# Influence of relativistic rotation on the equation of state of gluodynamics

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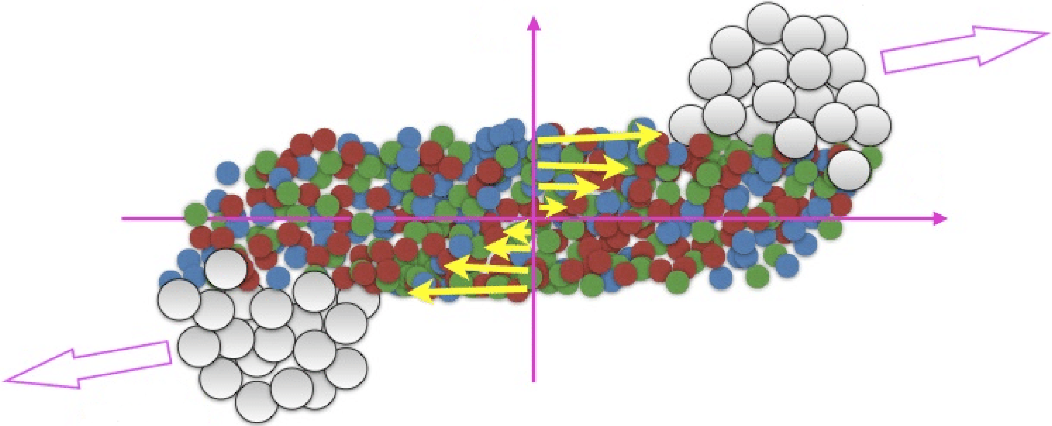
# Motivation



# Metric in rotating coordinates

$$x \equiv (x_0,\ x_1,\ x_2,\ x_3) = (t,\ r\sin\varphi,\ r\cos\varphi,\ z) \tag{1}$$

$$t = t_{\text{lab}}, \quad r = r_{\text{lab}}, \quad z = z_{\text{lab}}, \quad \varphi \sim (\varphi_{\text{lab}} - \Omega t) \tag{2}$$

$$g_{\mu\nu}^{(\text{lab})} = \eta_{\mu\nu} = \text{diag}(1,\ -1,\ -1,\ -1) \tag{3}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 - r^2\Omega^2) dt^2 - 2r^2\Omega dt d\varphi - dr^2 - r^2 d\varphi^2 - dz^2 \tag{4}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{5}$$

# Free energy density

$$E = E^{(\mathrm{lab})} - J\Omega \tag{6}$$

$$F = E - TS \tag{7}$$

$$J = -\left(\frac{\partial F}{\partial \Omega}\right)_T \tag{8}$$

$$Z = \int \mathcal{D}[U]\, e^{-S_E[U]} \tag{9}$$

$$F = -T \log Z \tag{10}$$

$$f = \frac{F}{V} \tag{11}$$

# EoS expansion coefficients

$$\frac{f(T, v_I, \dots)}{T^4} = c_0(T, \dots) + c_2(T, \dots) v_I^2 + c_4(T, \dots) v_I^4 + \dots \tag{12}$$

$$F = VT^4 \frac{f}{T^4} = VT^4 \left[c_0 - c_2 v^2 + c_4 v^4 + \dots\right] \tag{13}$$

$$J = -\partial_\Omega F = VT^4 c_2 \frac{N_r^2 a^2}{2} \Omega - O(\Omega^3) \tag{14}$$

$$I = \partial_\Omega J\big|_{\Omega=0} = VT^4 c_2 \frac{N_r^2 a^2}{2} = \boxed{\frac{1}{2} c_2 T^4 V L_r^2 = I} \tag{15}$$

# Gluodynamics in rotating coordinates

$$S_G = \int d^4x \, \sqrt{\det g_{\alpha\beta}} \, \frac{1}{2g_{\rm YM}^2} \, g^{\mu\nu} g^{\rho\sigma} \, \mathrm{tr} F_{\mu\rho} F_{\nu\sigma} \tag{16}$$

$$S_G = \frac{1}{g_{\rm YM}^2} \int d^4x \, \mathrm{tr}[(1-r^2\Omega^2)F_{xy}F_{xy} + (1-y^2\Omega^2)F_{xz}F_{xz} + (1-x^2\Omega^2)F_{yz}F_{yz} + F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + \\ + F_{z\tau}F_{z\tau} - 2iy\Omega(F_{xy}F_{y\tau} + F_{xz}F_{z\tau}) + 2ix\Omega(F_{yx}F_{x\tau} + F_{yz}F_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}] \tag{17}$$

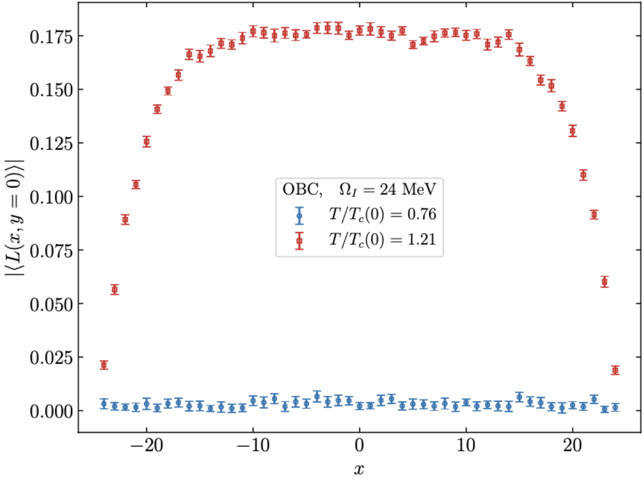
Simulations with a sign problem:

- Analytical continuation $\Omega_I = -i\Omega$
- Expansion coefficients at $\Omega = 0$

# Boundary conditions

- PBC are used for T and Z directions.
- PBC or OBC are used for X,Y directions.
- Typical lattice sizes: $40^4$ and $\{4 \ldots 8\} \times 40^3$.
- $N_t^4$ numerical factor practically forbids high $N_t$.
- Screening of BC due to high mass of glueballs.

# Screening of boundary conditions



# Lattice action density

$$S_G = -\beta \sum_{p\in\Lambda} s(p,\ N_t,\ N_r,\ N_z,\ v(p),\ Y) \tag{18}$$

$$s = \frac{1}{|\Lambda|} \sum_{p\in\Lambda} s(p,\ \ldots) \tag{19}$$

$$v_I = \frac{N_r}{2} a\Omega_I \tag{20}$$

$$\Delta s(\beta,\ \ldots) = \left[ s(\beta, N_t,\ \ldots) - s(\beta, N_t^{(\infty)},\ \ldots) \right] \tag{21}$$

# Free energy density from lattice data

$$s = \frac{1}{Z}\int \mathcal{D}[U]\,\frac{1}{|\Lambda|}\partial_\beta e^{-S_G[U]} = \frac{1}{|\Lambda|}\frac{\partial \log Z(\beta, N_t, N_r, N_z, v_I, Y)}{\partial\beta} \tag{22}$$

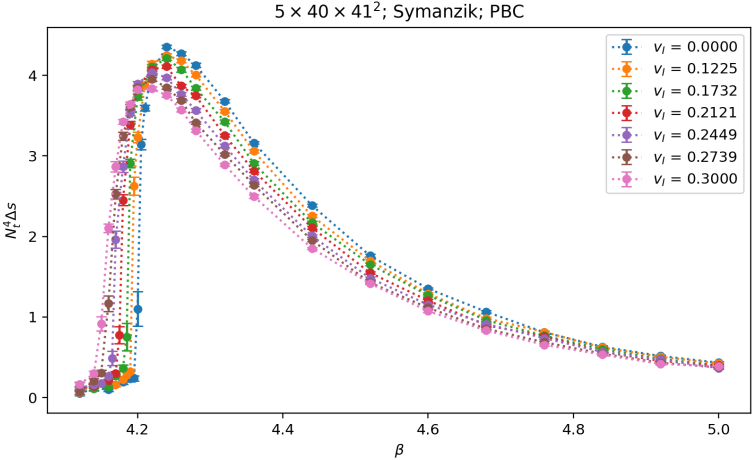
$$\frac{f(\beta, v_I, N_t, \ldots)}{T} = -\frac{1}{V}\log Z(\beta, v_I, N_t, \ldots) =$$

$$= -\frac{1}{N_r^2 N_z a^3(\beta)}\int_0^\beta d\tilde\beta\,(\partial_\beta \log Z)(\tilde\beta, v_I, N_t, \ldots) =$$

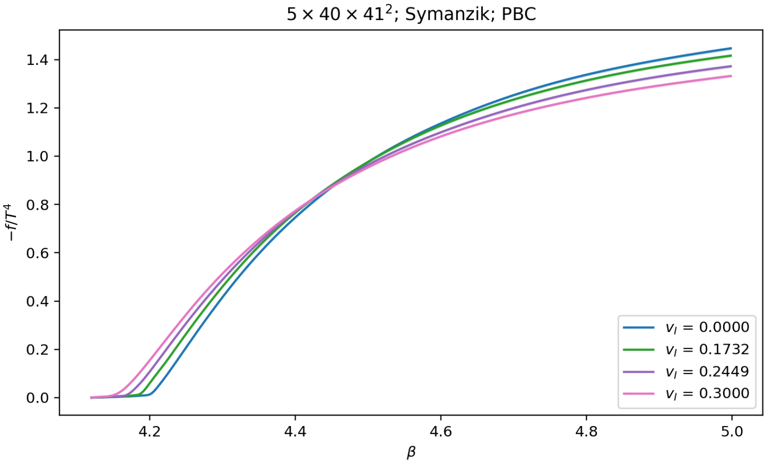
$$= -N_t^4\,T^3\int_0^\beta d\tilde\beta\; s(\tilde\beta, v_I, N_t, \ldots)$$

$$\boxed{\frac{f(\beta, v_I, N_t, \ldots)}{T^4} = -N_t^4\int_0^\beta d\tilde\beta\,\left[s(\tilde\beta, v_I, N_t, \ldots) - s(\tilde\beta, v_I, N_t \to \infty, \ldots)\right]} \tag{23}$$

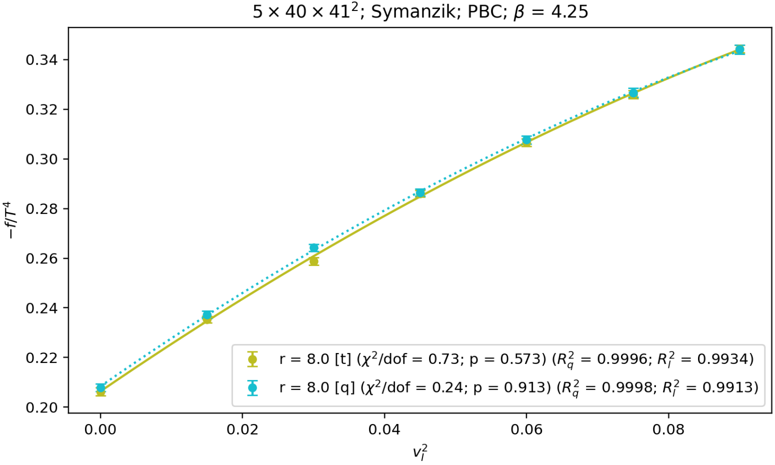
# Lattice action density



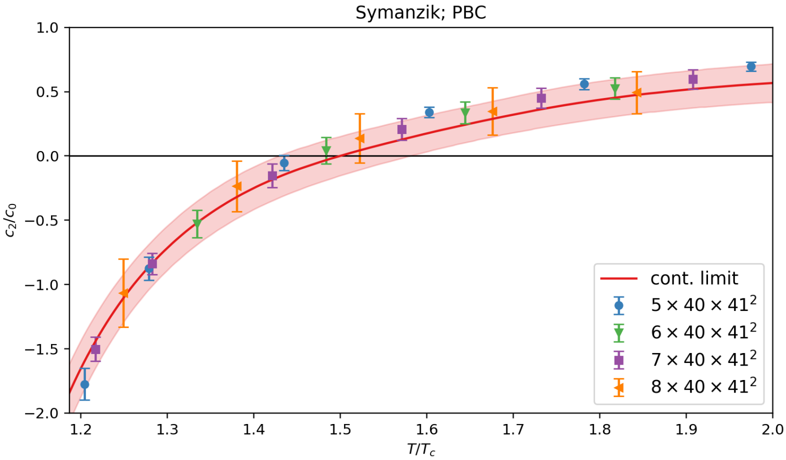
# EoS from lattice data



# Fitting EoS


$5 \times 40 \times 41^2$; Symanzik; PBC; $\beta = 4.25$
$-f/T^4$
0.34
0.32
0.30
0.28
0.26
0.24
0.22
0.20
0.00
0.02
0.04
0.06
0.08
$v_I^2$
r = 8.0 [t] ($\chi^2$/dof = 0.73; p = 0.573) ($R_q^2$ = 0.9996; $R_I^2$ = 0.9934)
r = 8.0 [q] ($\chi^2$/dof = 0.24; p = 0.913) ($R_q^2$ = 0.9998; $R_I^2$ = 0.9913)

# EoS expansion coefficients. Results



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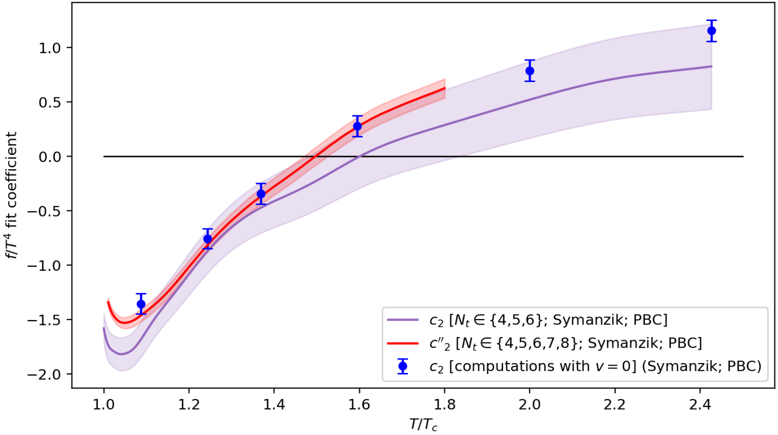

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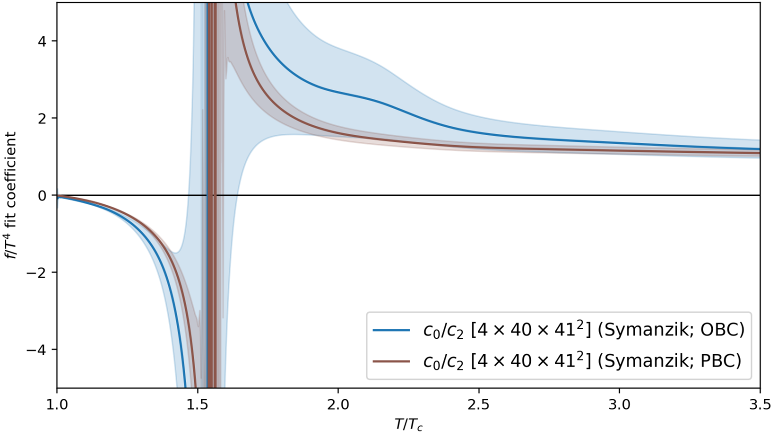

# Moment of inertia and scale anomaly

$$I(T) = -T^4 \int_0^T \frac{dT'}{T'} \frac{\langle T^\mu_\mu \rangle^{(2)}(T')}{T'^4} \tag{24}$$

$$\langle T^\mu_\mu \rangle^{(2k)}(T) = \frac{1}{(2k)!} \left[ \frac{\partial^{2k}}{\partial v_I^{2k}} \langle T^\mu_\mu \rangle (T, v_I) \right] \Bigg|_{v_I=0} \tag{25}$$

$$I = I_{\text{fluct}} + I_{\text{cond}} \tag{26}$$

# Conclusions

- Method for studying the dependence of the EoS of gluodynamics on rotation was introduced.
- Below a certain temperature negative moment of inertia suggests an instability of rigidly rotating plasma at these temperatures.
- We expect qualitatively the same results for QCD with dynamical quarks.

# Conclusions

Thanks for attention!