A new RMF based quark-nuclear matter EoS for applications in astrophysics and heavy-ion collisions

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Introduction

what is this about?

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- Equation of State

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  $\rightarrow$ relation between thermodynamic quantities (e.g. baryon particle density n, temperature T, baryon asymmetry $\delta = \frac{n_\mathrm{p} - n_\mathrm{n}}{n}$, inner energy density $\varepsilon \dots$) in an equilibrated system. e.g.:

$$p = p\left(T, n, \delta\right)$$

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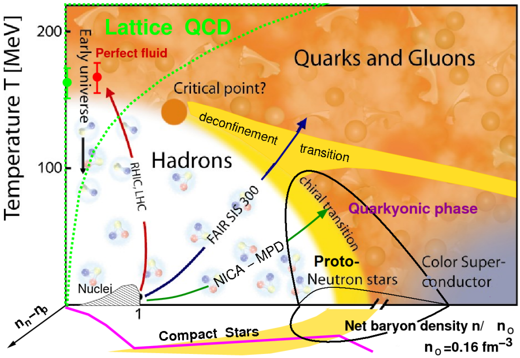
- this work?

  → is about the developement of an hybrid EoS, which contains a hadronic and a quark EOS and constructs an appropriate phase transition.

Outline

Location in the QCD phasediagram



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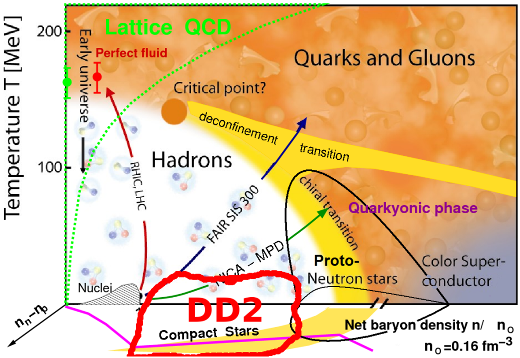

S. Typel, Phys. Rev. C **71**, 064301 (2005). (low temperature)

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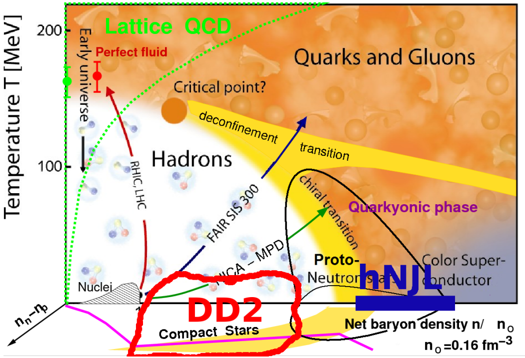

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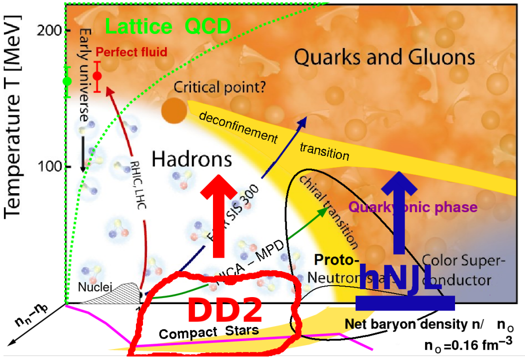

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Extend to finite/higher temperatures

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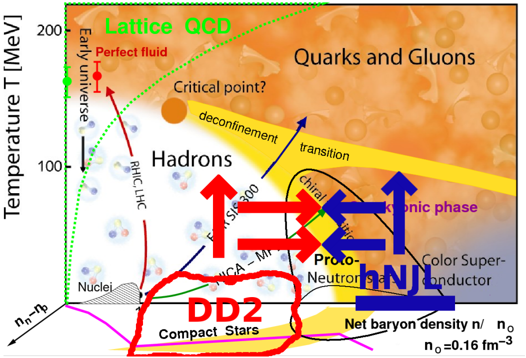

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Extend to finite/higher temperatures

improve behaviour near phase transition

Hadronic EOS

Hadronic EOS

Relativistic Mean Field approximation
DD2 Parametrisation by Stefan Typel

Hadronic EOS

Relativistic Mean Field: DD2

- Lagrangian

$$\mathscr{L}_{\mathrm{DD2}} = \overline{\psi}[\gamma^{\mu} i\partial_{\mu} - m]\psi$$

Hadronic EOS

Relativistic Mean Field: DD2

- Lagrangian

$$\begin{aligned}\mathscr{L}_{\mathrm{DD2}} = & \overline{\psi}[\gamma^\mu i\partial_\mu - m]\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 \\ & - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}\vec{H}_{\mu\nu}\cdot\vec{H}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu\end{aligned}$$

Hadronic EOS

Relativistic Mean Field: DD2

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$$
\begin{aligned}
\mathscr{L}_{\text{DD2}} = & \overline{\psi}[\gamma^{\mu}(i\partial_{\mu} - \Gamma_{\omega}\omega_{\mu} + \Gamma_{\rho}\vec{\tau}\vec{\rho}_{\mu}) - (m - \Gamma_{\sigma}\sigma)]\psi \\
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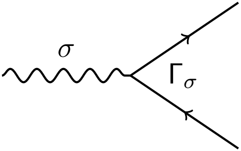

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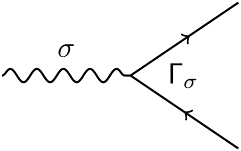

→ Euler-Lagrange-equations → baryon-/meson-equations

$$[\gamma^\mu (i\partial_\mu - \Sigma_\mu) - (m - \Sigma)]\,\psi = 0$$

$$m_\sigma^2 \sigma = \Gamma_\sigma n^S \, ; \quad m_\sigma^2 \omega = \Gamma_\omega n \, ; \quad m_\sigma^2 \rho = \Gamma_\rho n^I$$

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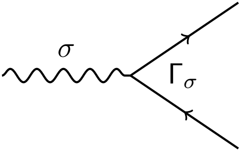

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→ mass-/energy-shifts

$$\Sigma = \Gamma_\sigma \sigma \; ; \quad \Sigma_0^i = \Gamma_\omega \omega + \tau_3^i \Gamma_\rho \rho + \Sigma_{\mathrm{R}}^i \; ; \qquad \Gamma_j = \Gamma_j(n)$$

Hadronic EOS

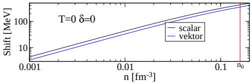
Relativistic Mean Field: DD2

- Relativistic energy dispersion relation

$$e_i(k) = \sqrt{[m - \Sigma(n,\delta,T)]^2 + k^2} + \Sigma_0^i(n,\delta,T) \qquad (1)$$

- With the quasiparticle energies, the chemical potentials μ_i follow from solving

$$n_i = \frac{1}{\pi^2}\int_0^\infty \mathrm{d}k \, \frac{k^2}{\exp\{[e_i(k) - \mu_i]/T\} + 1} \, . \qquad (2)$$



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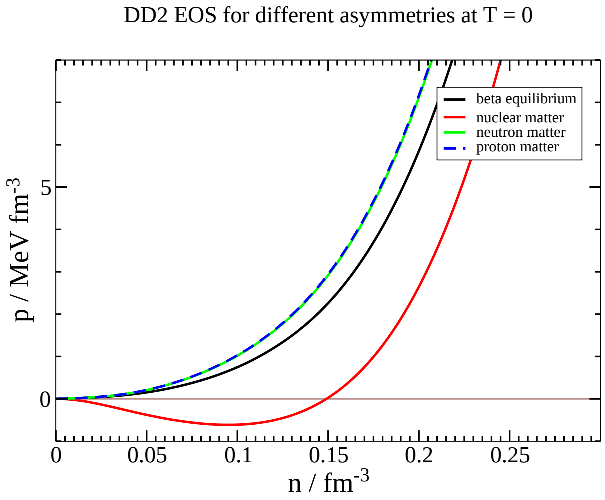

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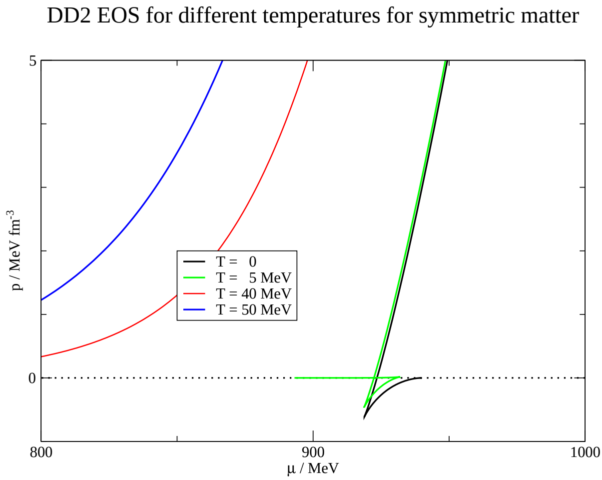

Hadronic EOS
open questions/tasks

- antiparticles are omitted in current calculations
  - $\rightarrow$ need to be included for higher temperatures
- more particles should be included (e.g. Pions, Kaons . . .)
  - $\rightarrow$ statistical model
- higher density corrections for phase transition
  - $\rightarrow$ excluded volume or Pauli-blocking

Quark EOS

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higher order Nambu-Jona-Lasinio (NJL) model
by Sanjin Benic

Quark EOS
higher order Nambu-Jona-Lasinio model

- Grand canonical potential (-density)

$$\Omega = U - 2N_c \sum_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ E + T \ln \left[1 + \mathrm{e}^{-\beta(E - \tilde{\mu}_q)} \right] + T \ln \left[1 + \mathrm{e}^{-\beta(E + \tilde{\mu}_q)} \right] \right\} + \Omega_0$$

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- with the quantities

$$E = \sqrt{p^2 + M^2}$$

$$U = \frac{g_{20}}{\Lambda^2}\sigma^2 + 3\frac{g_{40}}{\Lambda^8}\sigma^4 - 3\frac{g_{22}}{\Lambda^8}\sigma^2\omega^2 - \frac{g_{02}}{\Lambda^2}\omega^2 - 3\frac{g_{04}}{\Lambda^8}\omega^4$$

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$$\tilde{\mu}_q = \mu_q - \Delta E \quad ; \quad \Delta E = 2\frac{g_{02}}{\Lambda^2}\omega + 4\frac{g_{04}}{\Lambda^8}\omega^3 \cancel{+ 2\frac{g_{22}}{\Lambda^8}\sigma^2\omega}$$

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higher order Nambu-Jona-Lasinio model

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- leads to the gap equations

$$\sigma = n^{\mathrm{S}}(T, \{\mu_q\}, \sigma, \omega)$$
$$\omega = n^{\mathrm{V}}(T, \{\mu_q\}, \sigma, \omega) = n$$

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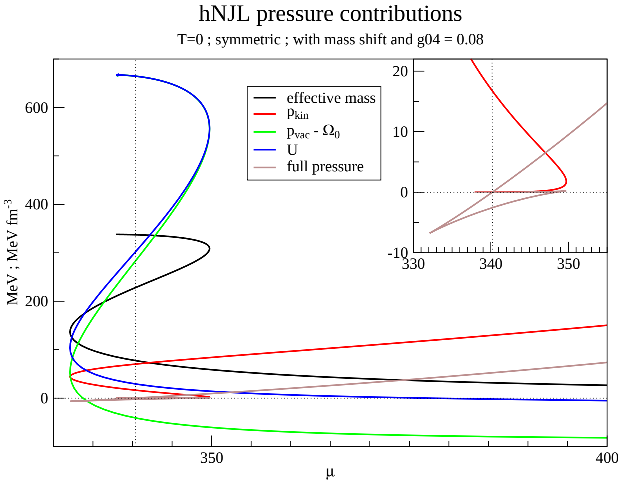


$$p = p_{\rm kin} + p_{\rm vac} - U - \Omega_0$$

Quark EOS

open questions/tasks

- finite temperature is included
- no isovector mesons like δ and ρ
  $\rightarrow$ needed for asymmetric behaviour
- gluon sector?
  $\rightarrow$ modelling by bag constant
- corrections near phase transition
  $\rightarrow$ available volume, cluster formation, ...

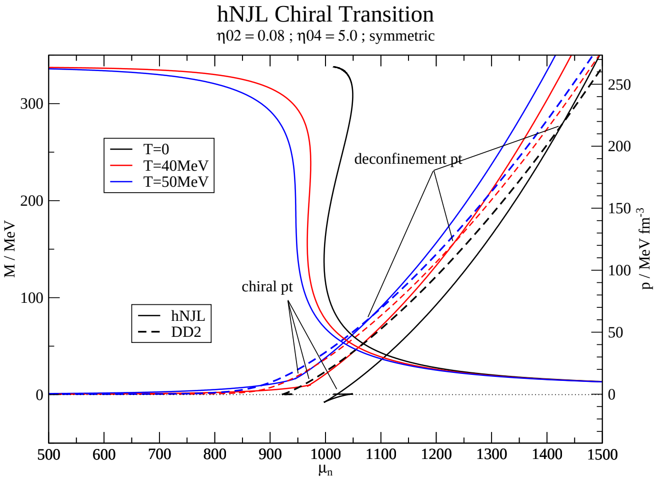
Phase transitions

Phase transitions

different approaches for constructing a hadron-quark transition

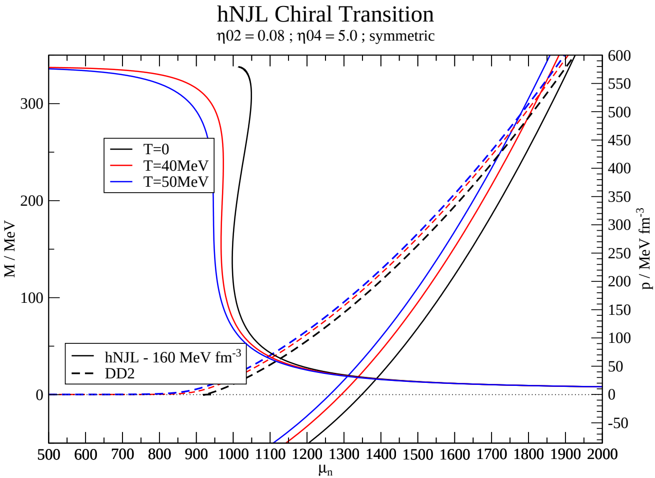
Phase transition

bare models



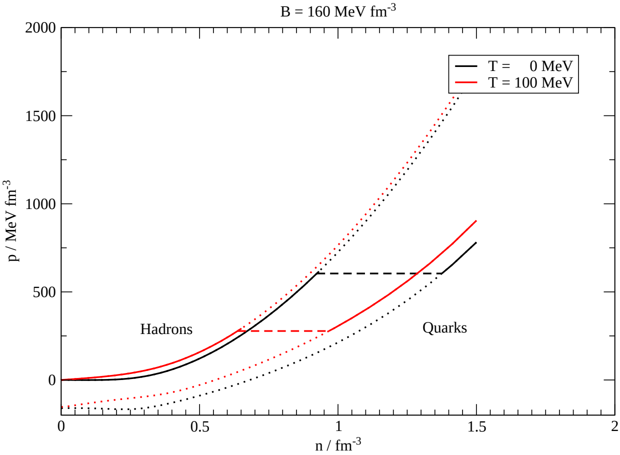
Phase transition

including bag constant



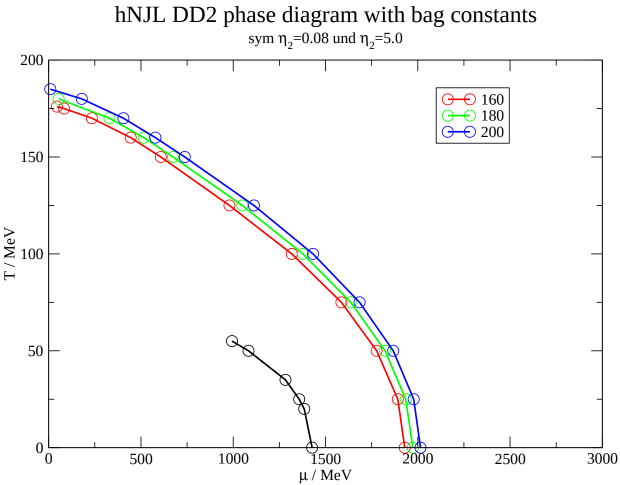
Phase transition

over density



Phase transition

phase diagram



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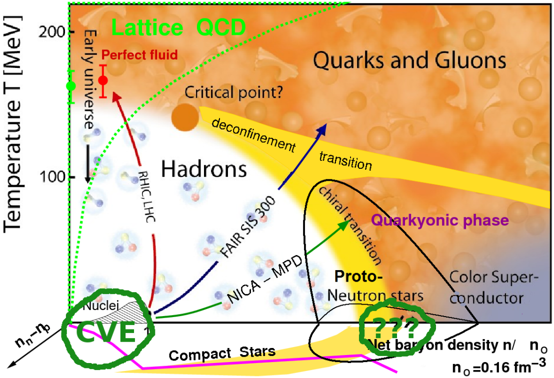
$\Rightarrow$ **still shipload of work to do!**

Cluster Virial Expansion
with medium modifications



"Cluster virial expantion and quasiparticle approach" → Poster 25
Describe the formation of nuclear clusters under consideration of in-medium effects

Thank you for your attention.