Nuclear structure and nucleon-nucleon interaction

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It has not been possible, yet, to construct interactions that could satisfy simultaneously three basic conditions:
(A). To be realistic, i.e., consistent with the nucleon-nucleon (NN) phase shifts.
(B). To ensure good saturation properties, i.e., correct binding energies at the observed radii.
(C). to provide good spectroscopy.

Given a sufficiently smooth Hamiltonian H, it can be separated as

$$H = H_m + H_M \tag{1}$$

Only the monopole field Hm is affected by spherical Hartree-Fock variation. Therefore it is entirely responsible for global saturation properties and single-particle behavior.

Conditions (A) and (C), as well as (B) and (C), are mutually compatible. An elementary argument explains the situation. The observed nuclear radii $R = 1.2 \cdot A^{1/3}$ fm imply average interparticle distances of some 2.4 fm, and therefore the nucleons "see" predominantly the medium range of the potential. This is a region that is well understood theoretically and well described by the realistic forces.

Since one of the major problems of nuclear physics is that realistic forces have bad saturation properties and since H_m is in charge of them it must be treated phenomenologically. Using realistic forces it was shown, that H_M can be devided in the following parts

$$H_M = H_C + H_R, \tag{2}$$

where H_C - collective part (quadrupole, octupole, hexadecapole), and H_R can be considered as a random matrix.

EDF -why relativistic approach? Strong spin-orbit interaction in nuclei

Spin-orbit coupling estimated by analogy to atomic physics have a correct sign within nuclei. However, its value is too small.

$$V_{so} = \frac{\hbar}{m^2c^2}\frac{1}{r}\frac{\partial V}{\partial r}\vec{s}\cdot\vec{l} \equiv \lambda\frac{1}{r}\frac{\partial V}{\partial r}\vec{s}\cdot\vec{l}$$

With $\lambda = 0.044 fm^2$. In nucleus $\lambda = 0.5 fm^2$.

Why spin-orbit interaction in nuclei is so strong?

Dirac equation

$$\left(\gamma^{\mu}(cp_{\mu} + V_{\mu}) + Mc^2 - S\right)\Psi = 0$$
$$V_{\mu} = (V_0, \vec{V}) \qquad (3)$$

In stationary case

$$H = \alpha(c\vec{p}) + \vec{V}) + V_0$$
$$+\beta(Mc^2 - S) \qquad (4)$$

$$\Psi = \begin{pmatrix} g \\ f \end{pmatrix}$$

$$\begin{pmatrix} Mc^2 + V_0 - S & \vec{\sigma}(\vec{p} - \vec{V}) \\ \vec{\sigma}(\vec{p} - \vec{V}) & -Mc^2 + V_0 + S \end{pmatrix} \begin{pmatrix} g \\ f \end{pmatrix} = E \begin{pmatrix} g \\ f \end{pmatrix}$$

S - scalar potential
$V_0(\vec{r})$- vector potential (time-like), $\vec{V}(\vec{r})$ - vector potential (space-like), $W_+ = S + V_0$

$$f(\vec{r}) = \frac{1}{E + 2Mc^2 - W_+}(\vec{\sigma} \cdot \vec{p})g(\vec{r})$$

$$E = Mc^2 + \varepsilon$$

Assuming that

$$|\varepsilon| \ll 2\left(Mc^2 - \frac{1}{2}(V_0 + S)\right) \equiv 2\tilde{M}c^2 \qquad (5)$$

we get

$$\left(\vec{p}\frac{1}{2\tilde{M}c^2}\vec{p} + \frac{\hbar^2}{4\tilde{M}^2c^2}\frac{1}{r}\frac{\partial(V_0+S)}{\partial r}\vec{l}\cdot\vec{s} + (V_0 - S)\right) g(\vec{r}) \approx \varepsilon g(\vec{r})$$

$V_0 - S = -50$ MeV
$V_0 + S = 700 - 800$ MeV

From the QCD sum rule it follows that $S/V_0 \approx 1.1$ with 20% accuracy.

No nuclear collapse occurs in RMF due to specific relativistic effect:

$$\rho_s(\vec{r}) = \sum_{i=1}^{A} \left(|f_i(\vec{r})|^2 - |g_i(\vec{r})|^2\right),$$

$$\rho_v(\vec{r}) = \sum_{i=1}^{A} \left(|f_i(\vec{r})|^2 + |g_i(\vec{r})|^2\right)$$

$$\rho_s = \rho_v - 2\sum_{i=1}^{A} g_i^2 \approx \rho_v - \frac{1}{m_{eff}} \sum_{i=1}^{A} |\nabla f_i|^2 \approx \rho_v - 2\tau_{kin}.$$

Present formulation of the Relativistic Mean Field theory (P.Ring, Progr.Part.Nucl.Phys. 37, 193 (1996). RMF)

RMF is a phenomenological approach to solving the nuclear problem of many bodies. This approach is Lorentz-invariant approach, and nucleons are treated as point particles. Nucleons in this approach interact by exchanging mesons. The number of mesons, their quantum numbers, the magnitude of their masses and the coupling constants are determined to better reproduce the experimental data.

Present formulation of the Relativistic Mean Field theory (RMF)

Only as few mesons as possible are included.
π: J=0, T=1 and $P = -1$.
σ: J=0, T=0.
ω: J=1, T=0.
ρ: J=1, T=1
Since pions have negative parity, the corresponding average nuclear field does not preserve parity, which contradicts what happens in real nuclei.

$$L = L_N + L_M + L_{int}, \quad L_M = L_\sigma + L_\omega + L_\rho + L_A,$$

$$L_N = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi, \quad L_\sigma = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right),$$

$$L_\omega = -\frac{1}{2}\left(\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu\right),$$

$$L_\rho = -\frac{1}{2}\left(\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} - \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu\right).$$

$$L_{int} = -g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu \omega^\mu \psi - g_\rho \bar{\psi}\gamma_\mu \vec{\tau}\vec{\rho}^\mu \psi - e\bar{\psi}\gamma_\mu A^\mu \psi.$$

The Lagrangian stated above leads to the following equation for nucleons, which in static approximation takes the form:

$$\left(-i\vec{\alpha}\cdot\nabla + \beta(M+S) + V\right)\psi_i = \varepsilon_i\psi_i$$

$$\begin{aligned} V(\vec{r}) &= g_\omega\omega^0(\vec{r}) + g_\rho\tau_3\rho_3^0(\vec{r}) + eA^0(\vec{r}), \\ S(\vec{r}) &= g_\sigma\sigma(\vec{r}). \end{aligned}$$

Meson fields satisfy the following equations

$$(-\Delta + m_\sigma^2)\sigma = -g_\sigma \rho_s$$
$$(-\Delta + m_\omega^2)\omega^0 = g_\omega \rho_v$$
$$(-\Delta + m_\rho^2)\rho_3^0 = g_3 \rho_3$$
$$-\Delta \rho_c^0 = e\rho_c$$

where

$$\rho_s = \sum_i^A \bar{\psi}_i \psi_i, \quad \rho_v = \sum_i^A \psi_i^+ \psi_i,$$
$$\rho_3 = \sum_i^A \psi_i^+ \tau_3 \psi_i, \quad \rho_c = \sum_i^A \psi_i^+ \frac{1}{2}(1+\tau_3)\psi_i$$

1935-1960 „Fundamental" theory of nuclear forces

1960-2000 Different models of nuclear forces

1990-today Chiral EFT of nuclear forces

R.Machleidt, 2017.The nuclear forces : Meson theory versus effective field theory.

In 1935, Yukawa introduced the concept of massive particle exchange to explain the finite radius of nucleon-nucleon forces. Initially, it was assumed that this particle is a scalar boson, although the meson finally discovered in 1947/1948 turned out to be a pseudo-scalar (138 MeV). Therefore, since 1950, attempts began to build a pi-meson theory of nuclear forces.

This pion-meson theory had many problems and few successes for a reason that became clear later: pion dynamics is limited to a chiral concept that was not known in the 50s.

R.Machleidt, 2017. Different models of nuclear forces

In the early 60s, vector mesons were discovered:
ρ (770 MeV) and ω (782 MeV). This led to the creation of OBEP models that were very successful. It included half a dozen mesons, not all of which were important:
– σ - meson (500 MeV). Responsible for attraction, critical for explaining nucleon connectivity in the nucleus.
– ω - meson (782 MeV). Gives rise to strong repulsion at short distances and spin-orbital forces.
– π - meson (138 MeV). Responsible for long range and tensor forces.
– ρ - meson (770 MeV). "Cuts"pion tensor forces at short distances.

R.Machleidt, 2017. Different models of nuclear forces

To avoid problems with multinucleon exchange and higher order corrections, the models only allowed for single exchange of mesons (OBEP). In addition one would multiply the meson-nucleon vertices with formfactors to remove the singularities at short distances.

Relativistic OBEP (70s) describes the phases of nucleon-nucleon scattering to energies of the order of 1 GeV, having 35 parameters.

Problems of OBEP:
Width of σ - meson – 400-700 MeV, $c\tau$ =0.3 fm.
Can be considered as an imitating effect of the s-wave 2-pion exchange.
ω - meson. $\hbar/mc$ = 0.25 fm. Two-pion exchange describes asymptotics of the ω and ρ exchange.

"All this could be the happy end of the theory of nuclear forces, but with the QCD becoming an authoritative theory of strong interactions, the mesonic theory was lowered to the level of a model".

Nuclear physics is a collection of models.This is unsatisfactory in view of the traditional goal of theoretical physics, namely, to develop theories that are unifying and fundamental. However, the gap between the present nuclear models and the ideal goal is so wide that there is no hope to overcome it any time soon. This is where the notions of effectiveness (effective theories) enter the picture.

For the EFT construction the following steps need to be taken:

1. There is a large gap in the hadron spectrum between the masses of π and ω, ρ mesons. Thus, there are are two energy scales: low-energy scale $Q \sim m_\pi$ and high-energy scale $\Lambda_\chi \sim m_\rho$. Expansion parameter Q/Λ_χ can be used.

2. Degrees of freedom of EFT active at low-energy scale: π, N.

3. Recognize the relevant symmetries (chiral symmetry which is broken).

4. Build the most general Lagrangian consistent with these symmetries. Since the interactions of π (Goldstone boson) must vanish at zero momentum transfer and in the chiral limit $(m_\pi \rightarrow 0)$ the expansion of Lagrangian is organized in powers of derivatives and pion mass.

5. Do an expansion in Q/Λ_χ.

Weinberg: Despite the fact that we are in the region of strong interactions, the connection of the Goldstone boson with other particles and between themselves is weak, as the final result of the low-energy theory.
This coupling resembles a coupling through a derivative.

For low-energy observables, partial waves with $L \leq 2$ are most important. To describe them, knowledge of the NN potential over short distances is important. In the meson theory, NN interaction is described by heavy meson exchange

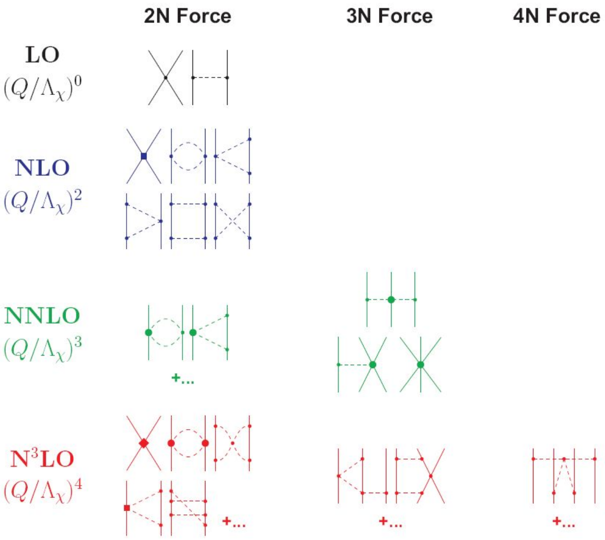
$$\int d^3q \frac{\exp(i\vec{q}\cdot\vec{r})}{m_\omega^2 + Q^2} \sim \frac{\exp(-m_\omega r)}{r} \tag{6}$$

At expansion by degrees of small moment Q

$$\frac{1}{m_\omega^2 + Q^2} \approx \frac{1}{m_\omega^2}\left(1 - \frac{Q^2}{m_\omega^2} + \frac{Q^4}{m_\omega^4} - ...\right) \tag{7}$$

Terms in parentheses act directly between nucleons. They called contact terms. Contact terms play an important role in renormalization. Contact terms absorb infinities and remove scale and cutoff dependence. Thus, in EFT short range NN interaction is described by contact terms which are constrained by parity, time-reversal, but not by chiral symmetries.

ChEFT diagrams



In 2002, it was established that the calculation of NN potential should be carried out up to N^3LO order. In this case, the same accuracy is achieved in describing the nucleon-nucleon phases at energies not higher than 300 MeV as with meson potentials, which, however, describe phases up to 1 GeV.

In this case, the number of EFT potential parameters is 24, and the mesonic - 35.

Thus, with the help of EFT, the substantiation of the mesonic NN potential was actually obtained.

Структура ядра и киральная эффективная теория поля.

1.It is considered established that NNN forces must be considered to predict the nuclear structure.

2. In the initial phase, the 3NFs were typically adjusted in A=3 and/or the A=4 systems and the ab initio calculations were driven up to the oxygen region. It turned out that for A < 16 the ground-state energies and radii are predicted about right, no matter what type of chiral or phenomenological potentials were applied (local, nonlocal, soft, hard, etc.) and what the details of the 3NF adjustments to few-body systems were. It may be suggestive to perceive the α-particle substruture of ^{16}O to be part of the explanation.

The picture changed, when it became possible to move up to medium-mass nuclei (e. g., the calcium or even the tin regions). Large variations of the predictions now occurred depending on what forces were used, and cases of severe underbinding as well as of substantial overbinding were observed. Ever since the nuclear structure community understands that the ab initio explanation of intermediate and heavy nuclei is a severe, still unsolved, problem.

K. Hebeler, Phys. Rept. 890, 1 (2021).

A seemingly successfull interaction for the intermediate mass region appears to be the force that is commonly denoted by 1.8/2.0(EM) („Magic force"), which is a similarity renormalization group (SRG) evolved version of the NNNLO 2NF complemented by a NNLO 3NF adjusted to the triton binding energy and the charge radius of 4He. With this force, the ground-state energies all the way up to the tin isotopes are reproduced perfectly, but with charge radii being on the smaller side. Nuclear matter saturation is also reproduced reasonably well, with a slightly too high saturation density. However, these calculations are not consistently ab initio, because the 2NF of 1.8/2.0(EM) is SRG evolved, while the 3NF is not. Still, this force is providing clues for how to get the intermediate and heavy mass region right.

Thus, in the follow-up, there have been attempts to get the medium-mass nuclei under control by means of more consistent ab initio calculations. Of the various efforts, let us single out three, which demonstrate in more detail what the problems are.

J. Hoppe, C. Drischler, K. Hebeler, A. Schwenk, and J. Simonis, Phys. Rev. C **100**, 024318 (2019); C.Drishler, K.Hebeler, and A.Schwenk, Phys.Rev.Lett. **122**, 042501 (2019).

Recently developed soft chiral 2NF at NNLO and NNNLO were picked up and complemented with 3NF at NNLO and NNNLO, respectively, to fit the triton binding energy and nuclear matter saturation. These forces were then applied in calculations of finite nuclei up to 68Ni predicting underbinding and slightly too large radii.

T. Hüther, K. Vobig, K. Hebeler, R. Machleidt, and R. Roth, Phys. Lett. B **808**, 135651 (2020)

In a separate study, the same 2NFs were employed, but with the 3NFs now adjusted to the triton and ^{16}O ground-state energies. The interactions so obtained reproduce accurately experimental energies and proton radii of nuclei up to 78Ni. However, when the 2NF plus 3NF combinations are utilized in nuclear matter, then dramatic overbinding and no saturation at reasonable densities is obtained. Obviously, there is a problem with achieving simultaneously reasonable results for nuclear matter and medium mass nuclei: nuclear matter is saturated right, but nuclei are underbound or nuclei are bound accurately, but nuclear matter is overbound.

The path to the calculations of the binding energies of heavy nuclei passes through the calculations of the binding energies of nuclear matter. The attempts to explain nuclear matter saturation have a long history. The modern view is that the 3NF is essential to obtain saturation. In this scenario, the 2NF substantially overbinds nuclear matter, while the 3NF contribution is repulsive and strongly density-dependent leading to saturation at the appropriate energy and density.

When 2NF+3NF комбинация from [C.Drishler, K.Hebeler, and A.Schwenk, PRL 122, 042501 (2019)] was applied in calculations of binding energies of nuclei up to Ni isotopes there was obtained underbinding of the ground state energies. On the other hand, the 2NF + 3NF combination, known as 1.8/2.0 or Magic, which correctly describes the saturation of nuclear matter, reproduces nuclear binding energies up to Sn. The difference between these two cases lies in the 2NF part of these forces. 2NF contribution to the binding energy of nuclear matter in the case of 1.8/2.0 forces significantly exceeds the expected result. On the other hand, the contribution to the binding energy of nuclear matter 2NF a member of the first of the forces considered, only slightly exceeds the expected result.

This shows that a considerable overbinding of nuclear matter by the 2NF is necessary to correctly bind intermediate-mass nuclei, when 3NFs at NNLO are applied.

W. G. Jiang, et al., Phys. Rev. C **102**, 054301 (2020).

In recent work by the Göteborg-Oak Ridge (GO) group the authors present an NNLO model including Delta-isobars that apparently overcomes the above problem. With this model, they obtain accurate binding energies and radii for a range of nuclei from A = 16 to A = 132, and provide accurate equations of state for nuclear matter. However, the accuracy of the NN part of these interactions is not checked against NN data. Another aspect of interest (not investigated) is if the inclusion of Delta-degrees of freedom leads to a higher degree of softness. Note that the successful "Magic"1.8/2.0(EM) potential is very soft since it is SRG evolved. Moreover, a recent study, which investigated the essential elements of nuclear binding using nuclear lattice simulations, has come to the conclusion that proper nuclear matter saturation requires a considerable amount of non-locality in the NN interaction implying a high degree of softness.

Binding energies of medium mass nuclei. Guthenburg-Oak Ridge group potential (GO) (2020).

The predictions by the original GO potentials for the nuclear matter binding are similar to the Magic one, which explains the results of the GO potentials for nuclei up to A = 132, similar to that with Magic. On the other hand this potential has serious problems with scattering data description. When describing pp-scattering at energies below 100 MeV, it turned out that the χ^2 for these potentials is more than 3 times higher than the result obtained more than 60 years ago with the Hamada-Johnston potential. For that reason this potential where corrected. The refitted potentials are less attractive than the original GO versions and they produce underbinding in intermediate-mass nuclei. The question we wish to address is then why, after refit to proper accuracy, this potential lost attraction?

Binding energies of medium mass nuclei. Guthenburg-Oak Ridge group potential (GO) (2020).

The T-matrix is essentially the sum of two terms: the central force term, V_C, and the second order in tensor force V_T . A potential with a strong V_T will produce a large (attractive) second order term and, hence, go along with a weaker (attractive) central force; as compared to a weak tensor force potential, where the lack of attraction by the second order term has to be compensated by a stronger (attractive) central force.

The G-matrix equation differs from the T-matrix equation in two ways: First, the Pauli projector, which prevents scattering into occupied states and, thus, cuts out the low-momentum spectrum. Second, the single-particle spectrum in nuclear matter, which enhances the energy denominator, thereby decreasing the integrand.

Guthenburg-Oak Ridge group potential (GO) (2020).

Both medium effects reduce the size of the (attractive) integral term. Thus, large tensor force potentials undergo a larger reduction of attraction from these medium effects than weak tensor force potentials. This explains the well-known fact that NN potentials with a weaker tensor force yield more attractive results when applied in nuclear few- and many-body systems as compared to their strong tensor force counterparts. The GO potentials have a very weak tensor force. To agree with the empirical information on phases, the tensor force has to be stronger, like in the case of the refitted potential.
To summarize, when the parameters of the GO potentials are corrected to obtain a realistic fit of the scattering data, the favorable predictions for intermediate-mass nuclei are very likely to disappear, as did the extra attraction in nuclear matter.

Extraordinary nonlocality of the 1.8/2.0 Magic potential, which is explained by the use of Similarity Renorm. group is a source of additional attraction manifested in calculations of binding energies. This leads to the fact that the term of the 2nd order in tensor interaction is unusually small, and therefore the central potential is large and attractive. This degree of non-locality cannot currently be achieved by any conventional chiral potential.

The problem of describing the properties of medium mass nuclei remains unresolved.