Various corners of QCD and 2 color QCD phase diagrams

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БАЗИС

Фонд развития теоретической физики и математики

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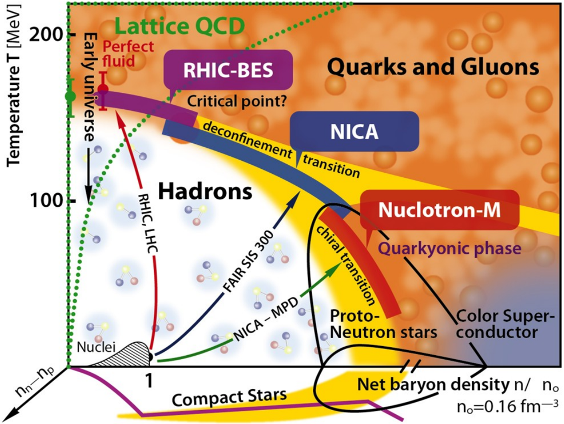
  БАЗИС

  Фонд развития теоретической физики и математики

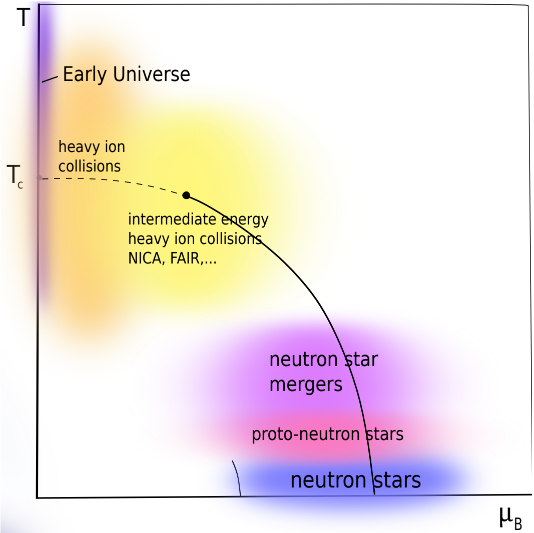
QCD Phase Diagram



QCD Dhase Diagram

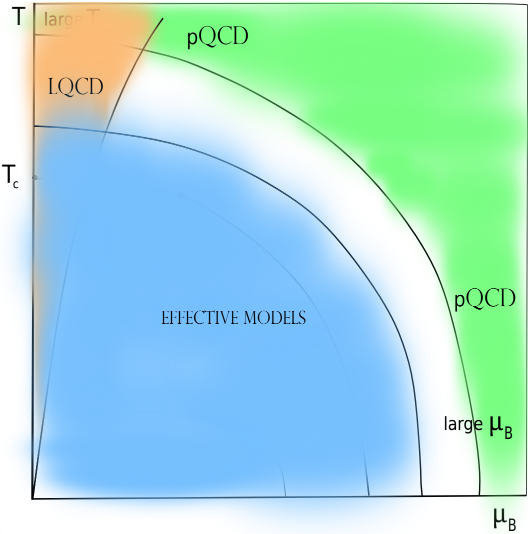
QCD at T and μ
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- .....



NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

$$\mu, T < 600\,\mathrm{MeV}$$

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

- **QCD phase diagram**

  with **different chemical potentials**

  and matter content including **chiral imbalance**

- **QC_2D phase diagram** and **diquark condensation** phenomenon

  with different chemical potentials, including μ_5

- **QCD phase diagram** and **color superconductivity phenomenon**

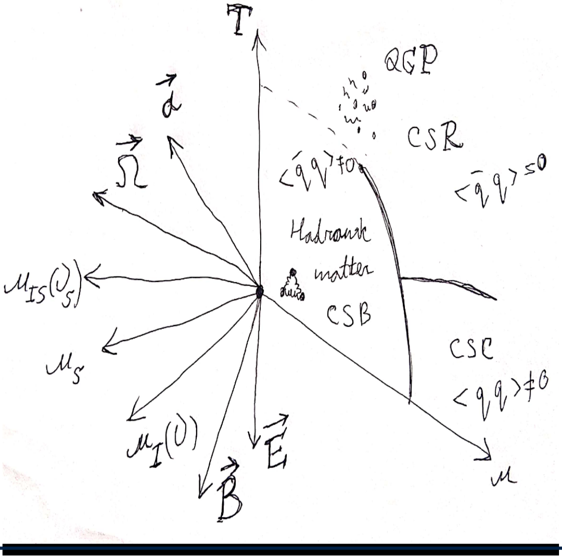
  with **different chemical potentials**

  and matter content including **chiral imbalance**

More external conditions to QCD

More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields *(see talk by A. N. Tawfik)*
- rotation of the system $\vec{\Omega}$ *A. Roenko, D. Sychev and G. Prokhorov*
- acceleration $\vec{a}$ *G. Prokhorov*
- finite size effects (finite volume and boundary conditions)



More external conditions to QCD

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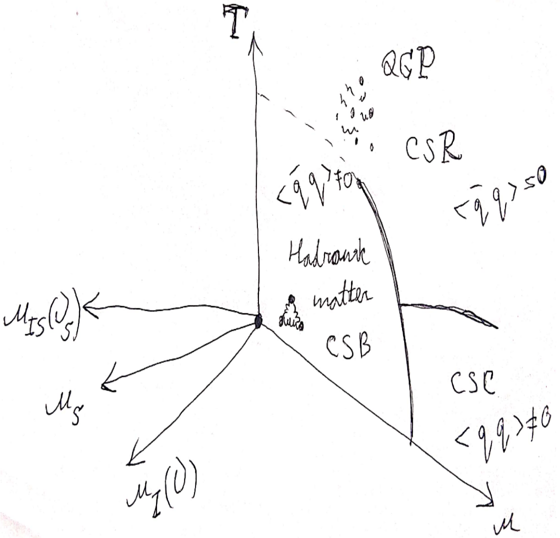


Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Different types of chemical potentials

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad\qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

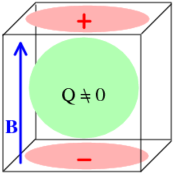
chiral (axial) chemical potential

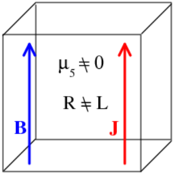
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

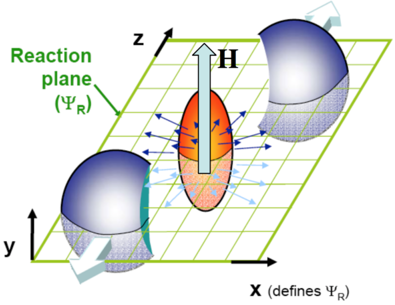
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

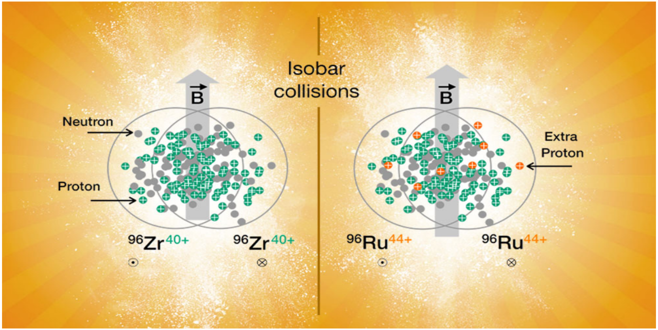




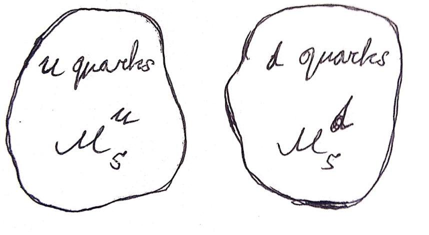


$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



The first blind analysis results isoba run have been recently released by the STAR Collaboration. Under the pre-defined assumption of identical background in RuRu and ZrZr, the results are **inconsistent with the presence of CME**, as well as with all existing theoretical models (whether including CME or not). However **the observed difference of backgrounds** must be taken into account **before any physical conclusion is drawn**.



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

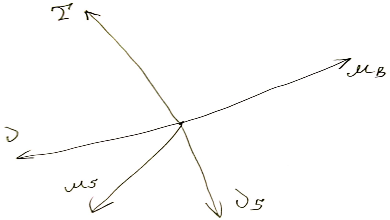
$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

- Chiral isospin imbalance and chiral imbalance μ_{I5} and μ_5 can be generated in parallel magnetic and electric fileds $\vec{E} \,||\, \vec{B}$

- Chiral imbalance could appear in dense matter
  - Chiral separation effect
    (Thanks for the idea to Igor Shovkovy)
  - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$



- **Duality between CSB and PC** has been found in effective model of QCD, 3 color, without **diquark condensation** phenomenon

- **Additional dualities** have been found in **QC_2D phase diagram**. There has been shown that the phase diagram have a highly symmetric structure

- **QCD phase diagram** has been studied and **color superconductivity phenomenon** and interesting qualitative features has been revealed

Recall that in NJL model **without color superconductivity phenomenon** there have been found **dualities**

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

Dualities

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

Duality in the phase portrait

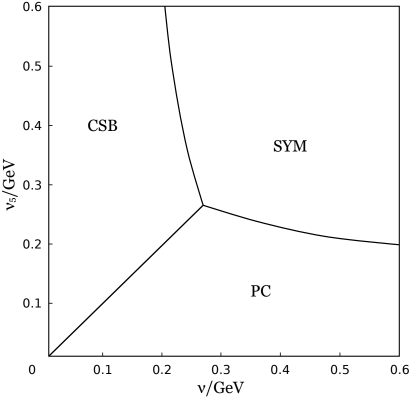


Figure: NJL model results

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

Inhomogeneous phases (case)

Homogeneous case

$\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_\pm(x)\rangle = \pi, \quad \langle\pi_3(x)\rangle = 0.$$

schematic (ν_5, μ)-phase diagram

- exchange axis ν to the axis ν_5,
- rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

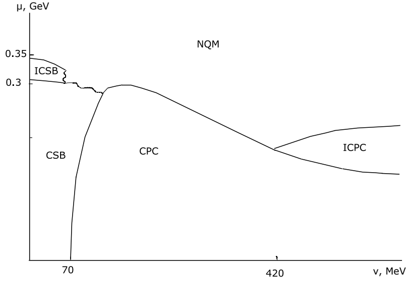


Figure: (ν, μ)-phase diagram

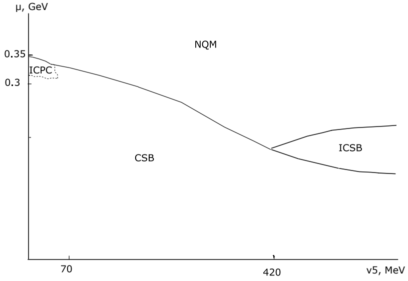


Figure: (ν_5, μ)-phase diagram

Two colour QCD case

QC_2D

There are a lot similarities:

- similar phase transitions:

  confinement/deconfinement, chiral symmetry breaking/restoration at large T *and* μ

- A lot of physical quantities coincide with some accuracy

  Critical temperature, shear viscosity etc.

There is **no sign problem** in SU(2) case

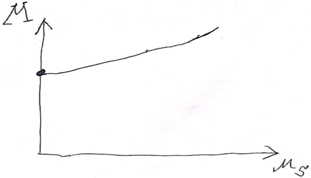
$$(Det(D(\mu)))^\dagger = Det(D(\mu))$$

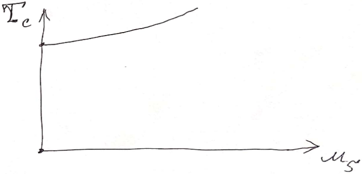
and lattice simulations at non-zero baryon density are possible

It is a great playground for studying dense matter

Catalysis of chiral symmetry beaking

QCD at non-zero μ_5

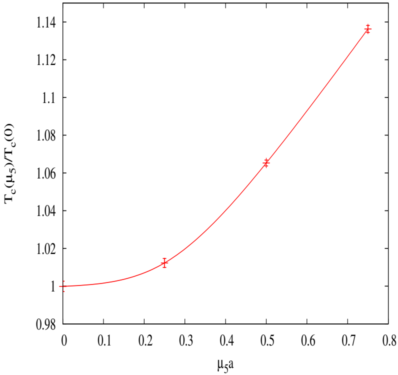




catalysis of CSB by chiral imbalance:

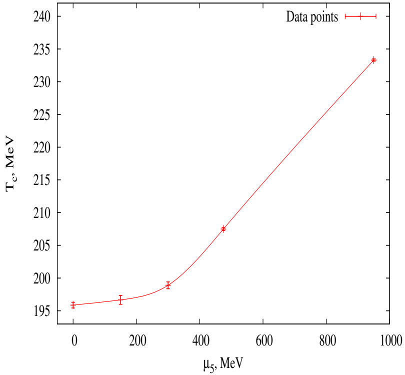
- increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)



V. Braguta, A. Kotov et al, JHEP 1506, 094 (2015),PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, Phys. Rev. D 93, 034509 (2016), arXiv:1512.05873 [hep-lat]

Phase diagram of QC_2D

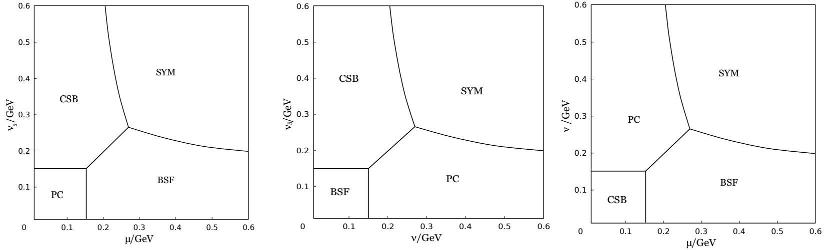
Condensates and phases

$M = \langle \sigma(x) \rangle \sim \langle \bar{q}q \rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle \pi_1(x) \rangle = \langle \bar{q}\gamma^5\tau_1 q \rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle \Delta(x) \rangle = \langle qq \rangle = \langle q^T C\gamma^5\sigma_2\tau_2 q \rangle,$ BSF phase: $\Delta \neq 0.$

Dualities in QC_2D



$$(a) \qquad \mathcal{D}_1: \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

J. Andersen, T. Brauner, D. T. Son, M. Stephanov, J. Kogut, ...

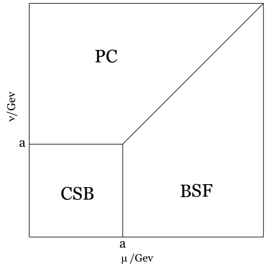
$$(b) \qquad \mathcal{D}_3: \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

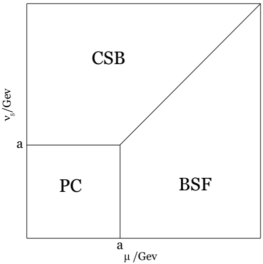
$$(c) \qquad \mathcal{D}_2: \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

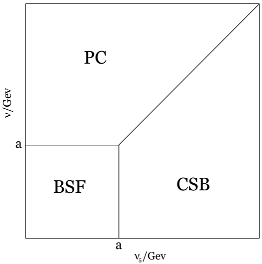
Each chemical potential is connected in one-to-one correspondence with some phenomenon (condensation)

- Baryon density μ $\Longleftrightarrow$ diquark condensation
- Isospin imbalance ν $\Longleftrightarrow$ pion condensation
- Chiral imbalance ν_5 $\Longleftrightarrow$ chiral symmetry breaking

One to one correspondence

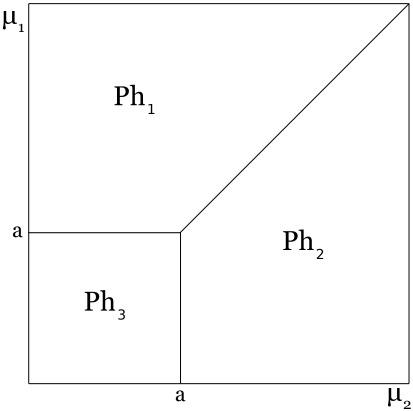


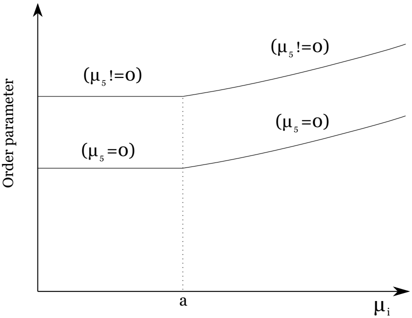




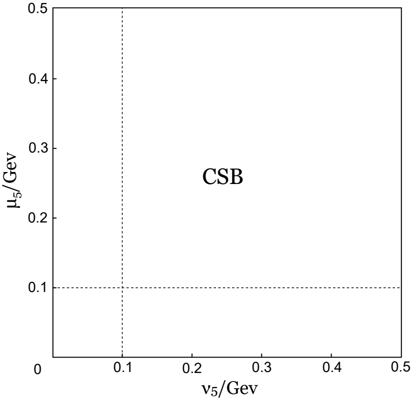
$\mu \longrightarrow \text{BSF},$ $\nu \longrightarrow \text{PC},$ $\nu_5 \longrightarrow \text{CSB}$

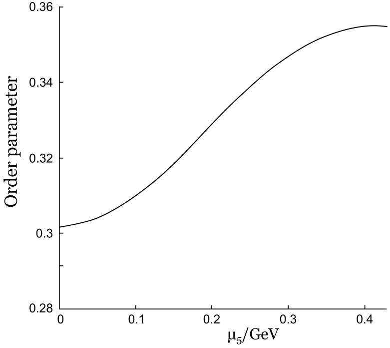
Universal catalysis effect of chiral imbalance

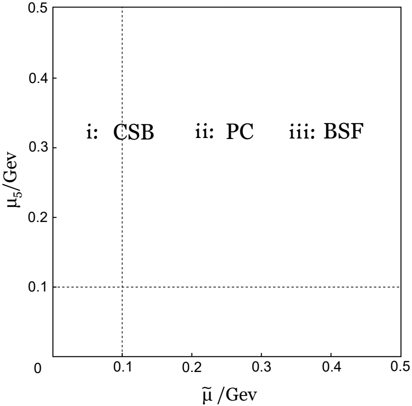


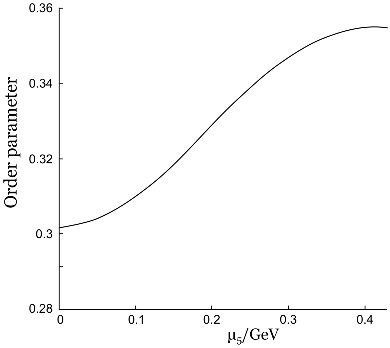


Universal catalysis effect of chiral imbalance

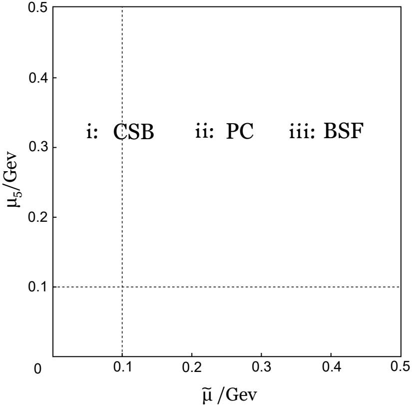


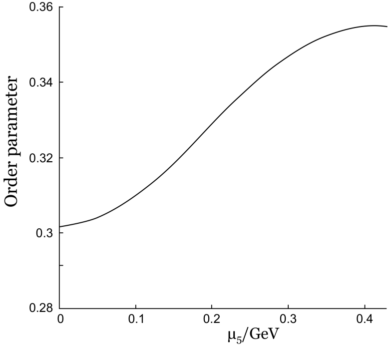



i: CSB
ii: PC
iii: BSF
μ_5/Gev
$\tilde{\mu}$ /Gev
0
0.1
0.2
0.3
0.4
0.5


Order parameter
μ_5/GeV
0.28
0.3
0.32
0.34
0.36
0
0.1
0.2
0.3
0.4

Chameleon property (mimicry) of μ_5

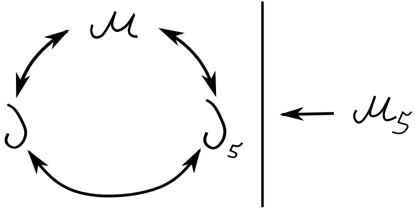


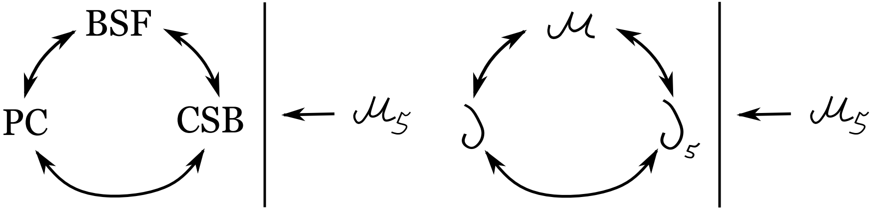


Chameleon nature of chiral imbalance μ_5

μ_5 mimics other chemical potentials μ, ν, ν_5

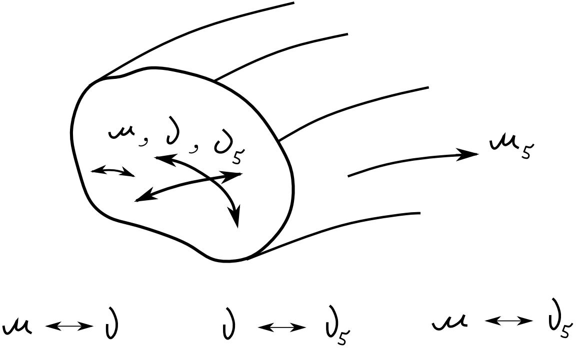
Chiral imbalance μ_5 does not participate in dual transformations



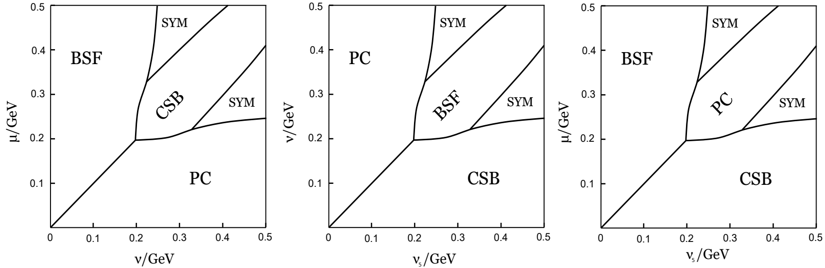


Chiral imbalance μ_5 does not participate in dual transformations

The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction

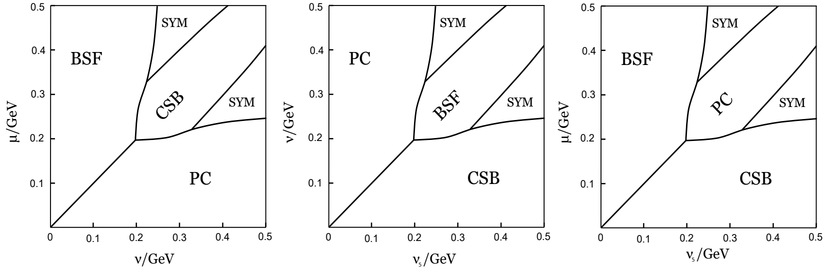


Dualities in the large values regime



All phase diagrams are dually connected

Phase structure in the large values regime

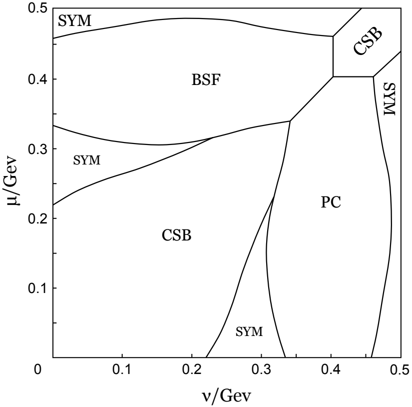


Chiral imbalance μ_5 could universally trigger all the phenomena

Diquark condensation at $\mu = 0$

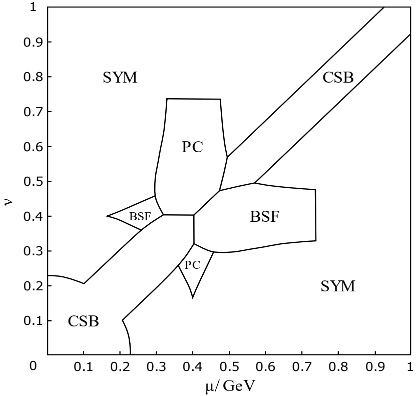
Chiral imbalance μ_5 leads to several rather peculiar phases in the system, e. g. the **diquark condensation** in the region of the phase diagram at $\mu = 0$

It was known that μ_5 leads to pion condensation in dense quark matter with zero $\nu = 0$ in SU(3) case and in SU(2) as well



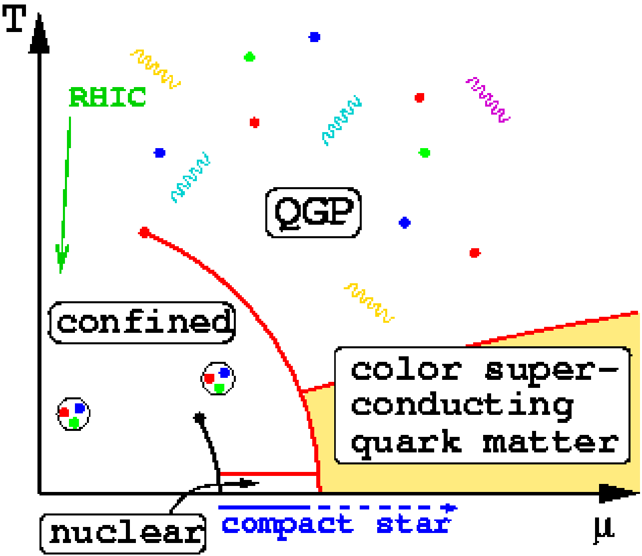
PC_d phase and diquark condensation

- PC_d phase has been predicted without possibility of diquark condensation
- Diquark condensation can take over the PC_d phase
- In two colour case diquark condensation is in a sense even stronger than in three colour case and starts from $\mu > 0$



PC_d phase is unaffected by BSF phase in two color case. Maybe one can infer that it is the case also for 3 color QCD

Phase diagram of QCD and color superconductivity at non-zero chiral imbalance


T
RHIC
QGP
confined
color super-
conducting
quark matter
nuclear
compact star
μ

The Lagrangian of three color NJL model

$$L = \bar{q}\Big[\gamma^{\nu} i\partial_{\nu} - m\Big]q + G\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\Big] +$$

$$+H \sum_{A=2,5,7} [\overline{q^c} i\gamma^5 \tau_2 \lambda_A q][\bar{q} i\gamma^5 \tau_2 \lambda_A q^c]$$

$$
\begin{aligned}
\mathcal{L} &= \bar{q}\Big[\gamma^{\nu} i\partial_{\nu} + \mathcal{M}\gamma^{0} - \sigma - m - i\gamma^{5}\vec{\pi}\vec{\tau}\Big]q - \frac{1}{4G}\Big[\sigma\sigma + \vec{\pi}^{2}\Big] \\
&- \frac{1}{4H}\Delta_A^{*}\Delta_A - \frac{\Delta_A^{*}}{2}[\overline{q^{c}} i\gamma^{5}\tau_2\lambda_A q] - \frac{\Delta_{A'}}{2}[\bar{q} i\gamma^{5}\tau_2\lambda_{A'} q^{c}]
\end{aligned}
$$

the equations of motion for bosonic fields, which take the form

$$\sigma(x) = -2G(\bar{q}q), \quad \pi_a(x) = -2G(\bar{q}i\gamma^5\tau_a q),$$

$$\Delta_A(x) = -2H(\overline{q^c}i\gamma^5\tau_2\lambda_A q), \quad \Delta^*_A(x) = -2H(\bar{q}i\gamma^5\tau_2\lambda_A q^c)$$

the mesonic fields $\sigma(x), \pi_a(x)$ are real quantities, i. e. $(\sigma(x))^\dagger = \sigma(x), \quad (\pi_a(x))^\dagger = \pi_a(x)$, but all diquark fields $\Delta_A(x)$ are complex scalars, so $(\Delta_A(x))^\dagger = \Delta^*_A(x)$.

Clearly, the real $\sigma(x)$ and $\pi_a(x)$ fields are color singlets, whereas scalar diquarks $\Delta_A(x)$ form a color antitriplet $\bar{3}_c$ of the $SU(3)_c$ group. Note that the auxiliary bosonic field $\pi_3(x)$ corresponds to real $\pi^0(x)$ meson, whereas the physical $\pi^\pm(x)$-meson fields are the following combinations of the composite fields (**??**), $\pi^\pm(x) = (\pi_1(x) \mp i\pi_2(x))/\sqrt{2}$. If some of the scalar diquark fields have a nonzero ground state expectation value, i. e. $\langle\Delta_A(x)\rangle \neq 0$, the color symmetry of the model is spontaneously broken down.

the Lagrangian and the effective action are invariant under the color $SU(3)_c$ group, hence the TDP depends on the combination

$$\Delta_2\Delta_2^* + \Delta_5\Delta_5^* + \Delta_7\Delta_7^* \equiv \Delta^2,$$

where Δ is a real quantity.

There are only three order parameters

$$M = \langle\sigma(x)\rangle = -2G\langle\bar{q}q\rangle, \quad \pi = \langle\pi_1(x)\rangle = -2G\langle\bar{q}i\gamma^5\tau_1 q\rangle,$$

$$\Delta = \langle\Delta(x)\rangle = -2H\langle\overline{q^c}i\gamma^5\tau_2\lambda_2 q\rangle$$

Condensates and phases

$M = \langle \sigma(x) \rangle \sim \langle \bar{q} q \rangle \neq 0,$ CSB phase:

$\pi = \langle \pi_1(x) \rangle = \langle \bar{q} \gamma^5 \tau_1 q \rangle \neq 0,$ PC phase: $\pi_1 \neq 0$

$\Delta = \langle \Delta(x) \rangle = \langle q q \rangle \neq 0,$ CSC phase: $\Delta \neq 0$

$$m_\pi,\ f_\pi,\ \langle \overline{q} q \rangle \longrightarrow \text{quark-antiquark coupling } G$$

H is not precisely determined

If the quark-antiquark interaction has been constrained empirically, the most natural solution is to determine the quark-quark coupling constants empirically, too. Unfortunately, the analog to the meson spectrum would be a diquark spectrum, which of course does not exist in nature

The most natural fit is

$$H = \frac{3}{4}G = 0.75G$$

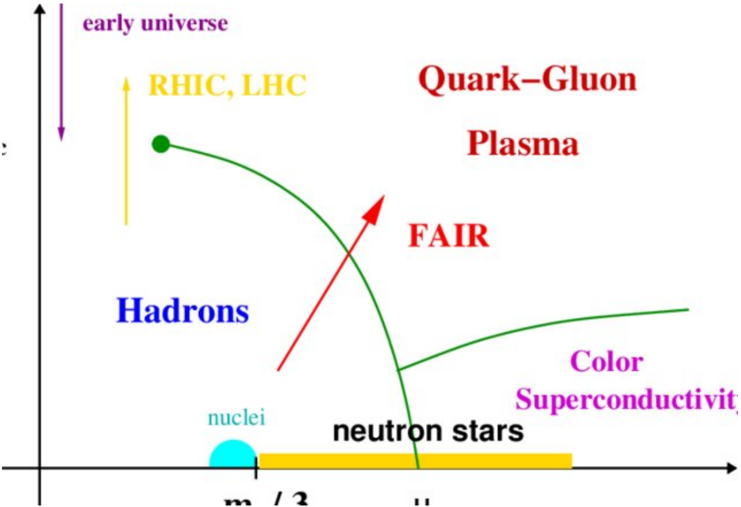
- from Fiertz transform
- or from reasonable value of condensate

But we can use $0 < H < G$

If we one consider unphysical twice as strong diquark channel

$$H = \frac{3}{2}G = 1.5\,G$$

It will be very instructive later


early universe
RHIC, LHC
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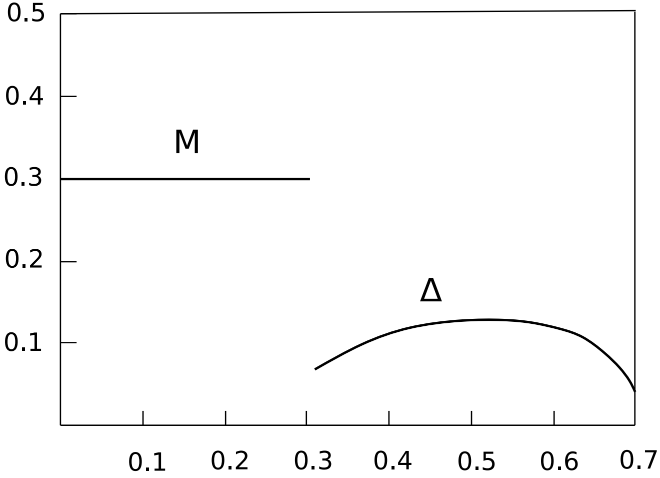
0.5
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M
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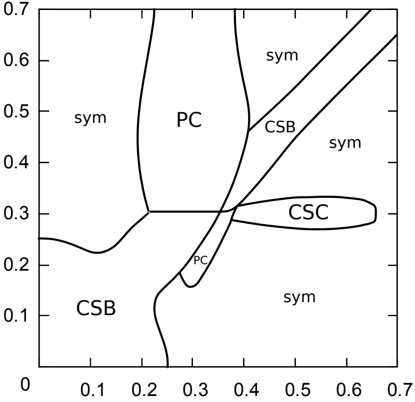
0.7

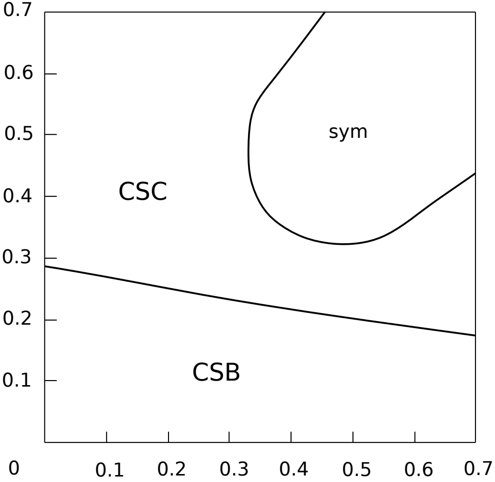
PC_d phase and diquark condensation

- PC_d phase at non-zero ν_5 has been predicted without possibility of diquark condensation
- Diquark condensation could take over the PC_d phase
- PC_d phase is unaffected by CSB phase in two color case.



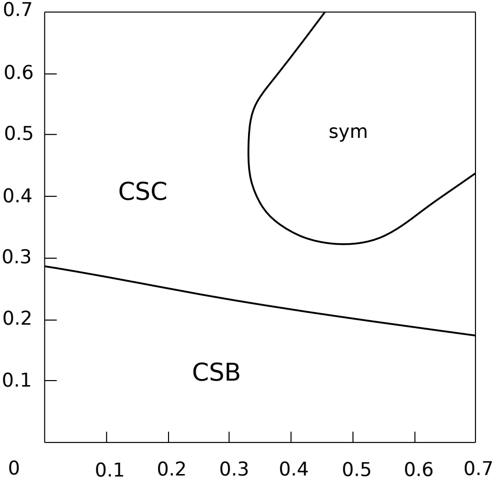
PC_d phase is unaffected by BSF phase in three color case.

Phase structure: (μ_5, μ)-phase diagram



Chiral imbalance μ_5 *facilitates the generation of color superconductivity*

Phase structure: (μ_5, μ)-phase diagram



Chiral imbalance μ_5 *facilitates the generation of color superconductivity*

Two regularization schemes have been used but further clarification is required

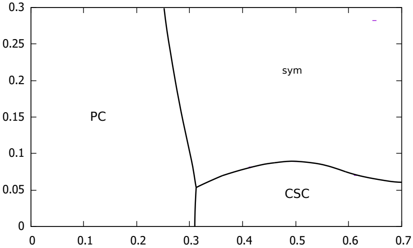


Figure: $\nu_5 = 0$

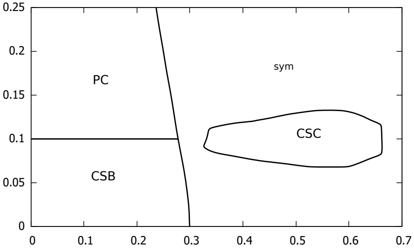
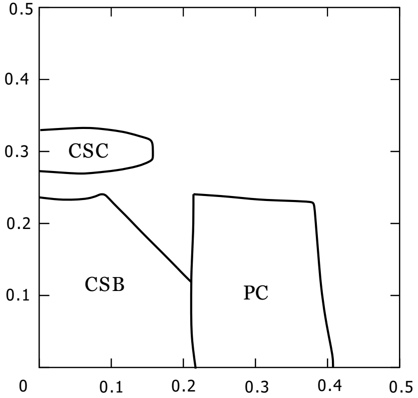


Figure: $\nu_5 = 0.1$

Chiral imbalance μ_5 leads to the **diquark condensation** in the region of the phase diagram at $\mu = 0$ in three color case



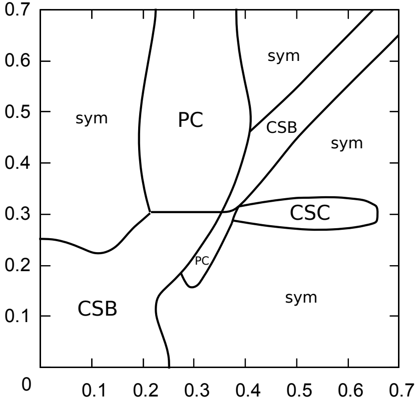
Qualitative dual properties

with color superconductivity phenomenon

in three color case

One can consider two regimes

- **physical** $H = \frac{3}{4}G = 0.75G$ or around
- **unphysical** $H = \frac{3}{4}G = 1.5G$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CSB
sym
CSC
PC
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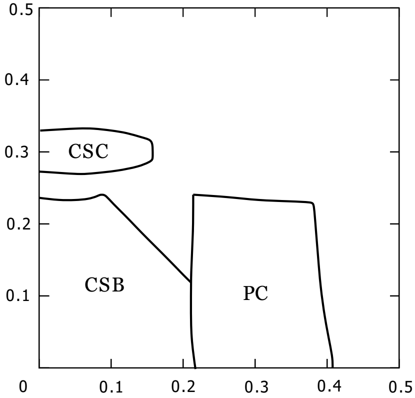
CSB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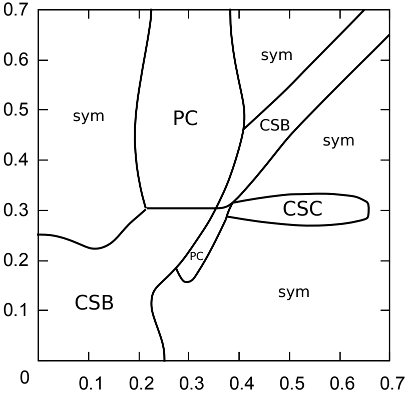
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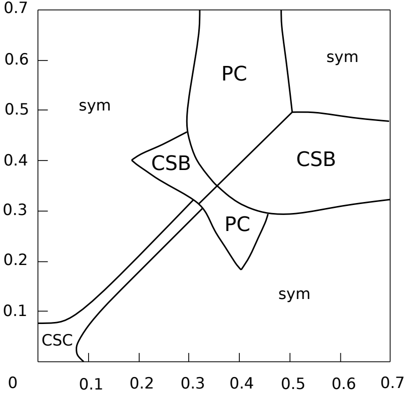
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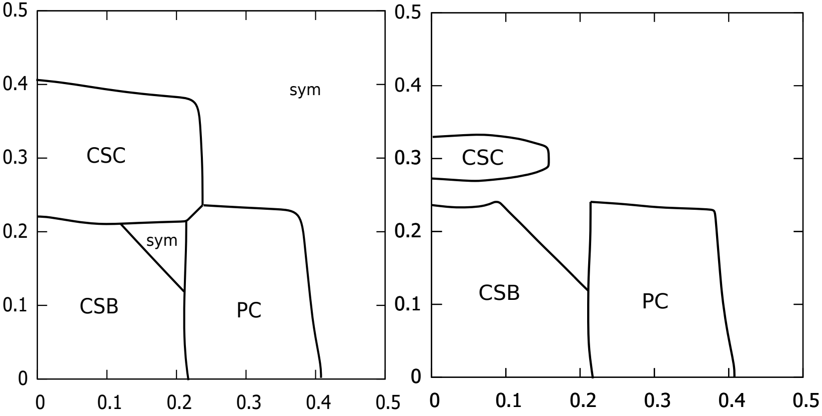
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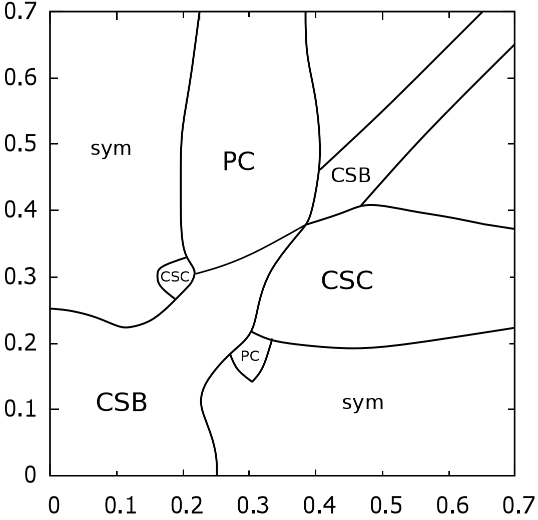
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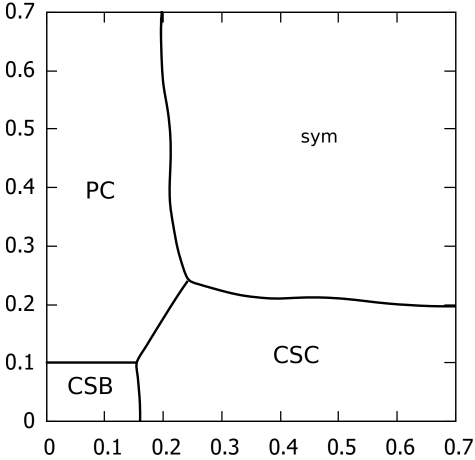
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CSB

The TDP projections have the following structure

$$F_1\,(M,\mu_i) = f(M,\mu_i)$$

$$F_2\,(\pi,\mu_i) = \mathcal{D}_3 f(\pi,\mu_i) = \mathcal{D}_1 F_3(\pi,\mu_i) + \bar{g}(T,\mu_i)$$

$$F_3\,(\Delta,\mu_i) = \mathcal{D}_2 F_1\,(\Delta,\mu_i) + \bar{f}(\mu_i)$$

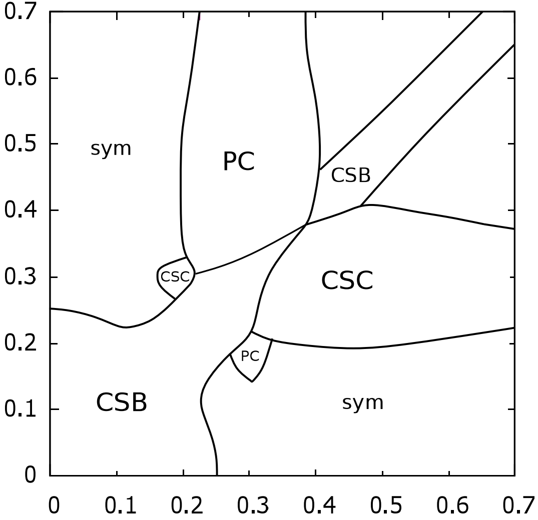
$$= \mathcal{D}_1\big(\mathcal{D}_3 f(M,\mu_i)\big) + \bar{f}_1(T,\mu_i)$$

Gap equations are dual with respect to each other so the condensates

$$\frac{\partial F_1\left(M,\mu_i\right)}{\partial M}=0$$

$$\frac{\partial F_2\left(\pi,\mu_i\right)}{\partial \pi}=0$$

$$\frac{\partial F_3\left(\Delta,\mu_i\right)}{\partial \Delta}=0$$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0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
sym
PC
CSB
CSC
CSC
PC
CSB
sym

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the mean field approximation

Dualities are connected with Pauli-Gursey group

Dualities were found in

- In the framework of NJL model beyond mean field

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Duality was found in

- In the framework of NJL model beyond mean field or at all orders of N_c approximation
- In QCD non-pertubartively (at the level of Lagrangian)

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied in two color color case
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality has been shown non-perturbarively in QCD