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Modified Fayans functional. Description of nuclear ground state properties and spin-isospin response.

Fine tuning of the Fayans energy density functional.
The constraints are given by nuclear g.s. properties and EOS.

Testing the calibrated functional
on isovector nuclear characteristics:
giant Gamow-Teller and spin-dipole resonances, magnetic moments e.t.c.

The aim.

Fine tuning of the Fayans functionals DF3-a, DF3-f .

Varying of previously unused ***isovector volume parameter h_{-2}*** ***.***

(There were not enough data for fitting before.)

The aim:

- *To study an impact of* ***h_{-2}*** *on EOS of the SNM and PNM, specifically on S(ρ) , L (ρ).*

The question is:

- *Would varying the h_{-2} allow us to meet the constraints obtained from the estimates of { J, L} parameters of nuclear EOS .*

These constraints has been obtained recently from:

1. *The neutron skin thickness of 208Pb, as measured in the PREX-II experiments*
   + "ab initio" χETF calculations of the nuclear g.s. properties
   + astrophysical observations :
   NS radii and gravitational observations data: LIGO, VIRGO, NICER .
2. *Additional condition: the energy of the giant E1 resonance 208Pb :*
   Ex = 14.2 ± 0.2 MeV

EOS for sub-nuclear matter, symmetric nuclear matter (SNM), nuclei , pure neutron matter (PNM) :

$$E(\rho_p + \rho_n, \delta) / A$$

Total energy / per nucleon as a function of

$\rho = \rho_p + \rho_n$ – *total barionic density ,*

$\delta = (\rho_p - \rho_n) / \rho$ - *isospin asymmetry*

SNM ($\delta = 0$) PNM ($\delta = 1$)

Nuclear EOS can be constructed microscopically from Energy -Density Functional (EDF) theory or χEFT…

In EDF approach, an equilibrium state of dense matter (if any) is found self-consistently (for each density ρ) by minimization of the functional $E(\rho_0)/A \rightarrow \min \{ \varepsilon (\rho, \delta)\}$

Fayans functionals FaNDF^0, DF3-a, DF3-f
The main feature: fractional density dependence

$$\mathcal{E} = \frac{3}{5}\varepsilon_p(\rho_p)\rho_p + \frac{3}{5}\varepsilon_n(\rho_n)\rho_n + \frac{C_0}{4}\left[a_+\frac{1-h_1^+(\rho/\rho_0)^\sigma}{1+h_2^+(\rho/\rho_0)^\sigma}\rho^2 + a_-\frac{1-h_1^-\rho/\rho_0}{1+h_2^-\rho/\rho_0}\rho^2\delta^2\right].$$

DF3-a and DF3-f - have less parameters than in FaNDF^0 and fractional-linear density dependence , σ =1 (linear Pade approximant).

Here ***a*** $^{+,-}$ ***h*** $^{+,-}_{1,2}$ *are iso-scalar (-vector) parameters of volume part of* $\mathcal{E}$.

Notice that, for ***h -$_2$ = h+$_2$=0,*** *the volume part reduces to the form of Skyrme EDFs.*
Previously - not enough data for constraining the ***h -$_2$ in DF3 family*** *.*

1. *FANDF0 S.A. Fayans, JETP Lett. 68 (1995)*
2. *DF3-a E.E. Saperstein, S.V. Tolokonnikov Phys.At.Nucl. 77 (2010).*
3. *DF3-f I.N.B, S.V. Tolokonnikov Phys. At. Phys. At. Nucl. 83 (2019).*

Here, $\rho_0 = 2(k_F^0)^3/3\pi^2$ being the equilibrium symmetric nuclear matter density.

The coefficient $C_0 = (dn/d\varepsilon_F)^{-1} = \pi^2/(k_F^0 m)$ is the inverse density of states at the Fermi surface.

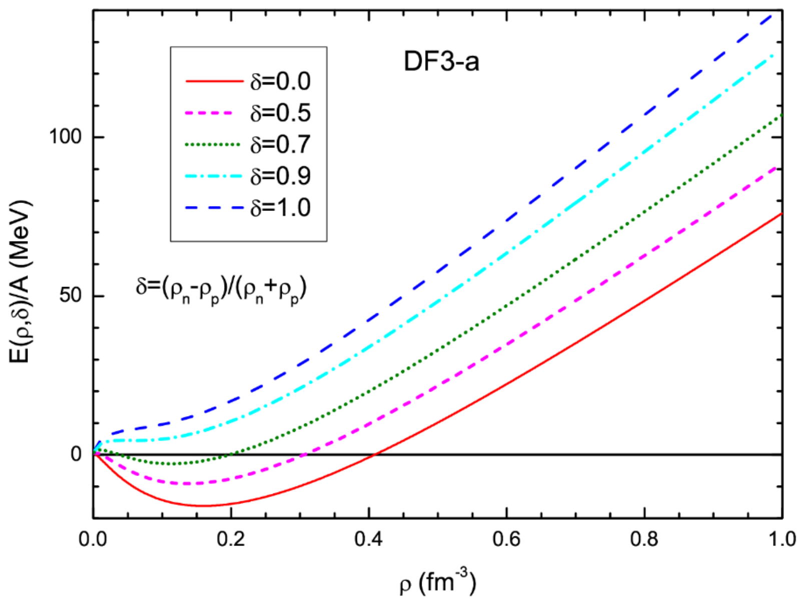
The power parameter $\sigma = 1/3$ is chosen in the FaNDF^0 functional, in contrast to the case of DF3-f, where $\sigma = 1$.

DF3-a
σ =1

$E(\rho, \delta)/A = E(\rho_p, \rho_n)/\rho$

$$E(\rho,\delta)/A = \varepsilon_{0\mathrm{F}} \left\{ \frac{3}{10} \left(\frac{\rho}{\rho_0}\right)^{2/3} \left[(1-\delta)^{5/3} + (1+\delta)^{5/3}\right] + \right.$$

$$\left. + \frac{1}{3} a_- \frac{1 - h_1^-(\rho/\rho_0)}{1 + h_2^-(\rho/\rho_0)} \left(\frac{\rho}{\rho_0}\right) \delta^2 \right\} .$$



δ<< 1

$$E(\rho,\delta)/A = E_{SNM}(\rho)/A + S(\rho)\delta^2 + \ldots ,$$

$$S(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho,\delta)/A}{\partial \delta^2} |_{\delta=0}$$

$$S(\rho) = \frac{1}{3} \varepsilon_{0\mathrm{F}} \left[\left(\frac{\rho}{\rho_0}\right)^{2/3} + a_- \frac{1 - h_1^-(\rho/\rho_0)}{1 + h_2^-(\rho/\rho_0)} \left(\frac{\rho}{\rho_0}\right) \right]$$

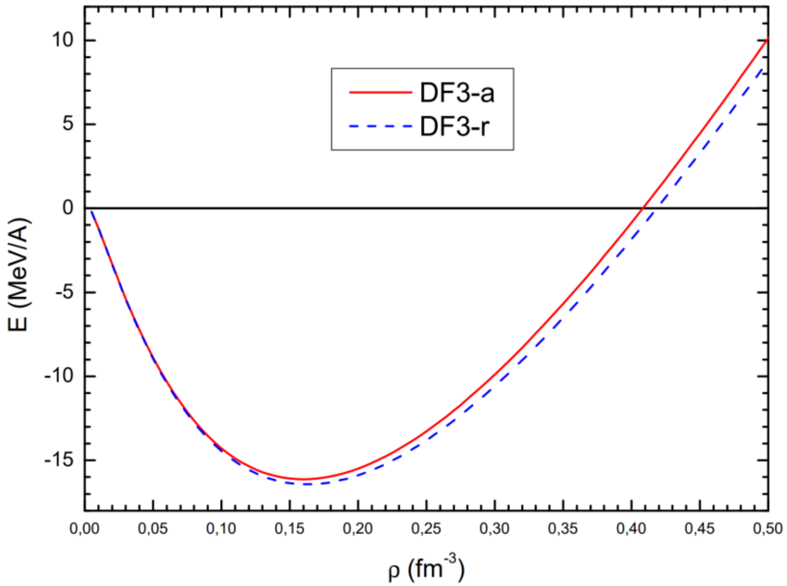
RELATIVISTIC CORRECTION for EOS
(valid for $y = h^* k_F / mc \sim 1$)

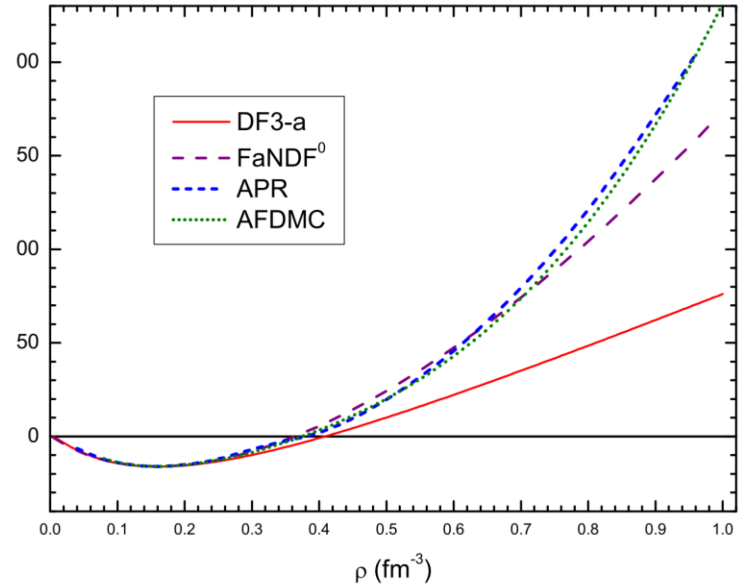
$E_{kin}(\rho) / A = mc^2 [\, g(y) / 8y^3 - 1]$

$g(y) = 3y(2y^2 - 1) \sqrt{(1 + y^2)} - 3 \ln \sqrt{(y + 1 + y^2)}$

$y = h^* k_F / mc \sim \rho/2$

Here $k_F^3 / 3\pi^2 = \rho/2$ *(SNM) ;*





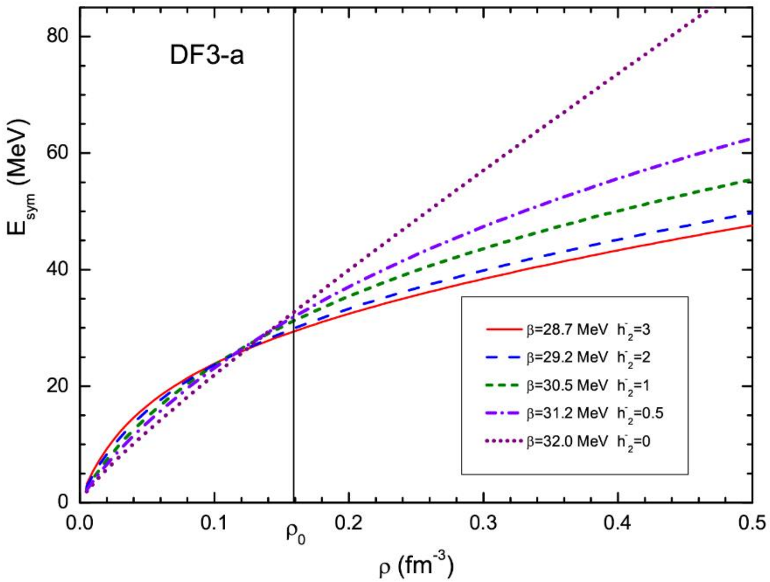
FANDF0 S.A. Fayans, JETP Lett.68 (1998).

DF3-a S.V. Tolokonnikov, E.E. Saperstein, Phys.At.Nucl (2010).

APR A.Akmal, V.R. Panharipande, D. Ravenhall Phys.Rev. C59 (1998).

AFDMC S. Gandol, A. Yu. Illarionov, K. E. Schmidt, F. Pederiva, and S. Fantoni, Phys. Rev. C 79, 054005 (2009).

$$E_{sym}(\rho) = E(\rho,1)/N - E(\rho,0)/A = \varepsilon_{0F}\left\{\frac{3}{5}\left(2^{2/3}-1\right)\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{1}{3}a_{-}\frac{1-h_1^{-}(\rho/\rho_0)}{1+h_2^{-}(\rho/\rho_0)}\left(\frac{\rho}{\rho_0}\right)\right\}. \quad (7)$$



DF3-a: J=S(ρ_0) ~ 30.0 MeV
E_{sym} near ρ_0 is not that sensitive to h^-_2

$$L(\rho) = 3\rho\frac{\partial E_{sym}(\rho)}{\partial\rho}$$

L= L (ρ_0) - "slope parameter" of E_{sym} (ρ) is known to be (linearly) correlated with
ΔRnp - neutron skin thickness !

The "slope" L could be derived from ΔRnp (208Pb) .
*Parity violating (e,e') data - Jefferson Lab exp. **PREX-II***

ΔRnp (^{208} Pb) = 0.283 ± 0.071 fm → L = 106 ± 37 MeV

D. Adhikari et.al. Phys.Rev.Lett 126, 172502 (2021)

PREX-II data **$L = 106 \pm 37$ MeV** , **ΔR_{np} (^{208}Pb) = 0.283 ± 0.071 fm**

D. Adhikari et.al. Phys.Rev.Lett 126, 172502 (2021)

are in tension with the set of nuclear structure and astro-data:

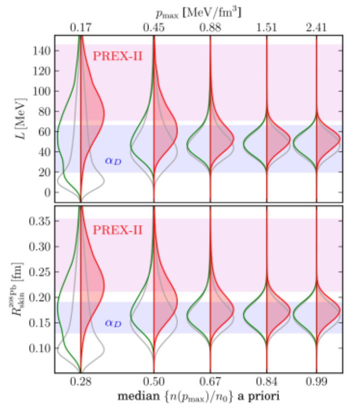


FIG. 2. Prior (gray, unshaded), Astro posterior (green, left-unshaded), and Astro + PREX-II posterior (red, right-shaded) distributions for L (top) and R_{skin}^{208Pb} (bottom) as a function of the maximum pressure (top axis) or density (bottom axis) up to which we trust theoretical nuclear-physics predictions from χEFT (see text for details). Shaded bands show the approximate 68% credible region from PREX-II [19] (pink) and from Ref. [13] based on the electric dipole polarizability α_D (light blue).

ETTERS **127,** 232501 (2021)

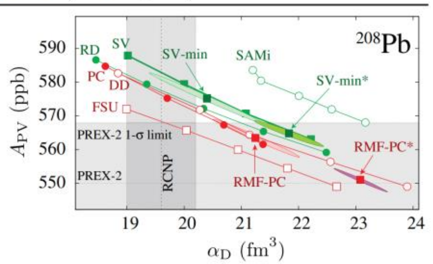


FIG. 2. A_{PV} versus α_D in ^{208}Pb for a set of covariant (red) and nonrelativistic (green) EDFs. Sets with systematically varied symmetry energy J are connected by lines. (Note that α_D increases as a function of J.) The SV-min, SV-min*, RMF-PC, and RMF-PC* results are shown together with their 1-sigma error ellipses. The experimental values of α_D [8,13] and A_{PV} [1] are indicated together with their 1-sigma error bars.

1. *Constraints based on PREX-II, NS masses, LIGO/Virgo, NICER + χEFT+ αD:*

$L = 59 \pm 16$ MeV

ΔR_{np} (208Pb) = 0.19 ± 0.07fm

J. Lattimer at S@INT, Seattle, 2021.

2.*Analysis of on-parametric EOS with Gauss Processes*

L = 49+14-15 MeV,

ΔR_{np}(208) = 0.17 ± 0.004 fm

J = 32.7+1.9-1.8

R.Essick , I. Tews , P. Landry , A. Schwenk, P.R.L 127, (2021).

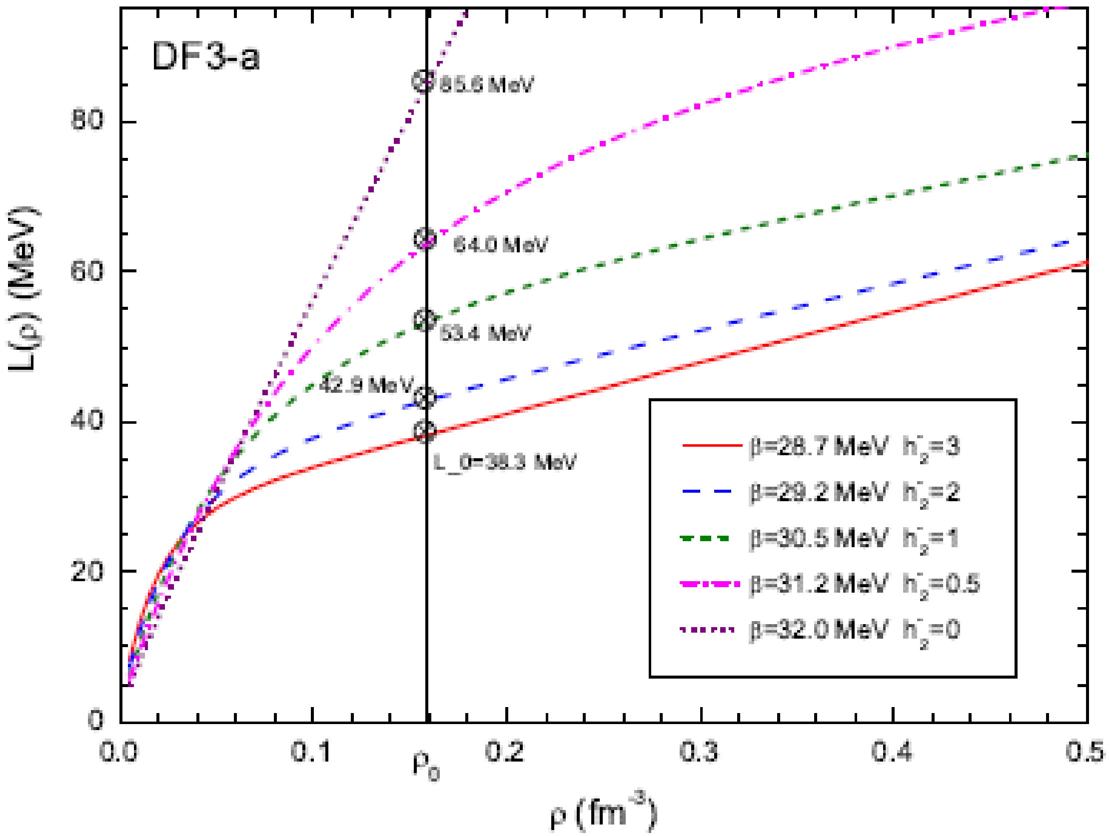
3. *Skyrme (SV, DD, PC, SAMi) calibrated on B(A,Z), Rch and αD.* ***If the quantified value of A_{PV} + αD were added :***

$L = 54 \pm 8$ MeV,

ΔR_{np}(208) = 0.19 ± 0.02 fm

P-G. Reinhard et.al . Phys.Rev.Lett. 127 (2021)

L (ρ, h2-)

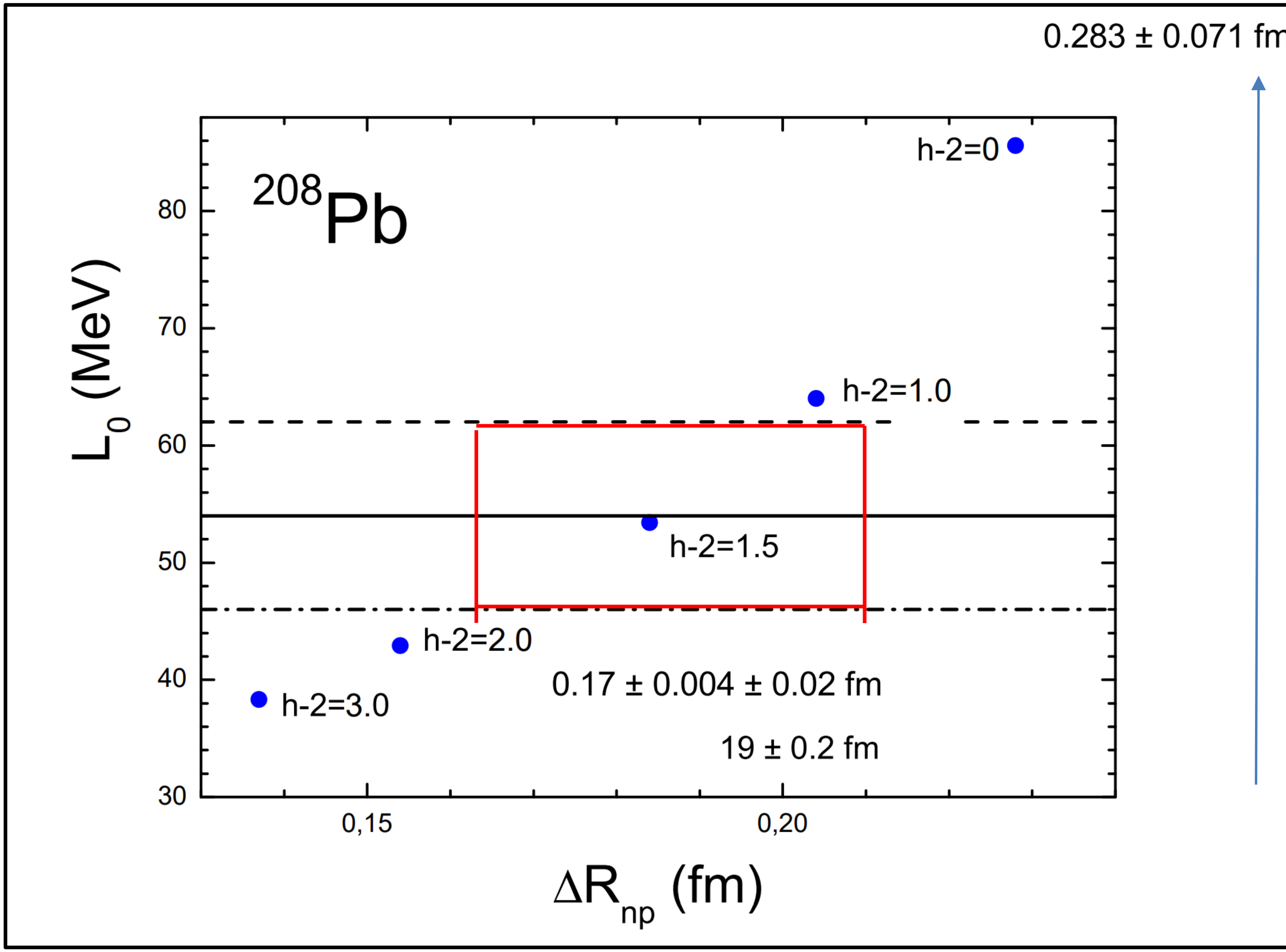


L =59 ± 16 MeV → ***h-2 = 0.5 - 2.0***

J. Lattimer in "S@INT,2021

L =58 ± 8 MeV → ***h-2 = 1.0 – 2.0***

P-G. Reinhard et.al . Phys.Rev.Lett. 127 (2021)



*L and ΔRnp (PREX II) = 0.283 ± 0.071 fm - **no match***

$L(\rho_0)$ = 58 ± 8 MeV

ΔRnp (PREX+Astro+χETF) = 0.19 ± 0.2 fm

*can be met at **$1.0 < h_{2-} < 2.0$***

Giant electric dipole resonance E (GDR,208Pb) = 14.2 + - 0.2 MeV
DF3-f

Таблица I: Расчет с функционалом DF3 для различных значений параметра h_2^-.

$\omega_{GDR} = \sqrt{m_3/m_1}$, m_1, m_1 — первый и третий моменты силовой функции GDR.

h_2^-	β (MeV)	f_{in}^-	f_{ex}^-	f_{surf}^-	ω_{GDR} (^{208}Pb) (MeV)	$L(\rho_0)$ (MeV)	ΔR_{np} (^{208}Pb) (fm)	ΔR_{np} (^{48}Ca) (fm)
0	32.0	0.808	0.808	0.808	12.80	85.6	0.228	0.192
0.5	31.2	0.775	1.163	0.969	13.37	64.0	0.204	0.180
1	30.5	0.747	1.494	1.115	13.73	53.4	0.184	0.170
2	29.2	0.694	2.080	1.387	14.11	42.9	0.154	0.154
3	28.7	0.673	2.693	1.687	14.41	38.3	0.137	0.143

h-2 = 1.0 -1.5

For h-2 = 1.5 **Rnp (208Pb) = 0.171 fm**
208 Pb *Estimated* Rnp = 0.17 ± 0.004 fm
Ex th E1 = 14.0 MeV
Ex exp E1 = 14.2 ± 0.2 MeV

For h-2 = 1.5 **Rnp (48Ca) = 0.160 fm**
48 Ca *CREX Rnp = 0.121 + 0.026 - 0.024*
p+(A,Z) Rnp= 0.158 ± 0.023±0.012 (T.Wakasa et al)

FANDF^0 *Rnp (208Pb) = 0.134 fm*

FANDF^0 *Rnp (48Ca) = 0.154 fm*

Conclusion - I

- *Previously unused isovector volume parameter h_{-2} of DF3-a, f functionals is varied. We keep the same quality of the DF3 fits to densities, nuclear masses, single-particle levels, charge radii.*

- *The slope parameter* ***$L=L(\rho_0)$*** *of symmetry energy $S(\rho)$ is found to be very sensitive to h_{-2} .*

- *h-2 can be fixed within rather narrow interval of* ***1 .0 – 1.5***
- ***in order to simultaneously describe***

- the nuclear EOS parameters {J, ***L}*** *estimated by P-G Reinhard (PRL127 2021);*
- together with the experimental energy of the E1 giant dipole resonance.

- *Notice, that for such h-2, the neutron skin ΔRnp (208Pb) found in the PREX II*
- *can't be described*

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Nuclear charge-exchange response within modified Fayans functional.

Applying the Fayans functional (calibrated by constraints from the giant E1 resonance and EOS) to the GT and spin-dipole resonances.

INFINUM -2023, JINR, Dubna 27.02-03.03. 2023

Nuclear charge-exchange response.

The EDF is the same as in the
$E(\rho, \delta)/A$

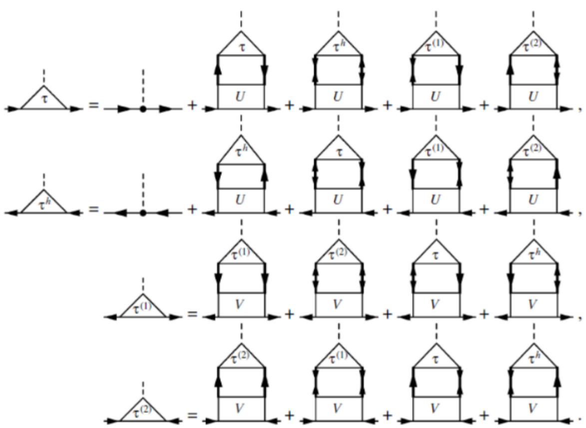
$$F^{\omega}{}_{\tau\tau} = \frac{\delta^2 E}{\delta\rho^{\tau}\delta\rho^{\tau}} \qquad \text{IAR} \qquad F^{\xi}{}_{\tau\tau} = \frac{\delta^2 E}{\delta\nu^{\tau}\delta\nu^{\tau}}$$

GT , SD

$$F^{\omega} = F_0 + F_{\pi} + F_{\rho}$$

$$F^{\xi} = g'_{\xi}\,(\tau_1\tau_2)$$

(T=0, pn-dynamic pairing)



Full basis CQRPA

$$\mathcal{F}^{\omega} = \mathcal{F}_0 + \mathcal{F}_{\pi} + \mathcal{F}_{\rho}.$$

where $C_0 = (dn/d\varepsilon_F)^{-1} = 300$ MeV fm^3 is a normalization constant, $\boldsymbol{\tau}$ are the isospin Pauli matrices, and g and g' are phenomenological parameters (the latter is known as the Migdal force).

The pion-exchange term has the form

$$\mathcal{F}_{\pi} = -\frac{4\pi \tilde{f}_{\pi}^2}{m_{\pi}^2}\frac{(\boldsymbol{\sigma}_1\mathbf{k})(\boldsymbol{\sigma}_2\mathbf{k})}{m_{\pi}^2 + k^2 + \Pi_{\Delta}}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2),$$

The rho-meson term is taken in the form [24]

$$\mathcal{F}_{\rho} = \frac{4\pi \tilde{f}_{\rho}^2}{m_{\rho}^2}\frac{[\boldsymbol{\sigma}_1 \times \mathbf{k}][\boldsymbol{\sigma}_2 \times \mathbf{k}]}{m_{\rho}^2 + k^2}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2),$$

C-RPA. Gamow-Teller Resonance in 208Pb revisited.

In pnRPA GTR max.energy is defined by the balance of the repulsive and attractive components of the amplitude F :

1) repulsion
$g'_{ph} > 0$
Notice: $\Delta g' \sim \Delta \rho_{nucl}$

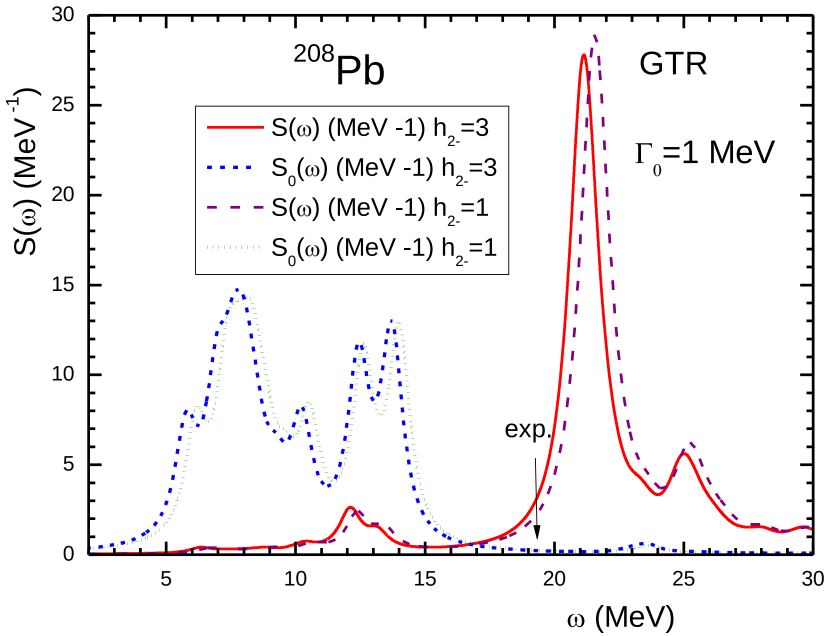
2) attraction
$f\pi < 0$ ($f\rho > 0$)
g'critical π –cond.~ 0.6
RPA collapse

$g'_{ph} \sim 1.1$ ~~$g'_{pp} \sim -0.3$~~
*C_0=300 MeV*fm^3*

$f\pi = -1.45\,(1-2\xi_{s\pi})^2$
$f\rho = 2.64\,(1-2\xi_{s\rho})^2$
$\xi_s = \xi_{s\pi} = \xi_{s\rho}$

New
h-2 changes volume isovector part of DF3-f

It shifts the GT max. downward



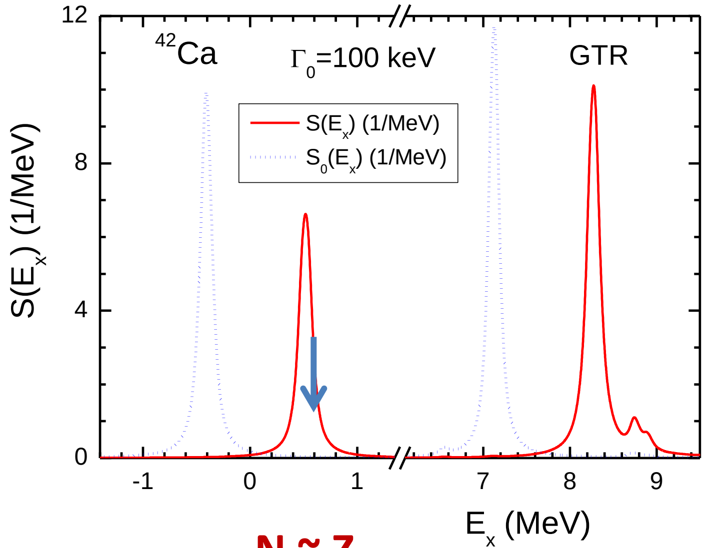
The result is:
Δω (th – exp) ~ +1.5 -1.8 MeV
for h-2 ~ 1.0 -1.5

The rest ~ - 1.5 MeV are "reserved"for the QPC effect Δω < 0 which is not included

Nuclei with pairing. Low-energy Super GT state in 42Ca

Y. Fujita, H. Fujita, et.al.
Phys.Rev.Lett. 112, 112502 (2014}
GT transition strength can also be concentrated in the lowest Jπ = 1+ GT state
Low-energy Super GT (LeSGT) state.
SU(4) –symmetry.

Initial even- even nuclei *have the structure of LS-closed-shell core nucleus + 2 neutrons (or 2 protons)" : they are* ***either Tz = +1 or –1 mirror nuclei*** *, the final nuclei - Tz = 0*
40 Ca core → 42Ca (p,n)42Sc.



N ~ Z

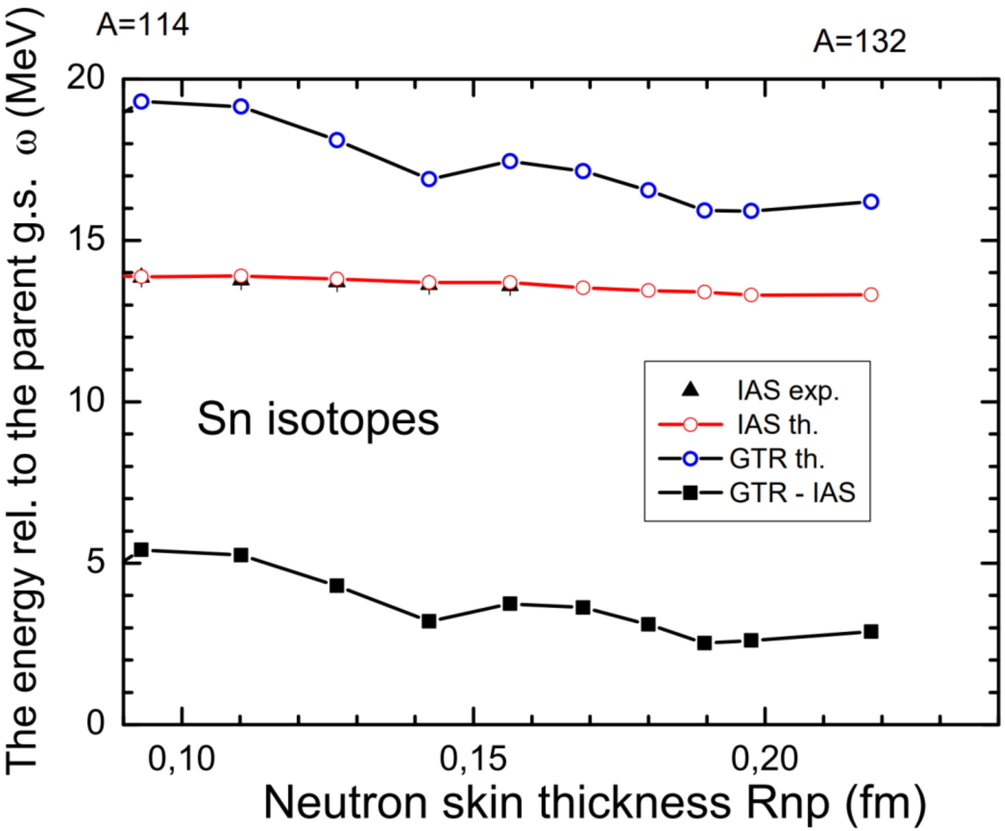
1nf7/2->1pf7/2 , ***1nf7/2->1pf5/2***

LeSGT strength is sensitive both to T=0, S=1 dynamic pairing (g'_ksi= - 0.3)
and to h-2 parameter

In (3He,t) reaction
Y. Fujita, H. Fujita, et.al.
Phys.Rev.Lett. 112, 112502 (2014}
No "sharp" GTR found at Ex<12 MeV … !
Strong fragmentation of GTR. ->
Background problems – multistep processes.

For GTR quasiparticle-phonon coupling is important!

E_{GTR}-E_{IAR} is indirectly related with Rnp and J, L



The energies of giant resonances calculated within the ***same E/A from DF3-f*** *together with <r2>.*
(and Q_2, μ…)

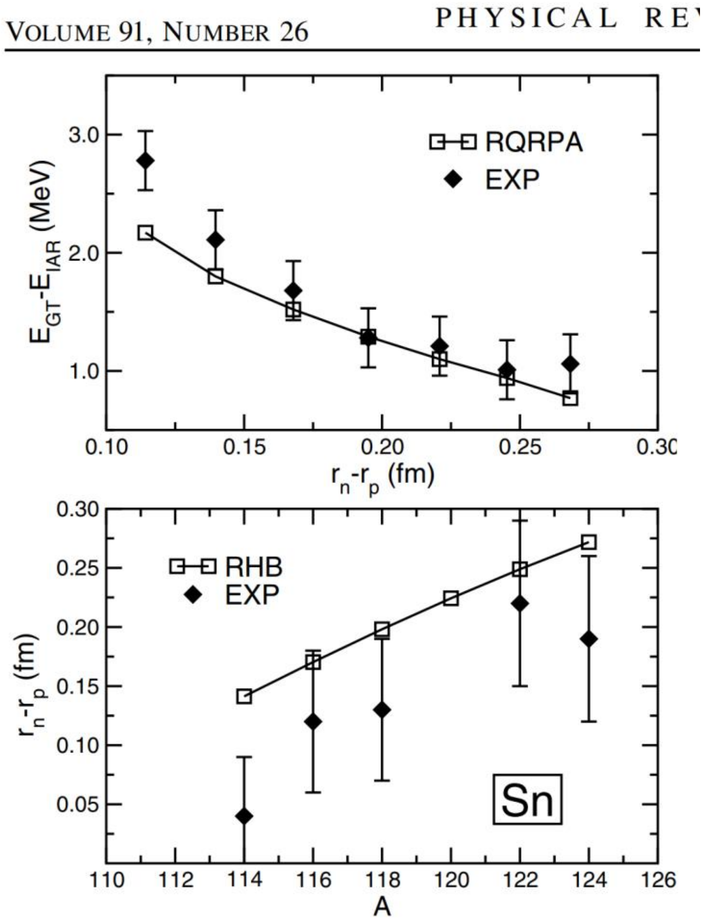


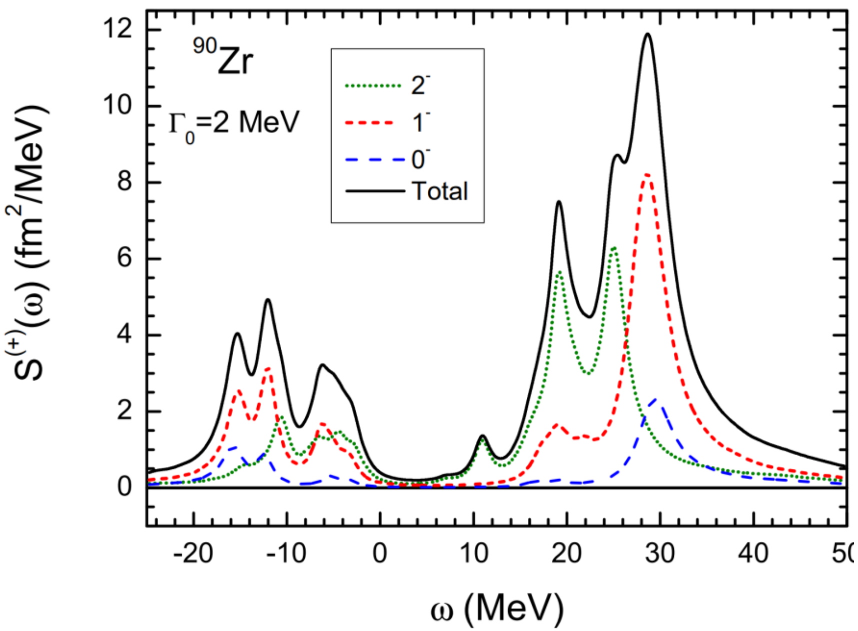
FIG. 2. The proton-neutron RQRPA and experimental [22] differences between the excitation energies of the GTR and IAS as a function of the calculated differences between the rms radii of the neutron and proton density distributions of even-even Sn isotopes (upper panel). In the lower panel the calculated differences $r_n - r_p$ are compared with experimental data [4].

D. Vretenar et.al.

Neutron skin thickness of 90Zr and symmetry energy constrained by charge exchange spin-dipole excitations

$$\hat{S}_\lambda^{\pm} = \Sigma_i\ \tau_i^{\pm} r_i\ [\sigma \times Y_1(\hat{r}_i)]_\lambda, \quad \Delta L=1\ \ \Delta J=0\text{-}2, \quad 90Zr \quad 0^-, 1^-, 2^-$$

$$S_- - S_+ = \Sigma_\lambda (2\lambda+1)/4\pi\ \left(N\langle r^2\rangle_n - Z\langle r^2\rangle_p \right) = 9/4\pi\ \left(N\langle r^2\rangle_n - Z\langle r^2\rangle_p \right)$$



y the charge... Chin. Phys. C **47**, 024102 (2023

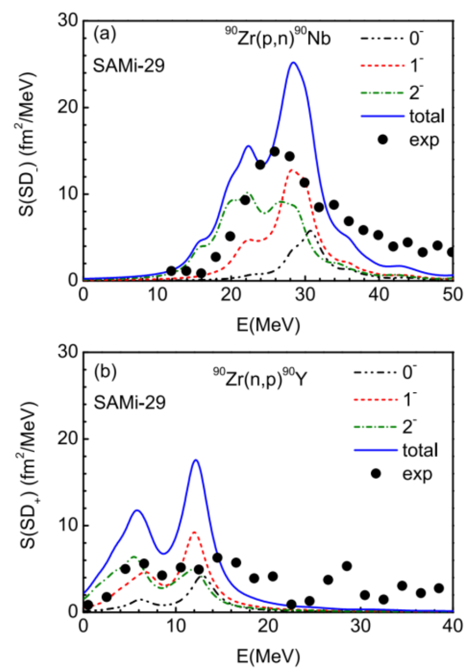


Fig. 1. (color online) SD strength distributions for S(SD$_-$) (a) and S(SD$_+$) (b) calculated in the *pn*-RPA with SAMi-29 interactions. The $\lambda^\pi = 0^-, 1^-, 2^-$ components and the total strengths are shown. The experimental data obtained from Refs. [42, 43] are shown as black symbols.

$$\sigma_{exp}(\theta,\omega) = \Sigma_{\Delta J\pi}\ a_{\Delta J\pi}\ \sigma_{DWIA\ \Delta J\pi}(\theta,\omega)$$

σ_{DWIA} - calculated

$a_{\Delta J\pi}$ fitted to exp. Σ

A constraint from the extracted Rnp leads to
J = 29.2 ± 2.6 MeV and L = 53.3 ± 28.2 MeV.

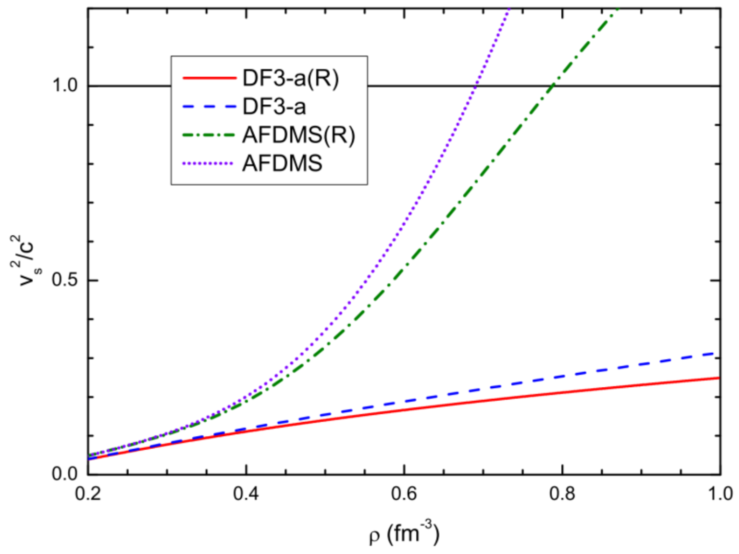
Conclusions - II

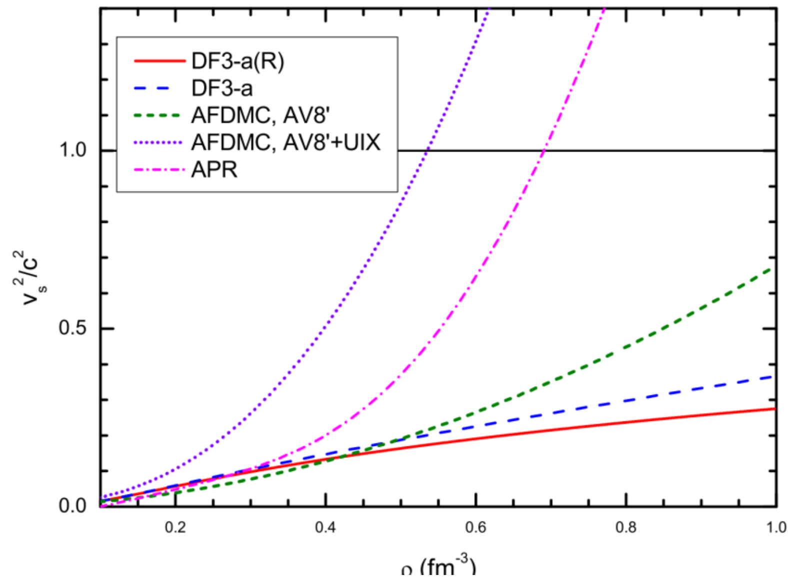
The DF3-a,f functionals are calibrated by the constraints imposed by nuclear EOS and Rnp values .

- *Using the newly fitted volume isovector parameter h-2 improves description of the Gamow-Teller resonances.*
- *Additional constraints on the neutron-skin thickness Rnp can be obtained, in principle, from spin-dipole resonance sum rule.*
- *The EOS consistent with the DF3-a,f functionals calibrated in such a way will be used for modeling of neutron star mergers (see A.V. Yudin et.al).*

Acknowledgments

V_s^2 / c^2 SNM



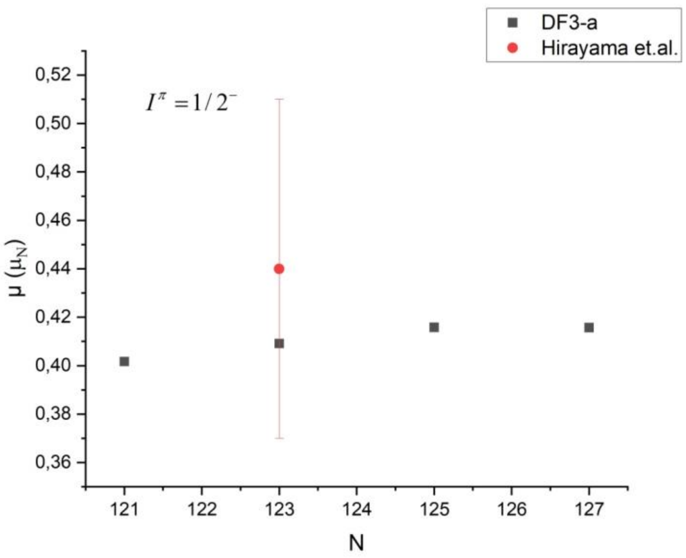


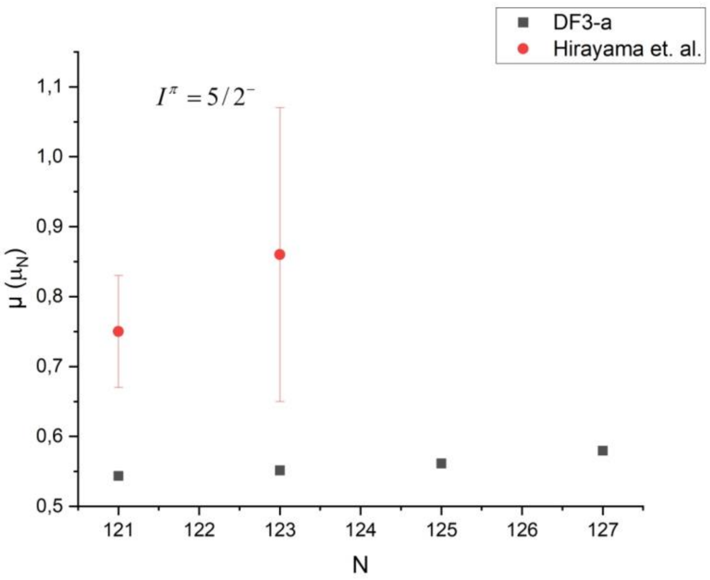
Ratio of hydrodynamic sound speed to speed of light for DF3-a , FANDF0, APR and AFDMC. (R) - with rel. correction incl.

APR A.Akmal, V.R. Panharipande, D. Ravenhall Phys.Rev. C59 (1998)

AFDMC S. Gandolfi, A. Yu. Illarionov, K. E. Schmidt, F. Pederiva, and S. Fantoni, Phys. Rev. C 79, 054005 (2009); D. Lonanrdoni et.al.

$_{78}$Pt isotopes
Magnetic moments for allowed set of {J/π} g.s.
Prediction within DF3-a.





For uniform system few useful EOS parametrizations exist.
The simplest one is a quadratic expansion on δ^2
Valid at $\delta \ll 1$, $\rho < 2\rho$

$$E(\rho,\delta)/A = E_{SNM}(\rho,0)/A + S(\rho)\delta^2 + \ \ldots$$

Symmetry energy → $S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)/A}{\partial\delta^2}|_{\delta=0}$

Expansion parameters J, L near equilibrium density
$\rho_0 = 0.164(7)$ fm-3

$J = S(\rho_0)$ - symmetry energy parameter,

$L = 3\rho \frac{\partial}{\partial\rho} Esym(\rho)_{|\rho0}$ - gradient (or slope) parameter

L is correlated with ΔRnp - neutron skin

J, L are derived from the nuclear properties : masses, charge radii …
and astrophysical measurements .
Accuracy is still insufficient.

GTR max. energy in pnQRPA is defined by :

g'ph>0 (repulsion) g'pp<0 and fπ<0 (attraction);

Additional effects:

h-2 < 0 (mostly Landau fragmentation)

Beyond QRPA effect of qp-Phonon – coupling is more important

In R-QRPA attraction may also be induced by time-like part of isovector-pseudovector coupling

$-\alpha_{PV} < 0$

$$V_{PV} = -\alpha_{PV}\delta(\vec{r}_1 - \vec{r}_2)\left(\gamma_0\gamma_5\gamma_\mu\vec{\tau}\right)^{(1)}\left(\gamma_0\gamma_5\gamma^\mu\vec{\tau}\right)^{(2)}$$

$$V_{PV} \sim -\alpha_{PV}\,\Sigma_L\,[\sigma_S\,Y_L]_J\,\vec{\tau}$$

D. Vale, Y.F.Niu, N.Paar, Phys.Rev. C 100, (2022)

qp-Phonon coupling was not included,

But still E (GTR) was adjusted to exp.energy w(208Pb)=19.2 MeV