

# Bayesian analysis of hybrid EoS constraints with mass-radius data for compact stars

Recent results described in arXiv:1506.07755

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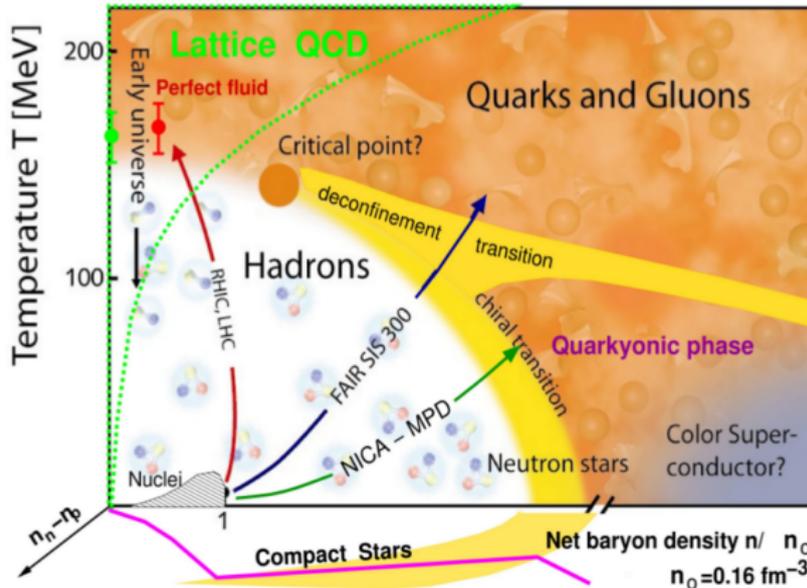
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# What if we have twins

## Important questions

- Does hybrid neutron star exist?
- Does CEP exist on QCD phase diagram?

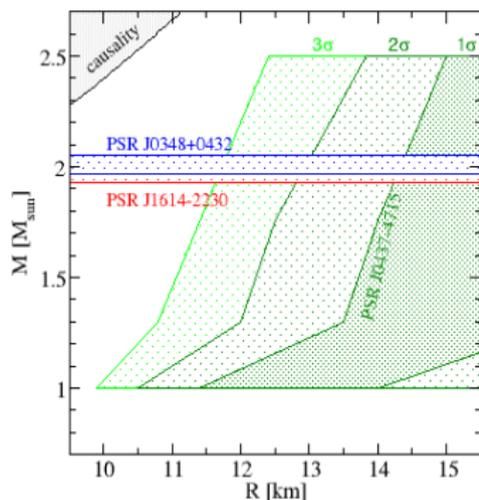
# Existence of CEP at the QCD Phase Diagram



Topic for discussion!

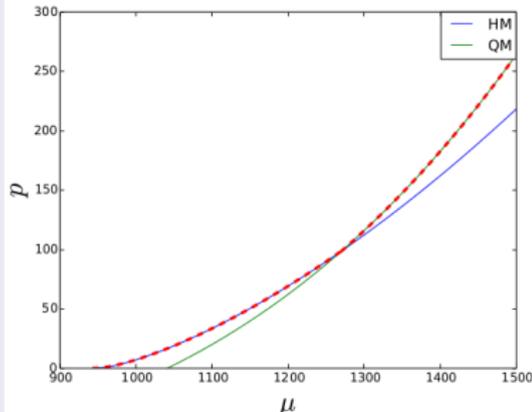
## Mass and Radius Constraints

Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. *Ast. J.* **762**, 96) and PSR J0348+0432 (Antoniadis *et al.* *Sci.* **340**, 6131) correspondingly.

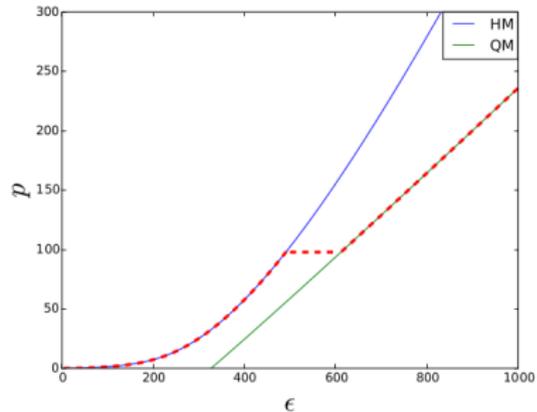


# Maxwell Construction

## Maxwell construction of hybrid EoS



(A)  $p(\mu)$  functions.



(B)  $p(\epsilon)$  functions.

$$p(\mu) = \max [p_H(\mu), p_Q(\mu)]$$

# Hadronic matter EoS

## Excluded-volume DD2

DD2 relativistic meanfield EoS was chosen for the analysis with an excluded volume correction **by S. Typel**.

This correction is applied at suprasaturation densities. In order to quantify the excluded volume effects, we introduce the closest packing parameter

$$\nu = 100 \times n_\nu \text{ fm}^3$$

where  $n_\nu$  is the closest packing density.

The parameter  $\nu$  was varied from 33 to 100, there were  $N_1 = 29$  numbers of this parameter.

# Quark matter EoS

## Quark matter EoS with $\eta_4$ parameter

The quark matter was modelling by a two flavor Nambu-Jona-Lasinio (NJL) model with 8-quark interactions in the scalar and the vector channel **by S. Benic** [arXiv:1503.09145].

The  $\eta_2$  – 4-quark vector couplings parameter was fixed (to describe hybrid stars with masses larger than  $2M_{sun}$  [arXiv:1401.5380]) and the  $\eta_4$  – 8-quark vector NJL couplings parameter was varied from 0 to 30 with step 1 (so,  $N_2 = 31$ ).

# Vector of Parameters

## Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values  $\vec{\pi} = \{\nu, \eta_4\}$ :

$$\vec{\pi}_i = \{\nu_{(k)}, \eta_{4(l)}\},$$

$i = 1 \dots N$  (here  $N = N_1 \times N_2$ ),  $i = N_2 \times k + l$  and  
 $k = 0 \dots N_1 - 1$ ,  $l = 0 \dots N_2 - 1$

# Qualification of the EoS models from Observation

## Goal of the BA

To find posterior probabilities of the set of  $\pi_i$  taking into account the observational constraints.

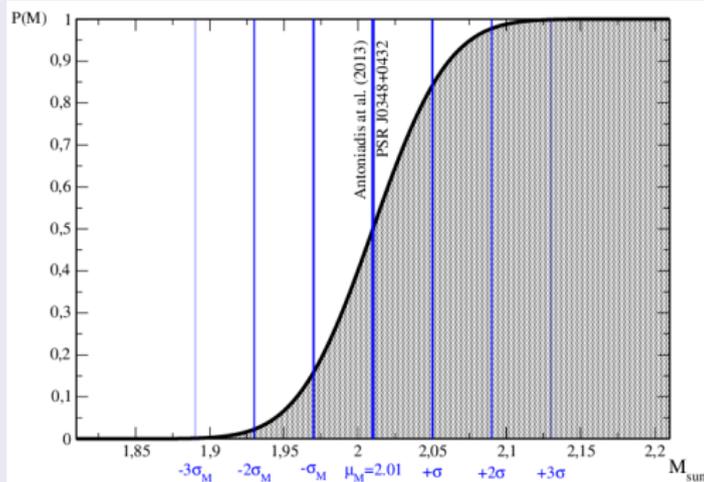
## Unification of priori probabilities

$$P(\pi_i) = 1/N \quad \text{for} \quad \forall i = 0..N - 1.$$

# Calculation of Probabilities

## Probability of Corresponding to Mass Constraint for $\pi_j$

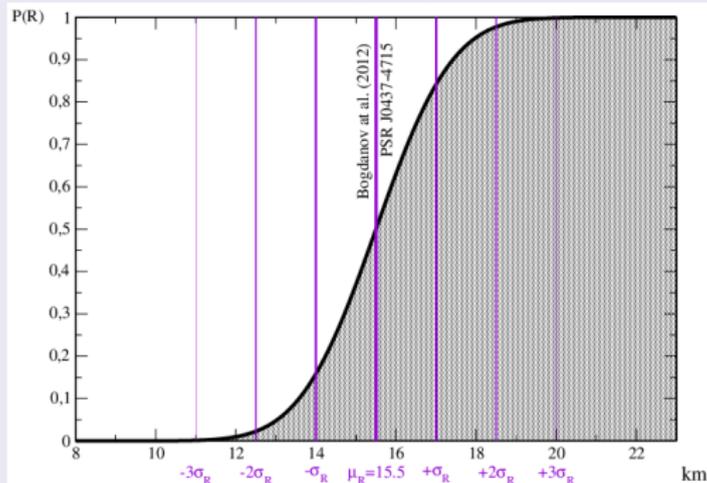
$P(E_A | \pi_j) = \Phi(M_i, \mu_A, \sigma_A)$ , here  $M_i$  is max mass given by  $\pi_j$ .  
 $\mu_A = 2.01 M_\odot$  and  $\sigma_A = 0.04 M_\odot$  [Antoniadis et al. Sci. 340].



# Calculation of Probabilities

## Probability of Corresponding to Radius Constraint for $\pi_i$

$P(E_B | \pi_i) = \Phi(R_i, \mu_B, \sigma_B)$ , here  $R_i$  is max radius given by  $\pi_i$ .  
 $\mu_B = 15.5$  km and  $\sigma_B = 1.5$  km [Bogdanov/Hambaryan et al.].



## Calculation of Probabilities

### Probability of All Constraints for $\pi_j$

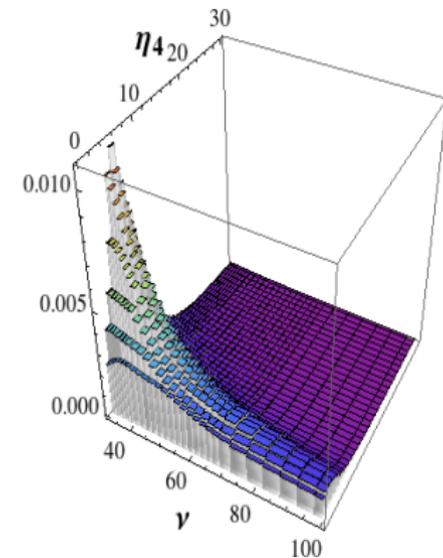
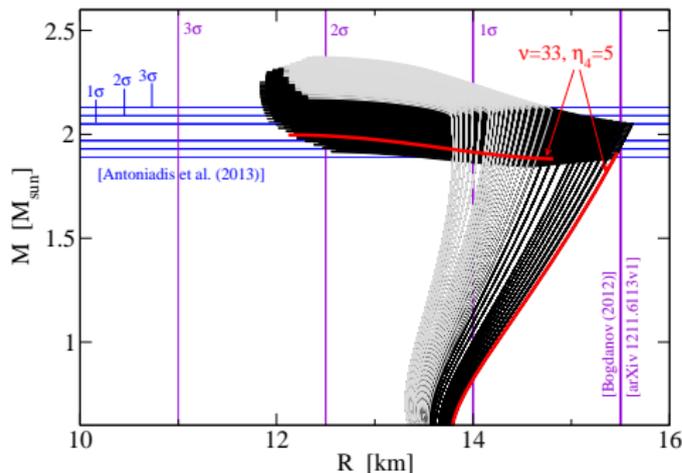
Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

$$P(E|\pi_j) = P(E_A|\pi_j) \times P(E_B|\pi_j)$$

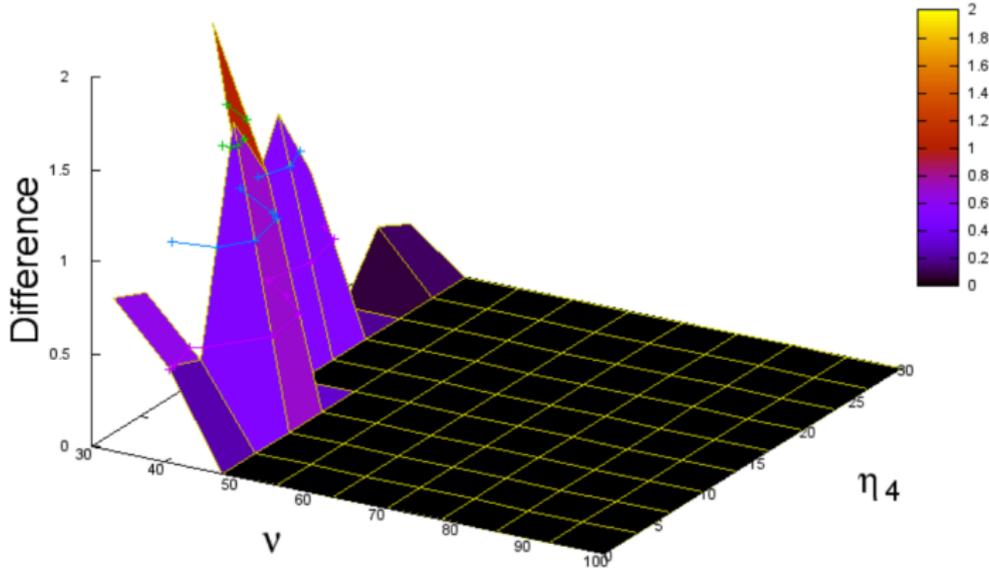
### Calculation of *a posteriori* Probabilities of $\pi_j$

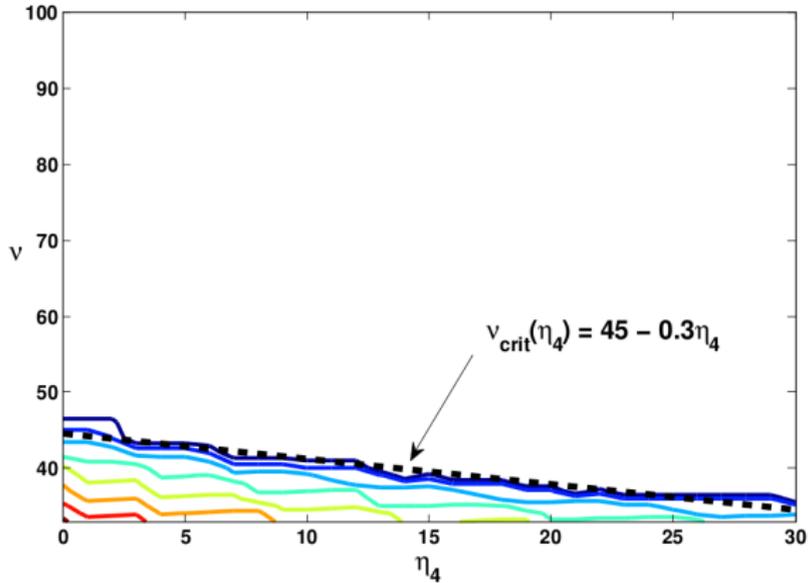
Now, we can calculate posterior probability of  $\pi_j$ :

$$P(\pi_j|E) = \frac{P(E|\pi_j) P(\pi_j)}{\sum_{j=0}^{N-1} P(E|\pi_j) P(\pi_j)}$$



Bayesian analysis of the Maxwell HEOs models based on excluded volume DD2 and NJL8.





# Conclusions

- The most probable models exhibit high-mass twin star configurations with quite distinguishable radii, differing by about 2 km.
- The region of the most probable models in the two-dimensional parameter space is sufficiently narrow, covering the ranges  $33 < \nu < 38$  and  $3 < \eta_4 < 7$ .
- The existence of the horizontal branch signals a strong first order deconfinement phase transition and is a feature accessible to verification by observation. To that end, **at least for two high-mass pulsars** with masses  $\sim 2 M_{\odot}$  (like PSR J1614-2230 and PSR J0348+0432) **the radii should be measured** to sufficient accuracy and turn out to be significantly different.

“Now let us travel into future. It is year **2017**, some new, reliable NS radius measurement methods are discovered and were used to find the size of two most massive pulsars, which still are PSR J0348+0432 and PSR J1614-2230. **The community was shocked** when received the results of observations: one radius is  $13 \pm 0.5$  km, while the other is  $11 \pm 0.5$  km!”

– *Michał Sokołowski*, Master Thesis, 2014