Strange to non-strange ratios in the SU(3) NJL-like models

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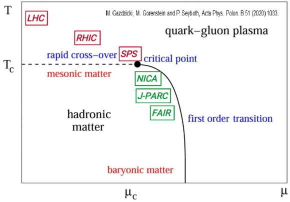
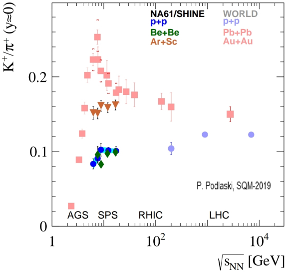
Dubna, March 2, 2023

Preface





- Onset of deconfinement (possible signal): characteristic enhanced production of pions ⇒ suppression of the strangeness-to-pion ratio (a jump to plateau is a signal of deconfinement and QGP formation (SMES M. Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B30, 2705 (1999)).
- Low energies: first order transition (?) the quick increase at low energies is a result of the partial chiral symmetry restoration (A. Palmese, et al. PRC 94, 044912 (2016)- PHSD; K. Bugaev - statistical model; J. Nayak - microscopic model).
- The system-size dependence of the K/π ratio (was shown by PHSD group)) was found by NA69/SHINE collaboration
- The fireball creation (?)

SU(3) PNJL model

The Lagrangian:

$$
\begin{aligned}
\mathcal{L} &= \bar{q}\,(\,i\gamma^{\mu} D_{\mu} \,-\, \hat{m} - \gamma_0\mu)\,q\, + \\
&+\; \frac{1}{2} G_S \sum_{a=0}^{8} [\,(\,\bar{q}\,\lambda^a\,q\,)^2 \,+\, (\,\bar{q}\,i\,\gamma_5\,\lambda^a\,q\,)^2\,] - \\
&-\; \frac{1}{2} G_V \sum_{a=0}^{8} [(\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_\mu i\gamma_5\lambda^a q)^2] \\
&-\; \sum_{\alpha} G^{\alpha}_{\text{diq}} \sum_{i,j} \left(\bar{q}_a\gamma_m u\Gamma^i_{\alpha} q^C_b\right) \left(\bar{q}^C_d\gamma^m u\Gamma^j_{\alpha} q_e\right) \varepsilon^{abc}\varepsilon^{de}_{c} \\
&-\; K\,\{\det[\bar{q}\,(\,1\,+\,\gamma_5\,)\,q\,] + \det[\bar{q}\,(\,1\,-\,\gamma_5\,)\,q\,]\,\} - \mathcal{U}(\Phi,\bar{\Phi};T)
\end{aligned}
$$

G_s G_v, K, G_{diq} - the coupling constants, parameters of the model.

$$
\tilde{G}_s(\Phi) = G_s[1 - \alpha_1\Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)].
$$
$$
\tilde{G}_v(\Phi) = G_v[1 - \alpha_1\Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)].
$$

The Polyakov loop and field Φ are determined as:

$\langle \ell(\vec{x}) \rangle = \frac{1}{N_c}\mathrm{Tr}_c L(\vec{x})$, $\Phi[A] = \langle\langle \ell \rangle\rangle$,

$L(\vec{x}) = \mathcal{P}\exp\left[i\int^{\beta} d\tau A_4(\vec{x},\tau)\right]$, $\beta = 1/T$

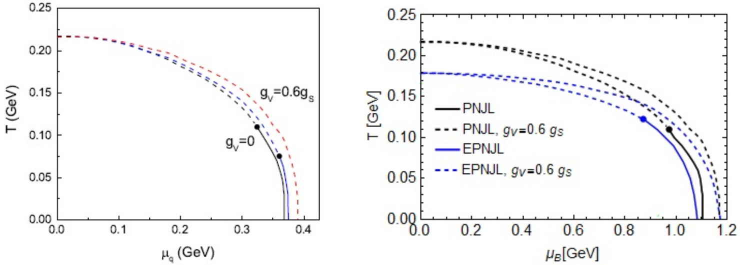
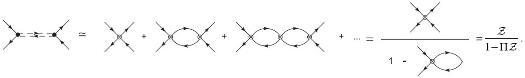


Figure 1: Phase diagrams for PNJL and EPNJL models for $G_v = 0, 0.6G_s$, $\mu_s = 0.5\mu_u$

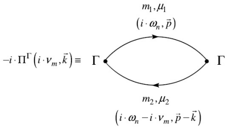
The meson masses: Bethe-Salpeter equation



The meson masses are defined by the Bethe-Salpeter equation at $\bar{P} = 0$

$$1 - P_{ij}\Pi^P_{ij}(P_0 = M, \bar{P} = \bar{0}) = 0 \ ,$$

and the polarization operator:



$$\Pi^P_{ij}(P_0) = 4\left((I^i_1 + I^j_1) - [P_0^2 - (m_i - m_j)^2]\ I^{ij}_2(P_0)\right) ,$$

with

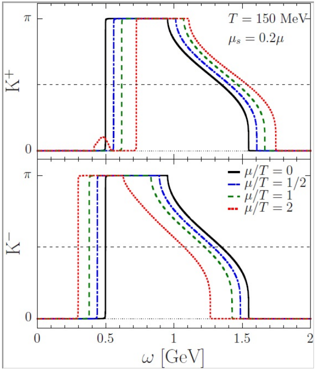
$$P_\pi = G_s + K\left<\bar{q}_s q_s\right> , \quad P_K = G_s + K\left<\bar{q}_u q_u\right>$$

where

$$I^i_1 = iN_c \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - m_i^2}, \quad I^{ij}_2(P_0) = iN_c \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2 - m_i^2)((p + P_0)^2 - m_j^2)}$$

When $T > T_{Mott}$ $(P_0 > m_i + m_j)$ the meson $\rightarrow$ the resonance state $\rightarrow$ $P_0 = M_M - 1/2i\Gamma_M$.

The meson masses: generalized Beth-Uhlenbeck approach



- The meson spectra beyond the mean field approximation can be obtained from the "polar" representation of complex meson propagator

$$\mathcal{S}_{ij}^{M}(\omega, \mathbf{q}) = |\mathcal{S}_{ij}^{M}(\omega, \mathbf{q})|\, \mathrm{e}^{i\delta_M(\omega, \mathbf{q})}, \qquad (1)$$

with the scattering mesonic phase shift

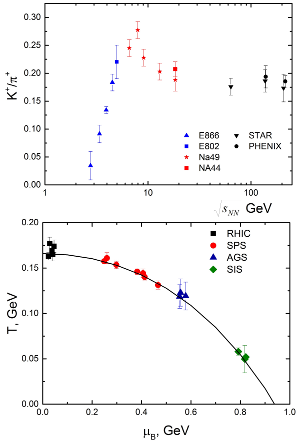
$$\delta_M(\omega, \bar{q}) = -arctan\{\frac{Im[\mathcal{S}_M(\omega - i\eta, \bar{q})]^{-1}}{Re[\mathcal{S}_M(\omega + i\eta, \bar{q})]^{-1}}\}$$

where

$$\mathcal{S}_M(\omega \pm i\eta, \bar{q}) = \frac{P_{ij}}{1 - 2P_{ij}\Pi_M(\omega \pm i\eta, \bar{q})} \qquad (2)$$

- the bound state appears at the energy, where the phase shift jumps up by the value π (Levinson's theorem, Wergieluk:2012gd).

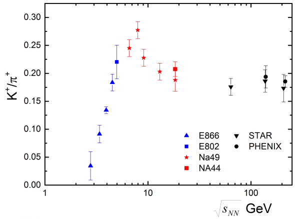
Can we apply the model to the 'horn' description?

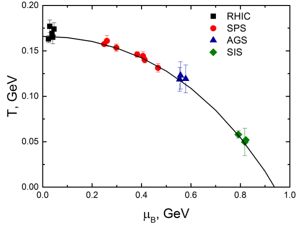


The model approach:

- all mesons were created during hadronization and we skip the rescattering, decays and so on..
- freeze-out line is coincide with the chiral phase transition line (it is an ansatz)
- Experiment: for each energy of collision we can find T^* and μ_B^* of the freeze-out
- Experiment: we can rescale the data as function of T^*/μ_B^*
- Theory: now we can calculate the kaon to pion ratio as a function T/μ_B where T and μ_b are chosen along the phase transition line.

Can we apply the model to the 'horn' description?





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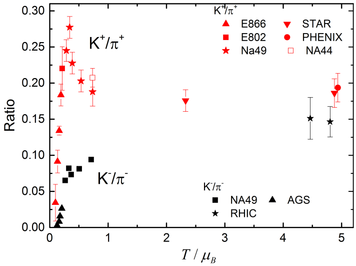
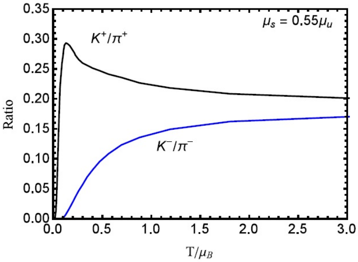
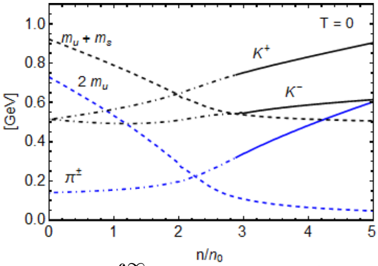


The theses were checked

- Splitting of kaons masses at high densities ⇒ the difference in the behaviour of the K/π at low energies;
- The peak depends on properties of the matter (strangeness neutrality, or chemical baryon potential of strange quark);





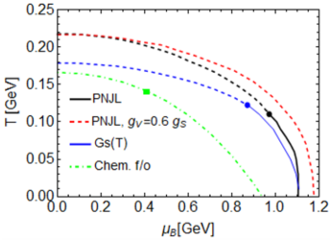
$$n_{K\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{K\pm}} \mp \mu_{K\pm})/T} - 1}, \quad n_{\pi\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{\pi\pm}} \mp \mu_{\pi\pm})/T} - 1}.$$

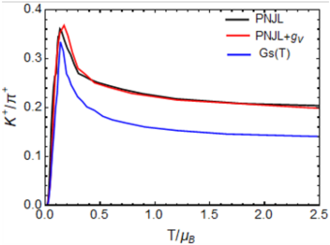
$\boxed{\mu_\pi = 0.135}$ (PLB 243, 181 (1990), Particles 3(2020) 29)

$\boxed{\mu_K = \mu_u - \mu_s}$ (see for example EPJ Web of Conferences 37, 09022 (2012)).

The theses were checked

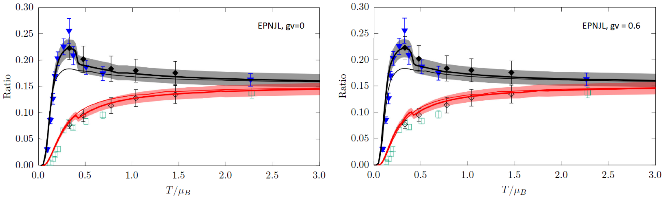
- Both K/π ratios are almost unaffected to the change of the order of the chiral phase from first order to crossover (but more sensitive to the slope of the phase diagram);





The theses were checked

- The sharpness of the "horn" is well explained by a Bose-enhanced pion production for $\sqrt{s_{NN}} > 8$ GeV;



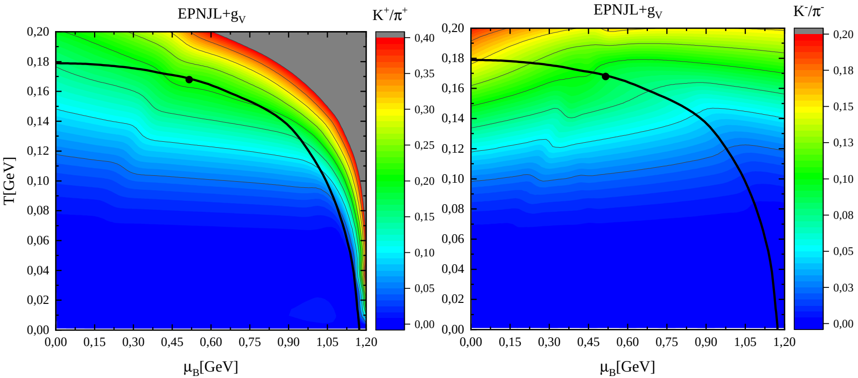
the non-equilibrium medium dependent chemical potential for pion and strange quark (Wood-Saxon form, $x = T/\mu_B$)

$$\mu_\pi(x) = \mu_\pi^{\rm min} + \frac{\mu_\pi^{\rm max} - \mu_\pi^{\rm min}}{1 + \exp(-(x - x_\pi^{\rm th})/\Delta x_\pi))},$$

$$\mu_s(x) = \frac{\mu_s^{\rm max}}{1 + \exp(-(x - x_s^{\rm th})/\Delta x_s))}.$$

parameters fitted to the experiment;

General overview



The baryon-to-pion ratio.

It is interesting to consider baryon-to-pion ratio in the PNJL model

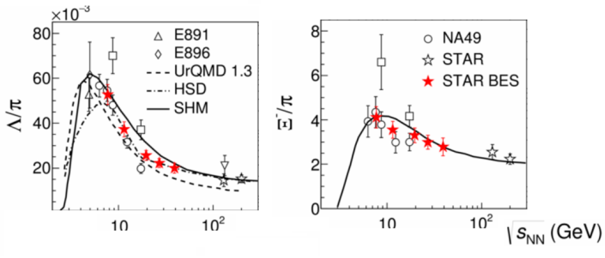


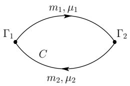
Figure 2: J. Cleymans, arXiv:0910.2128

Diquarks in PNJL model

Analogue for Bethe-Salpeter equation for diquarks (E. Blanquier J. Phys. G: Nucl. Part. Phys. 38 (2011) 105003):

$$1 - G_D \Pi_D^P(P_0 = M, \bar{P} = \bar{0}) = 0 \ ,$$

and the polarization operator:

$\Pi_{\rm diq} \equiv$ 

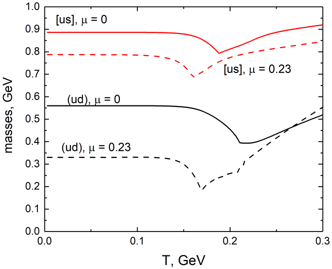
$$\Pi_{ij} = \int \frac{dp}{(2\pi)^4} tr\{S^{f_i}(\hat{q}_i, m_i)\Gamma S^{f_j C}(\hat{q}_j, m_j)\Gamma\},$$

$$\Pi_{ij} = \int \frac{dp}{(2\pi)^4} tr\{S^{f_i C}(\hat{q}_i, m_i)\Gamma S^{f_j}(\hat{q}_j, m_j)\Gamma\},$$

Γ	Meson type	Possible mesons	Diq. type	possible diq.
$i\gamma_5$	pseudoscalar	π, K	scalar	
1	scalar	σ, K_0^*	pseudoscalar	(ud), (us), (ds)
$\gamma^\mu i\gamma_5$	axial-vector	a_1^*, K_1^*	vector	
$i\gamma^\mu$	vector	ρ, K^*	axial-vector	[ud],[us],[ds],
				[uu],[dd],[ss]

The coupling constants for diquarks:

$$G_{\text{sc}}^{\text{diq}} = 0.7 - 0.75 G_{\text{s}},\, G_{\text{ax}} = G_{\text{sc}}^{\text{diq}}/4$$



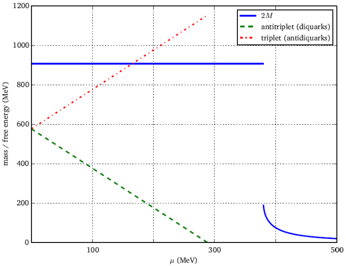
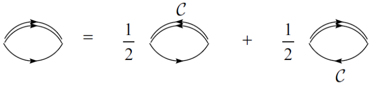


Figure 3: Diquark masses as function of T and μ_q and mass/free energy ratio (fig. taken from M. Buballa. Phys. Rept., (407):205?376,2005)

Baryons in PNJL model

Analogue for Bethe-Salpeter equation for baryons can be obtained supposing them quark-diquark system (E. Blanquier J. Phys. G: Nucl. Part. Phys. 38 (2011) 105003):

$$1 - Z\Pi_B^P(P_0 = M_B, \bar{P} = \bar{0}) = 0 \ ,$$



and the polarization operator (see for details Blanquier, Aichelin):

$$\Pi_{ij} = \int \frac{dp}{(2\pi)^4} tr\{S_q(\hat{q}_i, m_i)\Gamma S_D^C(\hat{q}_j, M_{Dj})\Gamma\},$$

$$\Pi_{ij} = \int \frac{dp}{(2\pi)^4} tr\{S_D(\hat{q}_i, M_{Di})\Gamma S_q^C(\hat{q}_j, m_j)\Gamma\},$$

Baryons in terms quark-diquark wave-functions

The earliest works showed that the baryon wave-function can be constructed in terms of quark-diquark wave-functions [Vogl:1991qt,Reinhardt1990

$$\Psi_{\text{octet}} \sim \sqrt{\frac{1}{2}}[(8_{mS}, 2_{mS}) + (8_{mA}, 2_{mA})], \quad (3)$$

where $8_{mS}, 8_{mA}$ - the mixed symmetric and mixed anti-symmetric octet in flavour space; $2_{mS}, 2_{mA}$ - spin symmetry.
The flavour octet wave-function is totally symmetric with respect to flavour $\otimes$ spin. The baryon masses can be computed as

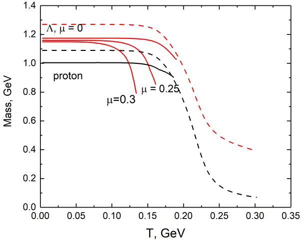
$$M(p) = \frac{1}{2}((ud) + [ud]) + m_u \quad (4)$$

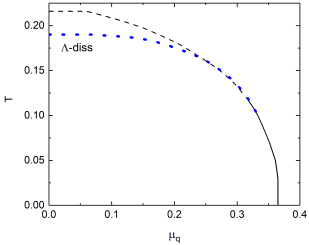
$$M(\Sigma) = \frac{1}{2}\left[(us) + \frac{1}{3}([us] + 2[ud])\right] + \frac{1}{3}(2m_u + m_s) \quad (5)$$

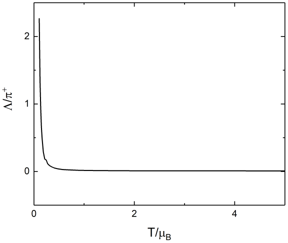
$$M(\Xi) = \frac{1}{2}\left[(us) + \frac{1}{3}([us] + 2[ss])\right] + \frac{1}{3}(2m_u + m_s) \quad (6)$$

$$M(\Lambda^0) = \frac{1}{2}\left[[us] + \frac{1}{3}((us) + 2(ud))\right] + \frac{1}{3}(2m_u + m_s), \quad (7)$$

The baryon dissociation: $T_{\text{dec}} = min\{T^{d}_{\text{bar}}, T^{Mott}_{\text{diquarks}}\}$.







Results and discussion

- Splitting of kaons masses at high densities $\Rightarrow$ the difference in the behaviour of the K/π at low energies;
- Both K/π ratios are almost unaffected to the change of the order of the chiral phase from first order to crossover (but more sensitive to the slope of the phase diagram);
- The sharpness of the "horn" is well explained by a Bose-enhanced pion production for $\sqrt{s_{NN}} > 8$ GeV;
- To describe the baryon-pion ratio the problem of description of the Bose condensate at finite density and correct description of diquarks and baryons should be solved (Blaschke, PoS,2014).


Спасибо
Gracias
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Obrigado
Спасибо
Dank U
Grazie
Ευχαριστώ
Danke
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Merci
Thank You
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Diolch
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Ngiyabonga
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