Holographic fishchain

Based on Gromov's arXiv:1903.10508

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From N=4 SYM to fishnet

- N=4 SYM considered to be integrable
- How to "simplify" the theory and study particular integrable parts of it? Twisting!

$$\mathrm{Tr}(\Phi_1[\Phi_2,\Phi_3]) \to \mathrm{Tr}(\Phi_1\Phi_2\Phi_3 e^{i\pi\beta} - \Phi_1\Phi_3\Phi_2 e^{-i\pi\beta})$$

$$\mathcal{L} = N_c \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu\phi_i^\dagger D_\mu\phi^i + i\bar{\psi}_{\dot{\alpha}\ A} D^{\dot{\alpha}\alpha}\psi^A_\alpha\right] + \mathcal{L}_{\text{int}}$$

$$\begin{aligned}\mathcal{L}_{\text{int}} = N_c g\ \mathrm{Tr}\Big[&\frac{g}{4}\{\phi_i^\dagger,\phi^i\}\{\phi_j^\dagger,\phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j \\ &- e^{-\frac{i}{2}\gamma_j^-}\,\bar{\psi}_j\phi^j\bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\,\bar{\psi}_4\phi^j\bar{\psi}_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k\phi^i\psi^j \\ &- e^{+\frac{i}{2}\gamma_j^-}\,\psi_4\phi_j^\dagger\psi_j + e^{-\frac{i}{2}\gamma_j^-}\,\psi_j\phi_j^\dagger\psi_4 + i\epsilon^{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar{\psi}_k\phi_i^\dagger\bar{\psi}_j\Big]\end{aligned}$$

From N=4 SYM to fishnets

- More simplifications: DS-limit: strong twist + weak coupling

$$\mathcal{L}_{\rm int} = N_c \,\mathrm{Tr}\Big[\xi_1^2\, \phi_2^\dagger\phi_3^\dagger\phi^2\phi^3 + \xi_2^2\, \phi_3^\dagger\phi_1^\dagger\phi^3\phi^1 + \xi_3^2\, \phi_1^\dagger\phi_2^\dagger\phi^1\phi^2 + i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3\phi_1^\dagger\bar\psi_2)$$
$$+\, i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1\phi_2^\dagger\bar\psi_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2\phi_3^\dagger\bar\psi_1)\Big].$$

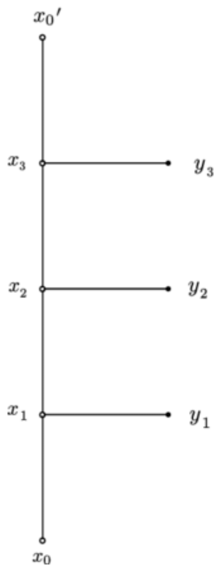
- MORE simplifications:
- $\xi_1 = \xi_2 = \xi_3$ beta-deformed SYM
- $\xi_1 = 0$ χ0-CFT
- $\xi_1 = \xi_2 = 0, \xi_3 \equiv \xi \neq 0$ biscalar fishnet **(the simplest case)**

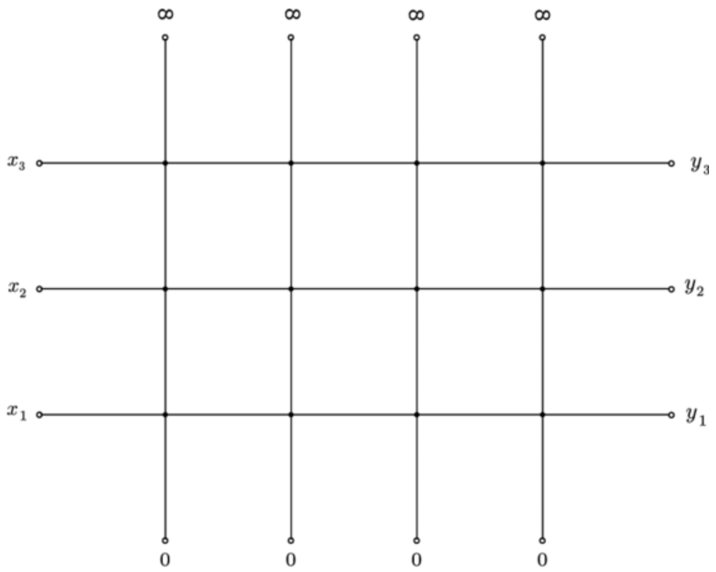
$$\mathcal{L}_{4d} = N\,\mathrm{tr}\left(|\partial\phi_1|^2 + |\partial\phi_2|^2 + (4\pi)^2\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)$$

Biscalar fishnet theory

- Conformal, non-unitary, solvable in the planar limit

$$\mathcal{L}_{4d} = N\,\mathrm{tr}\left(|\partial\phi_1|^2 + |\partial\phi_2|^2 + (4\pi)^2\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)$$





- Feynman rules:

$$\langle\phi_1(x_i)\phi_1^\dagger(y_i)\rangle = \frac{1}{(2\pi)^2(x_i - y_i)^2}$$

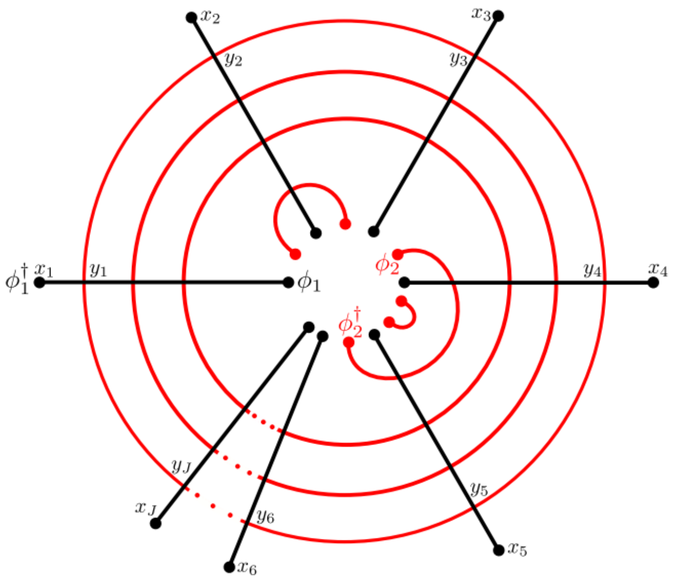
$$\langle\phi_2(y_i)\phi_2^\dagger(y_{i+1})\rangle = \frac{1}{(2\pi)^2(y_i - y_{y+1})^2}$$

$$(4\pi)^2\xi^2\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2$$

Biscalar fishnet theory

- Feynman diagram ("wheel-graph") for local operator:

$$\mathcal{O}_J = \phi_1^J (\phi_2^\dagger \phi_2)^n$$



- Graph-building operator:

$$\widehat{B}(\{\vec{y}_i\}_{i=1}^J, \{\vec{x}_j\}_{j=1}^J) = \prod_{i=1}^{J} \frac{(4\pi)^2 \xi^2}{(2\pi)^2(\vec{y}_i - \vec{y}_{i+1})^2 (2\pi)^2 (\vec{x}_i - \vec{y}_i)^2}$$

- All graph summation:

$$1 + \hat{B} + \hat{B}^2 + \ldots = \frac{1}{1 - \hat{B}}$$

- 2J-correlation function:

$$\langle x_1, \ldots, x_J | \frac{1}{1 - \hat{B}} | y_1, \ldots, y_J \rangle$$

Exact correlation function for J=2

- Conformal symmetry fixes form of eigenvectors (CFT wave function) of the graph-building operator

$$\Psi_{\Delta,S}(x_1,x_2) = \frac{1}{x_1^{\Delta-S} x_2^{\Delta-S} x_{12}^{S+2-\Delta}} (\text{tensor})^S$$

- Eigenvalue of the graph-building operator

$$\hat{B}_{\Delta,S}\Psi_{\Delta,S}(x_1,x_2) = E_{\Delta,S}\Psi_{\Delta,S}(x_1,x_2)$$

- So the correlation function can be represented as

$$\langle x_1,...,x_J| \frac{1}{1-\hat{B}_{\Delta,S}} |y_1,...,y_J\rangle =$$

$$= \sum_S \int d\Delta \frac{1}{1-E_{\Delta,S}} \langle x_1,...,x_J|\Psi_{\Delta,S}\rangle\langle\Psi_{\Delta,S}|y_1,...,y_J\rangle$$

- Poles given by:

$$E_{\Delta,S} = 1$$

Exact correlation function for J=2

- 4 pt correlation function can be written in the next form

conformal block

$$\mathcal{G}(x_1, x_2; y_1, y_2) = \sum_{\Delta,S} C_{\Delta,S}\, g_{\Delta,S}(u,v)$$

structure constant

- Spectral equation and its solutions:

$$(\nu^2 + S^2/4)(\nu^2 + (S+2)^2/4) = \xi^4 \quad \Rightarrow \quad \begin{aligned} \Delta_2(S) &= 2 + \sqrt{(S+1)^2 + 1 - 2\sqrt{(S+1)^2 + 4\xi^4}} \\ \Delta_4(S) &= 2 + \sqrt{(S+1)^2 + 1 + 2\sqrt{(S+1)^2 + 4\xi^4}} \end{aligned}$$

- Classical limit:

Spectrum:

$$\Delta = \sqrt{S^2 \pm 4\xi^2}$$

4pt correlation function:

$$\mathcal{G} \sim e^{-2i\xi\sqrt{\theta^2+\rho^2}}$$

$$u = \frac{4}{(\cos\theta - \cosh\rho)^2}$$

$$v = \frac{(\cos\theta + \cosh\rho)^2}{(\cos\theta - \cosh\rho)^2}$$

Derivation of dual

- Wave functions corresponding to dilatation operator are at the residues:

$$(\hat{B} - 1)\Psi = 0$$

Derivation of dual

- Wave functions corresponding to dilatation operator are at the residues:

$$(\hat{B}-1)\Psi = 0$$

$$\vec{p}_i = -i\frac{\partial}{\partial \vec{x}_i} \longrightarrow \prod_i \vec{p}_i^{\,2}$$

$$\prod_{i=1}^{J} \frac{\xi^2/\pi^2}{(\vec{y}_i - \vec{y}_{i+1})^2(\vec{x}_i - \vec{y}_i)^2} \longrightarrow \Box_i \frac{1}{(2\pi)^2(x_i - y_i)^2} = \delta(x_i - y_i)$$

$$\mathcal{H} \circ \Psi(x_i) = 0 \qquad \mathcal{H} = \prod_{i=0}^{J} p_i^2 - \prod_{i=0}^{J} \frac{4\xi^2}{(x_i - x_{i+1})^2}$$

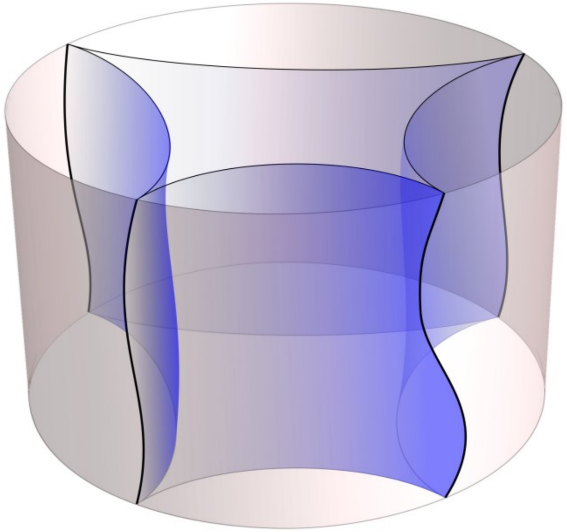
Quantum system with time-reparametrization symmetry?

Symmetries, EOMs, constraints

- Lagrangian at the classical level

$$L = \frac{2J-1}{2^{\frac{2J}{2J-1}}}\left(\frac{1}{\gamma}\prod_{i=1}^{J}\dot{\vec{x}}_i^2\right)^{\frac{1}{2J-1}} + \gamma\prod_{i=1}^{J}\frac{4\xi^2}{(\vec{x}_i-\vec{x}_{i+1})^2} \qquad S = \xi\int L\,dt = 2J\xi\int\left(\prod_{i=1}^{J}\frac{\dot{\vec{x}}_i^2}{(\vec{x}_i-\vec{x}_{i+1})^2}\right)^{\frac{1}{2J}} dt$$

As it is well known the conformal group in 4D coincides with the group of rotations in R(1,5), so we embed actions above to 6D space



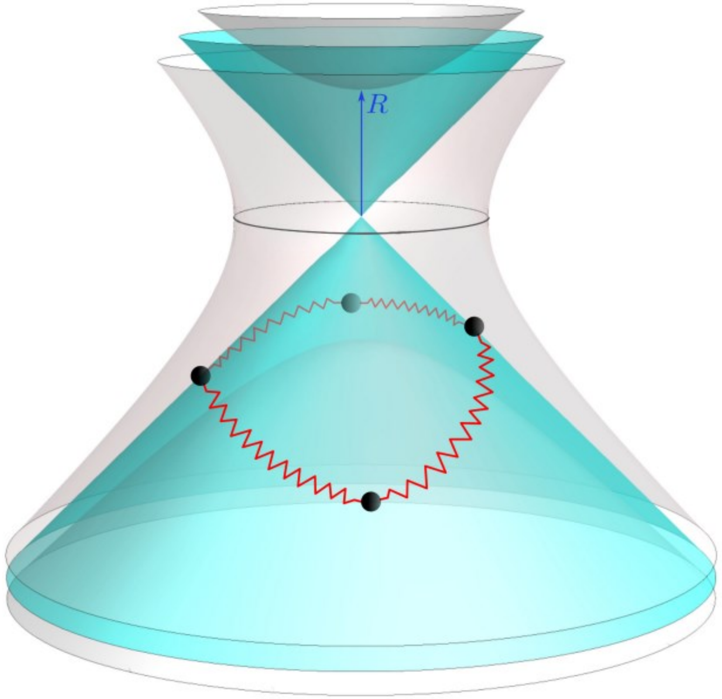
$$x_i^\mu \to X_i^M/X_i^+,\quad X_i^2=0,\quad X^+ = X_i^0 + X_i^{-1}$$

Fishnet action on projective lightcone: $$L = 2J\left(\prod_{i=1}^{J}\frac{\dot{X}_i\cdot\dot{X}_i}{-2X_i\cdot X_{i+1}}\right)^{\frac{1}{2J}}$$

Symmetries of the action:

1. Conformal symmetry
2. Time-reparametrisation symmetry
3. Time-dependent scaling
4. Translation along the chain
5. Gauge symmetry

Symmetries, EOMs, constraints



Virasoro-like constraints

$$\dot{X}_k^2 = 2\prod_i (-X_i \cdot X_{i+1}) \equiv \mathcal{L}$$

Polyakov-type action:

$$L = -\sum_i \left[\frac{\dot{X}_i^2}{2\alpha_i} + \eta_i X_i^2 + \gamma \prod_k (-X_k . X_{k+1})^{-\frac{1}{J}} \right]$$

EOM

$$\ddot{X}_i = 2\eta_i X_i - \frac{\mathcal{L}}{2}\left(\frac{X_{i+1}}{X_{i+1}.X_i} + \frac{X_{i-1}}{X_i.X_{i-1}} \right)$$

$$q_i^{MN} = 2\dot{X}_i^{[M} X_i^{N]} \quad j_i^{MN} = 2\frac{X_{i-1}^{[M} X_i^{N]}}{X_{i-1}.X_i} \quad \dot{q}_i = \frac{\mathcal{L}}{2}(j_{i+1} - j_i) \ ,$$

Nodes or SO(1,5)-charge and current density

Test (J=2 classical solutions)

- Fishnet:

Classical spectrum:

$$\Delta^2 = S^2 \mp 4\xi^2$$

4pt correlation function:

$$\mathcal{G} \sim \exp\left(-2i\xi\sqrt{\rho^2+\theta^2}\right)$$

$$u = \frac{4}{(\cos\theta - \cosh\rho)^2}$$

$$v = \frac{(\cos\theta+\cosh\rho)^2}{(\cos\theta-\cosh\rho)^2}$$

- Fishchain: define two 6D null-vectors

$$X_{1,2} = \frac{r}{\sqrt{2}}\left(\cosh s, \sinh s, \pm\cos\phi, \mp\sin\phi, 0, 0\right)$$

- Conserved charges:

$$S = \mp 4\xi\tau, \quad s = -i\tau\Delta/\xi,$$
$$\phi_1 = \tau S_1/\xi, \quad d\tau = \frac{dt}{r^2(t)}$$

Solution:

$$S^2 - \Delta^2 = \pm 4\xi^2$$

Integrability

- Zero curvature condition (like Toda chain):

$$\dot{\mathbb{L}}_i = \mathbb{V}_{i+1}.\mathbb{L}_i - \mathbb{L}_i.\mathbb{V}_i$$

$0 = j_{i+1} q_i^2 - j_i q_i^2$ + EOM

- a pair of spacelike and timelike connections (Lax pairs):

$$\mathbb{L}_i^{\mathbf{6}} = u^2 + u q_i + \frac{q_i^2}{2} \ , \qquad \mathbb{V}_i^{\mathbf{6}} = \frac{j_i}{u} \frac{\mathcal{L}}{2}$$

- Integral of motions will be given

$$\mathbb{T}(u) = \mathrm{tr}\left(\mathbb{L}_1(u) ... \mathbb{L}_J(u)\right)$$

$$\dot{\mathbb{T}}(u) = 0$$

Final remarks and conclusion

Conclusion:

- The fishchain can be derived explicitly from fishnet model
- It's conformal, integrable. Also the model reminds string theory
- It is dual to fishnets and live on lightcone

Further development

- One can quantise the model. Quantum fishchain lives in AdS5!
- Fishchain theory is applicable on gamma-deformed N=4 SYM (at least on U(1)-sector with some magnons)
- Open fishchain(open fishchain without periodical boundary conditions)

Open problems

- Fishchain for N=4 SYM?
- Continuum limit?
- Wilson lines?

Thanks!