Mott-hadron resonance gas in effective QCD and "toy" models

Aleksandr Dubinin

Institute of Theoretical Physics, University Wrocław, Poland

Collabaration: D. Blaschke, M. Buballa, L. Turko, G. Roepke, A. Radzhabov

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Outline

- Introduction
- Hadron resonance gas model
- Effective models for low-energy QCD
- Mean-field and fluctuations
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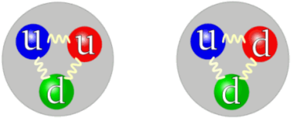
- Mott effect and temperature dependent hadron phase shifts
- Mott hadron resonance gas (MHRG)
- Conclusion

Introduction

QCD is the theory of strong interactions.

Basic elements: quarks and gluons:



Proton Neutron

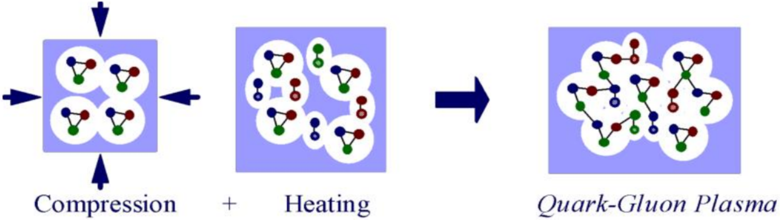
Quark composition of a proton and a neutron (diagrams from *Wikipedia*)

Properties of QCD

- Confinement of quarks ($r \sim 1$fm)
- Asymptotic freedom ($r \to 0$)
- Quark masses and chiral s.b.

Methods of analysis

- Effective theories
- Pertubative QCD
- Lattice QCD



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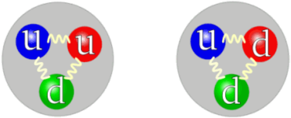

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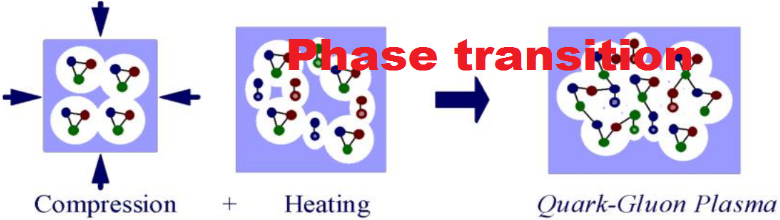
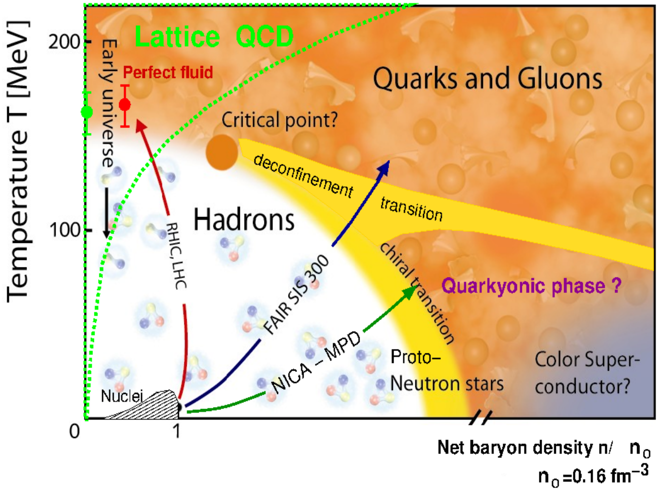


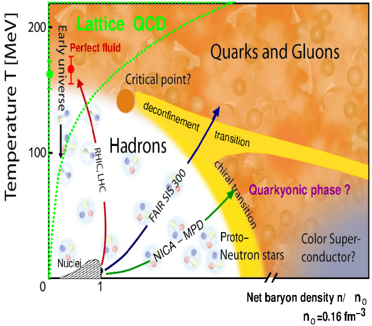
Introduction



http://inspirehep.net/record/823172/files/

Motivation

- Calculations in Lattice QCD are very difficult numerical computations which are carried out on supercomputers. LQCD results don't provide theoretical interpretations and are not applicable a large baryon chemical potential due to the sign problem.
- Effective low-energy theories and pertubative QCD are only applicable in distinct regions of the phase diagram. How to describe the phase transition between them is open.
- In the presentation, we construct a model for describing the phase transition.

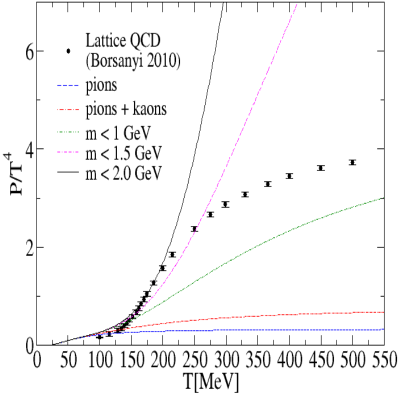


Therefore we need models which are easy to use and provide interpretable results!

Hadron resonance gas model

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, T \ln\left(1 \mp e^{-\sqrt{p^2+M_i^2}/T}\right),$$

Courtesy: Michal Naskre



HRG description of low-energy QCD breaks down at $T > 200$ MeV

PNJL as effective model for low-energy QCD

- The lagrangian NJL/PNJL

$$L_{PNJL} = \underbrace{\underbrace{\bar{q}\left(i\gamma^\mu D_\mu - m_0\right)q}_{\text{Dirac part}} + \underbrace{G_S\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right)}_{\text{Current}-\text{current interaction}}}_{L_{NJL}} - \underbrace{U\left(T, \Phi[A], \bar{\Phi}[A]\right)}_{\text{Polyakov loop potential}}$$

- Bosonize $\sigma(x) = G_s\left(\bar{q}(x)q(x)\right),\ \bar{\pi}(x) = G_s\left(\bar{q}(x)i\gamma_5\bar{\tau}q(x)\right)$

$$\Rightarrow L_{PNJL} = \bar{q}\left(i\gamma^\mu D_\mu - m_0 + (\sigma + i\gamma_5\bar{\tau}\bar{\pi})\right)q - (2G_S)^{-1}\left(\sigma^2 + \bar{\pi}^2\right) - U\left(T, \Phi[A], \bar{\Phi}[A]\right)$$

- Mean-field approximation

$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv 0$$

  - $S(\vec{x})$ time independent classical field

- Mean-field thermodynamical potential

$$\Omega_{MF}(T,\mu) = -\frac{1}{\beta V}\ln\int D\bar{q}Dq\exp\left(\int_{x\in[0,\beta]}dx\left(L_{MF} + \mu\bar{q}\gamma^0 q\right)\right)$$

Mean-field and fluctuations

- Mean-field Langrangian

$$L_{MF} = \bar{q}S^{-1}(x)q - \left(2G_S\right)^{-1}\sigma^2 - U\Big(T, \Phi[A], \bar{\Phi}[A]\Big),$$

  - quark fields can be integrated out!
- Inverse dressed propagator

$$S^{-1}(x) = i\gamma^\mu D_\mu - \underbrace{\left(m_0 + \sigma\right)}_{M}$$

- Constituent mass quark
  - found from minimize $\Omega_{MF} \to \sigma = 2G_S Tr \ln S^{-1}$
- Fluctuation part: Gaussian approximation
  - Expand the logarithm in MF thermodinamical potential

$$\Omega(T,\mu) = \Omega_{MF}(T,\mu) + \sum_X \Omega_X(T,\mu) + O[\phi_X^3]$$

Thermodynamic potential - propagators - phase shifts

Thermodynamic potential for bosonic degree of freedom (mode) X

$$\begin{aligned}\Omega_{\rm X}(T,\mu) &= \frac{1}{2}\frac{T}{V}Tr\ln S_{\rm X}^{-1}(iz_n,\mathbf{q}) = \frac{1}{2}d_{\rm X}\,T\sum_n\int\frac{\mathrm{d}^3q}{(2\pi)^3}\ln S_{\rm X}^{-1}(iz_n,\mathbf{q})\ , \\ &= -d_{\rm X}\,T\sum_n\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{1}{iz_n-\omega}\mathrm{Im}\ln S_{\rm X}^{-1}(\omega+i\eta,\mathbf{q})\ ,\end{aligned}$$

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Propagator = complex function → polar representation

$$S_{\rm X}^{-1}(iz_n,\mathbf{q}) = G_{\rm X}^{-1} - \Pi_{\rm X}(iz_n,\mathbf{q}) = |S_{\rm X}|{\rm e}^{i\delta_{\rm X}}\ ,\quad \delta_{\rm X}(\omega,\mathbf{q}) = -{\rm Im} \ln S_{\rm X}^{-1}(\omega-\mu_X+i\eta,\mathbf{q})$$

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&= -d_{\rm X} T \sum_n \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \, \frac{1}{iz_n - \omega} \mathrm{Im} \ln S_{\rm X}^{-1}(\omega + i\eta, \mathbf{q}) \,,
\end{aligned}
$$

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$$

Beth-Uhlenbeck formula for correlations (hadrons) in quark matter

[D. Blaschke, M. Buballa, A. Dubinin and G. Roepke, D. Zablocki, Annals Phys. 348 (2014) 228-255.]

$$
\begin{aligned}
\Omega_{\rm X}(T,\mu) &= d_{\rm X} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \, n_X^-(\omega) \delta_X(\omega,\mathbf{q}) \\
&= -d_{\rm X} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \, [1 + n_X^-(\omega) + n_X^+(\omega)] \delta_{\rm X}(\omega,\mathbf{q}) \\
&= d_{\rm X} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \left\{ \omega + T \ln\left(1 - \mathrm{e}^{-(\omega - \mu_{\rm X})/T}\right) \right. \\
&\quad \left. + T \ln\left(1 - \mathrm{e}^{-(\omega + \mu_{\rm X})/T}\right) \right\} \frac{d\delta_{\rm X}(\omega,\mathbf{q})}{d\omega} \,.
\end{aligned}
$$

The Mott transition, Levinson's theorem

Levinson's theorem in medium can be formulated as

[D. Blaschke, M. Buballa, A. Dubinin and G. Roepke,D. Zablocki, Ann. Phys. 348 (2014)], [A. Wergieluk et al., PPNL 10 (2013) 660], [A. Dubinin, D. Blaschke and Y. L. Kalinovsky, Acta Phys. Polon. Supp. 7 (2014)215]

$$\int_0^\infty d\omega \frac{1}{\pi}\frac{d\delta_i(\omega;T)}{d\omega} = 0 = \underbrace{\int_0^{\omega_{\rm thr}(T)} d\omega \frac{1}{\pi}\frac{d\delta_i(\omega;T)}{d\omega}}_{n_{B,i}(T)} + \underbrace{\frac{1}{\pi}\int_{\omega_{\rm thr}(T)}^\infty d\omega \frac{d\delta_i(\omega;T)}{d\omega}}_{\frac{1}{\pi}[\delta_i(\infty;T)-\delta_i(\omega_{\rm thr};T)]} ,$$

$$\delta_i(\omega_{thr},T) = \pi n_{B,i}(T)$$

at any given temperature T. Since under very general conditions holds $\delta_i(\infty;T) = 0$ for any temperature it follows that $\delta_i(\omega_{\rm thr};T) = \pi n_{B,i}(T)$, that decrementing the number of bound states in the channel i at the corresponding Mott temperature $T_{i,\rm Mott}$ has to be accompanied by a jump by π of the phase shift at threshold.

Mott hadron resonance gas (”toy”) model

Generic ansatz: mass spectrum in hot medium” below formulas: quark masses in medium: $m(T)$, $m_s(T)$ from PNJL” and i=pi, K, rho, N, ... hadron species

[D. Blaschke, A. Dubinin, M. Buballa, PRD 91(2015)12], [D. Blaschke, A. Dubinin,L. Turko, PPN 46(2015)10pp]

$$\boxed{M_i(T) = M_i(0) + \Gamma_i(T)}$$

$$\Gamma_i(T) = a\,(T - T_{\mathrm{Mott,i}})\Theta(T - T_{\mathrm{Mott,i}})\ ,$$

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$$\boxed{M_i(T_{\rm Mott,i}) = m_{\rm thr,i}(T_{\rm Mott,i})}$$

$$\Gamma_i(T) = a\,(T - T_{\rm Mott,i})\Theta(T - T_{\rm Mott,i})\ ,$$

$$m_{\rm thr,M}(T) = (2 - N_s)m(T) + N_s m_s(T)$$

$$m_{\rm thr,B}(T) = (3 - N_s)m(T) + N_s m_s(T)$$

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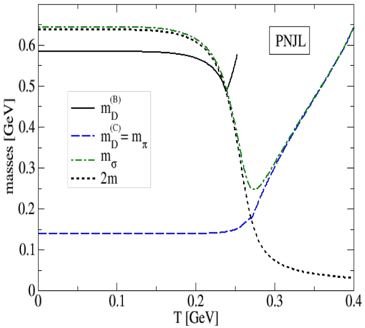
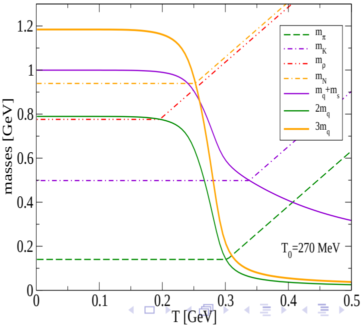
PNJL



"toy"



Mott effect and temperature dependent hadron phase shifts

[D. Blaschke, A. Dubinin,L. Turko, PPN 46(2015)10pp]

$$\begin{aligned}\delta_i(s;T) &= F(s)\left[\frac{\pi}{2}+\arctan\left(\frac{s-M_i^2(T)}{M_i(T)\Gamma_i(T)}\right)\right]\Bigg\{\theta(m^2_{\rm thr,i}-s) \\ &+ \theta(s-m^2_{\rm thr,i})\theta(m^2_{\rm thr,i}+N_i^2\Lambda^2-s)\left[\frac{m^2_{\rm thr,i}+N_i^2\Lambda^2-s}{N_i^2\Lambda^2}\right]\Bigg\},\end{aligned}$$

where the auxiliary function $F(s)=\sin(s/\Gamma^2)\Theta(\Gamma^2\pi/2-s)+\Theta(s-\Gamma^2\pi/2)$

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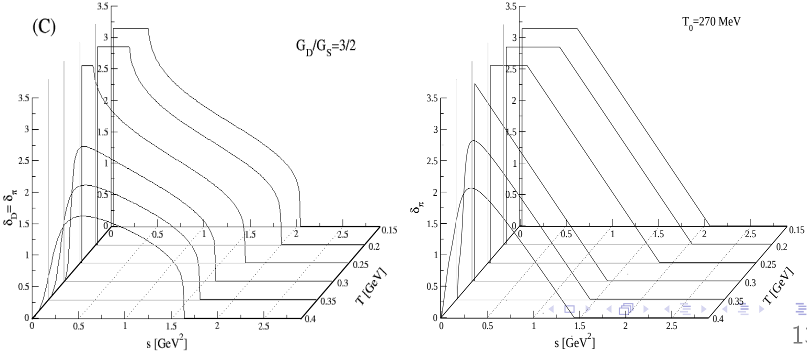
where the auxiliary function $F(s) = \sin(s/\Gamma^2)\Theta(\Gamma^2\pi/2 - s) + \Theta(s - \Gamma^2\pi/2)$

[D. Blaschke, A. Dubinin, M. Buballa, PRD 91(2015)12]



Mott hadron resonance gas (MHRG)

Partial pressure of two-particle correlation "i" in medium

[D. Blaschke, A. Dubinin,L. Turko, PPN 46(2015)10pp]

$$
\begin{aligned}
P_i(T) &= d_i \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega)\delta_i(\omega; T) \\
&= d_i \int_0^\infty \frac{dpp^2}{2\pi^2} \int_0^\infty \frac{ds}{2\pi} \frac{1}{\sqrt{p^2+s}} f_i(\sqrt{p^2+s})\delta_i(s; T),
\end{aligned}
$$

Here $f_i(\omega) = 1/[\exp(\omega/T) \mp 1]$, where the upper (lower) sign holds for mesons (baryons). Use the transformation $s = M^2$.

$$
P_i(T) = d_i \int_0^\infty \frac{dpp^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi} \frac{M}{\sqrt{p^2+M^2}} f_i(\sqrt{p^2+M^2})\delta_i(M^2; T).
$$

Generalized Beth-Uhlenbeck formula

$$
P_i(T) = \mp d_i \int_0^\infty \frac{dpp^2}{2\pi^2} \int_0^\infty dM\, T \ln(1 \mp e^{-\sqrt{p^2+M^2}/T}) \frac{1}{\pi} \frac{d\delta_i(M^2; T)}{dM} \;,
$$

$\Gamma_i(T) \to 0$ (and $m \to \infty$, quark confinement) , so that $\delta_i(M^2; T) = \pi\theta(M - M_i)$, we have $\pi^{-1} d\delta_i(M^2; T)/dM = \delta(M - M_i)$ so that the M-integration becomes trivial and gives

$$
P_i(T) = \mp d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, T \ln\left(1 \mp e^{-\sqrt{p^2+M_i^2}/T}\right),
$$

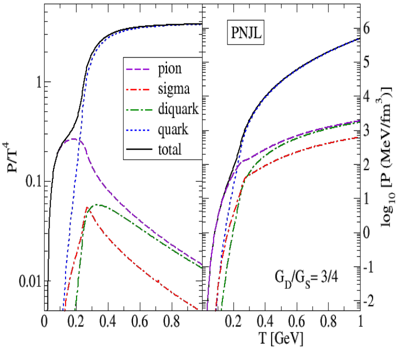
Mott hadron resonance gas (MHRG)

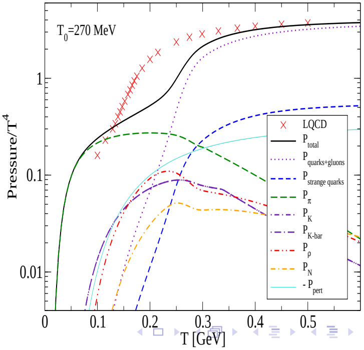
The thermodynamic potential, i.e. the pressure, of two-particle correlations in quark matter

[D. Blaschke, A. Dubinin , M. Buballa, PRD91(2015)12], [D. Blaschke, A. Dubinin,L. Turko, PPN 46(2015)10pp.]

$$\begin{aligned} P_i(T) &= d_i \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega)\delta_i(\omega; T) \\ &= d_i \int_0^\infty \frac{dpp^2}{2\pi^2} \int_0^\infty \frac{ds}{2\pi} \frac{1}{\sqrt{p^2+s}} f_i(\sqrt{p^2+s})\delta_i(s; T), \end{aligned}$$

$$P(T) = \sum_i P_i(T) + P_{PNJL}(T) + P_{pert}(T)$$





Conclusions

- *In this work an effective model is constructed which is capable of reproducing basic physical characteristics of the hadron resonance gas at low temperatures and embody the crucial effect of hadron dissociation by the Mott effect.*
- *The generalized Beth-Uhlenbeck form of the partial pressures is constructed for each hadronic channel.*
- *Numerical results show that the simplifying ansatz for the temperature dependence of both, the mass spectrum and the phase shifts of hadronic channels give results in qualitative agreement with recent ones from LQCD [1].*
- *The differences at the present level of sophistication of our effective model are well understood and basically two conclusions for its further development can be drawn. First, one can include higher lying hadronic states into the model. Second, and most importantly, the back reaction of the hadron resonance gas on the chiral condensate and thus to the quark masses and the continuum thresholds which derive from it must be taken into account.*

Phase shifts

[D. Blaschke, M. Buballa, A. Dubinin and G. Roepke, D. Zablocki, Annals Phys. 348 (2014) 228-255.]