In-medium heavy quarkonium from lattice QCD spectral functions

Alexander Rothkopf
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Heidelberg University

in **collaboration** with: Y.Burnier, O.Kaczmarek, S.Kim and P. Petreczky

References:

Y. Burnier, A.R.: Phys.Rev.Lett. 111 (2013) 182003

A. R., T. Hatsuda, S. Sasaki: Phys.Rev.Lett. 108 (2012) 162001

Y. Burnier, O. Kaczmarek, A. R.: Phys. Rev. Lett. 114 (**2015**) 082001

S.Kim, P. Petreczky, A.R.: Phys.Rev. D91 (**2015**) 054511

Motivation: Heavy-Ion Collisions

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- From RHIC to LHC: golden age of relativistic heavy-ion collision experiments

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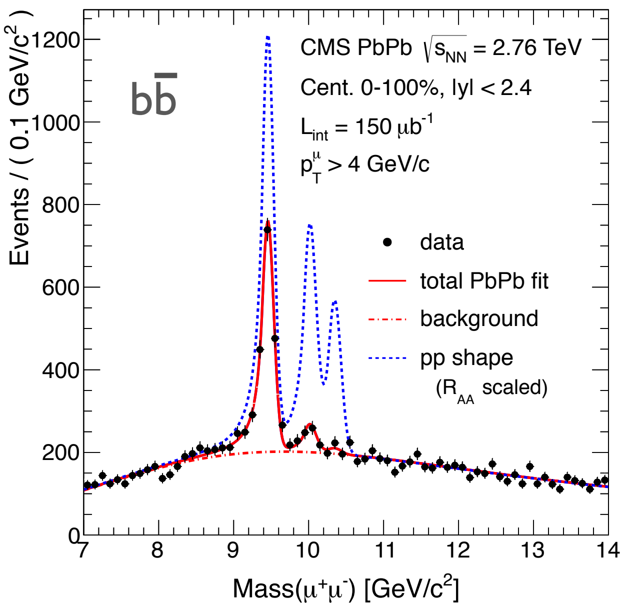
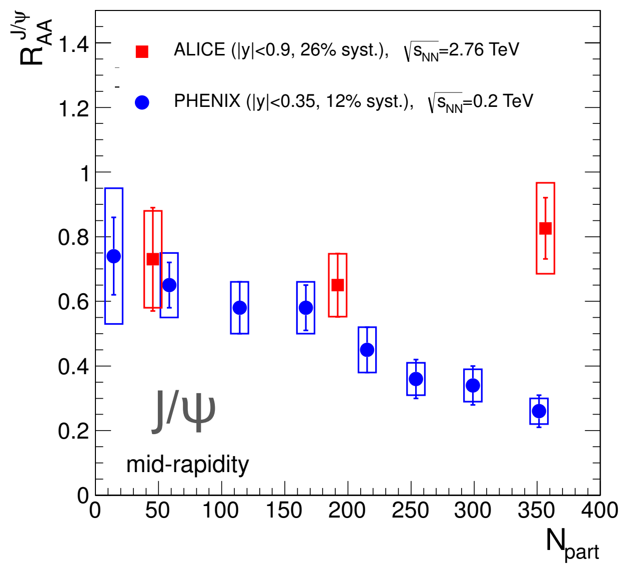


CMS, PRL 109 222301 (2012)



ALICE, PRL 109, 072301 (2012)
adapted from Andronic, et. al. Nucl.Phys.A904 (2013) 535c

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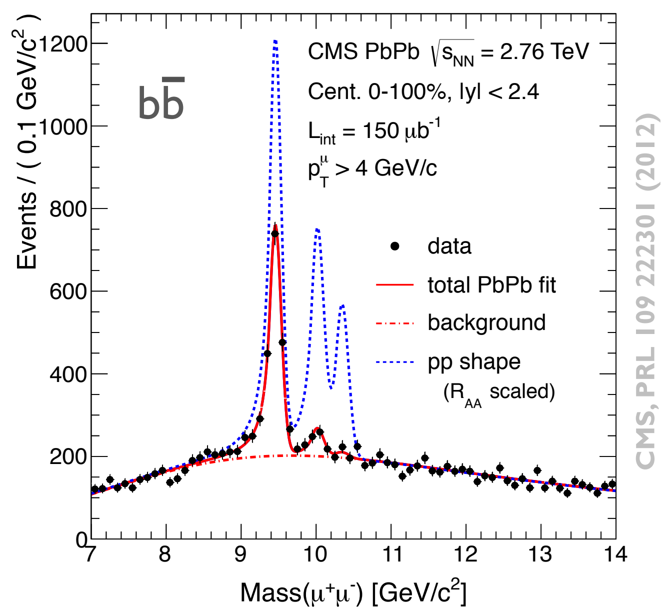
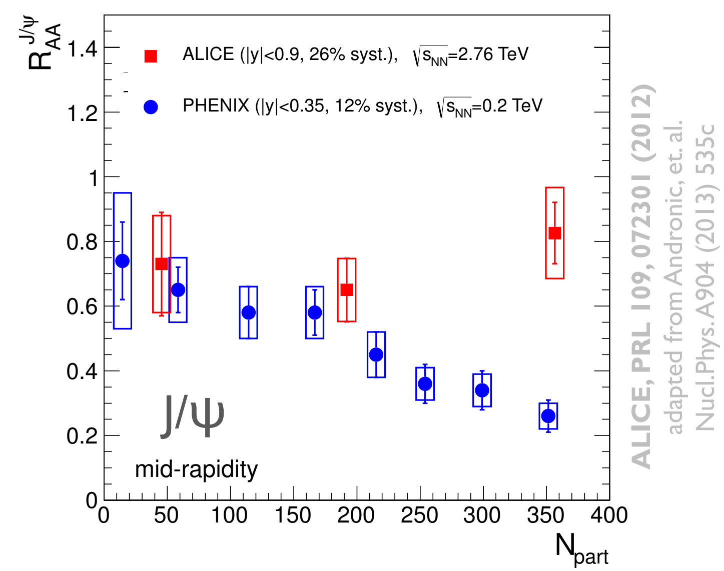
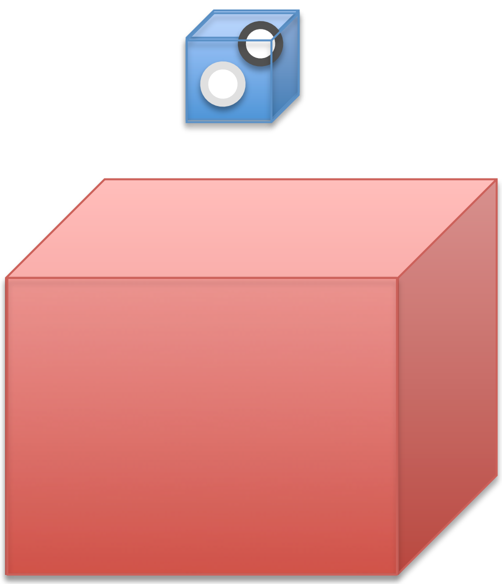




- Theory goal: 1st principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions

Two limits for in-medium $Q\bar{Q}$



T. Matsui and H. Satz: Phys.Lett. B178 (1986) 416

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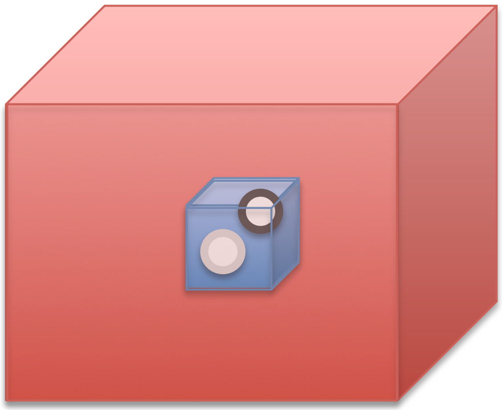

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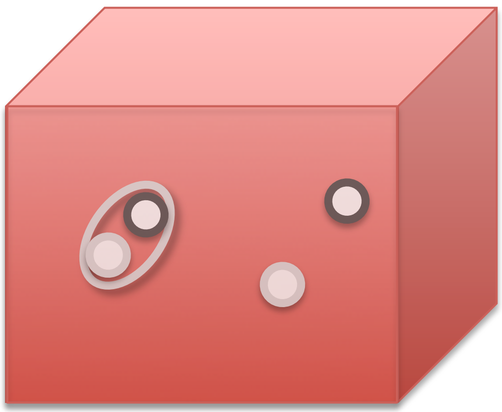

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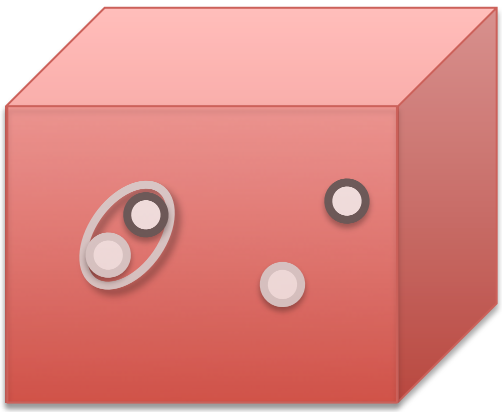

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Static: Kinetically equilibrated heavy quarks

presence of in-medium bound eigenstates?

modern approach: LATTICE QCD meson spectra

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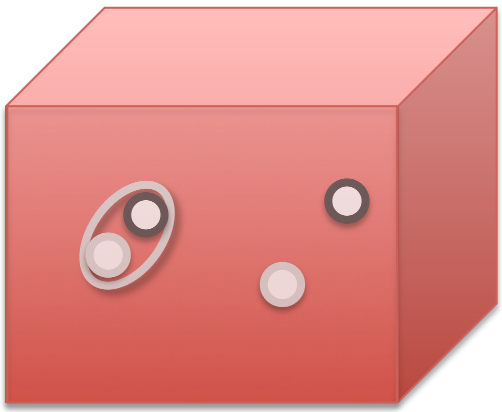

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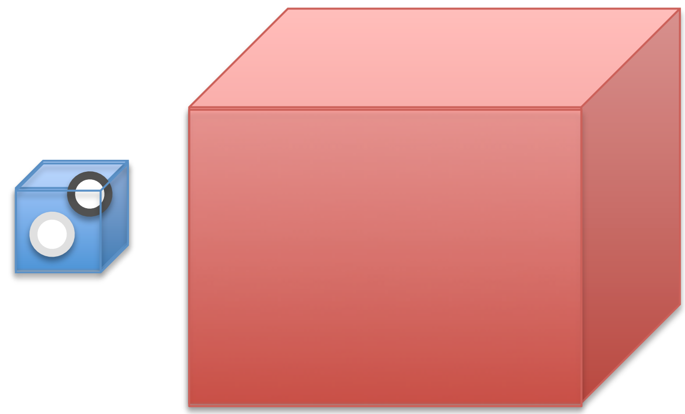

Quarkonium as Open-Quantum System
see e.g. Y. Akamatsu, A.R. PRD85 (2012) 105011

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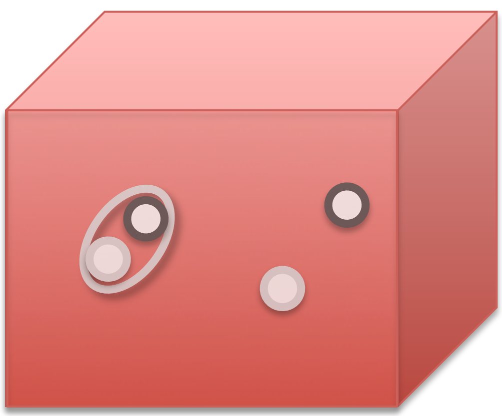

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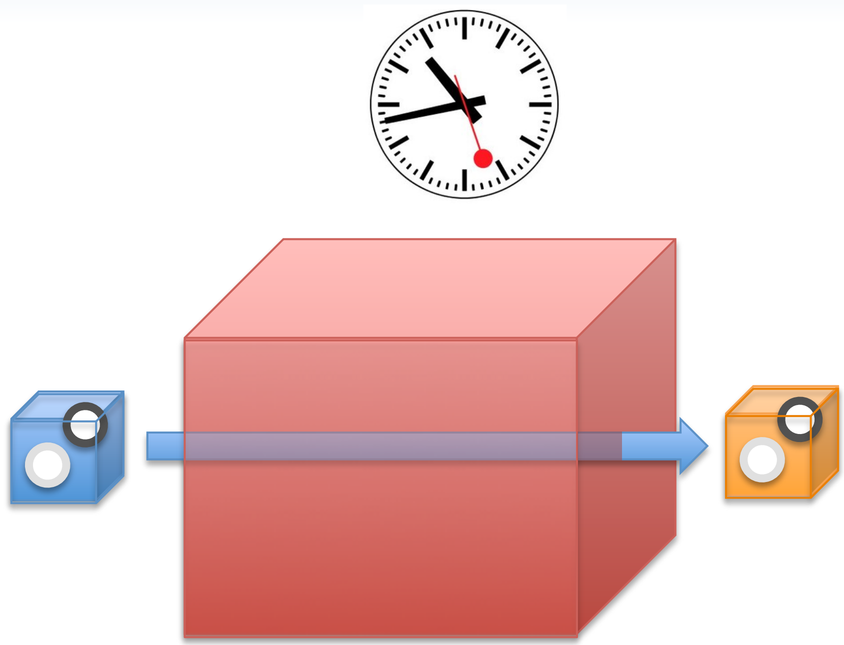

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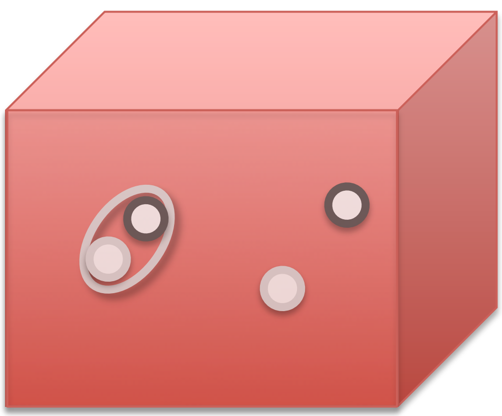

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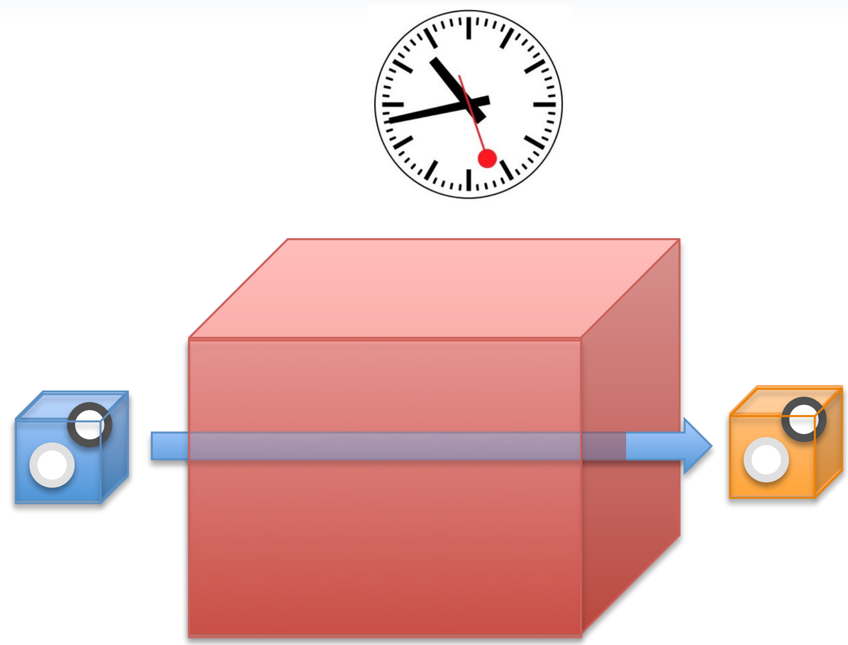

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Dynamical: real-time approach to equilibrium

redistribution of states over time?

LATTICE QCD based potential description

Y. Burnier, O. Kaczmarek, A. R.: Phys. Rev. Lett. 114, 082001 (2015)
A. R., T. Hatsuda, S. Sasaki: Phys.Rev.Lett. 108 162001 (2012)

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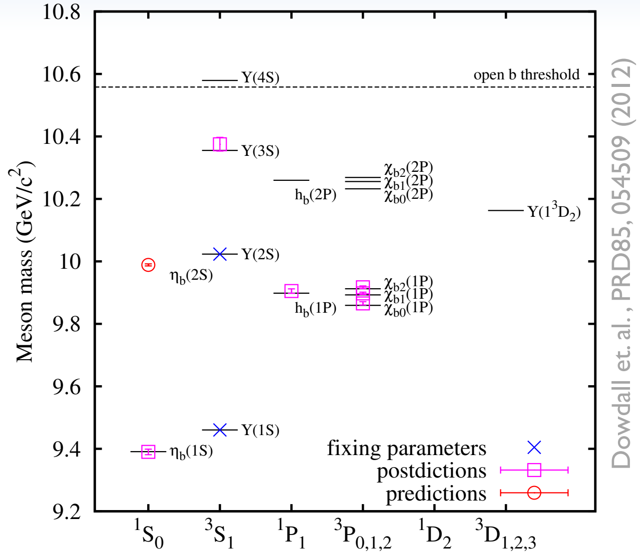
A robust tool: Lattice QCD

- Successful at T≈0: Quarkonium spectra
  - No modeling: starting point is discretized QCD action



Dowdall et. al., PRD85, 054509 (2012)

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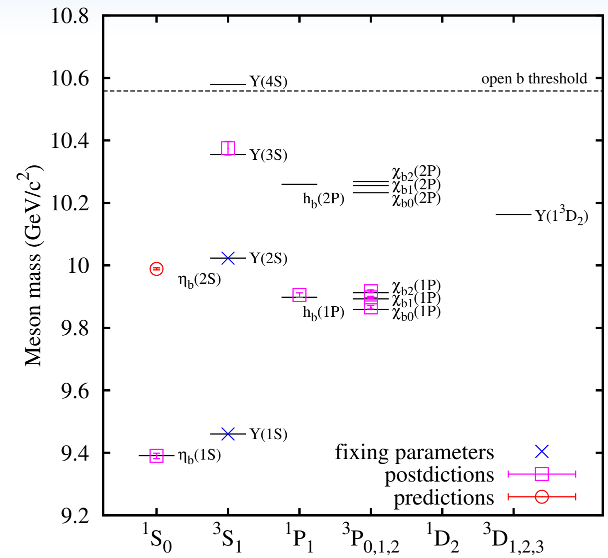

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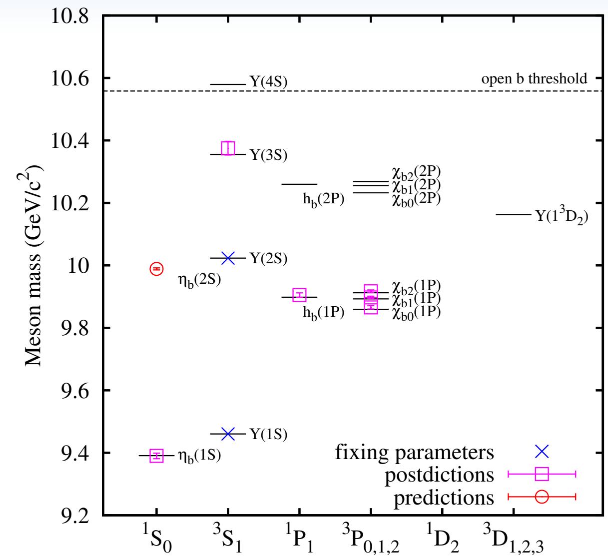

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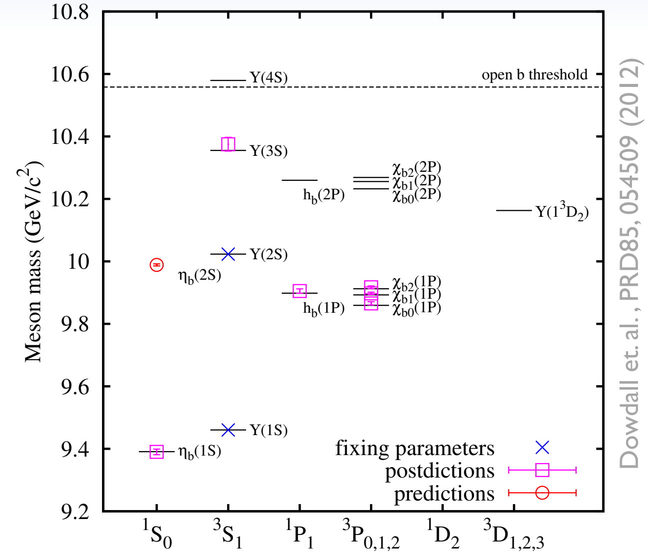
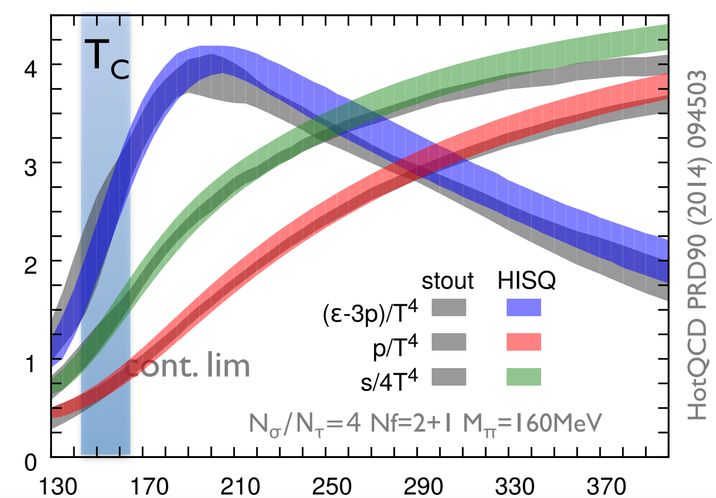


- Successful at T>0: QCD medium properties
  - (Pseudo)critical temperature: 154±9 MeV

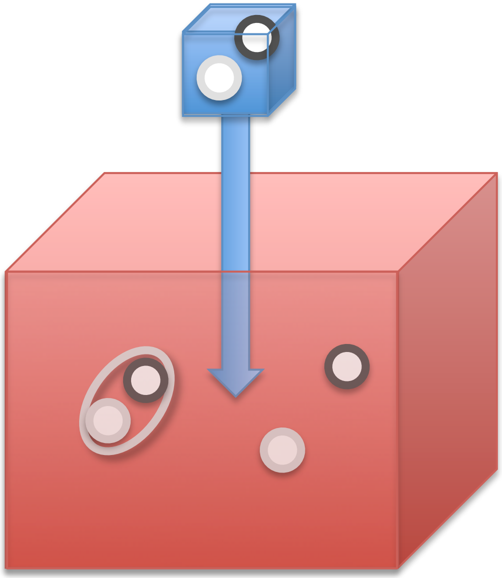
    WB JHEP 1009 (2010) 073 - HotQCD PRD85 (2012) 054503

  - Trace anomaly $\Theta^{\mu\mu} = \varepsilon - 3p$: strong coupling at T_C

    HotQCD PRD90 (2014) 094503 - WB PLB730 (2014) 99-104, see also tmfT PRD91 (2015) 7, 074504



In-medium $Q\bar{Q}$ part I



LATTICE QCD Bottomonium spectra

S.Kim, P. Petreczky, A.R.: Phys.Rev. D91 (2015) 054511

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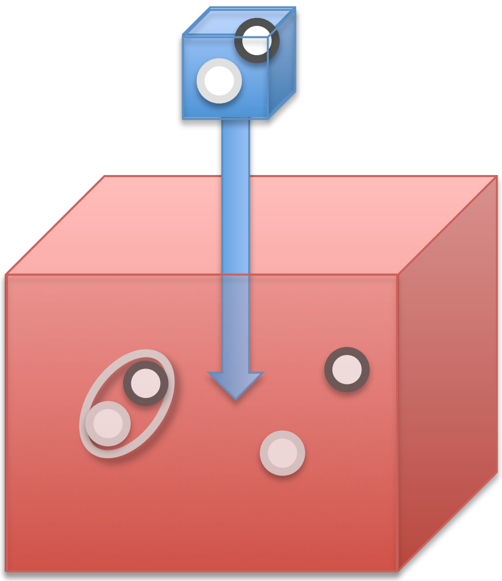

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PRACTICAL CHALLENGE: High cost if light and heavy d.o.f share the same spacetime grid

for a direct approach see e.g. H.T. Ding et. al. Phys.Rev. D86 (2012) 014509

$$a \ll \frac{1}{2m_b} \approx 0.02\mathrm{fm} \qquad \frac{1}{T} = N_\tau a \sim 1\mathrm{fm}$$

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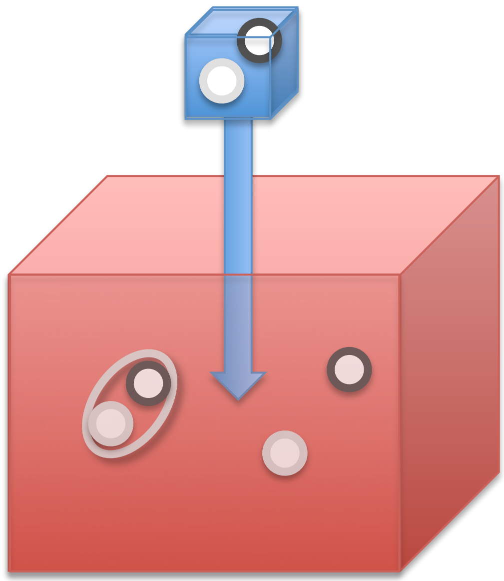

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Turn separation of scales into an advantage: effective field theory NRQCD

Thacker, Lepage Phys.Rev. D43 (1991) 196-208

Heavy Quarks on the Lattice

- Effective field theory from scale separation: $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

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Relativistic thermal field theory

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Dirac fields

$\bar{Q}(x), Q(x)$

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QCD	NRQCD
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$$L_{NRQCD} = \chi^\dagger\left(iD_t + \frac{D_i^2}{2M_Q} + \dots\right)\chi + \xi^\dagger(\dots)\xi$$

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$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{q}(\ldots)q$$

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- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau) = \langle\chi(\tau)\chi^\dagger(0)\rangle$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau)\left(1 - \frac{\mathbf{p}_{lat}^2}{4M_Q a} + \ldots\right)G(\mathbf{x}, \tau)$$

well behaved if $M_Q a > 1.5$

Davies, Thacker Phys.Rev. D45 (1992)

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- $^3S_1$ (Y) and $^3P_1$ (χ_{b1}) channel correlators $D(\tau)$ from heavy quark propagators $G(\tau)$

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) G_{x\tau} O^\dagger(\mathbf{x}_0, \tau_0) G^\dagger_{x\tau}\rangle_{med} \qquad O(^3S_1; x, \tau) = \sigma_i, \quad O(^3P_1; x, \tau) = \overleftrightarrow{\Delta}_i\sigma_j - \overleftrightarrow{\Delta}_j\sigma_i$$

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T>0 QCD with N_f=2+1 HISQ flavors

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- Light d.o.f. (gluons, u d s quarks) represented by realistic HotQCD lattices

A. Bazavov et. al., Phys. Rev. D 85 (2012) 054503

HotQCD HISQ/tree action $48^3 \times N_\tau$ $m_{u,d}/m_s = 0.05$ $T_C = 154(9)\text{MeV}$

β	6.664	6.700	6.740	6.770	6.800	6.840	6.880
$a[\text{fm}]$	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau = 12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
$a[\text{fm}]$	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
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- 48^3x12 with relatively light pions

M_π~ 161MeV and a **T_C=159±3MeV**

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- Important for the use with lattice NRQCD:

2.759 > $M_b a$ > 1.559

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a[fm]	0.1169	0.1130	0.1087	0.1057	0.1027	0.09893	0.09528
$M_b a$	2.759	2.667	2.566	2.495	2.424	2.335	2.249
$T/T_C(N_\tau = 12)$	0.911	0.944	0.980	1.008	1.038	1.078	1.119
β	6.910	6.950	6.990	7.030	7.100	7.150	7.280
a[fm]	0.09264	0.08925	0.086	0.08288	0.07772	0.07426	0.06603
$M_b a$	2.187	2.107	2.030	1.956	1.835	1.753	1.559
$T/T_C(N_\tau = 12)$	1.151	1.194	1.240	1.286	1.371	1.436	1.614

- 48^3x12 with relatively light pions

M_π~ 161MeV and a **T_C=159±3MeV**

- Important for the use with lattice NRQCD:

2.759 > M_ba > 1.559

- Temperature changed by variation of the lattice spacing 140MeV < T < 249MeV

For a study based on the fixed scale approach see: FASTSUM G. Aarts et. al. JHEP 1407 (2014) 097, JHEP 1111 (2011) 103

T>0 QCD with N_f=2+1 HISQ flavors

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- Light d.o.f. (gluons, u d s quarks) represented by realistic HotQCD lattices

A. Bazavov et. al., Phys. Rev. D 85 (2012) 054503

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- For calibration T≈0 configurations available at b=6.664, 6.8, 6.95, 7.28 (48^3x32,64)

A Novel Bayesian Approach

- Inversion of Laplace transform required to obtain spectra from correlators

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$

A Novel Bayesian Approach

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- Inversion of Laplace transform required to obtain spectra from correlators

$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

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  M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

  - Bayes theorem: Regularize the naïve χ^2 functional P[D|ρ] through a prior P[ρ|I]

$$P[\rho|D, I] \propto P[D|\rho]\; P[\rho|I]$$

A Novel Bayesian Approach

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Y.Burnier, A.R.
PRL 111 (2013) 18, 182003

A Novel Bayesian Approach

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$$\frac{\delta}{\delta\rho} P[\rho|D,I]\bigg|_{\rho=\rho^{BR}} = 0$$

  - No apriori restriction on the search space
  - In the following: constant default model m_l=const

T≈0 Bayesian Bottomonium Spectra

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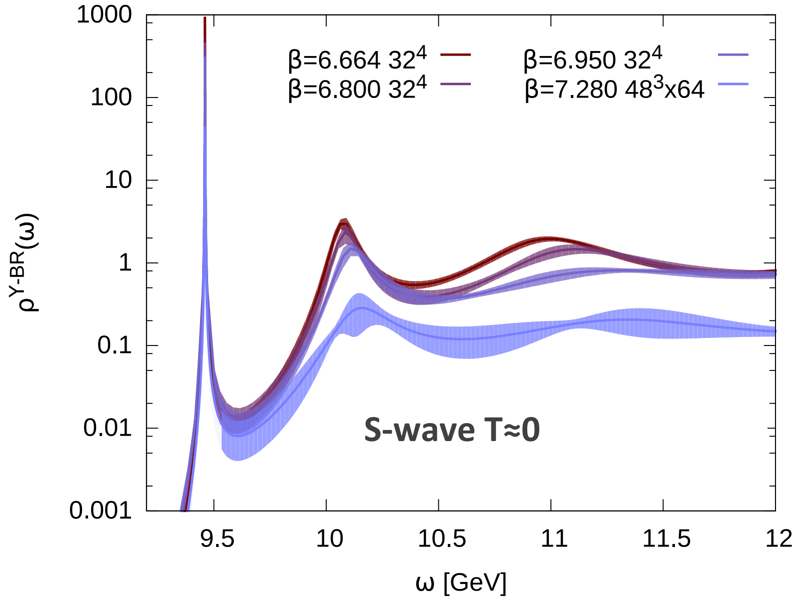
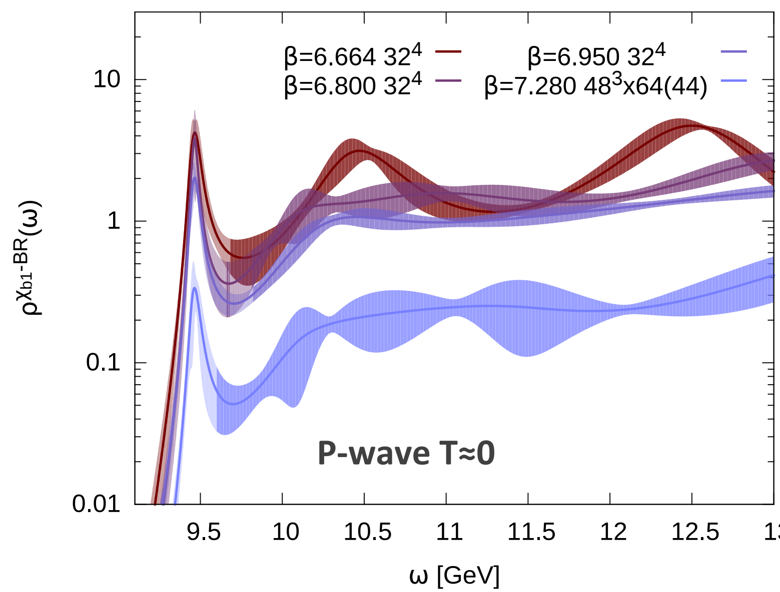
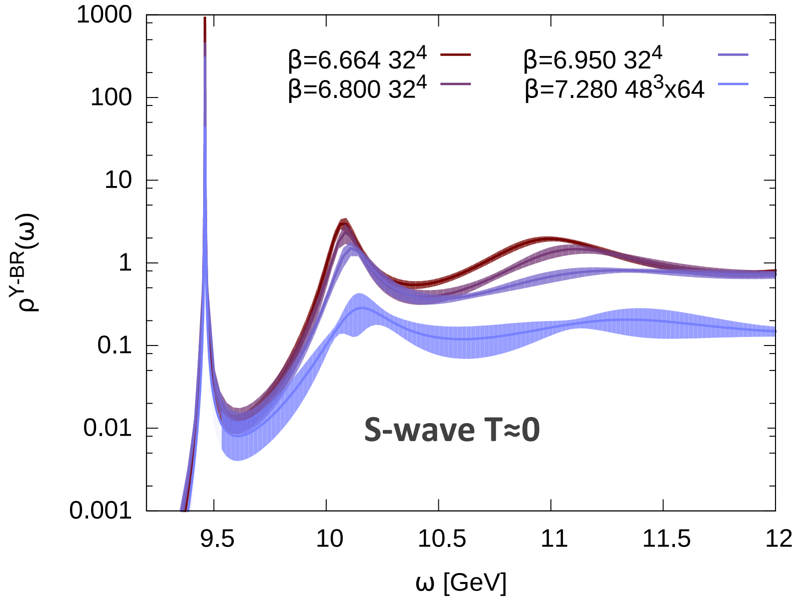
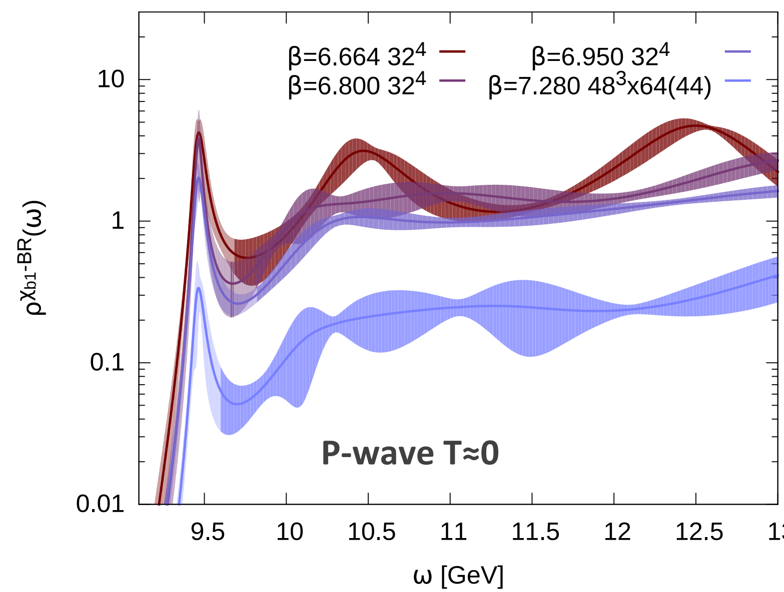




S.Kim, P.Petreczky, A.R.
Phys.Rev. D 91 (2015) 054511

T≈0 Bayesian Bottomonium Spectra





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T≈0 Bayesian Bottomonium Spectra

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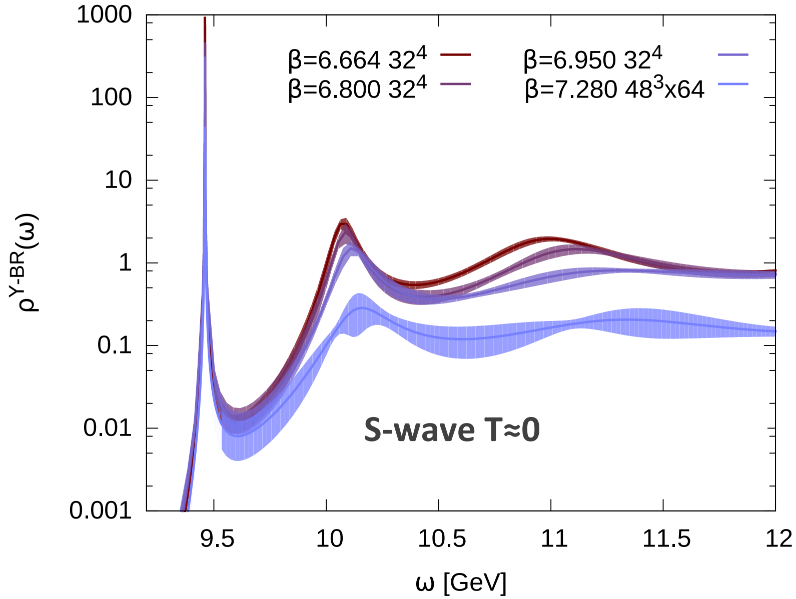

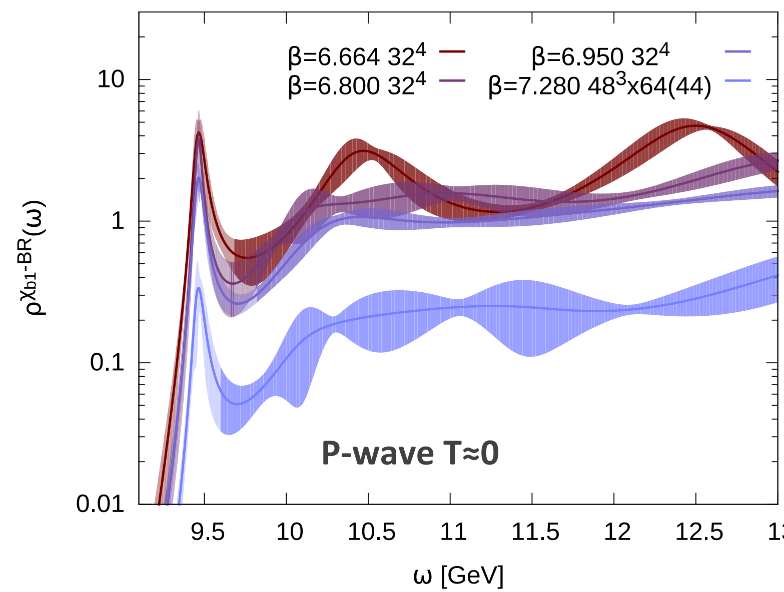

S.Kim, P.Petreczky, A.R.
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$$M^{\rm NRQCD}_{\chi_{b1}} = 9.917(3)\,{\rm GeV} \; > \; M^{\rm exp}_{\chi_{b1}(1P)} = 9.89278(26)(31)\,{\rm GeV}$$

T≈0 Bayesian Bottomonium Spectra

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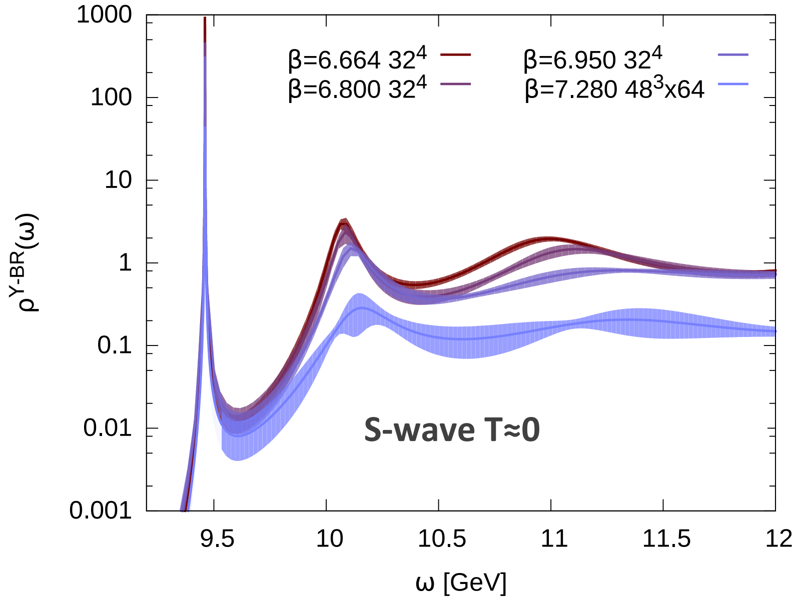

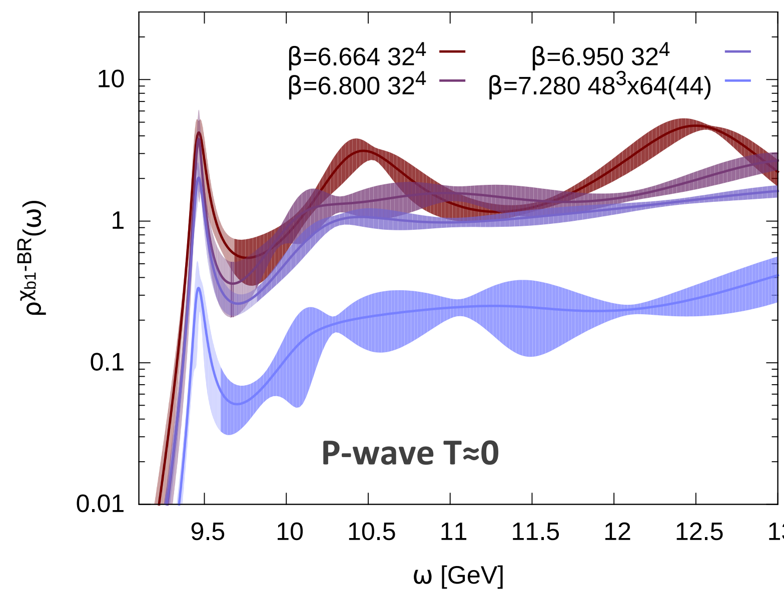

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- P-wave ground state broader: worse s/n ratio and smaller physical peak size

Reconstruction Accuracy: S-wave

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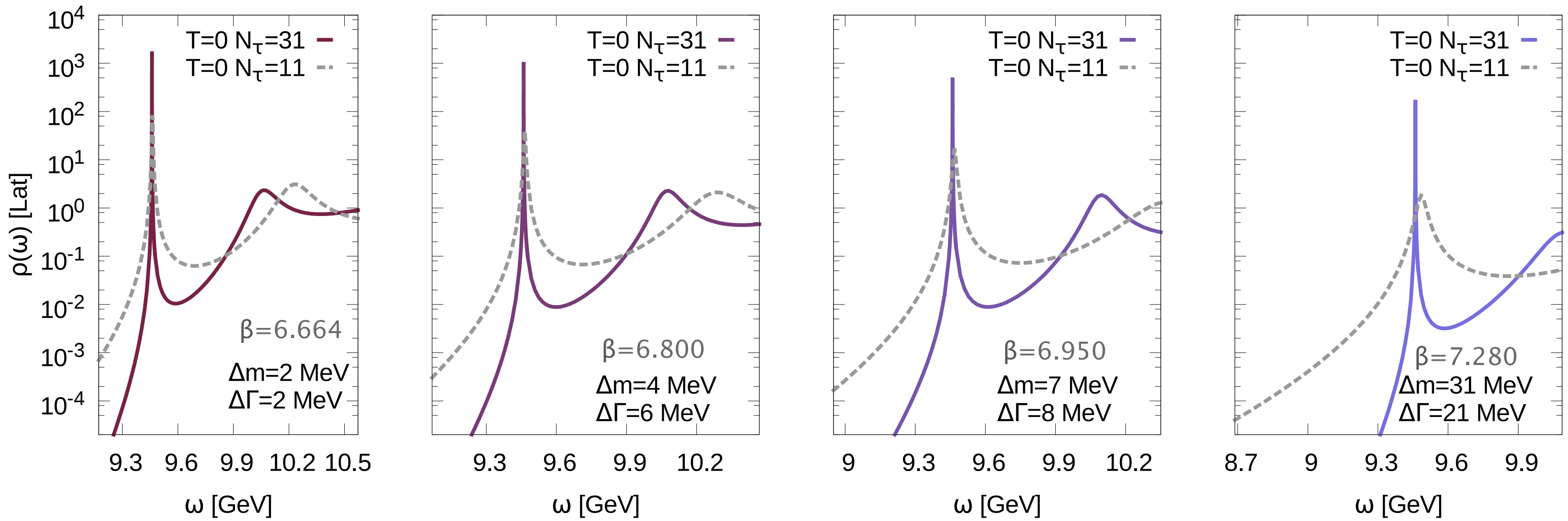

S.Kim, P.Petreczky, A.R.
Phys.Rev. D 91 (2015) 054511

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- How does accuracy suffer from limited available information at T>0 (Nτ=12) ?
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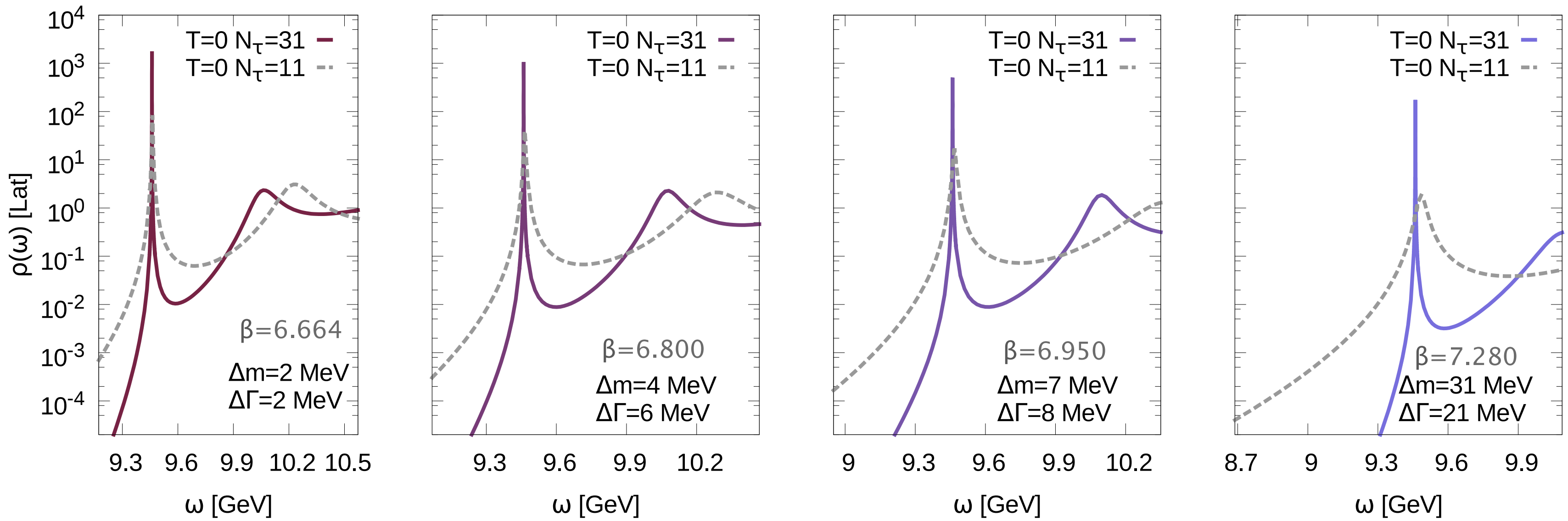

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Overall Limits:

$$\beta = 6.664 : \quad \Delta m_T < 2\text{MeV}, \quad \Delta\Gamma_T < 5\text{MeV}$$
$$\beta = 7.280 : \quad \Delta m_T < 40\text{MeV}, \quad \Delta\Gamma_T < 21\text{MeV}$$

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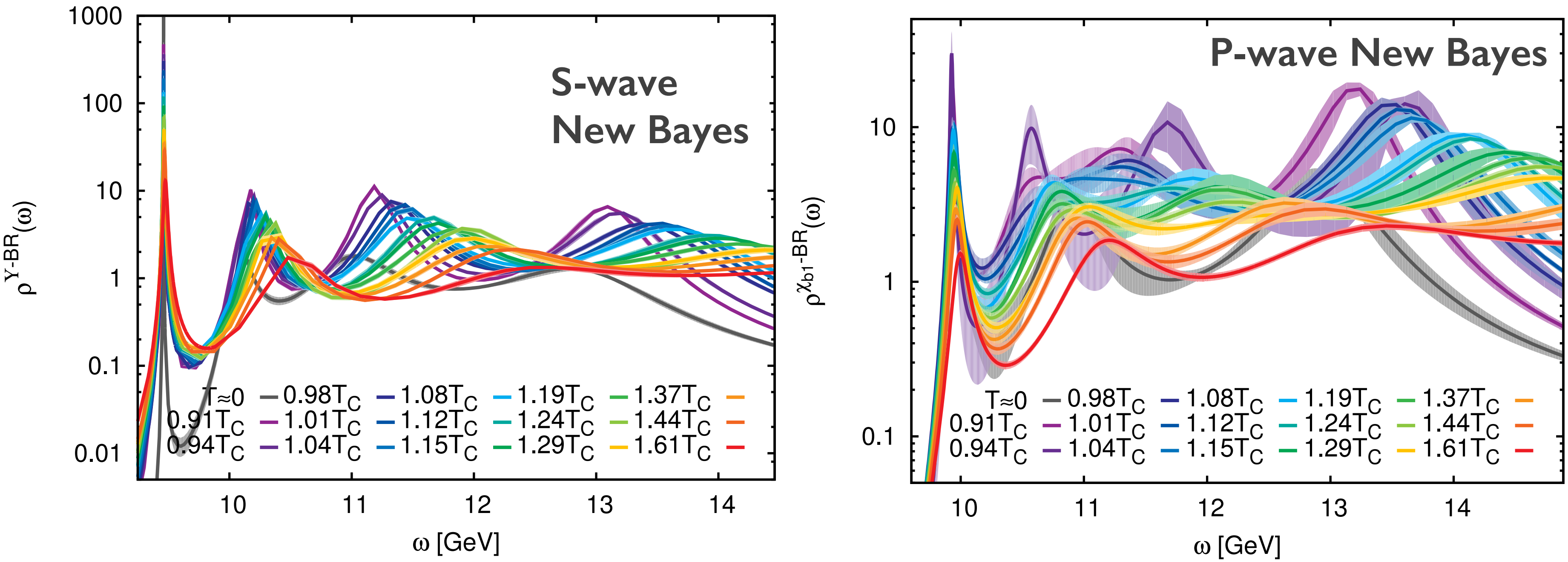
Spectral Functions At T>0



S.Kim, P.Petreczky, A.R. Phys.Rev. D 91 (2015) 054511

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N_ω=1200 I_ω=[-1,25] β^{num}=20 N_{jack}=10

m_l=const 512 bit precision, Δ tol=10^{-60}

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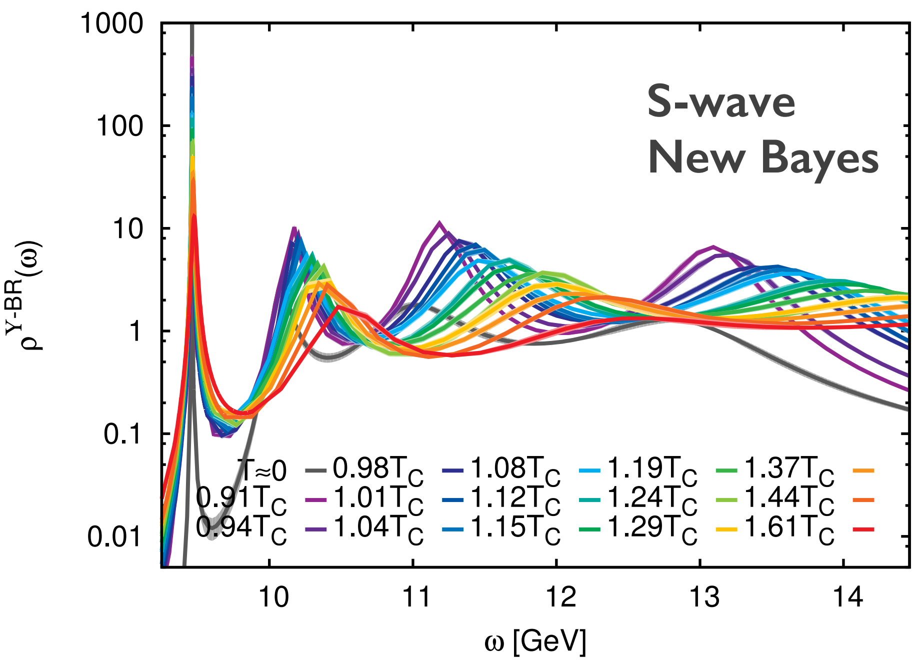

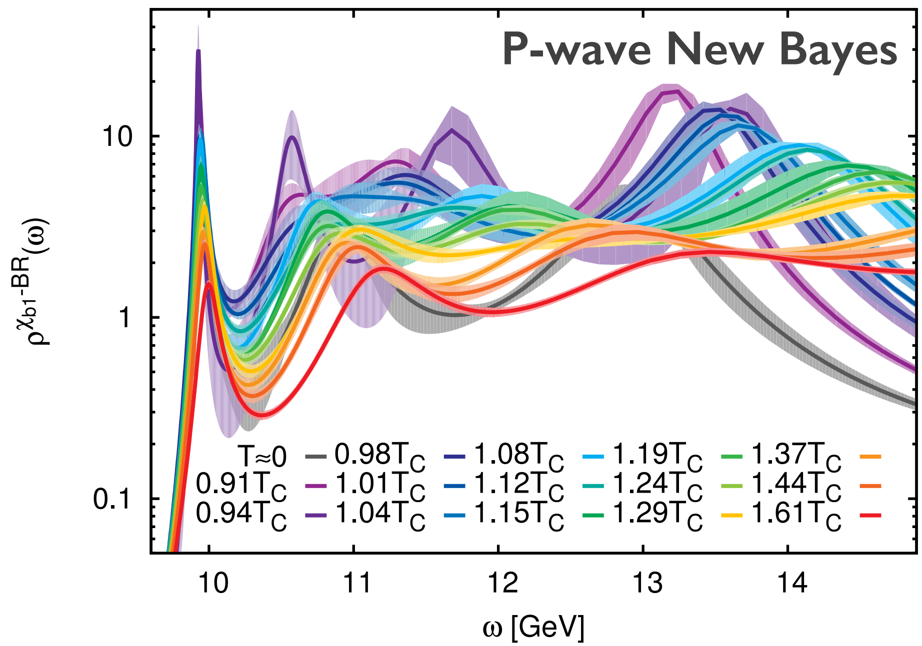

S.Kim, P.Petreczky, A.R. Phys.Rev. D 91 (2015) 054511

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- Naïve inspection by eye: S-wave ground state peak present up to 249MeV

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P-wave survival at T=249MeV

- Our strategy: systematic comparison to **non-interacting spectra**

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Analytically known, no peaked features

$$a_\tau E_{\mathbf{p}} = -\log\left(1 - \frac{\mathbf{p}^2_{\rm lat}}{8 M_b a_s}\right)$$

$$\rho_S(\omega) = \frac{4\pi N_c}{N_s^2} \sum_{\mathbf{p}} \delta(\omega - 2E_{\mathbf{p}})$$

G.Aarts et. al., **JHEP 1111 (2011) 103**

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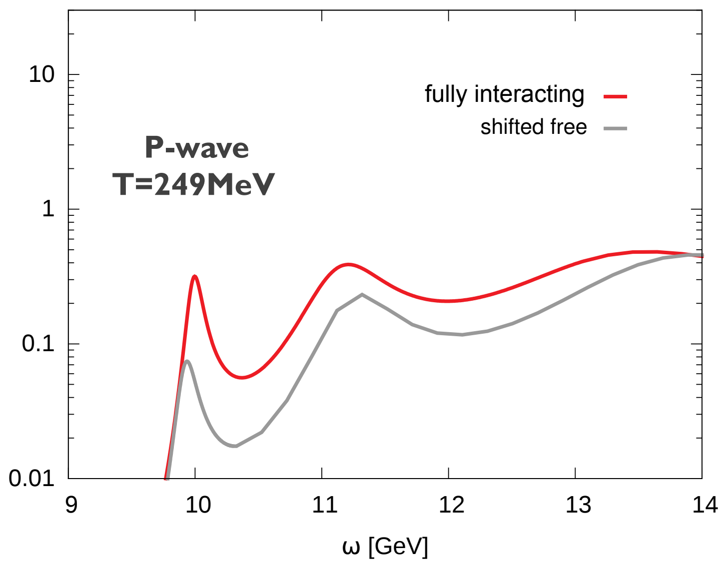
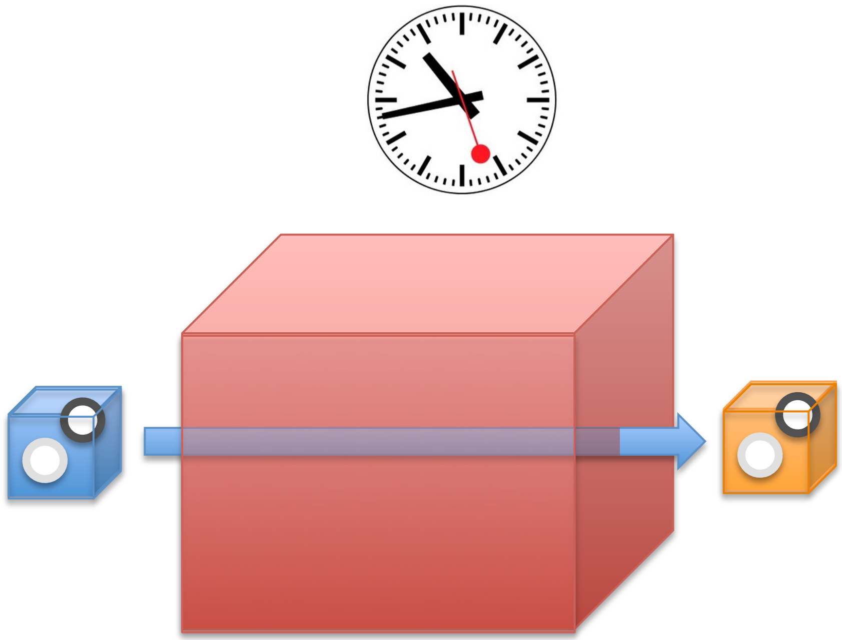


S.Kim, P.Petreczky, A.R.
Phys.Rev. D 91 (2015) 054511

- At T=249 MeV: Ground state peak stronger than numerical ringing by factor 3

In-medium $Q\bar{Q}$ part II

Conceptual Challenge: How to define the potential at finite temperature?



QQbar as Open-Quantum System

Lattice QCD based potential description

Y. Burnier, O. Kaczmarek, A. R.: Phys. Rev. Lett. 114, 082001 (2015)
A. R., T. Hatsuda, S. Sasaki: Phys. Rev. Lett. 108, 162001 (2012)

Defining the heavy quark potential

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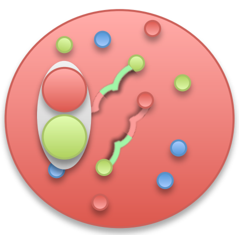
- Effective field theory

Brambilla et. al.
Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

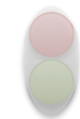


QCD
Dirac fields
$\bar{Q}(x), Q(x)$

NRQCD
Pauli fields
$\chi^\dagger(x), \chi(x)$

$\bar{q}(x), q(x), A^\mu(x)$

Quantum mechanics



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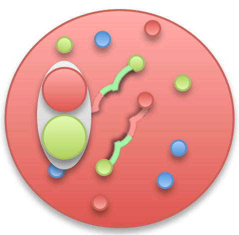
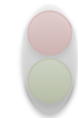


QCD → **NRQCD** → **pNRQCD** → Quantum mechanics

Dirac fields: $\bar{Q}(x), Q(x)$

Pauli fields: $\chi^{\dagger}(x), \chi(x)$

Singlet/Octet: $\psi_S(R,t), \psi_O(R,t)$



$\bar{q}(x), q(x), A^{\mu}(x)$

$$i\partial_t \psi_S = \left(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\right)\psi_S$$

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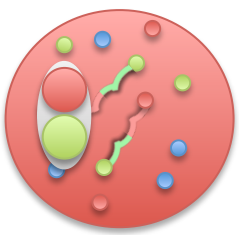

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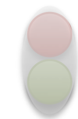
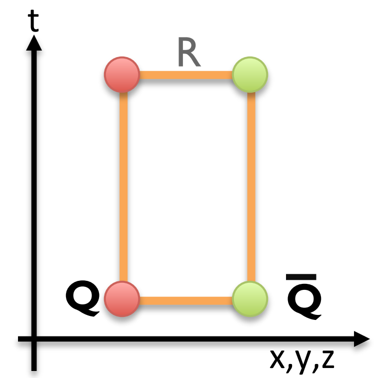


- Matching between QCD and pNRQCD in the static limit

$$\langle \psi_S(R,t)\psi_S^*(R,0)\rangle_{pNRQCD} \equiv W_\square(R,t) = \mathrm{Tr}\Big(\exp\Big[-i\int_\square dx^\mu A_\mu(x)\Big]\Big)$$



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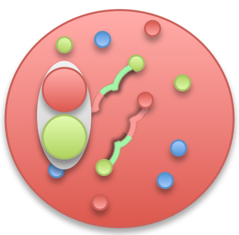

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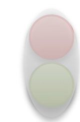

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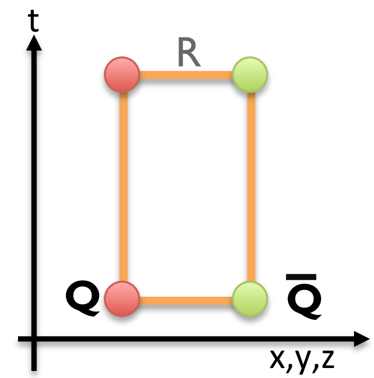

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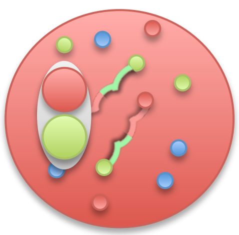
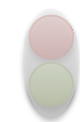


QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$	$\psi_S(R,t), \psi_O(R,t)$
$\bar{q}(x), q(x), A^\mu(x)$		$i\partial_t\psi_S = \left(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\right)\psi_S$

Quantum mechanics

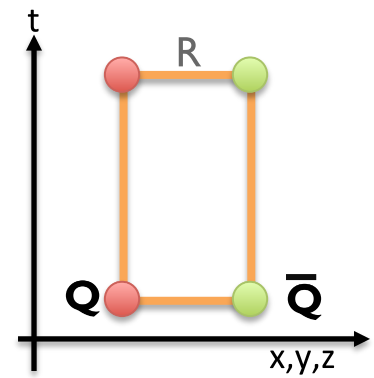


- Matching between QCD and pNRQCD in the static limit

$$\langle\psi_S(R,t)\psi_S^*(R,0)\rangle_{pNRQCD} \equiv W_\square(R,t) = \mathrm{Tr}\left(\exp\left[-i\int_\square dx^\mu A_\mu(x)\right]\right)$$

$$i\partial_t W_\square(R,t) \stackrel{t>>t_{med}}{=} V^{QCD}(R) W_\square(R,t)$$

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)}$$



Defining the heavy quark potential

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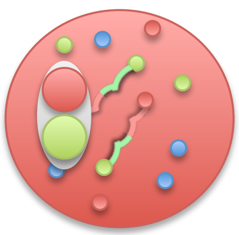
- Effective field theory

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory



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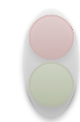

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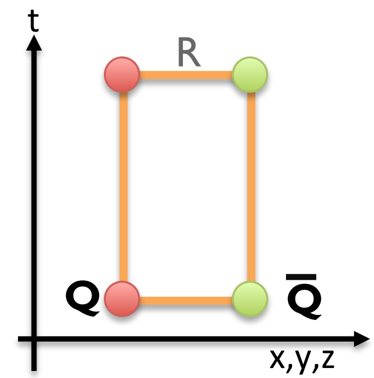

Im[V] first observed in Laine et al. JHEP03 (2007) 054;
For a discussion of Im[V] see e.g. A.R. JHEP 1404 (2014) 085

Extracting V^{QCD} from lattice QCD

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- On the lattice real-time observables not directly accessible!

Extracting V^{QCD} from lattice QCD

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- How to connect to the Euclidean domain: **spectral functions**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, \rho_\square(R,\omega)$$

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A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

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Bayesian spectral reconstruction

Y.Burnier, A.R. PRL 111 (2013) 182003

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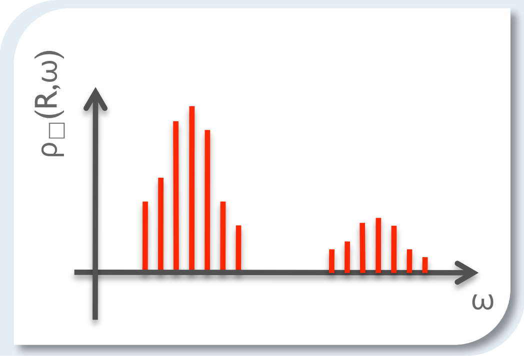

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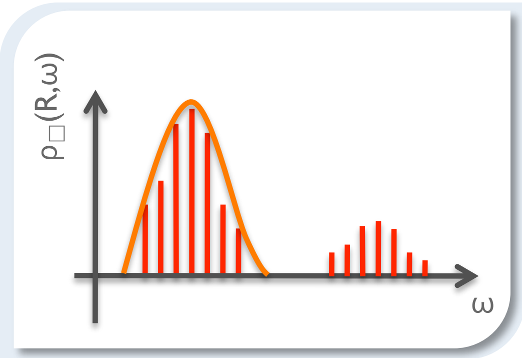

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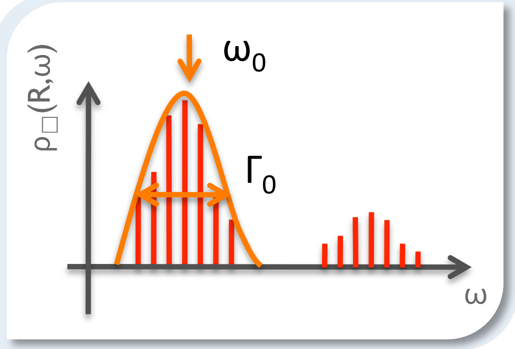

$$\rho_\square(R,\omega)=\frac{1}{\pi}e^{\gamma_1(R)}\frac{\Gamma_0(R)\cos[\gamma_2(R)]-(\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R)+(\omega_0(R)-\omega)^2}+\kappa_0(R)+\kappa_1(R)(\omega_0(R)-\omega)+\dots$$

Extracting V^{QCD} from lattice QCD

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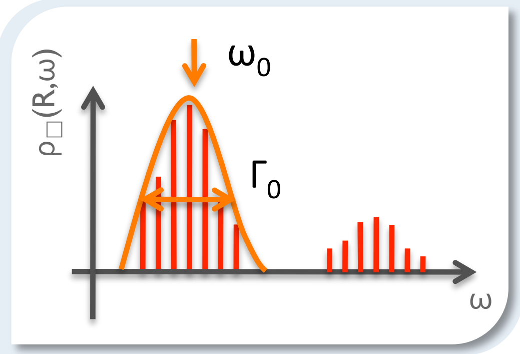

$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \dots$$

$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

Summary: V^{QCD} from the lattice

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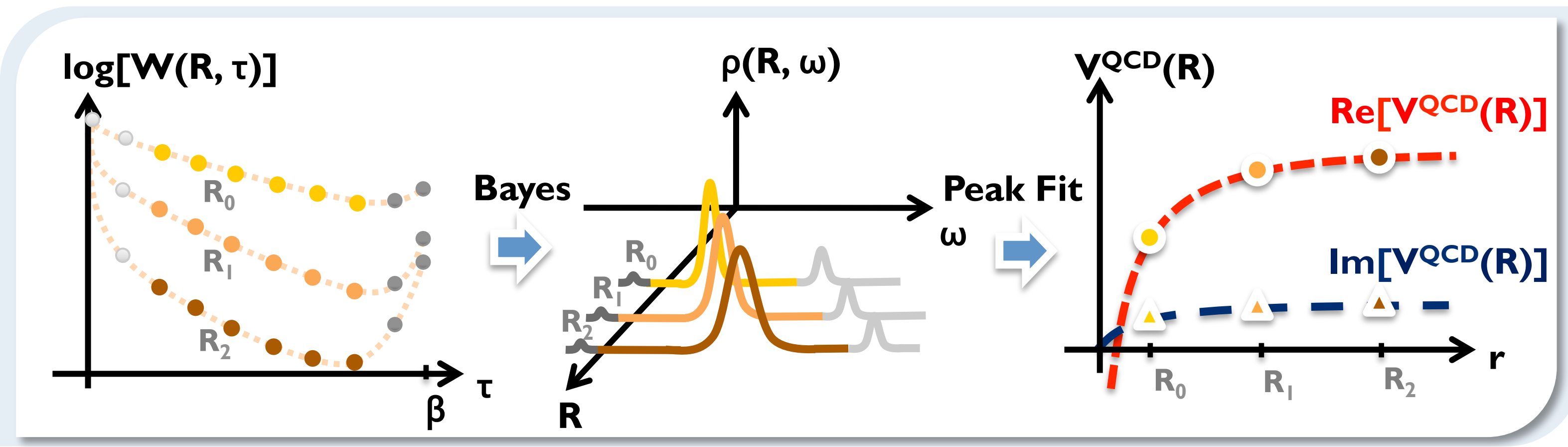
- From lattice QCD correlators to the complex heavy quark potential

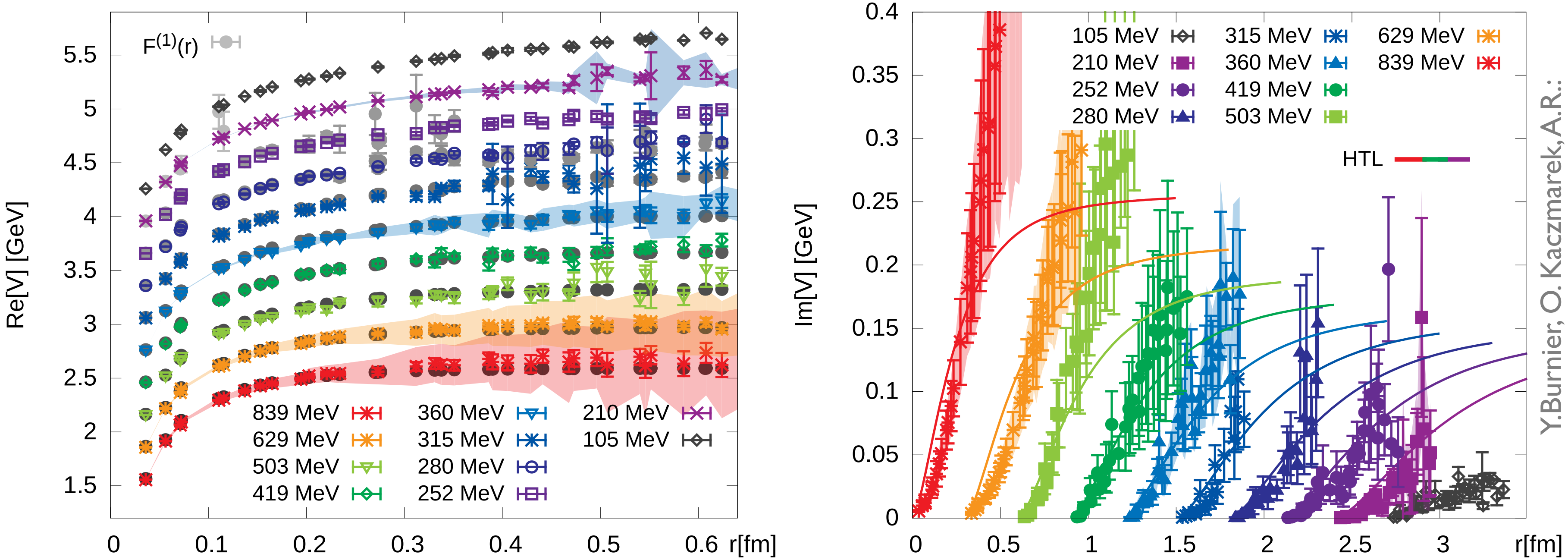
A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)



  - Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
    Practical reason: Absence of cusp divergences, hence less suppression along τ

V^{QCD} in quenched lattice QCD

Fixed scale approach: β=7.0 $\xi=a_s/a_\tau=4$ a_s=0.039fm N_τ=192-24



Y.Burnier, O. Kaczmarek, A.R.: Phys. Rev. Lett. 114 (2015) 082001

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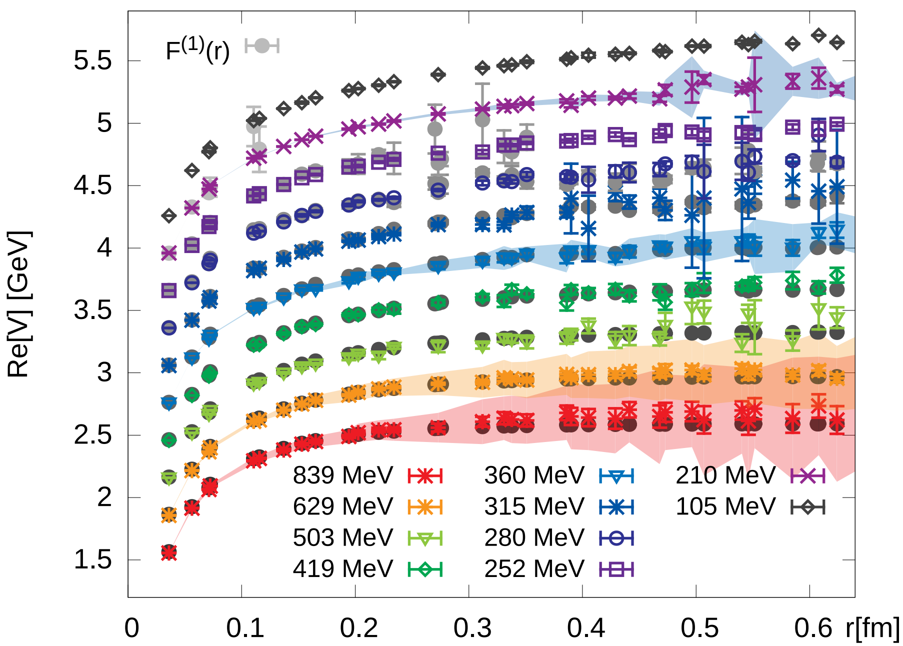

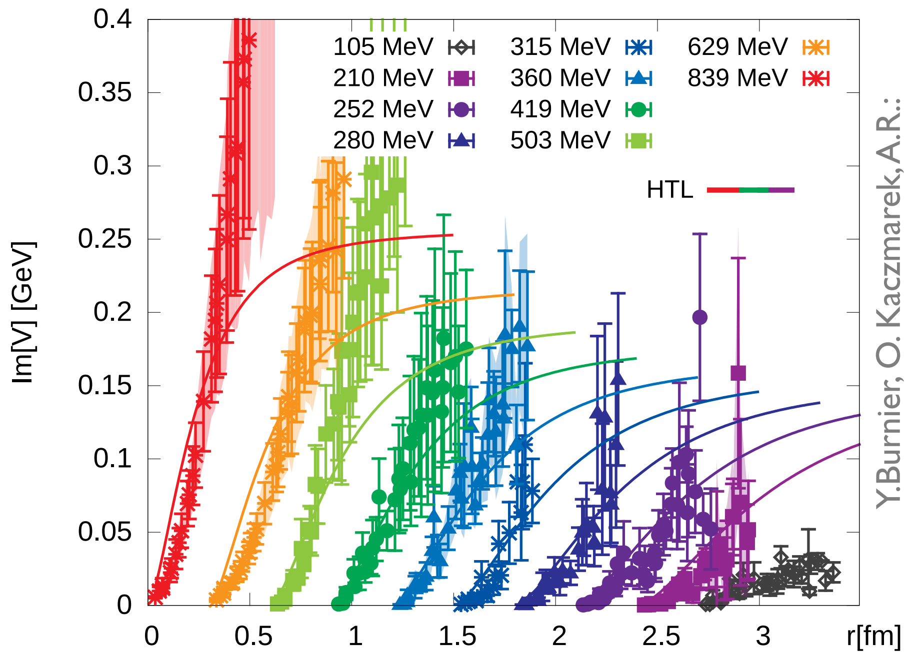

Y.Burnier, O. Kaczmarek, A.R.: Phys. Rev. Lett. 114 (2015) 082001

- $Re[V^{QCD}]$: smooth transition from confining to Debye screened behavior

V^{QCD} in quenched lattice QCD

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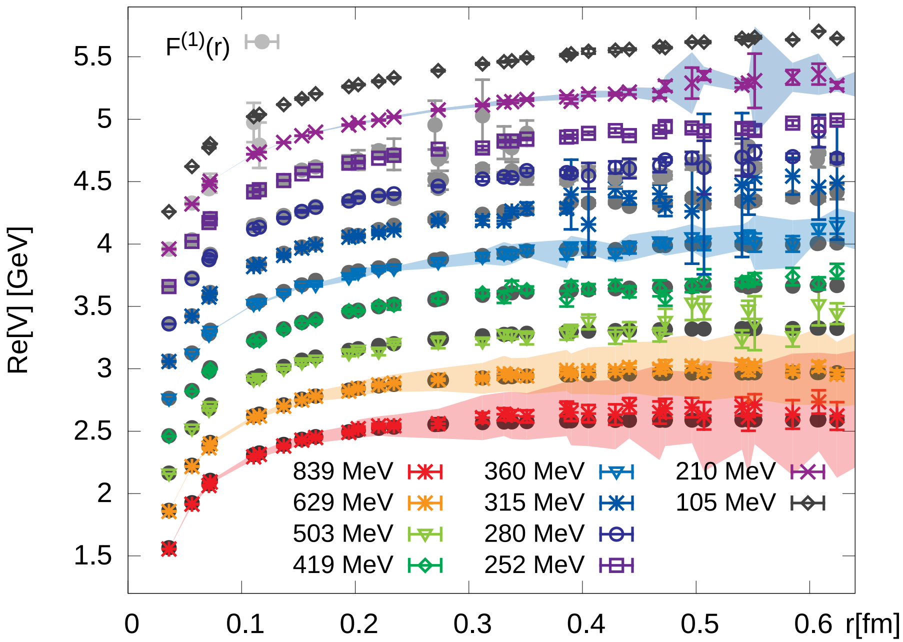

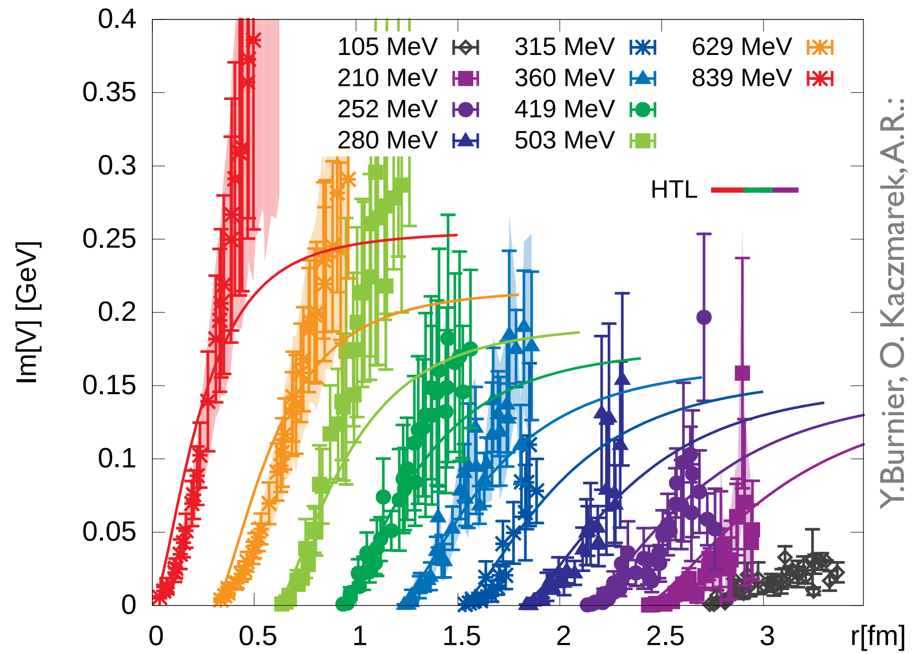

Y.Burnier, O. Kaczmarek, A.R.: Phys. Rev. Lett. 114 (2015) 082001

- $\mathrm{Re}[V^{QCD}]$: smooth transition from confining to Debye screened behavior
- First principles check: Color singlet free energies lie close to $\mathrm{Re}[V^{QCD}]$

$$F^{(1)}(R) = -\frac{1}{\beta}\log\left[W_{||}(R,\tau=\beta)\right]_{CG}$$

V^{QCD} in quenched lattice QCD

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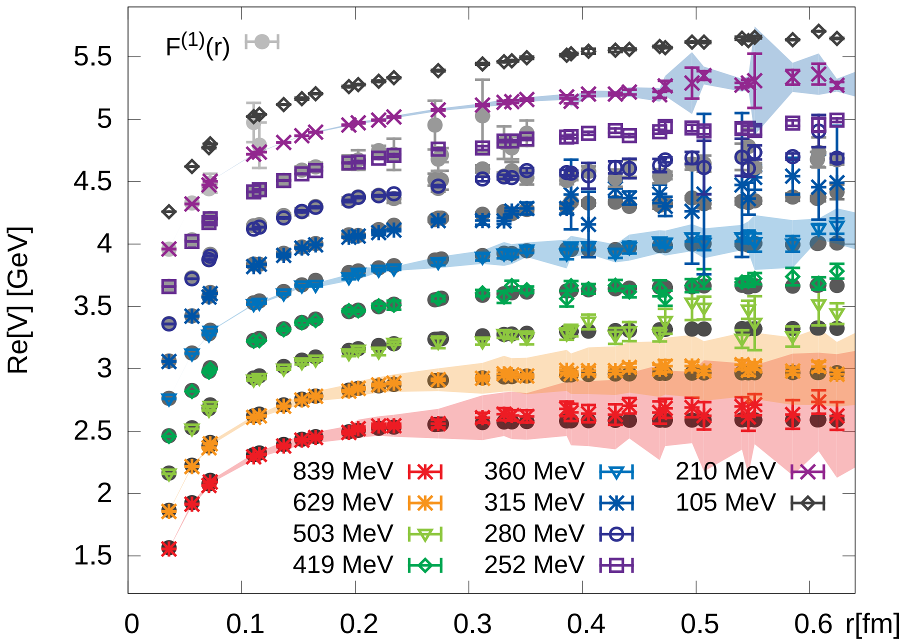

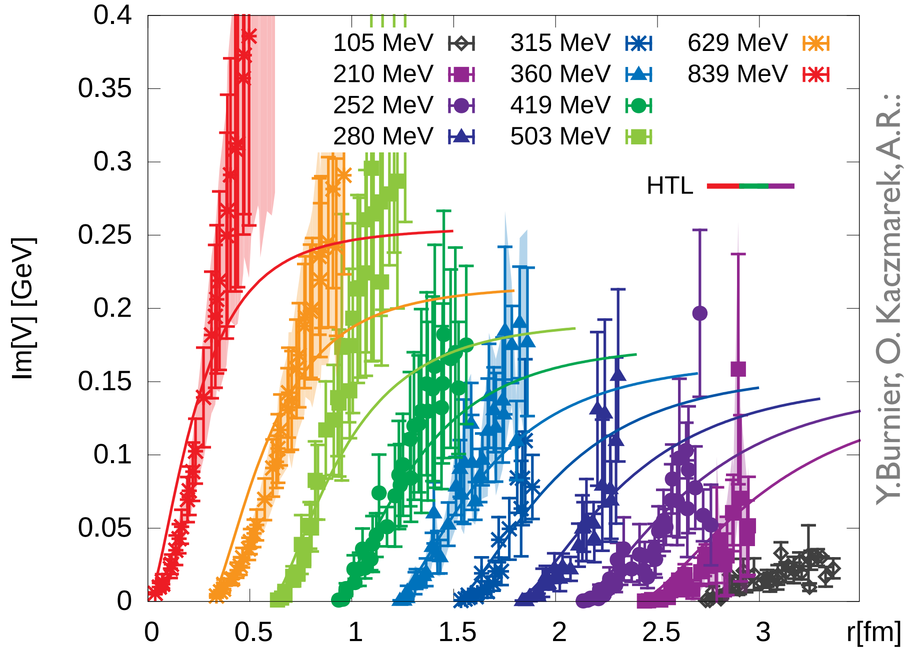

Y.Burnier, O. Kaczmarek, A.R.: Phys. Rev. Lett. 114 (2015) 082001

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- Im[V^{QCD}] for small R: same order of magnitude as in HTL perturbation theory

Re[V^{QCD}] in full lattice QCD

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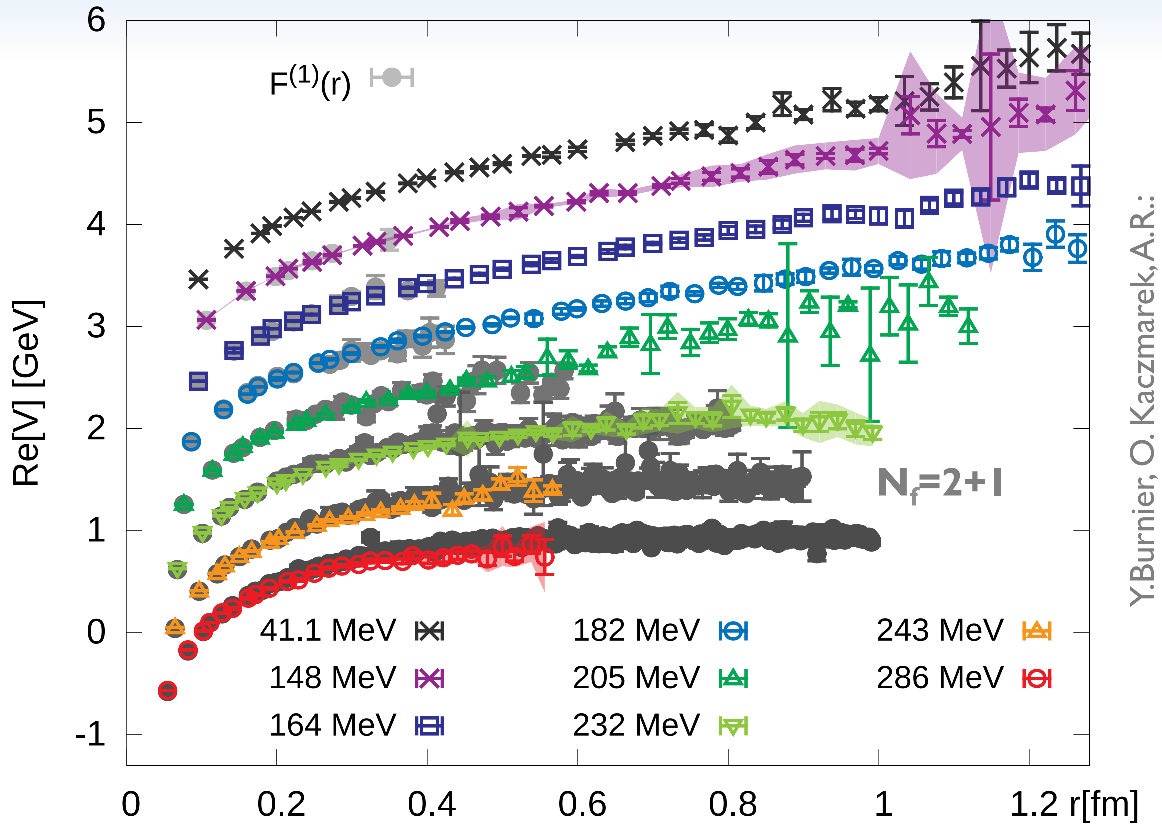
HotQCD Nf=2+1 lattices asqtad action
β=6.664 – 7.480 ξ=a_s/a_τ=1 N_τ=12
A. Bazavov et. al. PRD 85 (2012) 054503



Y.Burnier, O. Kaczmarek, A.R.:
Phys. Rev. Lett. 114 (2015) 082001

- Potential in the confining regime reliably extracted up to r=1fm (string breaking?)
- Qualitatively similar to quenched case (confinement to Debye screening)

Conclusions

- QCD Spectral functions provide multiple windows to in-medium $Q\bar{Q}$ physics

- New Bayesian spectral reconstruction improves their lattice QCD determination

- Bottomonium in a realistic thermal medium (HISQ - HotQCD)
  - N_τ=12 lattices give upper limits on in-medium modification
  - A systematic comparison between free and interacting spectra suggests:

    S-wave and P-wave ground state survive up to at least T=249MeV

- Effective field theory based potential for static quarks from T>0 QCD
  - No more need for modeling: QCD derived complex potential available
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    Re[V] smooth transition: confining to Debye screening, Im[V] of same order than HTL

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Благодарю вас за внимание - Thank you for your attention