Towards the quark–gluonic Equation of State including strange and charmed quarks with realistic masses

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Equation of State from Lattice QCD

All simulations at (almost) vanishing chemical potential (sign problem)

$N_f = 0$ (quenched approximation, no dynamical quarks):

- G. Boyd et al., Nucl.Phys. B **469**, 419 (1996)
- M. Okamoto et al. [*CP–PACS*], Phys. Rev. D **60**, 094510 (1999)
- S. Borsanyi et al. [*Wuppertal–Budapest*], JHEP **1207**, 056 (2012)

$N_f = 2$ (up/down quarks with equal masses):

- C.W. Bernard et al. [*MILC*], Phys. Rev. D **55**, 6861 (1997)
- A. Ali Khan et al [*CP–PACS*], Phys. Rev. D **64**, 074510 (2001)

$N_f = 2 + 1$ (strange quark included, unphysical light masses):

- T. Umeda et al. [*WHOT–QCD*], Phys. Rev. D **85**, 094508 (2012)
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$N_f = 2 + 1$ (realistic masses in staggered approximation):

- S. Borsanyi et al. [*Wuppertal–Budapest*], Phys. Lett. B **730**, 99 (2014)
- A. Bazavov et al. [*HotQCD*], Phys. Rev. D **90**, 094503 (2014)

Equation of State from Lattice QCD

At the moment simulations with strange and light quark at (almost) physical masses are available only with staggered fermions:

- rooting trick uncertainty
- check by more convenient fermion discretizations is desirable

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In this project we investigate EoS with $N_f = 2 + 1 + 1$ twisted mass Wilson formulation

- theoretically safe but computationally very demanding
  - light masses are still large
  - $m_\pi^\pm \simeq 470, 370, 260, 210$ MeV
- strange and charm quarks with realistic masses
- $O(a)$ improvement at maximal twist
- effects of dynamical charm on EoS are expected already at $T \gtrsim 350$ MeV
- crossover temperatures from several observables

Previous study with $N_f = 2$ twisted mass:

- F. Burger, E.-M. Ilgenfritz, M.P. Lombardo, M. Müller–Preussker [*tmfT*], Phys. Rev. D **91**, 074504 (2015)

Fixed–scale approach

Temperature on the lattice:

$$T = \frac{1}{a\,N_\tau}$$

Two approaches to study temperature dependence on practice:

- fixed N_τ (scans in coupling β)
- fixed scale $\beta(a)$, varying N_τ

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We employ fixed scale approach here

- T. Umeda et al. [*WHOT–QCD*], Phys. Rev. D **79**, 051501 (2009)
- significant economy on $T = 0$ simulations
- possibility to use fine lattices around T_c
- fixed spatial volume for wide temperature range

On the other hand:

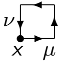
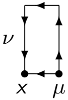
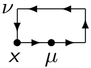
- large lattice artifacts at high T (small N_τ)
- higher statistics needed for low temperatures
- only discrete values of temperatures

The latter can be partially compensated by usage of odd N_τ lattices

Lattice setup

Iwasaki action with the tree–level Symanzik improvements:

$$S_g[U] = \beta\Big(c_0 \sum_P [1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,(U_P)] + c_1 \sum_R [1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,(U_R)]\Big)\,,$$

$U_P =$ , $U_R =$  $+$ , $c_0 = 3.648$, $c_1 = -0.331$

Wilson twisted mass fermionic action for light and heavy doublets:

$$S_f^l[U, \chi_l, \overline{\chi}_l] = \sum_{x,y} \overline{\chi}_l(x)\left[\delta_{x,y} - \kappa D_{\mathrm{W}}(x,y)[U] + 2i\kappa a \boldsymbol{\mu_l} \gamma_5 \delta_{x,y}\tau^3\right]\chi_l(y)\,,$$

$$S_f^h[U, \chi_h, \overline{\chi}_h] = \sum_{x,y} \overline{\chi}_h(x)\left[\delta_{x,y} - \kappa D_W(x,y)[U] + 2i\kappa \boldsymbol{\mu_\sigma} \gamma_5 \delta_{x,y}\tau^1 + 2\kappa \boldsymbol{\mu_\delta} \delta_{x,y}\tau^3\right]\chi_h(y)$$

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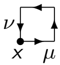
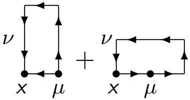
$U_P =$ 

, $U_R =$ 

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- hopping parameter $\kappa = \kappa_c(\beta)$ is at maximal twist $\Rightarrow$ automatic $O(a)$ improvement
- twisted–mass parameters μ_σ and μ_δ are tuned to the physical point (K and D masses, on the 10 % level)
- $\chi_{l,h} = \exp(-i\pi\gamma_5\tau^3/4)\psi_{l,h}$

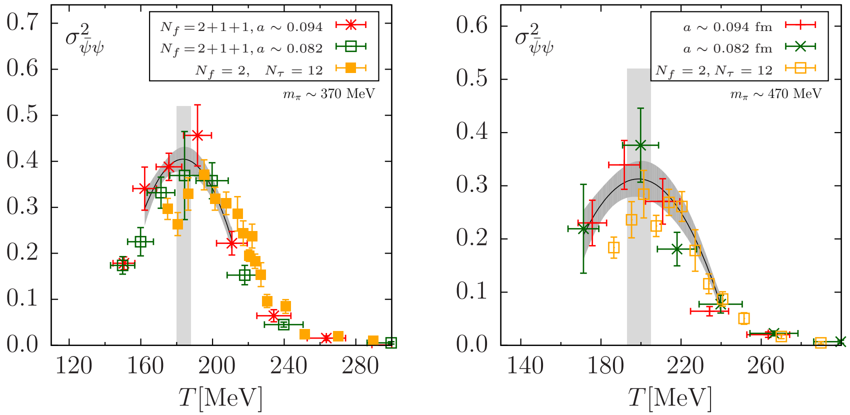
Lattice configurations

$T = 0$ (ETMC) nomenclature	$m_\pi^\pm$ [MeV]	$N_\tau \times N_\sigma^3$	statistics
A60.24	364(15)	$\{3,4,5,6,7,8,9,10,11,12\} \times 24^3$	2k-7k
		$\{13,14\} \times 32^3$	5k,27k
B55.32	372(17)	$\{3,4,5,6,7,8,10,11,12,13,14,15,16\} \times 32^3$	2k-27k
		$\{4,6,8,10,12\} \times 24^3$	1k-8k
D45.32	369(15)	$\{6,8,10,12,14,16\} \times 32^3$	1k-12k
A100.24s	466(19)	$\{4,6,8,10,12\} \times 24^3$	2k-4k
		$\{14\} \times 32^3$	1k
B85.24	465(21)	$\{4,6,8,10,12\} \times 24^3$	3k-4k
		$\{14\} \times 32^3$	1k
A30.32	261(11)	$\{4,6,8,10,12\} \times 32^3$	1k-5k
		$\{20\} \times 48^3$	4k
B25.32	256(12)	$\{4,6,8,10,12,14,16,18\} \times 40^3$	2k-8k
D15.48	213(9)	$\{4,6,8,10,12,14,16,18,20,24\} \times 48^3$	2k-6k

- zero–temperature configurations from ETMC Collaboration
- lattice spacings and other parameters are available in the literature:
  - e.g., C. Alexandrou et al. [*ETMC*], Phys. Rev. D **90**, 074501 (2014)

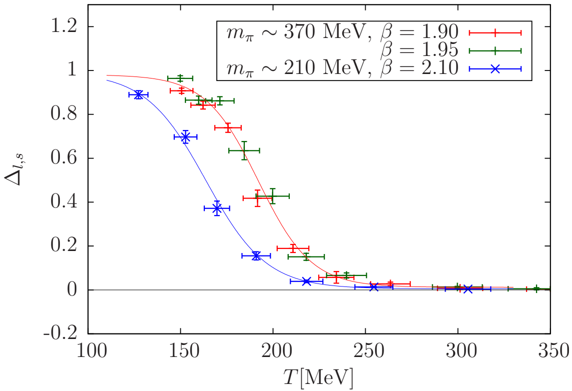
Pseudocritical temperature

$\sigma^2_{\bar\psi\psi} = \frac{V}{T}\left(\langle\bar\psi\psi^{\,2}\rangle - \langle\bar\psi\psi\rangle^2\right)$ — bare disconnected susceptibility



Pseudocritical temperature

$$\Delta_{l,s} = \frac{\langle\bar{\psi}\psi\rangle_l - \frac{\mu_l}{\mu_s}\langle\bar{\psi}\psi\rangle_s}{\langle\bar{\psi}\psi\rangle_l^{T=0} - \frac{\mu_l}{\mu_s}\langle\bar{\psi}\psi\rangle_s^{T=0}}$$ — renormalized condensate

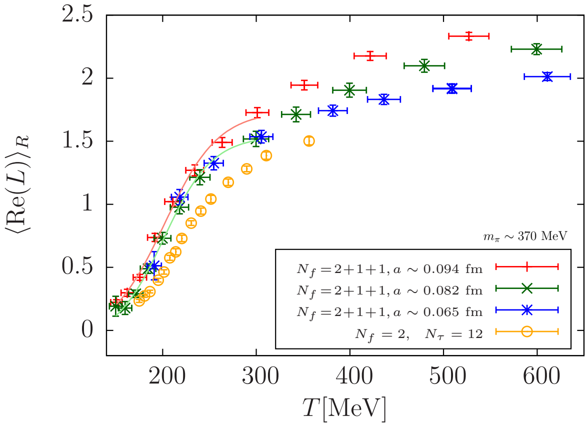


Fit to the tanh-function: $A + B\tanh[-C \times (T - T_c)]$

Pseudocritical temperature

Renormalized Polyakov loop:

$$\langle \mathrm{Re\ L} \rangle_R = \langle \mathrm{Re\ L} \rangle \exp[V(r_0)/2T]$$



Pseudocritical temperature — Summary

T_χ — bare disconnected susceptibility $\sigma^2_{\bar{\psi}\psi}$

T_Δ — renormalized condensate Δ_{ls}

T_{deconf} — Polyakov loop

$N_f = 2 + 1 + 1$:

$\sim m_\pi^\pm$	T_χ	T_Δ	T_{deconf}
210	152(5)	164(3)	–
260	170(5)	–	–
370	184(4)	192(2)	201(3)
470	199(6)	–	–

Compare with $N_f = 2$:

$\sim m_\pi^\pm$	T_χ	T_{deconf}
360	193(13)	219(3)(14)
430	208(14)	225(3)(14)

- F. Burger et al. [*tmfT*], Phys. Rev. D **91**, 074504 (2015)

Equation of State from the lattice

Pressure:

$$p = -\lim_{V \to \infty} f, \quad f = -\frac{T}{V} \ln \mathcal{Z}, \qquad p = \frac{1}{N_\tau N_\sigma^3} \ln \mathcal{Z}$$

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Only derivatives of partition function $\mathcal{Z}$ are accessible

$$T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\epsilon - 3p}{T^4}, \qquad \frac{p}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} d\tau \frac{\epsilon - 3p}{\tau^5}, \quad p(T_0) \approx 0$$

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Interaction measure (trace anomaly):

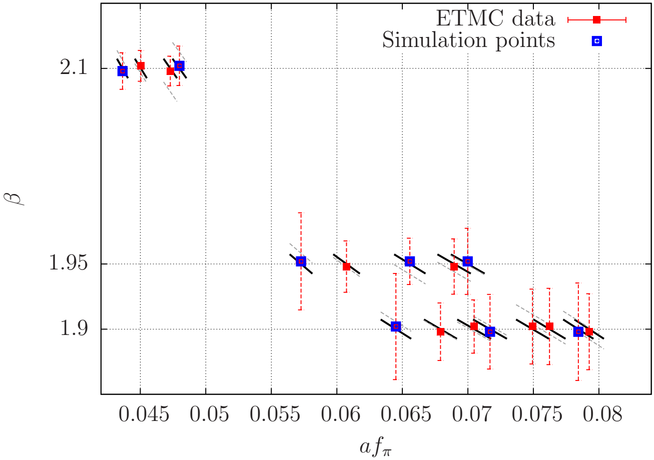
$$I(T) = \epsilon - 3p, \qquad \frac{I(T)}{T^4} = -\frac{1}{T^3 V}\left(\frac{d\ln\mathcal{Z}}{d\ln a}\right) = \frac{1}{T^3 V}\sum_i \frac{db_i}{da}\left\langle\frac{\partial S}{\partial b_i}\right\rangle$$

$$\frac{I(T)}{T^4} = N_\tau^4\Big\{\left(-a\frac{d\beta}{da}\right)\Big(\frac{c_0}{3}\langle\sum_P \mathrm{Re\,Tr}\,U_P\rangle_{\rm sub} + \frac{c_1}{3}\langle\sum_R \mathrm{Re\,Tr}\,U_R\rangle_{\rm sub} + \frac{\partial\kappa_c}{\partial\beta}\langle\bar\chi D_{\rm W}[U]\chi\rangle_{\rm sub}$$
$$-2a\mu_l\frac{\partial\kappa_c}{\partial\beta}\langle\bar\chi i\gamma_5\tau^3\chi\rangle_{\rm sub}\Big) + 2\kappa_c\left(a\frac{d(a\mu_l)}{da}\right)\langle\bar\chi i\gamma_5\tau^3\chi\rangle_{\rm sub} + \text{heavy terms}\Big\}$$

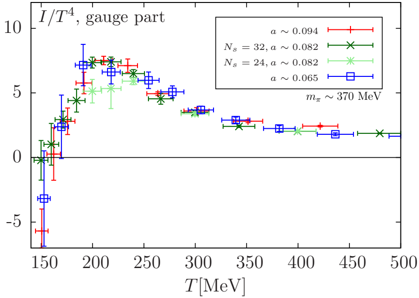
$$I(T) = I_{\rm gauge} + I_{\rm light} + I_{\rm heavy}$$

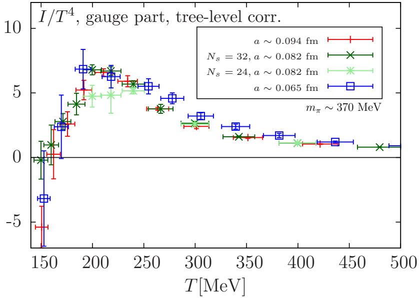
Beta functions

The generalized β-functions db_i/da are obtained from a global fit in $\pi^{\pm}$, K, and D masses, using $f_{\pi^{\pm}}$ to set the scale: $a\frac{db_i}{da} = (af_{\pi^{\pm}})\frac{db_i}{d(af_{\pi^{\pm}})}$



Equation of State





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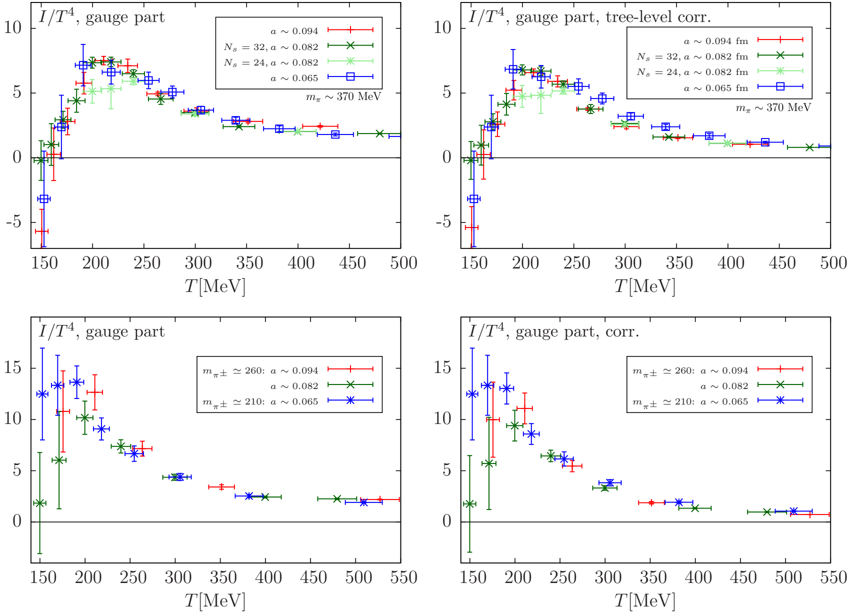
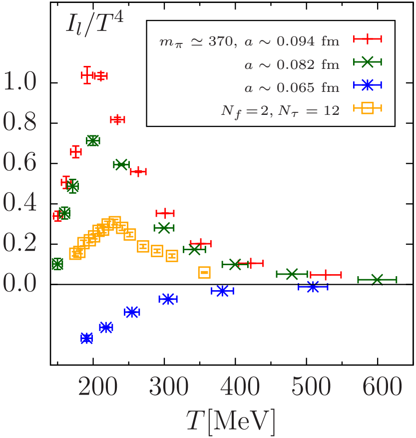
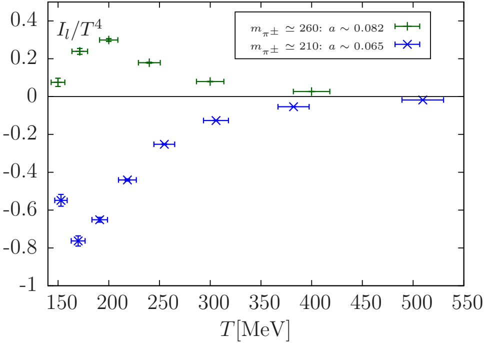


Equation of State

Contributions from light quarks action:





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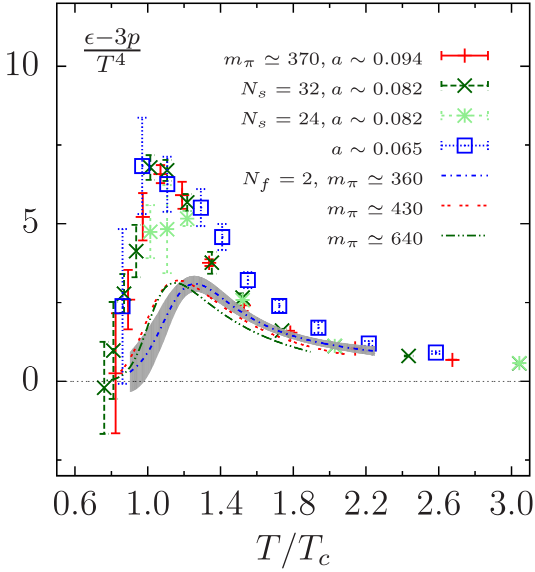

Conclusions & Outlook

- Pseudocritical temperatures from gluonic and fermionic observables
- Downshift of T_c with addition of s and c quarks, as well as approaching physical $m_{\pi^\pm}$
- "Gluonic" part of trace anomaly shown in details, including tree–level corrections
- Light quarks contribution (still preliminary)

In prospective:

- Corrections from heavy part of the action
- Higher statistics for lower $m_{\pi^\pm}$, and so for β–functions (still large uncertainty to EoS)
- Continuum limit

Thank you for attention!