

Quantum spin dynamics of quasi-one-dimensional Heisenberg-Ising magnets in a transverse field: confined spinons, E_8 spectrum, and quantum phase transitions

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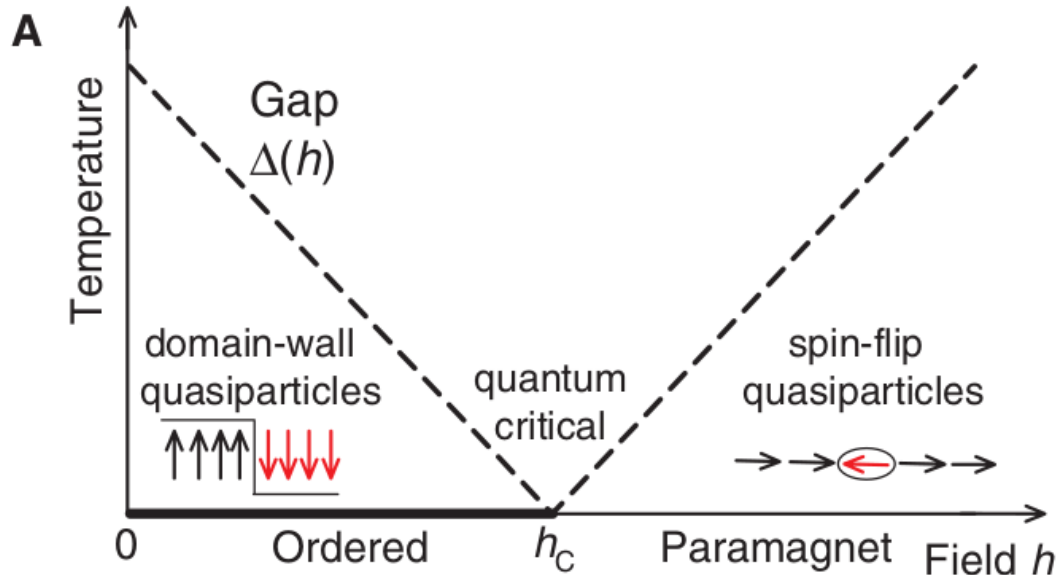
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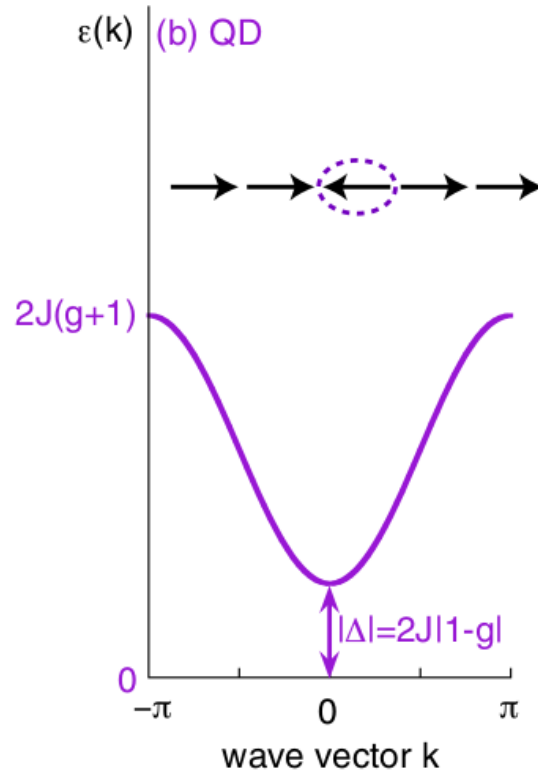
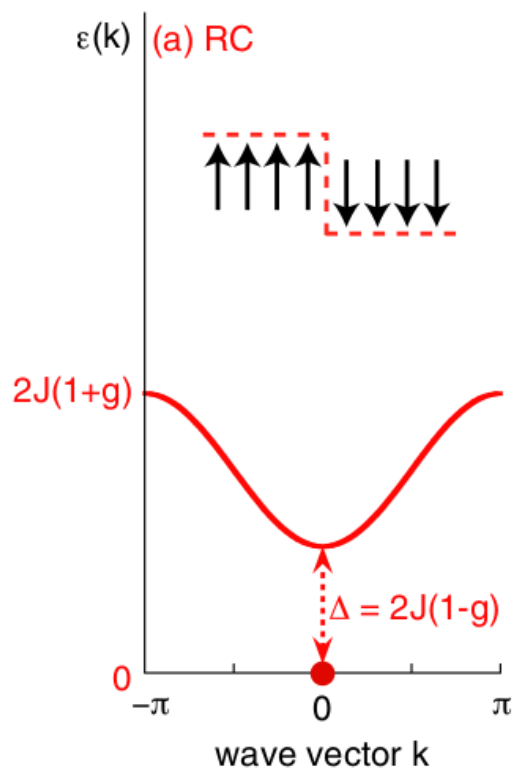
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1D Transverse-field Ising model

$$H = \sum_i -JS_i^z S_{i+1}^z - hS_i^x$$



1D Ising model

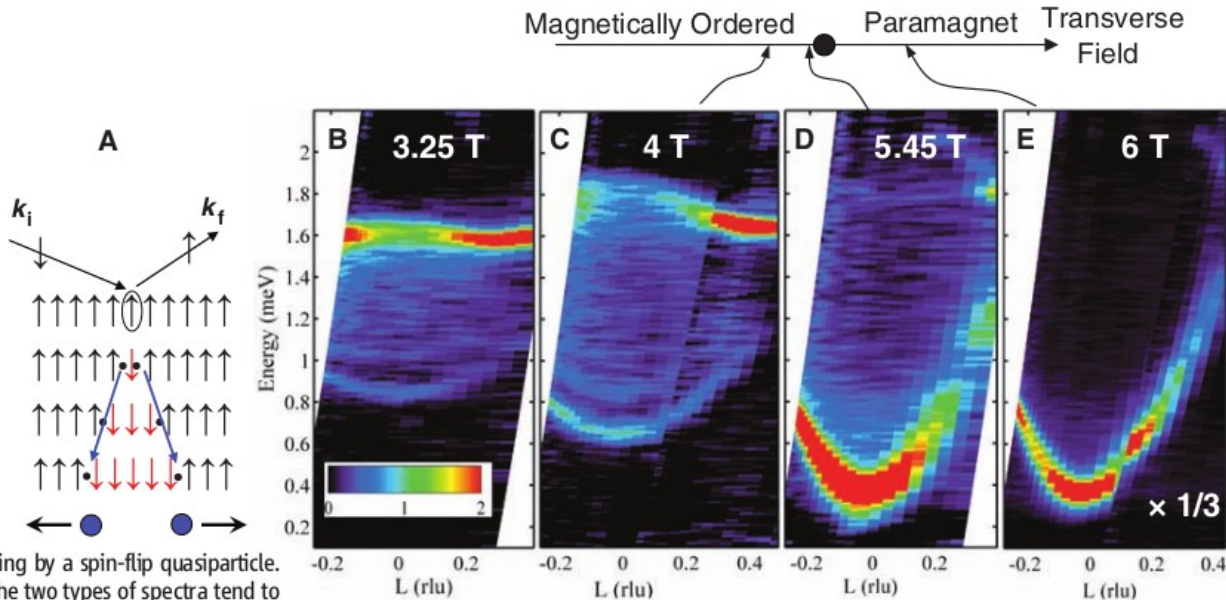


1D Ising model: INS

Fig. 2. (A) Cartoon of a neutron spin-flip scattering that creates a pair of independently propagating kinks in a ferromagnetically ordered chain.

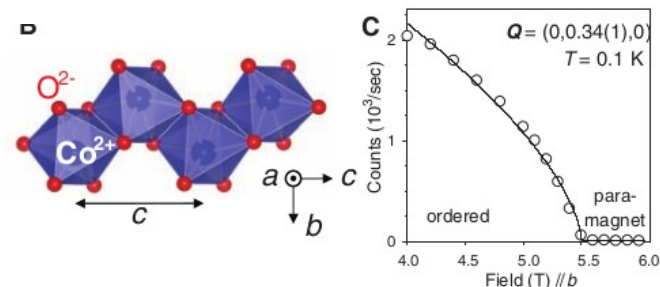
(B to E) Spin excitations in CoNb_2O_6 near the critical field as a function of wave vector along the chain (in rlu units of $2\pi/c$) and energy (18). In the ordered phase [(B) and (C)], excitations form a continuum due to scattering by pairs of kinks [as illustrated in (A)]; in the paramagnetic phase (E), a single dominant sharp mode occurs, due to scattering by a spin-flip quasiparticle.

Near the critical field (D), the two types of spectra tend to merge into one another. Intensities in (E) are multiplied by $1/3$ to make them comparable to the other panels.



Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry

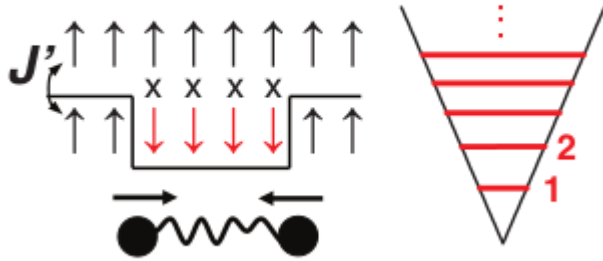
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Interchain coupling or longitudinal field

G

kink confinement



$$h_z = \sum_{\delta} J_{\delta} \langle \mathbf{S}^z \rangle$$

$$\lambda = 2h_z \langle \mathbf{S}^z \rangle / \tilde{c}$$

$$V(x) = \lambda|x|$$

$$-\frac{\hbar^2}{\mu} \frac{d^2 \varphi}{dx^2} + \lambda|x|\varphi = (E - 2E_0)\varphi$$

Kink confinement

$$-\frac{\hbar^2}{\mu} \frac{d^2\varphi}{dx^2} + \lambda|x|\varphi = (E - 2E_0)\varphi$$

$$E_i = 2E_0 + \zeta_i \lambda^{2/3} \left(\frac{\hbar^2}{\mu} \right)^{1/3}, \quad i = 1, 2, 3, \dots,$$

$$\text{Ai}(-\zeta_i) = 0 \quad \text{- Airy function}$$

THz Spectroscopy

$$\alpha \equiv \Delta\alpha(B) = \alpha(B, 2.7\text{K}) - \alpha(0\text{T}, 10\text{K}) = -\frac{1}{d} \ln \left[\frac{I(B, 2.7\text{K})}{I(0\text{T}, 10\text{K})} \right], \quad (5)$$

where $I(B, T)$ is the transmitted intensity in a given field B at a given temperature T , and d denotes the sample thickness. For this insulator with relatively weak absorption peaks of

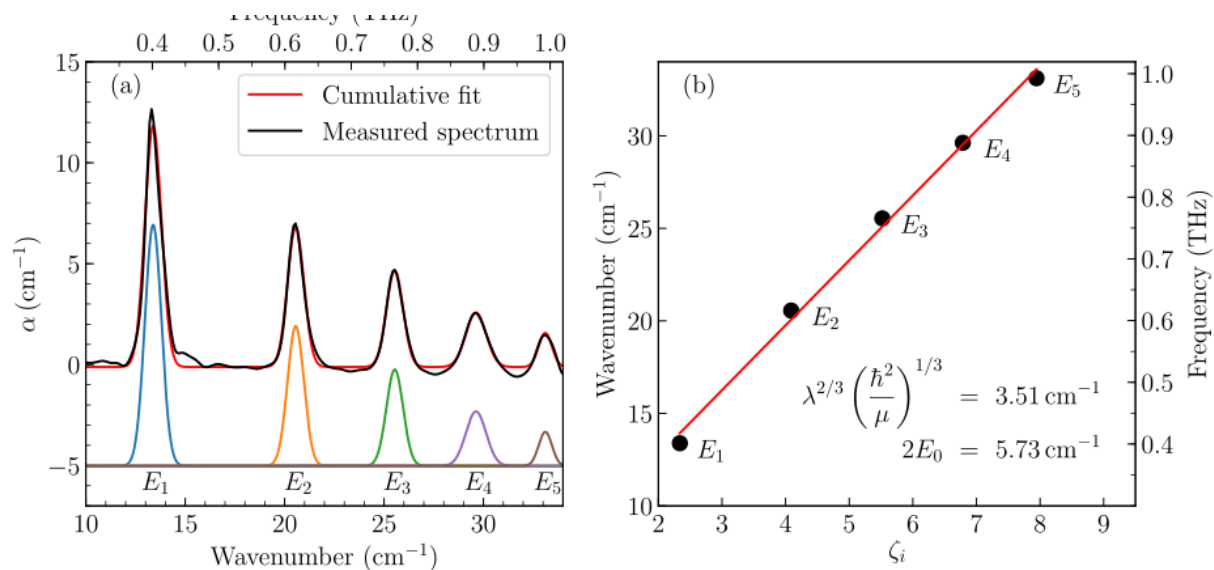


Figure 5. (a) Zero-field spectrum of $\text{BaCo}_2\text{V}_2\text{O}_8$ at 2.7 K below T_N . Fits of the five absorption peaks E_1 – E_5 are shown under the spectrum. The red line shows the cumulative

THz Spectroscopy

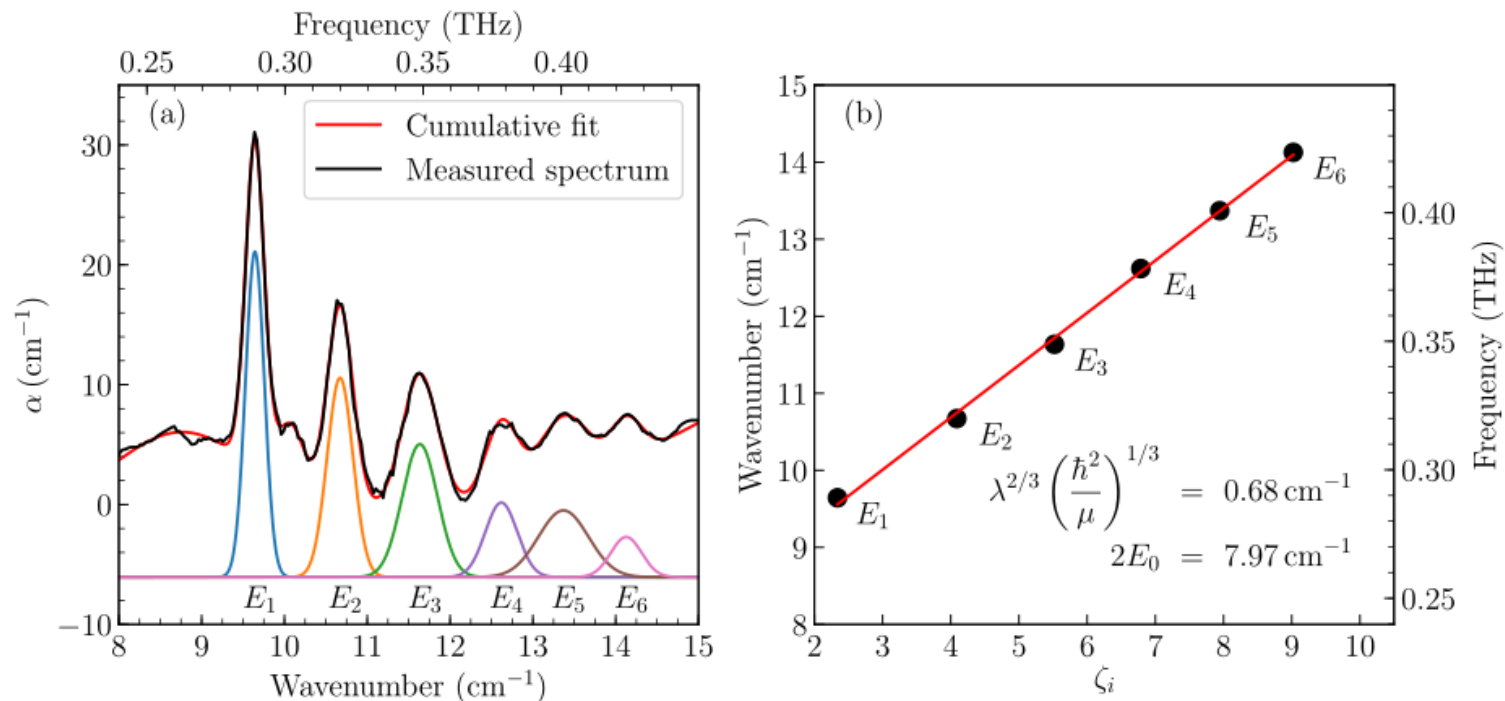
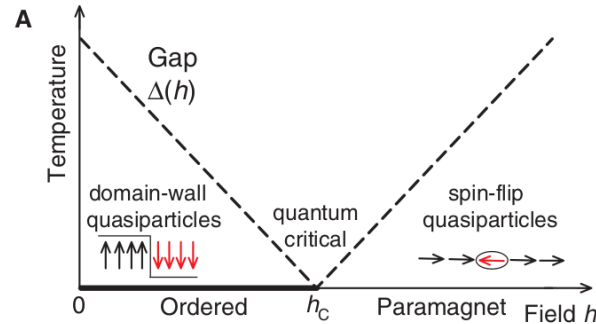


Figure 6. (a) Zero-field spectrum of CoNb_2O_6 at 250 mK below T_C . Fits of the six absorption peaks E_1 – E_6 are shown under the spectrum. The red line shows the cumulative absorption, which includes the six peaks and a wide absorption background. (b) Absorption peak energies follow a linear dependence on ζ_i , the negative zeros of the Airy function $A_i(-\zeta_i) = 0$, for $2E_0 = 7.97 \text{ cm}^{-1}$ and $\lambda^{2/3} \left(\frac{\hbar^2}{\mu} \right)^{1/3} = 0.68 \text{ cm}^{-1}$.

Critical point



$$H_{1/2}^{(1,2)} = H_{1/2} + h \int \sigma(x) d^2x$$

INTEGRALS OF MOTION AND S-MATRIX OF THE (SCALED) $T = T_c$ ISING MODEL WITH MAGNETIC FIELD

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Exactly solvable

$$H_{1/2}^{(1,2)} = H_{1/2} + h \int \sigma(x) d^2x$$

$$m_1 = m; \quad m_2 = 2m \cos \frac{\pi}{5}; \quad m_3 = 2m \cos \frac{\pi}{30};$$

$$m_4 = 2m_2 \cos \frac{7\pi}{30}; \quad m_5 = 2m_2 \cos \frac{2\pi}{15}; \quad m_6 = 2m_2 \cos \frac{\pi}{30};$$

$$m_7 = 4m_2 \cos \frac{\pi}{5} \cos \frac{7\pi}{30}; \quad m_8 = 4m_2 \cos \frac{\pi}{5} \cos \frac{2\pi}{15}.$$

$$\frac{m_2}{m_1} = \frac{\sqrt{5} + 1}{2}$$

The exact relations between the coupling constants and the masses of particles for the integrable perturbed Conformal Field Theories

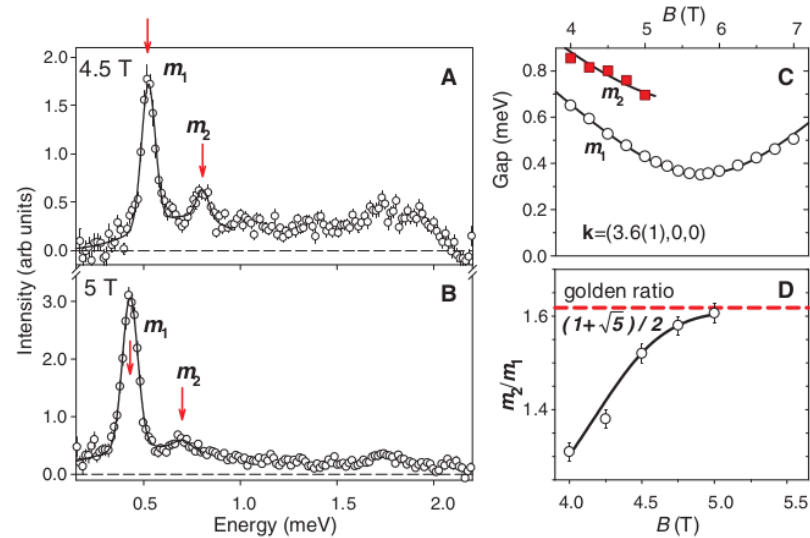
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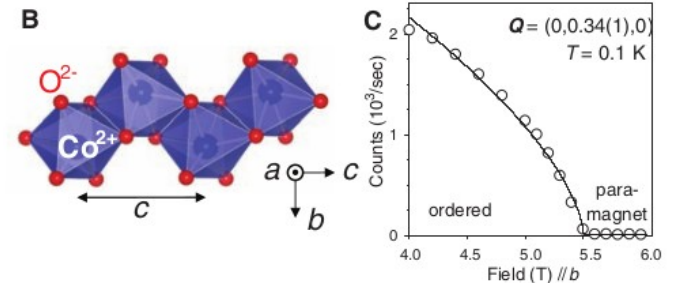
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E8 Golden ratio in INS



Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry

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E8 excitations

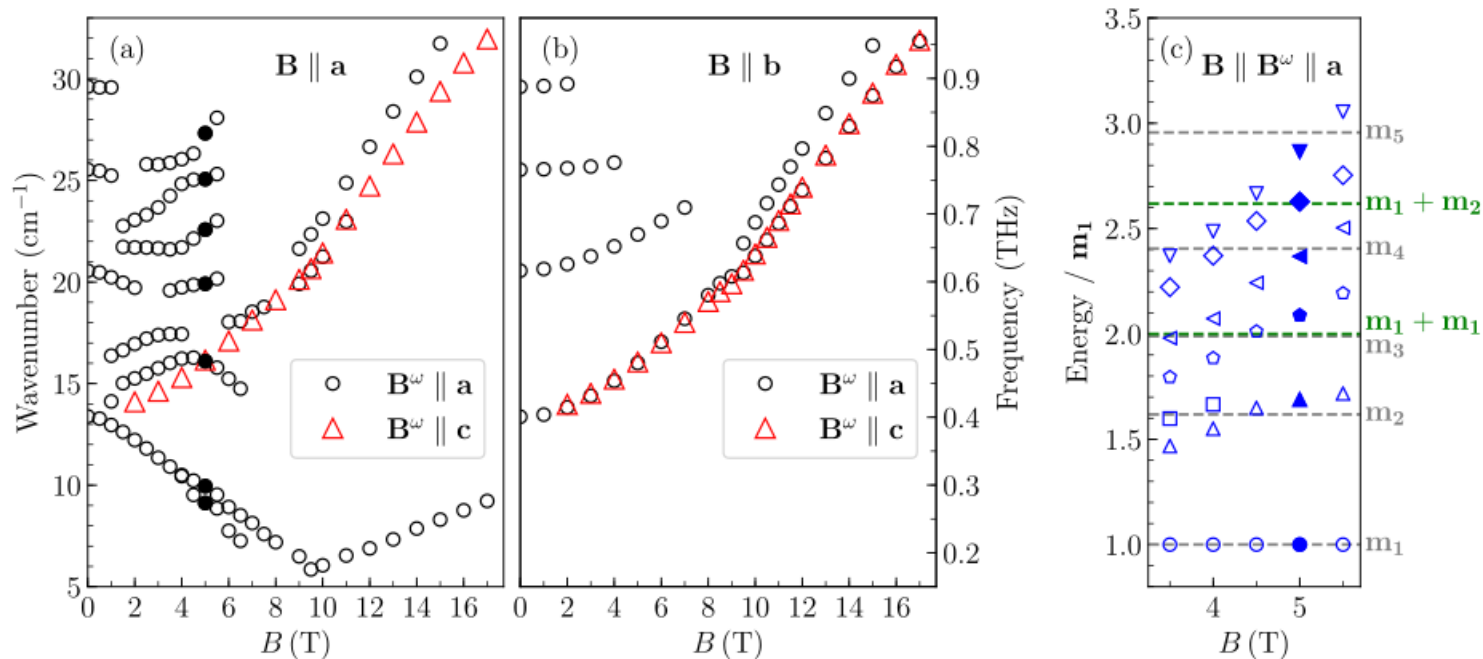


Figure 8. $\text{BaCo}_2\text{V}_2\text{O}_8$ absorption peak frequency dependence on the magnetic field at 2.7 K for the two field directions: (a) $\mathbf{B} \parallel \mathbf{a}$ and (b) $\mathbf{B} \parallel \mathbf{b}$. Solid black dots in (a) indicate the E_8 excitations at $B_{\perp}^{c,1D}$. (c) Ratios of the excitation energies with respect to the lowest-frequency excitation m_1 for the field direction $\mathbf{B} \parallel \mathbf{a}$ close to $B_{\perp}^{c,1D}$. The ratios approach theoretically predicted values for the E_8 excitations (marked with dashed lines) at 5 T.

E8 excitations

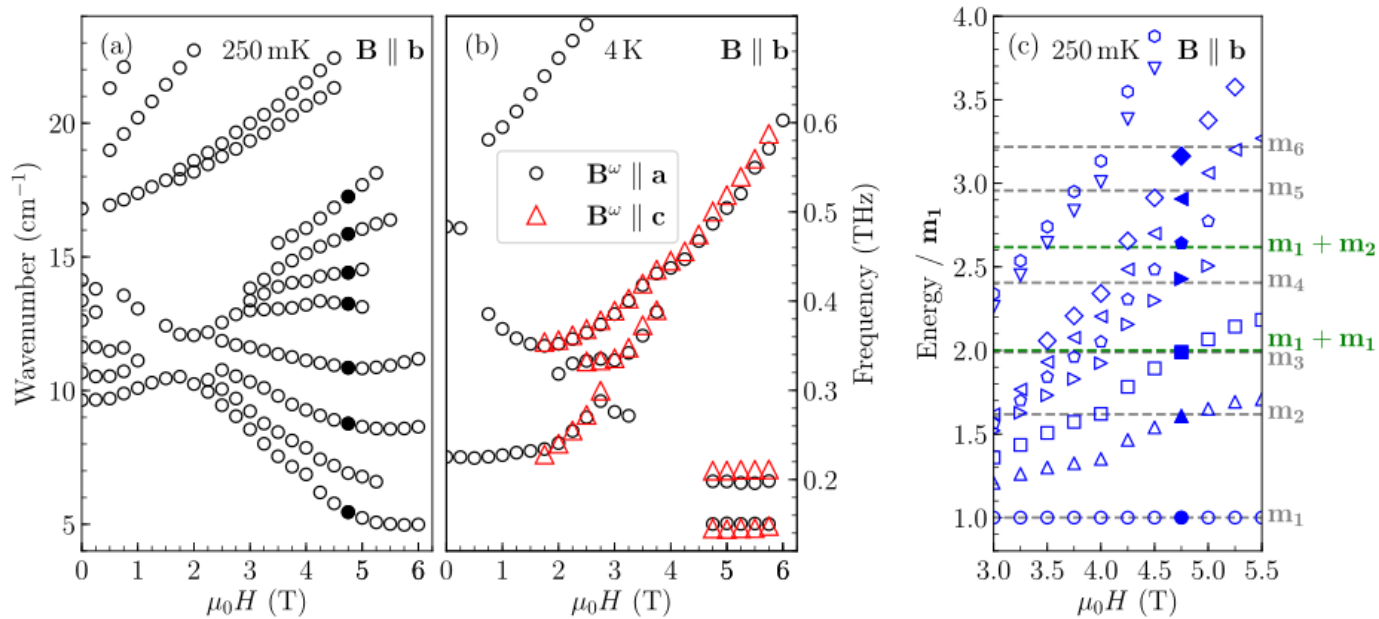


Figure 9. (a) CoNb_2O_6 absorption peak frequency dependence on the magnetic field $\mathbf{B} \parallel \mathbf{b}$ at 250 mK (see figure 4). Solid markers indicate the E_8 excitations at $B_\perp^{c,1D}$. (b) Resonance frequency as a function of the applied magnetic field at 4 K, which is above T_C . (c) Ratios of the excitation energies with respect to the lowest-frequency excitation m_1 for the field direction $\mathbf{B} \parallel \mathbf{b}$ close to $B_\perp^{c,1D}$ (see black dots in (a)). The ratios approach theoretically predicted values for the E_8 excitations (marked with dashed lines) at 4.75 T.