Invariants of angular distributions in description of Drell-Yan process data

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Outline

Introduction

Leptons are formed in decays of gauge bosons $Z \to l^- l^+$ produce in proton-proton collisions in the Drell-Yan process. Measurements of the angular distributions of this process provide a detailed method to test the Standard Model and to search for new physics.

Introduction

z-axis

In all reference frames, the angular coefficients are measured in the rest frame of the lepton pair. The angular distribution of decay depends on this pre-selected frame. Consequently, the outputs are inevitably dependent on the frame choice, so they are not convenient to work with.

Rotation-invariant observables which have been proposed in

- P. Faccioli, C. Lourenco, and J. Seixas, Rotation-invariant relations in vector meson decays. into fermion pairs, Phys. Rev. Lett. 105 (2010) 061601
- Y.-Q. Ma, J.-W. Qiu, and H. Zhang, arXiv:1703.04752 (2017).

provide much more powerful tool for data analysis.

We present an analysis of how these invariants change when the frame of reference changes.

INTRODUCTION

REFERENCE FRAMES

Collins–Soper (CS) frame

The z-axis is represented as a polar axis and is defined as the bisector of the angle between the momentum of one of the protons and the reverse momentum for the other proton.

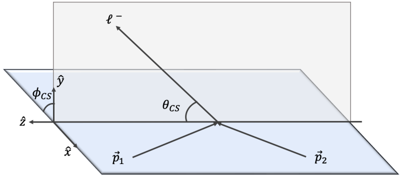


Figure. The Collins- Soper reference frame: the angles θ_{CS} and ϕ_{CS} are defined for a negatively charged lepton; $\hat{x}, \hat{y}, \hat{z}$–unique orthogonal coordinate axes of the system.

Gottfried–Jackson (GJ) frame

Z-axis is parallel to the axis of the incoming beam in the rest frame of the decaying particle and is directed in the direction of one of the protons in the boson rest frame.

Angular distribution in the Drell–Yan process

Let us consider a general expression of the angular distribution. In the literature, two types of parametrization are most commonly used.

- The first type, used in *theoretical* works:

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{3+\lambda}(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi +\\ +\frac{\nu}{2}\sin^2\theta\cos 2\phi + \rho\sin^2\theta\sin 2\phi +\\ +\sigma\sin 2\theta\sin\phi + 2A_\theta\cos\theta +\\ +2A_\phi\sin\theta\cos\phi + 2A_{\perp\phi}\sin\theta\sin\phi),\end{aligned} \tag{1}$$

where θ and ϕ are the polar and azimuthal angles of decay in the rest frame of the dilepton. The coefficients $\lambda, \mu, \nu, \rho, \sigma, A_\theta, A_\phi, A_{\perp\phi}$ carry information about the tensor polarization of the virtual photon.

Angular distribution in the Drell–Yan process

- The second type is used in *experimental* works:

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{16\pi}\Bigg(\left(1+\frac{A_0}{2}\right) + \left(1-\frac{3}{2}A_0\right)\cos^2\theta + \\ + A_1 \sin 2\theta \cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi + \\ + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\cos 2\phi + \\ + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\Bigg),\end{aligned} \tag{2}$$

where A_{0-7} is the complete set of polarization angle coefficients describing the angular distributions of leptons in the decay of Z–bosons.

Angular distribution in the Drell–Yan process

It is possible to express the parameters (1) through (2) by comparing the coefficients at the corresponding terms:

$$\lambda = \frac{2-3A_0}{2+A_0}, \qquad A_0 = \frac{4(1-\lambda)}{6+\nu} \tag{3}$$

$$\frac{\nu}{2} = \frac{A_2}{2+A_0}, \qquad A_1 = \frac{8\mu}{6+\nu} \tag{4}$$

$$\mu = \frac{2A_1}{2+A_0}, \qquad \frac{A_2}{2} = \frac{8\nu}{6+\nu} \tag{5}$$

$$\rho = \frac{2A_5}{2+A_0}, \qquad A_3 = \frac{16A_\phi}{6+\nu} \tag{6}$$

$$\sigma = \frac{2A_6}{2+A_0}, \qquad A_4 = \frac{16A_\theta}{6+\nu} \tag{7}$$

$$A_\theta = \frac{A_4}{2+A_0}, \qquad A_5 = \frac{8\rho}{6+\nu} \tag{8}$$

$$A_\phi = \frac{A_3}{2+A_0}, \qquad A_6 = \frac{8\sigma}{6+\nu} \tag{9}$$

$$A_{\perp\phi} = \frac{A_7}{2+A_0} \qquad A_7 = \frac{16A_{\perp\phi}}{6+\nu}; \tag{10}$$

Hadronic tensor

Cross section is proportional to the contraction of a hadronic $W^{\mu\nu}$ and lepton $L^{\mu\nu}$ tensors:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto W^{\mu\nu}L_{\mu\nu}. \tag{11}$$

Omitting the detailed derivation, let us use the result from [**1**]:

$$W^{ij} = \begin{pmatrix} \frac{1-\lambda}{2} & -\mu - iA_{\perp\phi} & -\sigma + iA_{\phi} \\ -\mu + iA_{\perp\phi} & \frac{1+\lambda-2\nu}{2} & -\rho - iA_{\theta} \\ -\sigma - iA_{\phi} & -\rho + iA_{\theta} & \frac{1+\lambda+2\nu}{2} \end{pmatrix}. \tag{12}$$

Normalization condition: $Tr\mathbf{W} = 1$.

Invariants

The density matrix (12) is associated with several rotation-invariant quantities proposed in:

- P. Faccioli, C. Lourenco, and J. Seixas, Rotation-invariant relations in vector meson decays. into fermion pairs, Phys. Rev. Lett. 105 (2010) 061601
- Y.-Q. Ma, J.-W. Qiu, and H. Zhang, arXiv:1703.04752 (2017).

$$U_1 = \frac{A_\theta^2 + A_\phi^2 + A_{\perp\theta\phi}^2}{(3+\lambda)^2}, \tag{13}$$

$$U_2 = \frac{\lambda^2 + 3(\frac{\nu^2}{4} + \mu^2 + \rho^2\sigma^2)}{(3+\lambda)^2}, \tag{14}$$

$$T = \frac{(\lambda + \frac{3}{2}\nu)(2\lambda^2 - 3\lambda\nu + 9\mu^2) + 9(\lambda\sigma^2 - 2\lambda\rho^2 + 6\mu\sigma\rho - \frac{3}{2}\nu\sigma^2)}{(3+\lambda)^3}. \tag{15}$$

Discussion

In this section we present the results of verification of rotational invariants using data obtained in NA10 experiment on the scattering of 252 GeV π^--mesons on a fixed tungsten target, in which only λ, μ, ν are measured, therefore, we neglect the rest of the coefficients.

- Phys.Rev.D 39 92-122, Experimental study of muon pairs produced by 252-GeV pions on tungsten (1989)

$$U_1 = 0; \quad U_2 = \frac{\lambda^2 + 3\left(\frac{\nu^2}{4} + \mu^2\right)}{(3+\lambda)^2}; \tag{16}$$

$$T = \frac{\left(\lambda + \frac{3\nu}{2}\right)\left(2\lambda^2 - 3\lambda\nu + 9\mu^2\right)}{(3+\lambda)^3}. \tag{17}$$

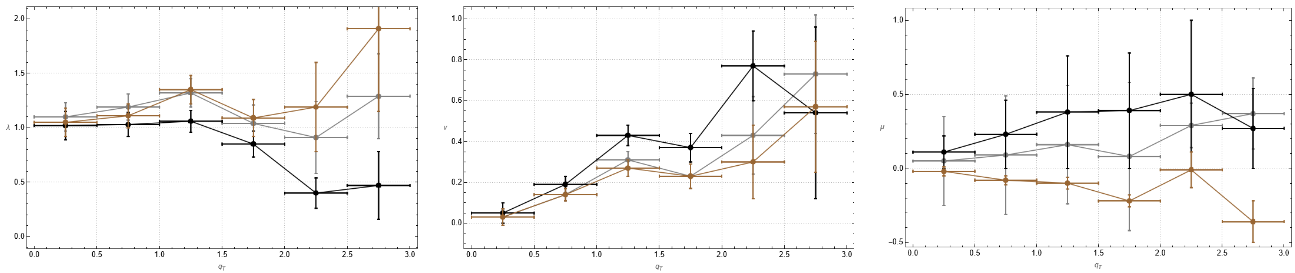
Discussion

We calculate the errors for rotational invariants as follows

$$\Delta U_2 = \partial_\lambda U_2 \cdot \Delta\lambda + \partial_\nu U_2 \cdot \Delta\nu + \partial_\mu U_2 \cdot \Delta\mu; \tag{18}$$

$$\Delta T = \partial_\lambda T \cdot \Delta\lambda + \partial_\nu T \cdot \Delta\nu + \partial_\mu T \cdot \Delta\mu. \tag{19}$$

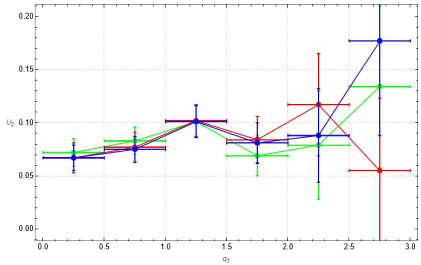
Discussion



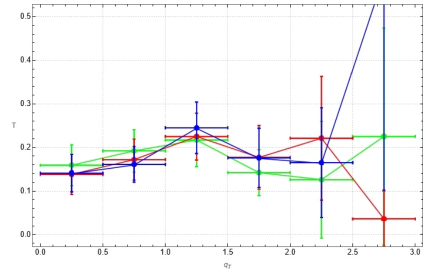
a) b) c)

Figure. Dependences of the parameters of angular distributions a) λ , b) ν, c) μ on the transverse momentum q_T in the Collins-Soper(black curve), Gottfried-Jackson (gray curve), u-channel (brown curve) reference frames.

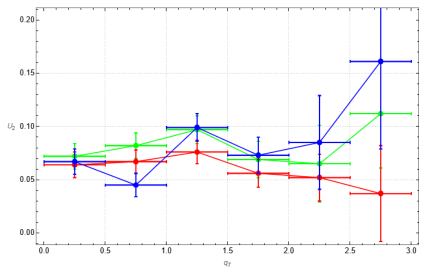
Discussion



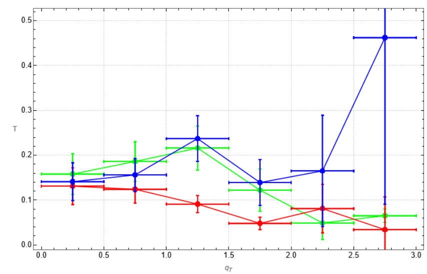
a)



b)



c)



d)

Figure. Dependences of rotational invariants a) U_2-invariant, b) T-invariant $\mu \neq 0$; c)U_2-invariant, d) T-invariant $\mu = 0$; on transverse momentum q_T in Collins-Soper (green curve), Gottfried-Jackson (red curve), u-channel (blue curve) reference frames.

Discussion

q_T[GeV]	$< \lambda >$	$< \nu >$	$< \mu >$	$< U_2 >$	$< T >$
0.25	1.061 ± 0.046	0.043 ± 0.011	0.061 ± 0.053	0.077 ± 0.003	0.158 ± 0.011
0.75	1.111 ± 0.085	0.165 ± 0.032	0.133 ± 0.094	0.088 ± 0.004	0.182 ± 0.016
1.25	1.247 ± 0.164	0.343 ± 0.081	0.213 ± 0.151	0.119 ± 0.001	0.233 ± 0.014
1.75	0.993 ± 0.197	0.284 ± 0.082	0.237 ± 0.162	0.085 ± 0.008	0.172 ± 0.019
2.25	0.831 ± 0.401	0.515 ± 0.246	0.271 ± 0.251	0.093 ± 0.019	0.175 ± 0.047
2.75	1.226 ± 0.723	0.618 ± 0.113	0.339 ± 0.063	0.125 ± 0.061	0.291 ± 0.281

a)

q_T[GeV]	$< \lambda >$	$< \nu >$	$< U_2 >$	$< T >$
0.25	1.061 ± 0.046	0.043 ± 0.011	0.027 ± 0.011	0.143 ± 0.019
0.75	1.111 ± 0.085	0.165 ± 0.032	0.029 ± 0.025	0.155 ± 0.031
1.25	1.247 ± 0.164	0.343 ± 0.081	0.091 ± 0.017	0.181 ± 0.089
1.75	0.993 ± 0.197	0.284 ± 0.082	0.066 ± 0.013	0.103 ± 0.048
2.25	0.831 ± 0.401	0.515 ± 0.246	0.067 ± 0.021	0.098 ± 0.059
2.75	1.226 ± 0.723	0.618 ± 0.113	0.103 ± 0.078	0.187 ± 0.372

b)

Table. Average values of coefficients of angular distributions and rotational invariants with standard deviation in different reference frames, a) $\mu \neq 0$, b) $\mu = 0$.

Conclusion

- Rotationally invariant quantities vary slightly when the reference frame is changed.
- Neglecting the coefficient μ in rotational invariants worsens this result.

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