Matching Hagedorn mass spectrum to Lattice QCD

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Strangeness in Quark Matter 2015, Dubna
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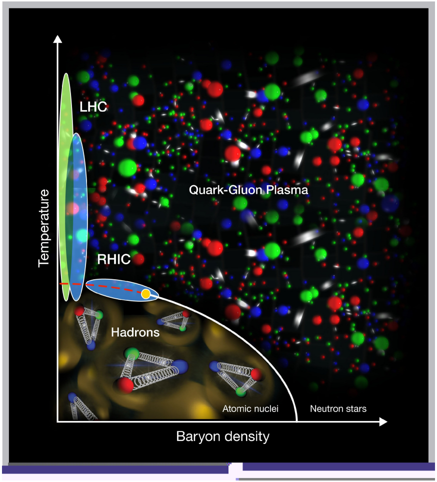
Outline

QCD phase diagram



EQUATION OF STATE

$\downarrow$

relevant degrees of freedom	hadrons and their resonances
$+$	$+$
interactions	point-like and independent species

$\downarrow$

HADRON RESONANCE GAS MODEL

Mass spectrum of hadronic matter

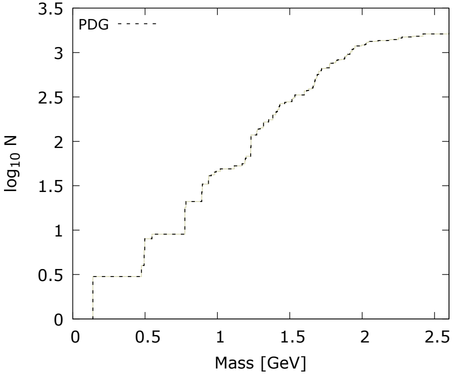
- discrete mass spectrum

$$\rho(m) = \sum_i d_i \delta(m - m_i)$$

- The same information can be stored in the cumulant

$$N(m) = \sum_i d_i \theta(m - m_i)$$

such that $\rho = \partial N / \partial m$

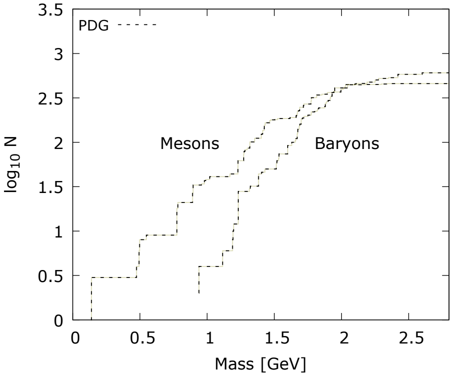


Mass spectrum of hadronic matter

One may decompose ρ into

- mesonic and baryonic sector

$$\rho = \rho^M + \rho^B$$



Mass spectrum of hadronic matter

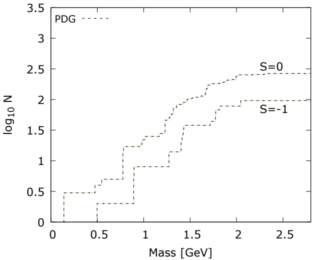
One may decompose ρ into

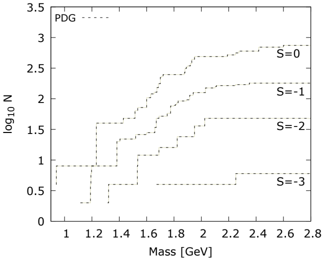
- mesonic and baryonic sector

$$\rho = \rho^M + \rho^B$$

- definite strangeness sectors

$$\begin{aligned} \rho^M &= \rho^M_{S=0} + \rho^M_{S=1} \\ \rho^B &= \rho^B_{S=0} + \rho^B_{S=1} + \rho^B_{S=2} + \rho^B_{S=3} \end{aligned}$$





Idea of ideal HRG

$$\ln Z \approx \sum_{i \in mes} \ln Z_i^M + \sum_{i \in bar} \ln Z_i^B$$

$$\ln Z_i^{M/B} = \pm \frac{d_i V}{2\pi} \int_0^\infty \mathrm{d}p \, p^2 \ln \left[1 \pm e^{\hat{\mu}_i} e^{-\hat{E}_i} \right]$$

$$\text{where } \hat{\mu}_i = B_i \hat{\mu}_B + S_i \hat{\mu}_S, \quad \hat{\mu} \equiv \mu / T, \quad \hat{E}_i \equiv E_i / T$$

Pressure (Boltzmann approximation)

$$P = \frac{1}{\pi^2} \sum_{i \in had} d_i m_i^2 T^2 K_2 \left(\frac{m_i}{T} \right) \cosh \hat{\mu}_i, \quad \hat{P} \equiv \frac{P}{T^4} \bigg|_{\hat{\mu}_B = \hat{\mu}_S = 0}$$

Fluctuations in ideal HRG

2^{nd} order correlations $\longrightarrow$ generalized susceptibilities

Generalized susceptibilities

$$\hat{\chi}_{xy} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_x \partial \hat{\mu}_y}, \qquad x, y = B, S$$

$$\hat{\chi}_{BB} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2\left(\frac{m_i}{T}\right) {B_i}^2 \qquad \text{baryons}$$

$$\hat{\chi}_{BS} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2\left(\frac{m_i}{T}\right) S_i B_i \qquad \text{strange baryons}$$

$$\hat{\chi}_{SS} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2\left(\frac{m_i}{T}\right) {S_i}^2 \qquad \text{strange hadrons}$$

Hagedorn mass spectrum

- the density of hadronic states increases exponentially

$$\rho(m) \sim m^{-a} e^{m/T_H}$$

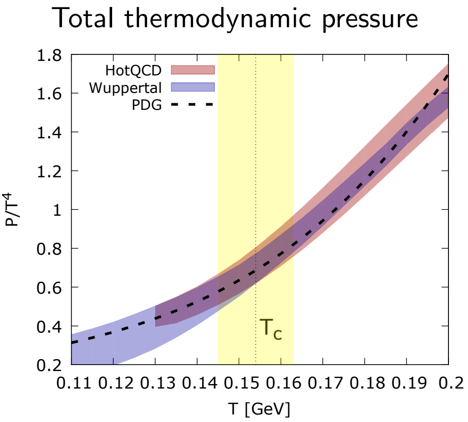
- existence of a limiting temperature, the Hagedorn temperature T_H, above which hadronic matter cannot exist.

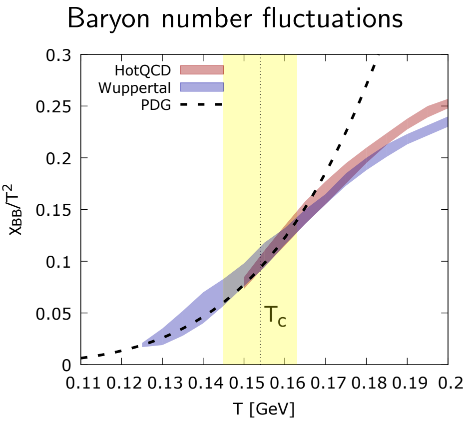
R. Hagedorn, Nuovo Cim. Suppl. **3**, 147 (1965)

- we use the following form

$$\rho^H(m) = \frac{A\, e^{m/T_H}}{(m^2 + m_0^2)^{5/4}}, \quad T_H \sim 180 \text{ MeV}$$

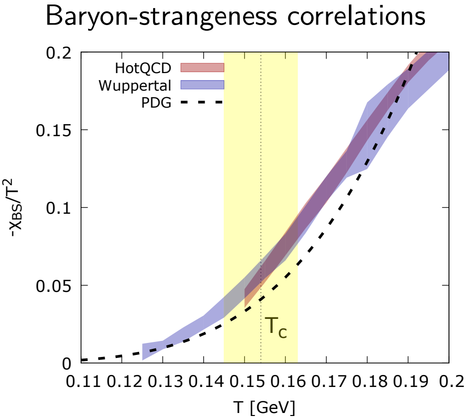
PDG vs Lattice QCD

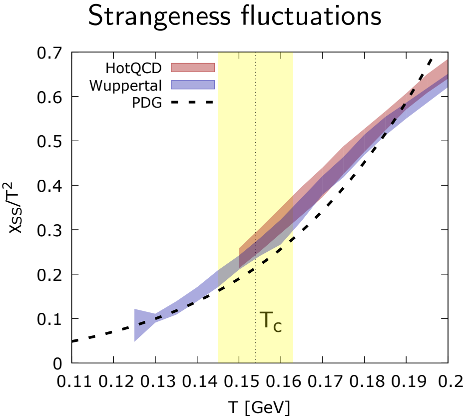




Wuppertal: arXiv:1309.5258v2, HotQCD: arXiv:1203.0784v2

PDG vs Lattice QCD





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Missing resonances

- $\hat{P}$ and $\hat{\chi}_{BB}$ $\longrightarrow$ fit LQCD results
- $\hat{\chi}_{BS}$ $\longrightarrow$ missing resonances in the strange baryonic sector
- $\hat{\chi}_{SS}$ $\longrightarrow$ missing resonances in the strange sector

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Known states are not sufficient

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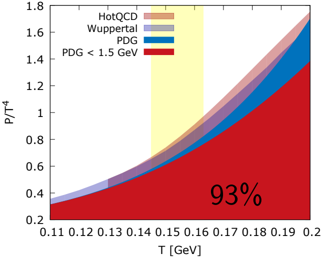
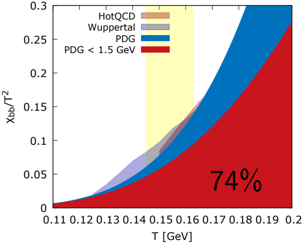
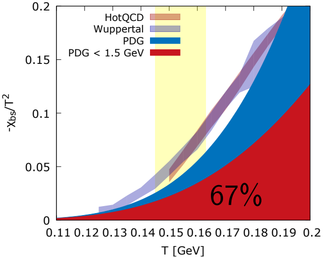
Goal:

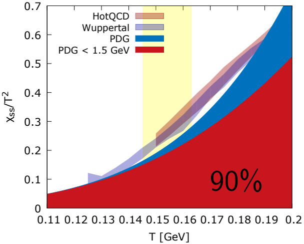
Identify the origin of the discrepancies in the strange hadronic sector of the HRG model

Importance of high-mass resonances at T = 0.15 GeV









Inconsistence of previous results

- different T_H for mesons and baryons based on cumulants
  - $T_H^M = 197$ MeV
  - $T_H^B = 141 \text{ MeV} < T_c \sim 155 \text{ MeV} \Rightarrow$ inconsistent with LQCD

W. Broniowski *et al*: Phys. Lett B **490**, 223 (2000),
Phys. Rev. D **70**, 117503 (2004)

Our key assumptions:

- the same crossover temperature $\Rightarrow$ the same T_H in all sectors,
- $T_H > T_c$ for the observables to be consistent with LQCD,
- quality of the fits is judged by the LQCD constraints.

Improved Hagedorn mass spectrum

Problem:

- cannot fit ρ^H in the low-mass region.

Solution:

- exclude ground states from the fit,
- start the fit from the first resonance m_x in given sector.

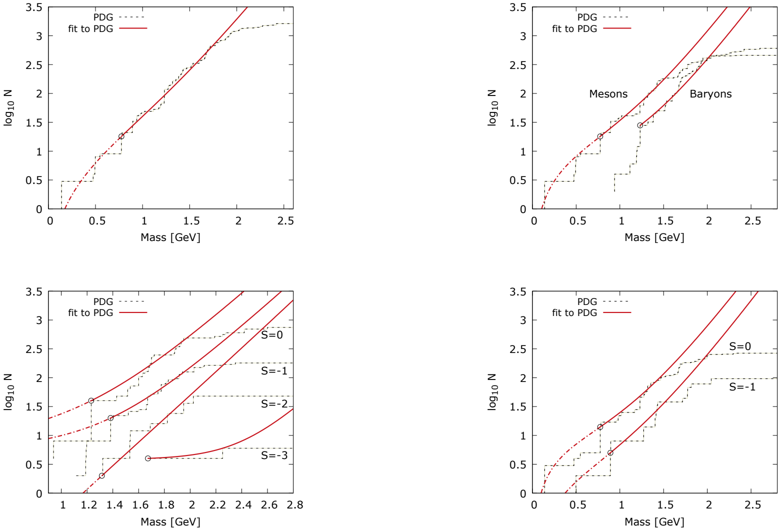
The improved Hagedorn spectrum reads

$$\rho(m) = \sum_{G.S.} d_i \delta(m - m_i) + \theta(m - m_x) \rho^H(m)$$

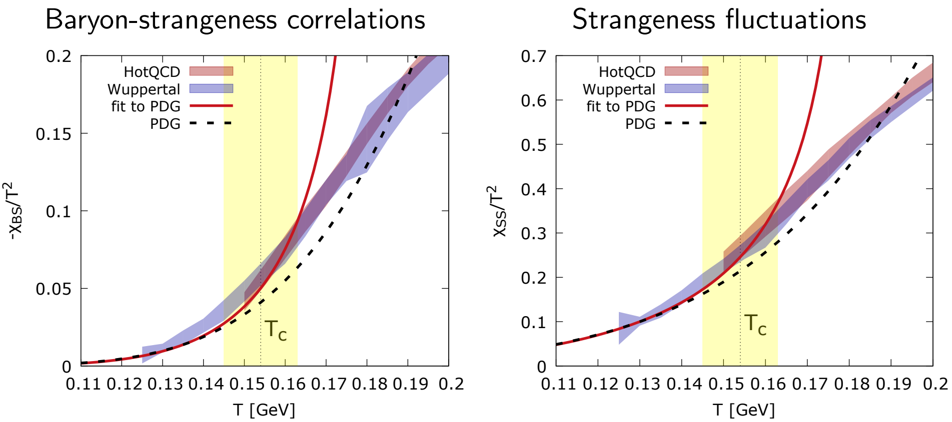
And the cumulant

$$N(m) = \sum_{G.S.} d_i \theta(m - m_i) + \theta(m - m_x) \int_{m_x}^{m} \mathrm{d}m \, \rho^H(m)$$

Fit to PDG cumulants

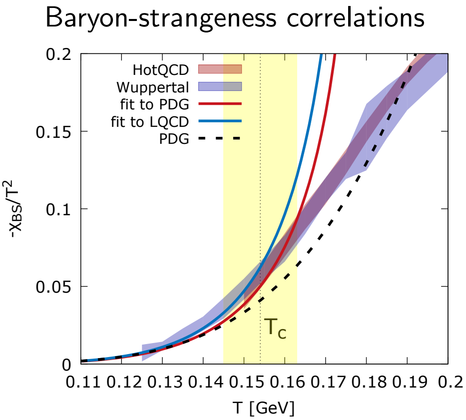


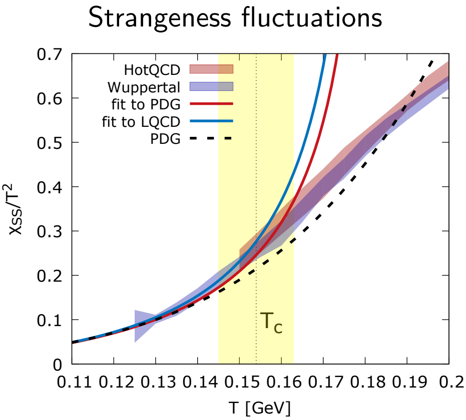
Heavy resonanes capture the difference only for high T



Wuppertal: arXiv:1309.5258v2, HotQCD: arXiv:1203.0784v2

Results → fit to observables





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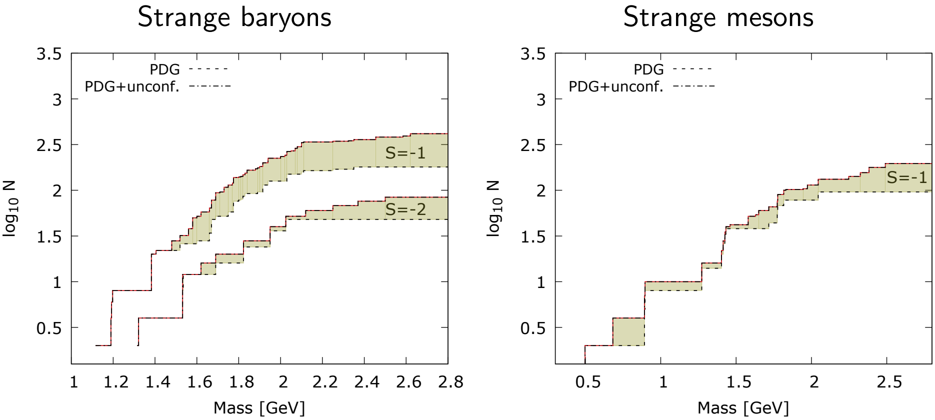
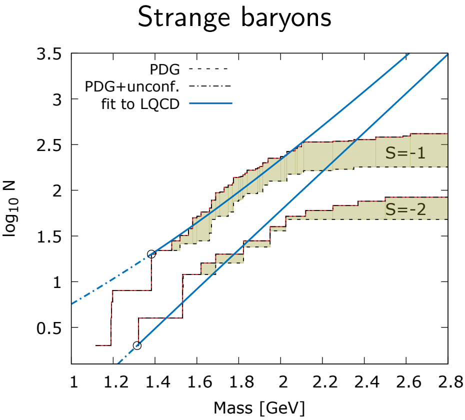
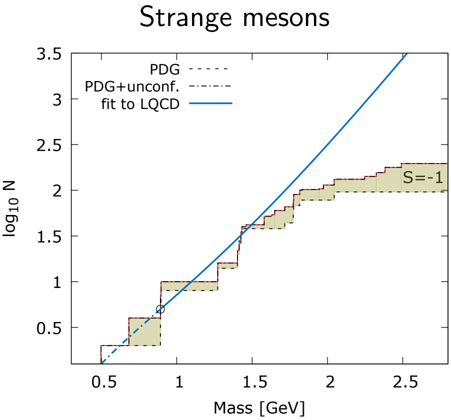
Results → mass spectrum



Results → mass spectrum





Conclusions

We addressed the problem of missing strange resonances in the HRG model

- It is possible to use common Hagedorn temperature for baryons and mesons
  - $T_H \leqslant T_c$ to comply with LQCD results,
  - functional form not suitable in the low-mass region;
- substantial contribution from high-mass strange states to the fluctuations near T_c
  - Hagedorn spectra are consistent with both observables and not yet established strange baryons spectra,
  - result for strange mesons spectrum $\longrightarrow$ extra undiscovered heavy states?
- new eperimental data and further LQCD studies would be extremely helpful in claryfing these issues.

The End