Lee-Yang zeros and Roberge-Weiss transition

Gerasimeniuk Nikolai
Pacific Quantum Center, FEFU

"Infinite and Finite Nuclear Matter (INFINUM-2023)", JINR

3 march 2023

Pacific Quantum Center (FEFU)

Introduction

In this project QCD phase structure at $T > T_{RW}$ was investigated by the Lee-Yang method in combination with canonical approach.

Outline:

- Problems
- Candidates for real QCD
- Canonical method and Lee-Yang's ideas
- Lee-Yang' zeros at $T > T_{RW}$
- Results
- Conclusion

PHYSICAL REVIEW D **107,** 014508 (2023)

Numerical study of the Roberge-Weiss transition

V. G. Bornyakov
Institute for High Energy Physics NRC "Kurchatov Institute," 142281 Protvino, Russia and National Research Center "Kurchatov Institute," 123182, Moscow, Russia

N. V. Gerasimeniuk, V. A. Goy, A. A. Korneev, and A. V. Molochkov
Pacific Quantum Center, Far Eastern Federal University, 690922 Vladivostok, Russia

A. Nakamura
RCNP, Osaka University, Osaka 567-0047, Japan and Pacific Quantum Center, Far Eastern Federal University, 690922 Vladivostok, Russia

R. N. Rogalyov
Institute for High Energy Physics NRC "Kurchatov Institute," 142281 Protvino, Russia

(Received 19 August 2022; accepted 8 December 2022; published 13 January 2023)

We study the Roberge-Weiss phase transition numerically. The phase transition is associated with the discontinuities in the quark-number density at specific values of imaginary quark chemical potential. We parametrize the quark-number density ρ_q by the polynomial fit function to compute the canonical partition functions. We demonstrate that this approach provides a good framework for analyzing lattice QCD data at finite density and a high temperature. We show numerically that, at high temperature, the Lee-Yang zeros lie on the negative real semiaxis provided that the high-quark-number contributions to the grand canonical partition function are taken into account. These Lee-Yang zeros have nonzero linear density, which signals the Roberge-Weiss phase transition. We demonstrate that this density agrees with the quark-number density discontinuity at the transition line.

DOI: 10.1103/PhysRevD.107.014508

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VOLUME 30, NUMBER 26 PHYSICAL REVIEW LETTERS 25 JUNE 1973

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 27 April 1973)

50 years of QCD!

Experiment:
Region of high energies and low densities: RHIC, LHC
Region of low energies and high densities: NICA, FAIR, J-PACK
Theory:
Effective models
Perturbation theory methods
Lattice simulations

Effective models not based on first principles.
Perturbation theory methods aren't applicable for a large coupling constant.
Lattice computing doesn't work at $\mu/T > 1$ (*sign problem*).

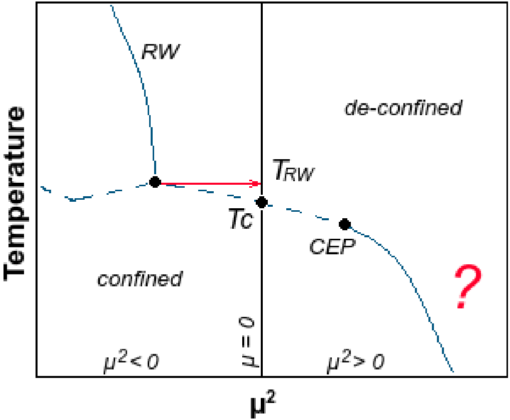
Candidates for QCD at real μ

Two types of candidates:
Based on complex simulations

- Complex Langevin method
- Lefschetz thimble

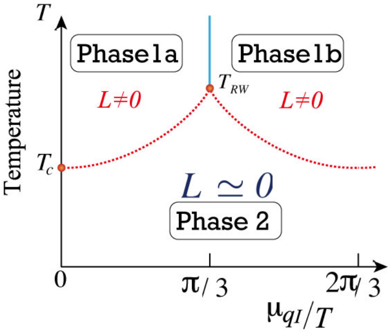
Based on meaningful lattice data

- Reweighting method
- Taylor expansion
- *Canonical method*



$$Z_{GC}(\mu_q) = \sum_n Z_C(n) e^{n\mu_q/T}, \quad Z_C(n) - ?$$

The canonical approach makes it possible to study the phase structure of QCD at finit density. The canonical approach is based on lattice data obtained in the maginary region of the chemical potential. This region is remarkable in that there is no sign problem for it, i.e. Monte-Carlo simulations are possible.



$$Z_{GC}(\mu_q) = \sum_{n=-\infty}^{\infty} Z_C(n) e^{n\mu_q/T}$$

For $\mu_q \to i\mu_{qI}$ (Fourier transformation)

$$\frac{\rho_{qI}}{T^3} = -\frac{1}{VT^2}\frac{\partial \ln Z_{GC}}{\partial \mu_{qI}} \approx \alpha_1 \frac{\mu_I}{T} + \alpha_3 \left(\frac{\mu_I}{T}\right)^3 \ldots$$

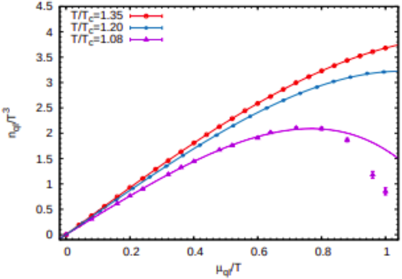
Algorithm:
$\rho_{qI} \to Z_{GC}(\mu_I) \to Z_C(n) \to Z_{GC}(\mu)$

Lattice calculation

Quark number density n_q can be computed numerically on a lattice for imaginary chemical potential by

$$\frac{\rho_{qI}}{T^3} = \frac{N_f}{VT^3}\frac{1}{Z_{GC}}\int \mathcal{D}U\,(det\Delta(\mu_q))^{N_f}\,Tr\left[\Delta^{-1}\frac{\partial\Delta}{\partial(\mu_q/T)}\right]e^{-S_g}$$

T/T_c	a_1	a_3	a_5
1.35(7)	4.671(2)	-0.991(4)	-
1.20(6)	4.409(6)	-1.032(31)	-0.165(32)
1.08(5)	3.880(17)	-1.62(21)	-0.59(0.47)



Iwasaki improved action: $S = S_g + S_q$

$S_g = -\beta\sum_{x,\mu\nu}\left(c_0 W^{1x1}_{\mu\nu}(x) + c_1 W^{1x2}_{\mu,\nu}(x)\right)$

$S_q = \sum_{f=u,d}\sum_{x,y}\overline{\psi}^f_x\Delta_{x,y}\psi^f_y$

V. G. Bornyakov [et. all], New approach to canonical partition functions computation in $N_f = 2$ lattice QCD at finite baryon density: Phys. Rev. D 95, 094506

In order to investigate the QCD phase structure, it suffices to study the behavior of the Lee-Yang zeros - roots of $Z_{GC}(\mu)$.
Nonanalyticity = phase transition?

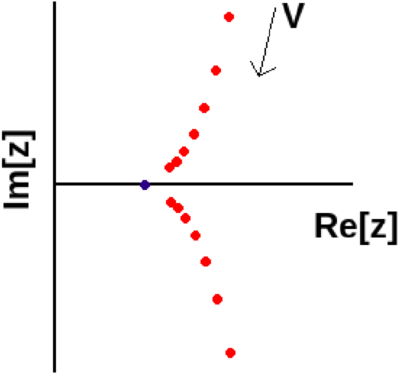
$$Z_{GC}(\mu_q) = \sum_{n=-N_{max}}^{N_{max}} Z_C(n) e^{n\mu_q/T}$$

$$Z_{GC}(\mu_q) \sim \prod_{n=1}^{2N_{max}} (\xi_q - \alpha_n), \quad \xi_q = e^{\mu_q/T}$$

$Z_C(n) = Z_C(-n)$ (*charge-parity inv.*)

$Z_C(n)$ *is positive and real*

α_n can't be real and positive!



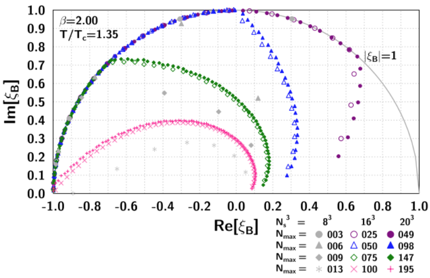
To study the phase structure of QCD, we also follow Lee-Yang's idea.

$$Z_{GC}(\mu_q) = \sum_{n=-N_{max}}^{N_{max}} Z_C(n)\xi_q^n = 0, \quad \xi_q = e^{\mu_q/T} = \xi_B^{1/3}$$

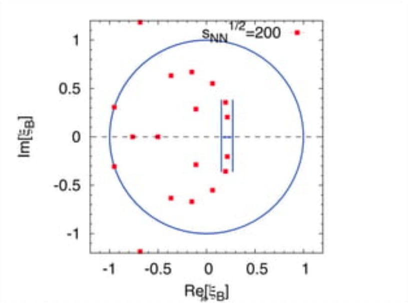
Free fermion gas in 1+1: $\frac{\mu_q}{T} = i\pi\frac{(2k+1)}{3} - \frac{3(2m+1)}{4V}$

Eff. model: CEM-bad, (arXiv:2103.07442), NJL-good, (arXiv:1905.10956)

Wakayama [et. all], Lee-Yang zeros in lattice QCD for searching phase transition points: arXiv:1802.02014



A.Nakamura, K.Nagata, Probing QCD Phase Structure by Baryon Multiplicity Distribution: arXiv:1305.0760



Canonical partition function

To approximate the integral for $Z_C(n)$ we use Laplace's method.

$$Z_C(n) = \int_0^{2\pi} \frac{d\theta_q}{2\pi} e^{-in\theta_q} Z_{GC}(\mu_{qI}) = \int_0^{2\pi} \frac{d\theta_q}{2\pi} e^{-in\theta_q} e^{-VT^3 \int_0^{\theta_q} d\theta_q' n_{qI}(\theta_q')}$$

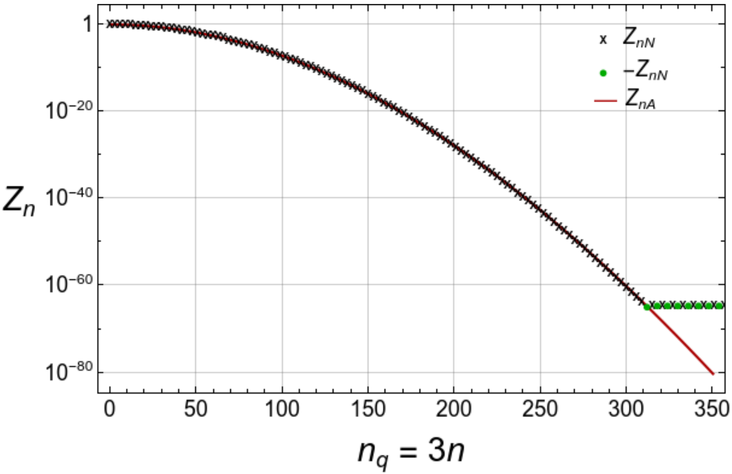
$$\lambda \gg 1, \quad \int_a^b g(x) e^{\lambda f(x)} dx \approx (i\, sign(f))^{1/2} g(s) e^{i\lambda f(s)} \sqrt{\frac{2\pi}{\lambda |f''(s)|}}$$

$$\nu = VT^3 \gg 1, \quad Z_C(n) = \sqrt{\frac{2\pi}{\nu F_n''(\theta_0)}} \exp\left[-\nu F_n(\theta_0)\right]$$

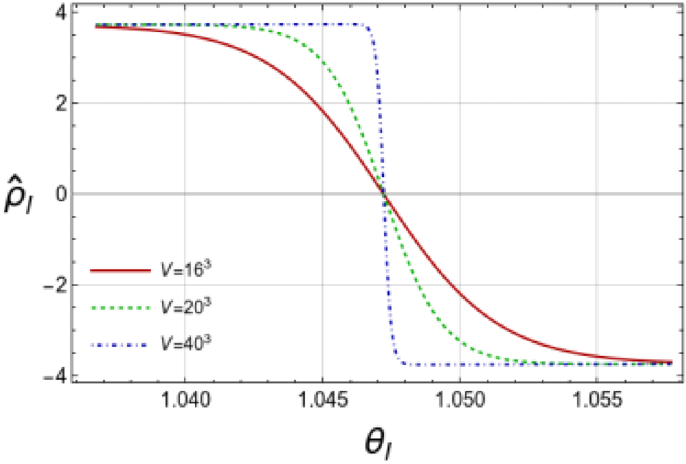
$$F_n(\theta) = -\rho_{qI}(n)\theta_q + \frac{1}{2} a_1 \theta_q^2 + \frac{1}{4} a_3 \theta_q^4$$

$$Z_{nA} = \frac{e^{-\nu F_n(\theta_0)}}{e^{-\nu F_0(\theta_0)}} \frac{\sqrt{F_0''(\theta_0)}}{\sqrt{F_n''(\theta_0)}}$$

Results for Z_{nA} and ρ_{qI}



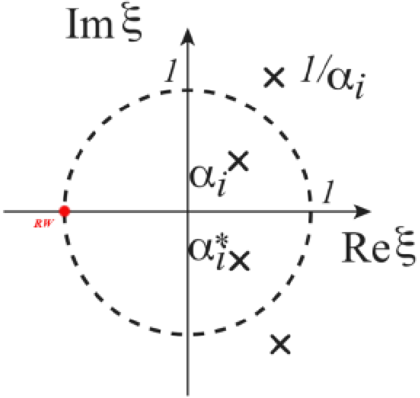
Results for Z_{nA} and ρ_{qI}

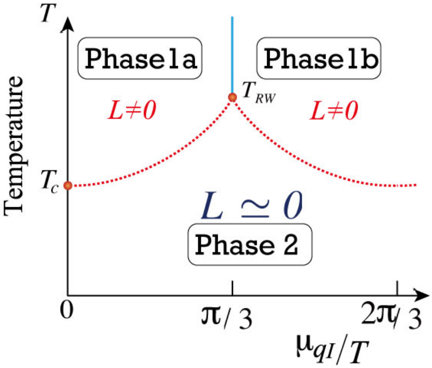


Lee-Yang zeros

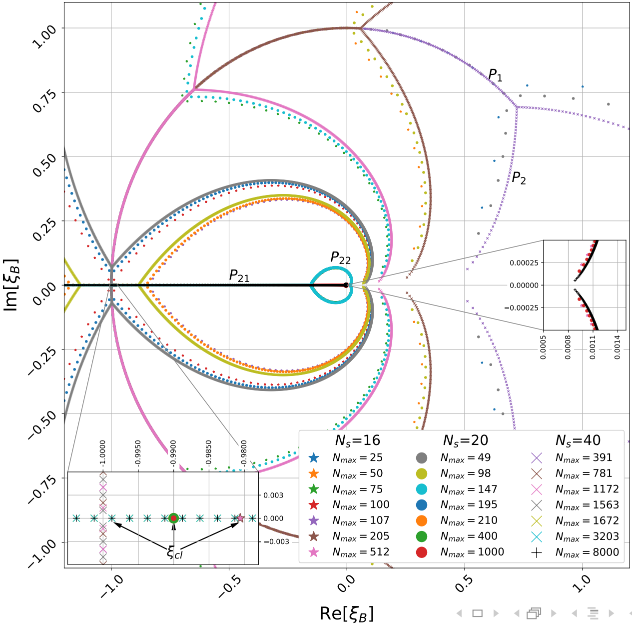
Lee-Yang zeros calculation based on Abert method, which is a variation of Newton's method, with the possibility of calculations in arbitrary precision.

$$Z_{GC}(\mu) = \xi^{-N_{max}} \sum_{n=0}^{2N_{max}} Z_C(n)\xi_B^n = 0, \quad \xi_B = \xi_q^3 = e^{3\mu_q/T}$$

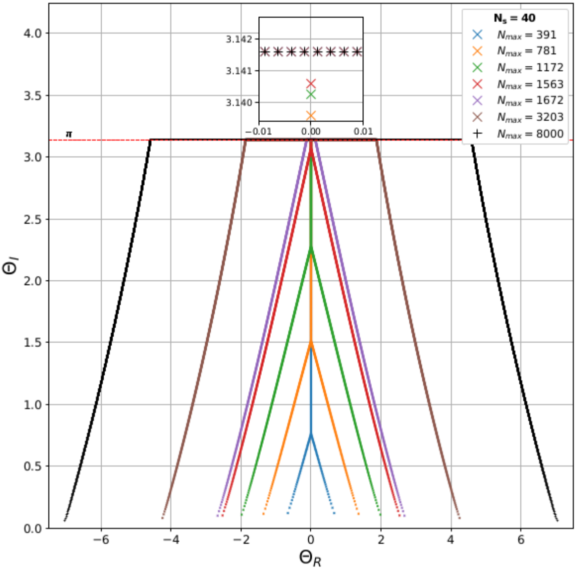




N_{max} dependence

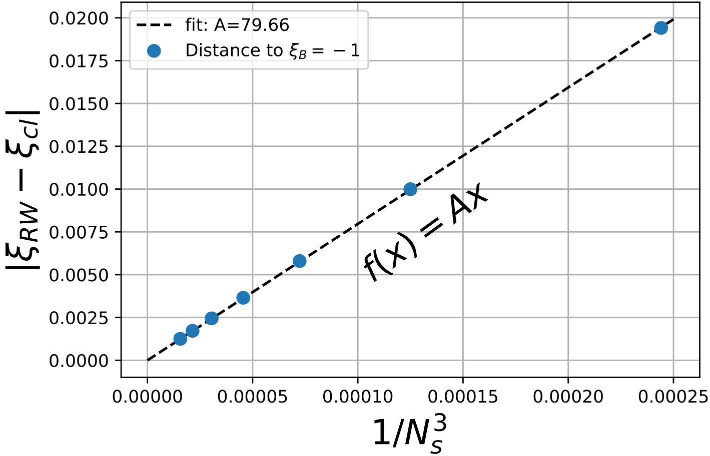


Lee-Yang zeros on μ/T-plane



V dependence

After making sure that at large N_{max} all Lee-Yang zeros lie on the negative axis $Re[\xi_B]$, the volume dependence was investigated.



The formula relating the LYZ density to the discontinuity $\Delta\rho_I$ in the average particle-number density was obtained by Lee and Yang. (DOI:10.1103/PhysRev.87.410)
Normalized density of LYZ on $Re[\xi_B] < 0$ axis

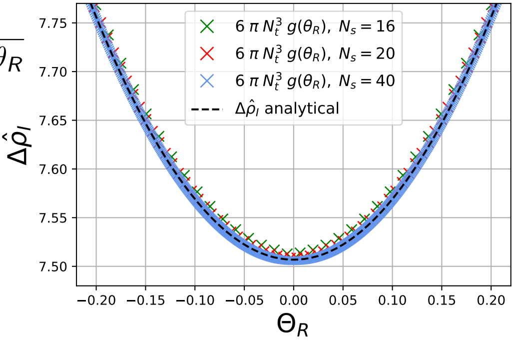
$$g(\theta_R) = \frac{1}{N_s^3}\frac{dN_{LYZ}(\theta_R)}{d\theta_R} \approx \frac{1}{N_s^3\ \Delta\theta_R}$$

Numerical:

$$\Delta\rho_I = 2\pi N_c N_t^3 g(\theta_R)$$

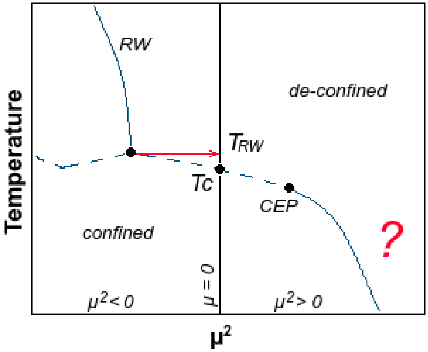
Analytical at $\theta = \frac{i\pi}{3} + \theta_R$:

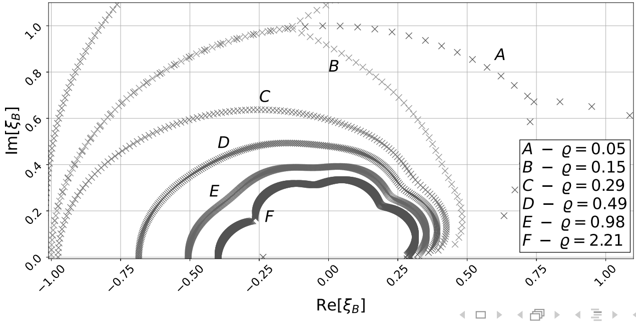
$$\Delta\rho_I = 2\left(a_1\frac{\pi}{3} + a_3\frac{\pi^3}{27} - a_3\pi\theta_R^2\right)$$



Conclusion

- We have studied $N_f = 2$ lattice QCD in the deconfinement phase above T_{RW} at $T/T_c = 1.35$.
- We demonstrated that Z_{nA} agree very well with values of Z_C where the latter are available and reproduce the input expression for the quark number density.
- In the study of Lee-Yang zeros, by increasing the maximum density, it was found that zeros appear on $Re[\xi_B] < 0$ and keep on it with N_{max} increasing.
- In the study of volume dependence, we show that in $V \to \infty$ there is nonzero density of LYZ in $\xi_B = -1$.
- We derived the relation between the quark number density discontinuity and the LYZ density and observed that our numerical results for the LYZ density nicely satisfy this relation.


RW
de-confined
T_{RW}
Tc
CEP
confined
?
$\mu = 0$
$\mu^2 < 0$
$\mu^2 > 0$
Temperature
μ^2


A
B
C
D
E
F
A − $\varrho = 0.05$
B − $\varrho = 0.15$
C − $\varrho = 0.29$
D − $\varrho = 0.49$
E − $\varrho = 0.98$
F − $\varrho = 2.21$
$\mathrm{Im}[\xi_B]$
$\mathrm{Re}[\xi_B]$
0.0
0.2
0.4
0.6
0.8
1.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00