Gravity and Nature of Nuclear Matter

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The presentation is organized as follows.

PRINCIPLE of GENERAL COVARIANCE

The Principle of General Covariance is known as fundamental principle of general relativity theory. All aspects of this principle can be formulated on the basis of two seminal papers Einstein and Grossmann (1913) and Einstein (1914).

It is evident that there is too intimate connection between gravity and the rest to be considered separately.

Hence, the Principle of General Covariance should be considered as the fundamental principle of nature as a whole and hence, Predictive General Covariant Physics.

I can not to give expanded formulation of the three aspects of the Principle of General Covariance here but for our goals it is sufficient to understand that the tensor fields and connections can only be used to

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formulate general covariant laws and principles of symmetry of nature. "General Covariant" everywhere means "accepted by the Principle of General Covariance."

The evident problem is how to explain Spin in the framework of the Principle of General Covariance. Answer gives general covariant Dirac equation.

The general covariant physics deals with the systems of functions of many variables and their classification. Hence, its only background structure is the reference space R^n. A point of R^n is defined as n-tuple of real numbers

$$x = (x^1, x^2, \cdots x^n), \quad - - \infty < x^i < \infty.$$

GENERAL COVARIANT INTERVAL and BILATERAL SYMMETRY

"What is the general covariant generalization of the concept of interval of the special theory relativity ?" The answer is given by the formula

$$ds^2 = \overline{g}_{ij} dx^i dx^j = (\frac{2t_i t_j}{g_{kl} t^k t^l} - g_{ij}) dx^i dx^j,$$

which can be found, for example in (S.W. Hawking and G.F.R. Ellis. The Large Scale Structure of Space–Time, 1973). From this formula it follows that

$$\overline{g}_{ij} = \frac{2t_i t_j}{g_{kl} t^k t^l} - g_{ij}, \quad t_i = g_{ij} t^j. \tag{1}$$

We see that in the general case the Einstein potential $\overline{g}_{ij}$ is the function a vector field t^i and a symmetrical tensor field g_{ij}, which defines the natural general covariant scalar product in the general covariant linear space of vector fields

$$(t|u) = g_{ij}t^i u^j, \quad (t|t) = g_{ij}t^i t^j \geq 0, \quad (t|t) = 0$$

if and only if $t^i = 0$.

Now our goal is to ensure the internal content of equation (1). The right– hand side of equation (1) contains fourteen unknown functions. It is clear that this number should be reduced to the ten unknown functions of the Einstein potential $\overline{g}_{ij}$. The first natural step is to put $t_i = \partial_i f, \quad t^i = g^{ij}t_j$. and after that to write the equation

$$g^{ij}\partial_i f \partial_j f = 1.$$

Important argument in favor of this equation will be presented below. Hence, for the Einstein potential we

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have

$$\overline{g}_{ij} = 2\partial_i f \partial_j f - g_{ij}, \quad t^i = g^{ij}\partial_j f, \quad g^{ij}\partial_i f \partial_j f = 1.$$

We will call $f(x)$ a scalar constituent of the Einstein potential and g_{ij} a Riemann constituent.
Now our goal is to discover more deep reason of the connection between indefinite quadratic form

$$\overline{g}_{ij} v^i v^j$$

and the positive definite quadratic form

$$g_{ij} v^i v^j.$$

The idea is to find a linear self–adjoint operator R^i_j such that relation

$$\overline{g}_{ij} v^i v^j = g_{ij} v^i R^j_k v^k$$

is fulfilled for any vector field v^i. We pay attention to that there is fundamental discrete symmetry of nature known as bilateral symmetry which is defined by a given vector and the well known scalar product in familiar vector algebra. On the ground of this association, we give the natural general covariant definition of the bilateral symmetry in the framework of the Principle of General Covariance as follows. A pair of vector fields v and $\overline{v}$ has general covariant bilateral symmetry with respect to the gradient of the scalar constituent of the Einstein potential $t^i = g^{ij}\partial_j$ if the sum of these fields is collinear to t^i and their difference is orthogonal to it, $\overline{v} + v = \lambda t, \quad (\overline{v}|t) = (v|t)$, where $(u|v)$ is a scalar product defined above. From this definition it follows that the connection between the right–hand sided and left–hand sided vector fields can be represented as a

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linear transformation (reflection) $\overline{v}^i = R^i_j v^j$, where

$$R^i_j = 2t^i t_j - \delta^i_j.$$

is needed operator.
On this reason we put forward an idea that general covariant bilateral symmetry is strict and fundamental symmetry of nature and, hence, in all natural physical processes a right-hand sided general covariant physical quantity always appears in pairs with the left-hand sided one .
We recognized that in general the fundamental concept of interval is clearly defined by the general covariant bilateral symmetry and the Lorentz group is byproduct of this symmetry. Moreover, now it is clear how to introduce the concepts of electric and magnetic fields in the framework of the Principle of General

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Covariance, derive the Maxwell equations and explain the nature of time.

GENERAL COVARIANT MAXWELL EQUATIONS

General covariant electric and magnetic fields. Let e_{ijkl} be antisymmetric Levi-Civita tensor associated with $g_{ij}, \quad e_{1234} = \sqrt{g}$, where $g = Det(g_{ij}) > 0$. For the contravariant antisymmetric tensor $e^{ijkl} = g^{im}g^{jn}g^{kr}g^{ls}e_{mnrs}, \quad e^{1234} = \sqrt{1/g}$. For the tensor of the electromagnetic field F_{ij} we put $\widetilde{F}_{ij} = \frac{1}{2}e_{ijkl}F^{kl}, \quad F^{kl} = g^{ki}g^{lj}F_{ij}$. General covariant electric and magnetic fields are introduced as follows

$$E_i = t^k F_{ik}, \quad H_i = t^k \widetilde{F}_{ik}.$$

This Equations can be inverted

$$F_{ij} = -t_i E_j + t_j E_i - e_{ijkl} t^k H^l.$$

Now we are ready to derive Maxwell equations for the general covariant electric and magnetic fields from the equations for the tensor of the electromagnetic field F_{ij}, which are considered in the general theory of relativity. Beforehand, it is important to formulate basic relations of the natural generalization of familiar vector algebra and vector calculus in general covariant form.

The scalar product $(A|B)$ of the vector fields A^i and B^i was defined above. The vector product $C = [A \times B]$ is defined as follows:

$$C^i = [A \times B]^i = e^{ijkl} t_j A_k B_l, \quad A_k = g_{kl} A^l.$$

We suppose that g_{ij} and t_i are known. It is evident that $[A \times B] + [B \times A] = 0$. The main relations of general covariant vector algebra

$$|[A \times B]| = |A||B| \sin\alpha, \quad [A \times [B \times C]] = B(A, C) - C(A, B$$

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are fulfilled. We also state that

$$[ABC] = [BCA] = [CAB], \quad [ABC] = (A|[B \times C]).$$

The basic differential operators of the general covariant vector calculus are defined quite naturally

$$\mathrm{div}\, A = \frac{1}{\sqrt{g}} \partial_i (\sqrt{g} A^i), \quad (\mathrm{grad}\, \phi)^i = g^{ij} \partial_j \phi$$

$$\mathrm{div}\, \mathrm{grad} \phi = \frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ij} \partial_j \phi) = \nabla_i \nabla^i \phi,$$

where ∇_i is a covariant derivative with respect to the Levi-Civita connection belonging to the g_{ij}.
The rotor of the vector field A is introduced as a vector product of the 4d operator ∇ and A

$$(\mathrm{rot}\, A)^i = [\nabla \times A]^i = e^{ijkl} t_j \partial_k A_l = \frac{1}{2} e^{ijkl} t_j (\partial_k A_l - \partial_l A_k).$$

We have

$$\operatorname{rot}\operatorname{grad}\phi = 0, \quad \operatorname{div}\operatorname{rot}A = 0.$$

Hence it follows that equations for the tensor of the electromagnetic field can be written as equations for the general covariant electric and magnetic fields

$$\operatorname{rot}E = -\frac{1}{\sqrt{g}}D_t(\sqrt{g}H), \quad \operatorname{rot}H = \frac{1}{\sqrt{g}}D_t(\sqrt{g}E),$$

$$\operatorname{div}E = 0, \quad \operatorname{div}H = 0,$$

$$(t|E) = 0, \quad (t|H) = 0,$$

where D_t is the operator of the Lie derivative

$$(D_tE)^i = t^k\partial_kE^i - E^k\partial_kt^i.$$

Comparing the general covariant Maxwell equations with original one we can conclude that the scalar constituent $f(x)$ of the Einstein potential has an exact relation to the enigma of natural time (an entity which belongs to nature itself) and the operator

$$D_t = t^k \partial_k$$

is general covariant analog of the operator $\partial/\partial t$.

DUALITY of NATURAL TIME and EXISTENCE of DUAL PARTICLES

We believe that now it is time to formulate adequate and key definitions of natural time.

Definition: a moment of natural time is a number that we put in correspondence to any point of the reference space R^4. Hence, a moment of time is defined by the equation

$$t = f(x^1, x^2, \cdots x^4) = f(x),$$

and $f(x)$ is identified with the scalar constituent of the Einstein potential

$$\overline{g}_{ij} = 2\partial_i f \partial_j f - g_{ij}, \quad t^i = g^{ij}\partial_j f, \quad g^{ij}\partial_i f \partial_j f = 1.$$

By definition, all points of the reference space that correspond to the same moment of time constitute physical space $S(t)$ or isochrone. Isochrone is defined by the equation

$$f(x^1, x^2, \cdots x^4) = f(x) = t = constant.$$

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The gradient of time is the vector field t with the components

$$t^i = (\nabla f)^i = g^{ij}\partial_j f = g^{ij} t_j,$$

which defines fundamental discrete internal symmetry-general covariant bilateral symmetry defined above. Since the operator

$$D_t = t^k \partial_k = g^{kl}\partial_l f(x)$$

is general covariant analog of the operator $\partial/\partial t$, then the general covariant analog of the evident commutation relation

$$[\partial/\partial t, t] = 1$$

is

$$[D_t, f(x)] = 1$$

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But the last commutation relation is fulfilled under the condition that

$$g^{ij}\partial_i f(x)\partial_j f(x) = 1.$$

The fundamental (from a physical point of view) observation reads that this equation has not only general solution but also a special solution known as the function of geodesic distance. This means that there are two different times in nature and, hence, two different kinds of natural dynamical processes.
To illustrate this far reaching general statement, we consider the four-dimensional reference space R^4 with the metric $dl^2 = g_{ij}dx^i dx^j = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2, \quad g_{ij} = \delta_{ij}$ and look for solutions to equation $g^{ij}\partial_i f(x)\partial_j f(x) = 1$ under

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this condition. We have

$$\left(\frac{\partial f}{\partial x^1}\right)^2 + \left(\frac{\partial f}{\partial x^2}\right)^2 + \left(\frac{\partial f}{\partial x^3}\right)^2 + \left(\frac{\partial f}{\partial x^4}\right)^2 = 1$$

and in accordance with our general statement exhibit a general solution

$$f(x) = a_i x^i = a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4,$$

where $a_i = t_i$ and hence $(a|a) = 1$ and a special solution $f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}$. For the gradient of time we have in the first case

$$t^i = a^i$$

and

$$t^i = x^i / \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}$$

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in the second one.
From the equations

$$f(x) = a_i x^i = a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 = t = constant,$$

and

$$f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2} = \tau = constant$$

we see that in one case physical space is the familiar three-dimensional Euclidian space E^3 and in the other new physical space is the three-dimensional sphere S^3. We can see that there are two different times. The physical (mass) points are to be identified with the points belonging to the three-dimensional Euclidian space E^3, but the points belonging to the 3d-sphere S^3 should be put in correspondence to the Spherical Tops. Indeed, the symmetries of the Euclidian space can be

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composed of translations and rotations and the symmetries of the 3d-sphere S^3 coincide with those of the Spherical Top

$$[L_j,\, L_k] = i\varepsilon_{jkl}L_l, \quad [\widetilde{L}_j,\, \widetilde{L}_k] = -i\varepsilon_{jkl}\widetilde{L}_l, \quad [L_j,\, \widetilde{L}_k] = 0.$$

$$L_1^2 + L_2^2 + L_3^2 = \widetilde{L}_1^2 + \widetilde{L}_2^2 + \widetilde{L}_3^2.$$

In other words, geometrical points in the Euclidian and spherical spaces have different physical meanings. The concept of Spherical Top can be reduced to the concept of point particle but in the dual time and dual physical space. A natural rotation is a motion in dual time. Hence, it is natural to put forward the idea of dual approach to the world of elementary particles which can explain the existence of leptons and dual leptons which can be identified with fundamental

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particles of nuclear matter.

GENERAL COVARIANT DIRAC EQUATION

When Goudsmit and Uhlenbeck proposed the spin hypothesis , they had in mind a mechanical picture of the Top. However, it was soon recognized that this idea is not connected with movement in space. Hence, the constructive idea of Kronig and Uhlenberck can be considered as concept of Intrinsic Top which can be put in correspondence to the quantum Spherical Top. In definite sense this means that two set of commuting Dirac gamma matrices should naturally appear in appropriate theory. This situation is realized in general covariant Dirac equation.

A set of the simplest (irreducible) geometrical quantities consists of scalar field $a(x)$, covariant vector field $a_i(x)$, and antisymmetric covariant tensor fields $a_{ij}(x)$, $a_{ijk}(x)$ and $a_{ijkl}(x)$, which can be called the general covariant Dirac field:

$$\mathbf{A} = (a, a_i, a_{ij}, a_{ijk}, a_{ijkl}), \quad i, j, k, l = 1, 2, 3, 4.$$

To define general covariant Dirac matrices (operators), let us consider the natural algebraic operators $\overline{\mathbf{A}} = \hat{\mathrm{E}}_{\mathbf{v}}\mathbf{A}$ and $\overline{\mathbf{A}} = \hat{\mathrm{I}}_{\mathbf{v}}\mathbf{A}$ defined by the vector field v^i as follows:

$$\hat{\mathrm{E}}_{\mathbf{v}} : \overline{\mathbf{A}} = \hat{\mathrm{E}}_{\mathbf{v}}\mathbf{A} = (0,\ v_i a,\ v_{[i}a_{j]},\ v_{[i}a_{jk]},\ v_{[i}a_{jkl]}),$$

$$\hat{\mathrm{I}}_{\mathbf{v}} : \overline{\mathbf{A}} = \hat{\mathrm{I}}_{\mathbf{v}}\mathbf{A} = (v^m a_m,\ v^m a_{mi},\ v^m a_{mij},\ v^m a_{mijk},\ 0),$$

where the square brackets $[\cdots]$ denote the process of alternation and $v_i = g_{ij}v^j$. For any vector fields v^i and

w^i we have $\hat{\mathrm{I}}_{\mathbf{v}}\hat{\mathrm{E}}_{\mathbf{w}} + \hat{\mathrm{E}}_{\mathbf{w}}\hat{\mathrm{I}}_{\mathbf{v}} = (\mathbf{v}|\mathbf{w}) \cdot \hat{\mathrm{E}}$, where $(\mathbf{v}|\mathbf{w}) = g_{ij}v^i w^j$ is a positive-definite scalar product. We mention also the evident relations $\hat{\mathrm{E}}_{\mathbf{v}}\hat{\mathrm{E}}_{\mathbf{w}} + \hat{\mathrm{E}}_{\mathbf{w}}\hat{\mathrm{E}}_{\mathbf{v}} = 0, \quad \hat{\mathrm{I}}_{\mathbf{v}}\hat{\mathrm{I}}_{\mathbf{w}} + \hat{\mathrm{I}}_{\mathbf{w}}\hat{\mathrm{I}}_{\mathbf{v}} = 0$. To complete, let us introduce the numerical diagonal operator $\hat{\mathrm{Z}}$ that is defined by the conditions $Z^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad Z^{i_1\cdots i_p}_{j_1\cdots j_p} = (-1)^p \delta^{i_1\cdots i_p}_{j_1\cdots j_p}$. From the definition of $\hat{\mathrm{Z}}$, we immediately have the following relations: $\hat{\mathrm{E}}_{\mathbf{v}}\hat{\mathrm{Z}} + \hat{\mathrm{Z}}\hat{\mathrm{E}}_{\mathbf{v}} = 0, \quad \hat{\mathrm{I}}_{\mathbf{v}}\hat{\mathrm{Z}} + \hat{\mathrm{Z}}\hat{\mathrm{I}}_{\mathbf{v}} = 0, \quad \hat{\mathrm{Z}}^2 = \hat{\mathrm{E}}$.
Now we introduce the fundamental operators:

$$Q_{\mathbf{v}} = \hat{\mathrm{E}}_{\mathbf{v}} - \hat{\mathrm{I}}_{\mathbf{v}}, \quad \widetilde{Q}_{\mathbf{v}} = (\hat{\mathrm{E}}_{\mathbf{v}} + \hat{\mathrm{I}}_{\mathbf{v}})\hat{\mathrm{Z}}$$

which define the two set of general covariant Dirac matrices.
From the definition of the operators $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$, it

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follows that

$$Q_{\mathbf{v}} Q_{\mathbf{w}} + Q_{\mathbf{w}} Q_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w})\cdot\hat{\mathrm{E}}, \quad \widetilde{Q}_{\mathbf{v}} \widetilde{Q}_{\mathbf{w}} + \widetilde{Q}_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w})\cdot$$

Besides, we also deduce one very important relation

$$\widetilde{Q}_{\mathbf{v}} Q_{\mathbf{w}} = Q_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}},$$

that is fulfilled at any vector fields **v** and **w**.
Now we can factorize vector field **v** and, hence, to write

$$Q_{\mathbf{v}} = v^i \Gamma_i,$$

$$\widetilde{Q}_{\mathbf{v}} = v^i \widetilde{\Gamma}_i$$

It is evident that for the general covariant Dirac gamma matrices Γ_i and $\widetilde{\Gamma}_i$ the following relations are fulfilled

$$\Gamma_i \Gamma_j + \Gamma_j \Gamma_i = - - 2 g_{ij} E, \quad \widetilde{\Gamma}_i \widetilde{\Gamma}_j + \widetilde{\Gamma}_j \widetilde{\Gamma}_i - -2 g_{ij} E$$

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$$\Gamma_i\widetilde{\Gamma}_j = \widetilde{\Gamma}_j\Gamma_i$$

General Covariant Dirac equation can be written in the different forms. Below it is written as system of four scalar and four vector equations:

$$\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu - q\phi\,\kappa &= \nabla_i \mathrm{M}^i - q\Phi_i \mathrm{K}^i + m\,\kappa \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa + q\phi\,\mu &= \nabla_i \mathrm{K}^i + q\Phi_i \mathrm{M}^i - m\,\mu \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda - q\phi\,\nu &= \nabla_i \mathrm{L}^i - q\Phi_i \mathrm{N}^i - m\,\nu \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu + q\phi\,\lambda &= \nabla_i \mathrm{N}^i + q\Phi_i \mathrm{L}^i + m\,\lambda
\end{aligned} \qquad (2)$$

$$\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{M}_i - q\phi\mathrm{K}_i &= (\mathrm{rot}\,\mathbf{N})_i + q[\mathbf{\Phi}\times\mathbf{L}]_i + \triangle_i\,\mu - \ldots \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{K}_i + q\phi\mathrm{M}_i &= -(\mathrm{rot}\,\mathbf{L})_i + q[\mathbf{\Phi}\times\mathbf{N}]_i + \triangle_i\,\kappa + \ldots \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{L}_i - q\phi\mathrm{N}_i &= (\mathrm{rot}\,\mathbf{K})_i + q[\mathbf{\Phi}\times\mathbf{M}]_i + \triangle_i\,\lambda - \ldots \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{N}_i + q\phi\mathrm{L}_i &= -(\mathrm{rot}\,\mathbf{M})_i + q[\mathbf{\Phi}\times\mathbf{K}]_i + \triangle_i\,\nu + \ldots
\end{aligned} \qquad (3)$$

where

$$\nabla_{\mathbf{t}} = t^i \nabla_i, \quad \varphi = \nabla_i t^i \quad \text{and} \quad \phi = t^i A_i$$

is a scalar potential of the electromagnetic field and

$$\Phi_i = A_i - \phi t_i, \quad t^i \Phi_i = 0$$

is its vector potential.
We also write this fundamental equations in the invariant form:

$$\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu - q\phi\,\kappa &= \operatorname{div}\mathbf{M} - q(\boldsymbol{\Phi}|\mathbf{K}) + m\,\kappa \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa + q\phi\,\mu &= \operatorname{div}\mathbf{K} + q(\boldsymbol{\Phi}|\mathbf{M}) - m\,\mu \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda - q\phi\,\nu &= \operatorname{div}\mathbf{L} - q(\boldsymbol{\Phi}|\mathbf{N}) - m\,\nu \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu + q\phi\,\lambda &= \operatorname{div}\mathbf{N} + q(\boldsymbol{\Phi}|\mathbf{L}) + m\,\lambda
\end{aligned} \qquad (4)$$

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{M} - q\phi\mathbf{K} &= (\mathrm{rot}\,\mathbf{N}) + q[\mathbf{\Phi}\times\mathbf{L}] + \mathrm{grad}\,\mu - q \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{K} + q\phi\mathbf{M} &= -(\mathrm{rot}\,\mathbf{L}) + q[\mathbf{\Phi}\times\mathbf{N}] + \mathrm{grad}\,\kappa + \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{L} - q\phi\mathbf{N} &= \mathrm{rot}\,\mathbf{K} + q[\mathbf{\Phi}\times\mathbf{M}] + \mathrm{grad}\,\lambda - q\mathbf{\Phi} \\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{N} + q\phi\mathbf{L} &= -\mathrm{rot}\,\mathbf{M} + q[\mathbf{\Phi}\times\mathbf{K}] + \mathrm{grad}\,\nu + q
\end{aligned}
\tag{5}
$$

Below we exhibit the Dirac equations describing quantum mechanics of dual particles with the spin one half:

$$
\begin{aligned}
\frac{1}{\tau}(D + \tfrac{3}{2})\kappa &= \mathrm{div}\,\mathbf{K} - m\,\mu \\
\frac{1}{\tau}(D + \tfrac{3}{2})\lambda &= \mathrm{div}\,\mathbf{L} - m\,\nu \\
\frac{1}{\tau}(D + \tfrac{3}{2})\mu &= \mathrm{div}\,\mathbf{M} + m\,\kappa \\
\frac{1}{\tau}(D + \tfrac{3}{2})\nu &= \mathrm{div}\,\mathbf{N} + m\,\lambda
\end{aligned}
\tag{6}
$$

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$$\begin{aligned}
\frac{1}{\tau}(D+\tfrac{3}{2})\mathbf{K} &= -\mathrm{rot}\,\mathbf{L} + \mathrm{grad}\,\kappa + m\,\mathbf{M}\\
\frac{1}{\tau}(D+\tfrac{3}{2})\mathbf{L} &= \mathrm{rot}\,\mathbf{K} + \mathrm{grad}\,\lambda + m\,\mathbf{N}\\
\frac{1}{\tau}(D+\tfrac{3}{2})\mathbf{M} &= \mathrm{rot}\,\mathbf{N} + \mathrm{grad}\,\mu - m\,\mathbf{K}\\
\frac{1}{\tau}(D+\tfrac{3}{2})\mathbf{N} &= -\mathrm{rot}\,\mathbf{M} + \mathrm{grad}\,\nu - m\,\mathbf{L},
\end{aligned} \qquad (7)$$

where $\tau = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}$, $D = x^i\partial_i$. The other operators are defined as follows:

$$(\mathrm{rot}\,\mathbf{M})^i = \frac{1}{\tau}e^{ijkl}x_j\partial_k M_l, \quad (i,\,j,\,k,\,l = 1,2,3,4)$$

where e^{ijkl}, is the Levi-Civita tensor normalized as follows $e_{1234} = \sqrt{g} = 1$;

$$(\mathrm{grad}\varphi)_i = \triangle_i\,\varphi, \quad \triangle_i = \partial_i - \frac{x_i}{\tau^2}D, \quad \mathrm{rot}\,\mathrm{grad} = 0, \quad \mathrm{div}$$

and $(\mathbf{x}|\mathbf{K}) = (\mathbf{x}|\mathbf{L}) = (\mathbf{x}|\mathbf{M}) = (\mathbf{x}|\mathbf{N}) = 0$. The Maxwell

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equations for dual photons read

$$\frac{1}{\tau}(D+2)\mathbf{H} = -\mathrm{rot}\,\mathbf{E}, \quad \frac{1}{\tau}(D+2)\mathbf{E} = \mathrm{rot}\,\mathbf{H}, \qquad (8)$$

$$(\mathbf{x}|\mathbf{E}) = (\mathbf{x}|\mathbf{H}) = 0, \quad \mathrm{div}\,\mathbf{E} = \mathrm{div}\,\mathbf{H} = 0, \qquad (9)$$

where

$$E_i = \frac{x^k}{\tau} F_{ik}, \quad H_i = \frac{x^k}{\tau} \overset{*}{F}_{ik}, \quad F_{ij} = \partial_i A_j - \partial_j A_i, \quad \overset{*}{F}_{ij} = \frac{1}{2}$$

It is natural to put forward conjecture that equations (7),(8) and (9),(10) are a key to discover the internal structure of nuclear matter.

CONCLUSION

1 Predictive Physics defined by the Principle of General Covariance states that there are particles with

properties of Spherical Top, which are characterized by the two angular momenta and live in the dual time on the three dimensional sphere.

2 It is possible to describe rotational motion of these particles on the fundamental (field–theoretical) level. (General covariant Dirac equation)

3 We put forward idea that the dual particles can explain nature of nuclear matter.

4 The general covariant Dirac equation has additional degrees of freedom and hence, existence of generations

$$(e, \nu_e),\ (\mu, \nu_\mu),\ (\tau, \nu_\tau),$$

can be considered as natural but the number of generation should be equal to four. The same conclusion is true for the dual particles.

5 The general covariant internal symmetry is $U(1)$ or $SU(2)xU(1)$. There are no evidences of the existence of general covariant $SU(3)$ internal symmetry in our disposal and that is why we cannot to identify dual leptons with quarks.

THANK YOU FOR ATTENTION