Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

Cancellation of the sigma mode in the thermal pion gas by quark Pauli blocking

D. Blaschke, A. Friesen, Yu. Kalinovsky

INFINUM-2023
02.03.2023

Content

1. Introduction
2. The interactiong pion gas
3. Analytic justification for the cancellation
4. Mesons at finite temperature in the Nambu – Jona-Lasinio model
5. Results and conclusion

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

Introduction

The thermodynamics of the hadron resonance gas (HRG) is of crucial importance for the interpretation of the results of the ab-initio evaluation of the QCD partition function by simulations of the lattice gauge theory as well as for the explanation of yields of hadrons produced in ultrarelativistic heavy-ion collisions at the chemical freeze-out.

This HRG model makes the simplifying assumption that instead of accounting for the interactions among hadrons one may just evaluate the statistical sum over all known resonances in the spectrum of hadrons as eigenstates of the QCD Hamiltonian.

While such a procedure would account for the attractive interaction leading to the formation of resonances in the spectrum, one may additionally account for repulsion by invoking an excluded volume of the hadrons treated.

A simple model system for the study of the interplay between attraction and repulsion in hot hadronic matter is the interacting pion gas at finite temperature.

One can use the Beth-Uhlenbeck approach with the well measured phase shifts of the pion-pion interactions in free space that allow to evaluate the second virial coefficient of the partition function.

The contributions from phase shifts in the attractive isospin-zero σ-meson channel (δ_0^0) and in the repulsive isospin-2 channel (δ_0^2) largely compensate each other in the partition function.

On the quark level of description, the repulsion in the $\pi\pi$ scattering is due to the quark exchange interaction between pions, represented on the quark one-loop level by the so-called box diagrams.

It can be shown that this quark exchange interaction, also denoted as quark Pauli blocking, leads to a repulsive phase shift well in accordance with the experimental data.

The inclusion of the σ-meson to the HRG gives a significant improvement of the statistical model description of the "horn" structure in the beam energy dependence of the kaon to pion ratio K^+/π^+.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

In the Beth-Uhlenbeck (or S-matrix) formulation of the thermodynamics of the interacting HRG a cancellation of the attractive δ_0^0 channel (σ-meson) against the repulsive δ_0^2 channel (Pauli blocking) occurs.
A sigma meson has to be considered as an important degree of freedom in the statistical model of particle production where a sudden (chemical) freeze-out of particle species occurs in the vicinity of the QCD chiral restoration/quark deconfinement temperature.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

When formulating the hadron resonance gas in the S-matrix formalism one has to take into account the medium effects on the scattering phase shifts, i.e. to apply the generalized Beth-Uhlenbeck approach.

This should then in particular take into account the effects of chiral symmetry restoration, namely that the sigma meson becomes degenerate with the pion above the chiral restoration transition that entails dropping quark masses.

Before the sigma meson mass becomes degenerate with that of the pion it has to cross the two-pion mass threshold where the strong two-pion decay channel of the sigma meson closes and it becomes a sharp resonance.

For a resonance at the threshold the scattering length approximation breaks down, while a Breit-Wigner ansatz for the phase shifts will be appropriate.

The dropping quark masses entail a Mott effect for both, the pion and the sigma.

This character change of the mesons from bound states to resonances in the continuum leads to the ceasing of the quark Pauli blocking effect and thus the resulting phase shift should turn to zero.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

The interactiong pion gas

The virial expansion of the grand canonical partition function of the system with the known S-matrix for two-particle scattering can be written as a virial expansion [1]

$$\mathrm{ln}Z = \mathrm{ln}Z_0 + \sum_{i_1,i_2} z_1^{i_1} z_2^{i_2} b(i_1, i_2),$$

where $z_j = \exp(\beta\mu_j)$ for $j = 1, 2$ and $b(i_1, i_2)$ is the second virial coefficient defined by the S-matrix with labels i_1, i_2 referring to a channel of the S-matrix with $i_1 + i_2$ particles in initial state.

[1] R. Venugopalan and M. Prakash, Nucl. Phys. A 546, 718 (1992); G. M. Welke, R. Venugopalan, and M. Prakash, Phys. Lett. B 245, 137 (1990).

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

$$b(i_1, i_2) = \frac{V}{4\pi i} \int \frac{d^3P}{(2\pi)^3} \int d\epsilon \exp(-\beta\sqrt{P^2+\epsilon^2}) \mathrm{Tr}_{i_1,i_2}\left[AS^{-1}\frac{\overleftrightarrow{\partial}}{\partial\epsilon}S \right].$$

Here $\beta = T^{-1}$ is the inverse temperature, V is the volume, P is the centre-of-mass (total) momentum and ϵ the energy of the two-particle system.

The symbol A denotes the symmetrization/antisymmetrization operator for a system of bosons/fermions.

The trace is taken over all combinations of particle number.

For a one-component system under the assumption that hadrons interact mainly via elastic collisions, the second virial coefficients can be simplified by chosing the representation of the S-matrix in terms of the two-particle phase shifts.

The lowest virial coefficient b_2 corresponds to the case $i_1 = i_2 = 1$.

$$b_2 = \frac{1}{2\pi^3\beta}\int_M^\infty d\epsilon\epsilon^2 K_2(\beta\epsilon)\sum_{l,I}{}' g_{l,I}\frac{\partial\delta_l^I(\epsilon)}{\partial\epsilon},$$

with the modified Bessel function K_2 and the degeneracy factor $g_{l,I} = (2l+1)(2I+1)$. M is the invariant mass of the interacting pair at threshold.
For given l the sum over I is restricted to values consistent with statistics.
In the limit that only the second virial coefficient is considered, the interaction pressure (as well as all the other thermodynamic variables) can be obtained in the form of a Beth-Uhlenbeck equation,

$$P_{int} = Tz^2 b_2 = \frac{z^2}{2\pi^3\beta^2}\int_M^\infty d\epsilon\epsilon^2 K_2(\beta\epsilon)\sum_{l,I}{}' g_{l,I}\frac{\partial\delta_l^I(\epsilon)}{\partial\epsilon}.$$

Content

Introduction

The interacting pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

For the one component pion gas, the total center of mass energy is chosen as $\epsilon = 2(\mathrm{q}^2 + \mathrm{M}_\pi^2)^{1/2}$ and the threshold mass is $\mathrm{M} = 2\mathrm{M}_\pi$. For the case $\delta_\mathrm{l}^\mathrm{I} \to 0$ at low energies $\epsilon \to \mathrm{M}$ the equation for the second virial coefficient b_2 can be simplified using integration by parts to

$$\mathrm{b}_2 = \frac{1}{2\pi^3}\int_{2\mathrm{M}_\pi}^{\infty} \mathrm{d}\epsilon\epsilon^2 \mathrm{K}_1(\beta\epsilon)\sum_{\mathrm{l,I}}{}' \mathrm{g}_{\mathrm{l,I}}\delta_\mathrm{l}^\mathrm{I}(\epsilon).$$

The corresponding pressure contribution from the two-particle interactions is

$$\mathrm{P}_\mathrm{int} = \mathrm{P}^{00} + \mathrm{P}^{02} + \mathrm{P}^{11} ,$$

where the partial pressure contributions at vanishing chemical potential $(\mathrm{z} = 1)$ are defined by the phase shift,

$$\mathrm{P}^{\mathrm{lI}} = \frac{\mathrm{g}_{\mathrm{l,I}}}{2\pi^3\beta}\int_{2\mathrm{M}_\pi}^{\infty} \mathrm{d}\epsilon\epsilon^2 \mathrm{K}_1(\beta\epsilon)\delta_\mathrm{l}^\mathrm{I}(\epsilon).$$

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

The phase shift in free space (vacuum) can be obtained from experiments and be compared with theoretical models for it. The low-energy $\pi\pi$ interaction includes the $\delta_0^0, \delta_1^1, \delta_0^2$ phase shifts. The repulsive isospin-2, S-wave phase shift can be well described in the scattering length approximation by $\delta_0^2 = -0.12\mathrm{q}/\mathrm{M}_\pi$. The phase shift δ_1^1 contains the ρ-meson resonance and δ_0^0 contains the σ-meson resonance. These resonant phase shifts can be chosen in the simple Breit-Wigner form,

$$\begin{aligned}\delta_0^0(\epsilon) &= \frac{\pi}{2} + \arctan\left(\frac{\epsilon - \mathrm{M}_\sigma}{\Gamma_\sigma/2}\right),\\ \delta_1^1(\epsilon) &= \frac{\pi}{2} + \arctan\left(\frac{\epsilon - \mathrm{M}_\rho}{\Gamma_\rho/2}\right).\end{aligned}$$

For the case $\mathrm{M}_\pi = 0.138$ GeV, $\mathrm{M}_\sigma = 5.8\ \mathrm{M}_\pi$, $\mathrm{M}_\rho = 5.53\ \mathrm{M}_\pi$ and

$$\begin{aligned}\Gamma_\sigma &= \beta_\sigma \mathrm{q},\\ \Gamma_\rho &= \beta_\rho \mathrm{q}\left(\frac{\mathrm{q}/\mathrm{M}_\pi}{1 + (\mathrm{q}/\mathrm{M}_\rho)^2}\right)^2,\end{aligned}$$

with $\beta_\sigma = 2.06$ and $\beta_\rho = 0.095$.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

In the work [2] the phase shift for $\mathrm{I} = 2$ low-energy $\pi\pi$ scattering was obtained within a diagrammatic approach to the quark exchange interaction between $\mathrm{q\bar{q}}$ mesons in the framework of a nonrelativistic potential model for their quark substructure [3] The phase shift, obtained by the so-called quark Born diagrams has the form

$$\sin\delta_0^2 = -\left\{ \frac{\alpha_\mathrm{s}}{9\lambda \mathrm{m}^2} \frac{\sqrt{\mathrm{q}^2 + \mathrm{M}_\pi^2}}{\mathrm{q}} \left(1 - \mathrm{e}^{-2\lambda \mathrm{q}^2} + \frac{16\lambda \mathrm{q}^2}{3\sqrt{3}} \mathrm{e}^{-4\lambda \mathrm{q}^2/3} \right) \right\}.$$

The parameter α_s is fixed to the value $\alpha_\mathrm{s}/\mathrm{m}^2 = 4.48\ \mathrm{GeV}^{-2}$ so that $\alpha_\mathrm{s} = 0.71$ for $\mathrm{m} = 0.40$ GeV. The parameter λ is related to the Gaussian wave function of the nucleon in the usual simple-harmonic-oscillator (SHO) model with the quark-model parameter $\beta_\mathrm{SHO} = 1/(2\sqrt{\lambda}) = 0.337$.

[2] T. Barnes and E. S. Swanson, Phys. Rev. D 46, 131 (1992).
[3] D. Blaschke and G. RBïopke, Phys. Lett. B 299, 332 (1993).

Content

Introduction

The interacting pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

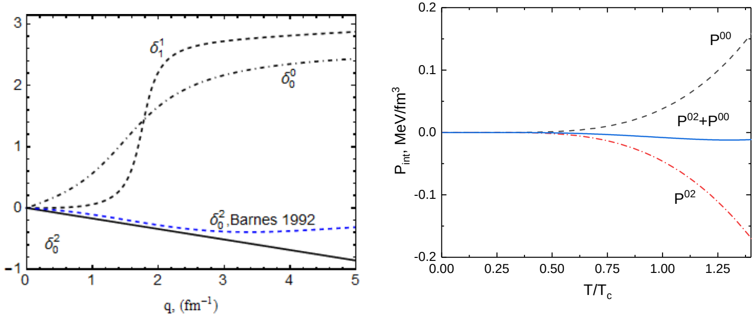

Figure 1: Left panel: $\pi\pi$-scattering phase shifts as functions of the total center-of-mass-momentum. The blue dashed line corresponds to the phase shift δ_0^2 calculated using Eq. (1). Right panel: Interaction contribution to the pressure according to the Beth-Uhlenbeck formula with the medium-independent phase shifts of the left panel (without the contribution from the ρ-meson channel). The almost perfect cancellation of the σ-meson (green dashed line) by the quark Pauli blocking (red dashed-dotted line) is demonstrated by the blue solid line.

Analytic justification for the cancellation

The fact of the σ cancellation at low energies and the robustness of this result at finite temperatures despite the in-medium modification of the phase shifts can be understood also analytically in the low-momentum expansion of the phase shifts of Eqs. (1) and (1) that introduces the scattering lengths $\mathrm{a}_\mathrm{l}^\mathrm{I}$,

$$\left.\frac{\mathrm{d}\delta_\mathrm{l}^\mathrm{i}}{\mathrm{dq}}\right|_{\mathrm{q}=0} \approx \mathrm{a}_\mathrm{l}^\mathrm{I}.$$

The σ meson scattering length is then

$$\mathrm{a}_0^0 = \frac{\beta_\sigma}{2(\mathrm{M}_\sigma - 2\mathrm{M}_\pi)}, \quad \beta_\sigma = 2.06,$$

which becomes singular in the vicinity of the chiral transition, when $\mathrm{M}_\sigma \to 2\mathrm{M}_\pi$ and the dominant $\sigma \to 2\pi$ decay channel closes so that the σ meson becomes a good resonance at the threshold. This is the limitation for the present considerations because the scattering length approximation breaks down in this case.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

The scattering length for the phase shift of the quark exchange process (quark Pauli blocking) from the Eq.(1) gives

$$a_0^2 = -\frac{2}{9}\left(1 + \frac{8}{3\sqrt{3}}\right)\alpha_s \frac{M_\pi}{m^2},$$

so that we obtain for the relevant ratio of scattering lengths

$$\begin{aligned}
\frac{a_0^0}{5a_0^2} &= \frac{1.03}{-2M_\pi + M_\sigma}\frac{1}{5}\left(-\frac{0.565\,\alpha_s M_\pi}{m^2}\right)^{-1} \\
&= -\frac{1.03}{-2M_\pi^2 + M_\sigma M_\pi}\frac{m^2}{2.82\alpha_s} \\
&= \frac{1.03}{2(M_\pi - M_\sigma/4)^2 - M_\sigma^2/8}\frac{m^2}{2.82\alpha_s} \approx -\frac{4m^2}{1.41\,\alpha_s M_\sigma^2},
\end{aligned}$$

where in the last step the approximation $(m_\pi - m_\sigma/4)^2 \approx 0$ has been used.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

The wanted result of the cancellation by quark Pauli blocking of the σ meson contribution to the thermodynamics of the pion gas is obtained when $\alpha_s \sim 0.7$.
Then, because $M_\sigma \approx 2m$, the result is $a_0^0/5a_0^2 = -0.99$, which means the total cancellation of contributions from $\sigma-$ channel by the contribution from the repulsive Pauli-blocking channel.
For the parameter set given by Welke [4] $\delta_0^0/5\delta_0^2 = -1.18$ and for the Weinberg results $a_0 = 0.158$ and $a_2 = -0.045$ follows $a_0^0/5a_0^2 = -0.708$ [5].

[4] G. M. Welke, R. Venugopalan, and M. Prakash, Phys. Lett. B 245, 137 (1990).

[5] S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)

Mesons at finite temperature in the Nambu – Jona-Lasinio model

We employ here the NJL model with two flavours of quarks defined by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm NJL} = \bar{\rm q}\left({\rm i}\gamma_\mu\partial^\mu - \hat{\rm m}_0\right){\rm q} \quad &+\quad {\rm G_s}\left[\left(\bar{\rm q}{\rm q}\right)^2 + \left(\bar{\rm q}{\rm i}\gamma_5\vec{\tau}{\rm q}\right)^2\right]\\ &-\quad {\rm G_v}\left[(\bar{\rm q}\gamma^\mu\tau^{\rm a}{\rm q})^2 + (\bar{\rm q}\gamma^\mu\gamma^5\tau^{\rm a}{\rm q})^2\right],\end{aligned}$$

with chirally symmetric four-quark interactions in the scalar, pseudoscalar, vector and axial-vector channels. $\rm G_s$ and $\rm G_v$ are the scalar and vector coupling constants, $\bar{\rm q}$ and q - the quark spinor fields, $\hat{\rm m}_0$ is the diagonal matrix of the current quark mass, $\hat{\rm m}_0 = {\rm diag}({\rm m}_{\rm u}^0, {\rm m}_{\rm d}^0)$ with ${\rm m}_{\rm u}^0 = {\rm m}_{\rm d}^0 = {\rm m}_0$, and $\overrightarrow{\tau}$ are the SU(2) Pauli matrices in flavor space with the components $\tau^{\rm a}({\rm a} = 1, 2, 3)$.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

In the mean-field approximation the constituent quark mass is obtained by solving the gap equation at the mean field level

$$m = m_0 + 8G_s N_c N_f \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \left[1 - f(E^+) - f(E^-)\right] ,$$

where the dependence on temperature and chemical potential is modeled in the Fermi-functions $f(E^\pm) = (1+e^{\beta E_p^\pm})^{-1}$ and the quark (antiquark) energy dispersion relation $E_p^\pm = E_p \pm \mu$.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

Mesons are considered as quark-antiquark bound states and their properties are described by the Bethe-Salpeter equation in the pole approximation

$$1 - 2\mathrm{G_s}\ \Pi_\mathrm{M}(\mathrm{k}^2)|_{\mathrm{k}^2=\mathrm{M}_\mathrm{M}^2} = 0 \ \ , \ \mathrm{M} = \pi, \sigma, \ldots$$

with the polarization operator $\Pi_\mathrm{M}(\mathrm{k}^2)$ determining the meson properties being defined as

$$\Pi_\mathrm{M}(\mathrm{k}^2) = \mathrm{i} \int \frac{\mathrm{d}^4\mathrm{p}}{(2\pi)^4}\ \mathrm{Tr}\ [\Gamma_\mathrm{M} \mathrm{S}(\mathrm{p}+\mathrm{k}) \Gamma_\mathrm{M} \mathrm{S}(\mathrm{p})] \,,$$

where the vertex factor Γ_M depends on the meson species $\mathrm{M} = \pi$, σ, ρ and a_1.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

For the pseudo-scalar π meson $\Gamma_\pi = i\gamma_5\tau^a$ and for the scalar σ meson $\Gamma_\sigma = 1\tau^a$; $S(q)$ is the quark propagator and the trace is being taken over color, flavor and spinor indices. For mesons at rest ($P = 0$) in the medium, these conditions correspond to the equations:

$$\begin{aligned} 1 + 8G_sN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \frac{E_p^2}{M_\pi^2 - 4E_p^2} \left(1 - f(E^+) - f(E^-)\right) &= 0, \\ 1 + 8G_sN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \frac{E_p^2 - m^2}{M_\sigma^2 - 4E_p^2} \left(1 - f(E^+) - f(E^-)\right) &= 0 \,. \end{aligned}$$

Generally, the pole mass equation can be extended to the vector and axial-vector case and the set of equations together with an equation for the vector meson is solved self-consistently. To simplify the calculations for the ρ meson, the relations suggested in the works [6] are used. The mass of the ρ-meson and its width can be calculated as

$$
\begin{aligned}
\mathrm{M}_\rho^2 &= \frac{\mathrm{g}_{\rho\mathrm{qq}}^2}{4\mathrm{G_v}}, \\
\mathrm{g}_{\rho\mathrm{qq}} &= \sqrt{6}\,\mathrm{g}_{\sigma\mathrm{qq}}, \\
\Gamma_{\rho\pi\pi} &= \frac{\mathrm{g}_{\rho\pi\pi}^2}{48\pi\mathrm{M}_\rho^2}\sqrt{\left(\mathrm{M}_\rho^2 - 4\mathrm{M}_\pi^2\right)^3}.
\end{aligned}
$$

[6] M. K. Volkov, D. Ebert, and M. Nagy, Int. J. Mod. Phys. A 13, 5443 (1998), M. K. Volkov and A. E. Radzhabov, Phys. Usp. 49, 551 (2006), D. Ebert, Y. L. Kalinovsky, L. Munchow, and M. K. Volkov, Int. J. Mod. Phys. A 8, 1295 (1993).

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

The decay width $\Gamma_{\sigma\pi\pi}$ within the NJL model is defined by the triangle Feynman diagram treating the sigma meson as quark-antiquark system [7]:

$$\Gamma_{\sigma\pi\pi} = \frac{3}{2}\ \frac{(2g_{\sigma qq}g^2_{\pi qq}A_{\sigma\pi\pi}(T,\mu))^2}{16\pi\ M_\sigma}\sqrt{1-\frac{4M^2_\pi}{M^2_\sigma}}\ ,$$

where the factor 3/2 takes into account the isospin conservation and $g_{\sigma qq}$ and $g_{\pi qq}$ are coupling constants. The amplitude of the triangle vertex $A_{\sigma\to\pi\pi}$ is

$$A_{\sigma\pi\pi} = \int \frac{d^4q}{(2\pi)^4}\ \mathrm{Tr}\{S(q)\ \Gamma_\pi\ S(q+P)\ \Gamma_\pi\ S(q)\}.$$

The kinematic factor $\sqrt{1-4M^2_\pi/M^2_\sigma}$ leads to the constraint $M_\sigma > 2M_\pi$, if this condition is broken, the decay $\sigma\to\pi\pi$ is forbidden and the σ meson becomes a good bound state of the $\pi\pi$ interaction.

[7] P. Zhuang and Z. Yang, Chin. Phys. Lett. 18, 344 (2001), A. V. Friesen, Y. L. Kalinovsky, and V. D. Toneev, Phys. Part. Nucl. Lett. 9, 1 (2012), 1104.2698.

Content

Introduction

The interacting pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

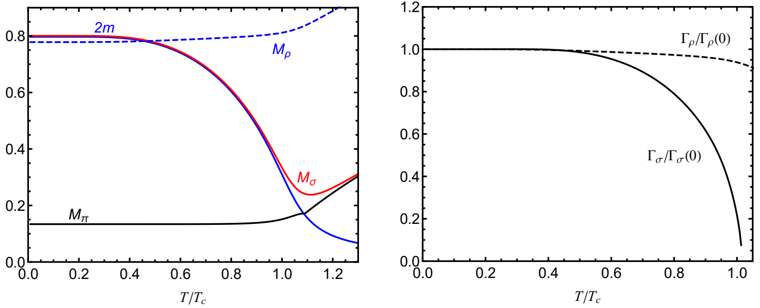

Figure 2: Left panel: the temperature dependence $\mathrm{M_i(T)}$ of mass spectra for mesons $(\mathrm{i} = \pi, \sigma, \rho)$ and doubled quark mass m(T)). Right panel: scaled temperature dependence of total decay widths $\Gamma_\mathrm{i}(\mathrm{T})/\Gamma_\mathrm{i}(0)$ for mesons $(\mathrm{i} = \sigma, \rho)$ calculated within NJL model.

Content

Introduction

The interacting pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

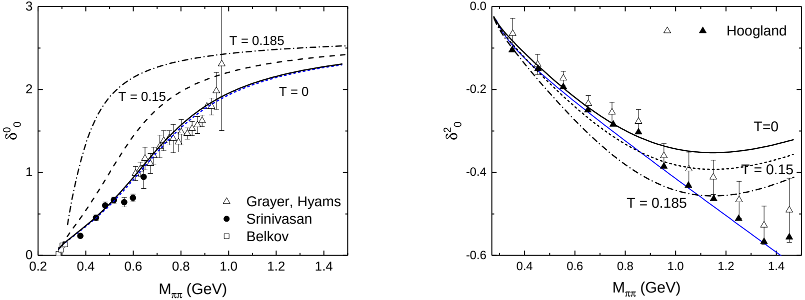

Figure 3: The phase shifts δ_0^0 (left panel) and δ_0^2 (right panel) for different temperatures T = 0, 0.15, 0.185 GeV. The solid blue line in the right panel corresponds to the scattering length approximation $\delta_0^2 = -0.12\mathrm{q}/\mathrm{M}_\pi$.

Experimental data are taken from [8] for δ_0^0 and from [9]

[8] A. A. Belkov, et.al., JETP Lett. 29, 597 (1979); G. Grayer et al., Nucl. Phys. B 75, 189 (1974); B. Hyams et al., Nucl. Phys. B 64, 134 (1973); V. Srinivasan et al., Phys. Rev. D 12, 681 (1975)

[9] W. Hoogland et al., Nucl. Phys. B 126, 109 (1977)

Content

Introduction

The interacting pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

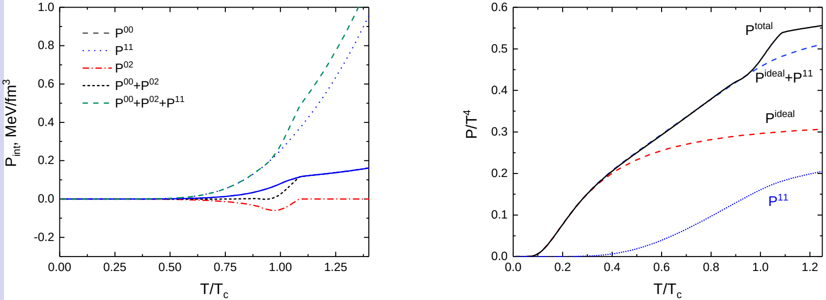

Figure 4: Left panel: Contributions to the pressure of the pion gas at second order of the virial expansion ($\pi\pi$ scattering) using the Beth-Uhlenbeck equation with medium dependent phase shifts. The quark Pauli blocking term P^{02} (blue solid line) is modified to take into account that it vanishes when the pion bound state is dissociated. Right panel: Pressure of the ideal pion gas (red dashed line), compared to the total pressure with all three interaction channels (black solid line) and with just the ρ-meson channel (blue dotted line). The ρ-meson contribution is also shown separately by the blue dotted line.

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

Results and conclusion

The contribution to the pressure from the S-wave channel is small due to an approximate cancellation of the attractive (I = 0) sigma resonance contribution against the repulsive (I = 2) contribution that is explained by quark Pauli blocking.
Using the simple Breit-Wigner approximation for the meson phase shifts [10] and the nonrelativistic potential model result [?] [11] for the phase shift in the repulsive channel (I=2) together with the temperature dependence of the mass spectra from the NJL model, we show that this cancellation appears not only in low-energy and low-temperature region as it was discussed in [12] but also takes place at finite temperature as long as $T \lesssim 0.75 T_c$.

[10] M. Welke, R. Venugopalan, and M. Prakash, Phys. Lett. B 245, 137 (1990).

[11] T. Barnes and E. S. Swanson, Phys. Rev. D 46, 131 (1992).

[12] W. Broniowski, F. Giacosa, and V. Begun, Phys. Rev. C 92, 034905 (2015)

Content

Introduction

The interactiong pion gas

Analytic justification for the cancellation

Mesons at finite temperature in the Nambu – Jona-Lasinio model

Results and conclusion

Table 1: The relation $a_0^0/(5a_2^0)$ in the frame of NJL model.

	T=0	T = 0.14	T = 0.16	T = 0.19	T = 0.2
a_0^0	0.148	0.155	0.164	0.202	0.237
a_2^0	-0.036	-0.037	-0.037	-0.041	-0.043
$a_0^0/(5a_2^0)$	-0.826	-0.85	-0.88	-0.996	-1.11

Results obtained in the Table 1 and results are shown in the Fig.4 show that the cancellation works also at finite temperature.

As can be seen from the equation for pressure, at high temperatures near the phase transition the difference $P^{00} + 5P^{20}$ becomes finite and σ-meson channel should be taken into account.

Thank you for attention