JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Lattice-QCD Studies of the H-Dibaryon

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Strangeness in Quark Matter

Joint Institute for Nuclear Research,Dubna, 6 July 2015

HELMHOLTZ ASSOCIATION Helmholtz Institute Mainz

PRISMA

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Introduction - The H-Dibaryon

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Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the Δ(1236) bind six quarks to form a stable, flavor-singlet (with strangeness of −2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in ΛΛ invariant-mass plots. Production and decay systematics of the H are discussed.

- MIT bag model predicts di-hyperon state (*H*) with

$$I = 0,\ S = -2,\ J^P = 0^+$$

and a mass of $m_H = 2150$ MeV

- H-Dibaryon must decay weakly

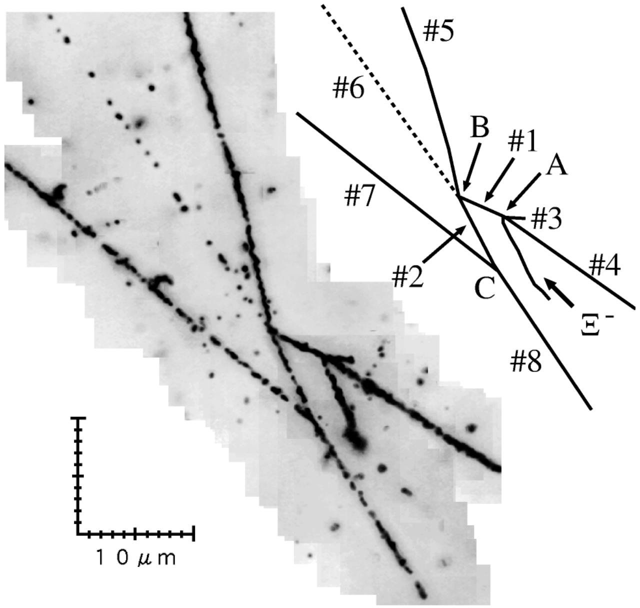
Experimental Searches

VOLUME 87, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 2001

Observation of a $^{\;6}_{\Lambda\Lambda}$He Double Hypernucleus (E373@KEK):

A double-hyperfragment event has been found in a hybrid-emulsion experiment. It is identified uniquely as the sequential decay of $^{\;6}_{\Lambda\Lambda}$He emitted from a Ξ^- hyperon nuclear capture at rest. The mass of $^{\;6}_{\Lambda\Lambda}$He and the Λ-Λ interaction energy $\Delta B_{\Lambda\Lambda}$ have been measured for the first time devoid of the ambiguities due to the possibilities of excited states. The value of $\Delta B_{\Lambda\Lambda}$ is $1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV. This demonstrates that the Λ-Λ interaction is weakly attractive.

“Nagara” event



Observation of a $^{\;6}_{\Lambda\Lambda}$He double-hypernucleus

Binding energy:

$$B_{\Lambda\Lambda} = 6.91 \pm 0.16\,\mathrm{MeV}$$

Absence of a strong decay $^{\;6}_{\Lambda\Lambda}\mathrm{He} \longrightarrow {}^4\mathrm{He} + H$

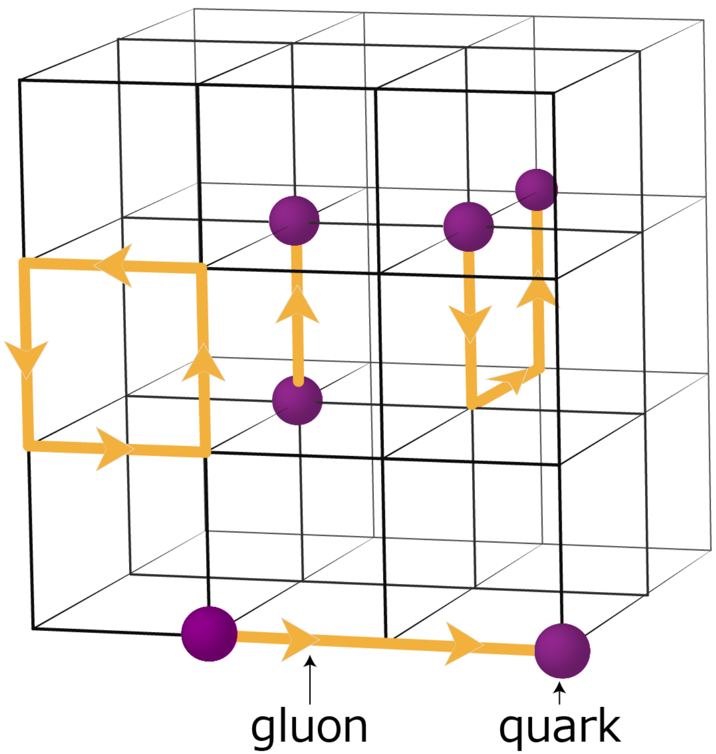
$$\Longrightarrow \quad m_H > 2m_\Lambda - B_{\Lambda\Lambda}$$

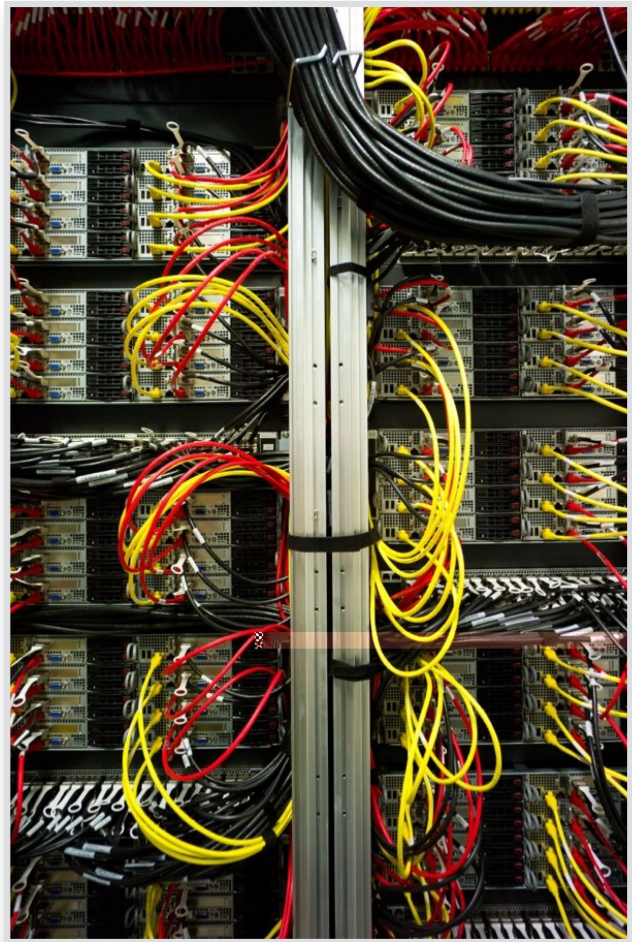
Current Status

❖ H-Dibaryon not firmly established

❖ Is a bound H-Dibaryon a consequence of QCD?

❖ Try "ab initio" technique: Lattice QCD





"Clover" @ HIM

Outline

I. The H-Dibaryon in Lattice QCD

II. Recent lattice calculations

III. Summary & Conclusions

Beyond Perturbation Theory: Lattice QCD

❖ Non-perturbative treatment; regularised Euclidean functional integrals

Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\mathrm{UV}}$

Finite volume: $L^3 \cdot T$

$$\langle \Omega \rangle = \frac{1}{Z} \int \prod_{x,\mu} dU_\mu(x)\, \Omega \prod_f \det\left(\not{D}^{\mathrm{lat}} + m_f\right) \mathrm{e}^{-S_{\mathrm{G}}[U]}$$

❖ Stochastic evaluation of $\langle \Omega \rangle$ via Markov process

Strong growth of numerical cost near physical m_u, m_d

❖ Pion mass, i.e. lightest mass in pseudoscalar channel:

$$\underset{(2001)}{\approx 500\,\mathrm{MeV}} \quad \longrightarrow \quad \underset{(2015)}{\approx 130-200\,\mathrm{MeV}}$$

Systematic effects

❖ Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}} = \left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

$\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

❖ Finite volume effects

- Empirically: $m_\pi L \geq 4$ sufficient for many purposes
- Could be more severe for multi-baryon systems
- Provide information on scattering phase shifts

❖ Unphysical quark masses

- Chiral extrapolation to physical values of m_u, m_d becomes obsolete

❖ Inefficient sampling of SU(3) group manifold

- Simulations become trapped in topological sectors as $a \to 0$
- Use open boundary conditions in time direction *[Lüscher & Schaefer, 2012]*

The H-Dibaryon in Lattice QCD

❖ Spectrum extracted from correlation functions:

$$\sum_{\vec{x},\vec{y}} \left\langle O_{\rm had}(y) O_{\rm had}^{\dagger}(x) \right\rangle = Z_0\, {\rm e}^{-E_0(y_0-x_0)} + Z_1\, {\rm e}^{-E_1(y_0-x_0)} + \ldots$$

$O_{\rm had}(x)$: interpolating operator for a given hadron;

projects on all states with the same quantum numbers

nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C \gamma_5 d^b \right) u^c$

The H-Dibaryon in Lattice QCD

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ground state energy

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The H-Dibaryon in Lattice QCD

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nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C \gamma_5 d^b \right) u^c$

❖ Ground state dominates at large Euclidean times: $y_0 - x_0 \to \infty$

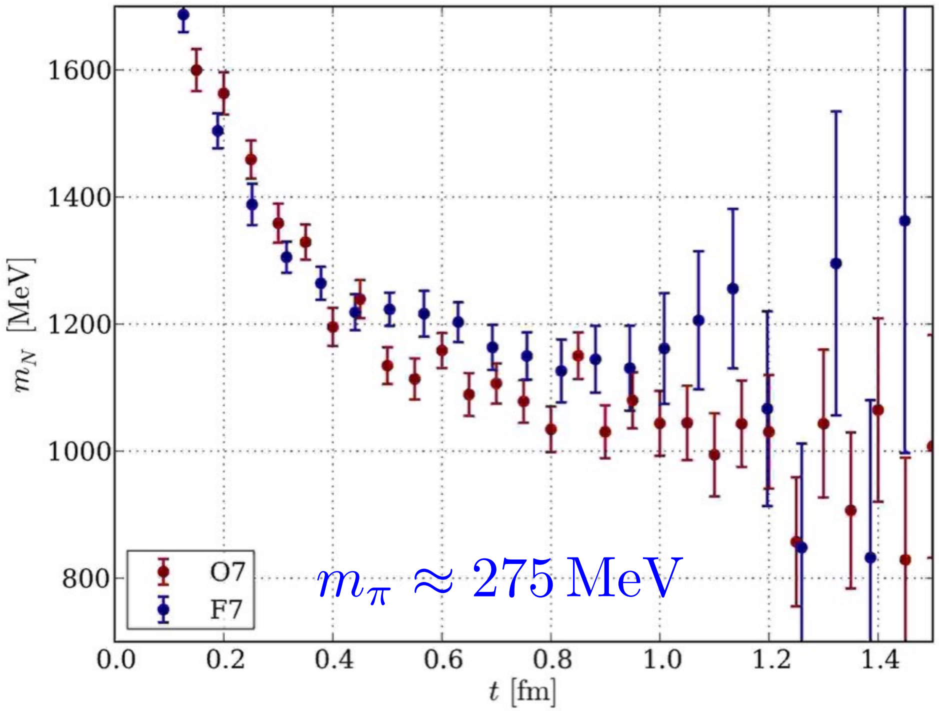
❖ Excited states are sub-leading contributions

The H-Dibaryon in Lattice QCD

❖ Noise problem of baryonic correlation functions:

Exponential growth of noise-to-signal ratio

Nucleon at rest: $R_{\mathrm{NS}}(x_0) = \mathrm{e}^{(m_{\mathrm{N}} - \frac{3}{2} m_\pi) x_0}$



[Capitani et al., arXiv:1504.04628]

❖ Excited state contributions die out slowly

❖ Ground state dominates only for $a \geq 0.5\,\mathrm{fm}$

❖ Precise calculations require very large statistics

The H-Dibaryon in Lattice QCD

- **Lüscher Method:** Finite-volume effects provide physical information

- Two-particle binding momentum:

$$p^2 = \frac{1}{4}(E^2 - \vec{P}\cdot\vec{P}) - m_\Lambda^2 \qquad E,\, m_\Lambda : \text{ determined in finite volume}$$

- Relation to scattering phase shifts in infinite volume:

$$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \qquad q = \frac{pL}{2\pi}$$

[Lüscher 1991, Rummukainen & Gottlieb 1995]

$$\mathcal{Z}_{0,0}(1,q^2) = \frac{1}{\sqrt{4\pi}}\left\{\sum_{q^2\neq n^2}^{\Lambda_n}\frac{1}{q^2-n^2} - 4\pi\Lambda_n\right\}$$

- Scattering amplitude: $\mathcal{A} \propto \dfrac{1}{p\cot\delta_0(p) - ip}$

Pole corresponds to binding energy in infinite volume

Recent lattice calculations: Overview

Collaboration	Method	$m_\pi\,[\mathrm{MeV}]$	N_f	References
HALQCD	Baryon-baryon potential; Nambu-Bethe-Salpeter wave function	$470 — 1170$	3	*Phys Rev Lett 106 (2011) 162002* *Nucl Phys A881 (2012) 28*
NPLQCD	Two-point correlation functions	806	3	*Phys Rev D87 (2013) 034506*
	Two-point correlation functions	230, 390	2+1	*Phys Rev Lett 106 (2011) 162001* *Mod Phys Lett A26 (2011) 2587*
Mainz	Two-point correlation functions	450, 1000	2	*PoS LATTICE2013 (2014) 440* *PoS LATTICE2014 (2015) 107*

❖ NPLQCD and HALQCD find bound H-dibaryon for $m_\pi \geq 400\,\mathrm{MeV}$

HALQCD Collaboration

❖ Obtain baryon-baryon potential from Nambu-Bethe-Salpeter amplitude computed on the lattice

$$G_4(\vec{r}, t - t_0) = \left\langle 0 \middle| (BB)^{(\alpha)}(\vec{r}, t)(\overline{BB})^{(\alpha)}(t_0) \middle| 0 \right\rangle = \phi(\vec{r}, t)\, \mathrm{e}^{-2M(t-t_0)}$$

$(BB)^{(\alpha)}(\vec{r}, t)$: 2-baryon interpolating operator; flavour irrep. α

$\phi(\vec{r}, t)$: NBS wave function

M : single baryon mass

❖ Determine potential via $$V(r) = \frac{[-H_0 - (\partial/\,\partial t)]\,\phi(\vec{r}, t)}{\phi(\vec{r}, t)}$$

❖ Solve the Schrödinger equation to determine binding energies and scattering phase shifts

HALQCD Collaboration

- ❖ Details of the calculation:

  $N_{\mathrm{f}} = 3$ i.e. mass-degenerate *u, d, s* quarks

  Single lattice spacing: $a = 0.121(2)\,\mathrm{fm}$

  5 pion masses in the range: $m_\pi = 469 - 1171\,\mathrm{MeV}$

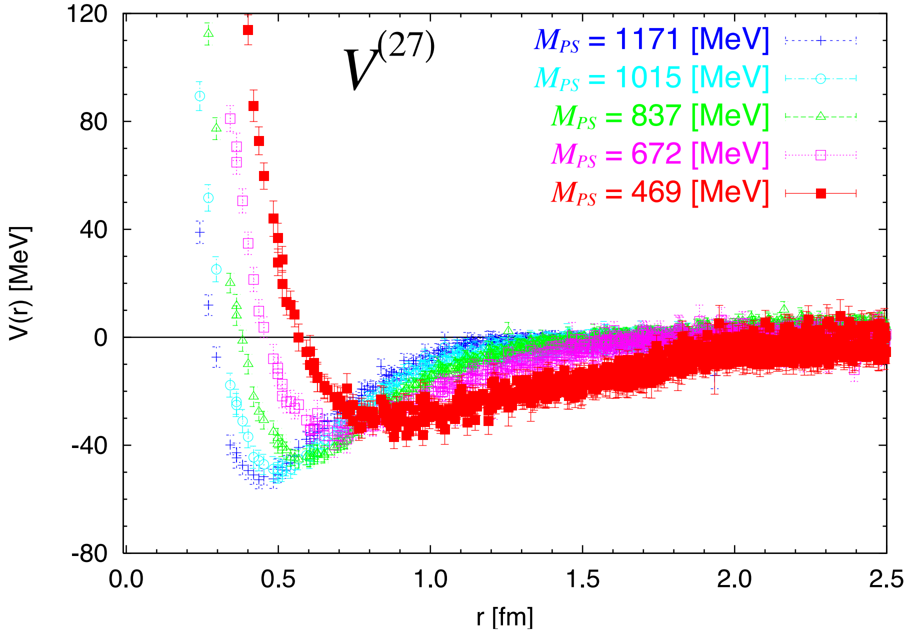
  Volumes: $L = 3.87,\ 2.90,\ 1.97\,\mathrm{fm}$

  Statistics: $O(500)$ gauge configurations per ensemble

  $O(8000)$ "measurements" for each pion mass

HALQCD Collaboration

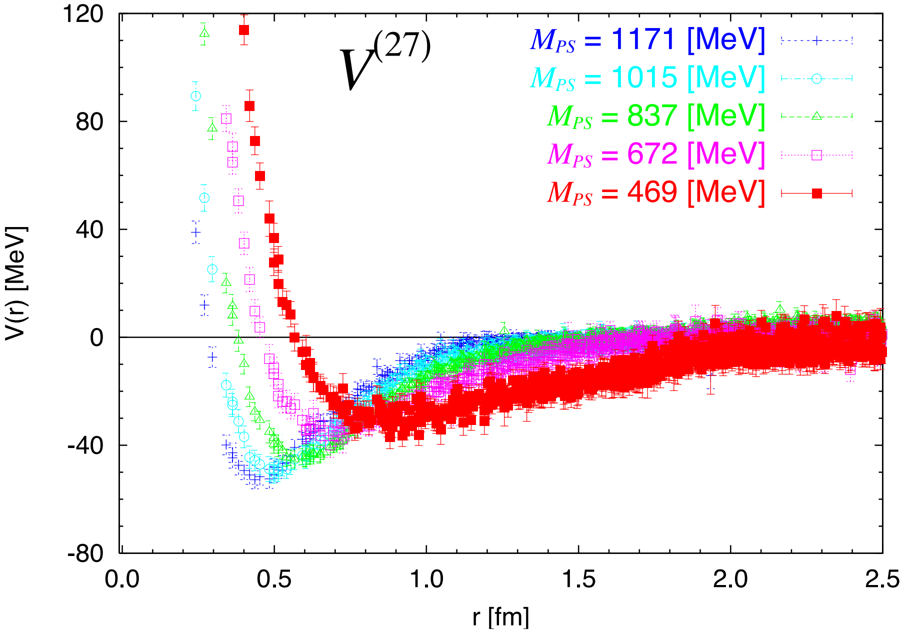
NN potential (27-plet)



- Repulsive core
- Attractive well at intermediate and long distances

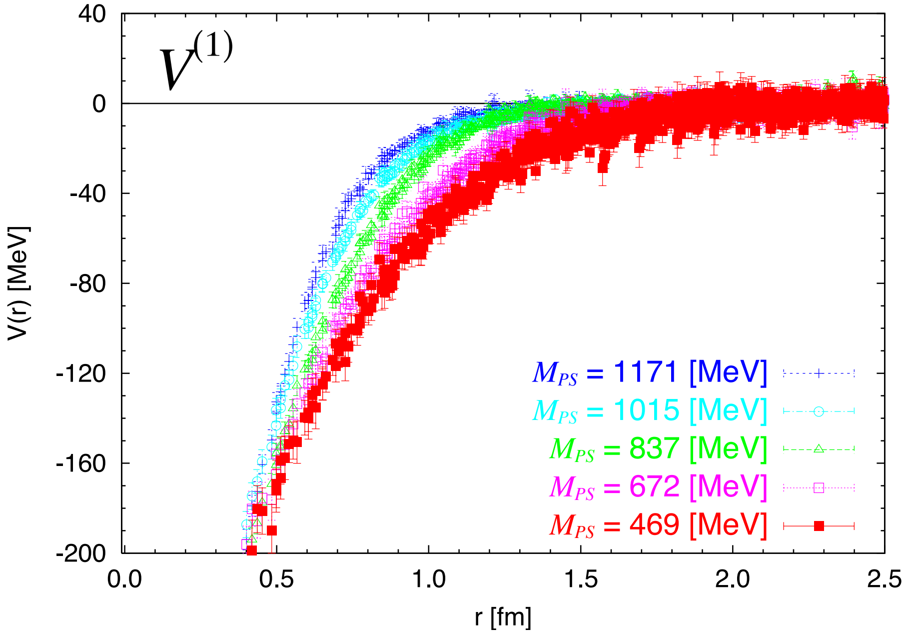
HALQCD Collaboration

NN potential (27-plet)



- Repulsive core
- Attractive well at intermediate and long distances

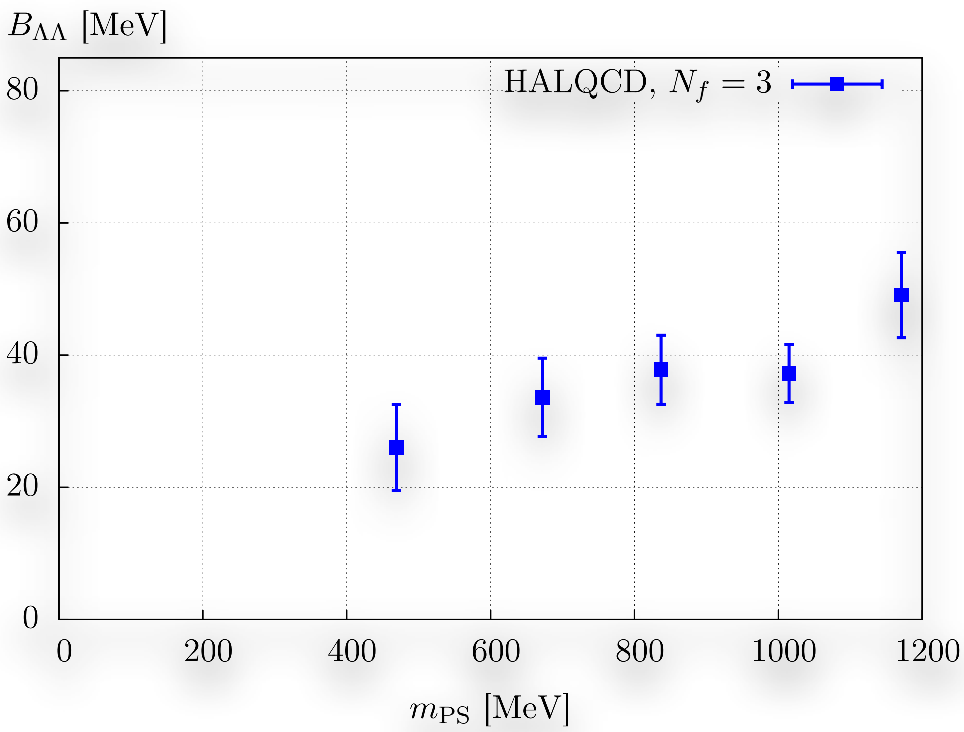
Flavour-singlet potential



- Attractive core
- Mild pion mass dependence
- Parameterise potential as input for Schrödinger equation

HALQCD Collaboration

❖ Chiral behaviour of H-dibaryon binding energies in SU(3) limit:



(statistical and systematic errors combined)

❖ Stronger attraction at smaller quark mass compensated by larger kinetic energy

❖ SU(3) breaking effects not accounted for

Variational method

❖ Consider set of interpolating operators: $O_i, \quad i = 1, \ldots, N$

Matrix correlator: $$C_{ij}(t) = \sum_{\vec{x}} \left\langle O_i(\vec{x}, t + t_0) O_j^\dagger(\vec{x}_0, t_0) \right\rangle$$

❖ Solve Generalised Eigenvalue Problem (GEVP):

$$\mathbf{C}(t + \Delta t) v_n(t) = \lambda_n(t) \mathbf{C}(t) v_n(t) \qquad E_{\text{eff}}(t) = -\partial_t \log \lambda_n(t)$$

❖ Operator basis:

- Apply "smearing" to interpolating operators at source and sink
- Use 6-quark and two-baryon operators
- Project onto irreducible representations in flavour space

Variational basis in the dibaryon channel

❖ Six-quark operators:

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}\left(s^a C\gamma_5 P_+ t^b\right)\left(v^l C\gamma_5 P_+ w^m\right)\left(r^k C\gamma_5 P_+ u^n\right)$$

$$H^{(\mathbf{1})} = \frac{1}{48}\left([sudsud] - [udusds] - [dudsus]\right)$$

$$H^{(\mathbf{27})} = \frac{1}{48\sqrt{3}}\left(2[sudsud] + [udusds] - [dudsus]\right)$$

❖ Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 t^j\right) r^k_\alpha$$

$$(BB)(\vec{p}_1, \vec{p}_2; t) = \sum_{\vec{x},\vec{y}} \mathrm{e}^{i\vec{p}_1\cdot\vec{x}}\mathrm{e}^{i\vec{p}_2\cdot\vec{y}} B_1(\vec{x},t) C\gamma_5 P_+ B_2(\vec{y},t)$$

Construct $(BB)^{(\mathbf{1})},\ (BB)^{(\mathbf{8})},\ (BB)^{(\mathbf{27})}$

NPLQCD Collaboration

- Compute energy levels in dibaryon channel from two-point correlation functions of multi-baryon operators:

$$O_{\rm H}^{\rm 2baryon} = \epsilon_{ijk} \left\{ \left(r^i C\gamma_5 P_+ s^j \right) t^k \right\} (\vec{x}, t)\, \epsilon_{lmn} \left\{ \left(u^l C\gamma_5 P_+ v^m \right) w^n \right\} (\vec{y}, t)$$

- Binding energy: $B_{\Lambda\Lambda} = 2m_\Lambda - E_{\Lambda\Lambda}$

- Details of the calculation: *[Beane et al, Phys Rev D87 (2013) 034506]*

  $N_{\rm f} = 3$ i.e. mass-degenerate *u, d, s* quarks

  Single lattice spacing: $a = 0.145(2)\,\mathrm{fm}$

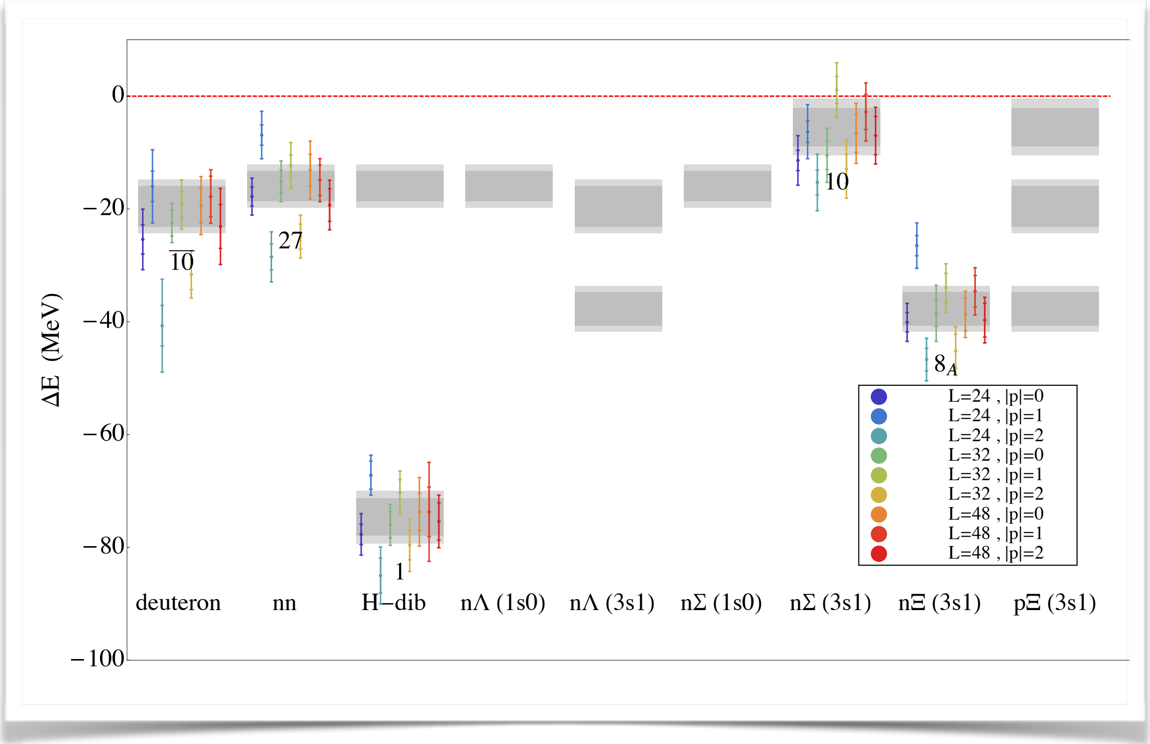
  Single pion mass: $m_\pi = 807\,\mathrm{MeV}$

  Volumes: $L = 3.4,\ 4.5,\ 6.7\,\mathrm{fm}$

  Statistics: $O(10^5)$ “measurements” per ensemble

NPLQCD Collaboration

❖ Binding energies of various two-baryon systems:



NPLQCD Collaboration

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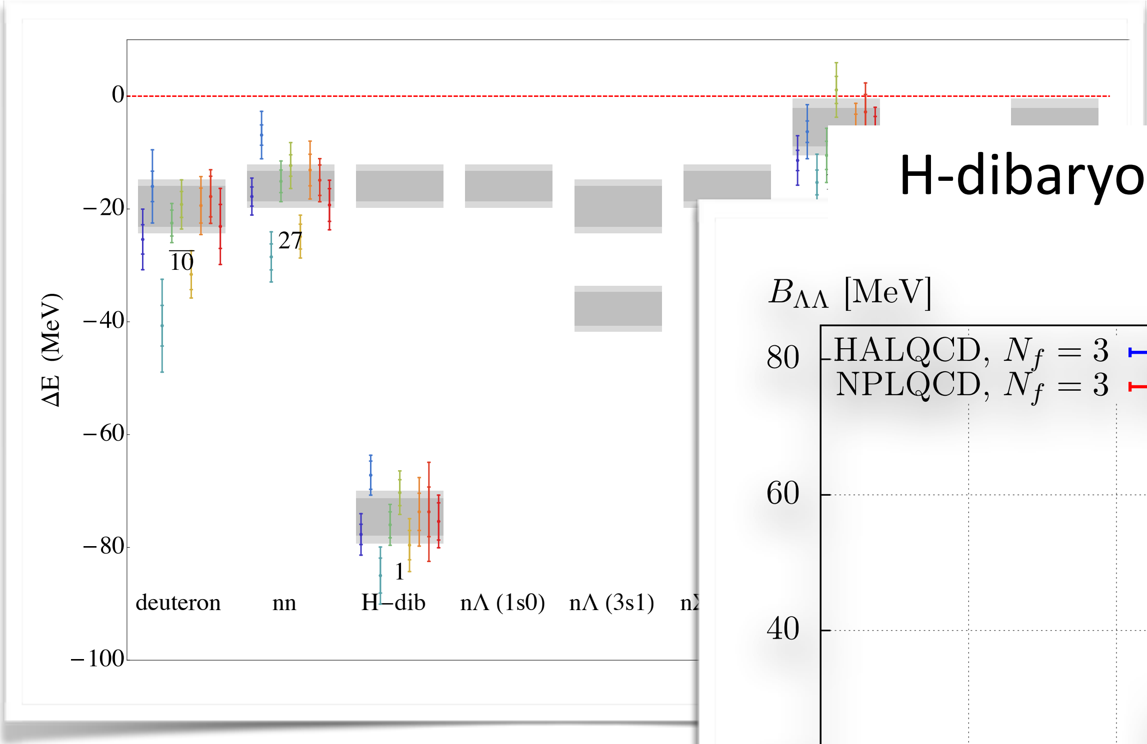
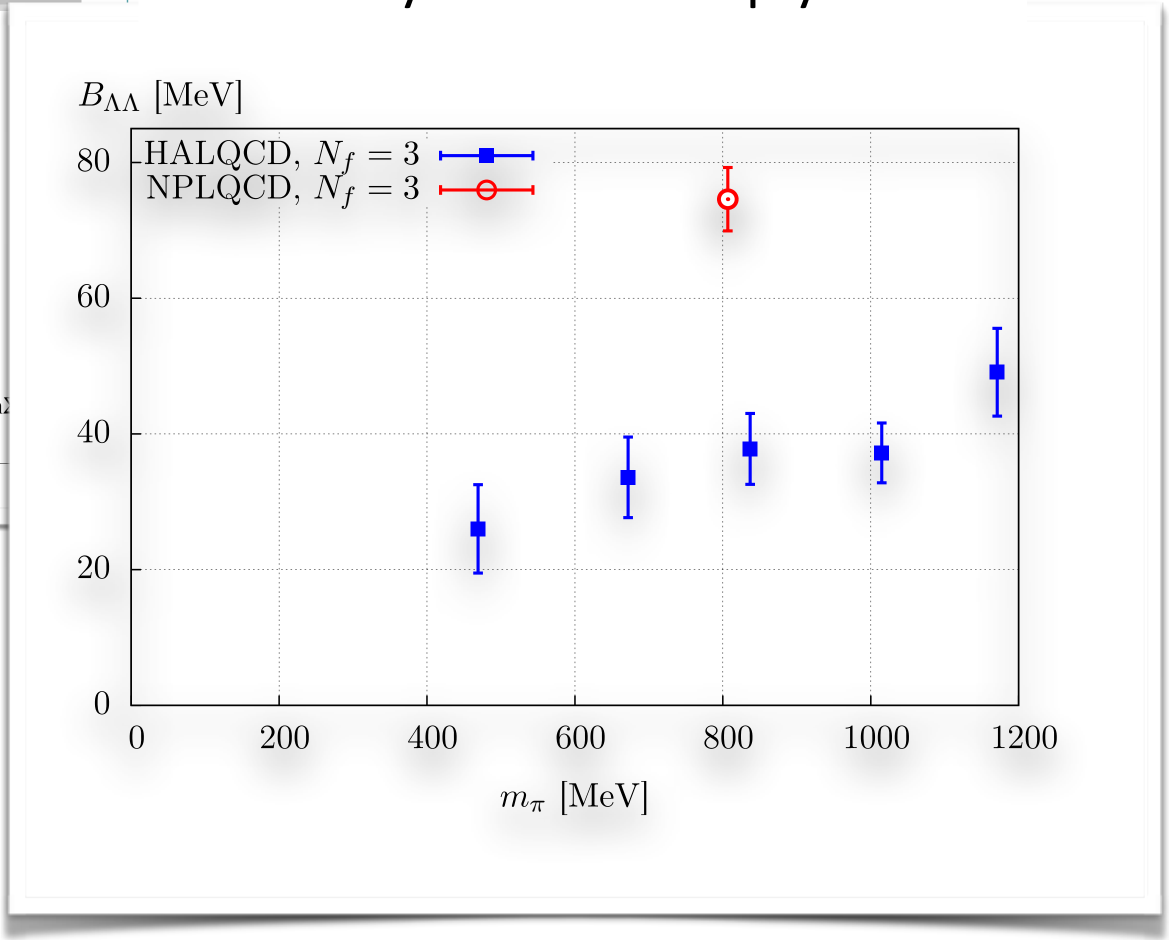


H-dibaryon more deeply bound:



NPLQCD Collaboration

[Beane et al, Phys Rev Lett (2011) 162001, Mod Phys Lett A26 (2011) 2587]

❖ Details of the calculation:

$N_f = 2+1,$ anisotropic action: $a_s/a_t = 3.50(3)$

Single lattice spacing: $a_s = 0.123(1)\,\mathrm{fm}$

Pion masses: $m_\pi = 389, 230\,\mathrm{MeV}$

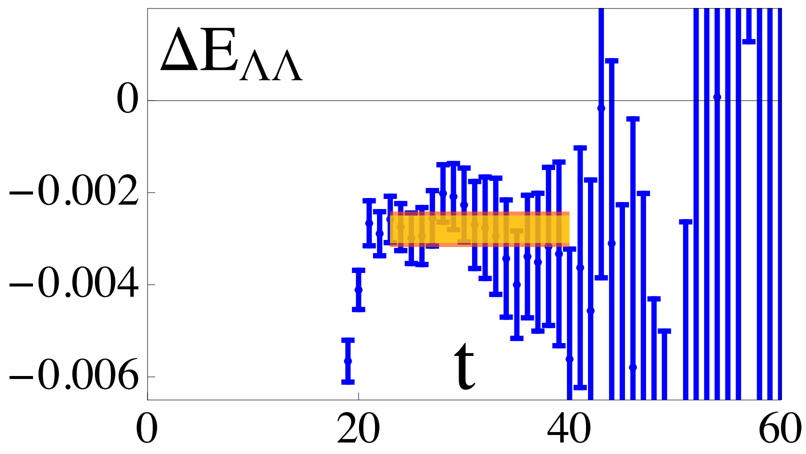
Volumes: $L = 3.9,\ 3.0,\ 2.5,\ 2.0\,\mathrm{fm}$

Statistics: $O(10^5)$ "measurements" per ensemble

❖ Variational approach:

source: six-quark operators

sink: two-baryon operators



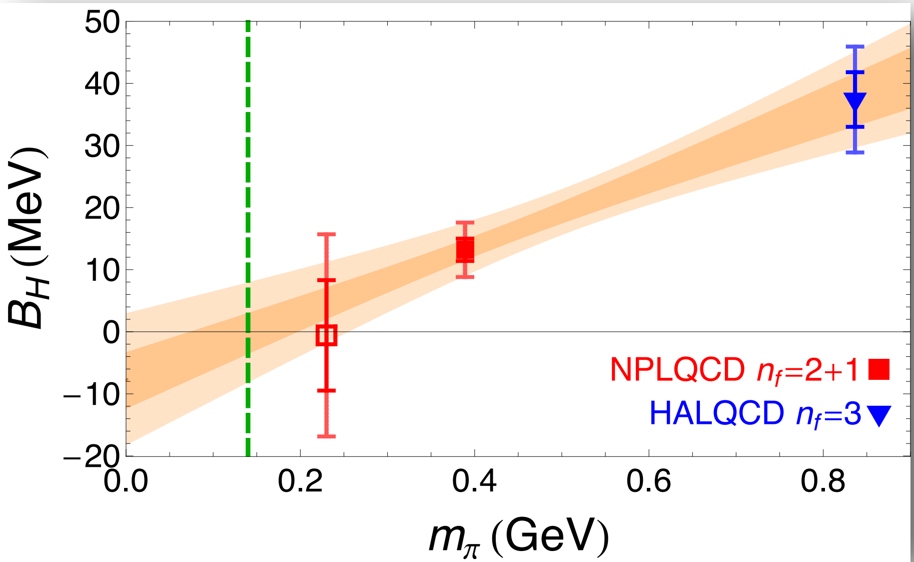
NPLQCD Collaboration

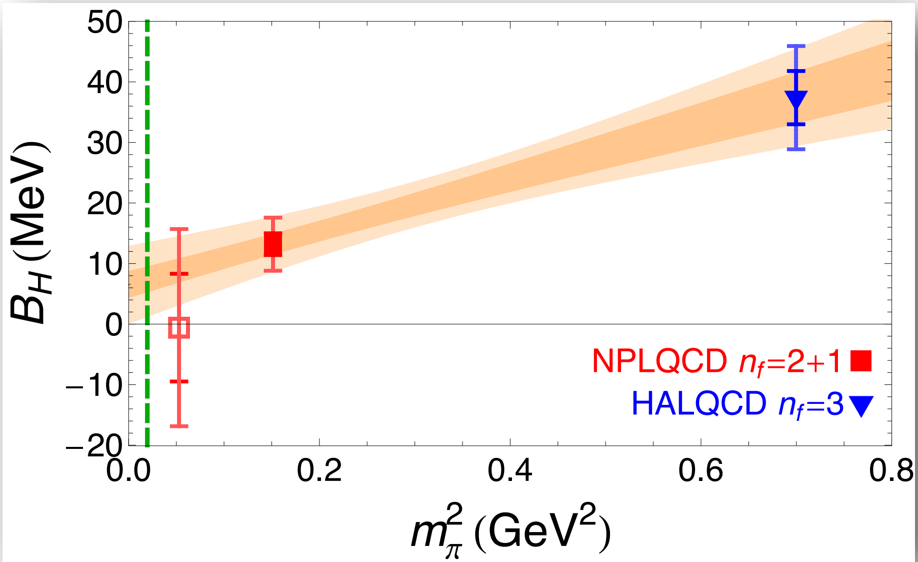
❖ Binding energy in infinite volume limit:

[Beane et al, Phys Rev Lett (2011) 162001, Mod Phys Lett A26 (2011) 2587]

$$B_{\Lambda\Lambda}^{\infty} = \begin{cases} 13.2(1.8)_{\rm stat}\,(4.0)_{\rm syst}\,\mathrm{MeV}, & m_\pi = 389\,\mathrm{MeV} \\ 0.6(8.9)_{\rm stat}\,(10.3)_{\rm syst}\,\mathrm{MeV}, & m_\pi = 230\,\mathrm{MeV} \end{cases}$$

❖ Chiral behaviour:





Mainz / CLS

- In collaboration with:

  A. Francis, J. Green, P. Junnarkar, Ch. Miao, T. Rae

  A. Francis at al., PoS LATTICE2013 (2014) 440, J. Green et al., PoS LATTICE2014 (2015) 107

- Details of the calculation:

  $N_\mathrm{f} = 2$ i.e. quenched strange quark; $\mathrm{O(a)}$ improved Wilson fermions

  Single lattice spacing: $a = 0.063\,\mathrm{fm}$

  Pion masses: $m_\pi = 457,\ 1000\,\mathrm{MeV}$

  Single volume: $L = 2.02\,\mathrm{fm}$

  Statistics: 43k, 125k "measurements"

Mainz / CLS

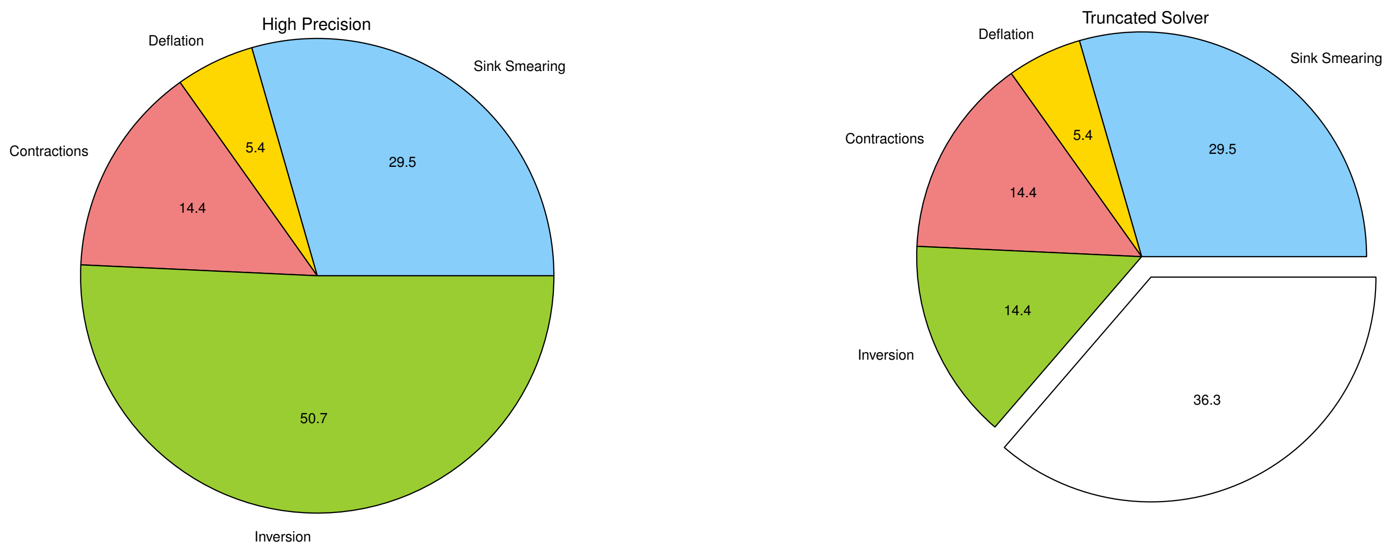
❖ **All-mode-averaging** *[Blum et al, Phys Rev D88 (2013) 094503*

Combine low-precision solver with bias correction:

$$O = O_{\vec{x}_0} - O^{(\mathrm{apprx})}_{\vec{x}_0} + \frac{1}{N_{\Delta\vec{x}}} \sum_{\Delta\vec{x}} O^{(\mathrm{apprx})}_{\vec{x}_0+\Delta\vec{x}}$$

Variance reduction:

$$2(1-r) + \frac{1}{N_{\Delta\vec{x}}}, \quad r = \mathrm{Corr}\left(O^{(\mathrm{apprx})}_{\vec{x}+\Delta\vec{x}}, O^{(\mathrm{apprx})}_{\vec{x}+\Delta\vec{x}'} \right)$$



Mainz / CLS

- **Ensemble "E1"**

  $m_\pi = 1000\,\mathrm{MeV} \approx m_\mathrm{K}$ i.e. SU(3)$_{\text{flavour}}$-symmetric

  GEVP set-up:

  source: 6-quark operator $H^{(\mathbf{1})}$ with two different smearing levels

  sink: 6-quark and multi-baryon operators

- **Ensemble "E5"**

  $m_\pi = 457\,\mathrm{MeV}$

  GEVP set-up:

  source: 6-quark operators $H^{(\mathbf{1})}$, $H^{(\mathbf{27})}$, different smearing levels

  sink: $H^{(\mathbf{1})}$ and $H^{(\mathbf{27})}$ or $(BB)^{(\mathbf{1})}$ and $(BB)^{(\mathbf{27})}$

- Consider different total momentum

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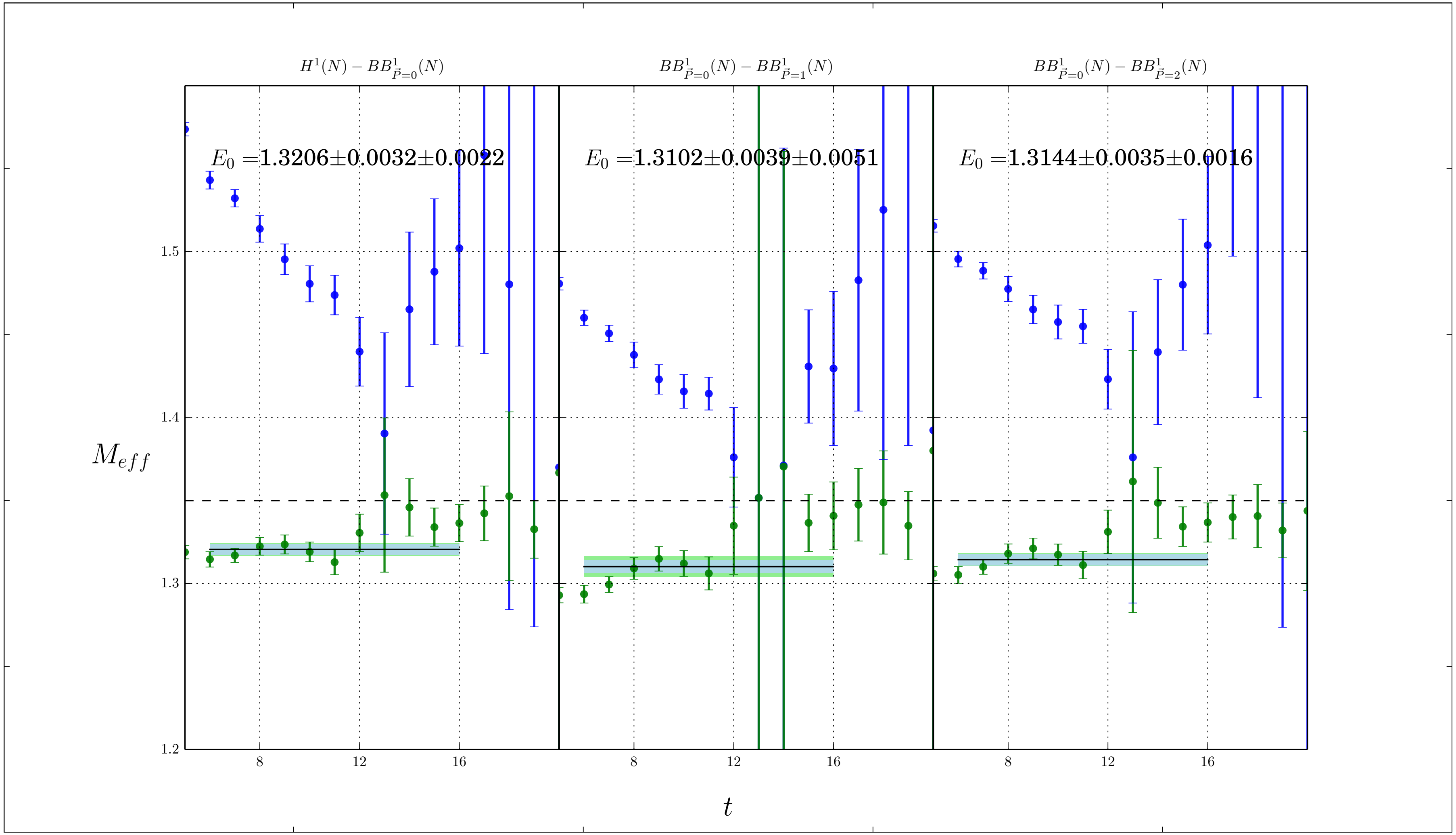
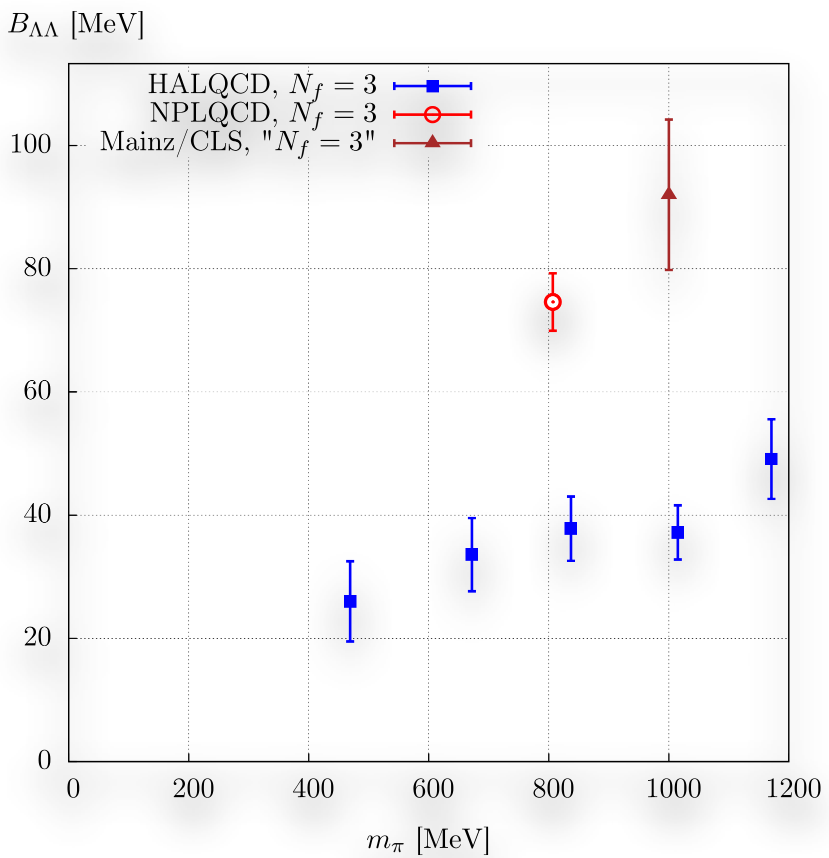
❖ **Ensemble "E1"**



❖ Bound state observed: $B_{\Lambda\Lambda} = 92 \pm 10 \pm 7\,\mathrm{MeV}$

Mainz / CLS

- Bound state observed: $B_{\Lambda\Lambda} = 92 \pm 10 \pm 7\,\mathrm{MeV}$
- Multi-baryon operators: better projection onto ground state
- Chiral behaviour:



- Caveat: binding energy estimated relative to non-interacting Λ's

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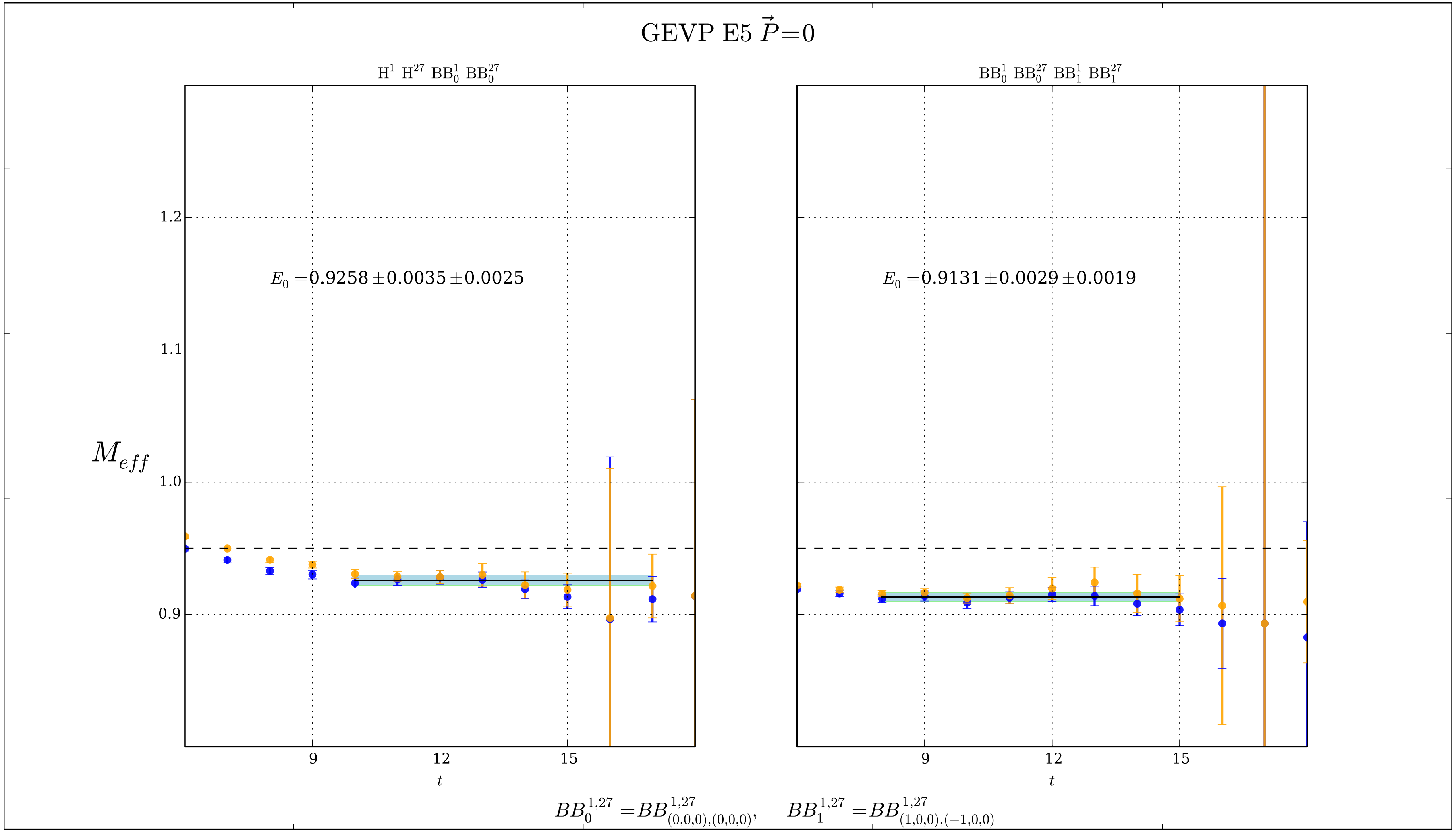
❖ Ensemble "E5"

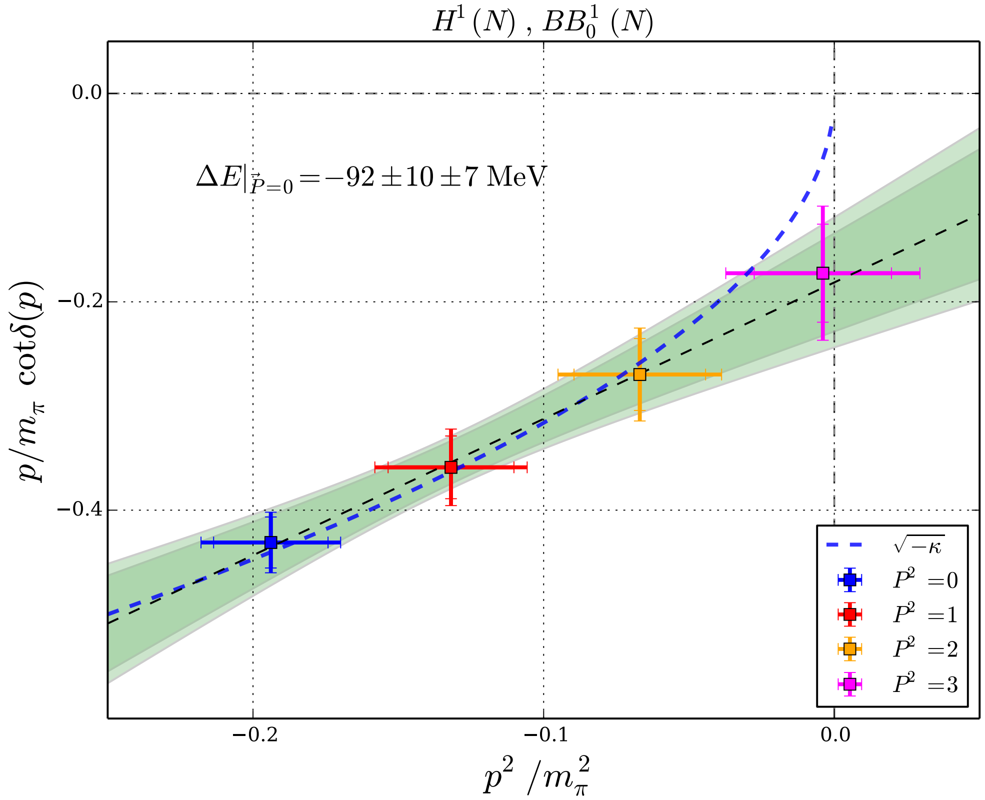


❖ Bound state observed: $B_{\Lambda\Lambda} = 77 \pm 11 \pm 7\,\mathrm{MeV}$

Scattering phase shifts

❖ Ensemble E1:

$$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$$



$$p^2 = \frac{1}{4}(E^2 - \vec{P}\cdot\vec{P}) - m_\Lambda^2$$

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p) - ip}$$

Preliminary!

❖ More data required for reliable determination of the pole

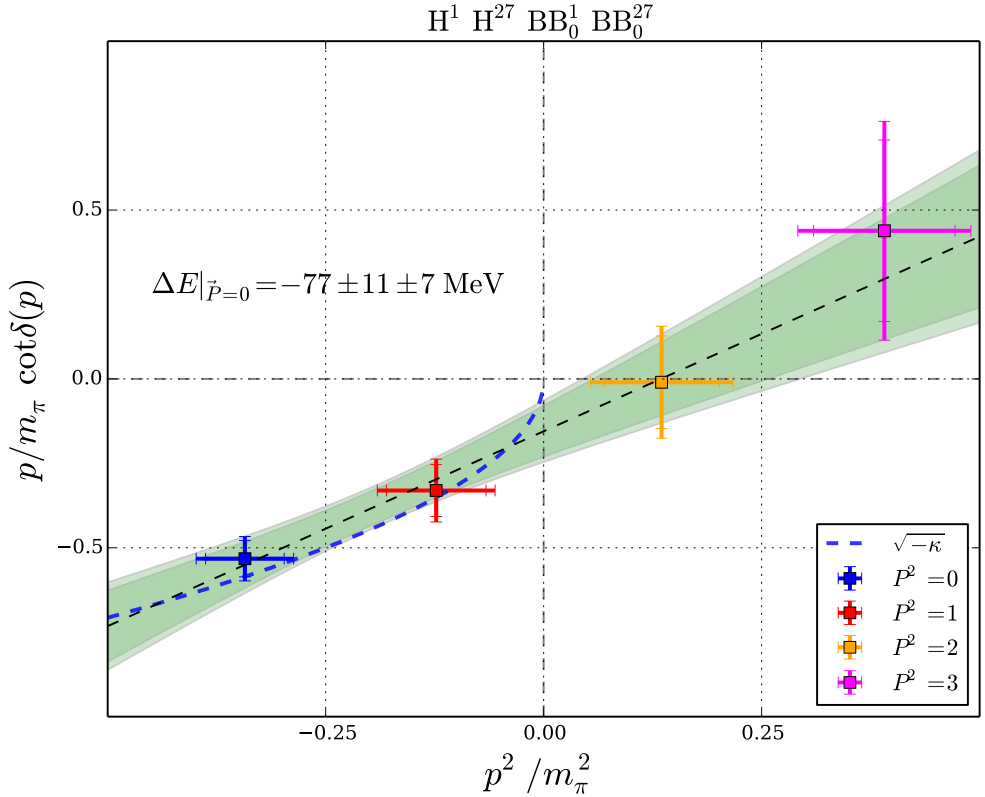
Scattering phase shifts

❖ Ensemble E5:

$$p \cot \delta_0(p) = \frac{2}{\gamma L \sqrt{\pi}} \mathcal{Z}_{0,0}(1, q^2), \quad q = \frac{pL}{2\pi}$$

$$p^2 = \frac{1}{4}(E^2 - \vec{P} \cdot \vec{P}) - m_\Lambda^2$$

$$\mathcal{A} \propto \frac{1}{p \cot \delta_0(p) - ip}$$



Preliminary!

❖ More data required for reliable determination of the pole

Summary and Conclusions

- Lattice calculations of the H-dibaryon binding energy technically demanding
- Bound H-dibaryon found for unphysically large pion masses
- Multi-baryon operators have good overlap onto ground state
- Chiral behaviour of binding energies to be investigated
- Need more and better lattice data to make use of finite-volume scattering formalism

Thank you!