JINR

Effects of partial restoration of chiral symmetry on particle production in heavy-ion collisions

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Plan:

- Motivation: chiral symmetry in massless QCD.

- Parity-doublet model (PDM).

- Giessen Boltzmann-Uehling-Uhlenbeck (GiBUU) model:
  relativistic mean field, collision term.

- Dynamics of particle production in central Au+Au collision at 1A GeV:
  N*(1535), η, ρ. Non-linear Walecka model vs PDM.

- Comparison with TAPS data on η and π^0 transverse mass spectra
  and HADES data on e^+e^- production.

- Summary and outlook.

Motivation

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{\substack{f=u,d,s,\\ c,b,t}} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu} \ , \quad a = 1, \ldots, 8$$

Gell-Mann matrices acting on color indices of quark fields

$$D_\mu \equiv \partial_\mu - ig \frac{\lambda_a^c}{2} \mathcal{A}_{\mu,a} \ , \quad G_{\mu\nu,a} \equiv \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c} \ .$$

SU(3) structure constants

- leave only light quarks (u,d) and neglect their masses

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{f=u,d} \bar{q}_f i\gamma^\mu D_\mu q_f - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu} \ ,$$

- introduce projecting operators:

$$P_R = \frac{1}{2}(1+\gamma_5) = P_R^\dagger, \quad P_L = \frac{1}{2}(1-\gamma_5) = P_L^\dagger, \quad P_R^2 = P_R, \quad P_L^2 = P_L, \quad P_L P_R = P_R P_L = 0,$$

$q_R = P_R q, \quad q_L = P_L q$ - right and left-handed components of the quark field

$$\mathcal{L}^0_{\mathrm{QCD}} = \sum_{f=u,d} (\bar{q}_{R,f} i\gamma^\mu D_\mu q_{R,f} + \bar{q}_{L,f} i\gamma^\mu D_\mu q_{L,f}) - \frac{1}{4} G_{\mu\nu,a} G_a^{\mu\nu}.$$

- invariant under global $SU(2)_R$ x $SU(2)_L$ x $U(1)_V$ flavor transformations

isospin Pauli matrices

$$\begin{aligned} U_R(\boldsymbol{\Theta_R}) &= e^{-i\boldsymbol{\Theta_R}\frac{\boldsymbol{\tau}}{2}} P_R + P_L \ , \\ U_L(\boldsymbol{\Theta_L}) &= P_R + e^{-i\boldsymbol{\Theta_L}\frac{\boldsymbol{\tau}}{2}} P_L \ , \\ U(\Theta) &= e^{-i\Theta} \end{aligned}$$

S. Scherer, arXiv:hep-ph/0210398

Vector and axial-vector transformations:

$$U_V(\mathbf{\Theta}) = U_R(\mathbf{\Theta})U_L(\mathbf{\Theta}) = e^{-i\mathbf{\Theta}\frac{\boldsymbol{\tau}}{2}} = e^{-i\mathbf{\Theta}\frac{\boldsymbol{\tau}}{2}}P_R + e^{-i\mathbf{\Theta}\frac{\boldsymbol{\tau}}{2}}P_L \ ,$$
$$U_A(\mathbf{\Theta}) = U_R(\mathbf{\Theta})U_L(-\mathbf{\Theta}) = e^{-i\mathbf{\Theta}\gamma_5\frac{\boldsymbol{\tau}}{2}} = e^{-i\mathbf{\Theta}\frac{\boldsymbol{\tau}}{2}}P_R + e^{i\mathbf{\Theta}\frac{\boldsymbol{\tau}}{2}}P_L \ .$$

Noether's theorem → conserved currents for classical fields:

$$\partial_\mu \boldsymbol{V}^\mu = 0 \ , \quad \boldsymbol{V}^\mu = \bar{q}_R\gamma^\mu\frac{\boldsymbol{\tau}}{2}q_R + \bar{q}_L\gamma^\mu\frac{\boldsymbol{\tau}}{2}q_L = \bar{q}\gamma^\mu\frac{\boldsymbol{\tau}}{2}q \ ,$$
$$\partial_\mu \boldsymbol{A}^\mu = 0 \ , \quad \boldsymbol{A}^\mu = \bar{q}_R\gamma^\mu\frac{\boldsymbol{\tau}}{2}q_R - \bar{q}_L\gamma^\mu\frac{\boldsymbol{\tau}}{2}q_L = \bar{q}\gamma^\mu\gamma_5\frac{\boldsymbol{\tau}}{2}q \ .$$

Axial charge is constant of motion:

$$\frac{d < \psi|\boldsymbol{Q}^A|\psi >}{dt} = 0 \ , \quad \boldsymbol{Q}^A(t) = \int d^3r \boldsymbol{A}^0(t,\vec{r}) = \int d^3r q^\dagger(t,\vec{r})\gamma_5\frac{\boldsymbol{\tau}}{2}q(t,\vec{r})$$

$$\hat{P}q(t,\vec{r}) = i\gamma^0 q(t,-\boldsymbol{r}) \ , \quad \hat{P}\boldsymbol{Q}^A = -\boldsymbol{Q}^A$$

$$[H^0_{\rm QCD}, \boldsymbol{Q}^A] = 0 \qquad |\psi' >= \boldsymbol{Q}^A|\psi >$$

$$H^0_{\rm QCD}|\psi >= E|\psi > \qquad H^0_{\rm QCD}|\psi' >= E|\psi' >$$

$$\hat{P}|\psi >= \eta|\psi > \qquad \hat{P}|\psi' >= -\eta|\psi' >$$

Parity-doublets, i.e. states with the same mass and spin, but opposite parity (Wigner-Weyl realization of chiral symmetry)

Lattice QCD calculations predict parity doubling if chiral symmetry is (artificially) restored

L. Y. Glozman, C. B. Lang, and M. Schrock, PRD 86, 014507 (2012) [arXiv:1205.4887];
G. Aarts, C. Allton, D. De Boni, S. Hands, B. Jäger, C. Praki, and J.-I. Skullerud,
JHEP 06, 034 (2017) [arXiv:1703.09246]

However, parity-doublets are not observed in the experiment.

Let us suggest that the ground state (vacuum) is invariant w/r to axial transformation, $\boldsymbol{Q}^A|0>=0.$

Then it follows from Coleman's theorem S. Coleman, J. Math. Phys. 7, 787 (1966) that the axial charge is conserved, $[H^0_{\mathrm{QCD}}, \boldsymbol{Q}^A] = 0.$

This would lead to the existence of parity-doublets.The conclusion is that the ground state is not invariant w/r to axial transformations, $\boldsymbol{Q}^A|0>\neq 0.$

Nambu-Goldstone realization of chiral symmetry (of QCD Hamiltonian): spontaneous breaking of chiral symmetry (of vacuum).

- massless pions generated by Q^A

- instead of parity-partner hadron one gets hadron+massless pion state

Equal-time commutation relation W. Weise, arXiv:nucl-th/0504087:

$$[Q_a^A, \bar{q}(x)\gamma_5\tau_b q(x)] = -\delta_{ab}\bar{q}(x)q(x), \quad \bar{q}(x)q(x) = \bar{u}u + \bar{d}d.$$

If $< 0|\bar{q}(x)q(x)|0 > \neq 0$ then $Q_a^A|0 > \neq 0.$

The scalar quark-antiquark condensate breaks chiral symmetry.

PCAC:
$$\partial_\mu A_1^\mu = i m_q \bar{q}\gamma_5\tau_1 q, \qquad m_q \equiv \frac{1}{2}(m_u + m_d)$$
$$\partial_\mu A_a^\mu = m_\pi^2 f_\pi \pi_a, \quad f_\pi \simeq 93\ \mathrm{MeV}$$ - pion decay constant

Gell-Mann, Oakes, Renner (GOR) relation: $-m_q < 0|\bar{q}q|0 >= f_\pi^2 m_\pi^2$

$$\frac{1}{2} < 0|\bar{q}q|0 > \simeq < 0|\bar{u}u|0 > \simeq < 0|\bar{d}d|0 > = -(1.5-3)\ \mathrm{fm}^{-3}$$

Hellmann-Feynman theorem T.D. Cohen et al., PRC 45, 1881 (1992):

$$\frac{< \bar{q}q >_{\rho_N}}{< \bar{q}q >_0} = 1 - \frac{\sigma_N}{f_\pi^2 m_\pi^2}\rho_N$$

$$\sigma_N = m_q \frac{dm_N}{dm_q} = m_q \int d^3x \left(< N|\,\bar{q}q\,|N > - < 0|\,\bar{q}q\,|0 >\right) = 40 - 60\ \mathrm{MeV}$$ - nucleon σ term

LQCD πN scatt. phen.

R. Gupta et al., arXiv:2105.12095

The tendency towards chiral symmetry restoration with increasing nucleon density.

Parity-doublet model (PDM)

C.E. DeTar, T. Kunihiro, PRD 39, 2805 (1989); D. Jido et al., NPA 671, 471 (2000); Prog. Theor. Phys. 106, 873 (2001)

- includes the nucleon and its negative parity partner state, N*(1535)
- keeps $SU(2)_R$ x $SU(2)_L$ symmetry on the hadronic level
- mirror assignment for the chiral symmetry basis states N_1 (P=+), N_2 (P=-):

$$
\begin{aligned}
N_{1R} &\to U_R N_{1R}, \quad N_{1L} \to U_L N_{1L}, \\
N_{2R} &\to U_L N_{2R}, \quad N_{2L} \to U_R N_{2L}.
\end{aligned}
$$

Lagrangian:

$$
\begin{aligned}
\mathcal{L} &= \bar{N}_1[i\partial + g_1(\sigma + i\gamma_5 \boldsymbol{\tau}\boldsymbol{\pi})]N_1 + \bar{N}_2[i\partial + g_2(\sigma - i\gamma_5 \boldsymbol{\tau}\boldsymbol{\pi})]N_2 \\
&\quad -m_0(\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1) + \mathcal{L}_{\text{mes}} + \underbrace{\mathcal{L}_0 + \mathcal{L}_1}_{} \ .
\end{aligned}
$$

Contact fermionic interactions
(short-range repulsion, needed for nuclear EOS)

Note: cross-product terms are chirally-invariant::

$$
\bar{N}_1\gamma_5 N_2 = \bar{N}_{1R}\gamma_5 N_{2L} + \bar{N}_{1L}\gamma_5 N_{2R} \to \bar{N}_{1R}U_R^\dagger\gamma_5 U_R N_{2L} + \bar{N}_{1L}U_L^\dagger\gamma_5 U_L N_{2R} = \bar{N}_1\gamma_5 N_2 \ .
$$

- mass eigenstates, i.e. nucleon and N*, are obtained by proper unitary transformation diagonalising the mass matrix

The meson Lagrangian Y. Motohiro et al., PRC 92, 025201 (2015); I.J. Shin et al., arXiv:1805.03402; M. Kim et al., PRC 101, 064614 (2020):

$$\begin{aligned}\mathcal{L}_{\rm mes} &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} \\ &+\frac{\bar{\mu}^2}{2}(\sigma^2+\boldsymbol{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\boldsymbol{\pi}^2)^2 + \frac{\lambda_6}{6}(\sigma^2+\boldsymbol{\pi}^2)^3 + \varepsilon\sigma\ .\end{aligned}$$

6-point interaction
(needed for realistic values of
incompressibility)

Explicit chiral symmetry breaking term
(to account for finite quark masses)

Isoscalar and isovector four-fermion NJL-type interaction terms:

$$\begin{aligned}\mathcal{L}_0 &= -G_0(\bar{N}_1\gamma^\mu N_1 + \bar{N}_2\gamma^\mu N_2)^2\ , \\ \mathcal{L}_1 &= -G_1[(\bar{N}_1\gamma^\mu\boldsymbol{\tau}N_1 + \bar{N}_2\gamma^\mu\boldsymbol{\tau}N_2)^2 + (\bar{N}_1\gamma^\mu\gamma_5\boldsymbol{\tau}N_1 - \bar{N}_2\gamma^\mu\gamma_5\boldsymbol{\tau}N_2)^2]\ .\end{aligned}$$

Bosonisation provides statistically equivalent description (Hubbard-Stratonovich transformation):

$$\begin{aligned}\mathcal{L}_0 &= \frac{m_\omega^2}{2}\omega^\mu\omega_\mu - g_\omega\omega_\mu(\bar{N}_1\gamma^\mu N_1 + \bar{N}_2\gamma^\mu N_2)\ , \\ \mathcal{L}_1 &= \frac{m_\rho^2}{2}(\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu + \boldsymbol{a}_1^\mu\boldsymbol{a}_{1\mu}) - g_\rho\bar{N}_1(\boldsymbol{\rho}^\mu - \gamma_5\boldsymbol{a}_1^\mu)\gamma_\mu\boldsymbol{\tau}N_1 - g_\rho\bar{N}_2(\boldsymbol{\rho}^\mu + \gamma_5\boldsymbol{a}_1^\mu)\gamma_\mu\boldsymbol{\tau}N_2\ ,\end{aligned}$$

with $m_\omega^2/g_\omega^2 = 1/(2G_0)$ and $m_\rho^2/g_\rho^2 = 1/(2G_1)$.

Note: mathematically equivalent to the non-linear Walecka model w/o space-time derivatives.
However, we assume that these auxiliary Hubbard fields do not coincide with physical massive vector meson fields.
They rather parameterize short-range QCD interactions.

Full model Lagrangian in the basis of mass eigenstates (physical basis) :

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \bar{N}_+[i\partial - m_+ - ig_{\pi N_+ N_+}\gamma_5 \boldsymbol{\tau\pi} - (g_\omega \omega^\mu + g_\rho \boldsymbol{\tau\rho}^\mu - g_{a_1}\gamma_5 \boldsymbol{\tau a}_1^\mu)\gamma_\mu]N_+ \\
& + \bar{N}_-[i\partial - m_- - ig_{\pi N_- N_-}\gamma_5 \boldsymbol{\tau\pi} - (g_\omega \omega^\mu + g_\rho \boldsymbol{\tau\rho}^\mu + g_{a_1}\gamma_5 \boldsymbol{\tau a}_1^\mu)\gamma_\mu]N_- \\
& + \frac{m_\omega^2}{2}\omega^\mu\omega_\mu + \frac{m_\rho^2}{2}(\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu + \boldsymbol{a}_1^\mu \boldsymbol{a}_{1\mu}) \\
& - ig_{\pi N_+ N_-}\bar{N}_+ \boldsymbol{\tau\pi} N_- + ig_{\pi N_+ N_-}\bar{N}_- \boldsymbol{\tau\pi} N_+ \\
& + g_{a_1 N_+ N_-}\bar{N}_+\gamma_\mu \boldsymbol{\tau a}_1^\mu N_- + g_{a_1 N_+ N_-}\bar{N}_-\gamma_\mu \boldsymbol{\tau a}_1^\mu N_+ + \mathcal{L}_{\rm mes} \ ,
\end{aligned}
$$

$$
m_\pm = \frac{1}{2}\left[\sqrt{\sigma^2(g_1+g_2)^2 + 4m_0^2} \pm \sigma(g_2 - g_1)\right] \ .
$$

Decreasing scalar field σ leads to the mass degeneracy of the parity partners, $m_\pm \to m_0$

Vacuum conditions: $\sigma(\rho_B = 0) \equiv \sigma_0 = f_\pi = 93\ \mathrm{MeV}$ (consistent with Goldberger-Treiman relation),

$m_+(\sigma_0) = m_N, \quad m_-(\sigma_0) = m_{N^*(1535)}.$ $\qquad g_{\pi NN} \simeq m_N/f_\pi$

In PDM σ has a meaning of a condensate field, i.e. $\dfrac{\sigma(\rho_B)}{\sigma_0} \simeq \dfrac{<\bar{q}q>_{\rho_B}}{<\bar{q}q>_0} = 1 - \dfrac{\sigma_N}{f_\pi^2 m_\pi^2}\rho_B$

Mean field approximation:

- pion mean field (P=-) disappears in nuclear matter ground state;

- isovector-axial vector $\mathbf{a}_1^\mu$ mean field vanishes in spin-saturated nuclear matter.

Lagrange's EoMs:

$$-\bar{\mu}^2\sigma + \lambda\sigma^3 - \lambda_6\sigma^5 - \varepsilon = -\sum_{i=\pm}\frac{\partial m_i}{\partial\sigma}\langle\bar{N}_i(x)N_i(x)\rangle \ ,$$

$$\omega^\nu(x) = \frac{g_\omega}{m_\omega^2}\sum_{i=\pm}\langle\bar{N}_i(x)\gamma^\nu N_i(x)\rangle \ ,$$

$$\boldsymbol{\rho}^\nu(x) = \frac{g_\rho}{m_\rho^2}\sum_{i=\pm}\langle\bar{N}_i(x)\gamma^\nu\boldsymbol{\tau}N_i(x)\rangle \ ,$$

In-medium Dirac equation:

$$[\gamma^\mu(i\partial_\mu - V_\mu) - m_\pm]N_\pm(x) = 0 \ , \quad V_\mu = g_\omega\omega_\mu + g_\rho\boldsymbol{\tau}\boldsymbol{\rho}_\mu \ .$$

Plane-wave solutions:

$$N_\pm \propto \exp(-ipx),$$

$$[\gamma^\mu p^*_\mu - m_\pm]N_\pm = 0 \ , \quad p^*_\mu \equiv p_\mu - (V_\mu)_{I_z I_z}, \quad I_z = \pm 1/2.$$

kinetic four-momentum

Dispersion relation (in-medium mass-shell condition): $(p^*)^2 - m_\pm^2 = 0$.

Table 1: The sets of parameters of the PDM.

	Set P3 [a]	Set 2 [b]
m_0 (MeV)	790	700
m_σ (MeV)	370.63	384.428
m_ω (MeV)	783	783
m_ρ (MeV)	—	776
g_ω	6.79	7.05508
g_ρ	0	4.07986
g_1	13.00	14.1708
g_2	6.97	7.76222
$\lambda_6 f_\pi^2$	0	15.7393
m_+ (MeV)	939	939
m_- (MeV)	1500	1535
K (MeV)	510.57	215

[a] D. Zschiesche et al., PRC 75, 055202 (2007);

[b] I.J. Shin et al., arXiv:1805.03402.

Harmonic oscillator decomposition near vacuum value $\sigma=f_\pi$:

$$\begin{aligned}\mathcal{H}_{\rm mes} &= -\frac{\bar{\mu}^2}{2}(\sigma^2+\boldsymbol{\pi}^2)+\frac{\lambda}{4}(\sigma^2+\boldsymbol{\pi}^2)^2-\frac{\lambda_6}{6}(\sigma^2+\boldsymbol{\pi}^2)^3-\varepsilon\sigma \\ &= \text{const}+\frac{m_\pi^2}{2}\boldsymbol{\pi}^2+\frac{m_\sigma^2}{2}\Delta\sigma^2+O(\boldsymbol{\pi}^2\Delta\sigma)+O(\Delta\sigma^3),\end{aligned}$$

$$\Delta\sigma=\sigma-f_\pi, \qquad \frac{\partial\mathcal{H}_{\rm mes}}{\partial\sigma}_{|\sigma=f_\pi}=0\ .$$

$$\Rightarrow \left\{\begin{aligned}\bar{\mu}^2 &= \frac{m_\sigma^2-3m_\pi^2}{2}+\lambda_6 f_\pi^4\ , \\ \lambda &= \frac{m_\sigma^2-m_\pi^2}{2f_\pi^2}+2\lambda_6 f_\pi^2\ , \\ \varepsilon &= m_\pi^2 f_\pi\ .\end{aligned}\right.$$

Infinite nuclear matter

Saturation conditions:

$$\frac{\partial \mathcal{E}(\rho_B)/\rho_B}{\partial \rho_B}_{|\rho_B=\rho_0} = 0 \ ,$$

$$\frac{\mathcal{E}(\rho_0)}{\rho_0} \simeq -16 \text{ MeV} \ , \ \rho_0 = 0.16 \text{ fm}^{-3},$$

$$\mathcal{E}(\rho_B) \equiv T^{00}(\rho_B) - T^{00}(0) - m_N \rho_B$$

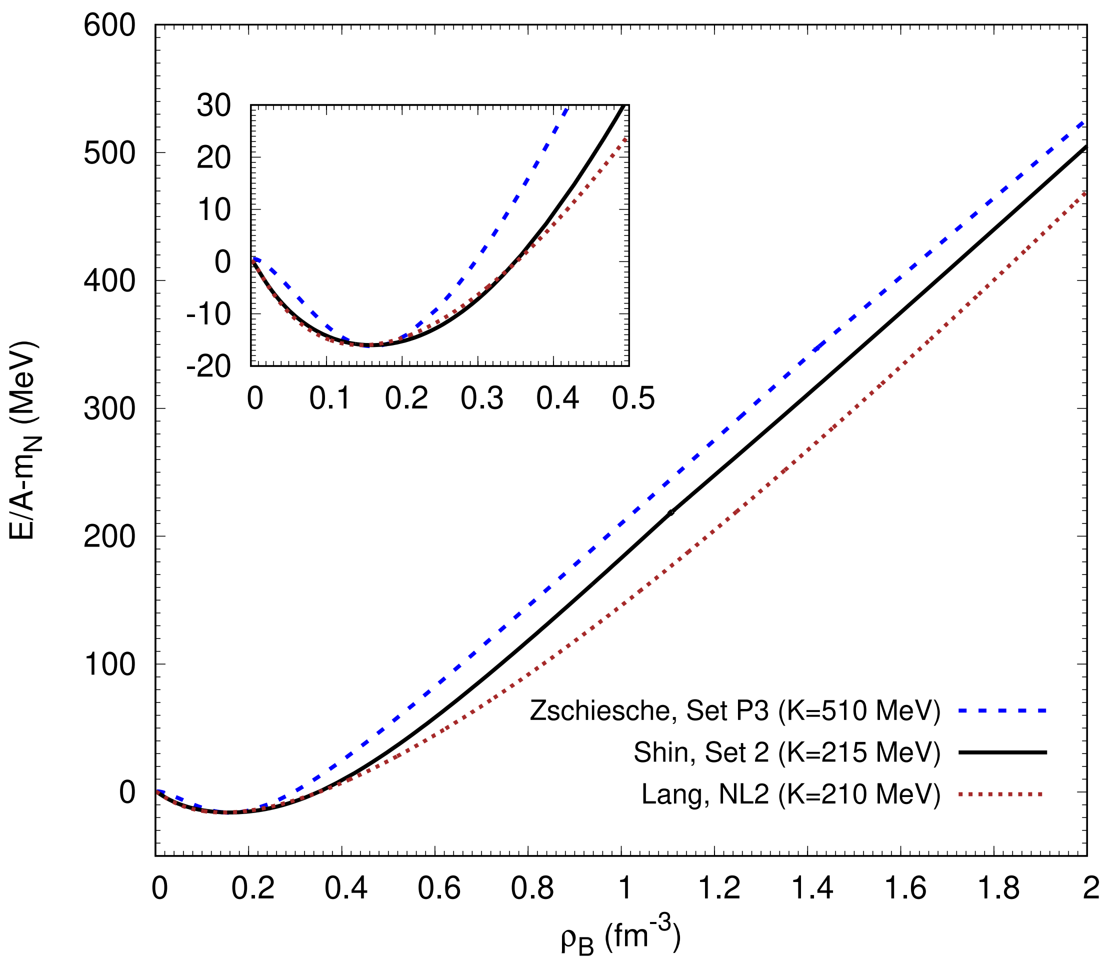
- non-relativistic energy density;

$$K = 9\rho_0^2 \frac{\partial^2 \mathcal{E}(\rho_B)/\rho_B}{\partial \rho_B^2}_{|\rho_B=\rho_0} \simeq 200 - 380 \text{ MeV}$$

- incompressibility.

from GMR frequencies

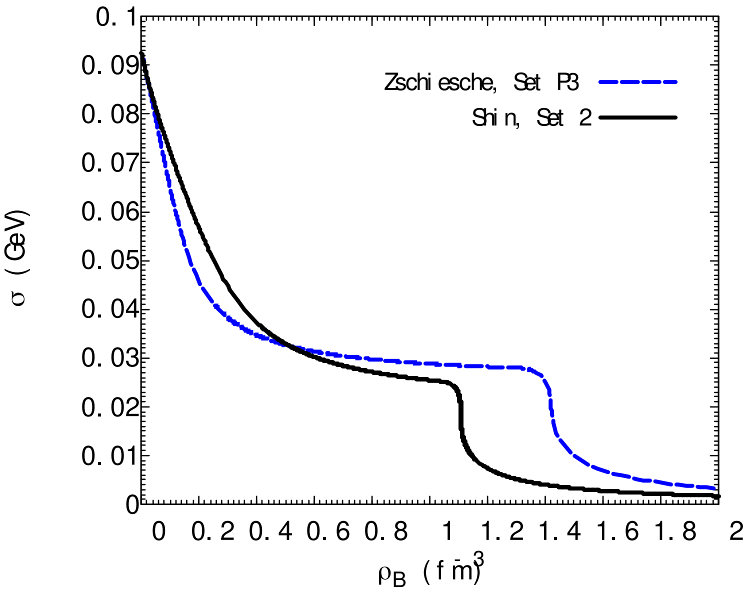
phen. upper limit

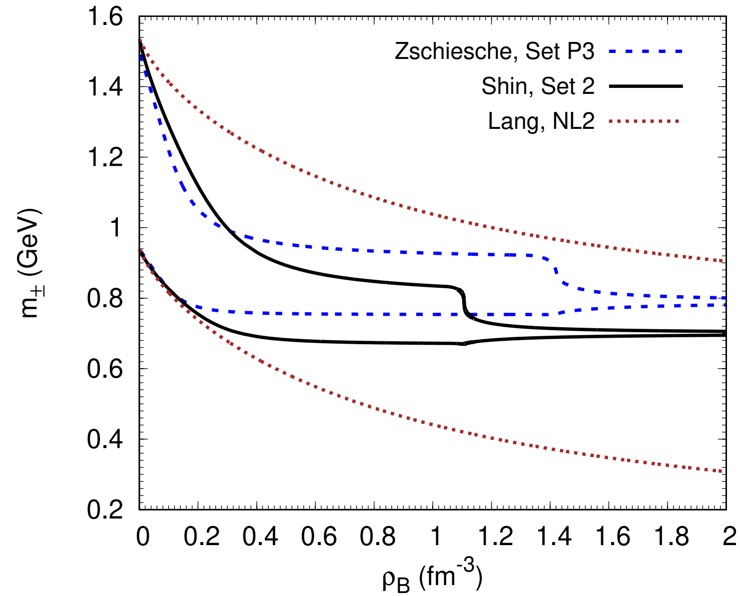


D. Zschiesche et al., PRC 75, 055202 (2007);
I.J. Shin et al., arXiv:1805.03402;
PDM

A. Lang et al., NPA 541, 507 (1992).
non-linear Walecka model

Set 2 and NL2 produce similar EoS at $\rho_B \leq 3\rho_0$ accessible in HIC at E_{lab}=1-2A GeV.





- The chiral 1-st order phase transition at baryon densities of 1-1.4 fm^{-3} unlikely to be reached before the transition to QGP will happen. However, a faster decrease of m_- at low densities may have observable consequences.

- Since $m_{N^*(1535)} - m_N - m_\eta \approx 50$ MeV the decay channel $N^*(1535) \rightarrow N\,\eta$ closes already at $\rho_B \approx 0.4\rho_0$ D. Jido, E.E. Kolomeitsev, H. Nagahiro, S. Hirenzaki, NPA 811, 158 (2008). Impact on the photoproduction of η-mesic nuclei. Also in qualitative agreement with observed $A^{2/3}$ dependence of η photoproduction M. Roebig-Landau et al., PLB 373, 45 (1996); H. Kim, D. Jido, M. Oka, NPA 640, 77 (1998).

- In HIC, however, the low-mass N*(1535)-resonances may increase their masses since the system expands.

Main idea: decreasing threshold of N N → N N*(1535) should enhance the production of N*(1535) in HIC. This should lead to larger η production via N*(1535) → N η.

GiBUU model

- solves the coupled system of kinetic equations for the baryons (N,N*,Δ,Λ,Σ,…), corresponding antibaryons ($\overline{N}$,$\overline{N}$*,$\overline{\Delta}$,$\overline{\Lambda}$,$\overline{\Sigma}$,…), and mesons (π,K,...)

- initializations for the lepton-, photon-, hadron-, and heavy-ion-induced reactions on nuclei

Open source code in Fortran 2003 downloadable from:

https://gibuu.hepforge.org/trac/wiki

Details of GiBUU: ***O. Buss et al., Phys. Rep. 512, 1 (2012).***

Kinetic equation with relativistic mean fields

Distribution function in phase space (r,p*)

Number of sort "j" particles = $\int \frac{g_s^j d^3r d^3p^*}{(2\pi)^3} f_j^*(x,\mathbf{p}^*)$

$$(p_0^*)^{-1}\left[p_\mu^*\partial^\mu + (p_\mu^*\mathcal{F}_j^{\alpha\mu} + m_j^*\partial^\alpha m_j^*)\frac{\partial}{\partial p^{*\alpha}}\right] \overbrace{f_j^*(x,\mathbf{p}^*)} = \underbrace{I_j[\{f^*\}]}\ , \qquad (*)$$

$\mu = 0,1,2,3, \quad \alpha = 1,2,3, \quad j = N,\bar{N},\Delta,\bar{\Delta},\Lambda,\bar{\Lambda},\pi,K,\ldots \qquad x \equiv (t,\mathbf{r})$

Collision term (includes Pauli blocking for outgoing nucleons)

m_j^* **- effective (Dirac) mass,**

$p^{*\mu} = p^\mu - V_j^\mu$ **- kinetic four-momentum with effective mass shell constraint** $p^{*\mu}p_\mu^* = m_j^{*2}$,

$V_j^\mu = g_{\omega j}\omega^\mu + g_{\rho j}\tau_j^3\rho^{3\mu} + q_j A^\mu$ **- vector field,** $\qquad \tau_j^3 = +(-)1 \text{ for } j = p,\bar{n}\ (\bar{p},n)$,

$\mathcal{F}_j^{\mu\nu} = \partial^\mu V_j^\nu - \partial^\nu V_j^\mu$ **- field tensor.**

- For momentum-independent fields Eq.(*) is equivalent to the BUU equation

$$(\partial_t + \nabla_{\mathbf{p}}\varepsilon_j\nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_j\nabla_{\mathbf{p}})f_j(x,\mathbf{p}) = I_j[\{f\}]$$

$$\varepsilon_j(x,\mathbf{p}) = V_j^0 + \sqrt{m_j^{*2} + \mathbf{p}_j^{*2}}\ , \quad f_j(x,\mathbf{p}) = f_j^*(x,\mathbf{p}^*)\ .$$

Direct derivations of relativistic kinetic equation:

Yu.B. Ivanov, NPA 474, 669 (1987);
B. Blättel, V. Koch, U. Mosel, Rept. Prog. Phys. 56, 1 (1993).

Baryon-baryon collisions:
For $\sqrt{s} < 4$ GeV: $NN \to NN$, $NN \leftrightarrow NR$, $NN \leftrightarrow \Delta\Delta$, $NR \to NR'$,
$NN \to NNM$ ($M = \pi,\ \omega,\ \phi$), $np \to d\eta$ (via $np\eta$ final state),
$BB \to BYK$, $BB \to NNK\bar{K}$ ($B = N,\ R$).
For $\sqrt{s} > 4$ GeV: $BB \to X$ (PYTHIA 6).

Meson-baryon collisions:
For $\sqrt{s} < 2.2$ GeV: $\pi N \to R$, MN ($M = \pi, \omega, \phi, \rho, \sigma, \eta$), $M\Delta$ ($M = \pi, \eta, \rho$),
$\pi N^*(1440)$, $K\Lambda$, $K\Sigma$, $\omega\pi N$, $\phi\pi N$, $K\bar{K}N$, $\Lambda K\pi$, $\Sigma K\pi$, $\pi\pi N$, $\pi\pi\pi N$;
$\omega N \to R$, πN, ωN, $\pi\pi N$, ΛK, ΣK;
$\rho N \to R$, πN, ΛK, ΣK;
$\sigma N \to R$, πN, σN;
$\eta N \to R$, πN, ΛK, ΣK;
$\phi N \to \phi N$, πN, $\pi\pi N$;
$KN \to KN$, $KN\pi$;
$\bar{K}N \to Y^*$, $\bar{K}N$, $Y\pi$, $Y^*\pi$, ΞK, $\Xi K\pi$;
$J/\psi N \to J/\psi N$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$, $ND\bar{D}$;
$\pi\Delta \to R$, $K\Lambda$, $\Sigma\Lambda$;
$\rho\Delta \to R$;
$\eta\Delta \to \pi N$;
$\pi N^*(1440) \to R$;
$\pi Y(Y^*) \to Y^*$, $\bar{K}N$;
$\eta\Lambda \to \Lambda^*$;
$K\Lambda \to R$, πN, $\pi\Delta$;
For $\sqrt{s} > 2.2$ GeV: $MB \to X$ (PYTHIA 6 and JETSET)

Meson-meson collisions:
$MM \to R,\ K\bar{K},\ K^*\bar{K},\ K\bar{K}^*\ (M = \pi, \eta, \eta', \sigma, \rho, \omega)$.

Baryon-antibaryon collisions:
$\bar{B}B \to$ mesons by statistical annihilation model
E.S. Golubeva et al., NPA 537, 393 (1992);
I.A. Pshenichnov, PhD thesis, INR Moscow (1998)
or by string model (if $|Z_{\rm tot}| > 1$ or total charm$\neq 0$ or total strangeness$\neq 0$),
$\bar{B}B \to \bar{B}B$ (EL and CEX), $\bar{N}N \leftrightarrow \bar{\Delta}N(\bar{N}\Delta)$ (for $\sqrt{s} < 2.38$ GeV)
or $\bar{B}B \to \bar{B}B +$ mesons by FRITIOF (for $\sqrt{s} > 2.38$ GeV),
$\bar{N}N \to \bar{\Lambda}\Lambda$, $\bar{N}(\bar{\Delta})N(\Delta) \to \bar{\Lambda}\Sigma(\bar{\Sigma}\Lambda)$, $\bar{N}(\bar{\Delta})N(\Delta) \to \bar{\Xi}\Xi$, $\bar{N}N \to \bar{\Omega}\Omega$,
$\bar{N}N \to J/\psi$.

$3 \to 2$ collisions: $NN\pi \to NN$.

$3 \to 3$ collisions: $NN\Delta \to NNN$.

$3 \to N$ collisions: optional.

Time evolution of Au+Au central collision at 1A GeV

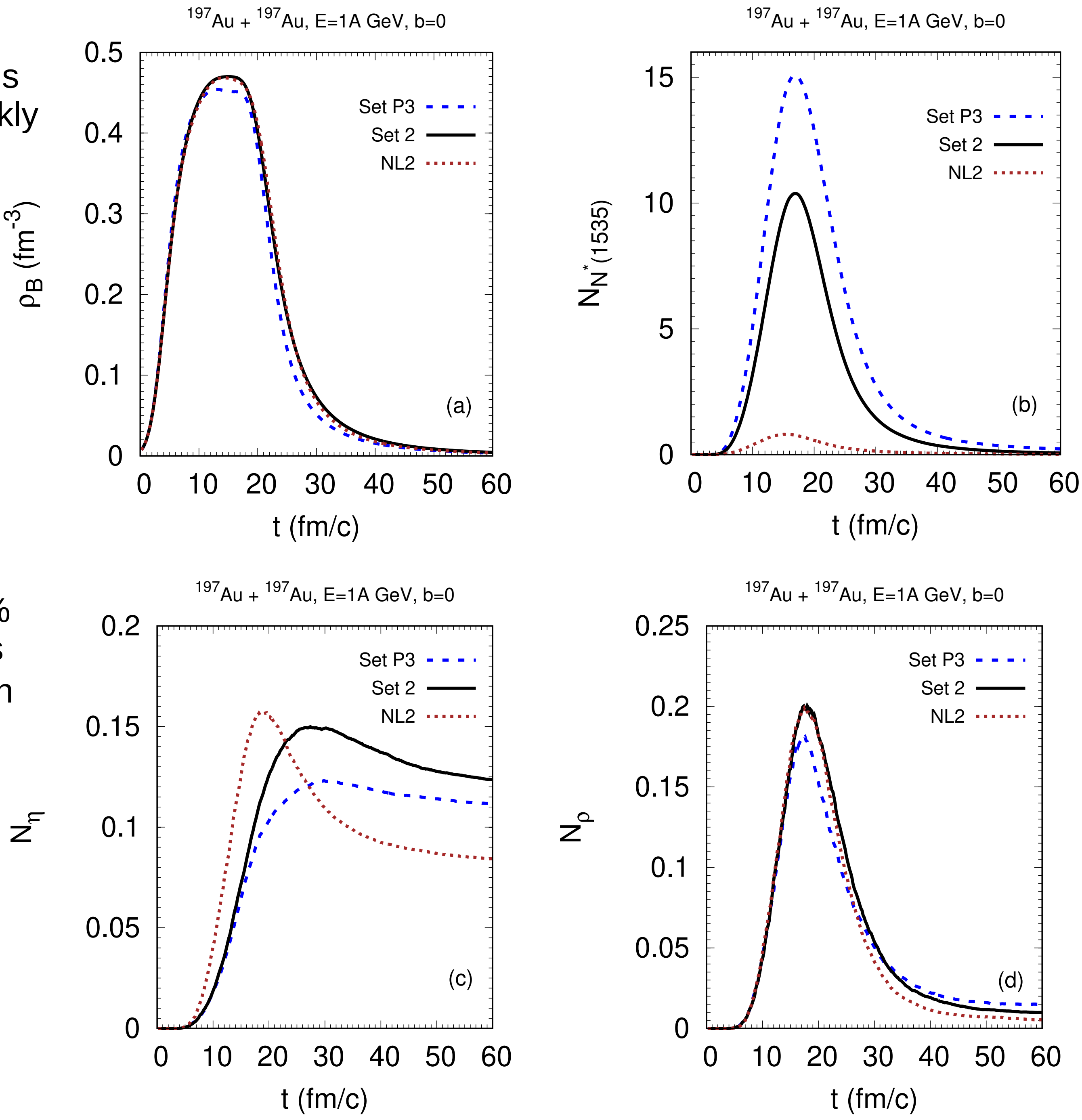
- Central baryon density depends on the used RMF set only weakly

- N*(1535) multiplicity with PDM is an order of magnitude larger than in NL2

- η multiplicity with PDM is ≈50% larger than wit NL2. Decreases at large times due to absorption

$$\eta N \to N^*(1535) \to \pi N.$$

- ρ multiplicity at large times is enhanced in PDM



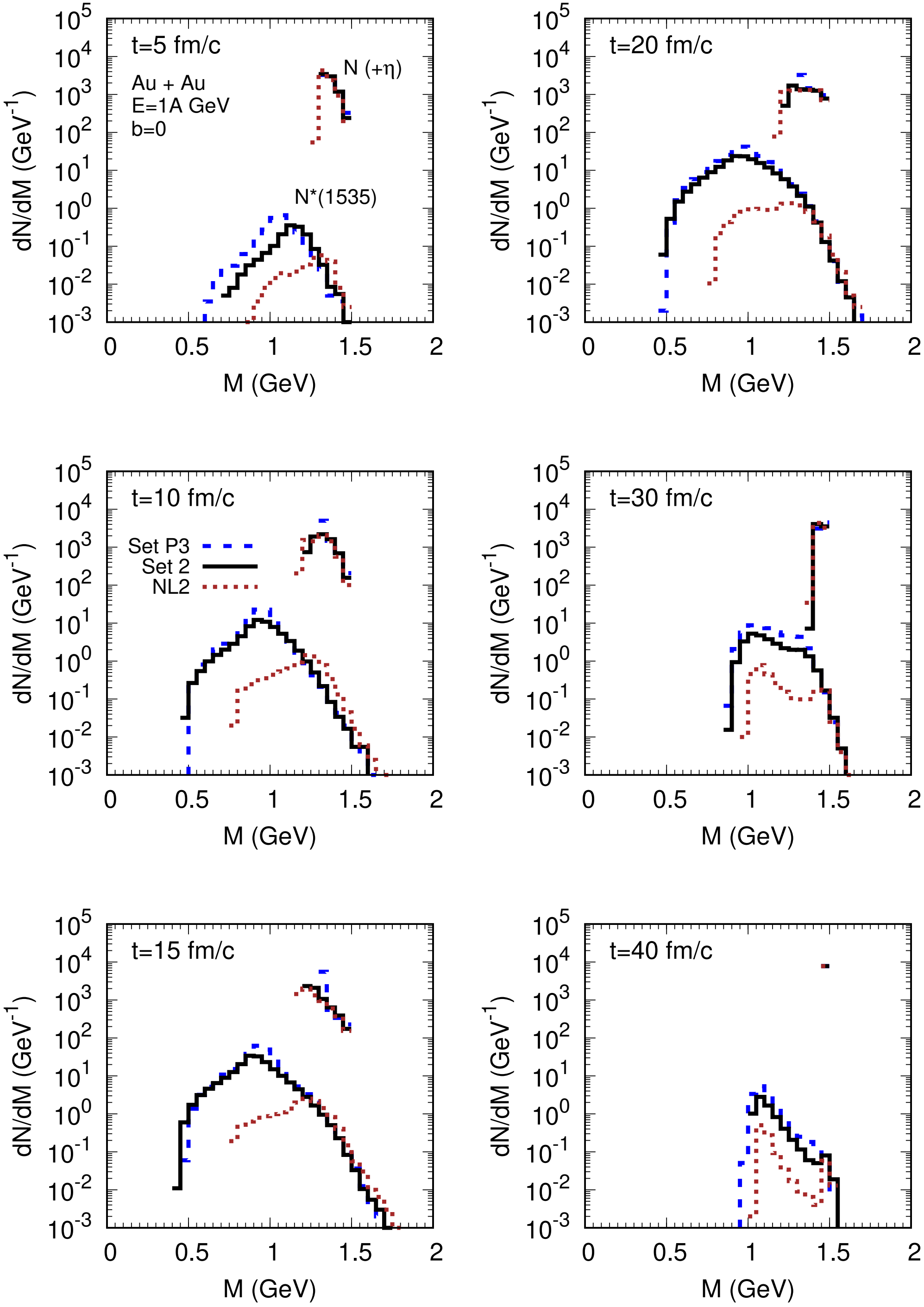
Invariant mass distributions of N(1535) and N*

(N distribution is right-shifted by the η mass)

- Excess of N*(1535) production at low inv. masses at t ≤ 20 fm/c with PDM

- At later times the system expands, N*(1535)'s migrate towards larger inv. masses and disappear via absorption N* N → NN and decays: N* → πN (51%), ηN (43%), ρN (3%), σN (1%), πN*(1440) (2%).

- Thus, most of the low-mass excess of N*(1535)'s goes into other channels than ηN.



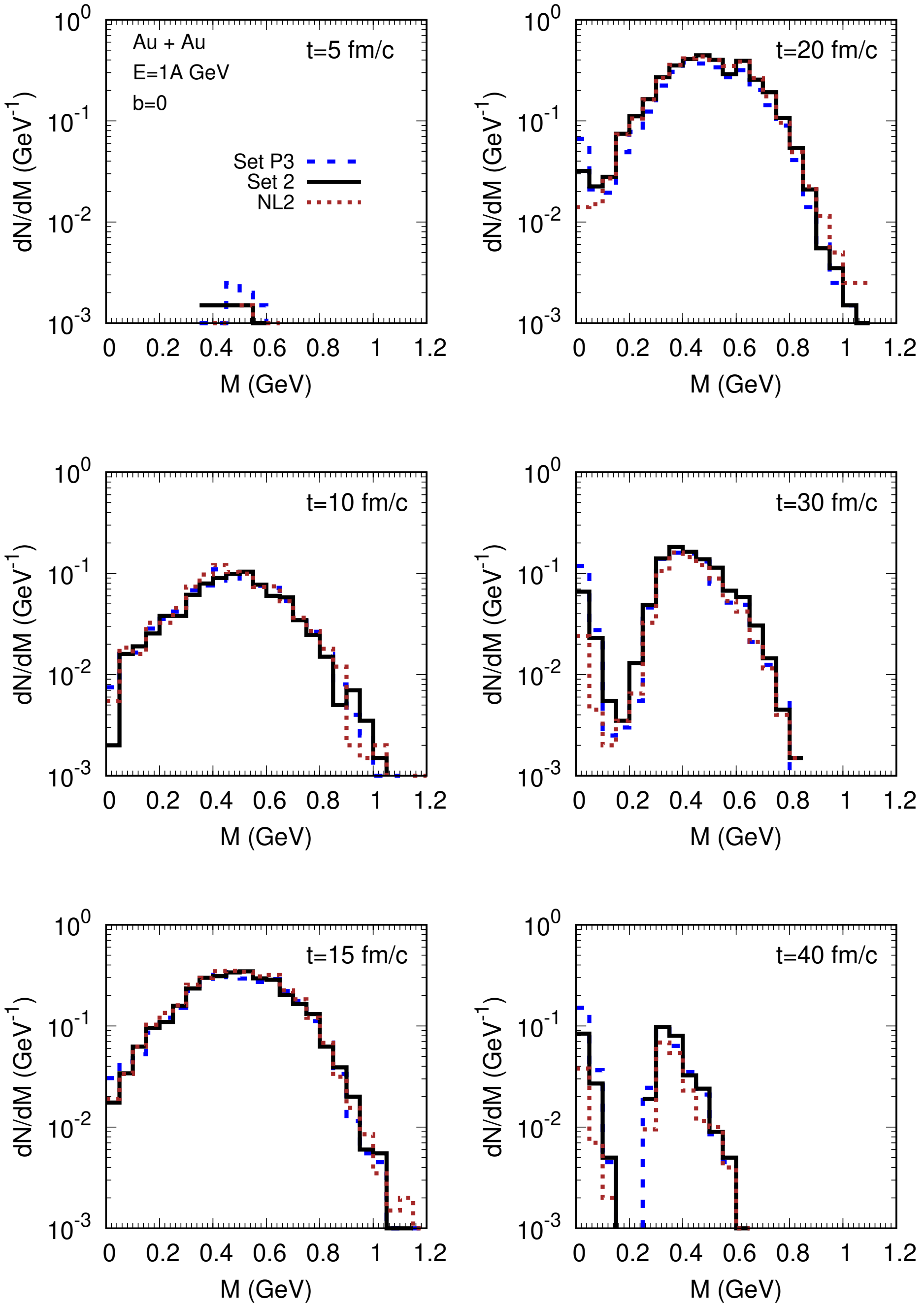
Invariant mass distribution of ρ meson

- N*(1520) → ρN contributes mostly.

- No difference between RMF sets for early times t ≤ 15 fm/c.

- At later times the excess of long-lived ρ's below ππ threshold is visible in PDM due to the contribution from N*(1535) → ρN.

Calculation is done with in-medium spectral function of ρ-meson, see **AL, U. Mosel, L. von Smekal, PRC 102, 064913 (2020) [arXiv:2009.11702]**



Comparison with experiment

Transverse mass spectra at midrapidity

In thermal equilibrium:

$$\frac{d^2\sigma}{dy m_t^2 dm_t} \propto e^{-m_t/T}$$

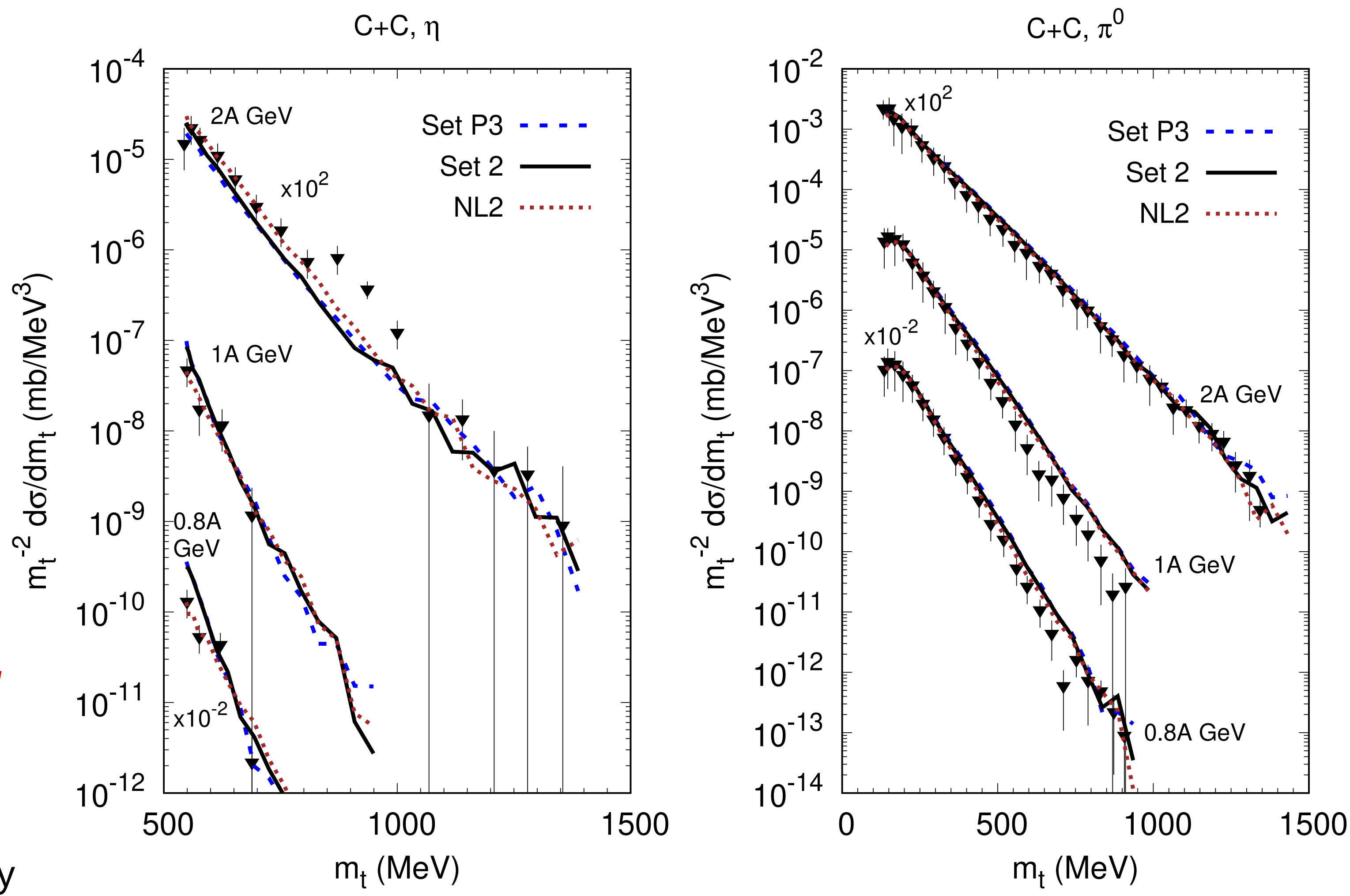
- m_t -scaling.

Small systems (like C+C) are likely to deviate from thermal equilibrium:

- cross sections become Important;

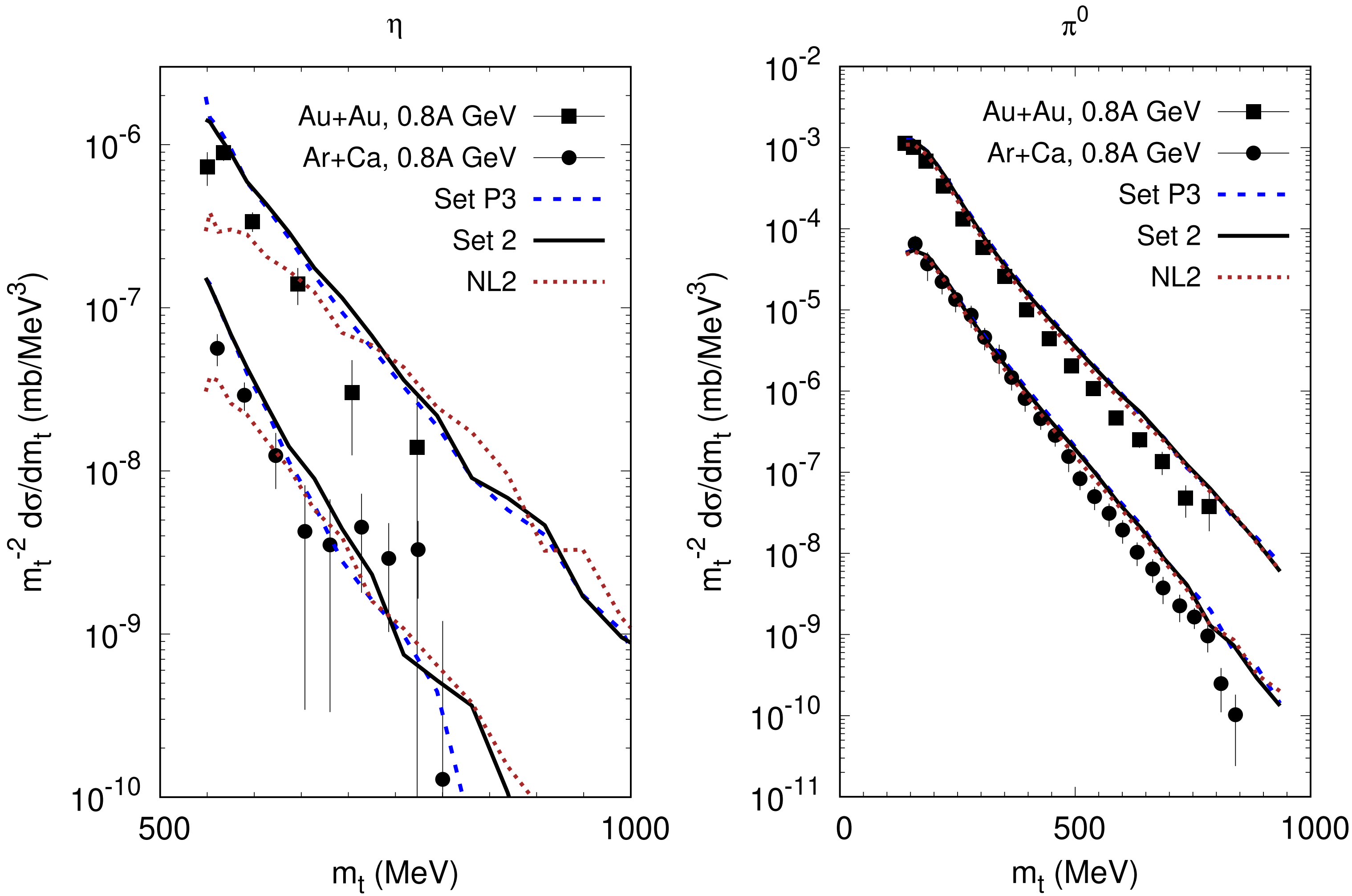
- lowering the threshold of NN → NN*(1535) in PDM enhances the yield of low-m_t η's at E_{lab}=0.8 and 1A GeV

Note: threshold beam energy for pp → ppη is 1.255 GeV



- no effect of RMF on π^0

Data: R. Averbeck et al. (TAPS), Z. Phys. A 359, 65 (1997)

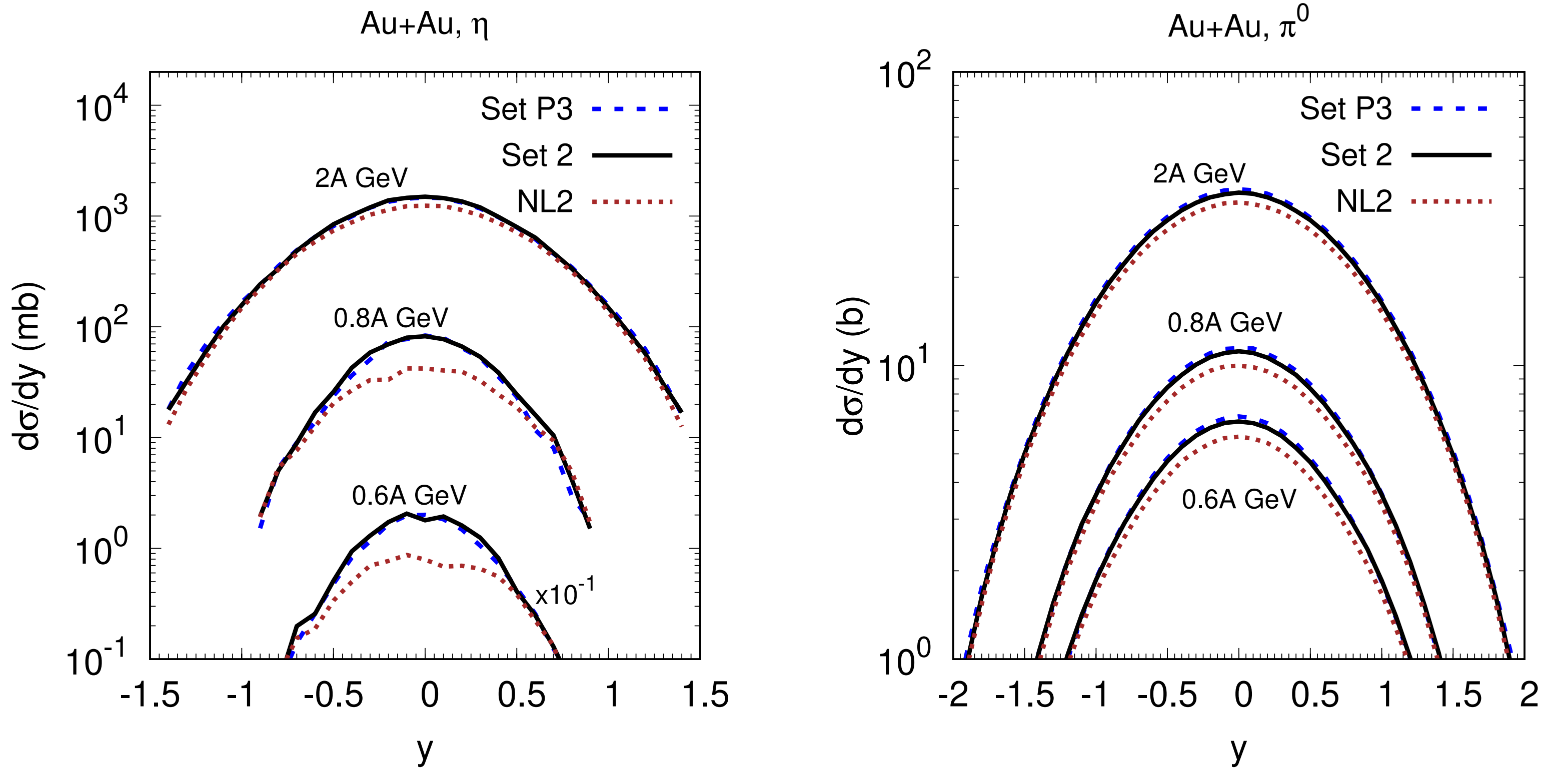


- For heavier systems the enhancement of η production at low m_t in PDM is more pronounced.

The effect of incompressibility is invisible: no difference between Set P3 (K=510 MeV) and Set 2(K=215 MeV). TAPS data seem to favor PDM (slopes better). No effect of RMF on pions.

Data Ar+Ca: A. Marin et al. (TAPS), PLB 409, 77 (1997);
Data Au+Au: A.R. Wolf et al. (TAPS), PRL 80, 5281 (1998)

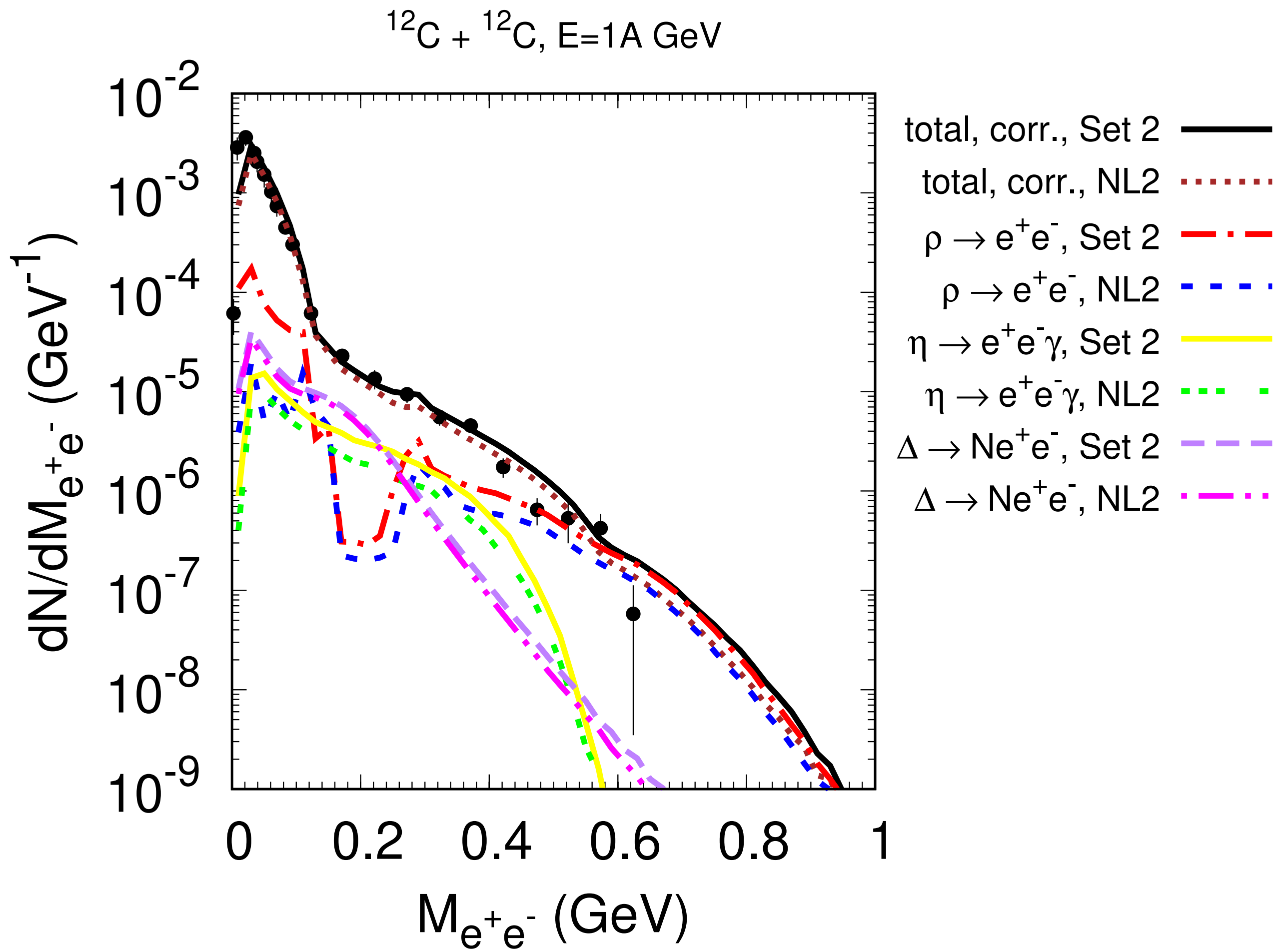
Rapidity spectra



- strongest effect at midrapidity, overall a factor of 2 enhancement of η production cross section at 0.6A GeV: 15.4±0.3 mb (PDM Set 2), 7.8±0.2 mb (Walecka NL2)

- pion production is also somewhat larger with PDM due N*(1535) → πN

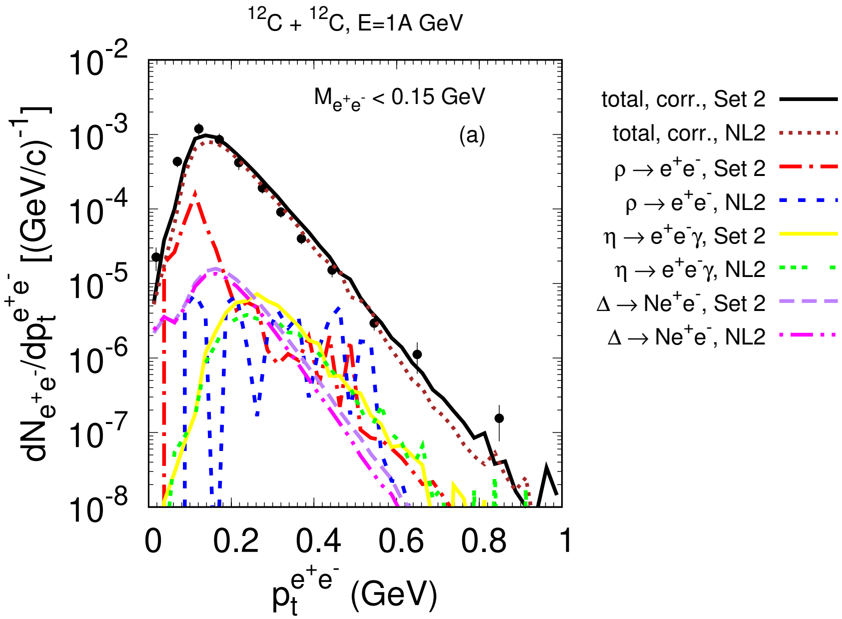
Dilepton (e^+e^-) production

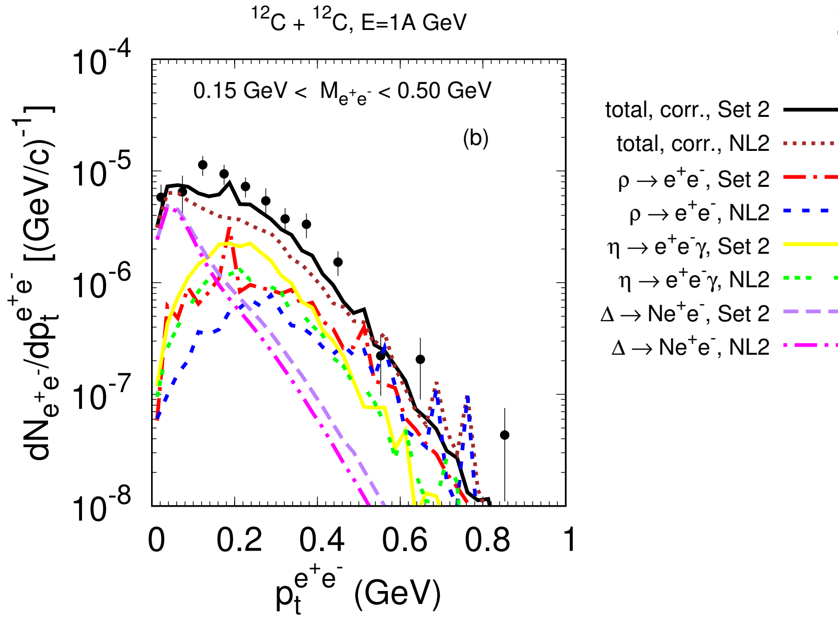


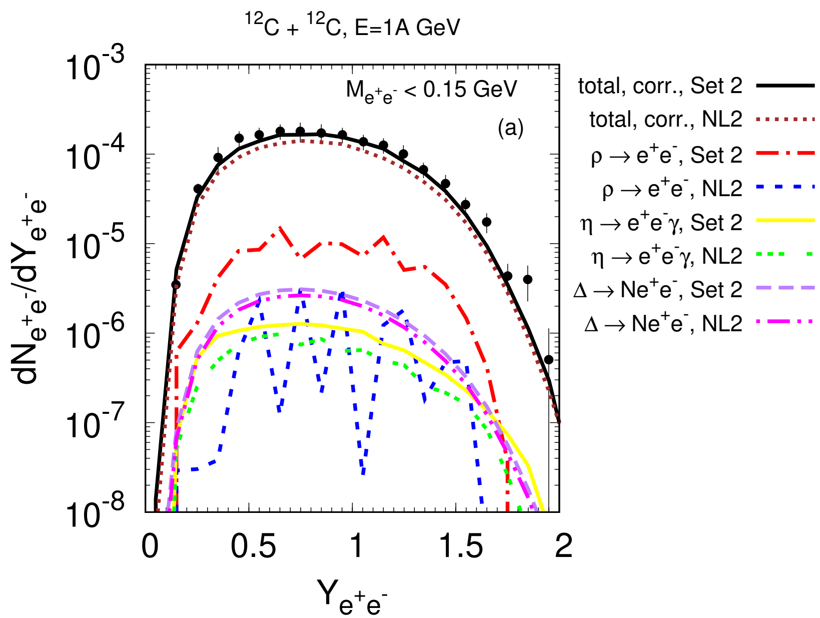
- Enhanced ρ direct,
η Dalitz, and Δ Dalitz
components with PDM (Set 2)

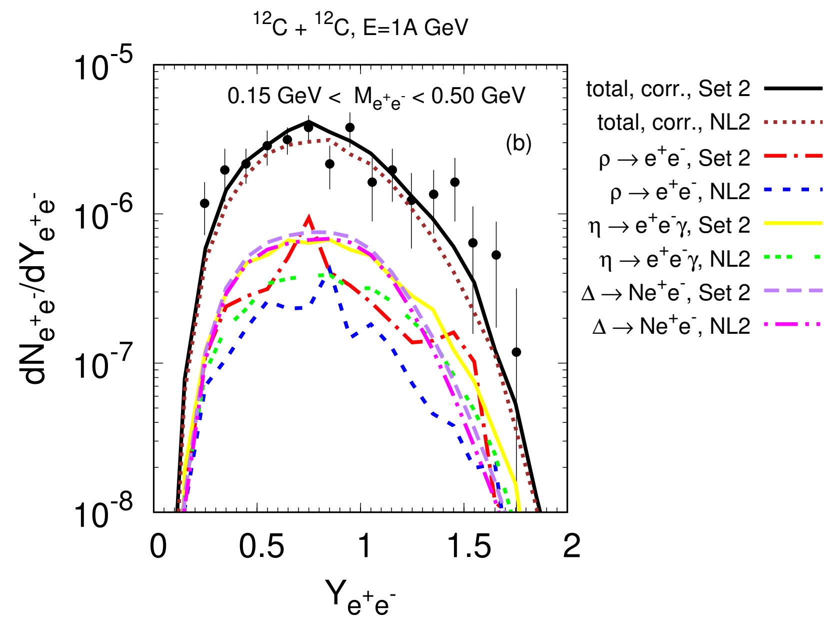
- Slightly larger yield at M_{e+e-}=0.15-0.5 GeV
with PDM (Set 2)

Data: ***G. Agakishiev et al. (HADES), PLB 663 (2008) 43***









ρ excess with PDM (Set 2) is stronger at low p_t and at midrapidity. However, hidden by π^0 Dalitz.

η excess in PDM (Set 2) is stronger at $p_t \approx 0.2$ GeV (due to photon recoil momentum) and at midrapidity

Data: ***Y. Pachmayer (HADES), PhD thesis, Frankfurt U. (2008).***

Summary

- PDM is implemented in the microscopic transport model GiBUU.

- Decreasing Dirac mass gap between N*(1535) and nucleon with increasing baryon density leads to the strong (10 times) enhancement of N*(1535) production at low inv. masses in Au+Au at 1A GeV central collisions.

- Enhanced low-m_t yield of η's with PDM. The effect is stronger at subthreshold beam energies (below 1.25 A GeV). The effects on dilepton inv. mass spectra are weak.

Outlook

- PDM provides relations between πNN*, πNN, and πN*N* coupling constants which can be tested in π(γ)N → πηN reactions close to threshold

- Chiral quartet model for J=3/2 sector to include Δ(1232), Δ(1700), N*(1520), and N*(1720), D. Jido et al., PRL 84, 3252 (2000)

- Nuclear structure applications of PDM: binding energies, charge radii, I.J. Shin et al., arXiv:1805.03402

- Astrophysical applications of PDM: neutron star properties, M. Marczenko et al., PRD 105, 103009 (2022)

Thank you for your attention

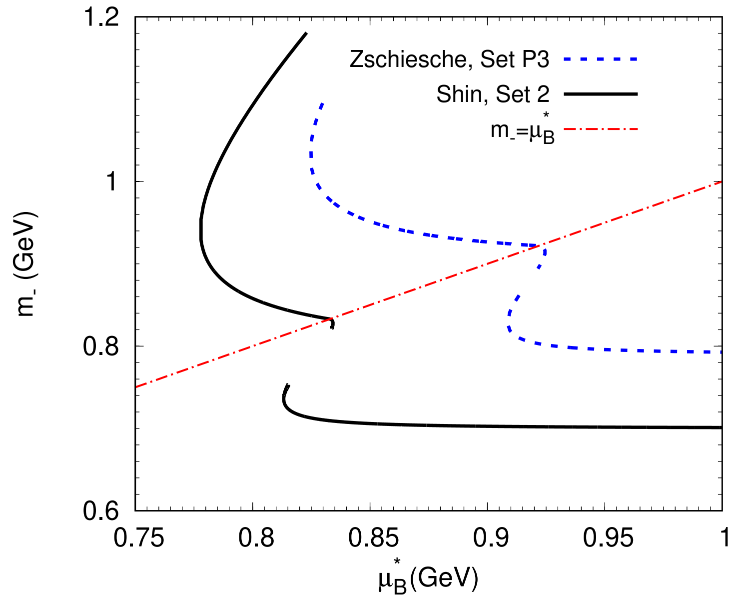
Backup

Chiral phase transition at T=0

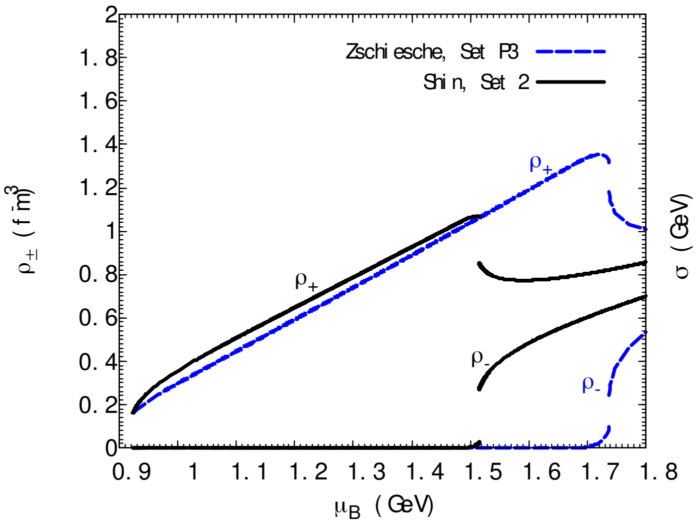
- In PDM at $\rho_B \approx 1 - 1.4$ fm^{-3} the Dirac mass of N*(1535) becomes less than the effective chemical potential:

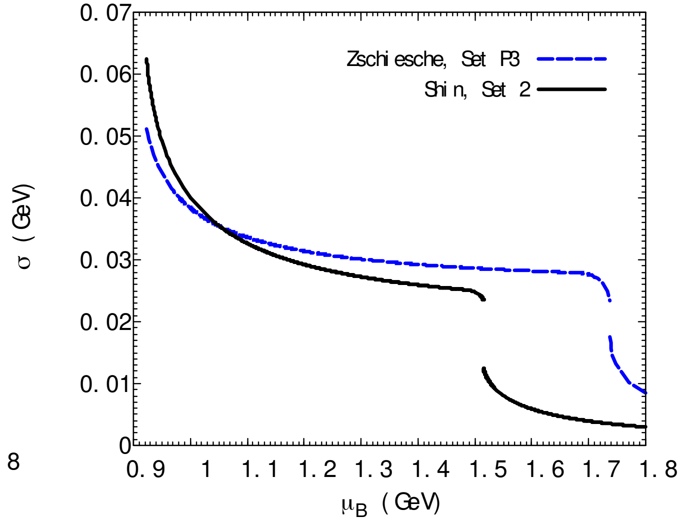
$$m_- < \mu_B^* = \mu_B - g_\omega \omega^0 \ .$$

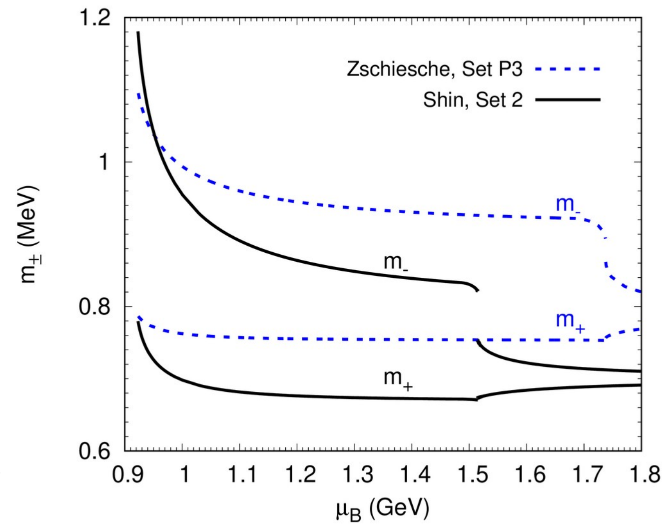
- Fermi sea of N*(1535) starts to fill-in, the partial densities and Dirac masses of nucleons and N*(1535) become close, σ-field quickly drops.



Line breaks: 1st order phase transition regions

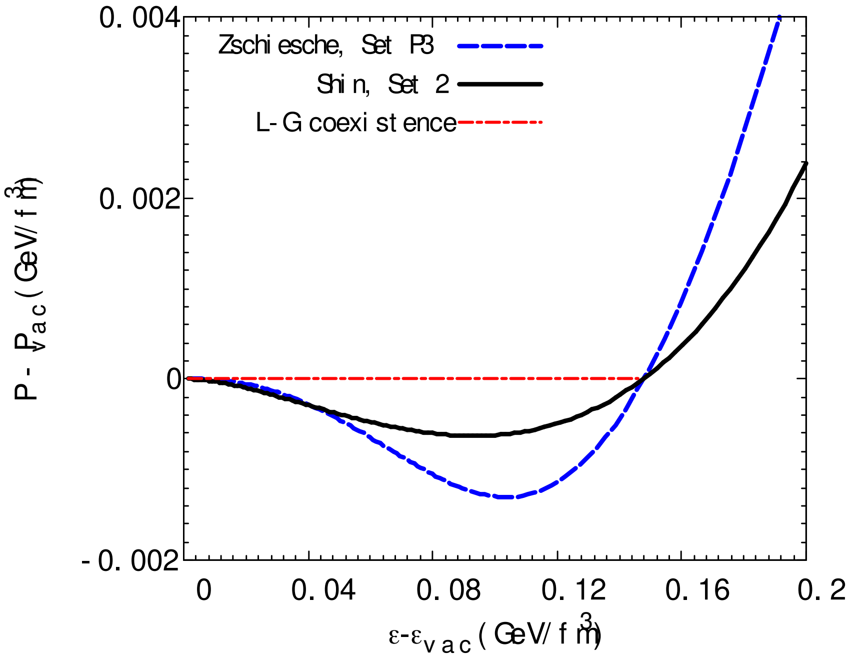


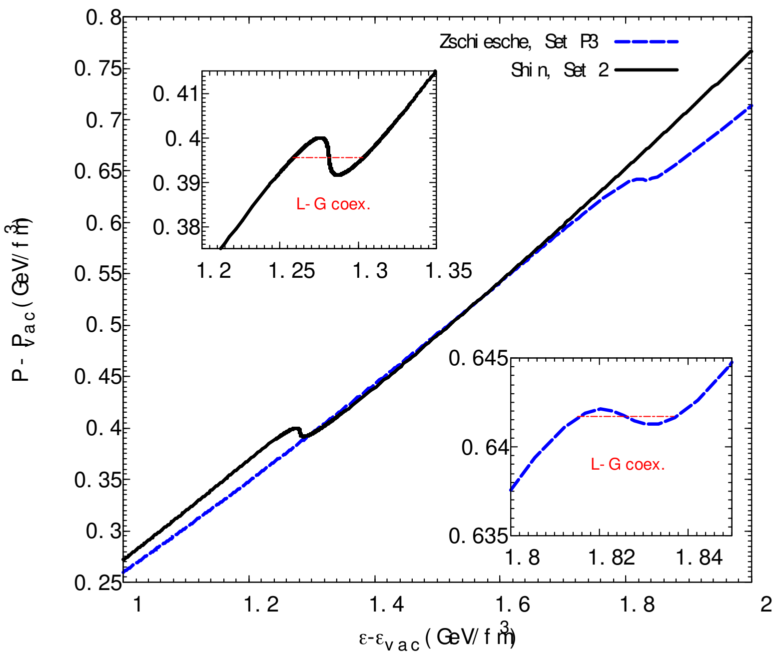




Liquid-gas coexistence regions at low and high densities from Gibbs conditions:

$$\mu_L = \mu_g$$
$$P_L = P_g \; .$$





Mass matrix diagonalization D. Jido et al., Prog. Theor. Phys. 106, 873 (2001):

$$(\bar{N}_1, \bar{N}_2)\left(\begin{array}{cc} -g_1\sigma & m_0\gamma_5 \\ -m_0\gamma_5 & -g_2\sigma \end{array}\right)\left(\begin{array}{c} N_1 \\ N_2 \end{array}\right) = (\bar{N}_+, \bar{N}_-)\left(\begin{array}{cc} m_+ & 0 \\ 0 & m_- \end{array}\right)\left(\begin{array}{c} N_+ \\ N_- \end{array}\right),$$

$$\left(\begin{array}{c} N_+ \\ N_- \end{array}\right) = \left(\begin{array}{cc} \cos\Theta & \gamma_5 \sin\Theta \\ -\gamma_5 \sin\Theta & \cos\Theta \end{array}\right)\left(\begin{array}{c} N_1 \\ N_2 \end{array}\right), \quad \tan 2\Theta = -\frac{2m_0}{\sigma(g_1+g_2)},$$

$$m_\pm = \frac{1}{2}\left[\sqrt{\sigma^2(g_1+g_2)^2 + 4m_0^2} \pm \sigma(g_2 - g_1)\right] .$$

Vacuum conditions:

Goldberger-Treiman relation →

$$\sigma \equiv \sigma_0 = f_\pi = 93 \text{ MeV},$$
$$m_+(\sigma_0) = m_N,$$
$$m_-(\sigma_0) = m_{N^*(1535)}.$$

Decreasing scalar field σ leads to the mass degeneracy of the parity partners, $m_\pm \to m_0$

Full model Lagrangian in the basis of mass eigenstates (physical basis) :

$$\begin{aligned}
\mathcal{L} &= \bar{N}_+[i\partial\!\!\!/ - m_+ - ig_{\pi N_+N_+}\gamma_5\boldsymbol{\tau\pi} - (g_\omega\omega^\mu + g_\rho\boldsymbol{\tau\rho}^\mu - g_{a_1}\gamma_5\boldsymbol{\tau a}_1^\mu)\gamma_\mu]N_+ \\
&\quad +\bar{N}_-[i\partial\!\!\!/ - m_- - ig_{\pi N_-N_-}\gamma_5\boldsymbol{\tau\pi} - (g_\omega\omega^\mu + g_\rho\boldsymbol{\tau\rho}^\mu + g_{a_1}\gamma_5\boldsymbol{\tau a}_1^\mu)\gamma_\mu]N_- \\
&\quad +\frac{m_\omega^2}{2}\omega^\mu\omega_\mu + \frac{m_\rho^2}{2}(\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu + \boldsymbol{a}_1^\mu\boldsymbol{a}_{1\mu}) \\
&\quad -ig_{\pi N_+N_-}\bar{N}_+\boldsymbol{\tau\pi}N_- + ig_{\pi N_+N_-}\bar{N}_-\boldsymbol{\tau\pi}N_+ \\
&\quad +g_{a_1N_+N_-}\bar{N}_+\gamma_\mu\boldsymbol{\tau a}_1^\mu N_- + g_{a_1N_+N_-}\bar{N}_-\gamma_\mu\boldsymbol{\tau a}_1^\mu N_+ + \mathcal{L}_{\mathrm{mes}} \ .
\end{aligned}$$

Coupling constants:

$$\begin{aligned}
g_{\pi N_+N_+} &= -g_1\cos^2\Theta - g_2\sin^2\Theta \ , \\
g_{\pi N_-N_-} &= g_2\cos^2\Theta + g_1\sin^2\Theta \ , \\
g_{\pi N_+N_-} &= \frac{g_1 - g_2}{2}\sin 2\Theta \ , \\
g_{a_1} &= g_\rho\cos 2\Theta \ , \\
g_{a_1N_+N_-} &= g_\rho\sin 2\Theta \ .
\end{aligned}$$

Non-diagonal transitions at finite mixing angle θ

Note: in the so-called “naive model”, i.e. when right (left) - handed components of N_1 and N_2 rotate in the same way, the coupling between N_+ and N_- completely disappears after the diagonalization.