INFINUM-2023, Dubna 27/02-03/03 2023

SOUND PROPAGATION IN a NEUTRON STAR with QUARK MATTER DROPLETS

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① Preliminary Remarks

Challenges at the intersection of quark matter (QM) and neutron stars (NSs) phenomena: $\;p = p(\varepsilon, T) \to p_{NS}(\varepsilon)$

(i) Derivation of the Equation of State (EoS) with hadron and quark degrees of freedom. Hadron-to-quark matter (HM-QM) transition

(ii) Determination of the speed of sound c_s behavior in the context of revolutinary discoveries of two-solar mass $2M_\odot$ NSs and merger events (breaking the conformal barriere $c_s^2 = 1/3$)

A recent breakthrough with problems (i) and (ii) has been a motivation for the present research:

E. Annala, T. Gorda, A. Kurkela, J. Nattila, A. Vuorinen
Nature Phys. 16 (2020) 907
>300 citations

An ensemble of 570 000 EoS s has been analyzed
Fits astrophysical multimessenger observations
Interpolation of the speed of sound $c_s^2 = \partial p / \partial \varepsilon$ in NS matter

The two-polytrope EoS which interpolate between the low- and high density limits

$$\gamma = \frac{d(\ln p)}{d(\ln \varepsilon)} = c_s^2 \, \frac{\varepsilon}{p} : \quad \begin{array}{ll} \gamma \lesssim 1.75 & QM \\ \gamma \gtrsim 2 & HM \end{array}$$

$\gamma = 1.75$ the deviding line between HM and QM
Transition at $\varepsilon \approx 400-700$ MeV/fm^3

(Textbook reminder $pV^\gamma = \text{const}$, $p = \text{const}\, \rho^\gamma$, $\left(\underset{\text{conformal}}{\varepsilon = 3p,\ \gamma = 1,\ c_s^2 = 1/3}\right)$)

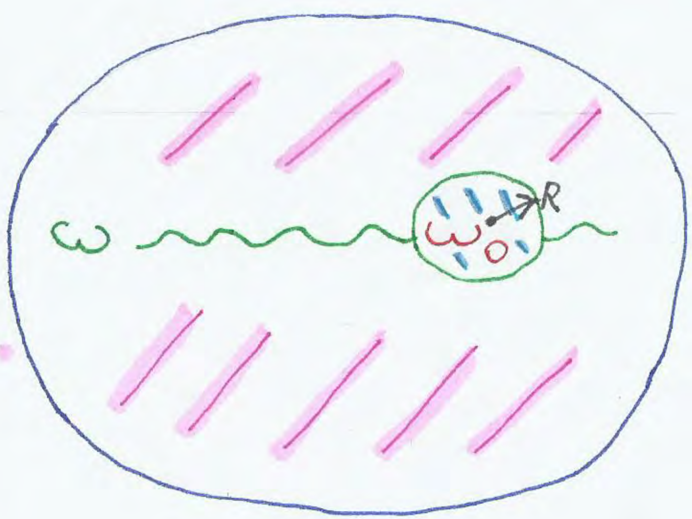
② QM droplet in HM bulk

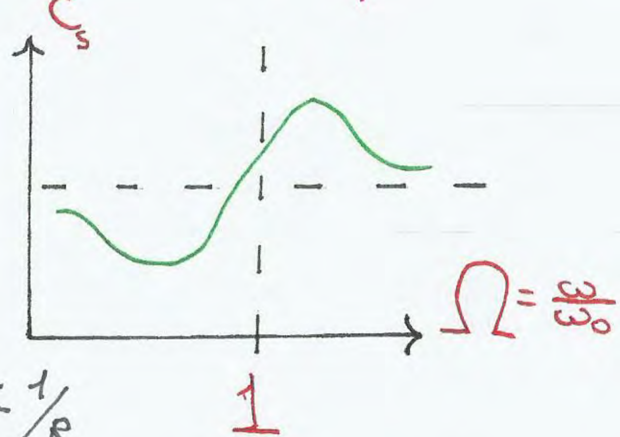
With two-polytrope EoS at hand two scenarious can be considered:

(i) QM drop inside the HM bulk

(ii) HM drop inside the QM bulk

Our focus is on (i) here. A number of authors contributed in one way or another: Glendenning, Pethick, Mishustin, Voskresensky, Shuryak, Lugones, Fogaca, ... Nucleation, percolation, spinodal decomposition. Our focal point is anomalous sound dispersion





$\omega_0 \propto 1/R$

How to describe the QM droplet in HM?

Lord Rayleigh suggested an answer 105 years ago.

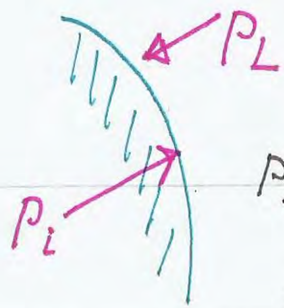
Rayleigh bubble equation — cavitation damage of ship propellers

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = 0, \quad \dot{R}(t) \sim (t_c - t)^{-3/5}$$ collapse

Oversimplified, only inertia forces

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(p_L - p_0 - P(t)\right)$$ Rayleigh-Plesset

bubbles of gas in liquid



$p_m \sin\omega t$
acoustic wave

$$p_L = p_i - \frac{2\sigma}{R} - 4\eta\frac{\dot{R}}{R}$$

surfac tension
depends on 1-st order, or crossover

shear viscosity

η(NS) subject to uncertainty

Yakovlev, Kolomeitsev
Shternin, Voskresensky
Shuryak, B.K.

- Bubble dynamics is a vast subject. Advanced eq-ns: Nolting-Neppiras, Prosperetti-Lezzi, relativistic R-Pl. Several monographs, $\sim 10^2$ papers. A lot of applications including QCD (Fogaca et al., Kharzeev-Shuryak), sonoluminescenting bubbles, collapse in QCD.

- Our task is to start from R-Pl eq-n and derive the <u>sound dispersion</u> pattern. Within the <u>two-polytrope</u> picture we have to define the physical parameters (p, ρ, η, R_0), insert them in the eq-ns and see what comes out of it.

Sound velocity and dispersion from R-Pl eq-n

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_0}\left(P_L - P_0 - P(t)\right)$$

$p_m \sin\omega t$
acoustic pressure

$\left.\begin{matrix}\rho_0 \\ P_0\end{matrix}\right\}$ ambient out of bubble

$$P_L = P_i - \cancel{\frac{2\sigma}{R}} - 4\eta\frac{\dot{R}}{R}$$

p_i

shear viscosity

Polytrope for QM bubble $\rightarrow P_i = P_{i0}\left(\frac{R_0}{R}\right)^{3\gamma} \simeq P_{i0}\left(1 - 3\gamma\frac{x}{R_0}\right)$

$x = R - R_0 \quad \gamma = 1.5 < \gamma_c = 1.75$

From R to $V = \frac{4}{3}\pi R^3$

$V = V_0 + z\,, \quad V = \left(\frac{V_0 + z}{V_0}\right)^{-\gamma}$

$$\ddot{z} + d\,\dot{z} + \omega_0^2 z = -\frac{4\pi R_0}{\rho_0} p_m \sin\omega t$$

oscillations with damping and external periodic driving

$d = \frac{4\eta}{\rho_0 R_0^2} \quad \omega_0^2 = \frac{3\gamma P_{i0}}{\rho_0 R_0^2}$

Dimensions $[\eta] = m^3 = \frac{g}{cm \cdot s}$

$[\rho] = m^4 \quad [d/\omega_0] = fm/R_0$

$\eta \sim 10^{15}\,\frac{g}{cm \cdot s}$ Kolomeitsev Voskresensky

η subject to uncertainties

$$\ddot{z} + \frac{4\eta}{\rho_0 R_0^2}\dot{z} + \frac{3\gamma p_{i0}}{\rho_0 R_0^2} z = -\frac{4\pi R_0}{\rho_0} p_m \sin\omega t$$

We need c_s^2 in terms of compressibility $\varkappa$

$$\varkappa = -\frac{1}{V}\frac{dV}{dp} = -\frac{\rho}{m}\frac{d(m/\rho)}{dp} = \frac{1}{\rho}\frac{1}{c_s^2} \rightarrow c_s = 1/\sqrt{\rho\varkappa}$$

$$z = Ae^{i\omega t}e^{i\varphi} = \tilde{z}e^{i\omega t}, \quad Ae^{i\omega t}e^{i\varphi}(-\omega^2 + i\omega d + \omega_0^2) = -p_m\frac{4\pi R_0}{\rho_0}$$

$$\left\{\begin{aligned} \tilde{z} &= Ae^{i\varphi} = -p_m \frac{V_0}{\gamma p_{i0}} \frac{1}{(1-\Omega^2 + i\delta\Omega)} \\ \varkappa &= -\frac{1}{V_0}\frac{\tilde{z}}{p_m} \end{aligned}\right\} \Rightarrow \varkappa = \frac{1}{\gamma p_{i0}} \frac{1}{(1-\Omega^2) + i\delta\Omega}$$

Two key parameters $\Omega = \frac{\omega}{\omega_0}$, $\delta = \frac{d}{\omega_0}$

$d = \frac{4\eta}{\rho_0 R_0^2}$ the viscous friction

$$\frac{\omega}{\omega_0} \sim \frac{R_0}{\lambda}$$

The Results

Mixed phase with $\beta = \dfrac{V_0}{V_{HM} + V_0}$, or $\beta = \dfrac{N V_0}{V_{HM} + N V_0}$

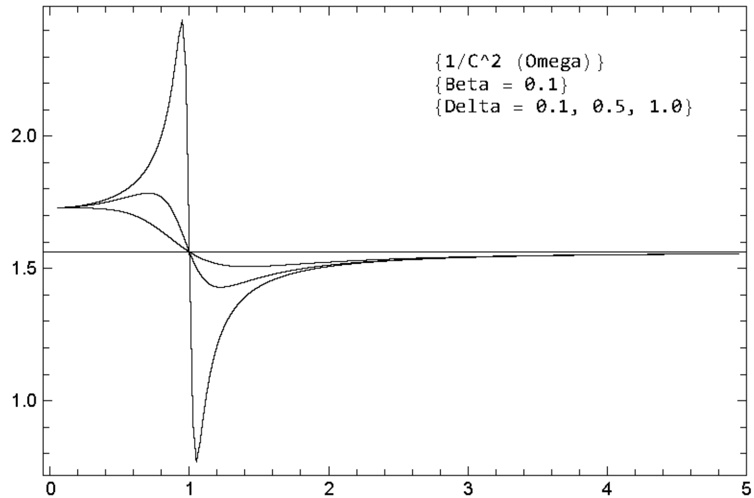
V_0 – the QM buble undisturbed volume

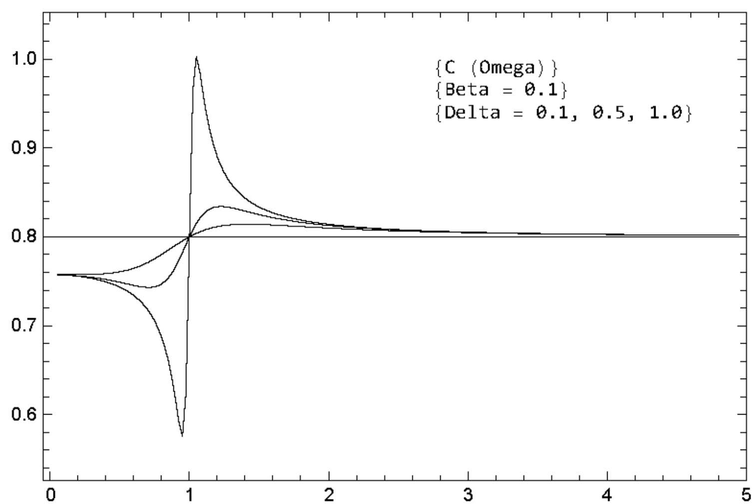
$$\frac{1}{C_s^2} = \frac{1}{C_0^2} + \beta \frac{\rho_0}{\gamma P_{i0}} \frac{1 - \Omega^2}{(1-\Omega^2)^2 + S^2 \Omega^2}, \quad C_0^2 = \frac{\gamma' p'}{\rho_0}$$

$$C_s \simeq \Big|_{\beta \ll 1} C_0 \left\{ 1 - \frac{1}{2} \beta \frac{C_0^2}{C_s^2} \frac{1 - \Omega^2}{(1-\Omega^2)^2 + S^2 \Omega^2} \right.$$

$\gamma(QM) < 1.75 \quad \gamma'(HM) > 1.75$

Our trial calculation with $\gamma = 1.5$, $\gamma' = 2$


{1/C^2 (Omega)}
{Beta = 0.1}
{Delta = 0.1, 0.5, 1.0}
2.0
1.5
1.0
0
1
2
3
4
5


{C (Omega)}
{Beta = 0.1}
{Delta = 0.1, 0.5, 1.0}
1.0
0.9
0.8
0.7
0.6
0
1
2
3
4
5

THE FINAL REMARKS

- The simple model based on a two-polytrope EoS has been presented for the propagation of the pressure wave in a bubbly quark-hadron phase
- The standard acoustic p-mode in NS with $\nu \simeq 1\,\mathrm{kHz}$ is far away from the resonance since $\lambda \gg R_0$
- In the limit $\Omega \to 0$, $\omega \ll \omega_0$ the bubble admixture reduces the speed of sound according to $c_s \simeq c_0 (1-\beta)$
- The resonance behavior is possible for the mixed phase fireball created in HIC

Thank you for attention