistic Particle and Ion Physics: Canonical or Grand Canonical?
R. Hagedorn and K. Redlich

Canonical aspects of strangeness enhancement
A. Tounsia, A. Mischkeb and K. Redlich hep-ph/0209284

$$\langle N_i \rangle = (2J_i+1)\frac{V}{(2\pi)^3}\int d^3p \frac{1}{\gamma_s^{-s_i}\exp[(E_i - \mu\cdot q_i)/T_{ch}] \pm 1}$$

The nature of V in elementary collisions is quite different from that in nuclear collisions and this can in effect lead to different behavior for strangeness production .

τ_q ,specifies a boost-invariant proper time at which local volume elements experience the transition from an initial state of frozen virtual partons to the partons which will eventually form hadrons



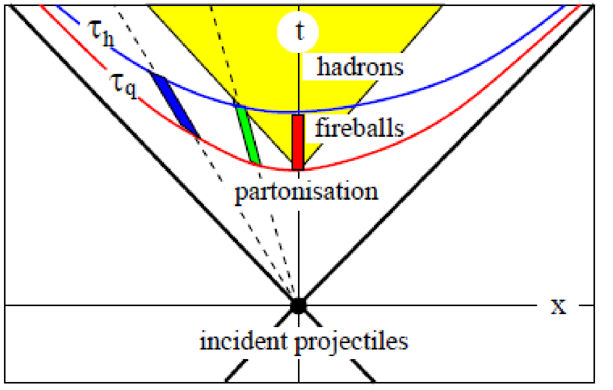
$$\sigma x_q = 2\sqrt{m_q^2 + k_T^2},$$

$$k_T = \sqrt{\pi\sigma/2},$$

$$x_q \simeq \sqrt{\frac{2\pi}{\sigma}} \simeq 1 \text{ fm},$$

arXiv:1310.6932
P.C. and H.Satz Causality Constraints on Hadron Production In High Energy Collisions

bubble of partonic medium of proper time τ
with $\tau_q < \tau < \tau_h$: fireball;
fireballs at different spatial
rapidities η



$$t = \tau \cosh \eta, \quad x = \tau \sinh \eta,$$

with transition lines

$$t^2 - x^2 = \tau^2$$

red fireball ($\eta = 0$) - causality region yellow
green fireball ($\eta = \eta_d$) - one common x-t point with red
blue fireball ($\eta > \eta_d$) - outside causality region of red

for $\eta > \eta_d$, with $\tanh \eta_d = (\tau_h^2 - \tau_q^2)/(\tau_h^2 + \tau_q^2)$

forward and backward fireballs are out of communication with central fireball examples:

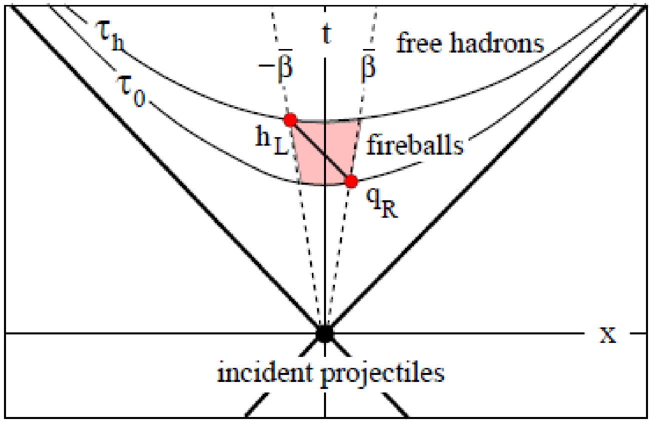
$\tau_q = 1$ fm, $\tau_h = 2$ fm $\rightarrow \eta_d = 0.7$

$\tau_q = 1$ fm, $\tau_h = 7$ fm $\rightarrow \eta_d = 2$

at RHIC and LHC, hadronisation occurs through causally disjoint fireballs

so far, have neglected spatial size:
what is the size of a fireball?
define through causal connectivity

require: the most separate points can still communicate



spatial diameter d of fireball in cms at hadronisation time

$$d = \sqrt{\frac{\tau_h}{\tau_q}(\tau_h - \tau_q)}$$

causal connection (and hence correlations) for hadron production at large rapidity intervals; ⇒ means that any correlations originated in the earlier partonisation stage.

examples for different hadronisation times:

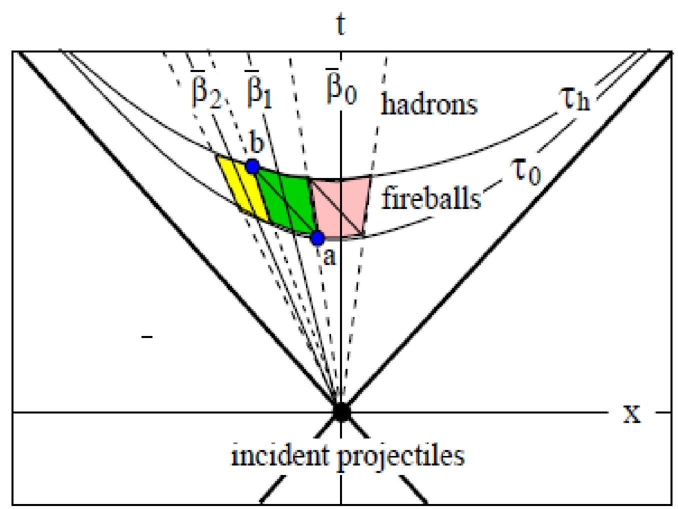
τ_h [fm]	β	η	$r = d/2$ [fm]
2	0.33	0.35	0.7
3	0.50	0.55	1.7
4	0.60	0.69	3.0
5	0.67	0.81	4.5

We now assume complete boost invariance: the collision leads to the production of identical fireballs at all rapidities, with identical formation and hadronisation times τ_q, τ_h in their respective rest frames. To study the causal connection of fireballs moving at dif-

denote average cms velocity of of central fireball by $\bar{\beta}_0 = 0$
can then partition production region into successive causally disjoint fireballs, of velocities

$$\bar{\beta}_n = \frac{\tau_h^{2n} - \tau_q^{2n}}{\tau_h^{2n} + \tau_q^{2n}}$$

$$n = 0, 1, 2, ...$$



one fireball - require that the spatially right-most point q_R at formation can send a signal to the spatially left-most point h_L at hadronisation; i.e., we require that the most separate points of the fireball can still communicate.

τ_h [fm]	β	η	r [fm]
2	0.33	0.35	0.7
3	0.50	0.55	1.7
4	0.60	0.69	3.0
5	0.67	0.81	4.5

Table 1: Velocity (β) and rapidity (η) limits of a fireball at rest in the center of mass, and its proper hadronisatin radius r, as given by eqs. 9 and 10, for a formation time $\tau_q = 1$ fm and different hadronisation times τ_h.

Hadronization

$$\langle N_i \rangle = (2J_i + 1)\frac{V}{(2\pi)^3}\int d^3p \, \frac{1}{\gamma_s^{-s_i} \exp[(E_i - \mu \cdot q_i)/T_{ch}] \pm 1}$$

V = volume of the equivalent global cluster

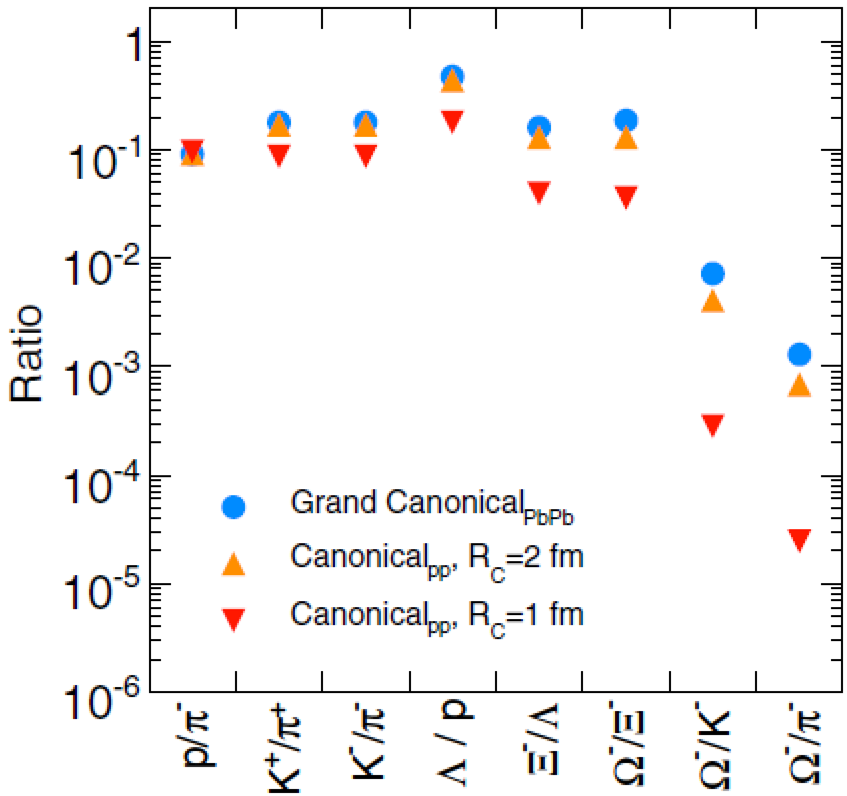
In elementary collisions, the clusters at rapidities sufficiently far apart are, as we have seen, causally disconnected, so that they cannot exchange information. Hence strangeness must be conserved locally; in pp collisions, for example, each cluster must have strangeness zero.

In high energy nuclear collisions the equivalent global cluster consists of the different clusters from the different nucleon-nucleon interactions at a common rapidity. At mid-rapiditiy, for example, we thus have the sum of the superimposed mid-rapidity clusters from the different nucleon-nucleon collisions, and these are all causally connected, allowing strangeness exchange and conservation between the different clusters

In elementary collisions, the clusters at rapidities sufficiently far apart are causally disconnected, so that they cannot exchange information. Hence strangeness must be conserved locally

→ **Correlation volume for strangeness**

Volume of the causally connected cluster



Statistical Thermodynamics in Relativistic Particle and Ion Physics: Canonical or Grand Canonical?

R. Hagedorn and K. Redlich Z. Phys. C - Particles and Fields 27, 541-551 (1985)

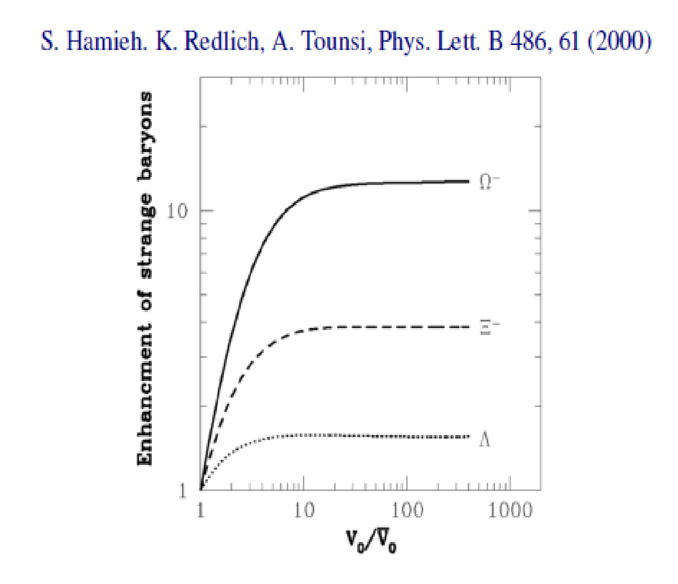
Canonical aspects of strangeness enhancement

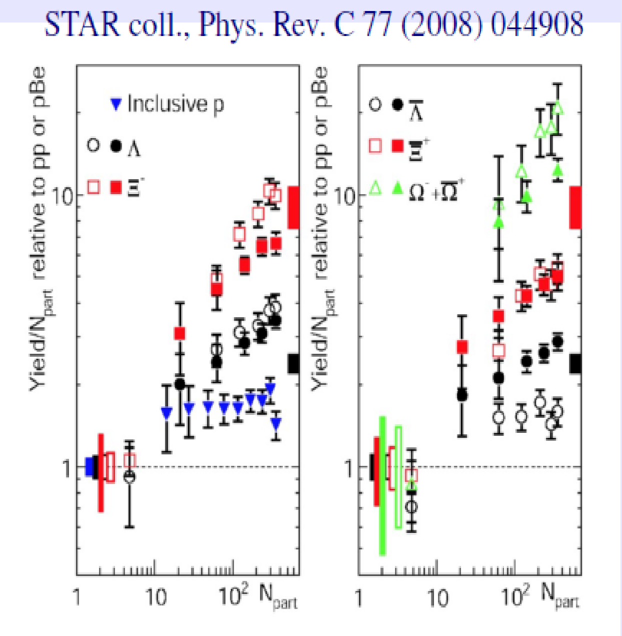
A. Tounsia, A. Mischkeb and K. Redlich hep-ph/0209284

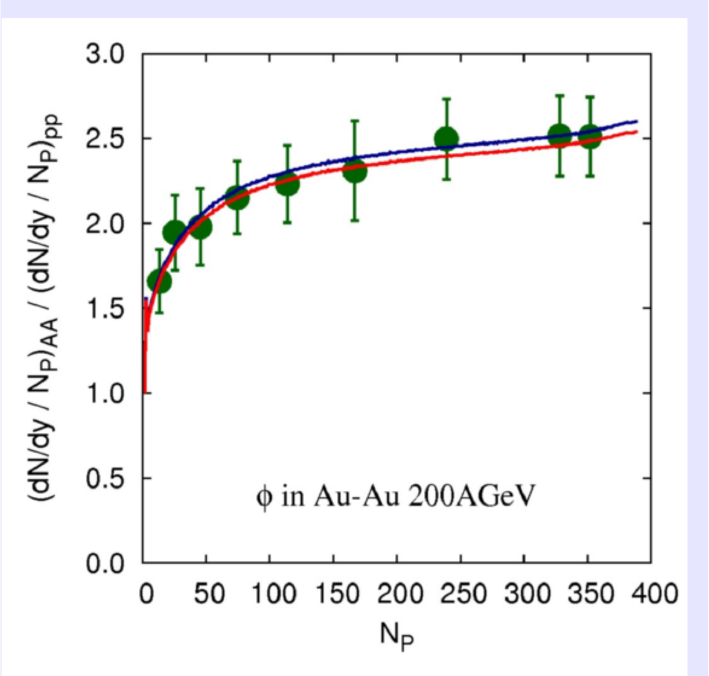
I Kraus, J Cleymans, H Oeschler and K Redlich

J. Phys. G: Nucl. Part. Phys. **37** (2010) 094021

Canonical suppression could be not enough…





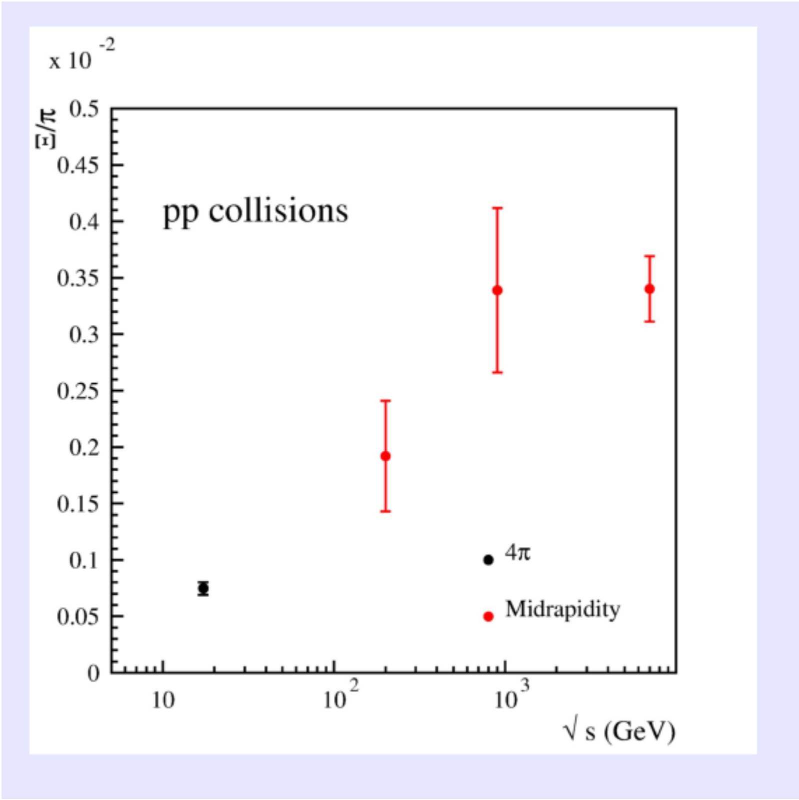


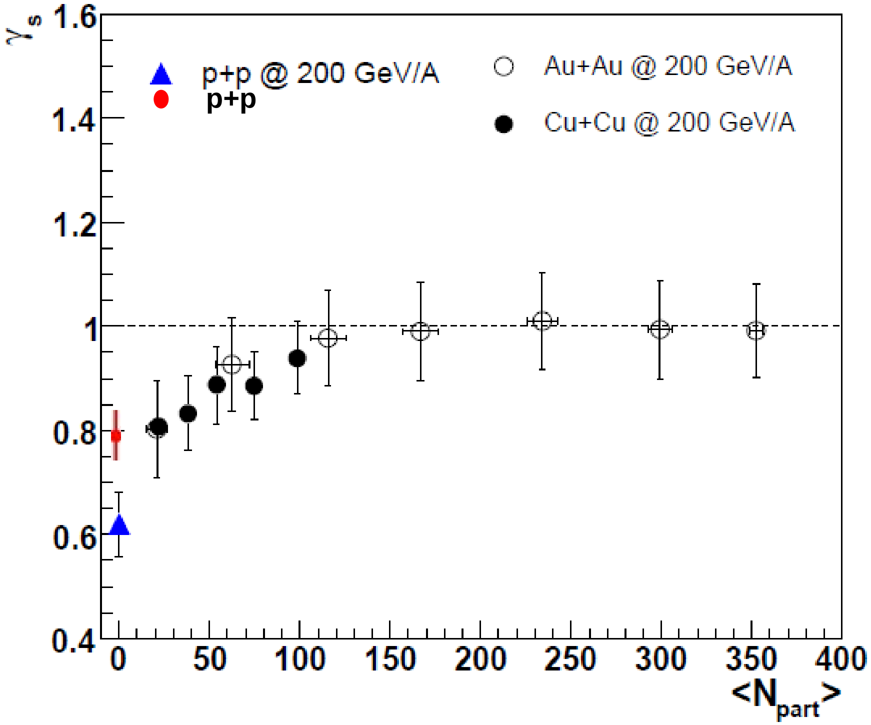
ϕ meson

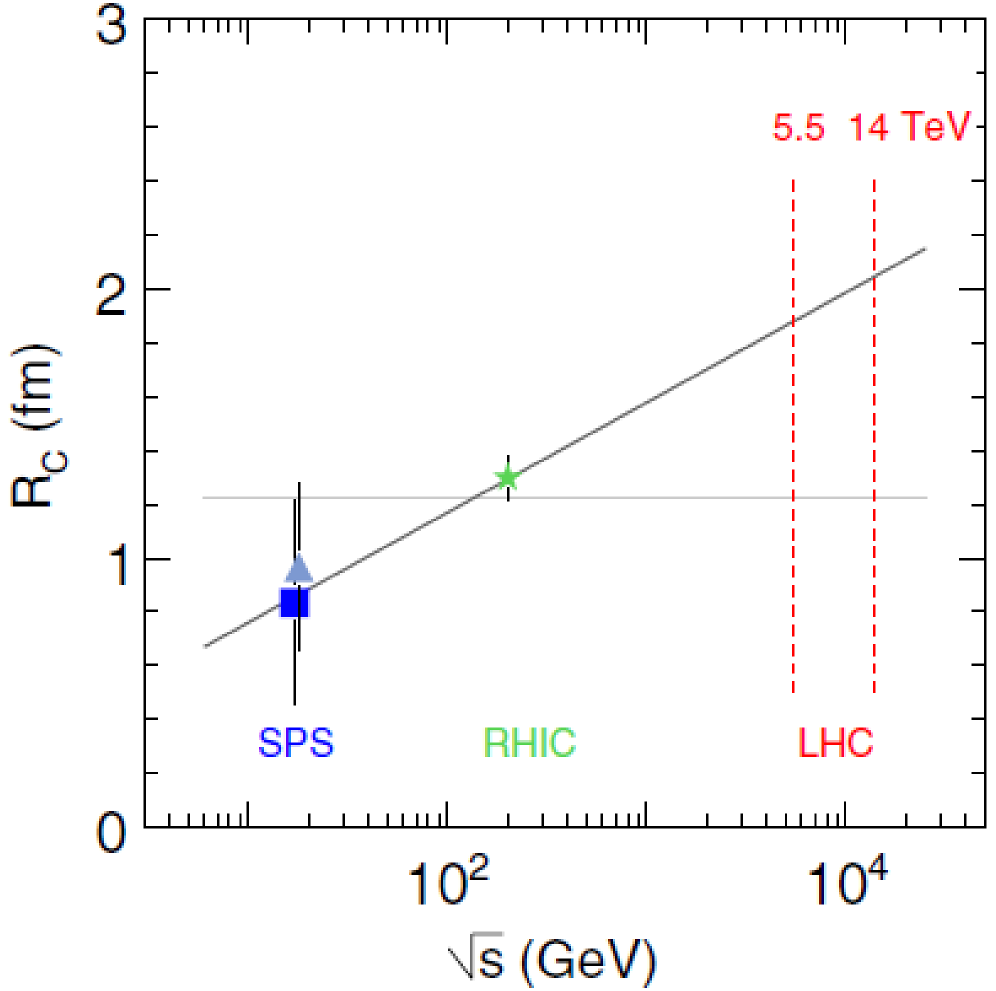
no canonical suppression

…. but it is in the data

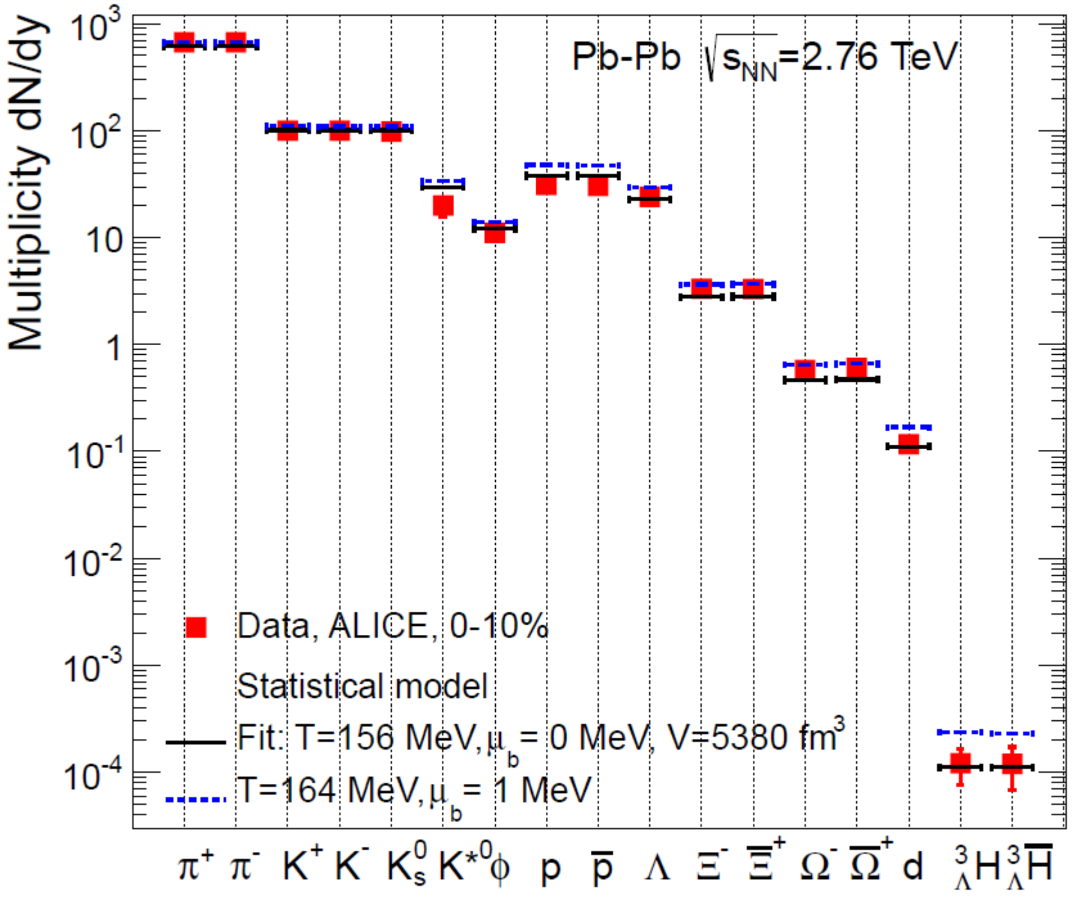
If in pp the energy is large enough to produce fireball with a large number of particles the volume increases and the canonical suppression decreases



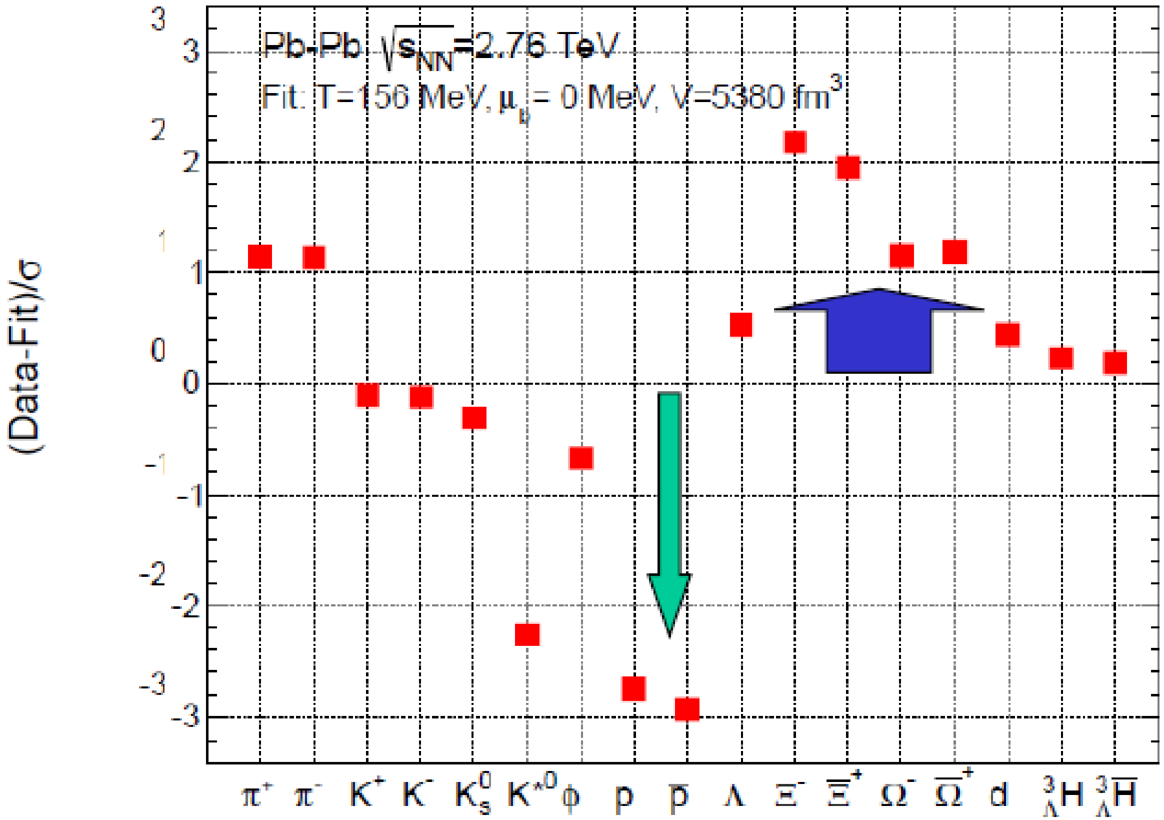



3
2
1
0
R_C (fm)
5.5 14 TeV
SPS
RHIC
LHC
10^2
10^4
$\sqrt{s}$ (GeV)

Fit statistical hadronization at LHC



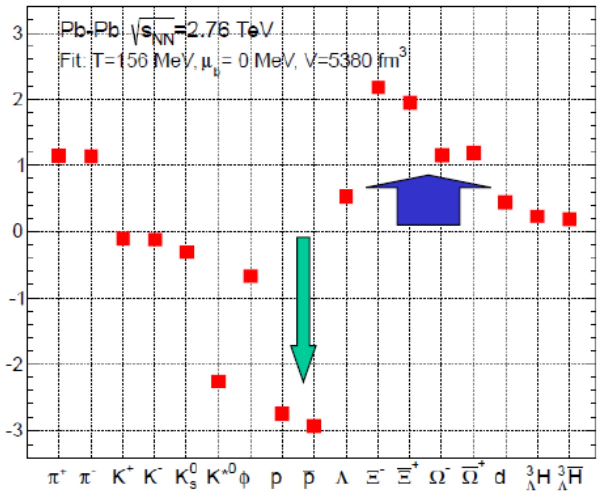
J Stachel, A Andronic, P Braun-Munzinger, K Redlich arxiv:1311.4662

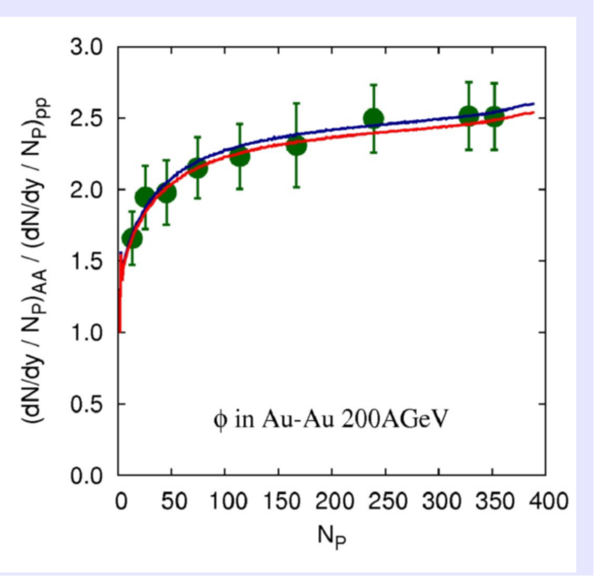


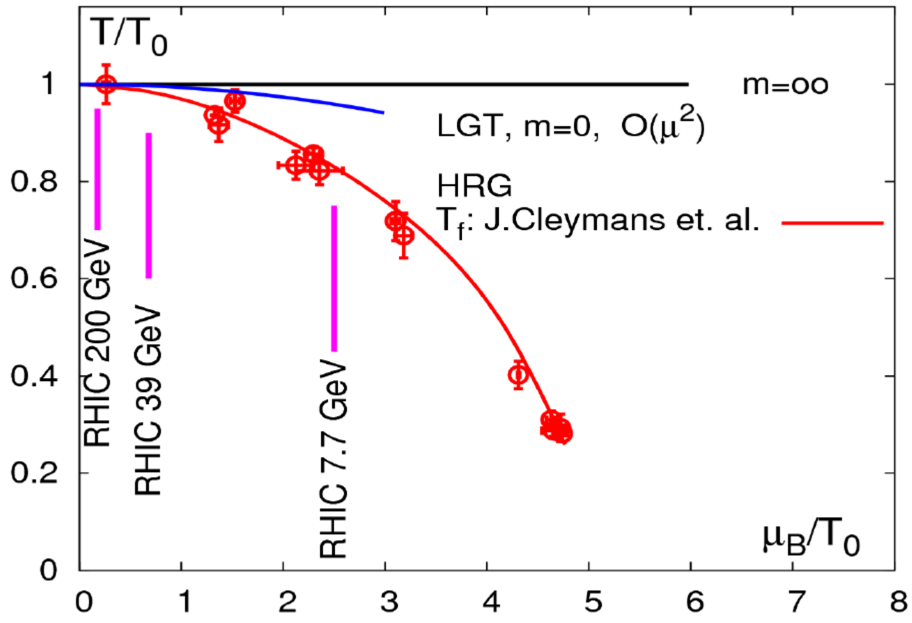
Pb-Pb $\sqrt{s_{NN}}$=2.76 TeV
Fit: T=156 MeV, μ_b = 0 MeV, V=5380 fm^3
(Data-Fit)/σ
3
2
1
0
-1
-2
-3
π^+ π^- K^+ K^- K^0_s K^{*0} φ p $\bar{p}$ Λ Ξ^- $\overline{\Xi}^+$ Ω^- $\overline{\Omega}^+$ d $^3_\Lambda H$ $^3_\Lambda \overline{H}$

φ meson

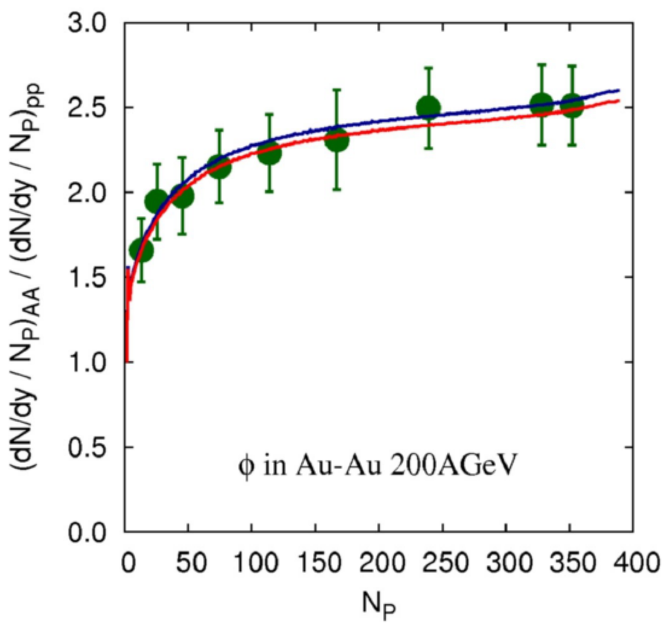
proton/pion







Extrapolated T-mu critical line is flatter than the CFO curve



Puzzles: Phi and Proton/pion ratio at LHC

ϕ meson

Succesfully described by the core-corona model (superposition of fully equilibrated core $\gamma_s=1$ and single NN collisions with $\gamma_s<1$)

F.B., J. Manninen, J. Phys. G. 35 (2008) 104013;
Phys. Lett. B 673 (2009) 19
J. Aichelin, K. Werner, Phys. Rev. C 79 (2009) 064907

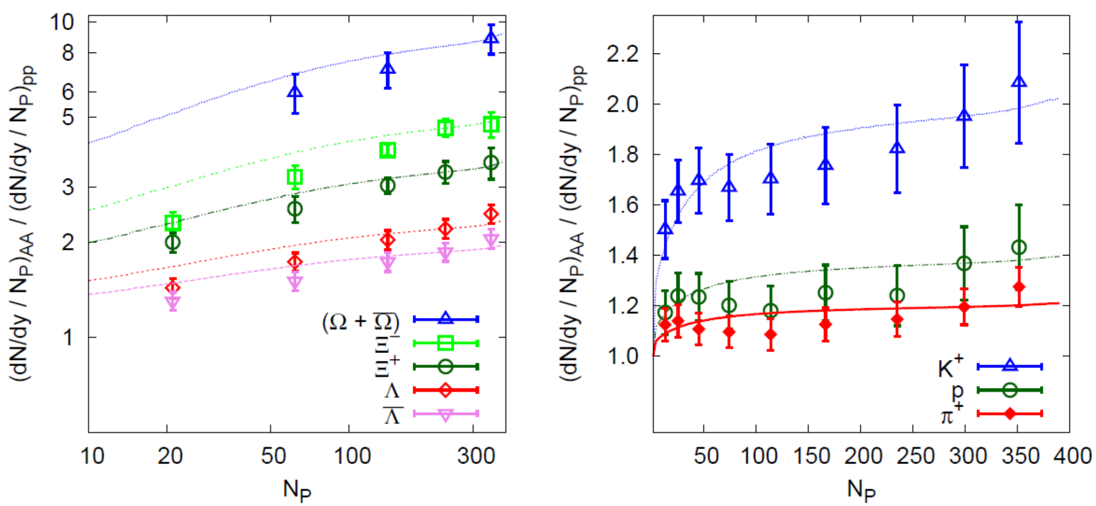
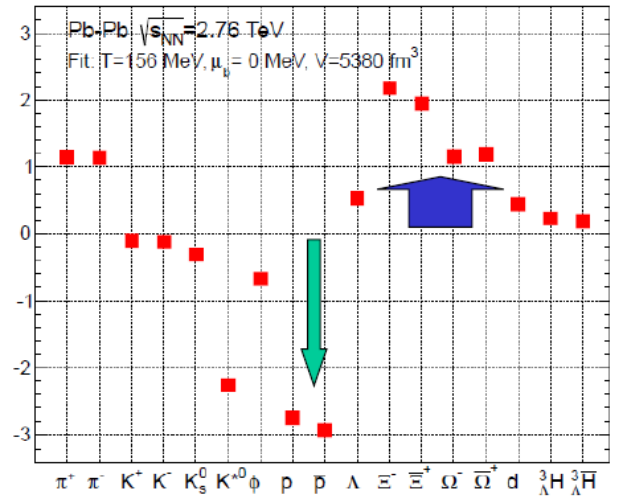


FIG. 3: **LEFT**: Ratio R_A (see text for definition) for hyperons measured in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The data points are from STAR [17] while the lines are calculated according to the Eq. (8) by fixing A in the most central bin.
RIGHT: The same quantity R_A for π^+, K^+ and p in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The data points are from STAR [25] while the lines are calculated according to the Eq. (8) by fixing A to the 2nd most central bin.

On the proton/pion puzzle

The proton and antiproton yields are under the statistical hadronization model by about 3 sigma



- incomplete hadronic spectrum in the statistical hadronization model
  - ***states above 3 GeV in mass (see later)***
  - ***lattice QCD simulations predict that numerous additional baryon resonances exist at low masses, partly with high spin and therefore degeneracy***

J Stachel, A Andronic, P Braun-Munzinger, K Redlich arxiv:1311.4662

Hagedorn states

At a temperature about 160 Mev, one populates the hadrons using multihadronic decay reactions driven through Hagedorn states.

$$\rho = Ae^{m/T_H}$$

$$n\pi \quad \leftrightarrow \quad HS \leftrightarrow n'\pi + X\bar{X}. \qquad X\bar{X} = p\bar{p}, K\bar{K}, \text{or } \Lambda\bar{\Lambda}.$$

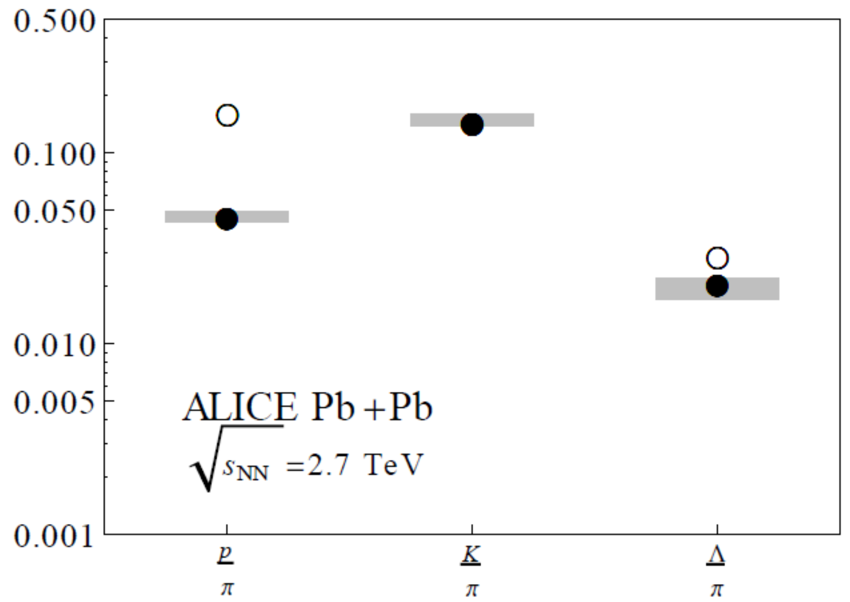


Figure 2. Comparison of the results of our extended mass spectrum model particle ratio calculations vs. experimental data points of ALICE at Pb+Pb LHC $\sqrt{s_{NN}} = 2.76$ TeV.

Nuclear Physics A 00 (2014) Jacquelyn Noronha-Hostler, Carsten Greiner

The solid black dots represent the situation where there are no initial protons, kaons, and lambdas in our system (while the pions and Hagedorn states begin in chemical equilibrium) whereas the outlined circles represent the scenario when all hadrons begin in chemical equilibrium

The Legacy of Rolf Hagedorn: Statistical Bootstrap and Ultimate Temperature – K.Redlich and H.Satz **arxiv: 1501.07523**

Hagedorn states

J. Noronha-Hostler, M. Beitel, C. Greiner, and I. Shovkovy arxiv : 0909.2908

relativistic particle *i* enclosed in volume $dN_i = 2m_i V \frac{d^4 p_i}{(2\pi)^3}\delta(p_i^2 - m_i^2)$

$$d\tilde{N}_i = 2m_i dm_i \tau_{\vec{C}_i}(m_i) V \frac{d^4 p_i}{(2\pi)^3}\delta(p_i^2 - m_i^2)$$

the mass degeneration → particle i carrying the quantum numbers denoted by $\vec{C}_i = (B_i, S_i, Q_i)$ with four-momentum between p_i and $p_i + dp_i$ might also take on different masses, whose distribution is given by the function $\tau_{\vec{C}_i}$.

Hagedorn just imagined that a heavy particle was somehow composed out of lighter ones, and these again in turn of still lighter ones, and so on, until one reached the pion as the lightest hadron. By combining heavy ones, you would get still heavier ones, again: and so on. The crucial input was that the composition law should be the same at each stage. Today we call that self-similarity

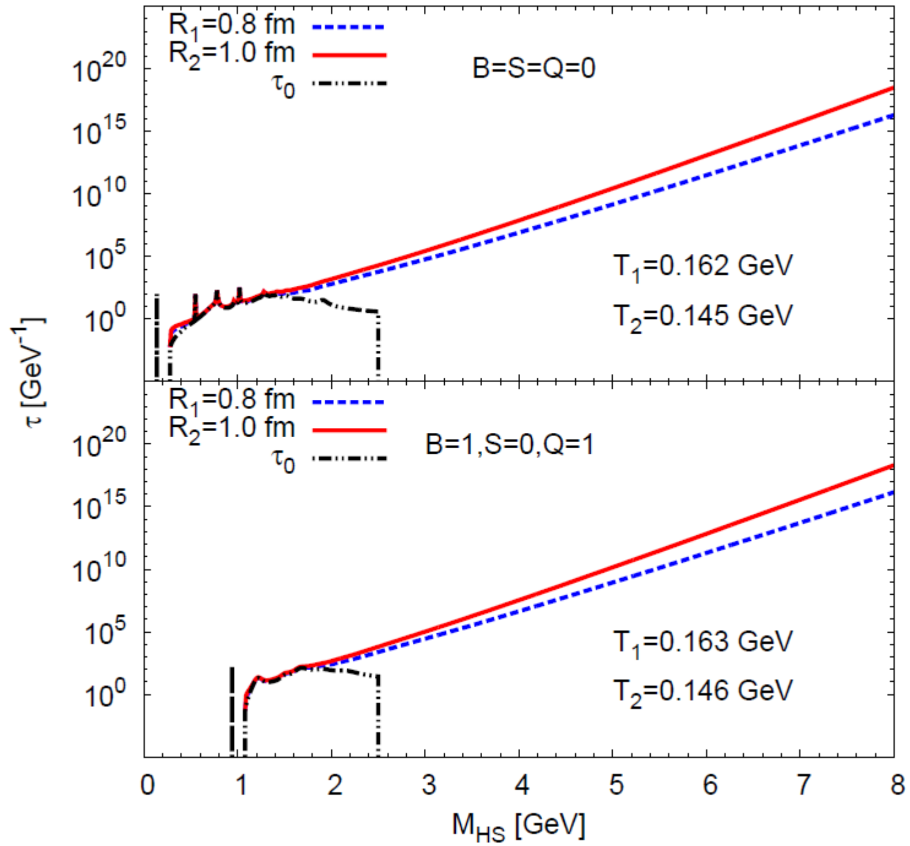
→ Bootstrap equation

M.Beitel,K.Gallmeister and C.Greiner PRC 90 045203 (2014)

$$\tau_{\vec{C}}(m) = \frac{R^3}{3\pi m} \sum_{\vec{C}_1,\vec{C}_2} \iint dm_1 dm_2\, \tau_{\vec{C}_1}(m_1)\, m_1 \times \tau_{\vec{C}_2}(m_2)\, m_2\, p_{cm}(m,m_1,m_2)\delta_{\vec{C},\vec{C}_1+\vec{C}_2},$$

$$p_{cm}(m,m_1,m_2) = \frac{1}{2m}\sqrt{\left(m^2 - m_1^2 - m_2^2\right)^2 - 4m_1^2 m_2^2},$$

R = radius of the hagedorn state



$$\tau_{\rm fit}(m) = A m^{-b} \exp(m/T_H)$$

$$T_H = 0.145 \text{ GeV for } R = 1.0 \text{ fm}$$

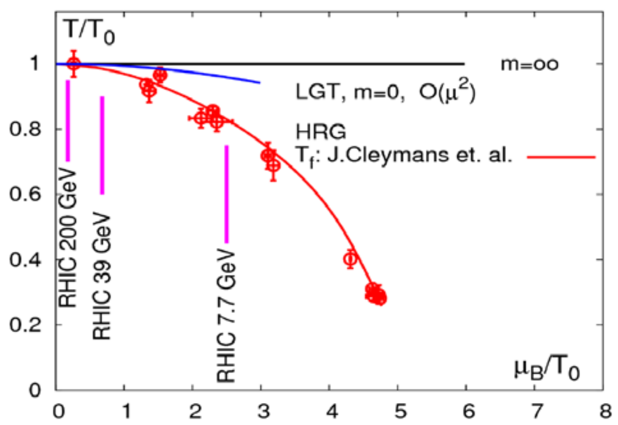
$$T_H = 0.162 \text{ GeV for } R = 0.8 \text{ fm}.$$

Inelastic reactions between hadrons occurring after hadronization at high energy

Hadron Formation in Relativistic Nuclear Collisions and the QCD Phase Diagram

Francesco Becattini, Marcus Bleicher, Thorsten Kollegger,Tim Schuster, Jan Steinheimer and Reinhard Stock

Phys. Rev. Lett. 111 (2013) 082302

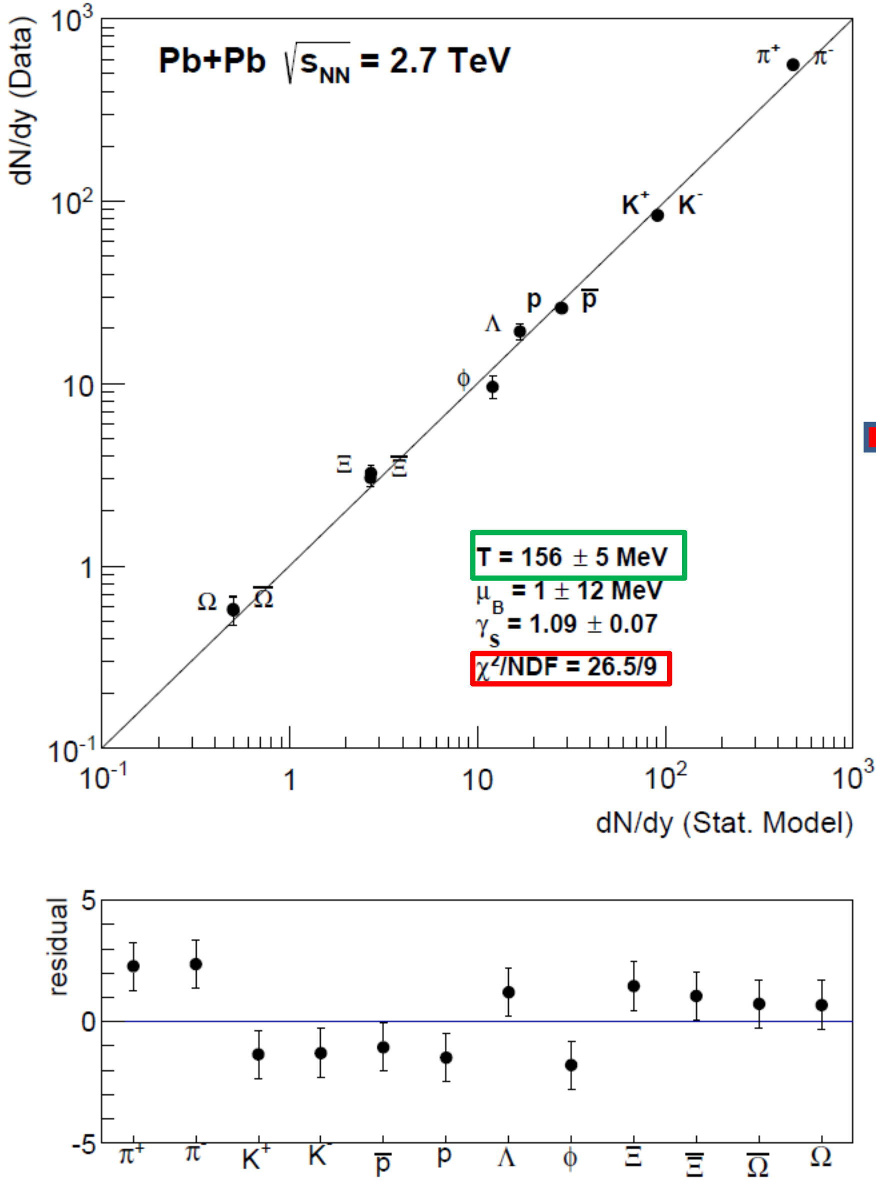


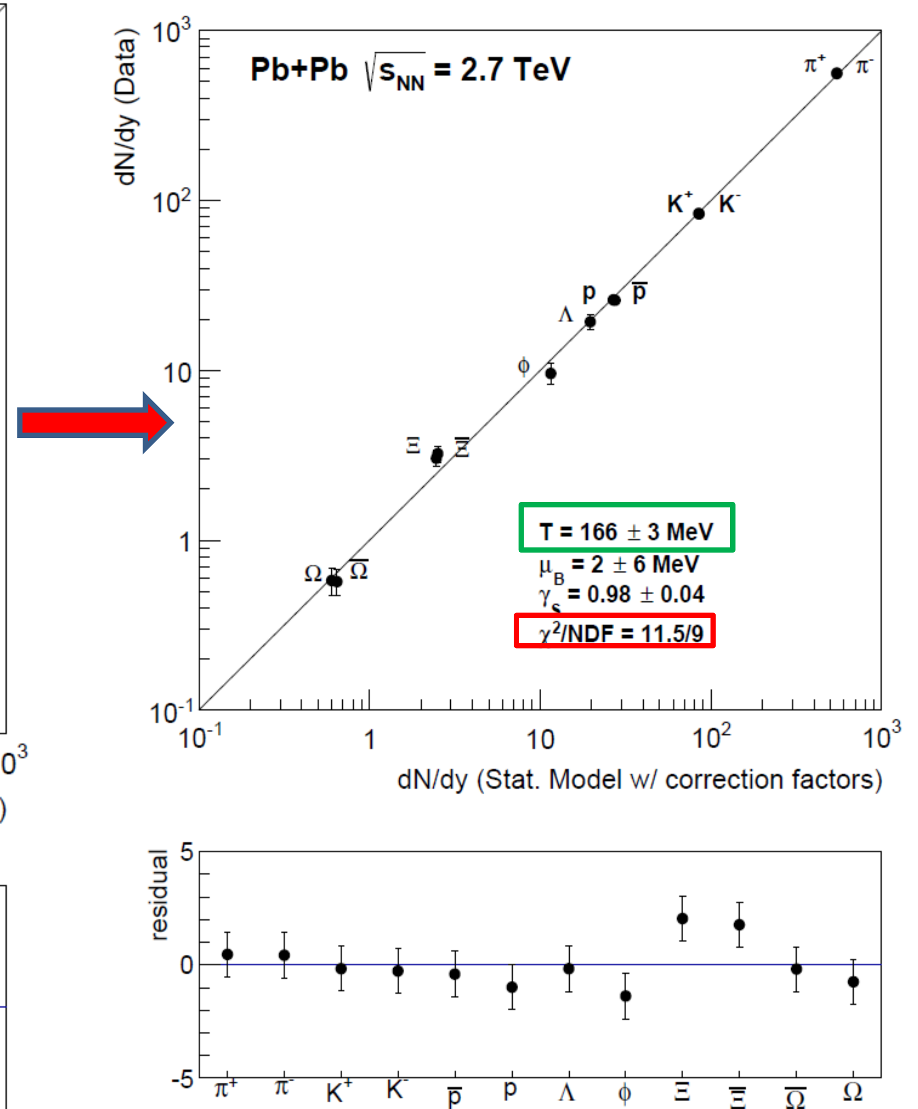
All previous determinations of its points in the (T, μ_B) plane, in the framework of SHM, have implicitly assumed that the primordial hadro-chemical equilibrium remains frozen-in throughout the final expansion phase.

The baryon-antibaryon annihilation and regeneration processes do not fall away with the onset of expansive cooling. Their final effect consists of a considerable distortion of the initial, post-hadronization equilibrium yield distribution, in the antibaryon and baryon sector which affects the outcome of the SHM analysis.

M.Bleicher
lec.1+2

The UrQMD (microscopic simulations that include inelastic processes) hadron/resonance cascade expansion stage is attached, as an “afterburner”. The outcome is again fitted by the SHM.





UrQMD study of the expansion phase effects.

Taking account of the yield changes in the baryon-antibaryon sector by applying UrQMDderived “survival factors” for each species, as a modification of the SM analysis, one recovers the primordial hadronization points.

	T (MeV)	μ_B (MeV)	γ_S	χ^2/NDF
Pb-Pb 20% central $\sqrt{s_{NN}} = 2.7$ TeV				
Std. fit	156 ± 5	1 ± 12	1.09 ± 0.07	$26.5/9$
Mod. fit	166 ± 3	2 ± 6	0.98 ± 0.04	$11.5/9$
Pb-Pb 5% central $\sqrt{s_{NN}} = 17.3$ GeV				
Std. fit	151 ± 4	266 ± 9	0.91 ± 0.05	$26.9/11$
Mod. fit	163 ± 4	250 ± 9	0.83 ± 0.04	$20.4/11$
Pb-Pb 5% central $\sqrt{s_{NN}} = 8.7$ GeV				
Std. fit	148 ± 4	385 ± 11	0.78 ± 0.06	$17.9/9$
Mod. fit	161 ± 6	376 ± 15	0.72 ± 0.06	$25.9/9$
Pb-Pb 5% central $\sqrt{s_{NN}} = 7.6$ GeV				
Std. fit	140 ± 1	437 ± 5	0.91 ± 0.01	$22.4/7$
Mod. fit	156 ± 5	426 ± 4	0.81 ± 0.00	$14.7/7$

Table 1: Results of the statistical model fits to LHC and SPS data.

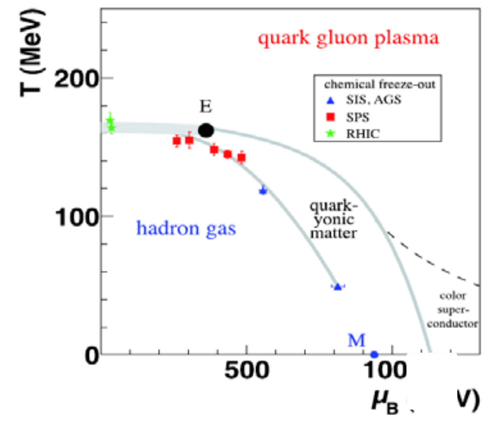


FIG. 1: (Color online) Sketch of the QCD phase diagram, including the hadronic freeze-out curve (see text).

However

Y.Pan ans S.Pratt PHYSICAL REVIEW C **89**, 044911 (2014)

regeneration

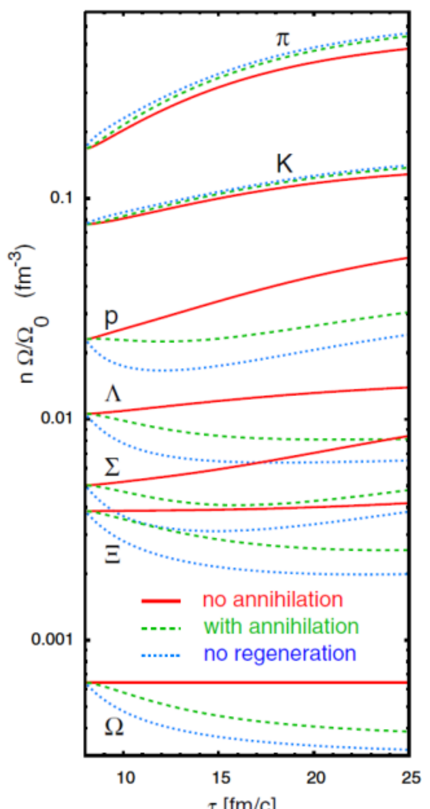


FIG. 1. (Color online) Hadronic densities, scaled by the volume $\Omega(\tau)/\Omega(\tau_0)$, for the π and K mesons, and for the p, Λ, Σ, Ξ, and Ω baryons. In the absence of annihilation, the proton yield increases due to the decay of resonances like the Δ. By adding annihilation, all baryon yields fall by $\sim$40%. Meson yields are modestly increased by the annihilation processes. If regeneration processes are ignored, the fraction of baryons that are annihilated increases to $\sim$50%.

1) ***Why do elementary high energy collisions***
show a statistical behavior?

2) ***Why is strangeness production universally suppressed***
in elementary collisions?

3) ***Why (almost) no strangeness suppression in nuclear collisions?***

4) ***Why hadron freeze-out for s/T^3 = 7 or E/N=1.08 Gev***

5) ***Why thermalization in so short time (0.5- 1 fm/c)***

Born in equilibrium ?

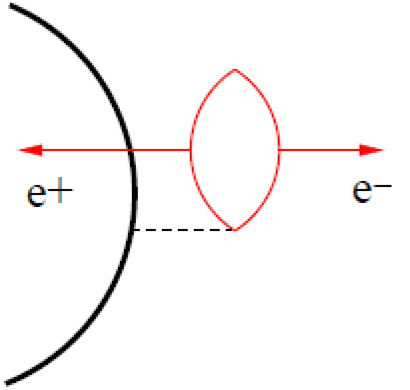
Is there another non-kinetic mechanism providing a common origin of the statistical features?

Recall

In Classical Black-hole particles are confined $\Rightarrow$ event horizon $\Rightarrow$ no communication with outside, but...Hawking radiation [Hawking 1975]

Quantum effect $\sim$ uncertainty principle $\longrightarrow$ vacuum fluctuation e^+e^- outside event horizon, with $\Delta E \Delta t \sim 1$ .If e^+ falls into black hole, then e^- can escape; equivalent:

e^- tunnels through event horizon



There is no information about state of system beyond event horizon; e^+ on one side, e^- on the other

$\Rightarrow$ Hawking radiation must be thermal

$$\frac{dN}{dk} \sim \exp\{-\frac{k}{T_{BH}}\}$$

with black hole temperature

$$T_{BH} = \frac{\hbar}{8\pi c GM}$$

relativistic quantum effect: disappears for $\hbar \to 0$ or $c \to \infty$

$\Rightarrow$ tunnelling through event horizon $\to$ thermal radiation

P.C. lect. 1+2
Kaempfer lect.1 +2

- Unruh relation [Unruh 1976]

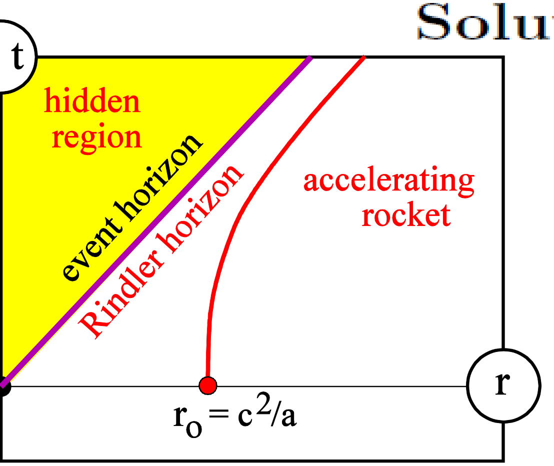
Event horizon arises for systems in uniform acceleration. For a mass m in uniform acceleration a

Rindler observer

$$\frac{d}{dt}\frac{mv}{\sqrt{1-v^2}} = F$$

where $v = dx/dt$, $F = ma$,

Solution: hyperbolic motion



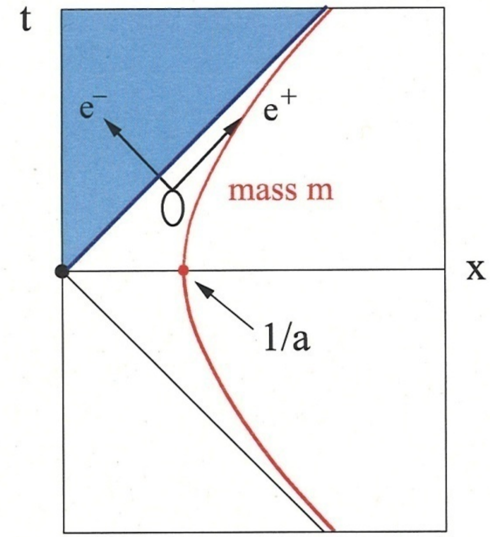
$$x = \frac{1}{a}\cosh a\tau$$

$$t = \frac{1}{a}\sinh a\tau$$

arXiv:0710.5373
The Unruh effect and its applications
Luis C. B. Crispino, Atsushi Higuchi, George E. A. Matsa

e^+ absorbed in detector on m

e^- disappears beyond event horizon



observer on m & observer in hidden region have incomplete information: $\Rightarrow$ each sees thermal radiation of

Unruh temperature $$T_U = \frac{\hbar a}{2\pi c} = \frac{\hbar F}{2m\pi c}$$

Uniform acceleration

Event Horizon

Universal thermal behavior

In QCD ?

Confinement $V \to \sigma r$

Physical vacuum ⟺ **Event horizon for colored constituents**

Thermal hadron production ⟺ **Hawking-Unruh radiation in QCD**

P.C., D.Kharzeev and H.Satz -- D.Kharzeev and Y.Tuchin (temperature)

Eur.Phys.J. C52 (2007) 187-201 **Nucl. Phys. A 753, 316 (2005)**

F.Becattini, P.C., J.Manninen and H.Satz (strangeness suppression in e+e-)

Eur.Phys.J. C56 (2008) 493-510

P.C. and H.Satz (strangeness enhancement in heavy ion collisions)

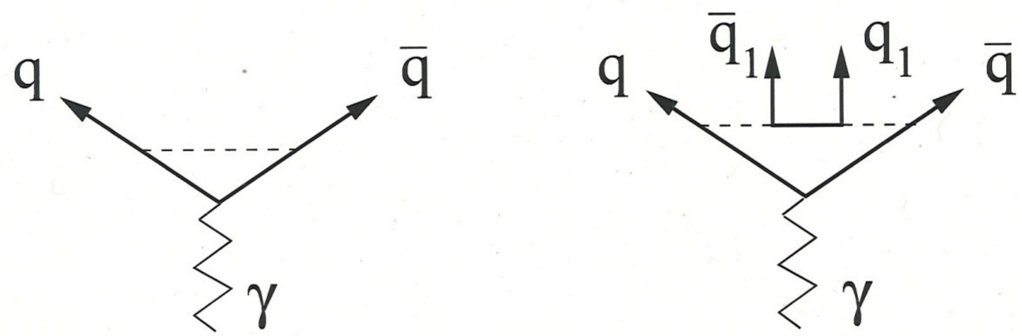
Adv.High Energy Phys. 2014 (2014) 376982

P.C., A. Iorio and H.Satz (entropy and freeze-out)

arXiv:1409.3104

Hadron production in e^+e^- annihilation:

"inside-outside cascade" [Bjorken 1976]



$q\bar{q}$ flux tube has thickness $r_T \simeq \sqrt{\dfrac{2}{\pi\sigma}}$

$q_1\bar{q}_1$ at rest in cms, but $k_T \simeq \dfrac{1}{r_T} \simeq \sqrt{\dfrac{\pi\sigma}{2}}$

$q\bar{q}$ separation at $q_1\bar{q}_1$ production $\sigma x(q\bar{q}) = 2\sqrt{m^2 + k_T^2}$

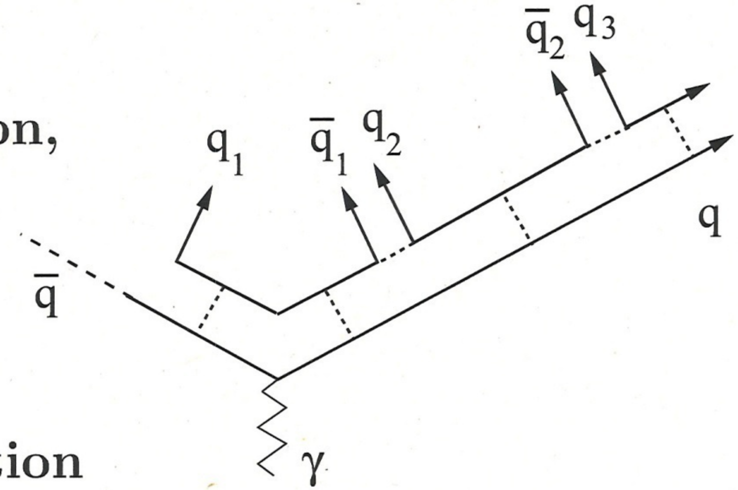
q_1 screens $\bar{q}$ from q, hence string breaking at

$$x_q \simeq \frac{2}{\sigma}\sqrt{m^2 + (\pi\sigma/2)} \simeq \sqrt{2\pi/\sigma} \simeq 1 \text{ fm}$$

new flux tubes $q\bar{q}_1$ and $\bar{q}q_1$
stretch $q_1\bar{q}_1$
to form new pair $q_2\bar{q}_2$

$$\sigma x(q_1\bar{q}_1) = 2\sqrt{m^2 + k_T^2}$$

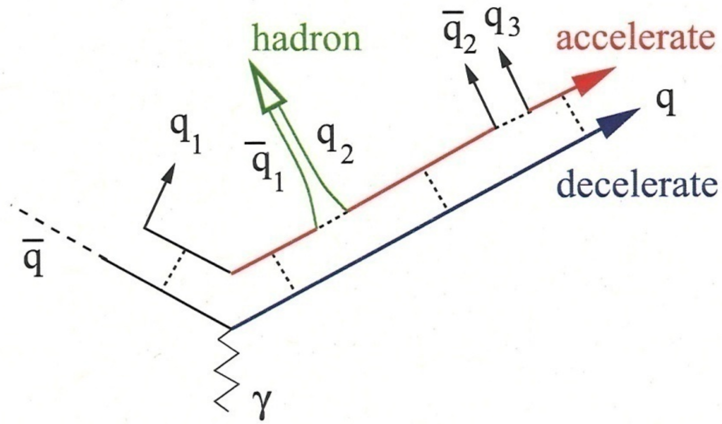
equivalent:
$\bar{q}_1$ reaches $q_1\bar{q}_1$ event horizon,
tunnels to become $\bar{q}_2$

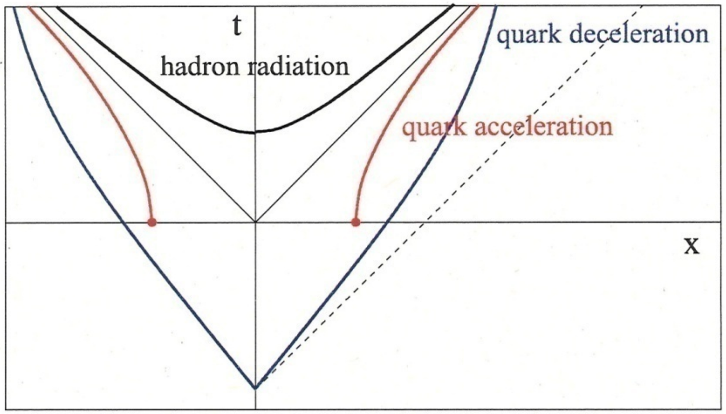


emission of hadron $\bar{q}_1 q_2$
as Hawking radiation

self-similar pattern:

- **screening**
- **string breaking**
- **tunnelling**
- **quark acceleration /deceleration**
- **Hawking radiation**





temperature of Hawking radiation: what acceleration?

$(\bar{q}_1 \to \bar{q}_2 \to \bar{q}_3 \to \ldots)$

$$a = F/m \;\Rightarrow\; a_q = \frac{\sigma}{w_q} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}}$$

string breaking & thickness determine $k_q \simeq \sqrt{\pi\sigma/2}$

$$\Rightarrow \quad a_q \simeq \frac{\sigma}{\sqrt{m_q^2 + (\sigma/2\pi)}}$$

for light quarks, $m_q \ll \sqrt{\sigma} \simeq 420$ MeV, hence

$$T = \frac{a}{2\pi} \simeq \sqrt{\frac{\sigma}{2\pi}} \simeq 170 \text{ MeV}$$

temperature of hadronic Hawking-Unruh radiation in QCD

1) ***Why do elementary high energy collisions* show a statistical behavior?**

2***) Why is strangeness production universally suppressed* in elementary collisions?**

3) ***Why (almost) no strangeness suppression in nuclear collisions?***

4) ***Why hadron freeze-out for s/T^3 = 7 or E/N=1.08 Gev***

5) ***Why thermalization in so short time (0.5- 1 fm/c)***

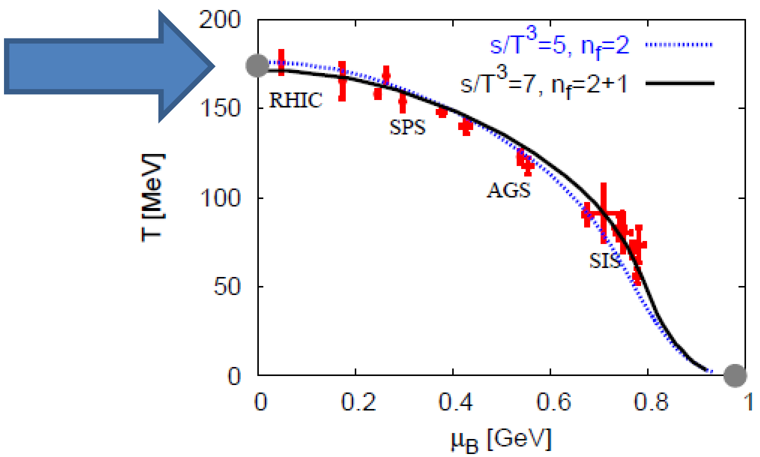
String breaking and E/N = 1.08 Gev

The energy of the pair produced by string breaking, i.e., of the newly formed hadron, is given by

$$E_h = \sigma R = \sqrt{2\pi\sigma}.$$

In the central rapidity region of high energy collisions, one has $\mu \simeq 0$, so that E_h is in fact the average energy $\langle E \rangle$ per hadron, with an average number $\langle N \rangle$ of newly produced hadrons. Hence we obtain

$$\frac{\langle E \rangle}{\langle N \rangle} = \sqrt{2\pi\sigma} \simeq 1.09 \pm 0.08,$$



Probing the States of Matter in QCD

Strangeness production is a fundamental tool to understand thermalization and hadronization

All abundances in high energy collisions , including those of strange hadrons, are indeed given by an ideal resonance gas

We thus find that high energy heavy ion collisions produce a medium which can be considered as hadronic matter in equilibrium, formed at the pseudo-critical hadronization temperature predicted by lattice QCD.

We want to find in the collision data some sign of the QCD transition, of something like critical behavior. For example, sufficiently close to a continuous transition, correlations appear at all scales because the correlation length diverges, and in QCD this must produce strong deviations from the ideal hadron gas behavior.

study the fluctuactions of conserved quantum numbers in an ideal hadron gas and compared this to both lattice results and heavy ion data.

Y. Hatta and M. A. Stephanov, Phys. Rev. Lett. **91**, 102003 (2003).
M. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
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M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009).
M. A. Stephanov, Phys. Rev. Lett. **107**, 052301 (2011).
F. Karsch and K. Redlich, Phys. Lett. B **695**, 136 (2011).
A. Bazavov, H. T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee and P. Petreczky, Phys. Rev. Lett. **109**, 192302 (2012).

C. Ratti *et al.* [Wuppertal-Budapest Collaboration], Nucl. Phys. A **855**, 253 (2011).
S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, Phys. Rev. Lett. **113**, 052301 (2014).

Example: baryon number

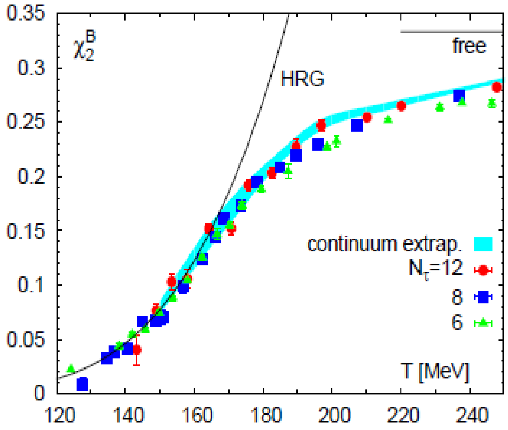
$$\frac{P(T,\mu)}{T^4} = \frac{1}{\pi^2}\sum_i d_i (m_i/T)^2 K_2(m_i/T)\cosh(B_i\mu/T),$$

$$\chi_B^{(n)}(T,\mu) = \frac{\partial^n (P/T^4)}{\partial(\mu/T)^n},$$

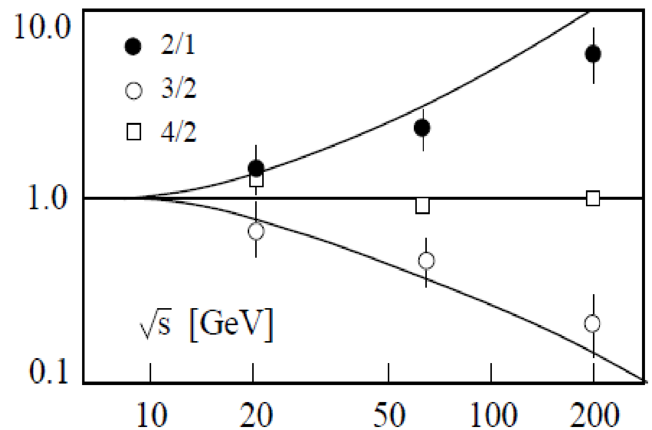
only baryons with $B = 1$ enter

$$\frac{\chi_B^{(3)}}{\chi_B^{(1)}} = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} = \frac{\chi_B^{(5)}}{\chi_B^{(3)}} = ... = 1$$

$$\frac{\chi_B^{(2)}}{\chi_B^{(1)}} = \coth(\mu/T),\ \frac{\chi_B^{(3)}}{\chi_B^{(2)}} = \tanh(\mu/T),\ \frac{\chi_B^{(4)}}{\chi_B^{(3)}} = \coth(\mu/T),$$



(a)



(b)

(a) Theory: lattice results (2+1 quark flavors, $\mu = 0$) for the second order baryon number cumulant, compared to the hadron resonance gas form. (b) Experiment: STAR data on baryon cumulant ratios, as function of collision energy $\sqrt{s}$, vs. hadron resonance gas predictions (solid lines).

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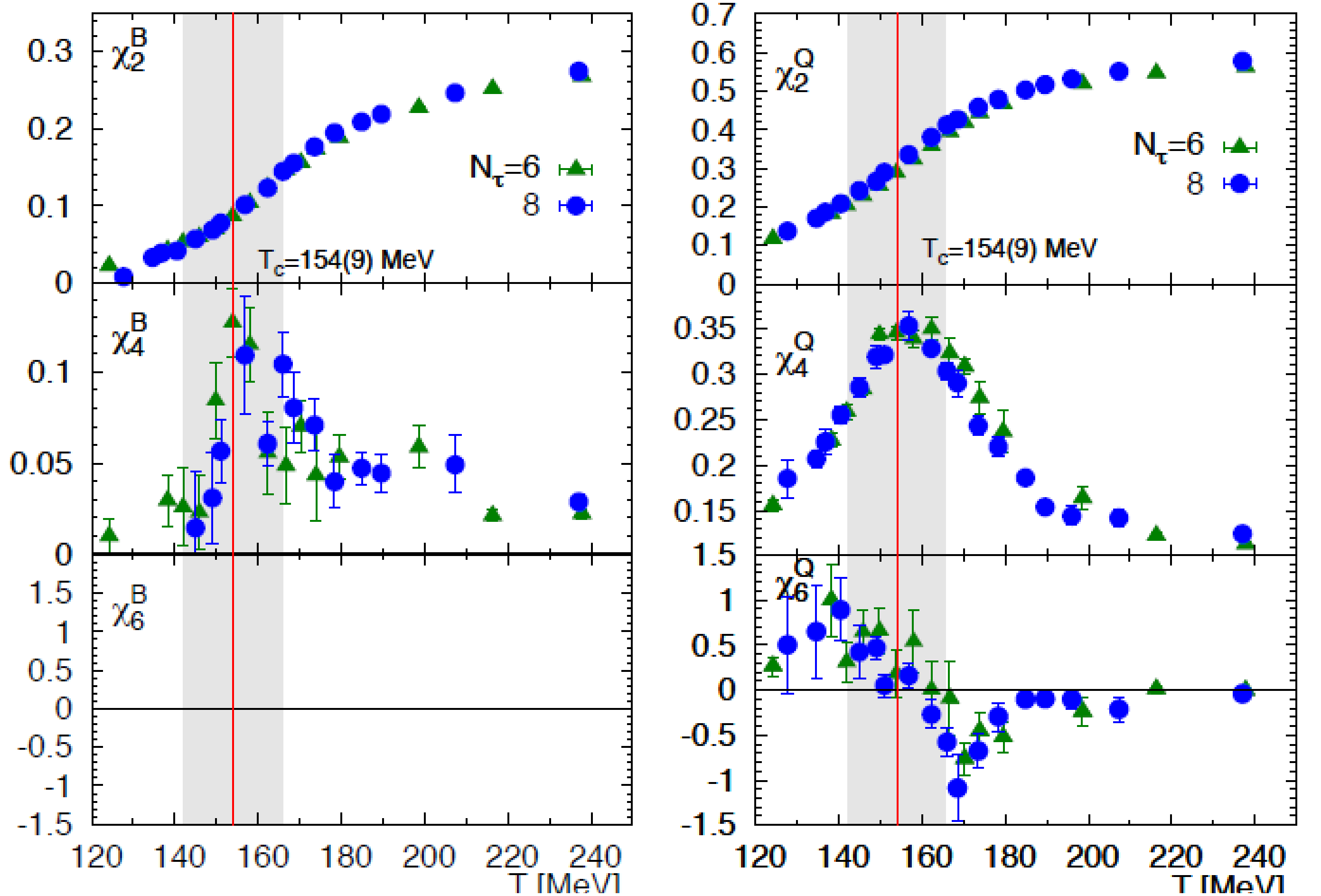
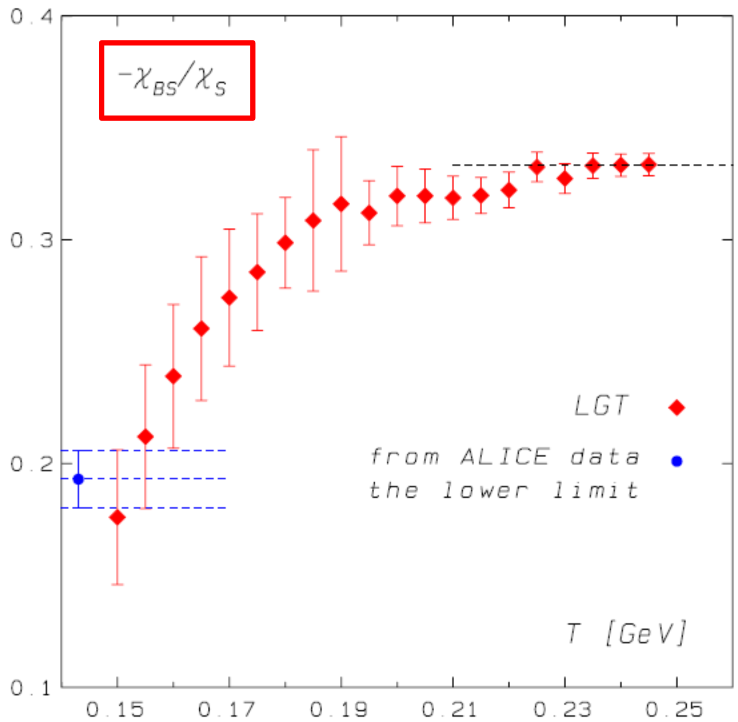


collisions. *E.g.*, for the diagonal fluctuations one obtains

$$
\begin{aligned}
\left(VT^3\right) \cdot \chi_2^X &= \left\langle (\delta N_X)^2 \right\rangle , \\
\left(VT^3\right) \cdot \chi_4^X &= \left\langle (\delta N_X)^4 \right\rangle - 3 \left\langle (\delta N_X)^2 \right\rangle^2 , \\
\left(VT^3\right) \cdot \chi_6^X &= \left\langle (\delta N_X)^6 \right\rangle - 15 \left\langle (\delta N_X)^4 \right\rangle \left\langle (\delta N_X)^2 \right\rangle + 30 \left\langle (\delta N_X)^2 \right\rangle^3 ,
\end{aligned}
$$

with $\delta N_X = N_X - \langle N_X \rangle$



$$\hat{\chi}_{NM} \equiv \frac{\chi_{NM}}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N \partial \hat{\mu}_M}$$

$$\hat{\chi}_N \equiv \frac{\chi_N}{T^2} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_N^2},$$

$$\hat{\chi}_N = \frac{1}{VT^3}(\langle N^2\rangle - \langle N\rangle^2).$$

Skellam distribution,

$$\hat{\chi}_N = \frac{\chi_N}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2(\langle N_n\rangle + \langle N_{-n}\rangle),$$

The LQCD results on temperature dependent ratio of baryon-strangeness correlation χ_{BS} and strangeness susceptibility from Ref. [24]. Also shown is a band for the lower limit on this ratio extracted from ALICE data from Eqs. (12) and (16). The horizontal line at high-T is an ideal gas value in a QGP.

P. Braun-Munzinger, A. Kalweit, K. Redlich and J. Stachel arxiv 1412.8614

Confronting fluctuations of conserved charges in central nuclear collisions at the LHC with predictions from Lattice QCD

Conclusions

- nuclear collisions at low baryon density produce a hadronic medium in thermal equilibrium at the confinement temperature found in lattice QCD;
- the critical behavior at the hadronization transition is encoded in fluctuations calculated in QCD, and these are in principle measurable for baryon number, charge and strangeness;
- the universal temperature at high energy suggests a universal mechanism underlying the Hadronization – a là Hawking-Unruh – where hadrons are born in equilibrium