Nuclear pasta in neutron stars

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UFSC

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Laboratoire de physique corpusculaire

Summary

- Neutron star & the pasta phase
- Effective relativistic mean field models
- Density and geometry fluctuations

  Phys.Rev.C 104 (2021) 2, L022801

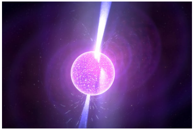
- Transport properties: relaxation time and electric conductivity

  MNRAS, accepted

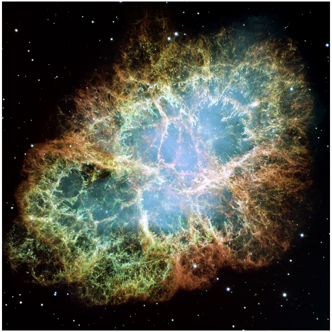
- Short-range correlations: effects in the pasta

  arXiv: 2211.14002

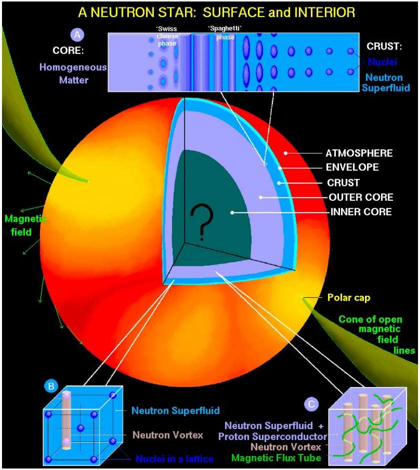
Neutron stars



Source: Gill, K., Scientific American (2019)

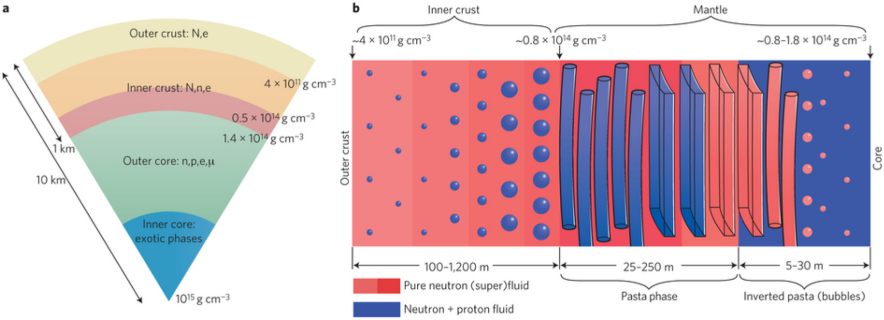


Hubble telescope



https://www.astro.umd.edu/∼miller/nstar.html

The inner crust



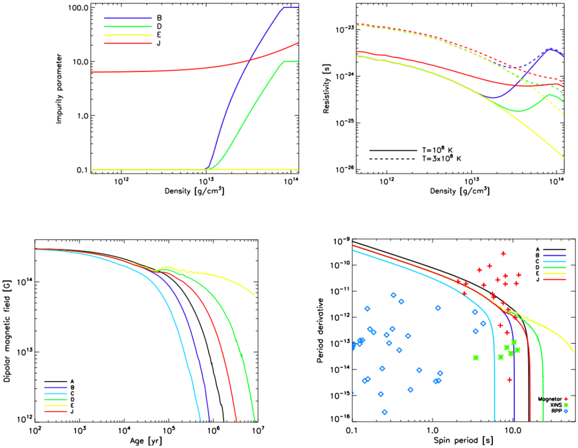
Source: Newton, W.G., Nature Phys. 9, p. 396–397 (2013)

$$\left.\begin{array}{l}\text{Thermo} - \text{magnetic evolution}\\ \text{Shear modes, Spin down \& GWs}\\ \text{Neutrino opacity in PNS}\end{array}\right\rangle \text{Sensitive to} \left\{\begin{array}{l}\text{Anisotropies}\\ \text{Impurities}\\ \text{Lattice}\end{array}\right.$$

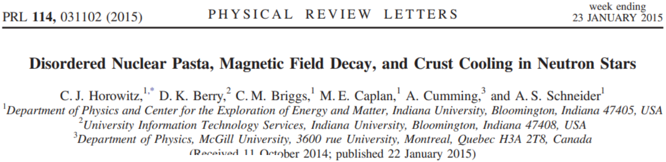
Role of impurities

A highly resistive layer within the crust of X-ray pulsars limits their spin periods

José A. Pons[1]★, Daniele Viganò[1] and Nanda Rea[2]



Role of impurities



PRL 114, 031102 (2015) PHYSICAL REVIEW LETTERS week ending 23 JANUARY 2015

Disordered Nuclear Pasta, Magnetic Field Decay, and Crust Cooling in Neutron Stars

C. J. Horowitz,[1,*] D. K. Berry,[2] C. M. Briggs,[1] M. E. Caplan,[1] A. Cumming,[3] and A. S. Schneider[1]
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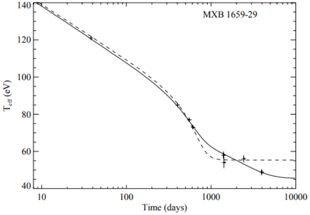
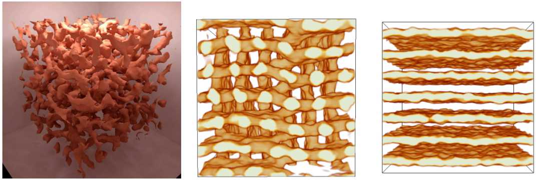
(Received 11 October 2014; published 22 January 2015)



Fixed impurity parameter of the order $Q_{\rm imp} \sim 1 - 50$.

$$Q_{\rm imp} = \sum_j (Z_j - \langle Z \rangle)^2$$

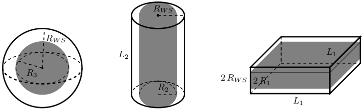


Source: Left: Horowitz et al., Phys. Rev. C 70, 065806 (2004); Right: Schneider et al., Phys. Rev. C 93, 065806 (2016)

One–component plasma

- The deformations are due to frustration: competition between the attractive nuclear interaction and the repulsive Coulomb force $(pp + pe + ee)$

$$E_S = S\sigma(Y_p, T) \qquad E_C = \frac{1}{2}\int_{\text{WS cell}} d^3\vec{r}\,\rho_{\text{ch}}(\vec{r})\,\phi(\vec{r})$$



Source: Pelicer, 2023.

$$\begin{cases} \mathcal{F}_{WS} = f\mathcal{F}_b^I + (1-f)\mathcal{F}_b^{II} + \beta\mathcal{F}_{\text{sc,d}} + \mathcal{F}_e \\ \rho_p = f\rho_p^I + (1-f)\rho_p^{II} \\ \rho_n = f\rho_n^I + (1-f)\rho_n^{II} \\ \rho_p = \rho_e \\ \{\rho_p^I, \rho_n^I, \rho_p^{II}, \rho_n^{II}, R_d, f\} \end{cases}$$

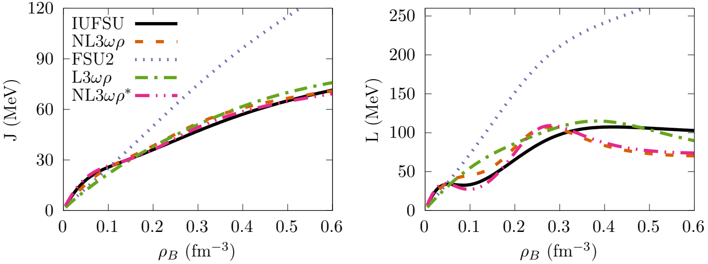
$$\mu_p^I = \mu_p^{II}, \quad \mu_n^I = \mu_n^{II}, \quad P^I = P^{II}$$

$$\mathcal{F}_{sc,d} = \frac{E_{S,d} + E_{c,d}}{V_{WS}} = \frac{d\sigma(Y_p, T)}{R_d} + 2\pi e^2 R_d^2 \left(\rho_p^I - \rho_p^{II}\right)^2 \Phi_d(\beta).$$

Effective RMF models

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho}.$$

$$\frac{E}{A} = \frac{\mathcal{E}_b}{\rho_B} = E_0 + \frac{1}{2}K\left(\frac{\rho_B - \rho_0}{3\rho_0}\right)^2 + (1 - 2Y_p)^2\,\mathcal{S}\left(\rho_B\right),$$

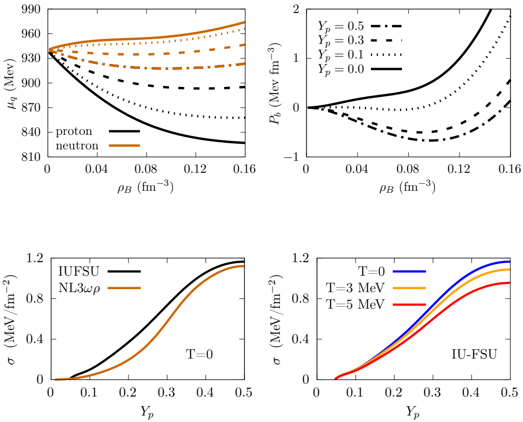


B/A (MeV)	J (MeV)	L (MeV)	K (MeV)	M_N^*	ρ_{sat} (fm^{-3})
15.8 - 16.5	28.6 - 34.4	30.6 -86.8	220 - 260	0.6 - 0.8	0.15 - 0.16

Based on Oertel, M. et al., Rev. Mod. Phys., v. 89, (2017)

Effective RMF models: parametrization

IUFSU

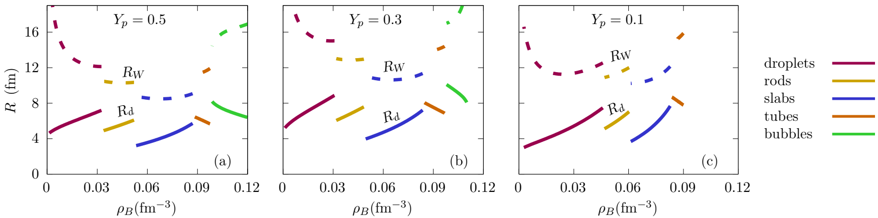


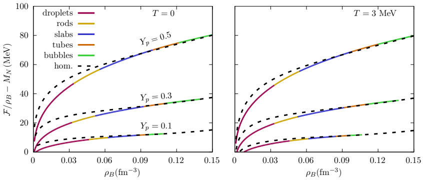
Params.	IUFSU
m_s (MeV)	491.5
m_v (MeV)	782.5
m_b (MeV)	763.0
g_s	9.971
g_v	13.032
g_b	13.590
κ	3.5695
λ	2.926
ξ	0.03
Λ	0.046
ρ_{sat} (fm^{-3})	0.155

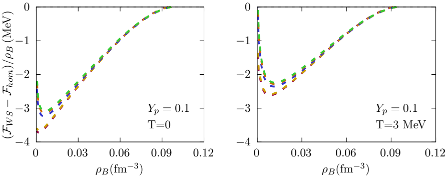
B/A (MeV)	J (MeV)	L (MeV)	K (MeV)	M_N^*	ρ_{sat} (fm^{-3})	M_{max}
-16.40	31.3	47.2	231.2	0.6	0.155	1.97 $M_\odot$

Fattoyev, F. J., et al. Phys. Rev. C 82, 055803 (2010)

One-component plasma: the pasta







- Small energy gap between pasta geometries

Multi–component approach

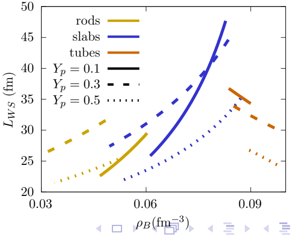
- Can we calculate the pasta impurity parameter with a RMF?

Gulminelli & Raduta, Phys. Rev. C 92, 055803 (2015)

$$Z = z_g^V z_e^V Z_{cl} \quad \Rightarrow \quad Z_{cl} = \sum_{\{n\}} \exp\left[-\beta V \sum_{N,d} n^{N,d} \tilde{\Omega}^{N,d}\right]$$

$$\tilde{\Omega}^{N,d} = V^N \quad \left[\mathcal{F}_b^N - \mathcal{F}_{b,g} + \mathcal{F}_{sc,d}^N - \mu_n\left(\rho_n^N - \rho_{ng}\right) - \mu_p\left(\rho_p^N - \rho_{pg}\right)\right] + \delta F^N$$

$$V_1^N = 2R_1^N\left(L_1^N\right)^2$$
$$= V_2^N = \pi\left(R_2^N\right)^2 L_2^N$$
$$= V_3^N = 4\pi\left(R_3^N\right)^3/3$$

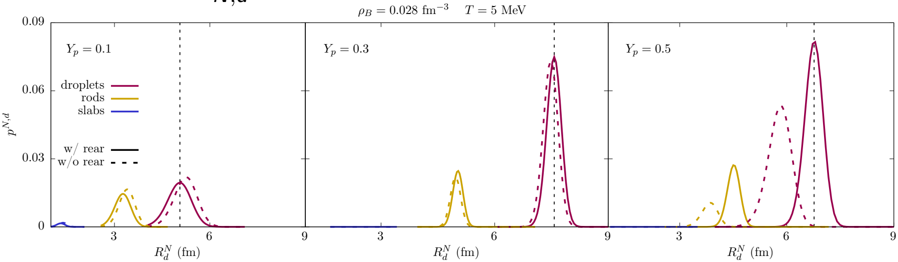


MCP: Rearrangement term

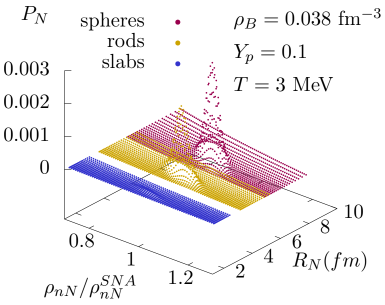
- Rearrangement: $\rho_p = \rho_e = \rho_{p\,WS} = f^N(\rho_p^N - \rho_{pg}) + \rho_{pg}$

$$\delta F^N = Z^N \left\langle \frac{f^M}{\rho_p^M - \rho_{pg}} \frac{\partial \mathcal{F}^M_{sc,d}}{\partial f^M} \right\rangle \approx \left[\frac{f}{\rho_p^I - \rho_{pg}} \frac{\partial \mathcal{F}_{sc,d}}{\partial f} \right]_{OCP}$$

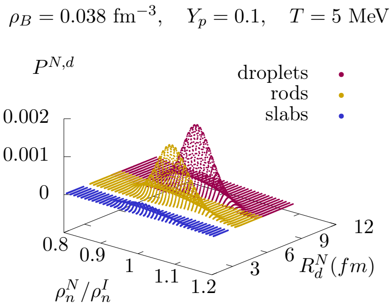
$$p^{N,d} = \frac{\exp\left(-\beta \tilde{\Omega}^{N,d}\right)}{\sum_{N,d} \exp\left(-\beta \tilde{\Omega}^{N,d}\right)}, \qquad \Bigg|\Bigg| \qquad P_d = \sum_N p^{N,d}$$



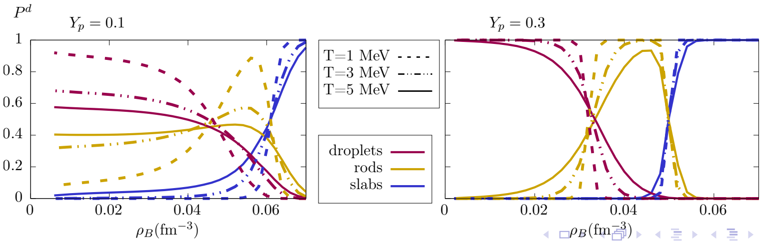
MCP: Results



P_N
spheres
rods
slabs
$\rho_B = 0.038$ fm^{-3}
$Y_p = 0.1$
$T = 3$ MeV
0.003
0.002
0.001
0
0.8
1
1.2
$\rho_{nN}/\rho_{nN}^{SNA}$
2
4
6
8
10
$R_N(fm)$



$\rho_B = 0.038$ fm^{-3}, $Y_p = 0.1$, $T = 5$ MeV
$P^{N,d}$
droplets
rods
slabs
0.002
0.001
0
0.8
0.9
1
1.1
1.2
ρ_n^N/ρ_n^I
3
6
9
12
$R_d^N(fm)$

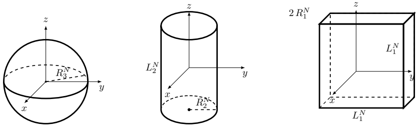


P^d
$Y_p = 0.1$
1
0.8
0.6
0.4
0.2
0
0.02
0.04
0.06
$\rho_B(\mathrm{fm}^{-3})$
T=1 MeV
T=3 MeV
T=5 MeV
droplets
rods
slabs
$Y_p = 0.3$
0
0.02
0.04
0.06
$\rho_B(\mathrm{fm}^{-3})$

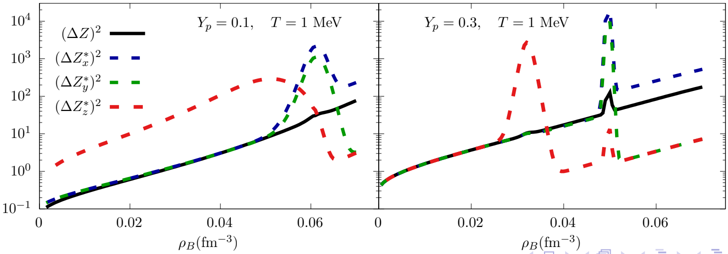
MCP: Impurity

$$Q_{\mathrm{imp}} = (\Delta Z)^2 = \sum_{N,d} p^{N,d} \left(Z^N - \langle Z^N \rangle \right)^2$$

$$Z^{*\,N}_{d,k} = Z^N \frac{S^N_{d,k}}{S^N_3}.$$



Axis	$S^N_{2,k}$	$S^N_{1,k}$
x	$\pi R^N_2 L^N_2$	$\left(L^N_1\right)^2$
y	$\pi R^N_2 L^N_2$	$R^N_1 L^N_1$
z	$\pi (R^N_2)^2$	$R^N_1 L^N_1$



Conductivity: Relaxation time approximation

- Main contribution to conductivity in the crust: *Electron–ion scattering*
  - Charge current

$$\mathbf{J}_e = -e \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, \mathbf{v}\, f(\mathbf{r},\mathbf{p}) = \hat{\boldsymbol{\sigma}} \mathbf{E}^*$$

  - Electron distribution function

$$f(\mathbf{r},\mathbf{p}) = f^0(\mathbf{r},\mathbf{p}) + \delta f(\mathbf{p}),$$

  - Linearized Boltzmann equation

$$\left(-\frac{\partial f_0}{\partial \epsilon_p}\right) \mathbf{v} \cdot \left[\frac{\partial \mu}{\partial \mathbf{x}} + e\mathbf{E} + \frac{\epsilon_p - \mu}{T}\frac{\partial T}{\partial \mathbf{x}}\right] - e(\mathbf{v}\times\mathbf{B})\frac{\partial \delta f}{\partial \mathbf{p}} = I[f],$$

  - Collision integral for elastic collisions

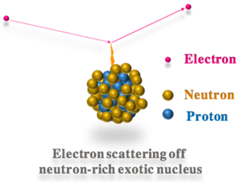
$$I[f] = 2\pi \int \frac{d^3\mathbf{p}\,'}{(2\pi)^3}\delta(\epsilon_p - \epsilon_{p'})W_{pp'}\left[\delta f(\mathbf{p'}) - \delta f(\mathbf{p})\right],$$

Conductivity: e-i scattering

$$W_{pp'} = \frac{e^2}{8\epsilon_F^2} \sum_{s,s'} \left| \bar{u}_{p's'} \gamma^0 u_{ps} \int d^3\mathbf{x} A_0(\mathbf{x}) e^{-i(\mathbf{p}-\mathbf{p'})\cdot\mathbf{x}} \right|^2$$

$$= \left(1 - \frac{q^2}{4\,\epsilon_F^2}\right) \left| \frac{4\pi Ze\, F_d(\mathbf{q})}{q^2 \varepsilon(q)} \right|^2 S(\mathbf{q}),$$

- $\mathbf{q} = \mathbf{p} - \mathbf{p'}$ & $q^2 = 2p_F^2(1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$
- Form factor ($F(\mathbf{q})$): ion finite size
- Structure factor ($S(\mathbf{q})$): particle correlations
  - Solid: Phonons
  - Liquid: Thermal ion-density fluctuations



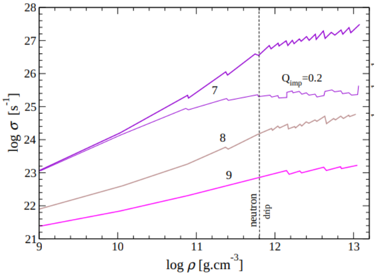
Source:Nuclear Science Group, Tohoku University (2017)

Isotropic transport

$$W_{pp'} = \sum_l w_l P_l\left(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}'\right)$$

$$\delta f(\mathbf{p}) = \sum_{l>0\, m} \delta f_{lm}\left(\epsilon_p\right) Y_l^m(\Omega_p).$$

$$I[f] = -\sum \delta f_{lm}(\epsilon_p)\nu_l(\epsilon_p) Y_l^m(\Omega_p),$$



Source: Chamel, 2008.

- Conductivity: $l = 1$: $I[f] = -\nu_1\, \delta f$

$$\sigma = \frac{n_e e^2}{m_e^* \nu_1}, \qquad \nu_1 = \frac{4\pi n_i e^4 Z^2}{v_F p_F^2} \int_0^{2p_F} \frac{dq}{q} \left(1 - \frac{q^2}{4\epsilon_F^2}\right) \frac{F^2(q)}{\varepsilon^2(q)} S(q)\,,$$

- Assumption of isotropy

Conductivity in the pasta: An intuitive proposal

Electron transport through nuclear pasta in magnetized neutron stars

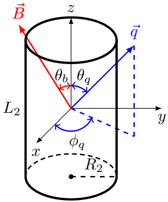
D. G. Yakovlev★

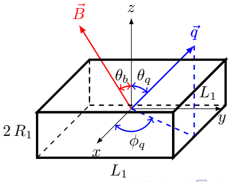
Ioffe Physical Technical Institute, 26 Politekhnicheskaya, St Petersburg 194021, Russia

$$I[f] = -\nu_1 \, \delta f \quad \Longrightarrow \quad -v_z \Phi_z \nu_a - \boldsymbol{v}_p \cdot \boldsymbol{\Phi}_p \nu_p,$$

- Provided expressions for the conductivity w/ magnetic field;
- ν_a/ν_p treated as a free parameter;

$$\hat{\sigma}_0 = \frac{e^2 n_e}{m_e^*} \begin{pmatrix} \nu_p^{-1} & 0 & 0 \\ 0 & \nu_p^{-1} & 0 \\ 0 & 0 & \nu_a^{-1} \end{pmatrix}$$





Anisotropic transport

$$W_{p\,p'}\left(\Omega_p, \Omega_{p'}, \epsilon_p\right) = \sum_{lm\,l'm'} \mathcal{W}_{lm\,l'm'}(\epsilon_p) Y_l^m(\Omega_p) Y_{l'}^{m'}(\Omega_{p'}).$$

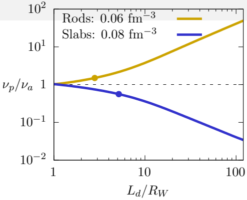
$$I[f] = -\sum_{lm,l'm'} \delta f_{lm}\left(\epsilon_p\right) \left[\nu\left(\epsilon_p\right)\right]_{lm}^{l'm'} Y_{l'}^{m'}\left(\Omega_p\right)$$

- $l = l' = 1$: $$\hat{\boldsymbol{\nu}} = \begin{pmatrix} \nu_{xx} & \nu_{xy} & \nu_{xz} \\ \nu_{yx} & \nu_{yy} & \nu_{yz} \\ \nu_{zx} & \nu_{zy} & \nu_{zz} \end{pmatrix} = \begin{pmatrix} \nu_p & 0 & 0 \\ 0 & \nu_p & 0 \\ 0 & 0 & \nu_a \end{pmatrix}$$

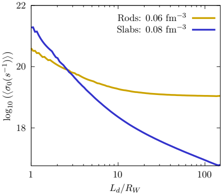
$$\nu_a\left(\epsilon_F\right) = \frac{12\pi n_i e^4 Z^2}{v_F p_F^2} \int_0^{2p_F} \frac{dq}{q} \left(1 - \frac{q^2}{4\epsilon_F^2}\right) \int \frac{d\Omega_q}{4\pi} \left|\frac{F_d(\mathbf{q})}{\varepsilon^2(q)}\right|^2 S(\mathbf{q}) \cos^2\theta_q$$

$$\nu_p\left(\epsilon_F\right) = \frac{12\pi n_i e^4 Z^2}{v_F p_F^2} \int_0^{2p_F} \frac{dq}{q} \left(1 - \frac{q^2}{4\epsilon_F^2}\right) \int \frac{d\Omega_q}{4\pi} \left|\frac{F_d(\mathbf{q})}{\varepsilon^2(q)}\right|^2 S(\mathbf{q}) \frac{1}{2} \sin^2\theta_q$$

Conductivity in the pasta



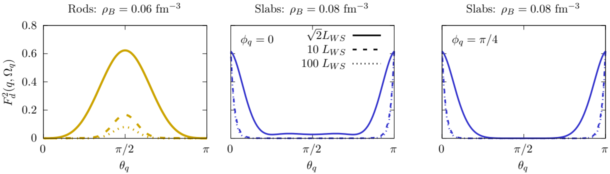
- Uncorrelated scatterings: $S(\mathbf{q}) \to 1$



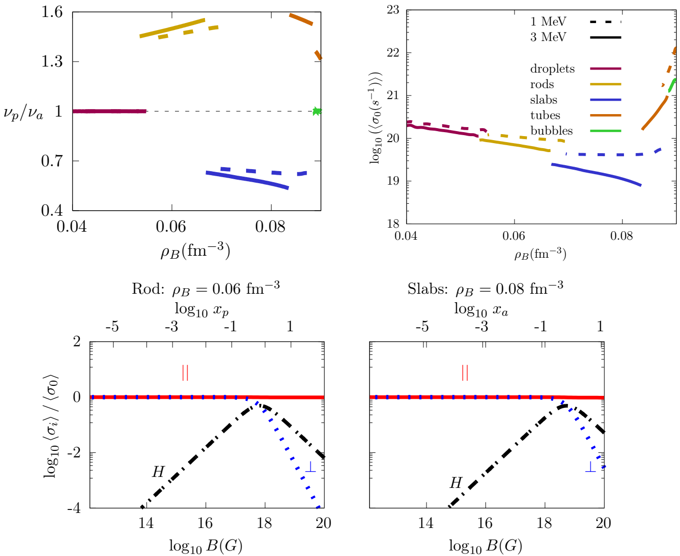
- Effective size: $L_d \approx \sqrt{2} L_{1\,\mathrm{WS}}$

$$2^{-\eta} \approx \frac{\delta n^2(2R_{W1})}{\delta n^2(R_{W1})} = \frac{\delta n^2(L_1^{eff})}{\delta n^2(L_{1\,\mathrm{WS}})}$$

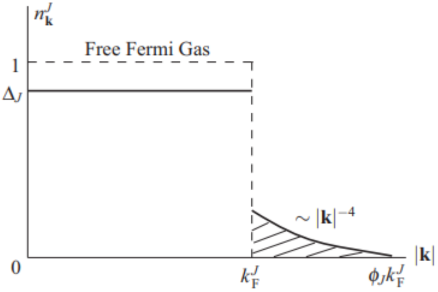
- $T \gtrsim 1$ MeV



Conductivity in the pasta



Short-range correlations

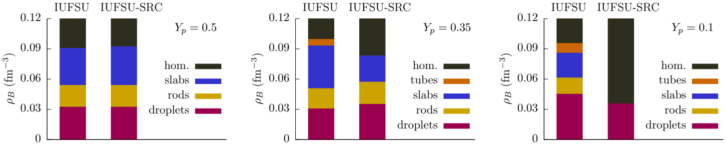


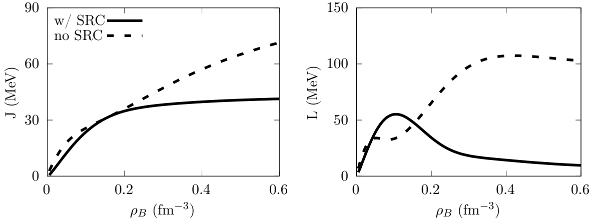
Source: B. J. Cai and B. A. Li, Phys. Rev. C 92, 011601(R) (2015)

	IUFSU-SRC
m_s (MeV)	491.5
m_v (MeV)	782.5
m_b (MeV)	763.0
g_s	10.132
g_v	11.867
g_b	15.551
κ	5.9113
λ	-179.28
ξ	0.03
Λ	0.0055
ρ_{sat} (fm^{-3})	0.155

$$f_{n,p}(k) = \begin{cases} \Delta_{n,p}, & 0 < k < k_{F\ n,p} \\ \dfrac{C_{n,p}\, k_{F\ n,p}^4}{k^4}, & k_{F\ n,p} < k < \phi_{n,p} k_{F\ n,p}, \end{cases}$$

SRCs & pasta





- SRCs $\Rightarrow$ no pasta in the RMF;
- The same results were obtained with the NL3 and FSU2r parametrizations;

Summary & Perspectives

- Inner crust of NS contains pasta $\rightarrow$ **anisotropy & impurity** affect cooling curve, spin period, neutrino opacity, radial oscillation and GW emission.
  - Pasta impurity with the RMF is very high, in accord to suggestions in the literature.
  - Analytical expressions for the axial and perpendicular collision rates of the electron–pasta contribution to conductivity were obtianed
  - How does the impurity affect transport in the pasta?
  - Structure factor of the pasta?
  - Better Z estimation
  - Viscosity calculation?
- SRCs in the RMF make the pasta disappear at low Y_p.
  - Changes in the surface tension?
  - Change in SRCs when considering asymmetric matter?

Thank you for watching!