Thermal effects on weak-interaction nuclear reactions in stellar conditions

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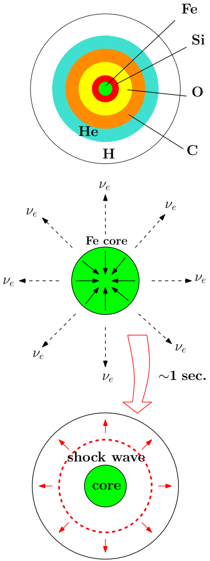


- Massive stars ($M \geq 10M_{\odot}$) at the end of their life have an onion like structure.
- When $M_{Core} \approx M_{Ch} = 1.44(2Y_e)^2 M_{\odot}$ the iron core starts to collapse ($Y_e = n_e/n_N$ is the electron fraction).
- Weak-interaction processes are **crucial** for supernova explosion mechanism:
  1. electron capture

$$e^- + A(N,Z) \to A(Z-1, N+1) + \nu_e$$

     reduces the electron gas pressure and determine M_{Core}.
  2. ν-nucleus reactions are important at $\rho \gtrsim 10^{11}$ g/cm^3:

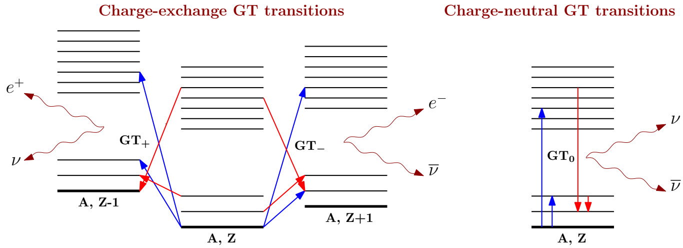
$$\nu + A(Z,N) \to A(Z,N) + \nu',$$

$$\nu + A(Z,N) \to A(Z+1, N-1) + e^-$$

     trap neutrinos inside the core and affect the energy transport.

In supernova $E_{e,\nu} \leq 30$ MeV and **Gamow-Teller** (1^+) transitions dominate the nuclear weak-interaction processes:

- $e^- + A(N,Z) \to A(Z-1, N+1) + \nu_e \qquad (GT_+ = \sum_i \sigma_i t_i^+)$;
- $\nu_e + A(Z,N) \to A(Z+1, N-1) + e^- \qquad (GT_- = \sum_i \sigma_i t_i^-)$;
- $\nu + A(Z,N) \to A(Z,N) + \nu' \qquad (GT_0 = \sum_i \sigma_i t_i^0)$.



In the supernova environment nuclear weak-interaction reactions take place at finite temperatures $T = 0.1 \div 5$ MeV ($0.86\,\mathrm{MeV} = 10^{10}$ K). Nuclear excited states are thermally populated according to Boltzmann distribution $p_i(T) \sim \exp(-E_i/T)$

$$\sigma(T, E_l) = Z(T)^{-1} \sum_i e^{-E_i/T} \sigma_i(E_l), \quad \lambda(T) = Z(T)^{-1} \sum_i e^{-E_i/T} \lambda_i$$

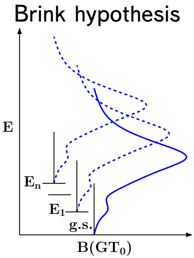
K. Langanke, G. Martinez-Pinedo, ADNDT 79 (2001) 1

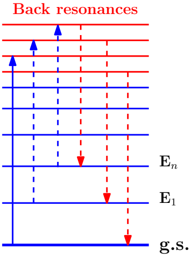
Shortcomings of SM calculations at $T \neq 0$:

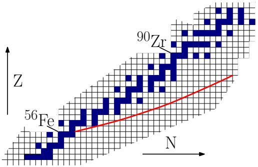
- Brink-Axel hypothesis is applied;
- the method of "back-resonances" is used;
- detailed balance principle is violated, i.e.

$$S(T, -E) \neq S(T, E) \exp(-E/T);$$

- the contribution of low- and negative-energy GT transitions is underestimated;
- to date SM calculations are limited by the iron-group nuclei (A=40-65)







Cross-section for semi-leptonic reaction on hot nucleus

$$\sigma(\varepsilon_l, T) = \sum_{if} p_i(T)\sigma_{if}(\varepsilon_l) = \frac{2G_F^2}{\hbar^4 c^4}\frac{\varepsilon_l}{p_l}\int_{-\infty}^{\varepsilon_l - m_{l'}c^2} dE\, \varepsilon_{l'} p_{l'} \int_{-1}^{1} d(\cos\theta)\left\{\sum_{J\geq 0}\eta_J^{CL}(E,T) + \sum_{J\geq 1}\eta_J^{T}(E,T)\right\},$$

where $\eta_J^{CL}(E,T)$ и $\eta_J^{T}(E,T)$ is a linear combination of spectral densities $S_{A,B}(E,T)$ of correlation functions for multipole operators of weak nuclear current

$$S_{A,B}(E,T) = \int \frac{dt}{2\pi} e^{iEt} \langle\langle A(t)B(0)\rangle\rangle, \qquad (A, B = M_J,\, L_J,\, T_J^{el},\, T_J^{mag}).$$

The explicit form of M_J, L_J, T_J^{el}, T_J^{mag} is obtained from the Donelly-Walecka theory.

Superoperators and calculation of spectral densities

Statistical average is given by $\langle\langle A(t)B(0)\rangle\rangle = \langle 0(T)|A(t)B(0)|0(T)\rangle$, and the thermal Hamiltonian $\mathcal{H} = H(\boldsymbol{a}^\dagger, \boldsymbol{a}) - H(\boldsymbol{a}^\dagger, \boldsymbol{a})$ determines spectral properties of hot nuclei:

$$S_{A,B}(E,T) = \sum_k \left\{\langle\mathcal{O}_k|B|0(T)\rangle\langle\mathcal{O}_k|A|0(T)\rangle^*\delta(E-\mathcal{E}_k) + \langle\widetilde{\mathcal{O}}_k|B|0(T)\rangle\langle\widetilde{\mathcal{O}}_k|A|0(T)\rangle^*\delta(E+\mathcal{E}_k)\right\},$$

where $|0(T)\rangle$ $(\mathcal{H}|0(T)\rangle = 0)$ is the thermal vacuum, $\mathcal{H}|\mathcal{O}_k\rangle = +\mathcal{E}_k|\mathcal{O}_k\rangle$ and $\mathcal{H}|\widetilde{\mathcal{O}}_k\rangle = -\mathcal{E}_k|\widetilde{\mathcal{O}}_k\rangle$.

SUPEROPERATORS – operators acting in the Liouville space.

Left and **right** fermionic creation and annihilation superoperators:

$$\mathbf{a}_k^\dagger |mn\rangle\!\rangle \leftrightarrow a_k^\dagger |m\rangle\langle n|, \quad \mathbf{a}_k^\dagger |mn\rangle\!\rangle \leftrightarrow \beta(m,n)\,|m\rangle\langle n|\, a_k,$$
$$\mathbf{a}_k |mn\rangle\!\rangle \leftrightarrow a_k |m\rangle\langle n|, \quad \mathbf{a}_k |mn\rangle\!\rangle \leftrightarrow \alpha(m,n)\,|m\rangle\langle n|\, a_k^\dagger.$$

If we demand that $\{\mathbf{a}_k, \mathbf{a}_{k'}\} = 0$, $\{\mathbf{a}_k, \mathbf{a}_{k'}^\dagger\} = \delta_{kk'}$ and $(\mathbf{a}_k)^\dagger = \mathbf{a}_k^\dagger$ then

$$\beta(m,n) = c(-1)^{m+n}, \quad \alpha(m,n) = c^*(-1)^{m+n+1} \text{ где } cc^* = 1.$$

If we put $c = i$, then the following properties are valid

- equation-of-motion: for $|O_k\rangle = \mathcal{O}_k^\dagger |0(T)\rangle$ and $\mathcal{O}_k |0(T)\rangle = 0$ we have

$$\langle 0(T)|[\delta O, [\mathcal{H}, \mathcal{O}_k^\dagger]]|0(T)\rangle = \mathcal{E}_k \langle 0(T)|[\delta O, \mathcal{O}_k^\dagger]|0(T)\rangle.$$

- thermal state condition

$$\langle \widetilde{O}_k|\, A\, |0(T)\rangle = e^{-\mathcal{E}_k/2T} \langle O_k|\, A^\dagger\, |0(T)\rangle^* \text{ and therefore } S_{B^\dagger A^\dagger}(-E,T) = e^{-E/T} S_{AB}(E,T).$$

Diagonalization of the thermal Hamiltonian

The nuclear hamiltonian $H = H_{sp} + H_{pair} + H_{ph}$, where H_{sp} and $H_{ph} = \sum_k h_k^\dagger h_k$ are obtained self-consistently from the Skyrme energy density functional .

The thermal Hamiltonian : $\mathcal{H} = H(\boldsymbol{a}^\dagger, \boldsymbol{a}) - H(\boldsymbol{a}^\dagger, \boldsymbol{a}) = \mathcal{H}_{sp} + \mathcal{H}_{pair} + \mathcal{H}_{ph}$.

1. Thermal quasiparticles:

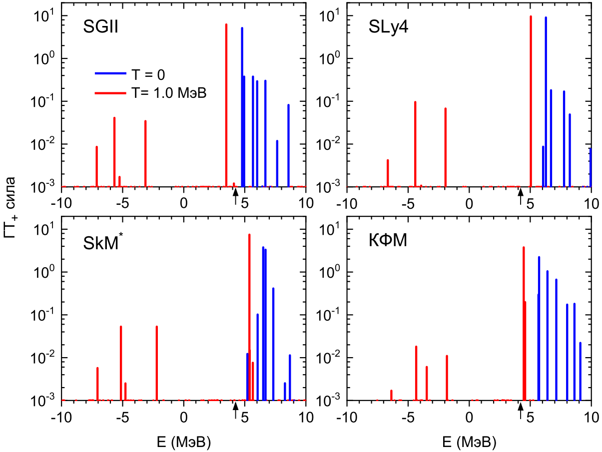
$$\mathcal{H}_{sp+pair} \approx \sum_{jm} \varepsilon_j(T)(\beta_{jm}^\dagger \beta_{jm} - \widetilde{\beta}_{jm}^\dagger \widetilde{\beta}_{jm})$$

2. Thermal phonons:

$$\mathcal{H} \approx \sum_{JMi} \omega_{Ji}(T)(Q_{JMi}^\dagger Q_{JMi} - \widetilde{Q}_{JMi}^\dagger \widetilde{Q}_{JMi}) + \mathcal{H}_{qph}, \quad Q_{JMi} |\phi_0(T)\rangle = \widetilde{Q}_{JMi} |\phi_0(T)\rangle = 0, \quad \text{where}$$

$$Q_{JMi}^\dagger = \sum_{12} \psi_{12} \beta_1^\dagger \beta_2^\dagger + \ldots + \sum_{12} \varphi_{12} \beta_1 \beta_2 + \ldots$$

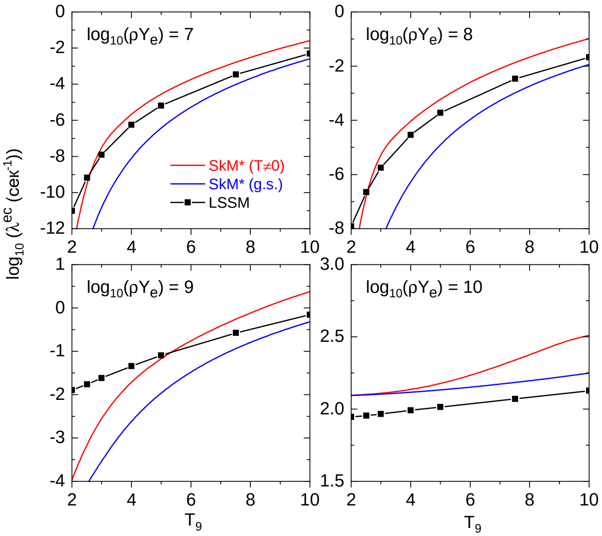
GT_+ strength distribution in ^{56}Fe at $T \neq 0$



The arrows indicate the zero-temperature EC threshold:

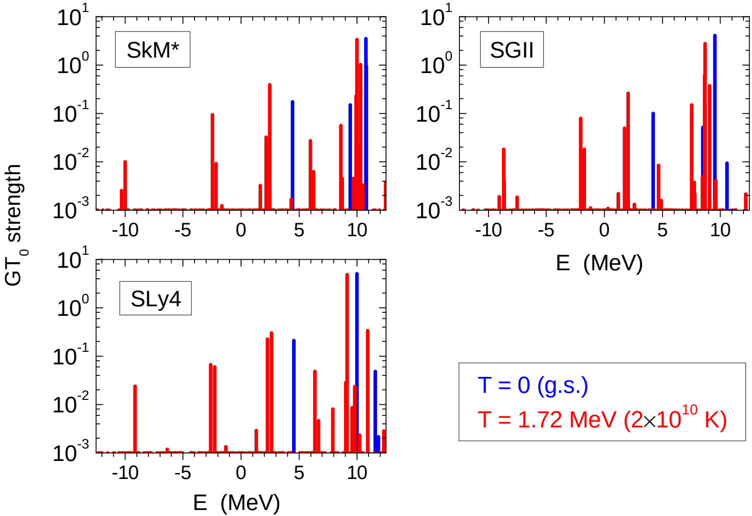
$Q = M(^{56}\mathrm{Mn}) - M(^{56}\mathrm{Fe}) = 4.2$ МэВ.

Electron capture rates for ^{56}Fe at $T \neq 0$

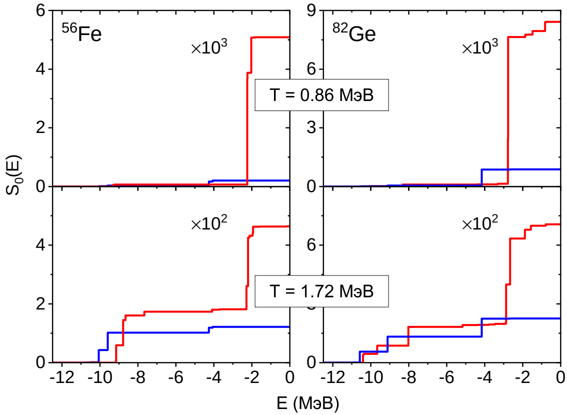


$T_9 = 10^9$ K (0.086 MeV), ρY_e is the electron gas density

GT_0 distribution in ^{56}Fe at $T \neq 0$



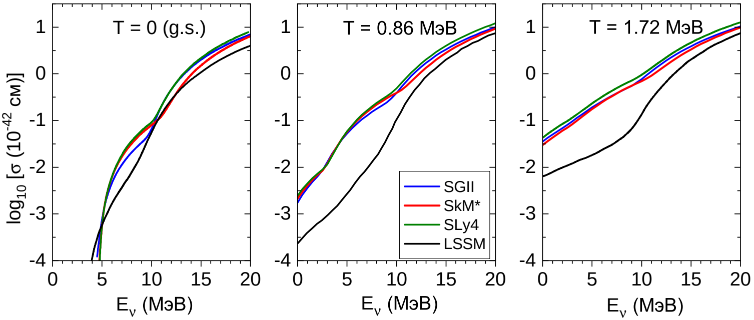
Detailed balance: $S_{\mathrm{GT}_0}(-E,T) = S_{\mathrm{GT}_0}(E,T)\exp\left(-\frac{E}{T}\right)$



Running sums $S_0(E,T) = \int\limits_{-\infty}^{E} S_{\mathrm{GT}_0}(E',T)dE'$ at $E < 0$ computed with and without the Brink-Axel hypothesis.

Neutrino inelastic scattering cross-sections for ^{56}Fe at $T \neq 0$

$\nu +^{56}\mathrm{Fe} \rightarrow \nu' +^{56}\mathrm{Fe}$

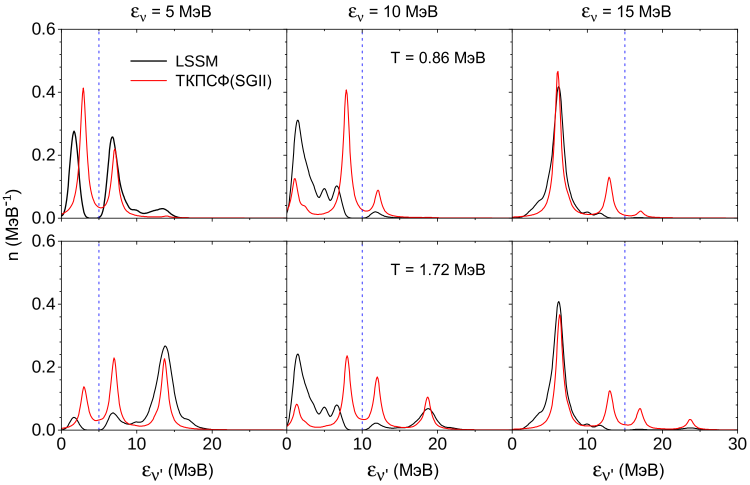


TQRPA : $\sigma(E_\nu, T) = \sigma_\uparrow(E_\nu, T) + \sigma_\downarrow(E_\nu, T)$

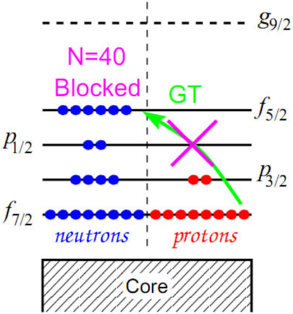
LSSM : $\sigma(E_\nu, T) = \sigma_{g.s}(E_\nu) + \sigma_\downarrow(E_\nu, T)$

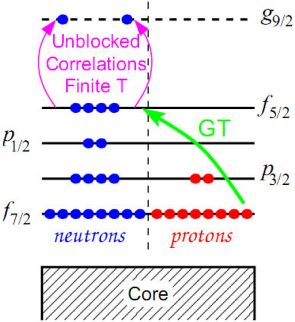
Thermal effects on neutrino spectrum

$\nu + ^{56}\mathrm{Fe} \rightarrow \nu' + ^{56}\mathrm{Fe}$



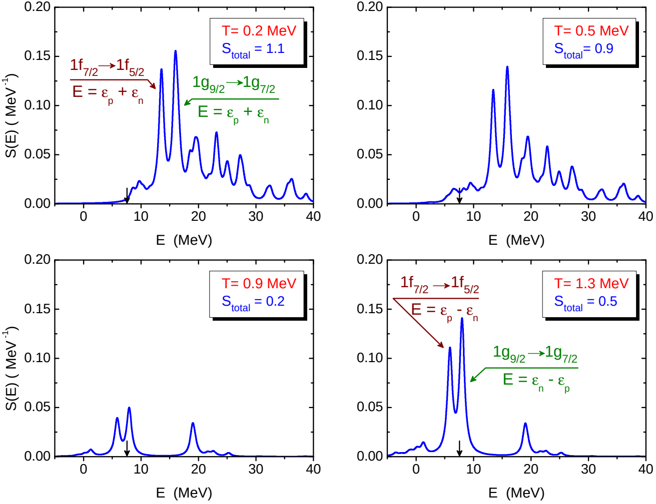
Neutron-rich nuclei ($N > 40, \quad Z < 40$)





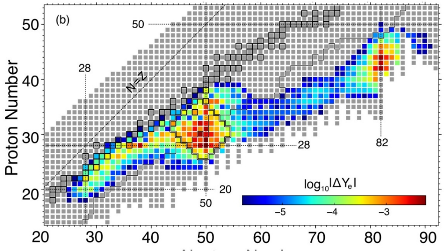
Unblocking mechanisms for GT_+ transitions: configurational mixing and thermal effects.

Thermal evolution of the GT_+ distribution in ^{76}Ge



A. Dzhioev et al, Phys. Rev. C **81** (2010) 015804

Ch. Sullivan et al, The Astrophysical Journal 816 (2016)



Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction up to neutrino trapping.

R. Titus et al, PRC 100 (2019)

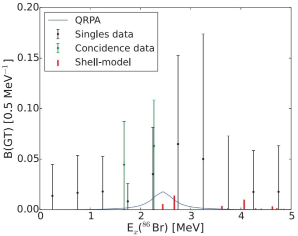


FIG. 5. Gamow-Teller strength distribution extracted from the ^{86}Kr(t, ^{3}He) data and comparison with shell-model and QRPA calculations, as described in the text.

J.C. Zamora et al, PRC100(R), 2019

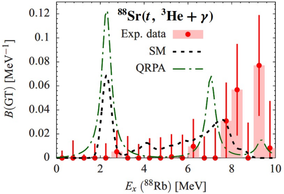
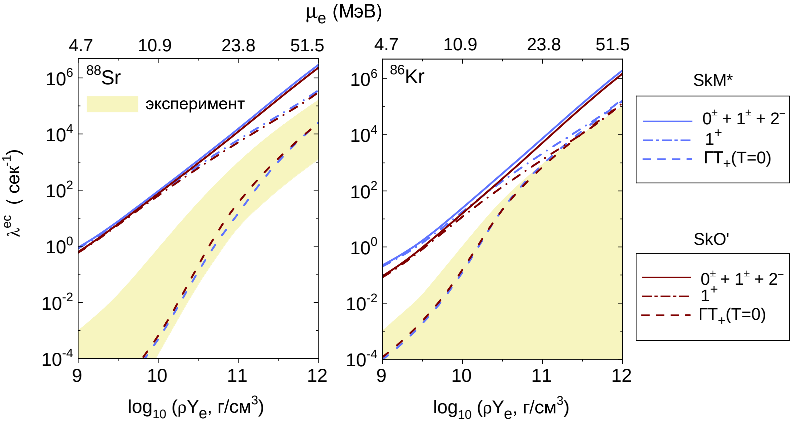


FIG. 2. B(GT) distribution extracted from MDA for $E_x <$ 10 MeV. The error bars denote only the statistical uncertainties.

Electron capture by $N = 50$ neutron-rich nuclei

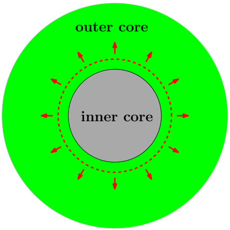


Electron capture rates at $T = 10^{10}$ K (0.86 MeV)

A. A. Dzhioev et al, PRC101 (2020)

Conclusion

1. A thermodynamically consistent approach to calculate cross-sections and rates of weak-interaction reactions with hot nuclei under astrophysical conditions is developed. The approach is based on the superoperator formalism.
2. An analysis of the temperature dependence of the GT strength function in ^{56}Fe and in a number of neutron-rich nuclei with $N \approx 50$ shows that the refusal to use the Brink-Axel hypothesis as well as the consistent implementation of the detailed balance principle noticeably increase the contribution of thermally excited nuclear states to the rates and cross sections of weak processes in comparison with the shell-model calculations. In particular, the enhanced contribution of de-excitation processes accelerates the temperature-induced growth of low-energy cross sections and rates.
3. It is found that temperature-induced weakening of pairing correlations drastically increases the temperature dependence of the energy and strength of unblocked Gamow-Teller $p \to n$ transitions in neutron-rich nuclei. It was shown that thermal effects are the main unblocking mechanism of low-energy Gamow-Teller transitions. For this reason the process of electron capture by nuclei in stellar matter is not interrupted at neutron-rich nuclei with N=50.
4. The enhanced contribution of charge-neutral transitions from thermally excited nuclear states broadens the energy spectrum of neutrinos scattered in the stellar matter in comparison with the results of shell-model calculations



$$\Delta M = M_{\text{iron core}} - M_{\text{inner core}}$$

- $M_{\text{iron core}} \approx M_{\text{Ch}} \sim Y_e^2$, therefore

$$e^- + (A,Z) \to (A,Z-1) + \nu_e$$
$$(A,Z) \to (A,Z+1) + e^- + \overline{\nu}_e$$

determine the **iron core** mass.

- ν-nucleus reactions become important at $\rho \geq 10^{11}$ g cm^{-3}:

$$\nu + (A,Z) \to (A,Z) + \nu$$
$$\nu + (A,Z) \to (A,Z) + \nu'$$
$$\nu_e + (A,Z) \to (A,Z-1) + e^-$$

trap neutrinos in the **inner core** and determine its mass:

$$M_{\text{inner core}} \sim Y_L^2, \text{ where } Y_L = Y_e + Y_\nu$$

- ν-nucleus reaction may alter the shock dynamics (preheating and melting the material ahead of the shock)