Heavy flavor transport

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Heavy Flavor in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why);
- Description of jet-quenching based on energy-degradation of external probes (high-p_T partons);

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NB At high-p_T the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed here, see M. Djordjevic talk)

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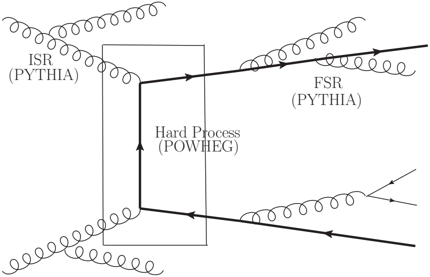
NB for realistic temperatures $g \sim 2$, so that one can wonder *whether a charm is really "heavy"*, at least in the initial stage of the evolution.

Simulating the initial hard production

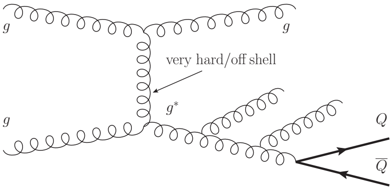


- Powerful pQCD tools[1] are available to simulate the initial $Q\overline{Q}$ production, interfacing the output of a NLO event-generator (POWHEG, MC@NLO) for the hard process with a parton-shower (PYTHIA, HERWIG) describing Initial and Final State Radiation and non-perturbative effects (intrinsic-k_T, MPI, hadronization)
- This provides a *fully exclusive information on the final state*

[1] For a systematic comparison (POWHEG vs MC@NLO vs FONLL): M. Cacciari *et al.*, JHEP 1210 (2012) 137.

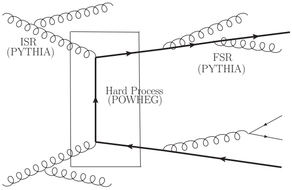
FONLL vs POWHEG+PS

FONLL



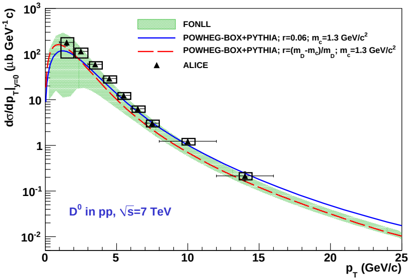
- It is a *calculation*
- It provides NLL accuracy, resumming large $\ln(p_T/M)$
- It includes processes missed by POWHEG (hard events with light partons)

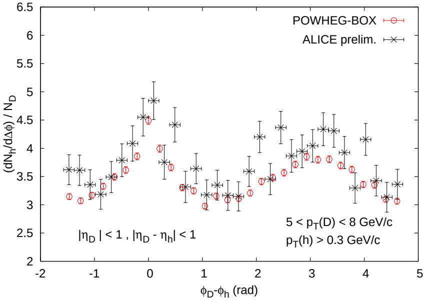
POWHEG+PS



- It is an *event generator*
- Results compatible with FONLL
- It is a more flexible tool, allowing to address more differential observables (e.g. $Q\overline{Q}$ correlations)

HF production in pp collisions: results

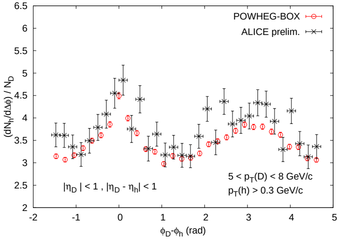


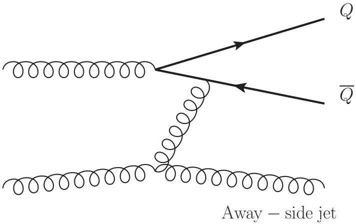


- Besides reproducing the inclusive p_T-spectra...[2]
- ...the POWHEG+PYTHIA setup allows also the comparison with $D-h$ correlation data, which start getting available.

[2] W.M. Alberico *et al*, Eur.Phys.J. C73 (2013) 2481

HF correlations: a caveat





- Due to the small BR direct $D - \overline{D}$ correlations (excluding forward LHCb data) have been so far out of reach;
- $Q\overline{Q}$ correlations *indirectly accessible* through $D - h$ and $e - h$ data;
- the latter however get also contribution from away-side light jets, which pose problems to the study of their medium-modification through simple heavy-flavor transport calculations

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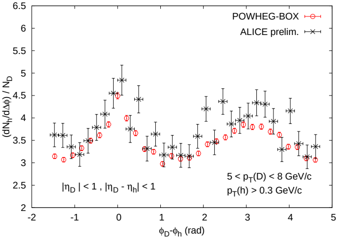

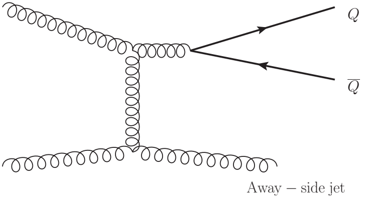

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HF in nucleus-nucleus collisions

- Transport calculations: a critical overview
- Towards a *precise determination* of the transport coefficients from QCD
- How close/far are heavy quarks go to/from thermalization? Are final (hadronic) observables able to answer this question?

HF in nucleus-nucleus collisions

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NB thermal equilibrium of HQ's at the end of the QGP phase is assumed in the description of hidden and open charm production within the Statistical Hadronization Model: answering this question may support or rule out such an hypothesis

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$[3]:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

  Neglecting $\mathbf{x}$-dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k}[\underbrace{w(\mathbf{p}+\mathbf{k}, \mathbf{k}) f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}}]$$

  $w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p} - \mathbf{k}$

[3] Approach adopted e.g. by the Catania and Nantes groups and for the whole medium in codes like BAMPS

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[4] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p},\mathbf{k}) f_Q(t,\mathbf{p})]$$

[4] B. Svetitsky, PRD 37, 2484 (1988)

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The Boltzmann equation reduces to the *Fokker-Planck equation* (approx. to be quantitatively tested!)

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*

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The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \,,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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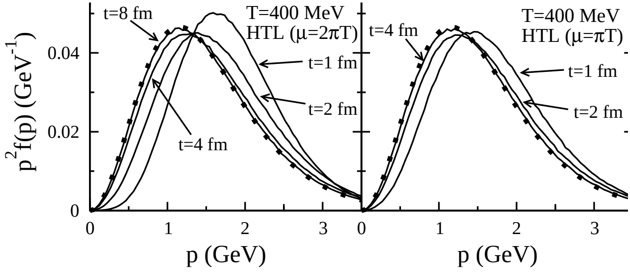
Transport coefficients (to *derive from theory*):

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\text{Ito}}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\parallel(p) - \kappa_\perp(p)}{v^2}\right]$$

fixed in order to assure approach to equilibrium (Einstein relation)

A first check: thermalization in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[5]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c and weak-coupling HTL transport coefficients)

[5]A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

The realistic case: expanding fireball

Within our POWLANG setup (POWHEG+LANGevin) the HQ evolution in heavy-ion collisions is simulated as follows

- $Q\overline{Q}$ pairs initially produced with the POWHEG-BOX package (with nPDFs) and distributed in the transverse plane according to $n_{\rm coll}(\mathbf{x}_\perp)$ from (optical) Glauber model;

[6] P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
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- update of the HQ momentum and position to be done at each step *in the local fluid rest-frame*
  - $u^\mu(x)$ used to perform the boost to the fluid rest-frame;
  - $T(x)$ used to set the value of the transport coefficients

  with $u^\mu(x)$ and $T(x)$ fields taken from the output of hydro codes[6];
- Procedure iterated until hadronization

---

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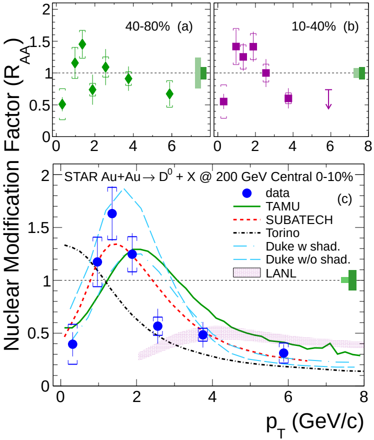
Results vs experimental data

D-mesons at low-p_T: STAR data compared to various model predictions (see the various talks).
Sharp peak ≈ 1.5 GeV in central ($0-10\%$) collisions:

- from charm radial flow?
- from coalescence with light quarks (included in some of the models)?

More in the following...



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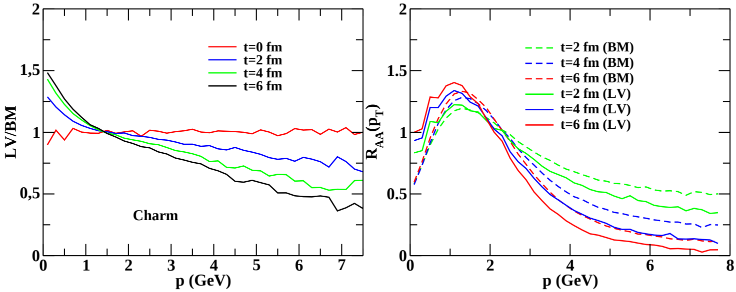

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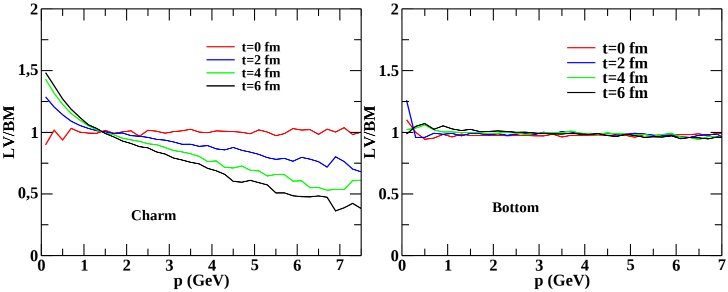

For beauty on the other hand Langevin≡Boltzmann!

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At the same time the Langevin/FP approach, although formally derived as a soft-scattering limit of the Boltzmann equation, can be considered *more general than the latter*, requiring simply the knowledge of a few transport coefficients (friction and diffusion) *meaningful even in a non-perturbative framework* and *not relying on quasi-particle picture* of the medium.

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Notice that, for the light quarks/gluons of the medium one has

- Thermal de Broglie wavelength: $\lambda_{\rm th} \sim 1/T$
- Mean free path: $\lambda_{\rm mfp} \sim 1/g^2 T$

In the weak-coupling regime one has $\lambda_{\rm th} \ll \lambda_{\rm mfp}$, so that between the relatively rare scatterings one has the propagation of *localized on-shell particles*. However as the coupling gets large $\lambda_{\rm th} \sim \lambda_{\rm mfp}$, the two scales are no longer well separated and a *picture based on on-shell distribution function* may be *no longer valid*: Kadanoff-Baym equations must be employed (see e.g. PHSD approach).

HF transport coefficients

In our POWLANG setup we implemented the results of

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- Non-perturbative calculations (lattice-QCD)

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Other approaches were followed in the literature

- Resonant scattering (Rapp *et al.*)
- AdS/CFT calculations
- ...

Transport coefficients: perturbative evaluation

It's the stage where the various models differ!

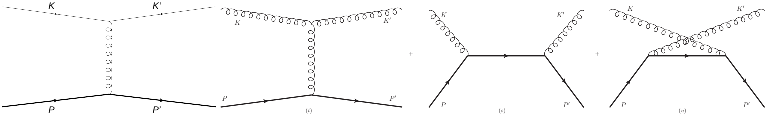
We account for the effect of $2 \rightarrow 2$ collisions in the medium

Intermediate cutoff $|t|^* \sim m_D^2$ [7] *separating the contributions of*

- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation
- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation (*resummation of medium effects*)

[7] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

Transport coefficients $\kappa_{T/L}(p)$: hard contribution

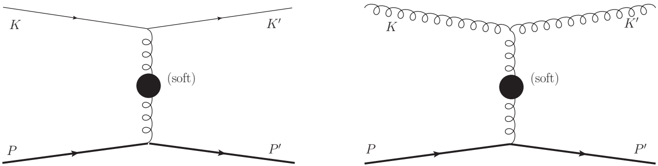


$$\kappa_T^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

$$\kappa_L^{g/q(\mathrm{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

where: $(|t| \equiv q^2 - \omega^2)$

Transport coefficients $\kappa_{T/L}(p)$: soft contribution



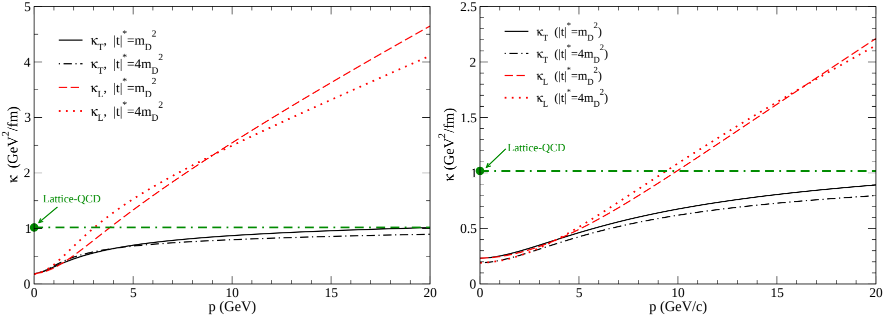
When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *b*lob represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.

Transport coefficients: numerical results

Combining together the hard and soft contributions...



...the dependence on the intermediate cutoff $|t|^*$ is very mild!

- *Strong momentum dependence* in the case of charm
- For beauty $\kappa_{T/L}$ stay closer and display a *milder growth* with p

Lattice-QCD transport coefficients: setup

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One consider the non-relativistic limit of the Langevin equation:

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\rm HQ} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt\, \underbrace{\langle F^i(t)F^i(0)\rangle_{\rm HQ}}_{\equiv D^>(t)}$$

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In the static limit the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x}) t^a Q(t,\mathbf{x}) \mathbf{E}^a(t,\mathbf{x})$$

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In the static limit the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x}) t^a Q(t,\mathbf{x}) \mathbf{E}^a(t,\mathbf{x})$$

κ is then given by the $\omega \to 0$ limit of the *spectral density* $\sigma(\omega)$ of the above E-field correlator

$$\kappa \equiv \lim_{\omega\to 0} \frac{D^>(\omega)}{3} \equiv \lim_{\omega\to 0} \frac{1}{3}\frac{\sigma(\omega)}{1-e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

Lattice-QCD transport coefficients: results

The spectral function $\sigma(\omega)$ has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = -\frac{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,0)]\rangle}$$

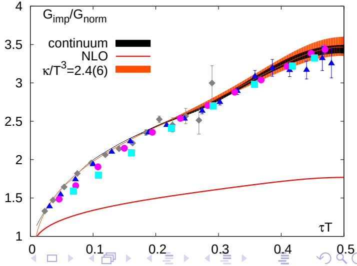
according to

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau-\beta/2)}{\sinh(\beta\omega/2)}\sigma(\omega)$$

One gets (D. Banerjee *et al.*, PRD 85 (2012) 014510; O. Kaczmarek *et al.*, PoS LATTICE2011 202 and NPA 931 (2014) 633)

$$\kappa/T^3 \approx 2.4(6)\ (quenched\ QCD,\ cont.lim.)$$

$\sim$3-5 times larger then the perturbative result (W.M. Alberico *et al*, EPJC 73 (2013) 2481). Challenge: approaching the continuum limit in full QCD



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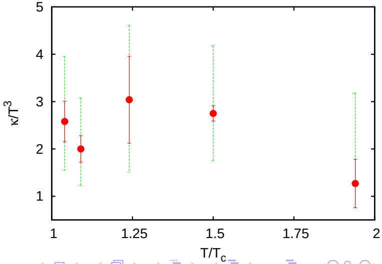

First message: look at beauty!

Measurements of beauty at low p_T (with future *detector upgrades*) in the next years will allow one to establish a link between first-principle theoretical predictions (e.g. l-QCD) and experimental observables:

- $M \gg gT$: Langevin equation equivalent to Boltzmann equation;
- $M \gg T$: static ($M = \infty$) l-QCD results more reliable for beauty
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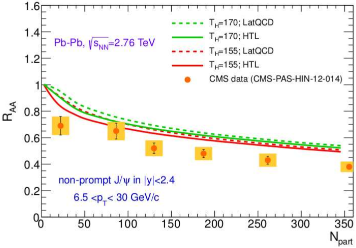

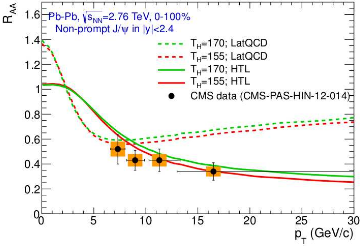

Measurements so far limited to non-prompt J/ψ's at quite high p_T (CMS data vs POWLANG results, EPJC 73 (2013) 2481)

HF in the POWLANG setup: recent developments

(Eur.Phys.J. C75 (2015) 3, 121 and work in progress)

The major novelty concerns the simulation of heavy-quark hadronization, which now can be performed via

- standard vacuum Fragmentation Functions
- recombination with thermal light partons

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The major novelty concerns the simulation of heavy-quark hadronization, which now can be performed via

- standard vacuum Fragmentation Functions
- recombination with thermal light partons

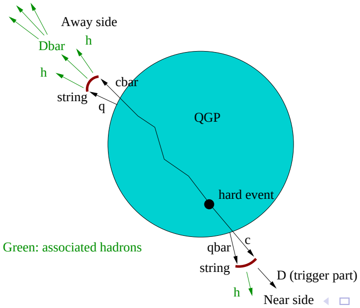
Presently, we are exploring the effects also in small systems (p-A collisions)

From quarks to hadrons

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective flow* of light quarks. We tried to estimate the effect through this model interfaced to our POWLANG transport code:

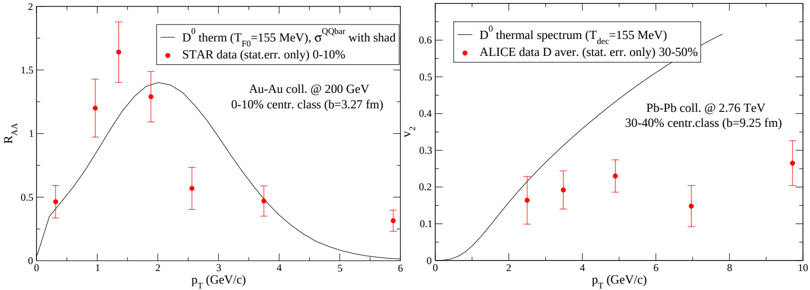
- At $T_{\rm dec}$ c-quarks coupled to light $\overline{q}$'s from a local *thermal distribution*, eventually *boosted* ($u^{\mu}_{\rm fluid} \neq 0$) to the lab frame;
- Strings are formed and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ($D + \pi + \dots$)

One can address the study of $D - h$ and $e - h$ correlations in AA collisions



From quarks to hadrons: effect on R_{AA} and v_2

Experimental data display a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization*



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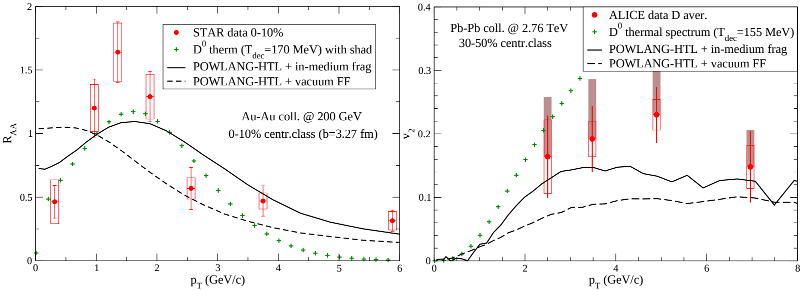
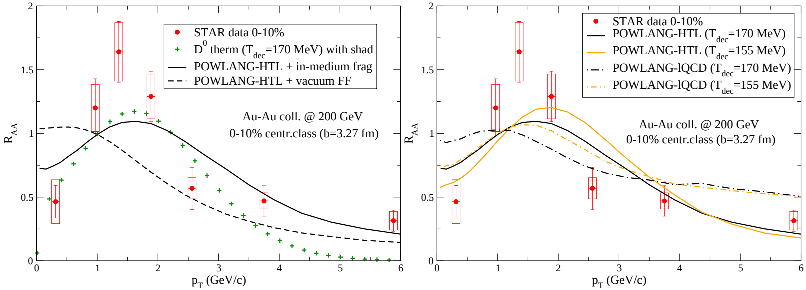


However, *comparing transport results with/without the boost* due to $u^{\mu}_{\rm fluid}$, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization.

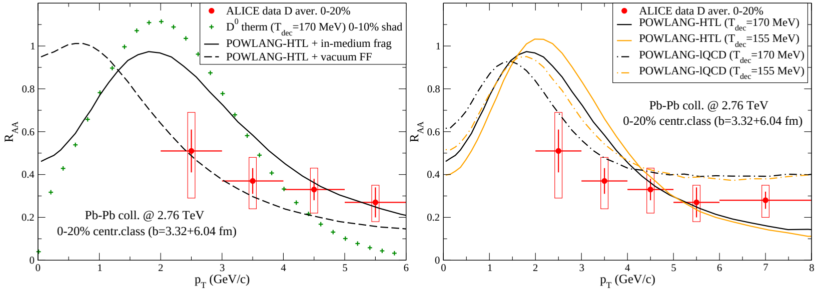
D-meson R_{AA} at RHIC



It is possible to perform a systematic study of different choices of

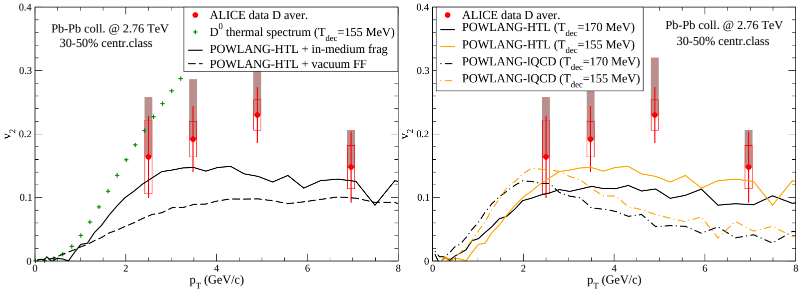
- Hadronization scheme (left panel)
- Transport coefficients (weak-coupling pQCD+HTL vs non-perturbative l-QCD) and decoupling temperature (right panel)

D-meson R_{AA} at LHC



Experimental data for central (0−20%) Pb-Pb collisions at LHC display a strong quenching, but – at least with the present bins and p_T range – don't show strong signatures of the bump from radial flow predicted by "thermal" and "transport + $Q\overline{q}_{\rm therm}$-string fragmentation" curves.

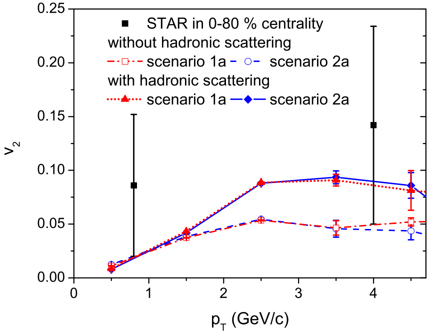
D-meson v_2 at LHC



Concerning D-meson v_2 in non-central $(30-50\%)$ Pb-Pb collisions:

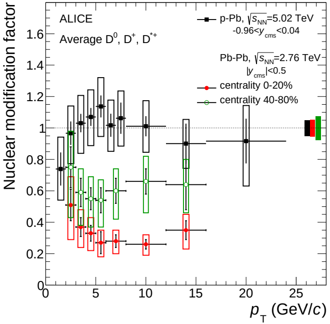
- $Q\overline{q}_{\rm therm}$-string fragmentation routine significantly improves our transport model predictions compared to the data;
- HTL curves with a *lower decoupling temperature* display the best agreement with ALICE data

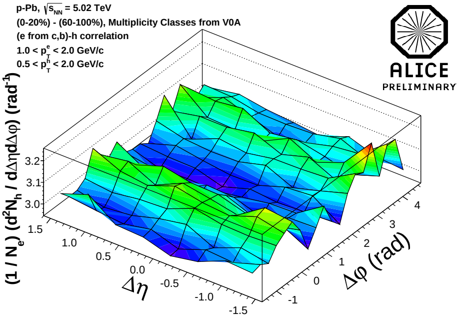
Room for hadronic rescattering?



- Although characterized by smaller values of the temperature and hence of the transport coefficients, in the late hadronic stage of the evolution the fireball is characterized by the maximum elliptic flow
- Including rescattering in the hadronic phase in transport models enhances the elliptic flow (see e.g. T. Song *et al.* arXiv:1503.03039)

Heavy Flavour in p-A: experimental indications



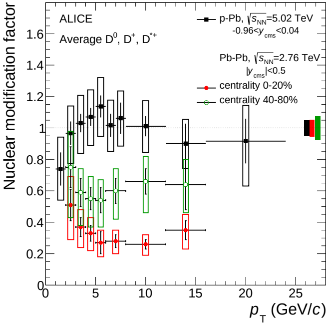


So far, experimental data don't allow one to draw firm conclusions

- D-meson $R_{AA} \approx 1$ over a wide p_T-range;
- e-h correlations provide *hints* of a double-ridge structure

How to reconcile the two observations?

Heavy Flavour in p-A: experimental indications

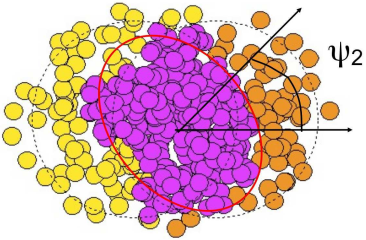


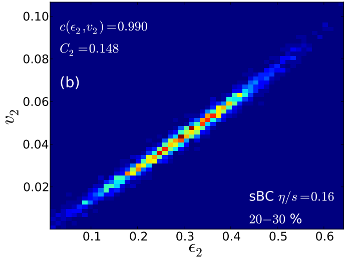
Medium modeling: event-by-event hydrodynamics

Event-by-event fluctuations (e.g. in the nucleon positions) leads to an initial *eccentricity*

$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2}\sum_{i=1}^{N_{\rm coll}} \exp\left[-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}\right] \quad \longrightarrow \quad \epsilon_2 = \frac{\sqrt{\{y^2-x^2\}^2+4\{xy\}^2}}{\{x^2+y^2\}}$$

which translates into a non-vanishing elliptic flow

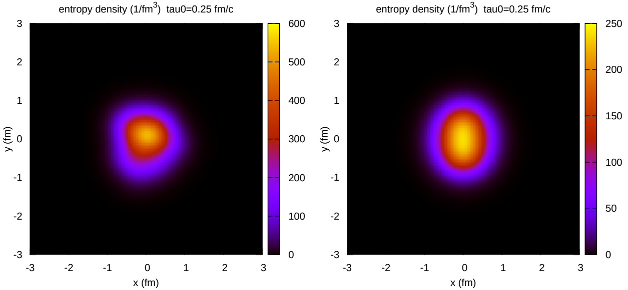




Notice the linear response to the initial eccentricity observed in event-by-event studies (Niemi *et al.*, PRC 87 (2013) 054901) of AA collisions. EbyE fluctuations at the origin of v_2 in p-A collisions!

Medium modeling for p-A collisions

A full event-by event hydro+transport study requires huge computing resources (time and storage). One can exploit the strong correlation $v_2 \sim \epsilon_2$ considering an *average background* obtained summing all the events of a given centrality class rotated of the *event-plane* angle ψ_2



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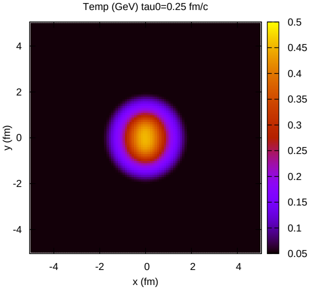
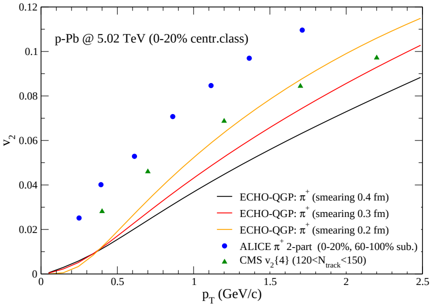
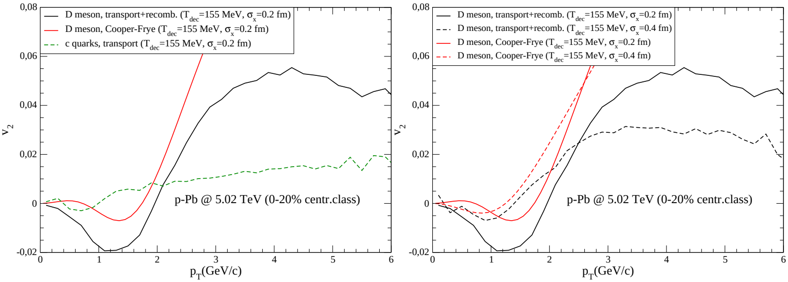




One can reproduce the light-hadron elliptic flow, although – with such a small system – there is a sensitivity to the smearing parameter ($\neq$ AA collisions): doing better would require knowing the proton structure

(simulations performed with ECHO-QGP: http://theory.fi.infn.it/echoqgp/)

HF transport in p-A collisions: preliminary results



First results with ECHO-QGP[8] + Langevin + in-medium hadronization:

- All the flow of D-mesons comes from the one of light partons;

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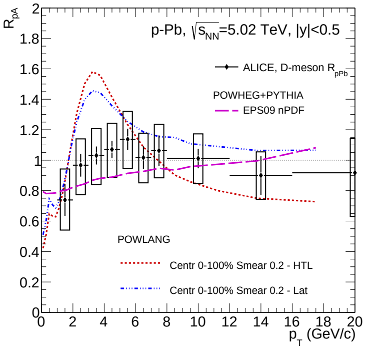

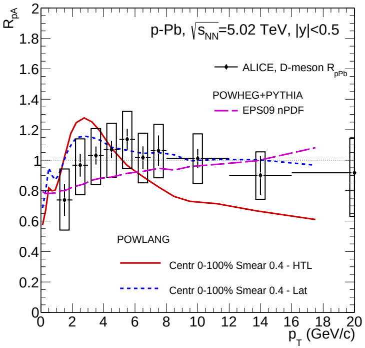

First results with ECHO-QGP[8] + Langevin + in-medium hadronization:

- All the flow of D-mesons comes from the one of light partons;
- Signatures of radial flow in the D-meson R_{pA}?

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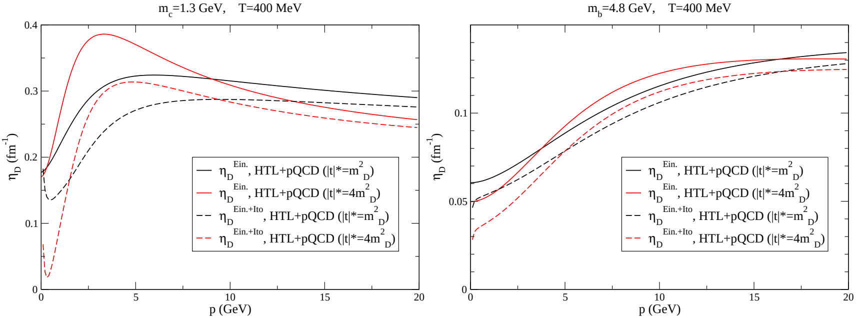
Summary and outlook

- I tried to give a critical overview on transport approaches based on the Boltzmann and Langevin/FP equations;
- I tried to stress the importance of future beauty measurements at low/moderate p_T to get information on the transport coefficients of the medium;
- For charm, observables are sensitive to what happens at hadronization. In general, models including recombination allows one to get an additional radial/elliptic flow, better reproducing the data;
- The possibility and importance of performing HF studies in small systems (p-Pb, d-Au...) was discussed and some preliminary results were shown;
- Rescattering in the hadronic phase can play a role;
- More differential observables (HF correlations) can be also addressed (see backup slides), although collecting sufficient statistics remains a challenge

Backup slides

Transport coefficients: numerical results

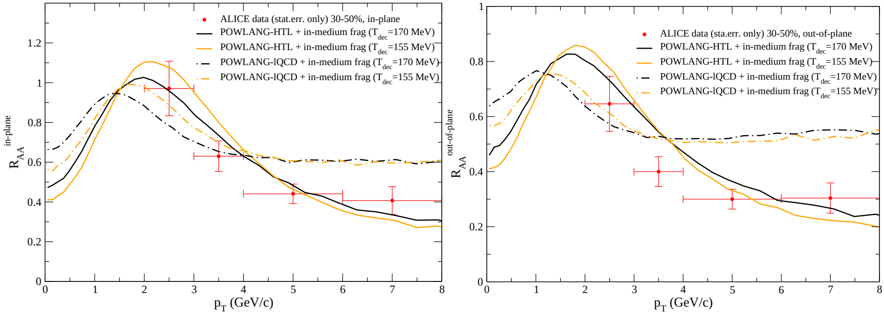
Combining together the hard and soft contributions...



...the dependence on the intermediate cutoff $|t|^*$ is very mild!

D meson R_{AA}: in-plane vs out-of-plane

One can study di R_{AA} in- and out-of-plane in non-central $(30-50\%)$ Pb-Pb collisions at LHC:



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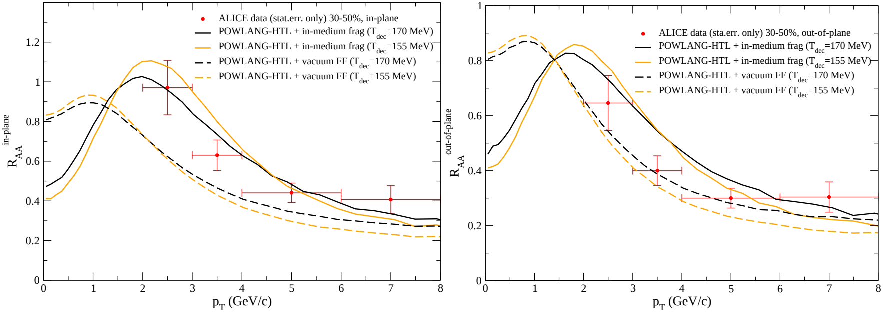
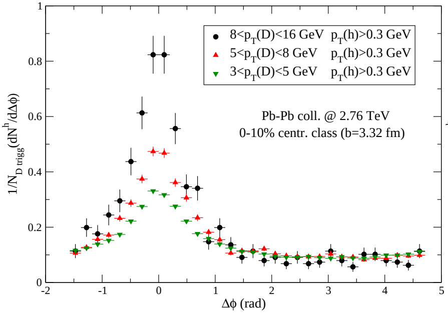
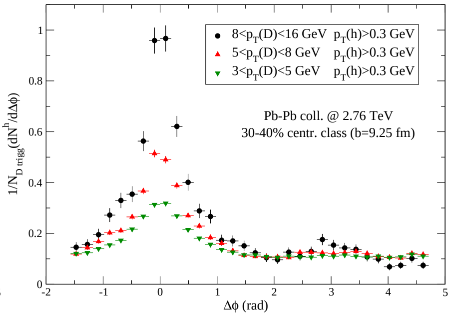


- Data better described by weak-coupling (pQCD+HTL) transport coefficients;
- $Q\overline{q}_{\rm therm}$-string fragmentation describes data slightly better than in-vacuum independent Fragmentation Functions.

Azimuthal correlations: $D-h$

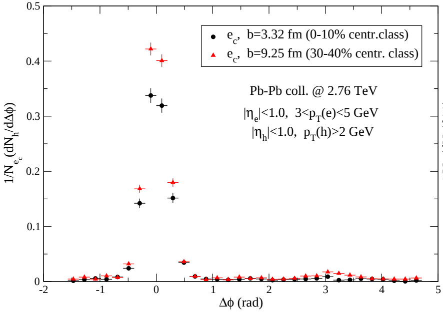


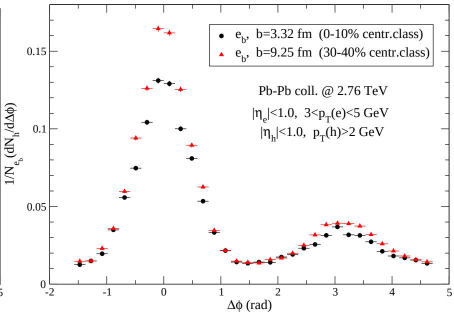


Away-side peak strongly suppressed
both in central and semi-central collisions

Azimuthal correlations: $e - h$

We plot the separate e_c (left) and e_b (right) contributions from charm and beauty decays





- charm away-side peak always strongly suppressed for any centrality and $p_T^{\rm ass}$ cut;
- beauty always-side peak suppressed but still visible, providing in principle a richer information.

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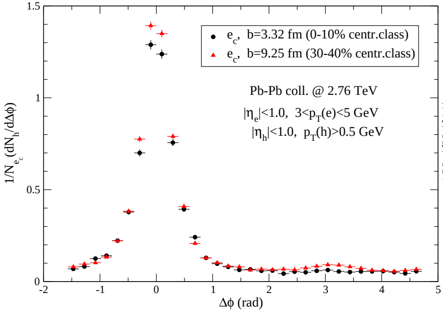

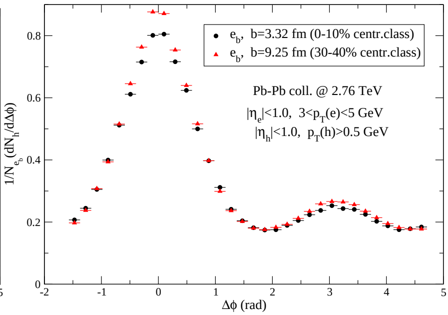

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