Electric dipole vorticity in nuclei

V.O. Nesterenko

Joint Institute for Nuclear Research, Dubna, Moscow region, Russia

in collaboration with:

P. Vishnevskiy,
BLTP JINR, Dubna; Inst. Nucl. Phys., Almaty, Kazakhstan
A. Repko
Inst. of Physics, Slovak Academy of Sciences, Bratislava, Slovakia
P.-G. Reinhard
Inst. of Theor. Physics II, University of Erlangen, Erlangen, Germany
J. Kvasil
Charles Univ, Praha, Czech Republic
P. von Neumann Cosel,
TU, Darmstadt, Germany

INFINUM-2023, 27.02-03.03.2023, BLTP JINR, Dubna, Russia

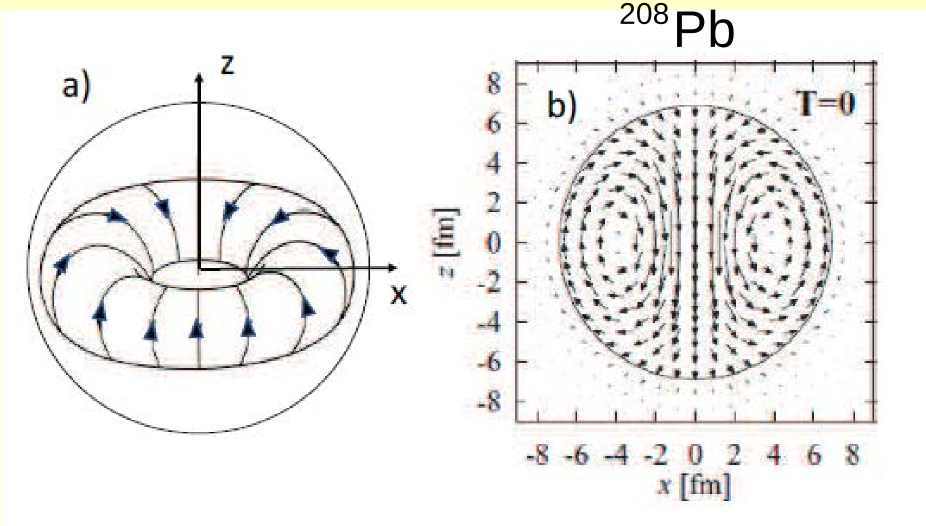
Vortical rings (VR)

Heavy Ion Collisions:

- vortex ring
- spinning

Yu.B. Ivanov, V.D. Toneev and A.A. Soldatov, Phys. At. Nucl. **83**, 179 (2020)

^{208}Pb



Toroidal dipole resonance in nuclei:

- confined vortical flow
- spinning-like oscillations
- ortex ring at each oscillation step
- unique example of intrinsic E1 vortical excitation

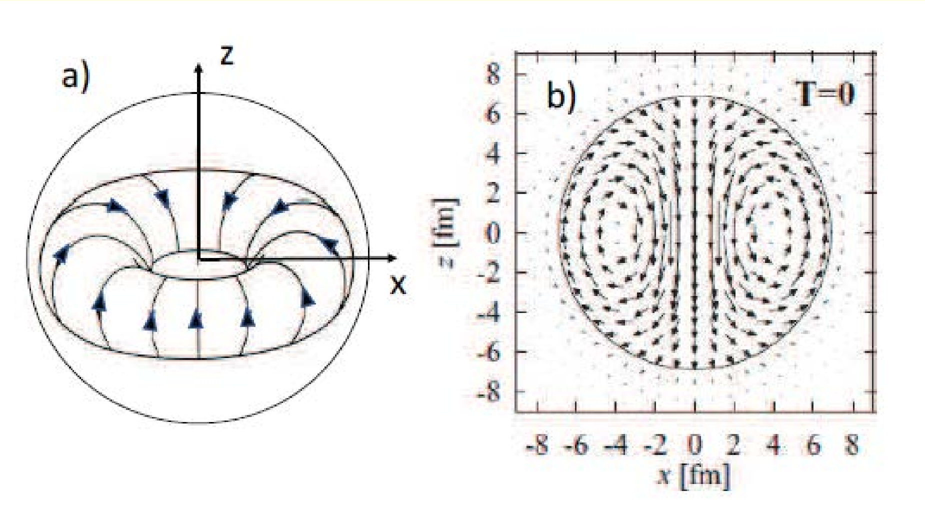
VON, J. Kvasil, A. Repko,W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).

Similar vortical flows in HIC and nuclei: mutual interest

Both vortical flows are Hill's vortex rings

Hill’s vortex ring (HIR)

- was suggested by Hill in 1894 as one of the exact solutions of Euler equations
- one of the most simple cases of a vortical flow
- looks like a torus-shaped vortex where the fluid mostly spins around an imaginary axis line in the form of the closed loop. Actually this is a vortex ring.
- exists in a limited volume, very stable, can be surrounded by another fluid and move along this fluid
- is the only possible form of a stable vortex, which may not consume the external energy
- has critical points at $R/\sqrt{2}$
- is plentiful in turbulent flows of liquids and gases, but are rarely noticed unless the motion of the fluid is revealed by suspended particles (smoke rings)



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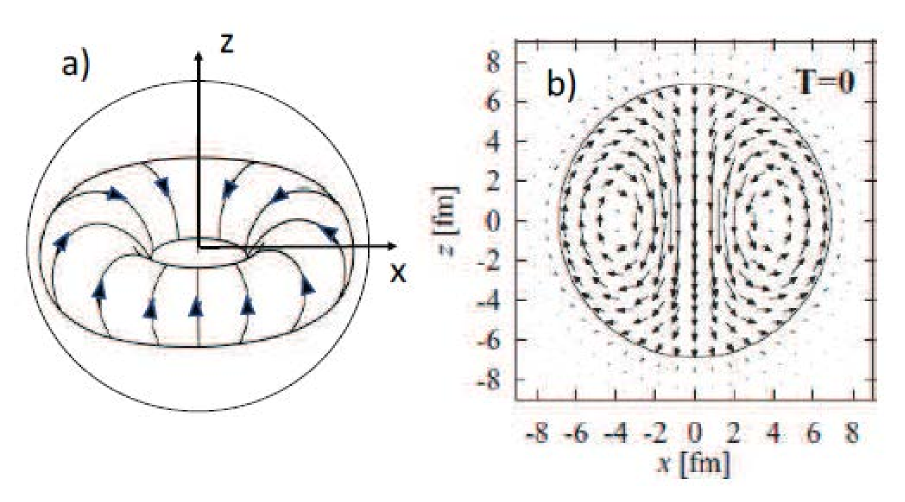


It would be strange if this elementary vortical mode would иу absent in nuclear dynamics.
In my talk, I will show that it must exist in nuclei as E1 toroidal mode.
Though it is not easy to identify it in experiment.

Content:

- Exotic E1 excitations: some basics.

J. Kvasil et al, PRC 84, 034303 (2011)

- Toroidal dipole resonance:

  - modern theoretical and experimental status

V.O. Nesterenko et al,
Phys. Atom. Nucl. 79, 842 (2016)

  - TDR vs PDR

A. Repko et al PRC, 87, 024305 (2013),
EPJA, 55, 242 (2019)

- Individual toroidal states in light nuclei

V.O. Nesterenko et al, PRL 120, 182501 (2018)
Y. Kanada-En'yo et al, PRC 95, 064319 (2017)

- Search of vortical states in (e,e'): 58Ni

V.O. Nesterenko et al, PRC 100, 064302 (2019).
V.O. Nesterenko, P.I. Vishnevskiy et al, to be submitted

- Reaction $(e,e'\gamma)$ to search toroidal states

- Conclusions and outlook

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes:

★ **V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).**
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
★ **E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).**
★ E.B.Balbutsev, I.V.Molodtsova, and A.V.Unzhakova, Europhys. Lett. 26, 499 (1994).
★ **S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).**
G.N. Afanasiev and Yu.P. Stepanovsky, J. Phys. A 28, 4565 (1995).
★ **I.N. Mikhailov, Ch. Brianson, P. Quentin, SJPN, 27, 303 (1996)**
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
★ **N.Ryezayeva, V.Yu. Ponomarev, et al, PRL 89, 272502 (2002).**
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).
★ **V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).**
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
S. Misicu, PRC 73, 024301 (2006).
X. Roca-Maza et al, PRC 85, 024601 (2012).
M. Urban, PRC, 85, 034322 (2012)
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007). review

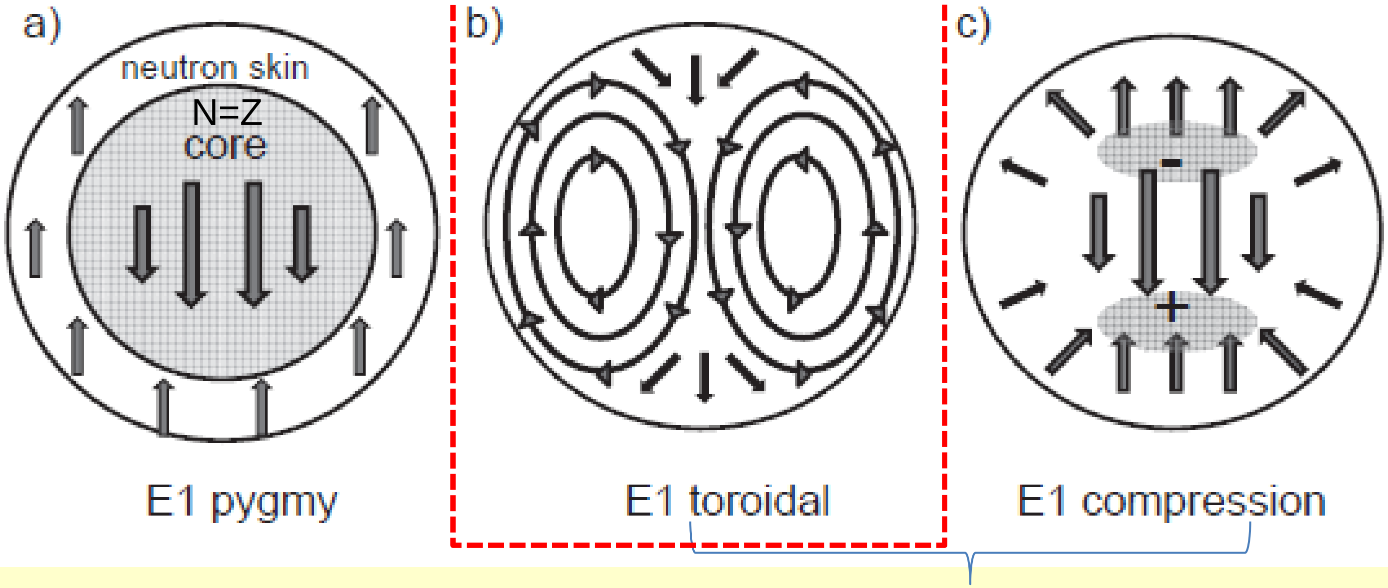
Exotic E1 excitations, intrinsic nuclear vorticity, vortical toroidal excitations

Exotic dipole resonances

R. Mohan et al (1971),

V.M. Dubovik (1975)
S.F. Semenko (1981)

M.N. Harakeh (1977)
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational

vortical

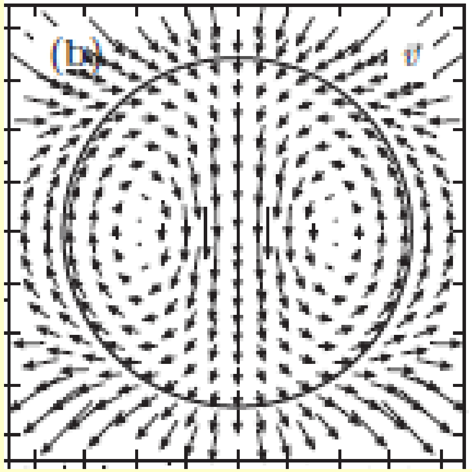
irrotational

$$E = 50 \div 60\, A^{-1/3} MeV \qquad E = 50 \div 70\, A^{-1/3} MeV \qquad E = 132\, A^{-1/3} MeV$$

Reviews:
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
VON, J. Kvasil, A. Repko,W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).

- Different kinds of dipole oscillations with fixed c.m.

Hydrodynamical vorticity: impact of nuclear surface



$$\vec{w}(\vec{r}) = \vec{\nabla}\times\vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$\vec{\nabla}\cdot\delta\vec{v} = \frac{\rho_0(\vec{\nabla}\cdot\delta\vec{j}_{nuc}) - \overbrace{\vec{\nabla}\rho_0\cdot\delta\vec{j}_{nuc}}^{\text{small}}}{\rho_0^2} \approx \frac{(\vec{\nabla}\cdot\delta\vec{j}_{nuc})}{\rho_0}$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$ - **continuity equation**

- impact of the surface to CE is small
- vortical (curl) currents do not contribute to CE

$$\vec{\nabla}\times\delta\vec{v} = \frac{\rho_0(\vec{\nabla}\times\delta\vec{j}_{nuc}) - \overbrace{\vec{\nabla}\rho_0\times\delta\vec{j}_{nuc}}^{\text{large}}}{\rho_0^2}$$

- impact of the surface to vorticity is large

Toroidal vortical mode appears in:

- nuclear current density

Following theorems of Helmholtz and Chandrasekhar/Moffat, the current distribution can be decomposed as

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\chi(\vec{r})]$$

electric moments

magnetic moments

electric **toroidal** moments

transversal

E1 GDR, compression

- Multipole electric operator (probe external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\,(j_{\lambda}(kr)Y_{\lambda\mu})]$$

$$j_{\lambda}(kr) = \frac{(kr)^{\lambda}}{(2\lambda+1)!!}[1-\frac{(kr)^{2}}{2(2\lambda+3)}+\ldots]$$

So, the toroidal operator is the second order term in long-wave expansion of the electric operator

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^{\lambda}Y_{\lambda\mu}$$

← standard electric operator
In long wave approximation

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)

Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \underbrace{\frac{5}{3}r < r^2 >_0}_{\text{cmc}}]\,\vec{Y}_{11\mu}(\hat{r})\cdot[\underbrace{\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{mainly vortical flow}}]$$

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}\,[\underbrace{\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{irrotational flow}}]$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

⇓

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

Toroidal and compression operators are coupled:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2\sqrt{3}c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot\vec{\nabla}\times(\vec{r}\times\vec{\nabla})[r^3 - \frac{5}{3}r < r^2 >_0]\,Y_{1\mu}(\hat{r})$$

- vorticity of the rotation of the compression field

Toroidal dipole resonance as a general feature of atomic nuclei

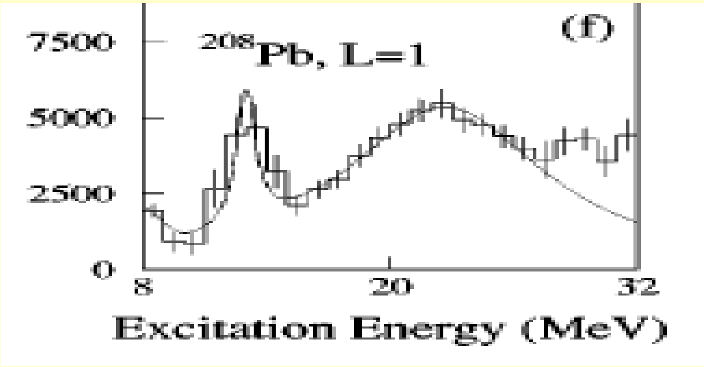
TDR and CDR constitute low- and high-energy ISGDR branches (?)

Experiment: (α,α')

Familiar treatment → LE (toroidal) HE (compression)

^{208}Pb

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004
M.Uchida et al, PRC 69, 051301(R) (2004)



TDR

CDR

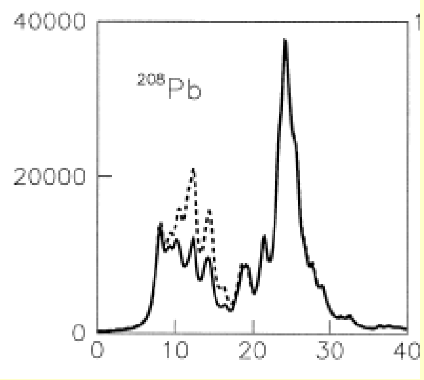
A.Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

There are also exp ISGDR data in

^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);



Perhaps Uchida observed at 10-17 MeV not TDR but CDR fraction coupled to TDR. Main TDR peak should lie lower at ~ 7-9 MeV.

The direct observation of TDR in (α,α') can be disputed in general since (α,α') is mainly determined by transition density while toroid mainly depends on the vortical transition current.

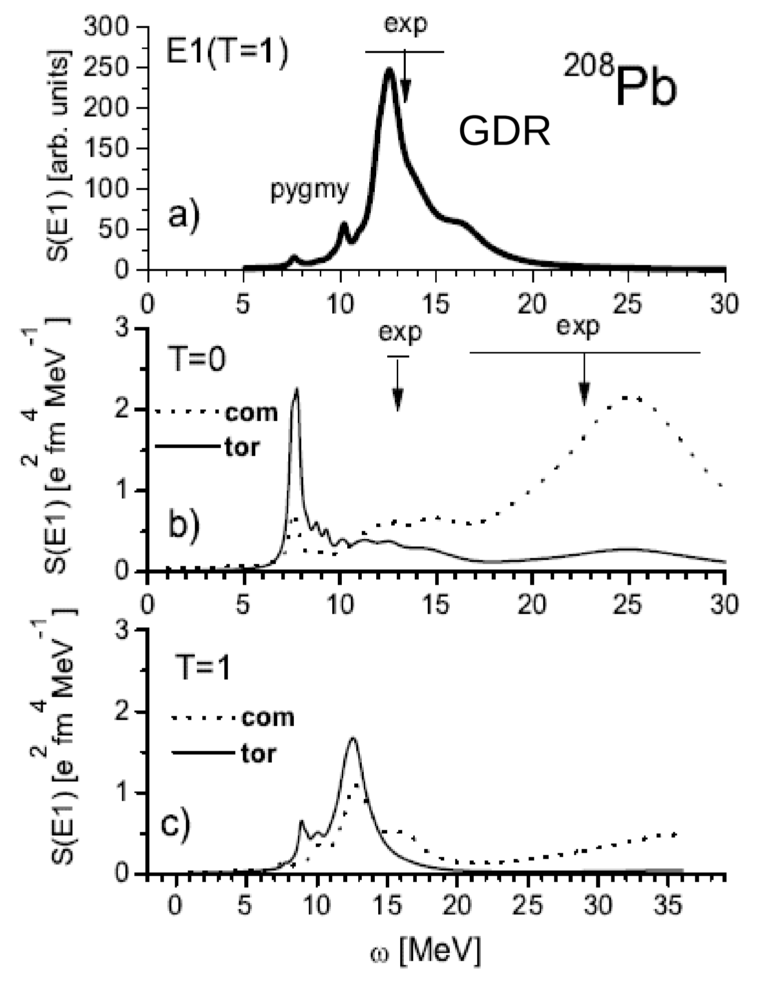
NEED IN NEW EXPERIMENTS!

Skyrme QRPA calculations

A. Repko, P.G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013)

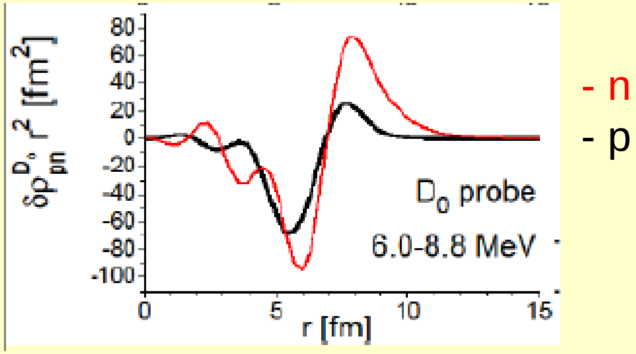
Strength functions

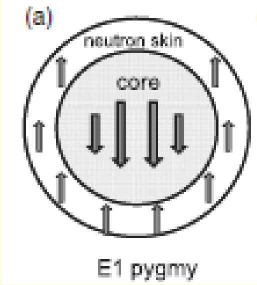
SLy6



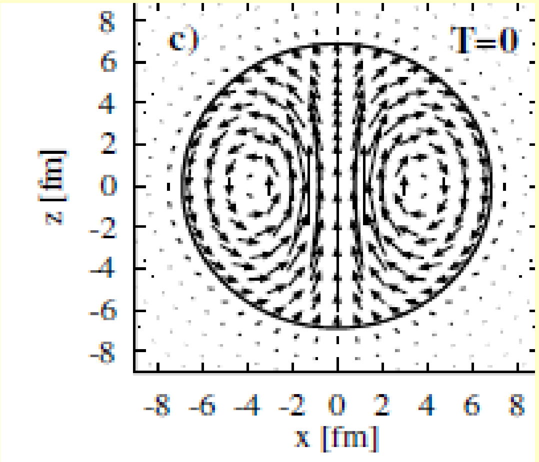
PDR region hosts TDR and CR!

Typical PDR transition density:



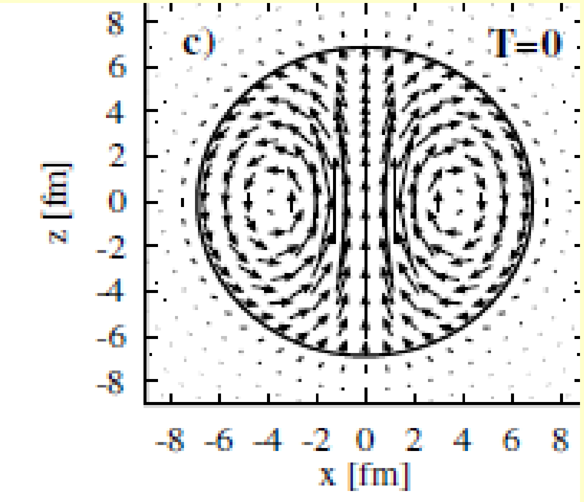


Nucleon current in the PDR region is mainly toroidal!

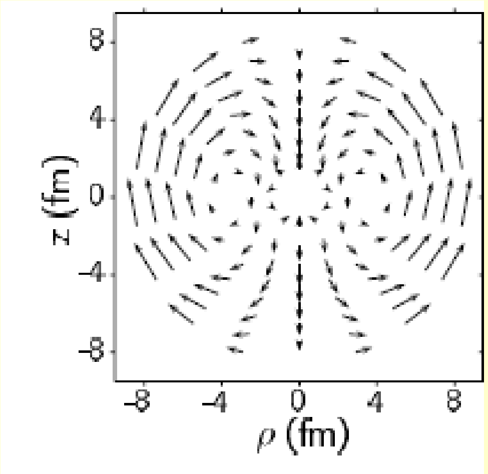


Toroidal flow in PDR energy region is obtained in various nuclei and within different models

Skyrme RPA: 208Pb
Repko, P.-G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013).

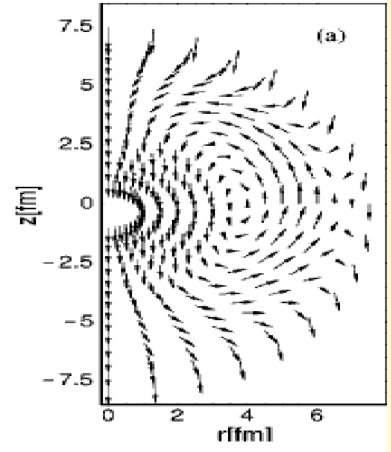


QPM: 208Pb
N.Ryezayeva et al,
PRL 89, 272502 (2002).



P. Papakonstantinou et al,
EPJA 47, 14 (2011).

Relativistic RPA: 116Sn
D. Vretenar et al,
PRC 65, 021301R (2002).



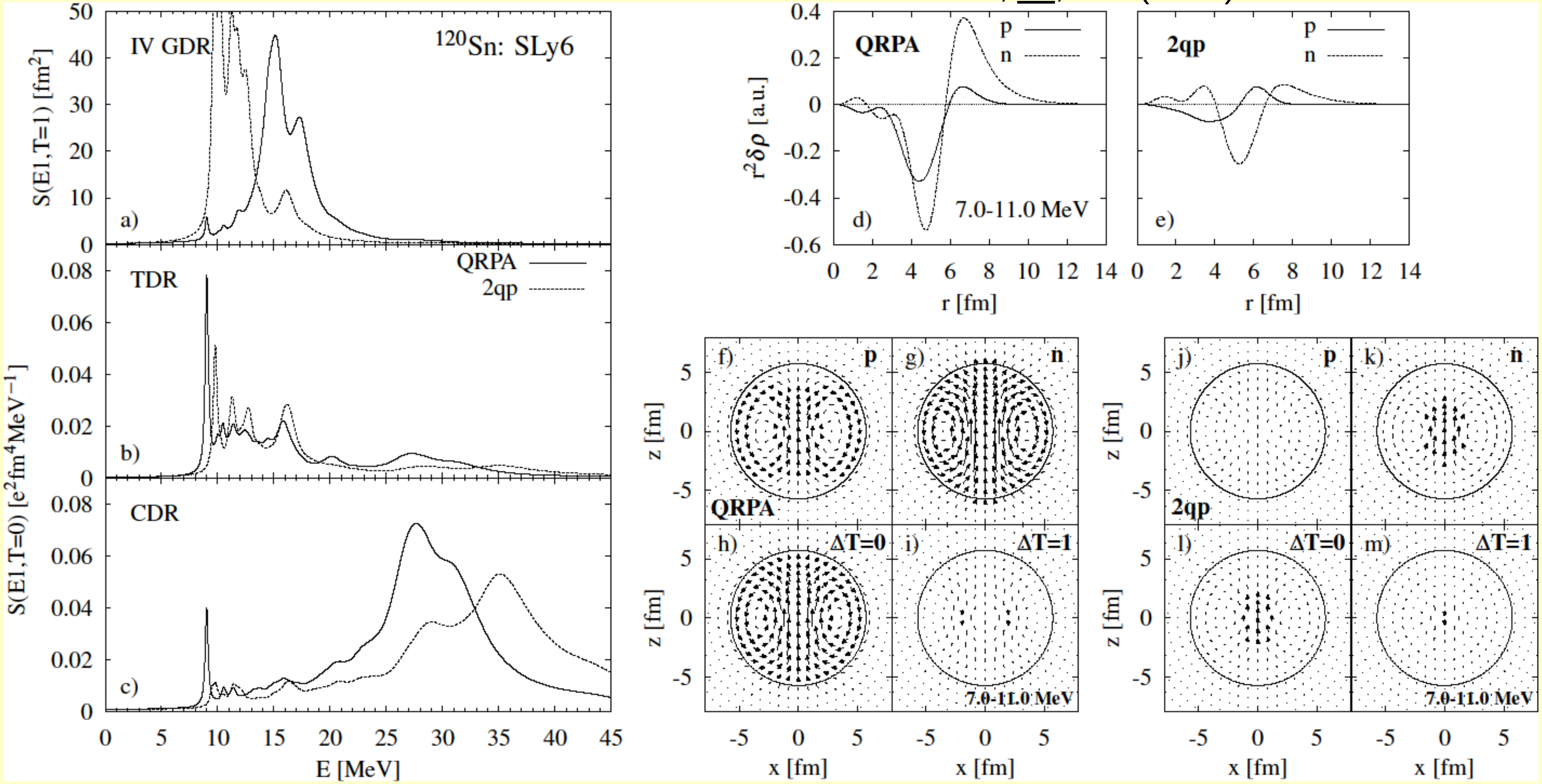
Similar results in Ca, Ni, Zr, Sn, Sm, Yb, U, ..

Nuclear vorticity was also earlier discussed in:

F.E. Serr, T.S. Dumitrescu, T. Suzuki, C.H. Dasso, NPA 404, 359 (1983),
D.G. Raventhall and J. Wambach, NPA 475, 468 (1987).

120Sn SLy6

A. Repko, VON, J. Kvasil and P.-G. Reinhard,
EPJA, 55, 242 (2019)



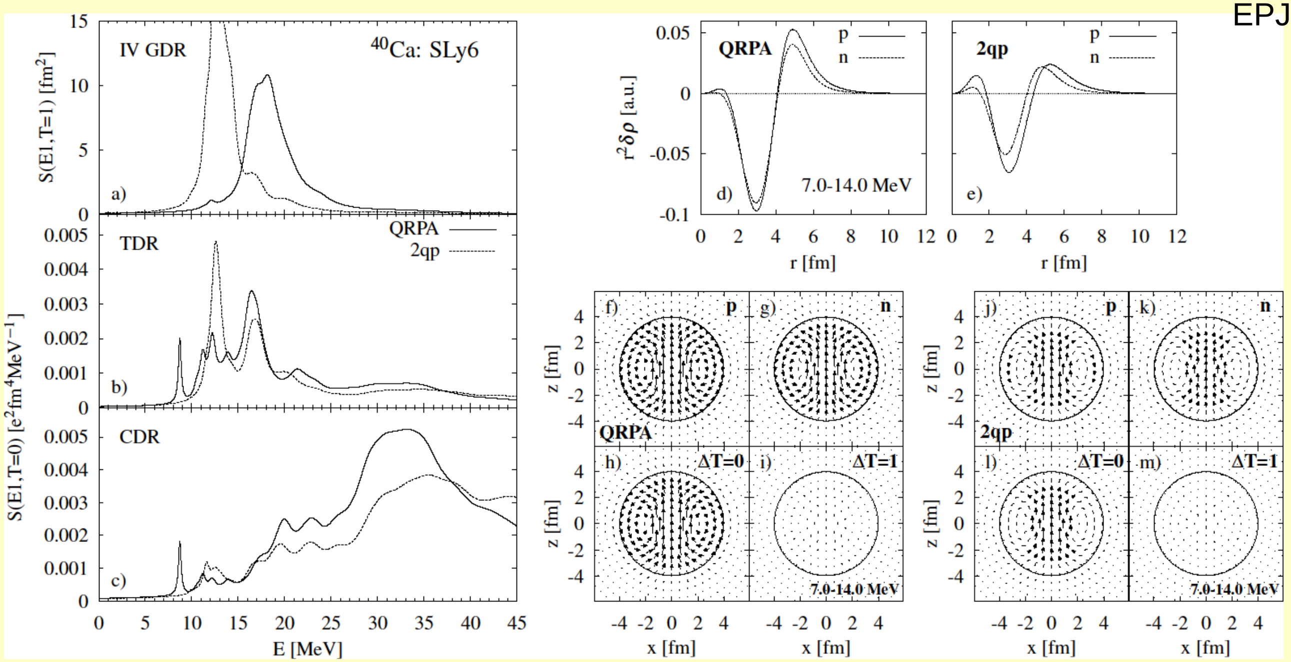
Strengths: TDR shares the same energy region with PDR and 2qp dipole strength
RPA transition densities: typical for PDR
RPA currents: clear IS toroidal flow with dominant neutron contribution

Is there one-to-one correspondence between transition densities and currents?

This can be checked through the continuity equation (CE): $-imE_\nu \, \delta\rho_\nu = \hbar^2 \vec{\nabla} \cdot \delta\vec{j}_\nu$

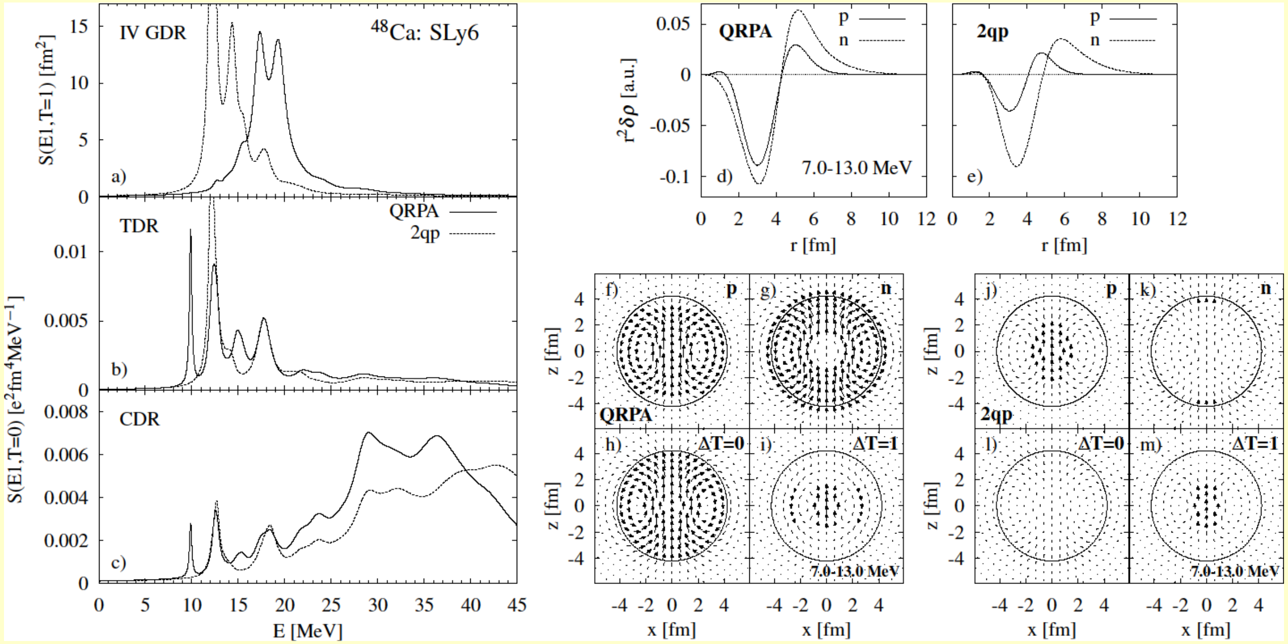
Only irrotational part of the current with non-zero divergence contributes to CE.

A. Repko, VON, J. Kvasil and P.-G. Reinhard,
EPJA, 55, 242 (2019)



40Ca, SLy6

- no PDR but clear TDR
- residual interaction
  enforces toroidal flow
- protons and neutrons
  equally contribute
- squeezed toroidal flow

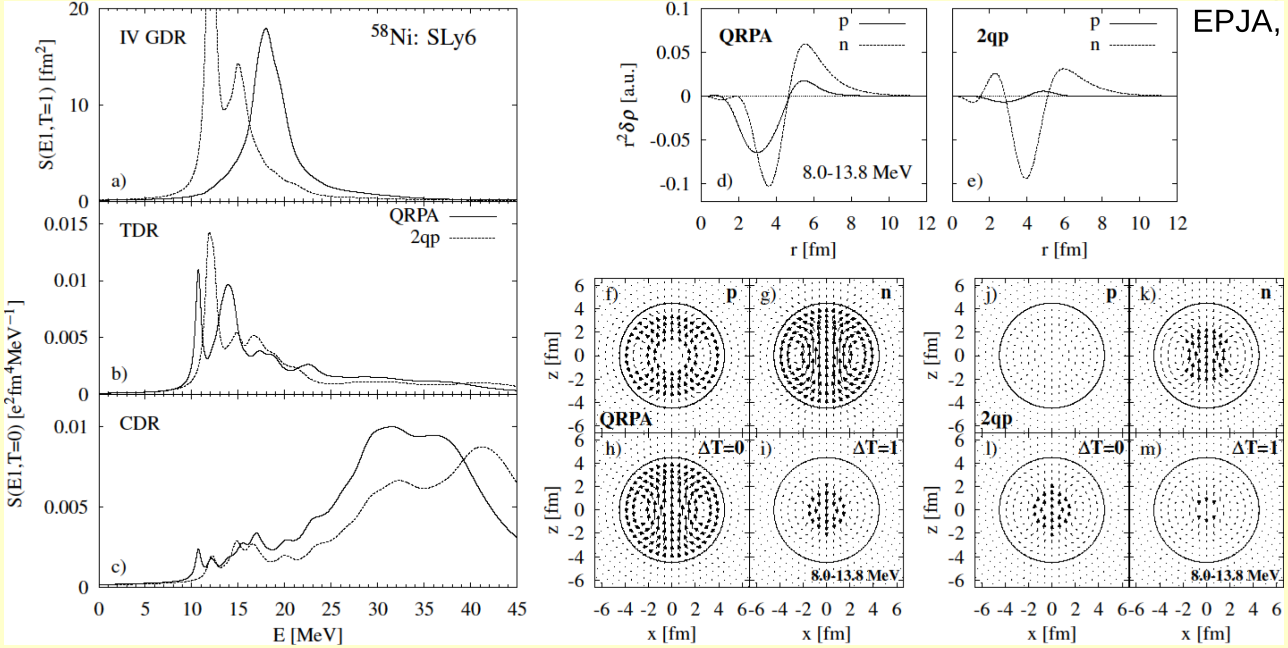


48Ca, SLy6

Basically the same results
as for 120Sn, SLy6.

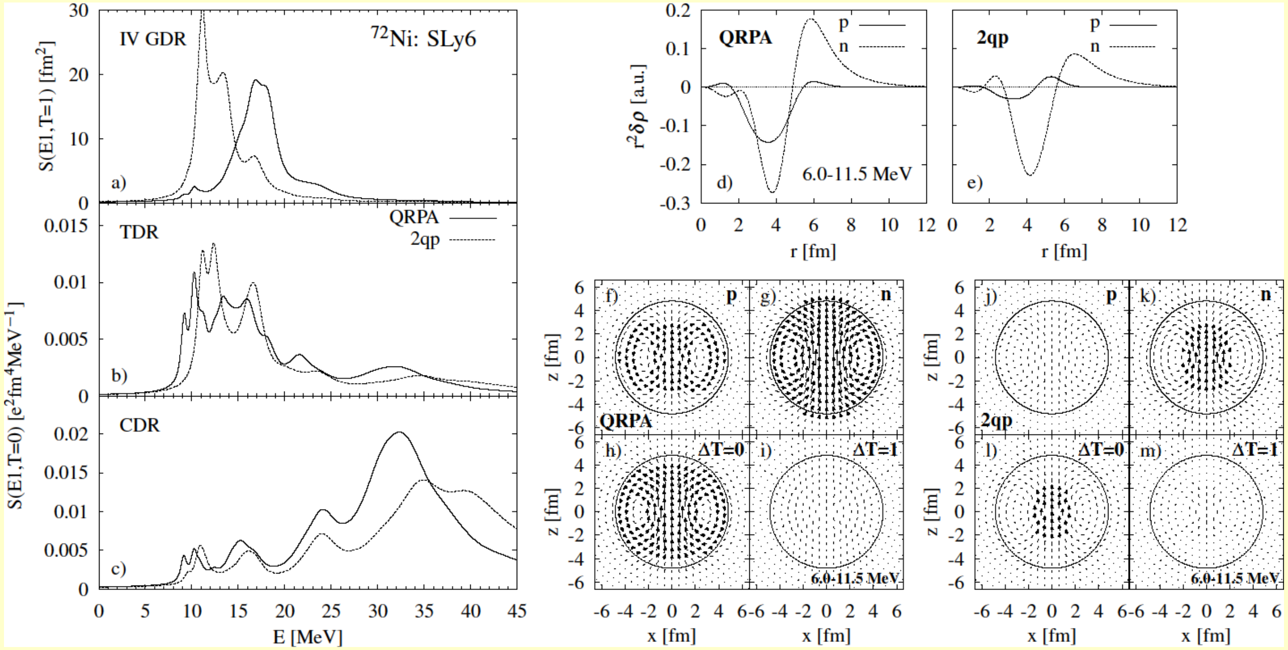
**Toroidal mode persists
In nuclei independently
on the neutron excess**

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



58Ni, SLy6

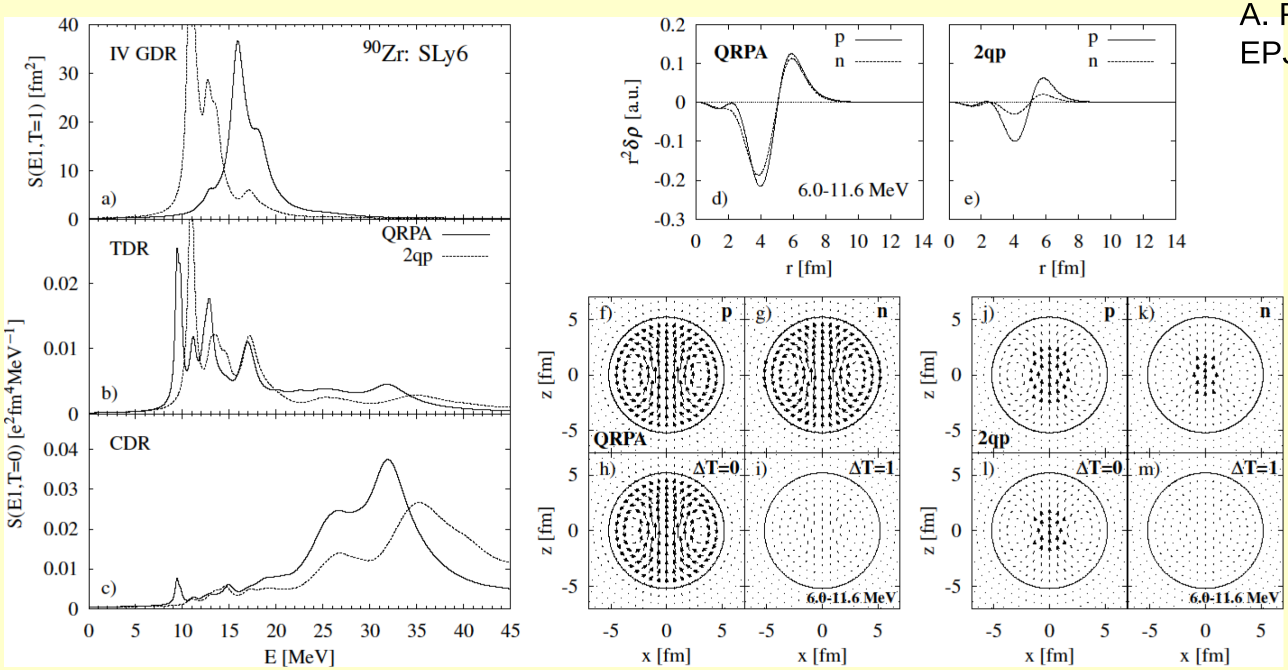
- small neutron excess, almost no PDR but strong TDR



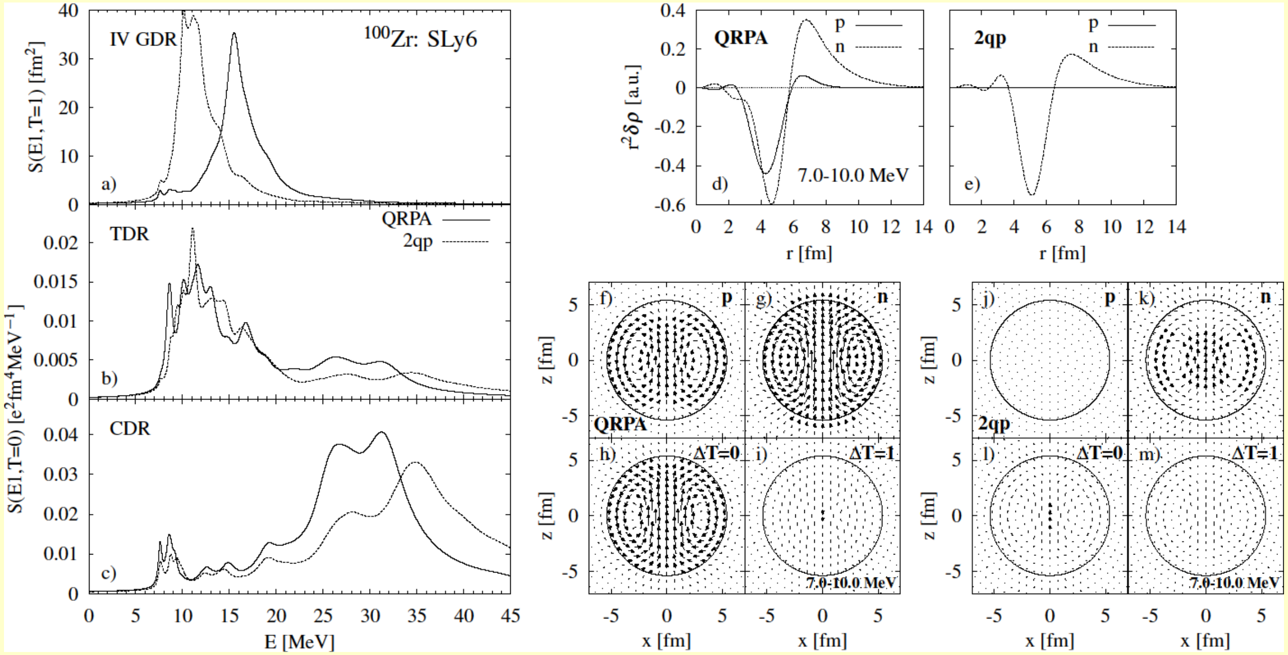
72Ni, SLy6

- large neutron excess
- dominant neutron, contribution to toroidal current,
- RPA enforces the flow
- basically the same results as for 120Sn, SLy6.

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



90Zr, SLy6



100Zr, SLy6

Basically the same results as for 120Sn, SLy6.

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)

Origin of E1 vortial toroidal strength

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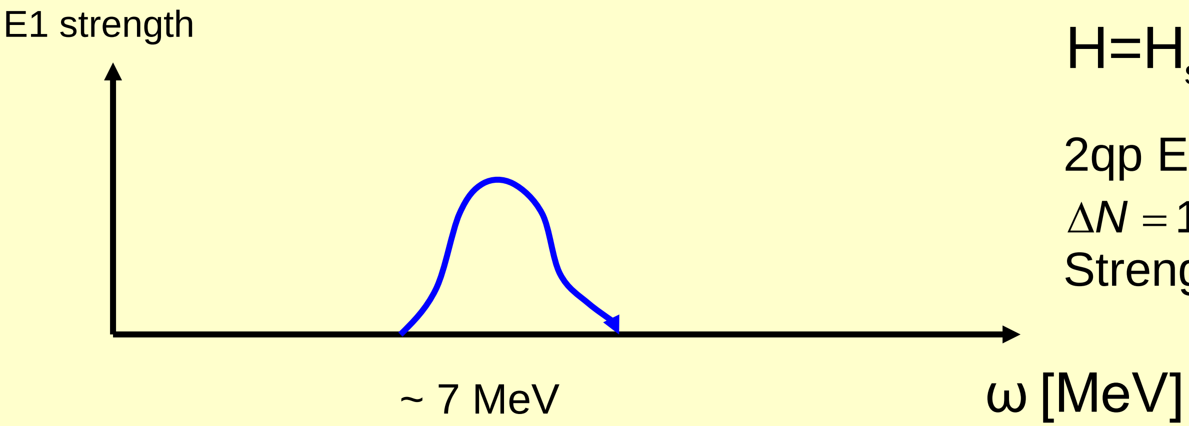
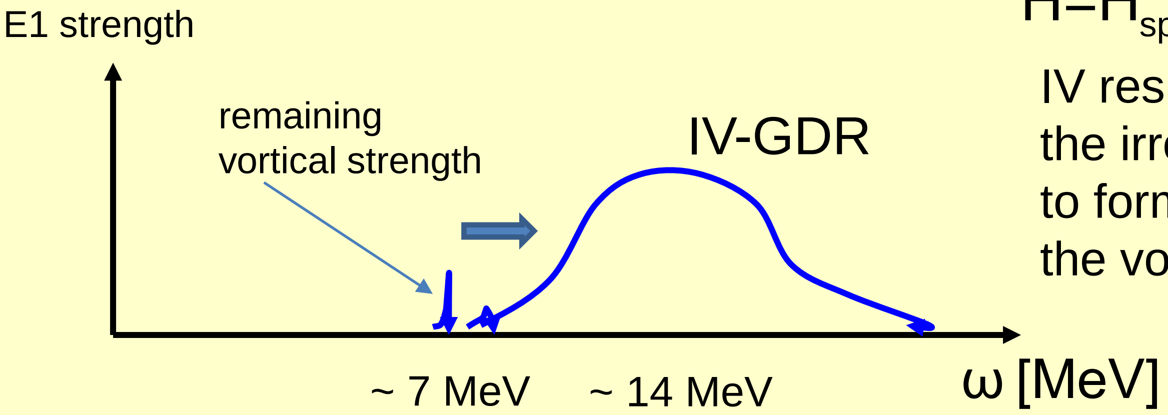
E1 toroidal strength must exist in all nuclei at the energy ~ E($\Delta N = 1$).



$H = H_{sp}$

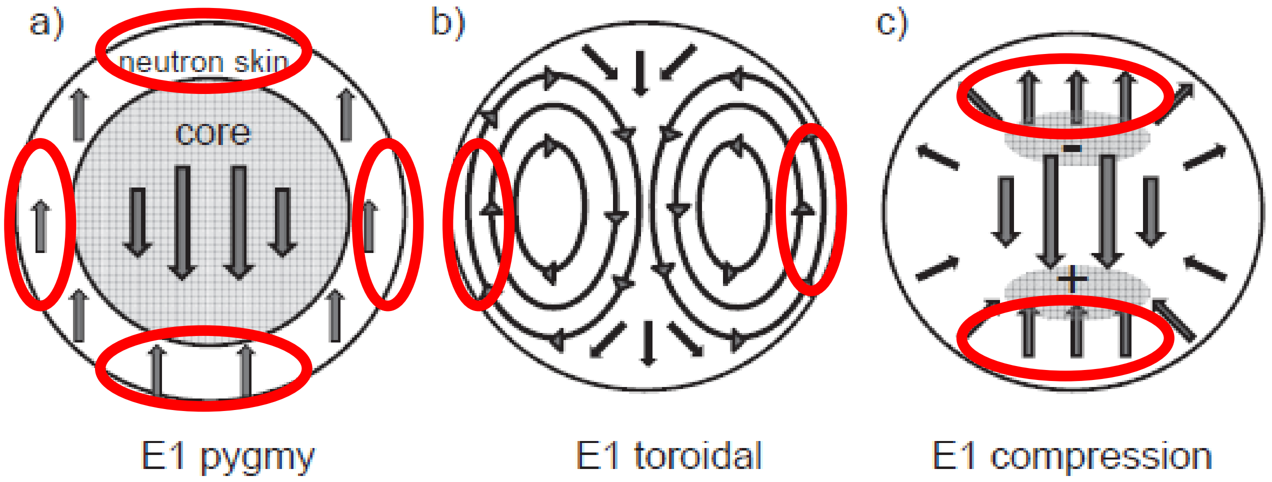
2qp E1 strength from
$\Delta N = 1$ dipole transitions
Strength is mainly irrotational

$H = H_{sp} + H_{res}$

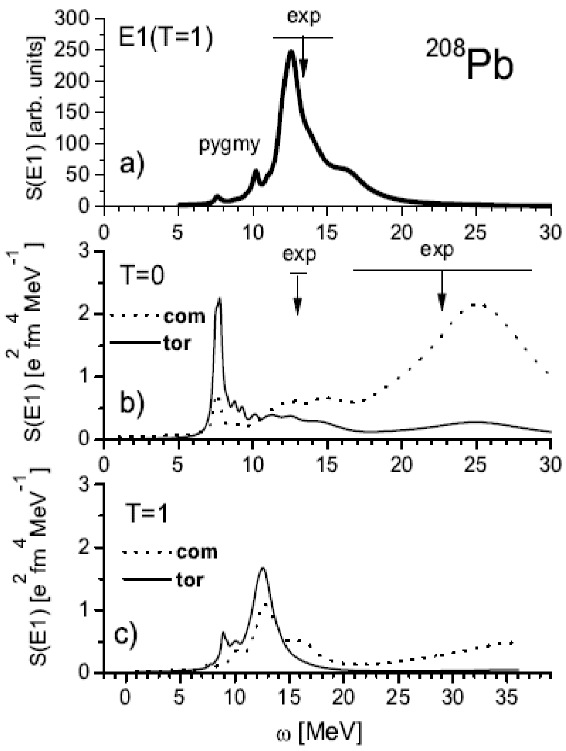


IV residual interaction upshifts
the irrotational E1 strength
to form GDR but leaves at 7 MeV
the vortical (toroidal) strength.

So TDR is indeed a general feature of atomic nuclei

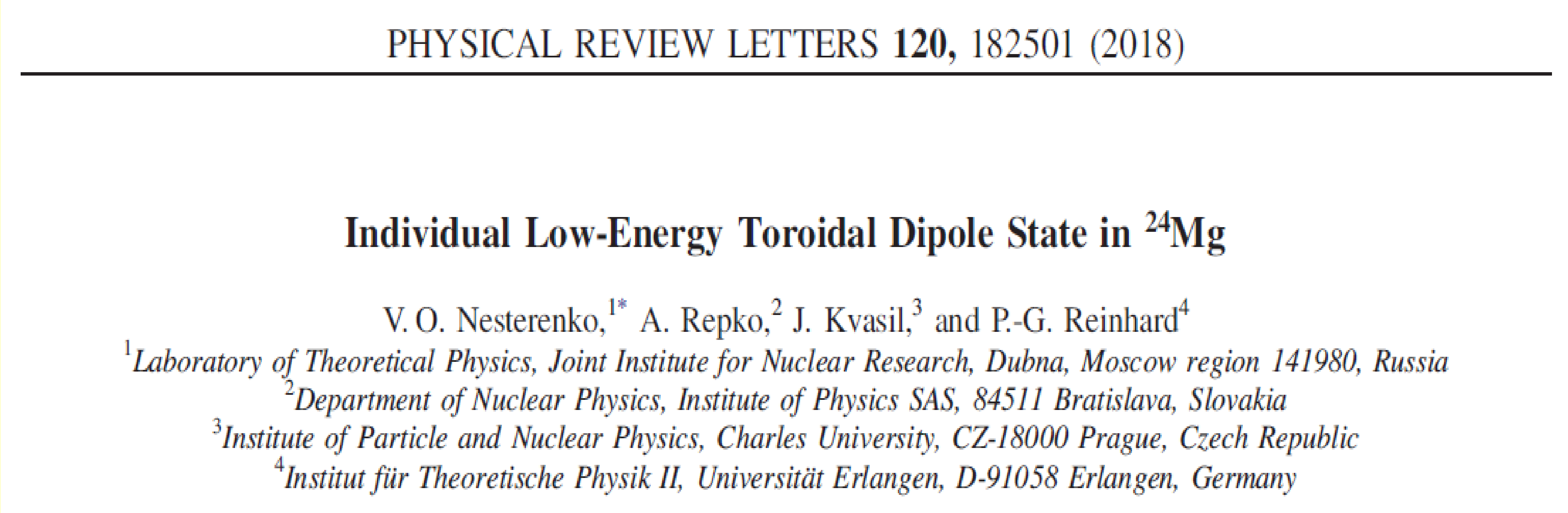


VON, J. Kvasil, A. Repko, W. Kleinig, P.-G. Reinhard, **Phys. Atom. Nucl.,** 79, 842 (2016).



PDR can be viewed as a local peripheral part of TDR and CDR

Individual toroidal states in light nuclei



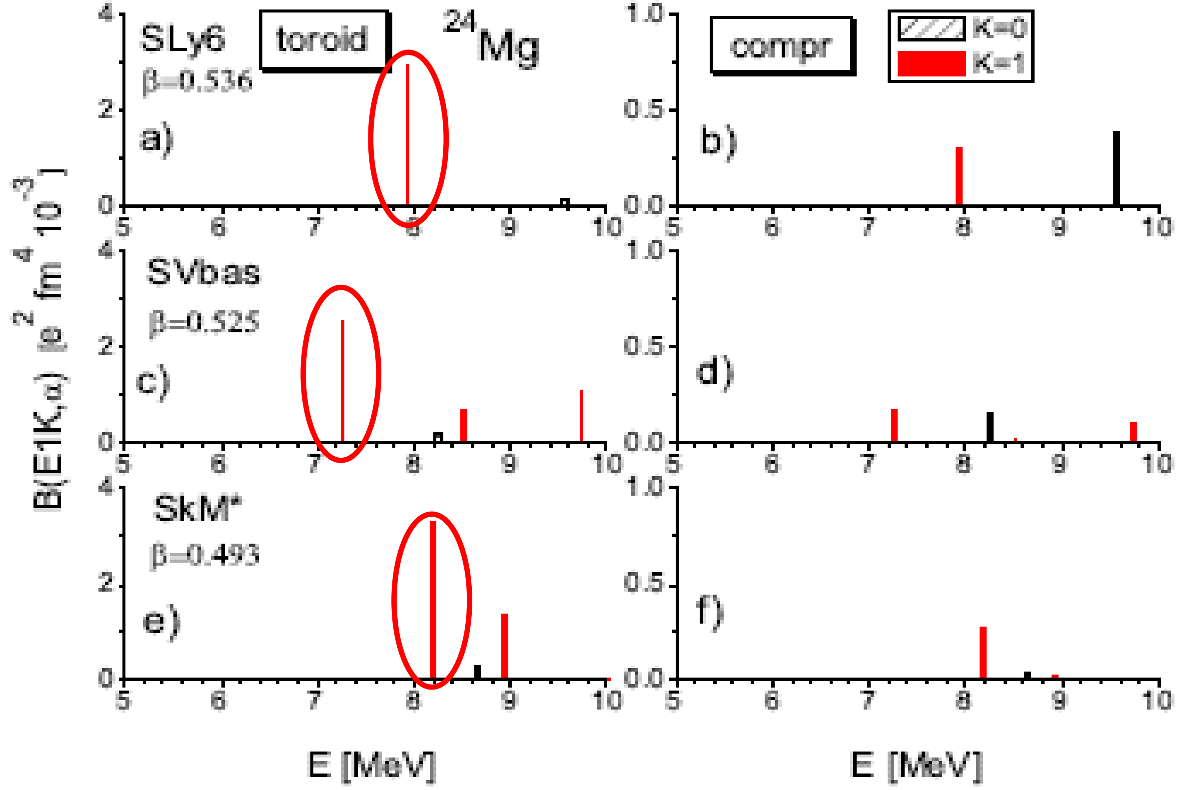
PHYSICAL REVIEW LETTERS **120,** 182501 (2018)

Individual Low-Energy Toroidal Dipole State in ^{24}Mg

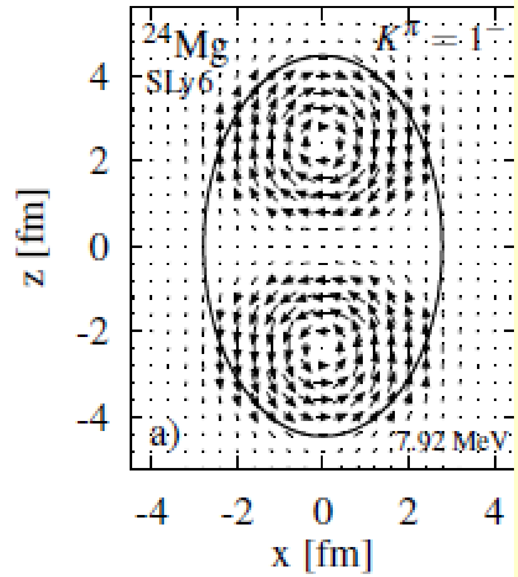
V. O. Nesterenko,[1*] A. Repko,[2] J. Kvasil,[3] and P.-G. Reinhard[4]
[1]*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia*
[2]*Department of Nuclear Physics, Institute of Physics SAS, 84511 Bratislava, Slovakia*
[3]*Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Prague, Czech Republic*
[4]*Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany*

^{24}Mg $\beta_2^{exp} = 0.605$

VON, A. Repko, J. Kvasil, P.-G. Reinhard, PRL 120, 182501 (2018)



QRPA results for
SLy6,
SVbas,
SkM*

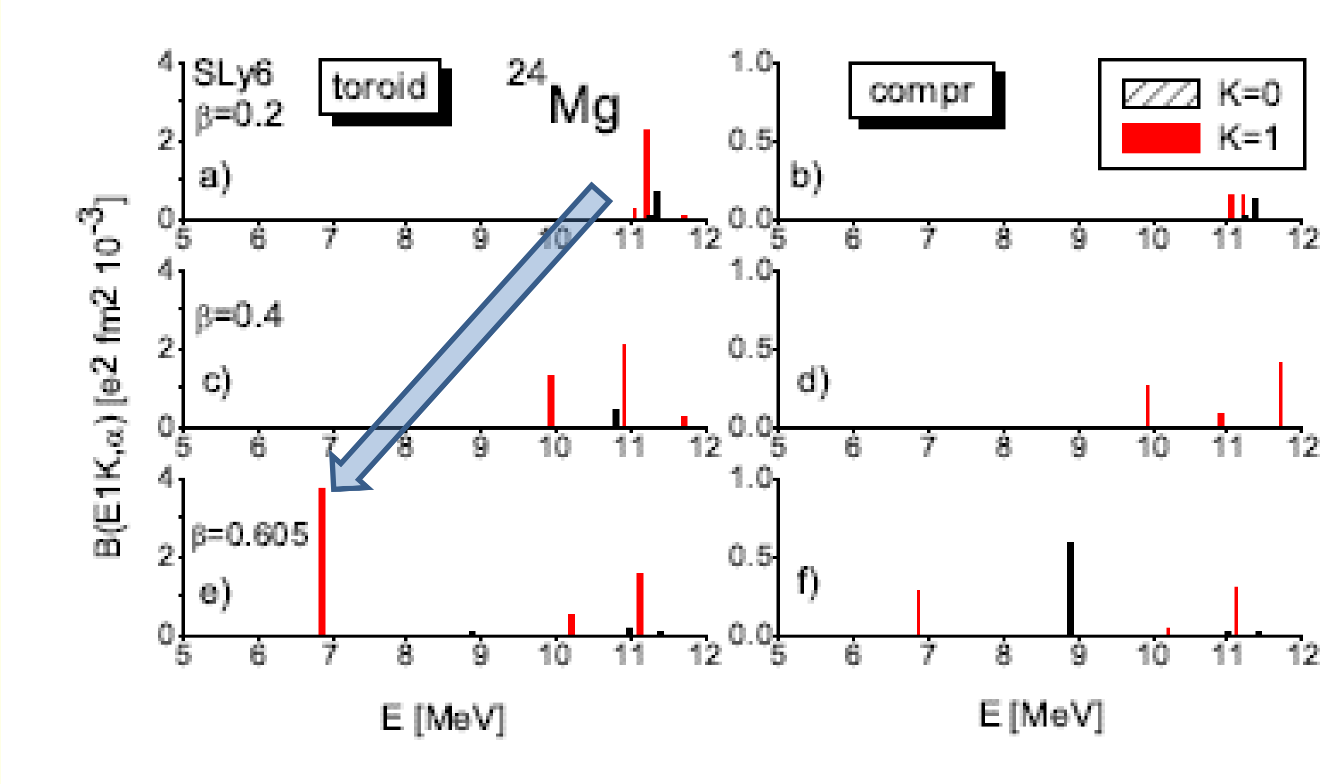


Persistence of the main result:
the **lowest** toroidal K=1 peak

The remarkable example of individual toroidal state!

Dependence on deformation

VON, A. Repko, J. Kvasil, P.-G. Reinhard,
PRL 120, 182501 (2018)



TS becomes lowest due to of the large axial prolate deformation.

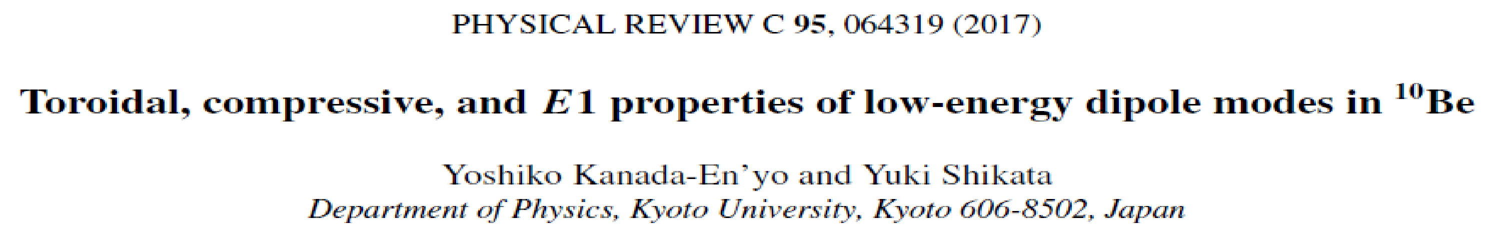
K=1 peak is:

- the lowest dipole state
- well separated from other states

To get individual lowest TS, two rigorous requirements should be held:

- huge prolate deformations
- sparse low-energy spectrum

This is just realized in light deformed nuclei



PHYSICAL REVIEW C 95, 064319 (2017)

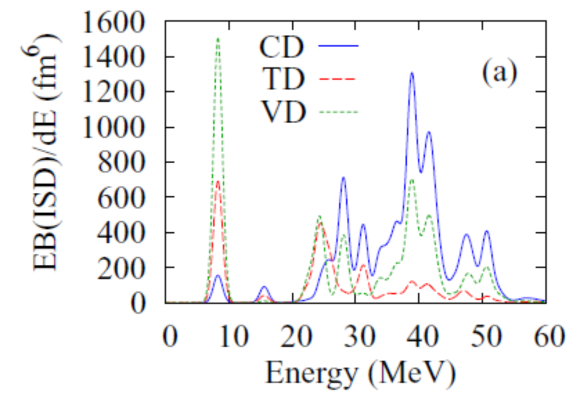
Toroidal, compressive, and $E1$ properties of low-energy dipole modes in ^{10}Be

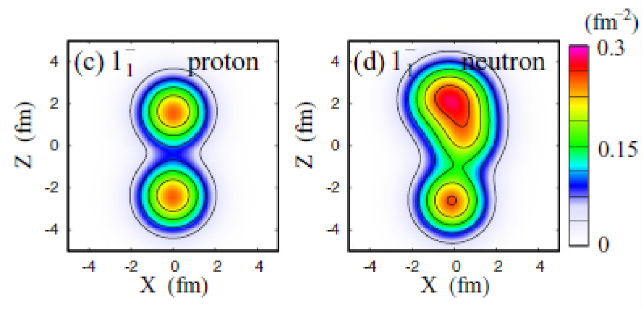
Yoshiko Kanada-En'yo and Yuki Shikata

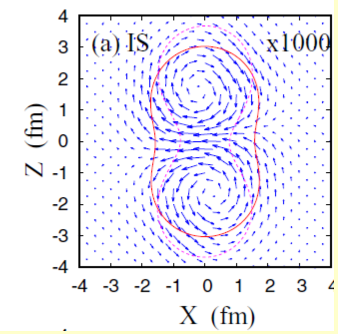
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

$^{10}\text{Be}={}^{6}\text{He}+\alpha$

Antisymmetrized molecular dynamics + generator coordinate method (AMD+GCM)
Cluster degrees + mean field







the lowest dipole state

$I^{\pi}K=1^{-}1_{1}$ is toroidal

Y. Kanada-En'yo, Y. Shikata, and H. Morita, Phys. Rev. C 97, 014303 (2018)
Y. Kanada-En'yo and H. Horiuchi, Front. Phys. 13, 132108 (2018)
Y. Kanada-En'yo, Y. Shikata, and H. Morita, PRC 97, 014303 (2018)
Y. Shikata, Y. Kanada-En'yo, and H.Morita, Prog. Theor. Exp. Phys. **2019**, 063D01 (2019).
Y. Kanada-En'yo and Y. Shikata, Phys. Rev. C 100, 014301 (2019).
Y. Shikata and Y. Kanada-En'yo, PRC, 103, 034312 (2021).
Y. Chiba, Y. Kanada-En'yo, and Y. Shikata, arXiv:1911.08734.

In AMD+GCM, toroidal states were found in ^{10}Be, ^{12}C, ^{16,18}O, ^{24}Mg

Interplay of cluster and vortical modes:

General fundamental problem:

modern theory and experiment are not yet able to propose reliable ways for identification of intrinsic vortical modes. This fundamental problem is still unresolved. The search of E1 toroidal mode could be the first step in this direction.

Search of TDR
in (e,e')

(e,e’): PWBA cross section

J. Heisenberg and H.P. Blok,
Ann. Rev. Nucl. Part. Sci, 33, 569 (1983),

$$\boxed{\sigma_{PWBA}(\theta,q)=\sigma_{Mott}(\theta,E_i)f_{rec}\{|F_E^C(q)|^2+(\frac{1}{2}+tg^2(\frac{\theta}{2})[|F_E^T(q)|^2+|F_M^T(q)|^2\}}$$

For $I^{\pi}=1^{-}$ states, $F_M^T(q)=0$

Toroidal contribution through $F_E^T(q)$

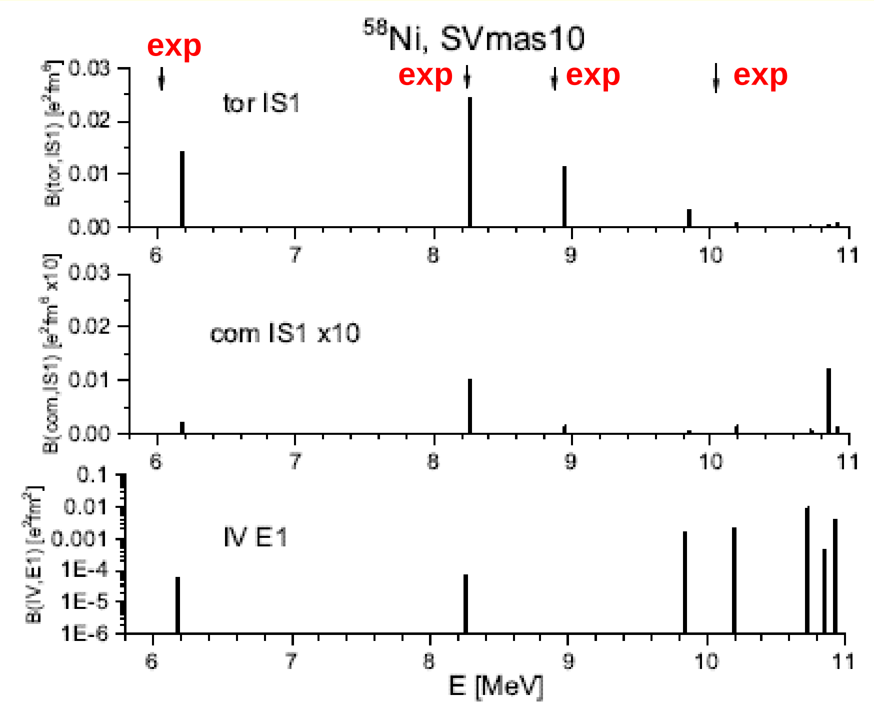
We need large transfer momenta to suppress contribution of $F_E^C(q)$ and enhance contributes of $F_E^T(q)$

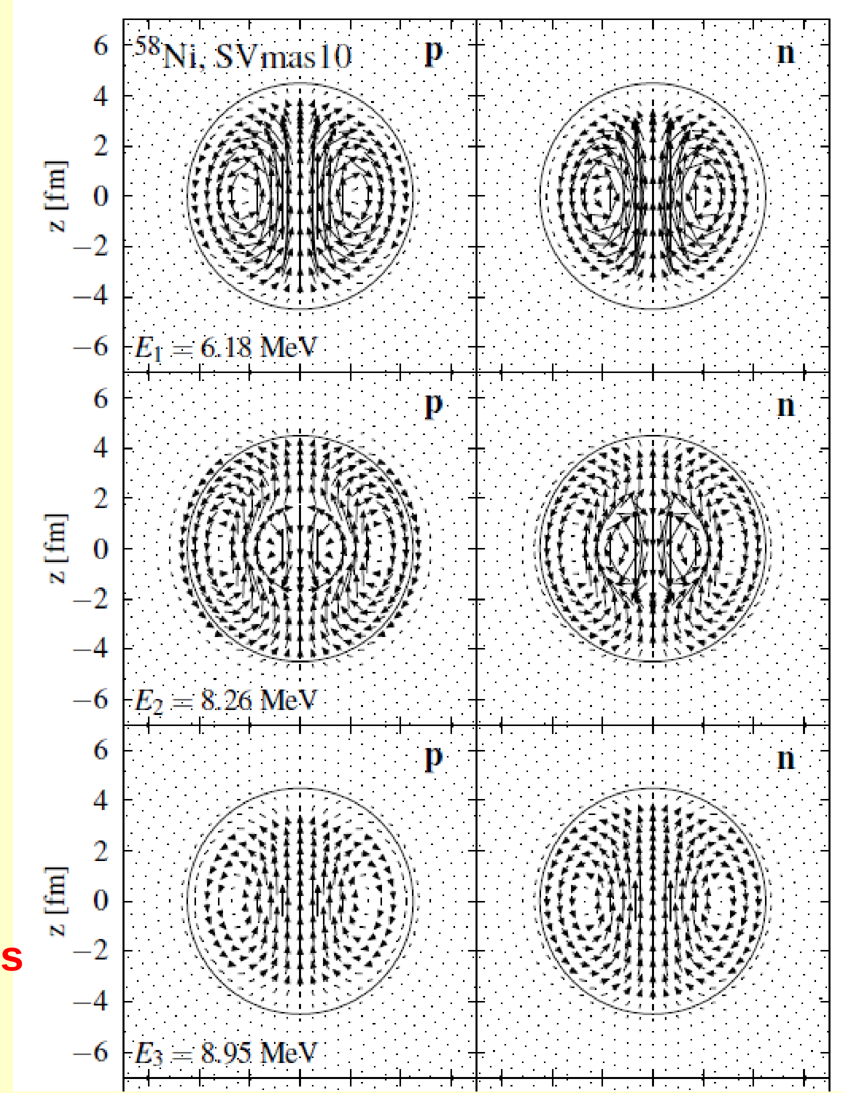
The available (e,e’) data for spherical 58Ni are for modest (Mettner) and large (Reitz) transfer momenta

Toroidal E1 states in spherical 58Ni

Old (e,e') experiments: W. Mettner, A. Richter et al, Nucl. Phys. A473, 160 (1987), Reitz
P. von Neumann Cosel (TU, Darmstadt) :

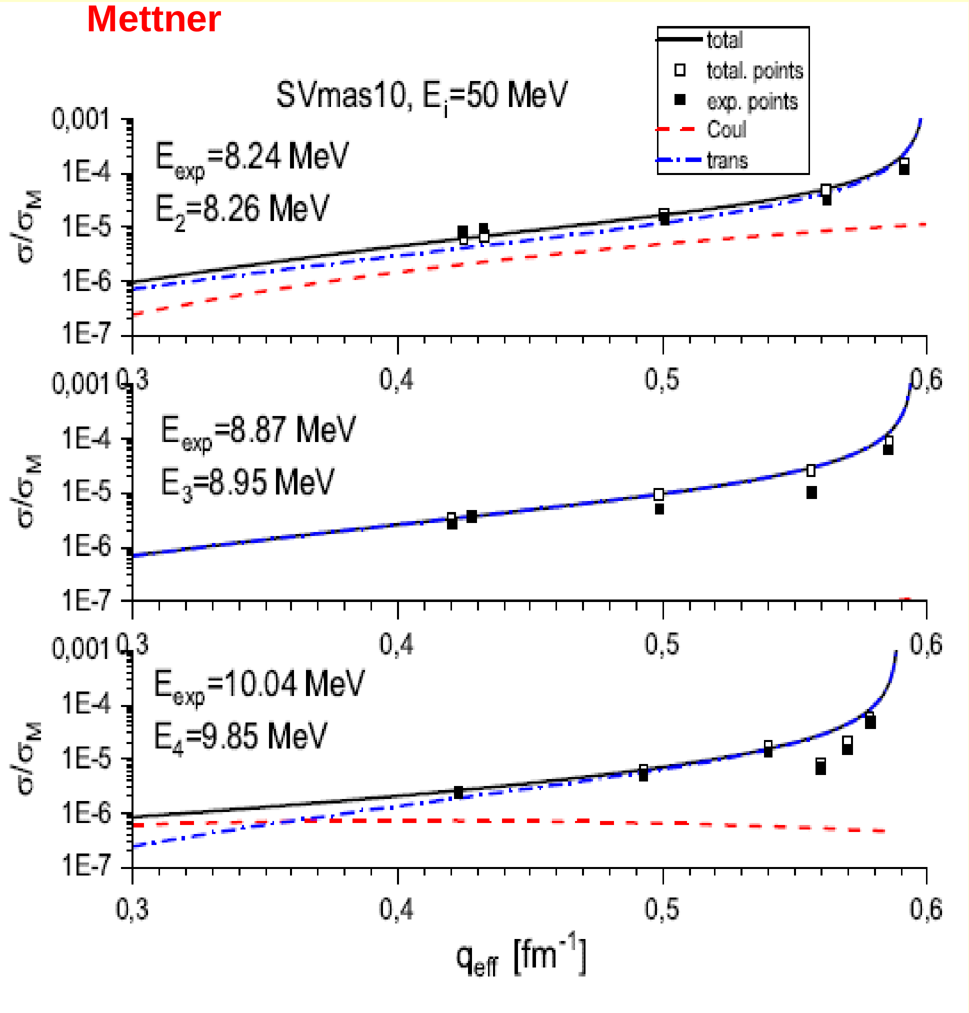
- large transversal form factors for some low-energy E1 states.
- exp. data from (g,g'), (p,p'), (α,α')
- to check if these states are toroidal?

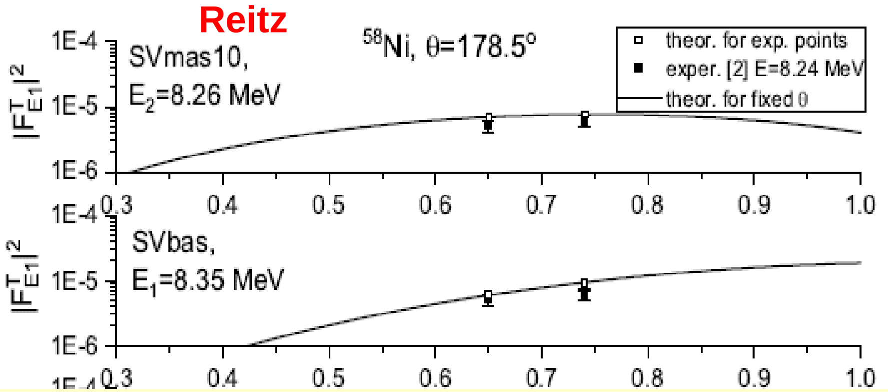


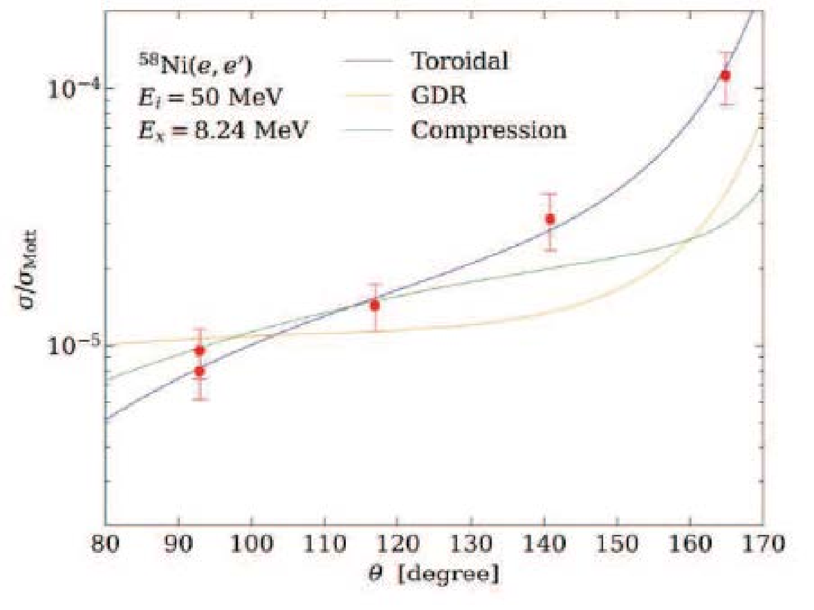


- **large E1 toroidal strength**
- **typical toroidal proton and neutron currents**

V.O. Nesterenko, P.I. Vishnevskiy, P.von Neumann Cosel, A.Repko. P.-G. Reinhard, J. Kvasil, to be submitted







The calculations well describe Mettner's and Reitz's (e,e') data

Just toroidal states (not GDR or compression) describe the slope in Mettner's data

This the first (e,e')-based confirmation of existence of E1 toroidal states in nuclei.

Next our step: $(e,e'\gamma)$

C.N. Papanikolas et al, PRL 54, 26 (1985):
first measurement of the relative FL/FT sign in 12C

$$\frac{d^4\sigma}{d\Omega_\gamma d\Omega_e\, d\omega\, dE_\gamma} = \sigma_{\rm Mott}\left(\frac{\Gamma_{\gamma f}}{\Gamma}\right)\Big\{V_L U_L |F_L(q)|^2 + V_T U_T |F_T(q)|^2 + V_I U_I \cos\phi_\gamma F_L(q) F_T(q) + V_S U_S \cos 2\phi_\gamma F_T(q) F_T(q)\Big\},$$

longitudinal-transversal interference terms allow to measure the relative sign of FL and FT.

In the long-wave approximation, Siegert's theorem gives the negative sign:

$$F_T(q) = -\frac{\omega}{q}\left(\frac{\lambda+1}{\lambda}\right)^{1/2} F_L(q) \quad \text{for} \quad q \to \omega.$$

For transversal toroidal E1 states, we expect the opposite sign. This could be a signature of the toroidal mode.
Besides, Hill's vortex ring should polarize the outcoming gamma quant (talk of Yu. Ivanov)

New $(e,e'\gamma)$ facilities in TU Darmstadt

Conclusions

- TDR is a remarkable example of the vortical intrinsic electric nuclear flow. TDR is the general feature of atomic nuclei, Exploration of the vortical flow in nuclei is yet very poor. Study of TDR can be a first important step in solution of this problem.

- Individual toroidal states (ITS) in light nuclei as a new way to explore vortical excitations. Interplay of cluster and vortical modes.

- First results: (e,e’)-based prediction of ITS in 58Ni.

- **Outlook:** search of ITS in $(e,e'\vec{\gamma})$, sum rules, similarities with HIC, …

Thank you for attention!

For light nuclei like 24Mg, we use:

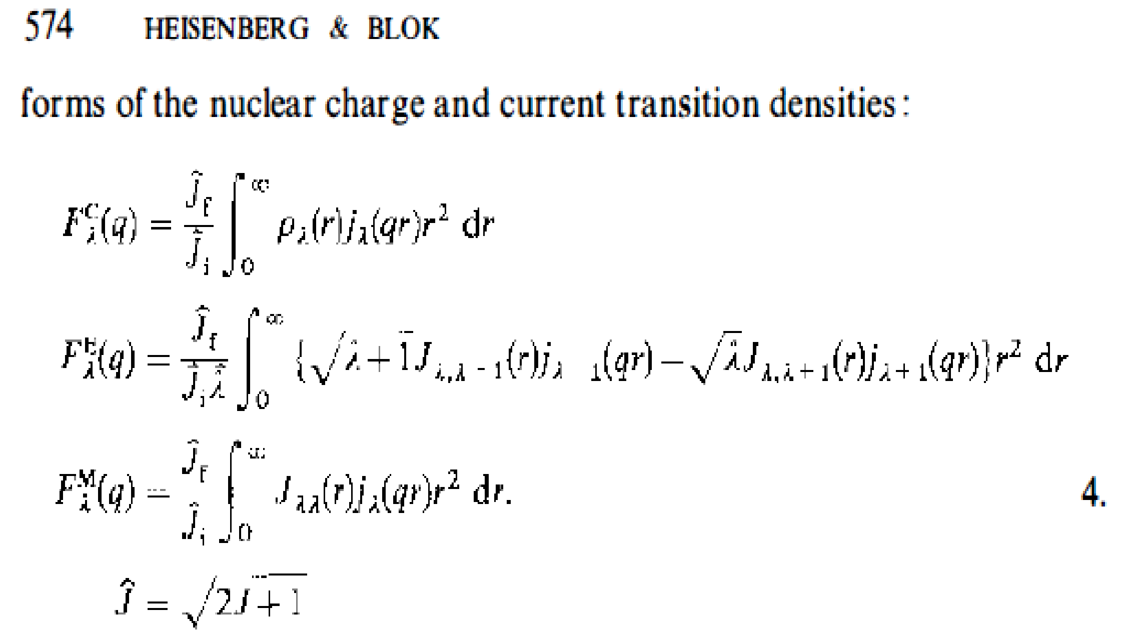
$$q_{eff} = q(1 + 1.5\frac{Z\alpha\hbar c}{E_i R})$$

- to take into account roughly the Coulomb distortions

$$R = 1.12\ A^{1/3} fm$$

$$f_{rec}(\theta, E_i) = 1$$

- no recoil



574 HEISENBERG & BLOK

forms of the nuclear charge and current transition densities:

$$F^{\mathrm{C}}_{\lambda}(q) = \frac{\hat{J}_{\mathrm{f}}}{\hat{J}_{\mathrm{i}}} \int_0^{\infty} \rho_{\lambda}(r) j_{\lambda}(qr) r^2 \,\mathrm{d}r$$

$$F^{\mathrm{E}}_{\lambda}(q) = \frac{\hat{J}_{\mathrm{f}}}{\hat{J}_{\mathrm{i}}\hat{\lambda}} \int_0^{\infty} \{\sqrt{\lambda+1} J_{\lambda,\lambda-1}(r) j_{\lambda\;1}(qr) - \sqrt{\lambda} J_{\lambda,\lambda+1}(r) j_{\lambda+1}(qr)\} r^2 \,\mathrm{d}r$$

$$F^{\mathrm{M}}_{\lambda}(q) = \frac{\hat{J}_{\mathrm{f}}}{\hat{J}_{\mathrm{i}}} \int_0^{\infty} J_{\lambda\lambda}(r) j_{\lambda}(qr) r^2 \,\mathrm{d}r. \qquad 4.$$

$$\hat{J} = \sqrt{2J+1}$$

Formfactors were calculated using transition densities and transition currents from Skyrme QRPA results

Conclusions

- TDR is a remarkable example of the vortical intrinsic electric nuclear flow. TDR is the general feature of atomic nuclei, Exploration of the vortical flow in nuclei is yet very poor. Study of TDR can be a first important step in solution of this problem.

- TDR vs PDR
  - **TDR coexists with PDR.
  - ** PDR picture (oscillations of the neutron excess against the core) is a rough imitation of the actual (basically toroidal) flow.
  - ** PDR is formed by a small irrotational fraction of mainly vortical dipole states.
  - ** PDR can be used as a doorway state in excitation of TDR.

- Individual toroidal states (ITS) in light nuclei as a new way to explore vortical excitations. Interplay of cluster and vortical modes.

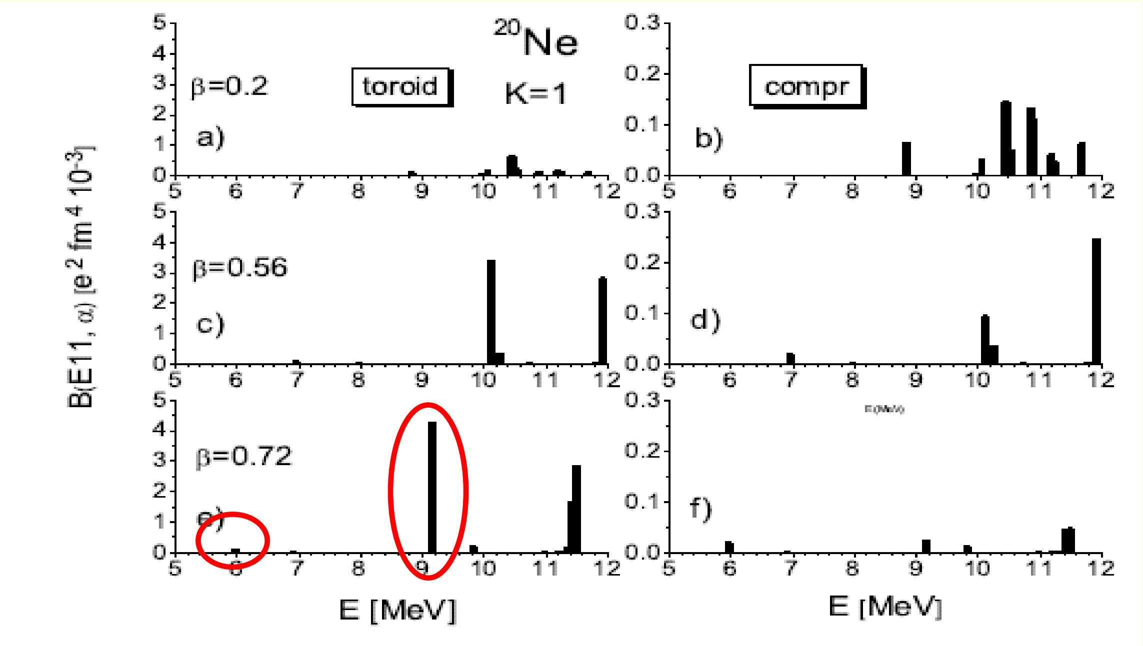
- First results: (e,e')-based prediction of ITS in 58Ni.

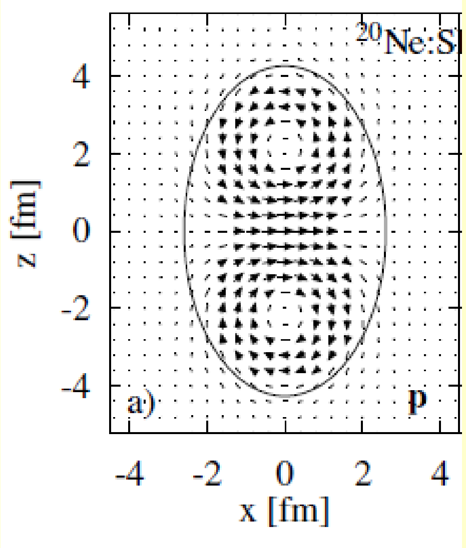
Outlook: search of ITS in $(e,e'\vec{\gamma})$, sum rules, similarities with HIC, …

20Ne

$$\beta_2^{exp} = 0.72$$

V.O. Nesterenko, J. Kvasil, A. Repko, and P.-G. Reinhard,
Eur. Phys. J. Web of Conf. 194, 03005 (2018) (2018)





TM: not lowest

P. Adsley, VON, M. Kimura, L.M. Donaldson, R. Neveling, et al, PRC 103, 044315 (2021)

Interplay of cluster and vortical modes in IS1 and IS0 states in light nuclei with a different deformation (prolate 24Mg, soft 26Mg, oblate 28Si).

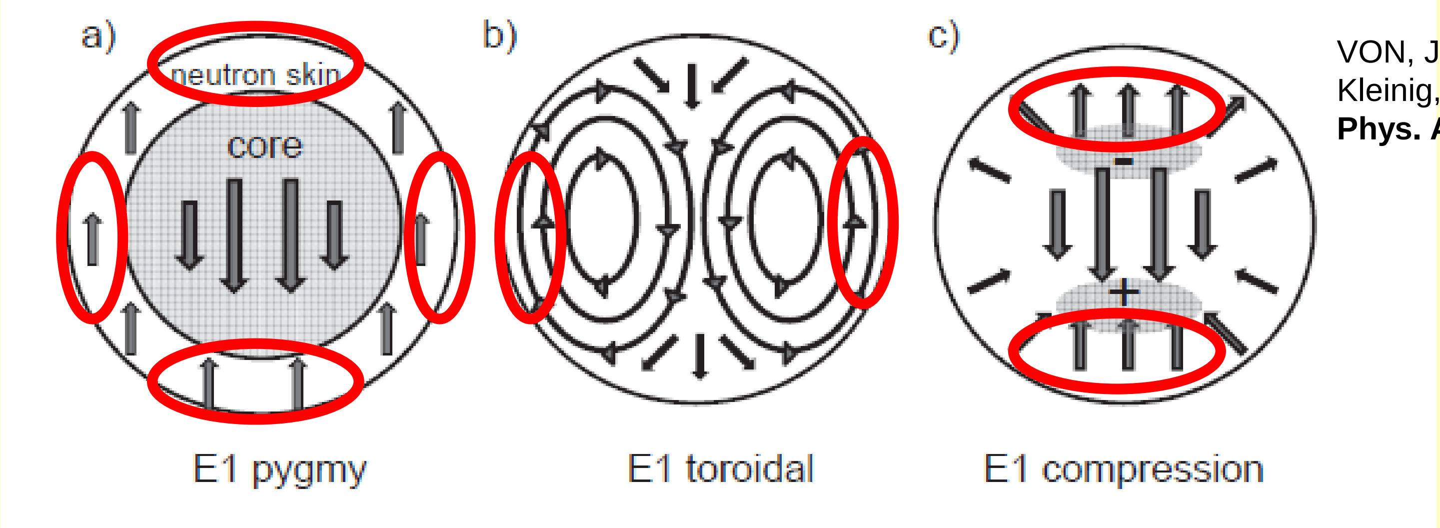
It was shown that low-energy vorticity is well localized in 24Mg, fragmented in 26Mg, and absent in 28Si.

Toroidal dipole rsonance
vs
pygmy dipole resonance

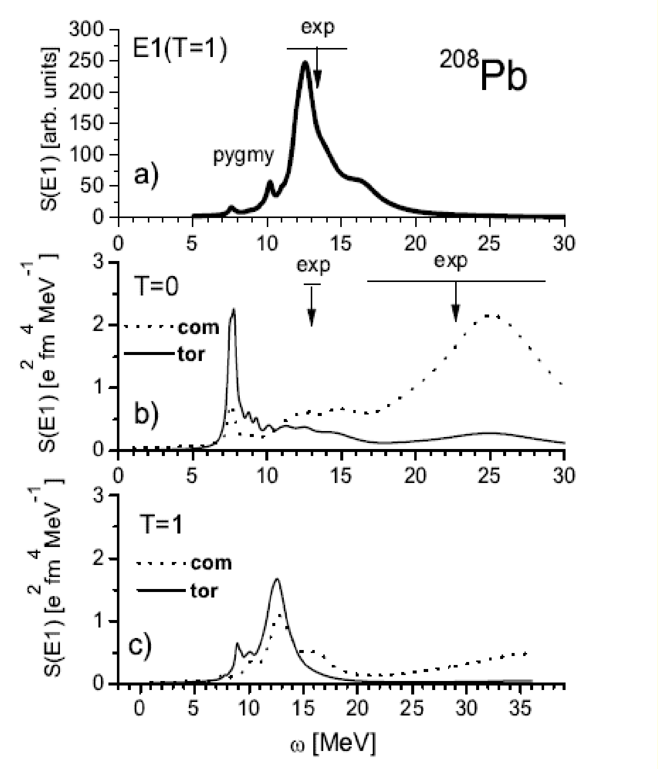
- PDR is extremely important for determination of the symmetry energy and various astrophysical reactions
- PDR and TDR are both E1 and lie in the same energy region
- So they should be related somehow

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil, Phys. Rev. C87, 024305 (2013)

A. Repko, V.O. Nesterenko, J. Kvasil, and P.-G. Reinhard, Eur. Phys. J. A 55, 242 (2019)



VON, J. Kvasil, A. Repko, W. Kleinig, P.-G. Reinhard, **Phys. Atom. Nucl.,** 79, 842 (2016).



PDR can be viewed as a local peripheral part of TDR and CDR

(e,e’): PWBA cross section

H. Theissen, Springer Tacts in Mord. Phy., 65 , 1 (1972)
J. Heisenberg and H.P. Blok, Ann. Rev. Nucl. Part. Sci, 33, 569 (1983),

$$\boxed{\sigma_{PWBA}(\theta,q)=\sigma_{Mott}(\theta,E_i)f_{rec}\{|F_E^C(q)|^2+(\frac{1}{2}+tg^2(\frac{\theta}{2})[|F_E^T(q)|^2+|F_M^T(q)|^2\}}$$

For $I^\pi=1^-$ states, $F_M^T(q)=0$ but $\hat{\vec{j}}^{\,q}_{mag}$ contributes to $F_E^T(q)$

Here we meet the problem: impact of the magnetization current

$$\hat{M}_{tor}(E1\mu)=\frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3+\frac{5}{3}r<r^2>_0]\,\vec{Y}_{11\mu}(\hat{r})\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

Nuclear current: sum of convective and magnetization terms

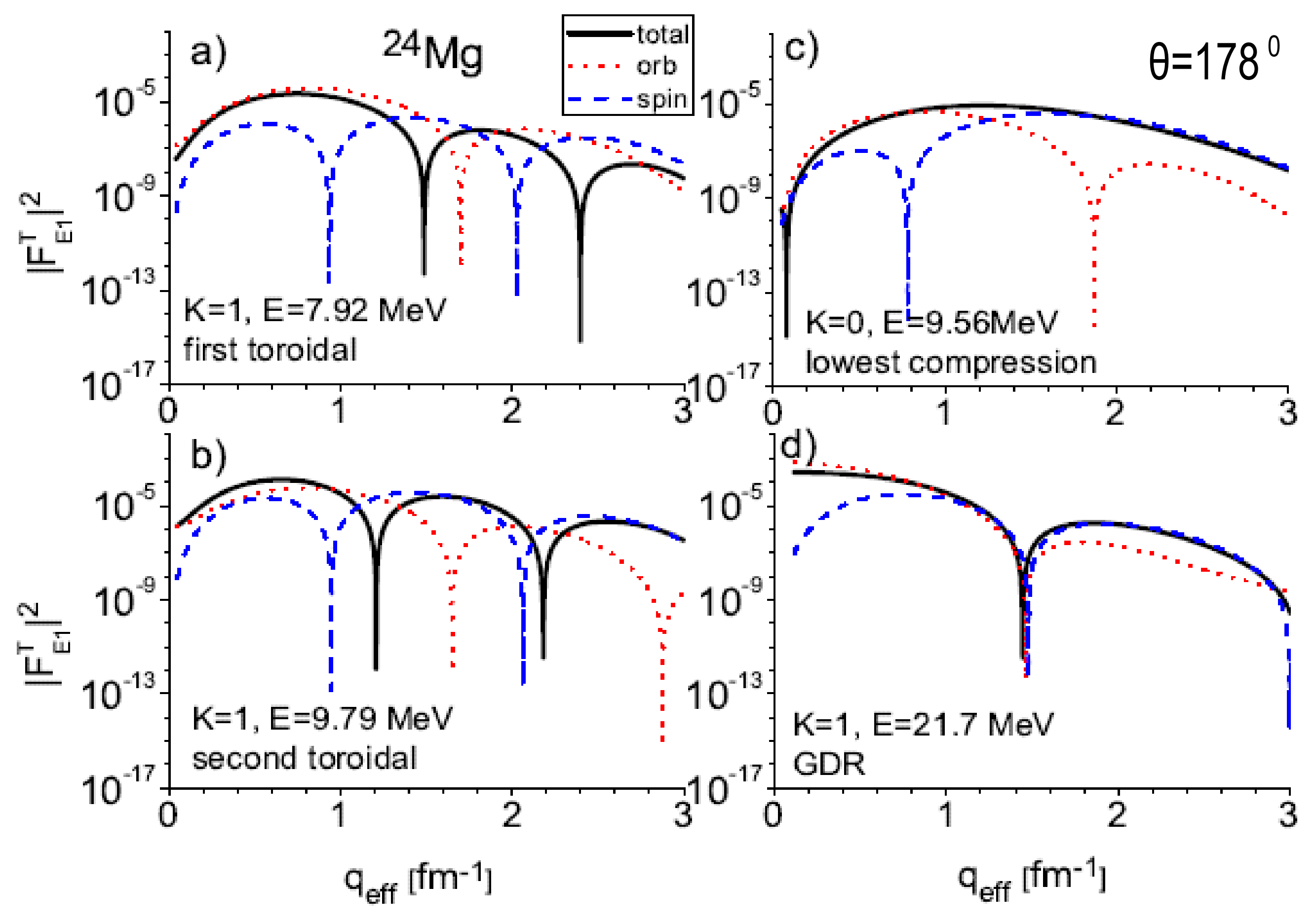
$$\hat{\vec{j}}_{nuc}(\vec{r})=\frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{\,q}_{con}(\vec{r})+\hat{\vec{j}}^{\,q}_{mag}(\vec{r}))$$

$$\hat{\vec{j}}^{\,q}_{con}(\vec{r})=-ie^q_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k-\overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right) \quad\longrightarrow\quad \text{toroidal flow}$$

$$\hat{\vec{j}}^{\,q}_{mag}(\vec{r})=\frac{g^q_s}{2}\gamma\sum_{k\ni q}\vec{\nabla}_k\times\hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k), \qquad \gamma=0.7 \quad\longrightarrow\quad \text{obstructive factor}$$

Transversal E1 form factor for different dipole states: orbital and spin contributions

V.O. Nesterenko et al,
PRC 100, 064302 (2019).



In toroidal states,
behavior of $|F^T_{E1}|^2$ is determined by the strong interference of $\vec{j}_C$ and $\vec{j}_M$ contributions. None of these contributions alone can describe $|F^T_{E1}|^2$!

A. Repko: decomposition in basis of solutions of vector Helmholtz equation

$$\vec{v}(\vec{r}) = \sum_{\lambda}\sum_{n=1}^{\infty}\sum_{\mu=-\lambda}^{+\lambda}\left[a_n^{(\lambda\mu)} j_{\lambda-1}(k_{n\lambda}r)\vec{Y}_{\lambda\mu}^{\lambda-1} + b_n^{(\lambda\mu)} j_{\lambda}(k_{n,\lambda-1}r)\vec{Y}_{\lambda\mu}^{\lambda} + c_n^{(\lambda\mu)} j_{\lambda+1}(k_{n\lambda}r)\vec{Y}_{\lambda\mu}^{\lambda+1}\right]\theta(R-r)$$

Flow of a sphere of radius R_1 inside the sphere of radius R

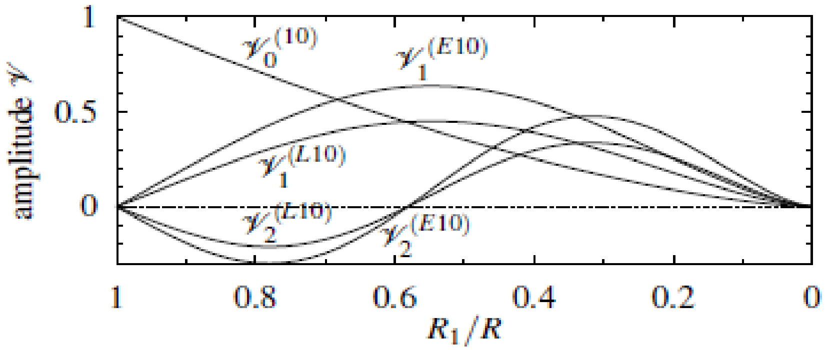


Figure 2: Decomposition of dipole mode $\vec{v} = \mathscr{V}_0^{(10)}\vec{u}_{10} + \sum_n(\mathscr{V}_n^{(L10)}\vec{v}_{n10}^{(L)} + \mathscr{V}_n^{(E10)}\vec{v}_{n10}^{(E)})$ with shrinking R_1.

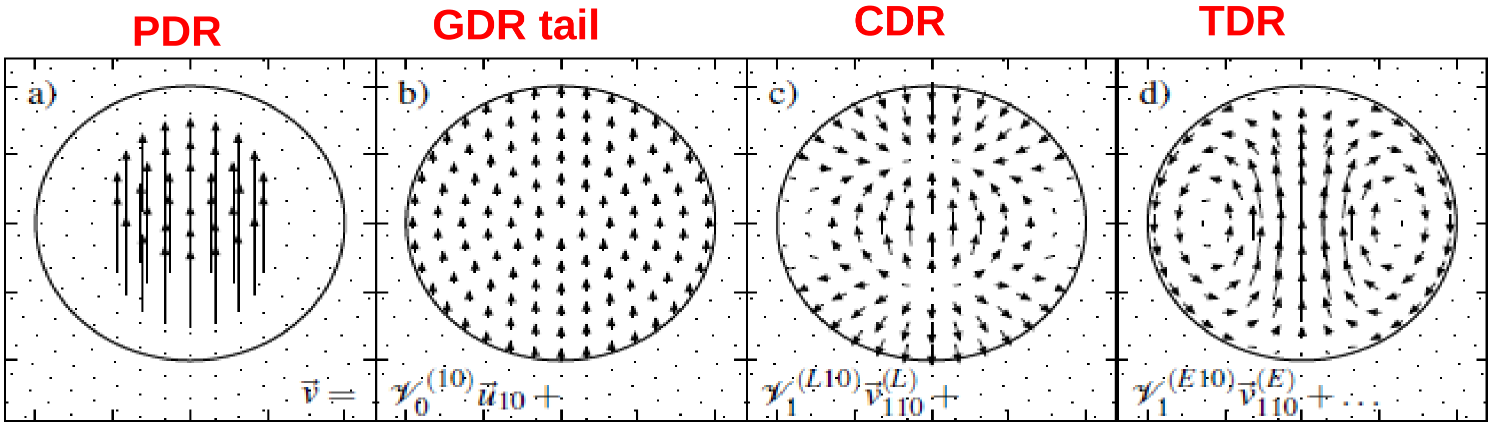
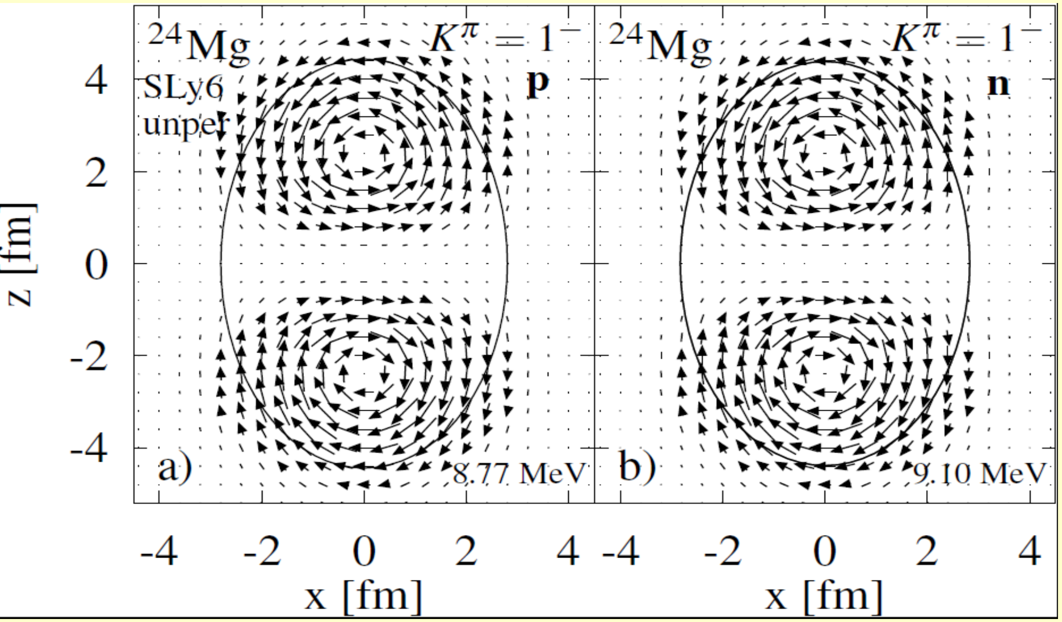


Figure 3: First terms in the decomposition of shrinked dipole zero-mode (i.e., a linear flow; $R_1 = R/2$)

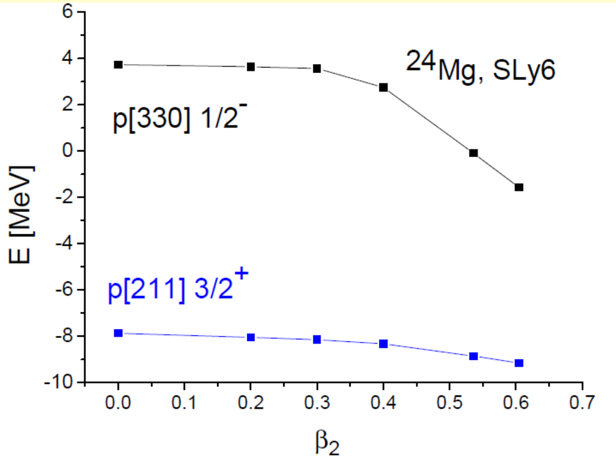
Toroidal flow: collective or 2qp origin?

flows of main 2qp configurations



$pp[211]\uparrow - [330]\uparrow$ (54%)

$nn[211]\uparrow - [330]\uparrow$ (39%)



Toroid is mainly 1ph (mean field) effect!

D.G. Ravenhall and J. Wambach,
NPA, 475, 468 (1987).

Explains the deformation effect in TS

The deformation-induced energy downshift is not universal.

Perhaps ^{24}Mg is one of very few nuclei where the toroidal mode is the lowest dipole K=1 state.

The model:

fully self-consistent Skyrme QRPA

1d, 2d QRPA (codes of A. Repko):

- fully self-consistent matrix RPA,
- ph- and pp-channels,
- cmc →

A. Repko, J. Kvasil, VON., and P.-G. Reinhard,
arXiv:1510.01248[nucl-th]
A. Repko , J. Kvasil, VON, W. Kleinig, P.-G. Reinhard
EPJA, 53, 221 (2017)

A. Repko, J. Kvasil, VON, PRC, 99, 044307 (2019)
J. Kvasil, A. Repko, VON, EPJA, 55, 213 (2019)

Mean field (SKYAX)

- 2D mesh in cylindrical coordinates
  - calculation box: 3R, mesh step 0.4 fm
  - sp levels up to +55 MeV

2qp basis (SLy6):

- 2qp states until ~ 100 MeV
- EWSR(E1,T=1), EWSR(E1,T=0): 97-100%

Pairing:

- volume monopole pairing
- BCS

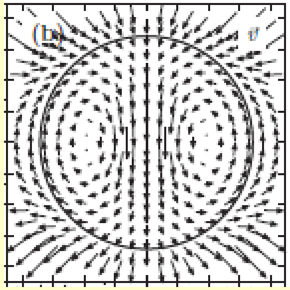
Matrix and separable QRPA:

Numerous calculations for:
GDR, PDR, GQR, GMR,
orbital and spin-flip M1,
low-energy states(2^+_γ , …)

Nuclei:
spherical and axial deformed,
from light to superheavy.

Two conceptions of nuclear vorticity : HD, RW

1. Hydrodynamical vorticity:



$$\vec{w}(\vec{r}) = \vec{\nabla}\times\vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$\vec{\nabla}\times\delta\vec{v} = \frac{\rho_0(\vec{\nabla}\times\delta\vec{j}_{nuc}) - \vec{\nabla}\rho_0\times\delta\vec{j}_{nuc}}{\rho_0^2}$$

2. RW vorticity

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$ - **continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}}[j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

$$\delta\vec{j}^{\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}}[\underbrace{j^{\nu}_{10}(r)}_{j_-}\vec{Y}^{*}_{10\mu} + \underbrace{j^{\nu}_{12}(r)}_{j_+}\vec{Y}^{*}_{12\mu}]$$

- **current transition density**

$$\boxed{j^{\nu}_{+}(r)}$$

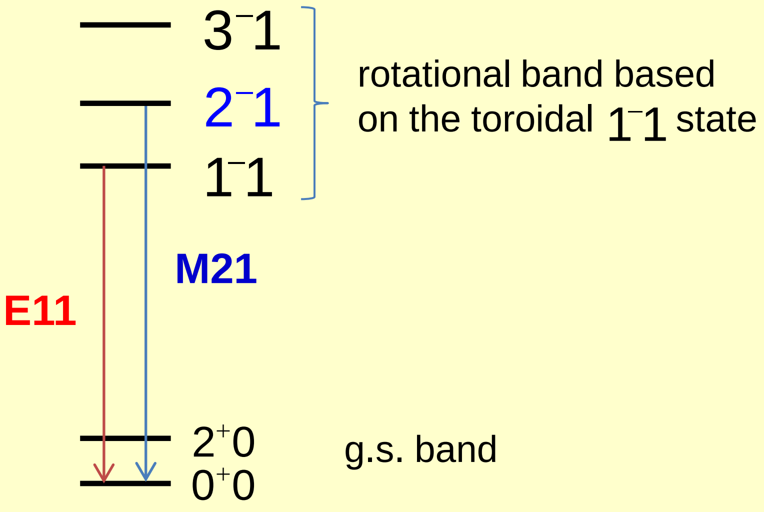
- independent part of charge-current distribution,
- decoupled from CE in the integral sense
- may be the measure of the vorticity

Two new factors to be used for identification of TM:
1) magnetic transitions (in deformed nuclei)
2) interference of the convection and spin nuclear currents

We know that the vortical orbital flow can be signified by enhanced magnetic transitions:

- M1 in scissors orbital mode,
- M2 in the twist orbital mode.

One may use this feature for the search of E1 TM in deformed nuclei:



rotational band based on the toroidal 1^-1 state

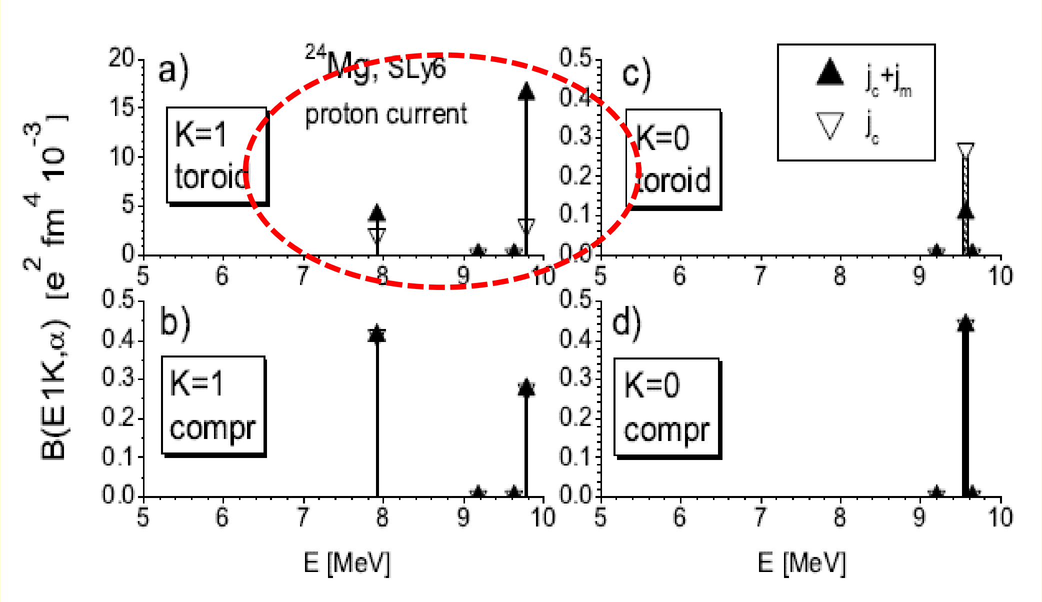
g.s. band

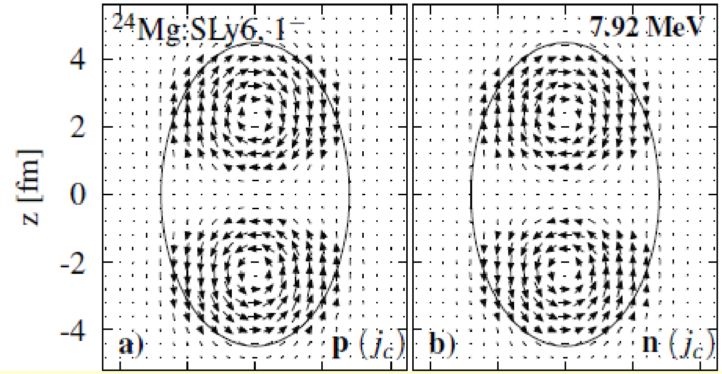
So perhaps TM can be characterized by enhanced M2(K=1) transitions

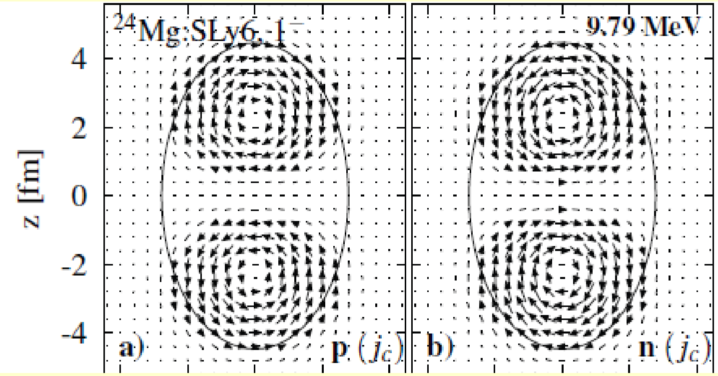
$$2^-1 \to 0^+0$$

and specific M2 form factors in (e,e')

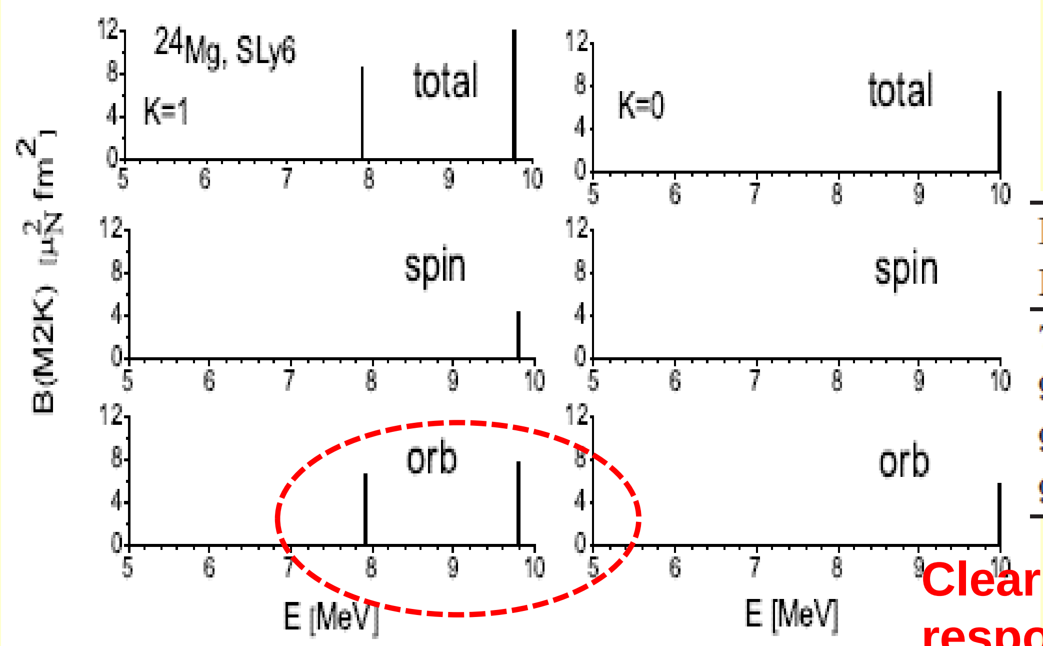
V.O. Nesterenko et al, PRC’19
arXiv: 1903.01348v2 [nucl-th]







$B(M21, 0^+0 \to 2^-1)$

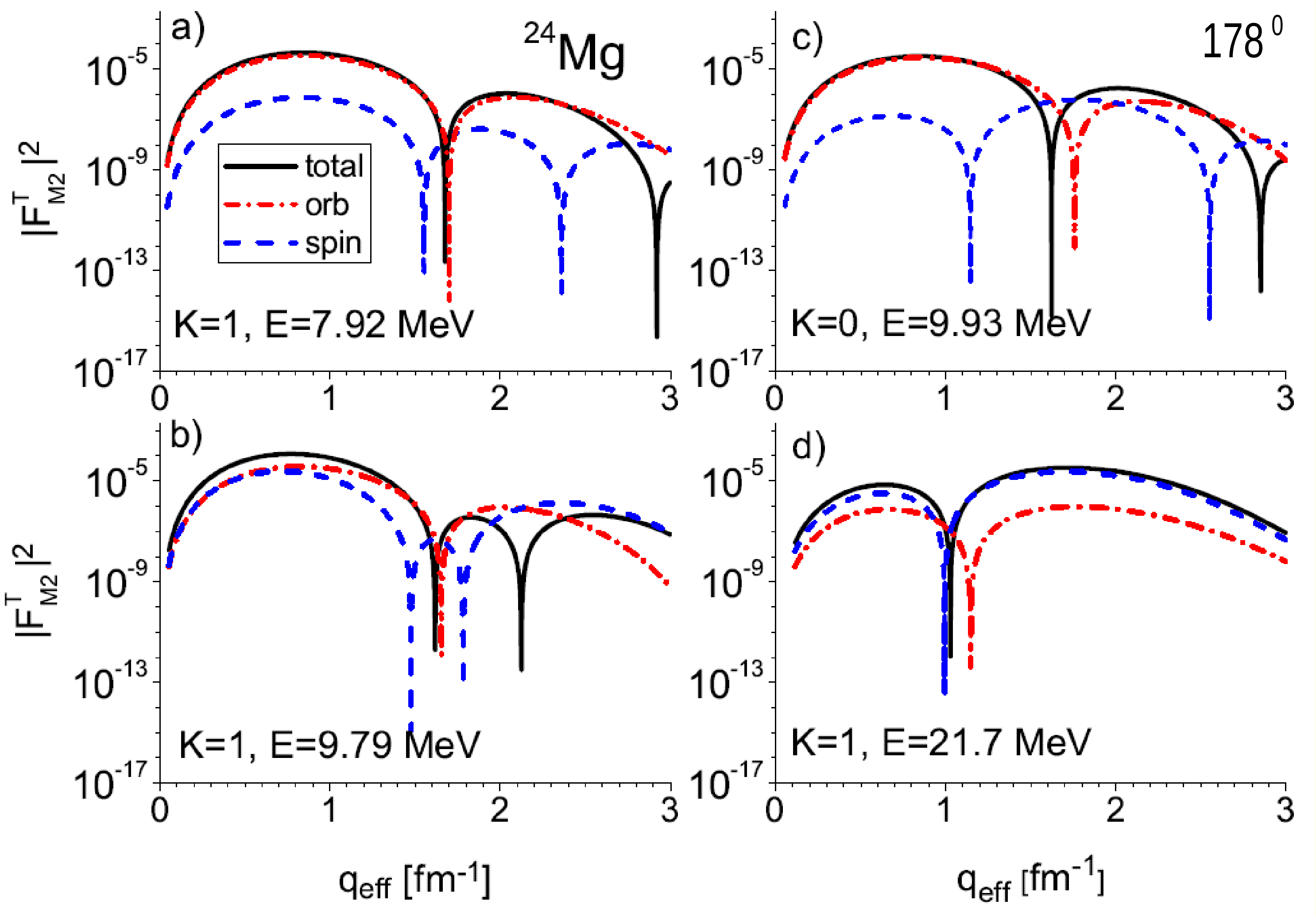


E MeV	K	$B(M2)_{\rm tot}$ W.u.	$B(M2)_{\rm spin}$ W.u	$B(M2)_{\rm orb}$ W.u.	$B(E1)$ W.u.
7.92	1	0.70	0.01	0.52	$3.2 \cdot 10^{-4}$
9.56	0	-	-	-	$2.4 \cdot 10^{-5}$
9.79	1	2.34	0.49	0.62	$4.2 \cdot 10^{-3}$
9.93	0	0.93	0.01	0.75	-

Clear correlation between toroidal E1(K=1) response and M2(K=1) response!

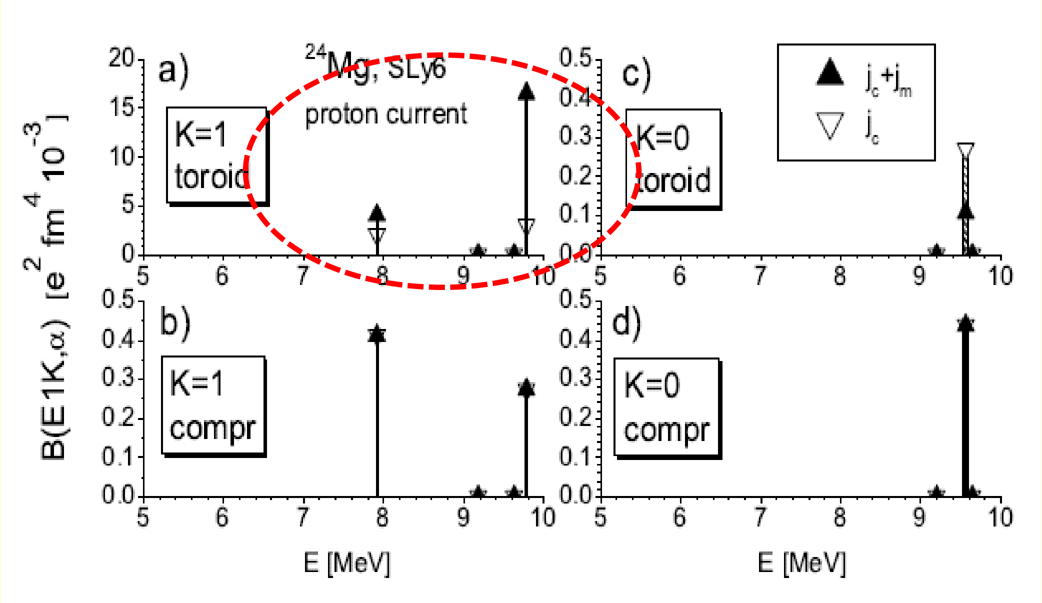
Transversal M2 form factor for different dipole states: orbital and spin contributions

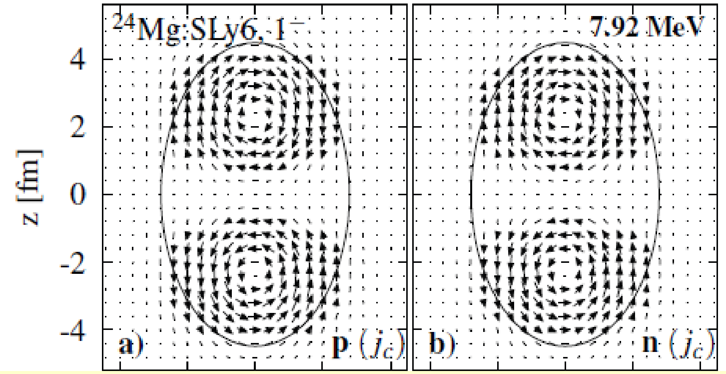
V.O. Nesterenko et al, PRC’19
arXiv: 1903.01348v2 [nucl-th]

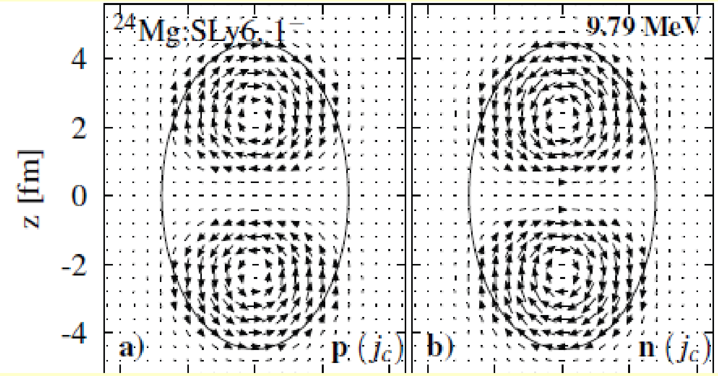


In toroidal states, M2 form factor is determined by the orbital contribution alone or by its interference pattern.

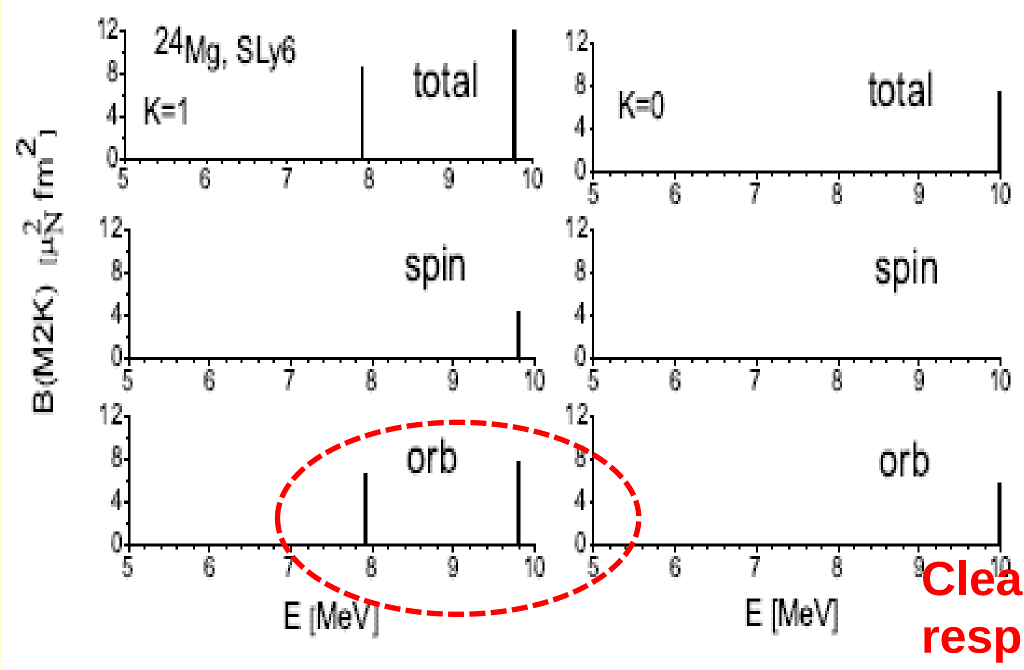
V.O. Nesterenko et al, PRC’19
arXiv: 1903.01348v2 [nucl-th]







$B(M21, 0^+0 \to 2^-1)$



E MeV	K	$B(M2)_{\rm tot}$ W.u.	$B(M2)_{\rm spin}$ W.u	$B(M2)_{\rm orb}$ W.u.	$B(E1)$ W.u.
7.92	1	0.70	0.01	0.52	$3.2 \cdot 10^{-4}$
9.56	0	-	-	-	$2.4 \cdot 10^{-5}$
9.79	1	2.34	0.49	0.62	$4.2 \cdot 10^{-3}$
9.93	0	0.93	0.01	0.75	-

Clear correlation between toroidal E1(K=1) response and M2(K=1) response!