Modeling of the entangled states transfer processes in microtubule tryptophan system

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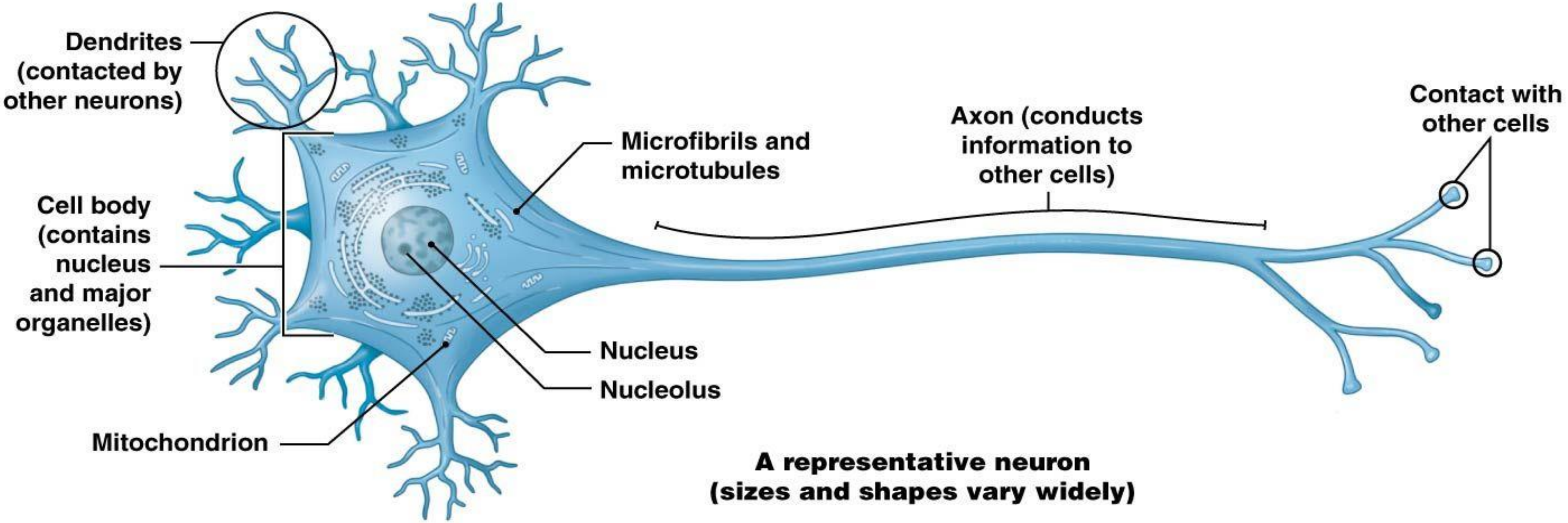
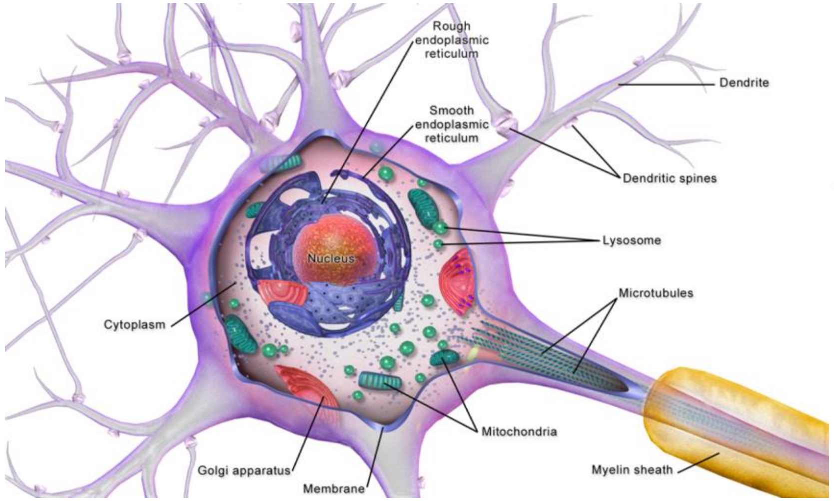
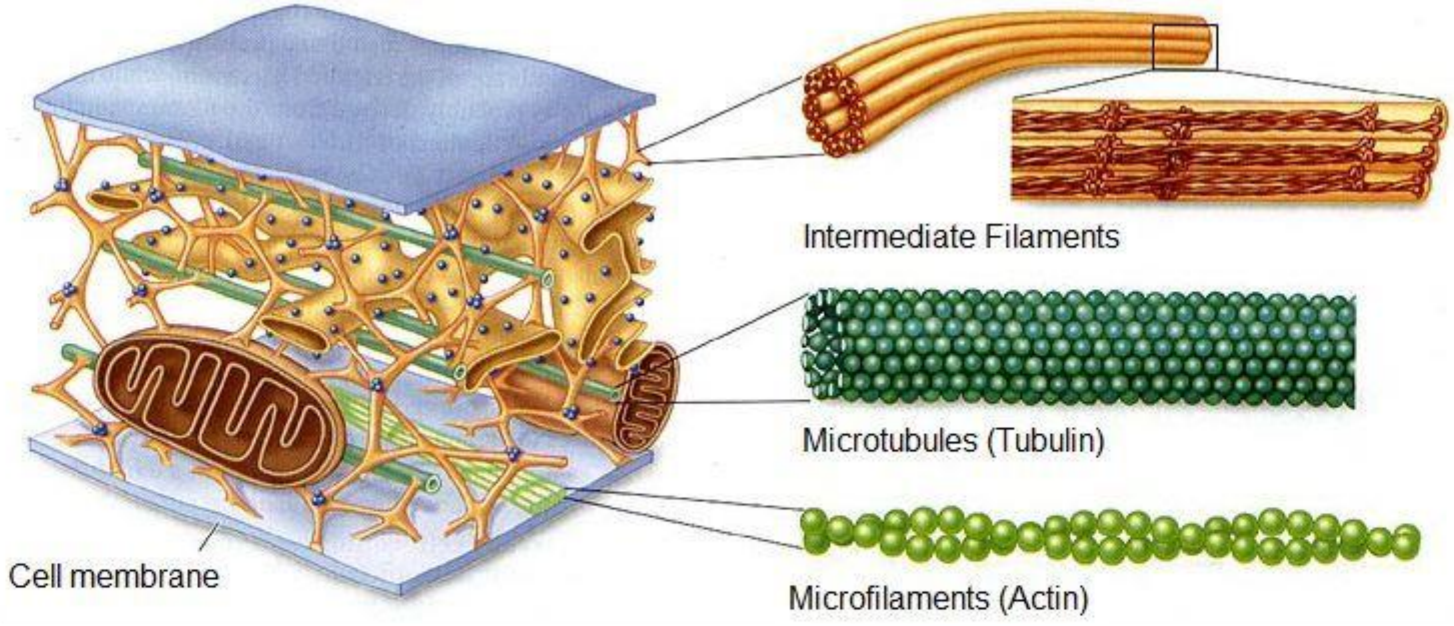
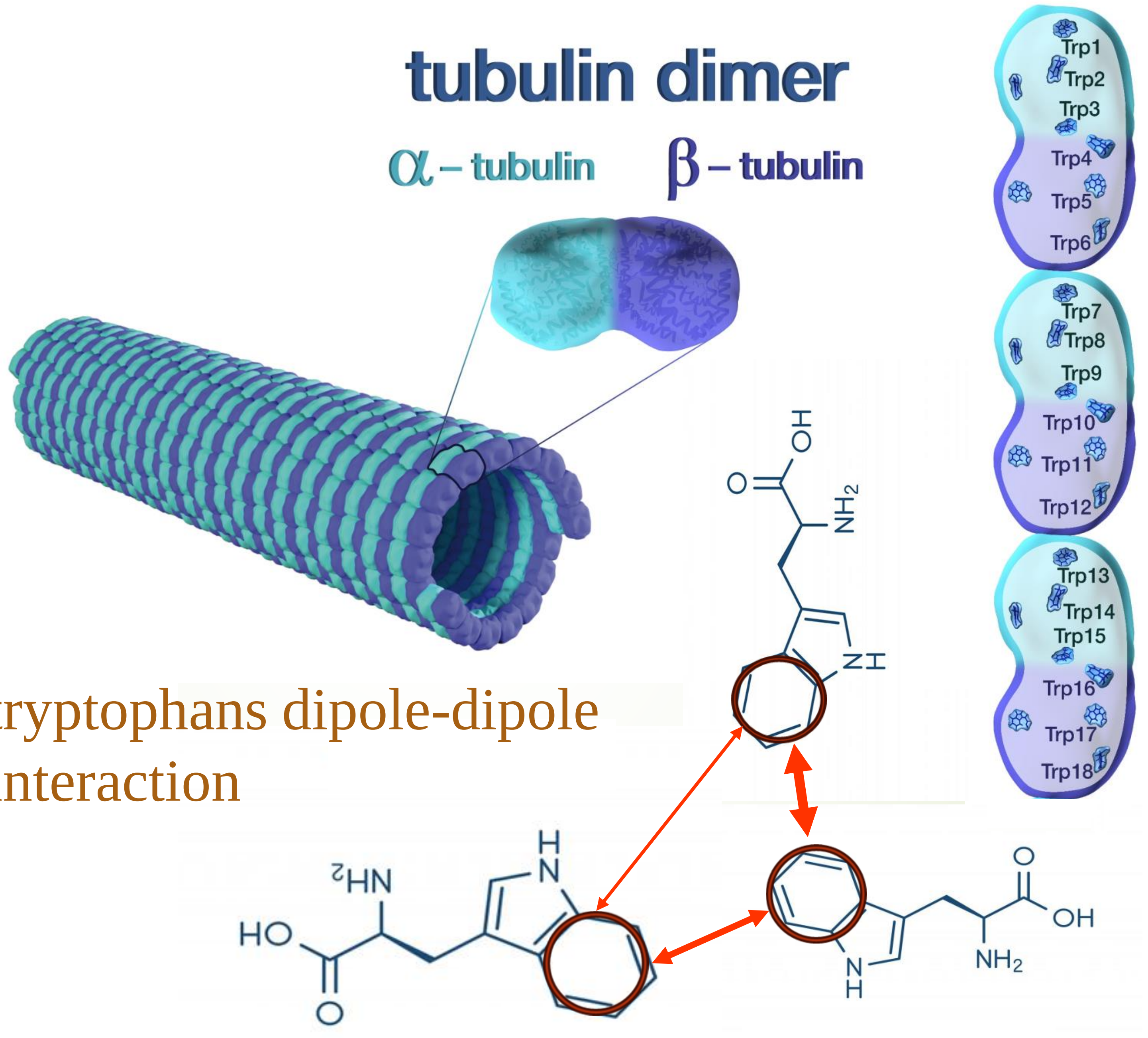
Microtubules

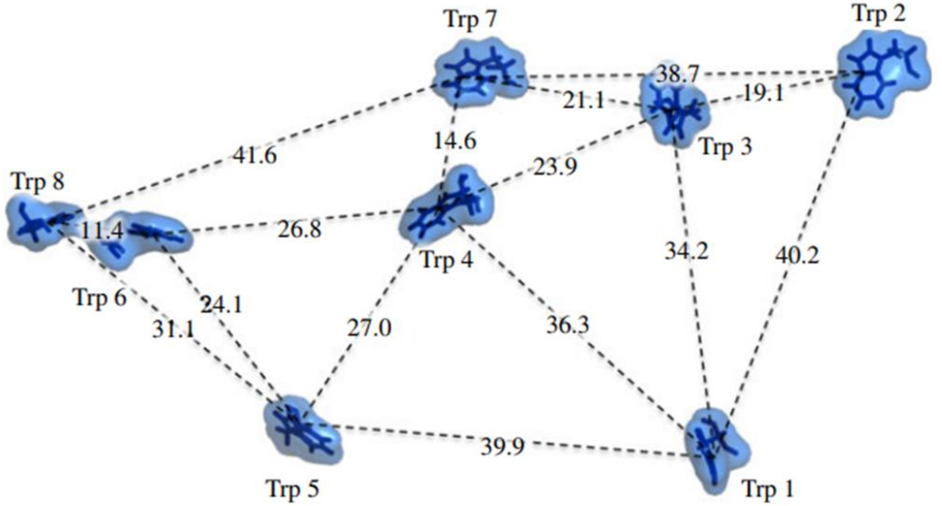

Dendrites
(contacted by
other neurons)
Cell body
(contains
nucleus
and major
organelles)
Mitochondrion
Microfibrils and
microtubules
Nucleus
Nucleolus
Axon (conducts
information to
other cells)
Contact with
other cells
A representative neuron
(sizes and shapes vary widely)

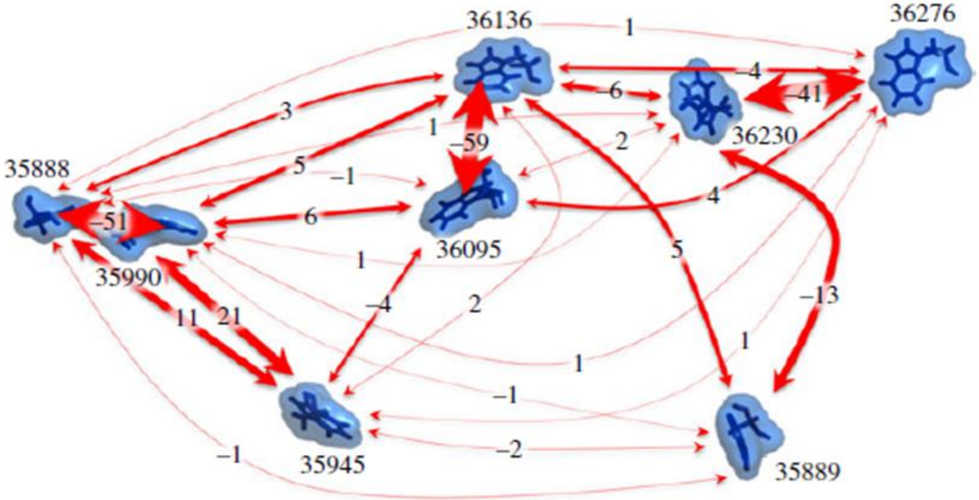

Rough
endoplasmic
reticulum
Smooth
endoplasmic
reticulum
Dendrite
Dendritic spines
Lysosome
Nucleus
Microtubules
Cytoplasm
Mitochondria
Golgi apparatus
Membrane
Myelin sheath


Intermediate Filaments
Microtubules (Tubulin)
Cell membrane
Microfilaments (Actin)



tryptophans dipole-dipole interaction





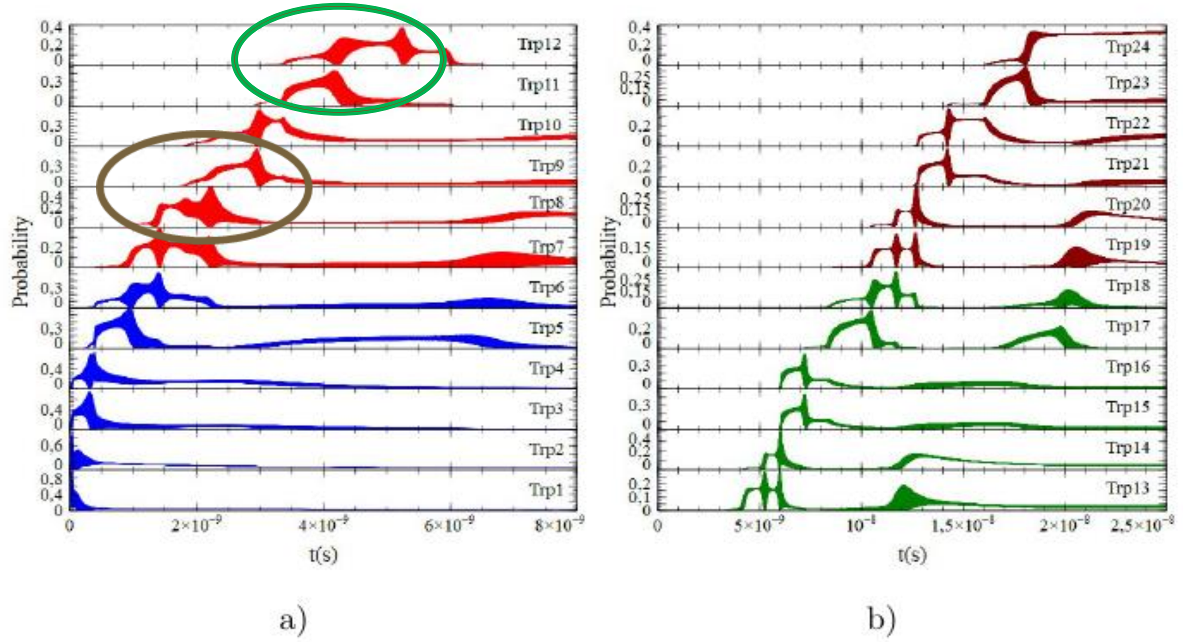
Craddock, T.J.A., Friesen, D., Mane, J., Hameroff, S., Tuszynski, J., 2014. Journal of the Royal Society Interface 11, 20140677.

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{H}_Q|\psi\rangle \qquad \hat{H}_Q = \sum_n \omega_n|n\rangle\langle n| + \sum_{n\neq n'} \nu_{n,n'}|n\rangle\langle n'|,$$

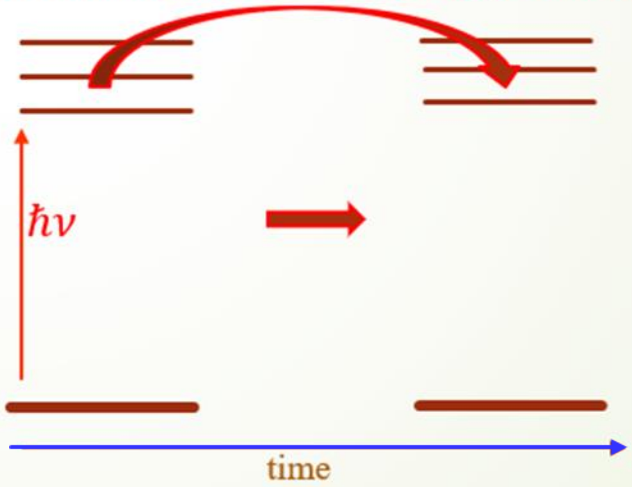
$$|\psi\rangle = \sum_n A_n(t)|n\rangle. \qquad \omega_n = \omega_n^0 + \kappa_n X_n.$$

$$M\frac{d^2X_n}{dt^2} = -\frac{\partial H}{\partial X_n} - \gamma\frac{dX_n}{dt} \qquad H = H_{Cl} + \langle\psi|\hat{H}_Q|\psi\rangle.$$

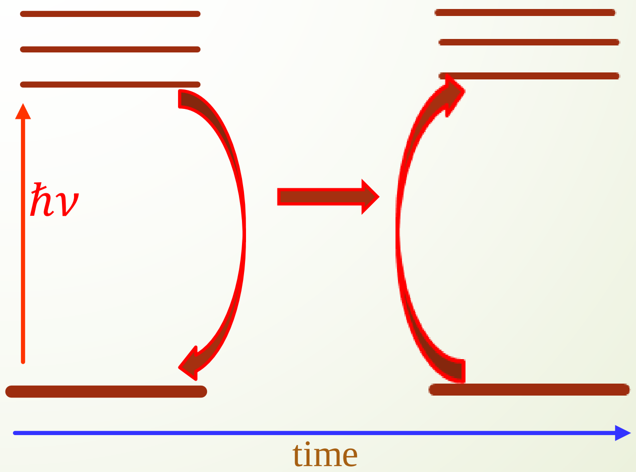
$$\begin{cases} i\dfrac{dA_n}{d\tilde{t}} = \omega_n A_n + \nu_{n,n+1}A_{n+1} + \nu_{n,n-1}A_{n-1} + \kappa_n W_n^2 X_n A_n, \\ \dfrac{d^2X_n}{d\tilde{t}^2} = -\gamma'\dfrac{dX_n}{d\tilde{t}} - W_n^2 X_n - |A_n|^2. \end{cases}$$



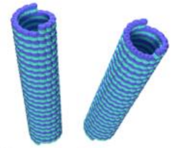
Exciton mechanism of energy transfer



Inductively resonant energy transfer



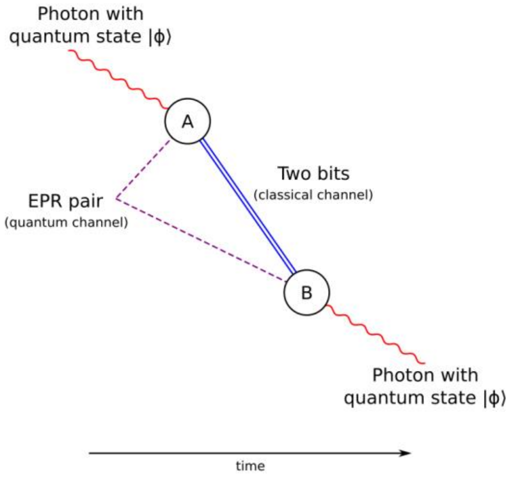
Energy transfer mechanisms in microtubules tryptophan system have been determined.



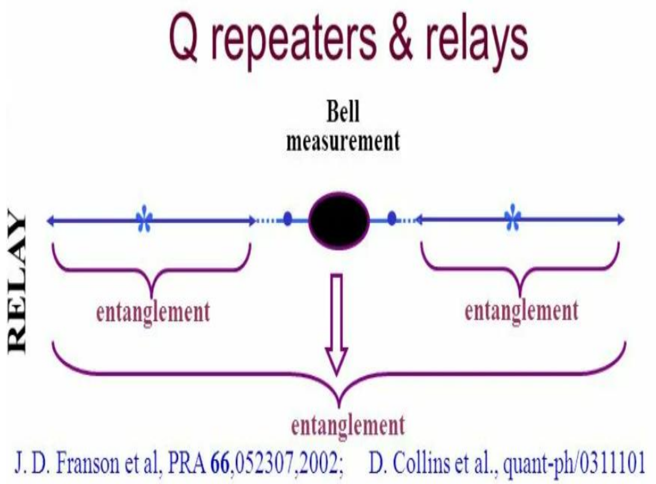
Microtubules

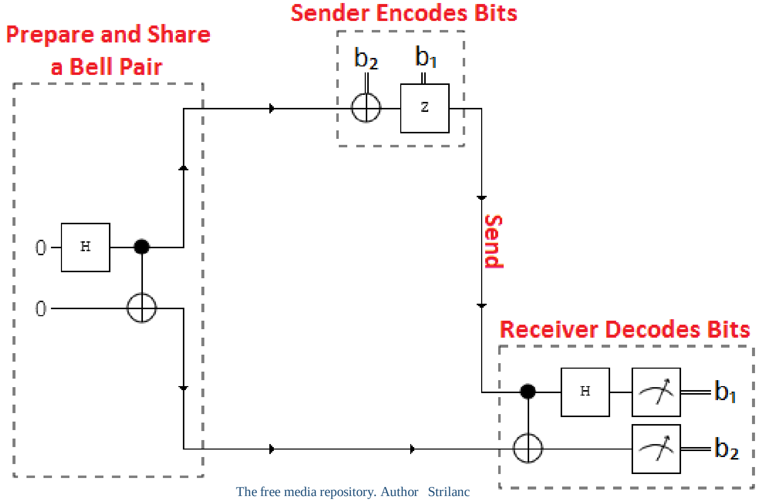
Entangled states are the basis of the theory of quantum communication:

quantum teleportation quantum repeater superdense coding



Modified form paper by Bennett et. al.,1993 Phys. Rev. Lett. V.70 p. 1895.





The free media repository. Author Strilanc

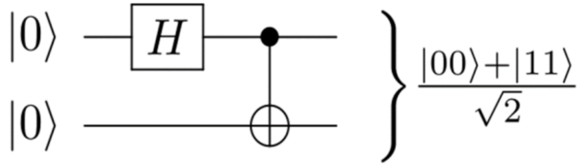
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

The pure Bell state is formed from two mixed states.



Entanglement has become the key ingredient in harnessing the power of quantum mechanics in technological applications such as quantum computing.

What does it mean to remain a density matrix?

$$\hat{\rho}^\dagger = \hat{\rho} \quad \mathrm{Tr}\hat{\rho} = \sum_n \rho_{nn} = 1 \quad \hat{\rho} = \sum_\nu \lambda_\nu |\nu\rangle\langle\nu|$$

$$\langle\chi|\hat{\rho}|\chi\rangle \geq 0 \quad \mathrm{Tr}(\hat{\rho})^2 = \sum_{nn'} |\rho_{nn'}|^2 \leq 1$$

Peres–Horodecki criterion

- A density matrix of two qubits is separable if and only if it remains a density matrix under partial transposition.

$$\hat{\varrho}^{PT} = \sum_{m,n}\sum_{\mu,\nu} \varrho^{PT}_{m\mu,n\nu}|m,\mu\rangle\langle n,\nu| = \sum_{m,n}\sum_{\mu,\nu} \varrho_{m\nu,n\mu}|m,\mu\rangle\langle n,\nu|$$

$$\hat{\varrho}^{PT} = \sum_n w_n \hat{\varrho}_{(1)n} \otimes \hat{\varrho}^T_{(2)n}$$

$$||\hat{\varrho}^{PT}|| = 1 .$$

$$||\hat{\varrho}^{PT}|| = 1 + 2\Big|\sum_i \lambda_i\Big| \equiv 1 + 2\mathcal{N}(\hat{\varrho})$$

Logarithmic negativity

$$E_\mathcal{N}(\hat{\varrho}) \equiv \log_2 ||\hat{\varrho}^{PT}|| = \log_2\left[|A_i(t)|^2+|A_j(t)|^2+\sqrt{(1-|A_i(t)|^2-|A_j(t)|^2)^2+4|A_i(t)|^2|A_j(t)|^2}\right].$$

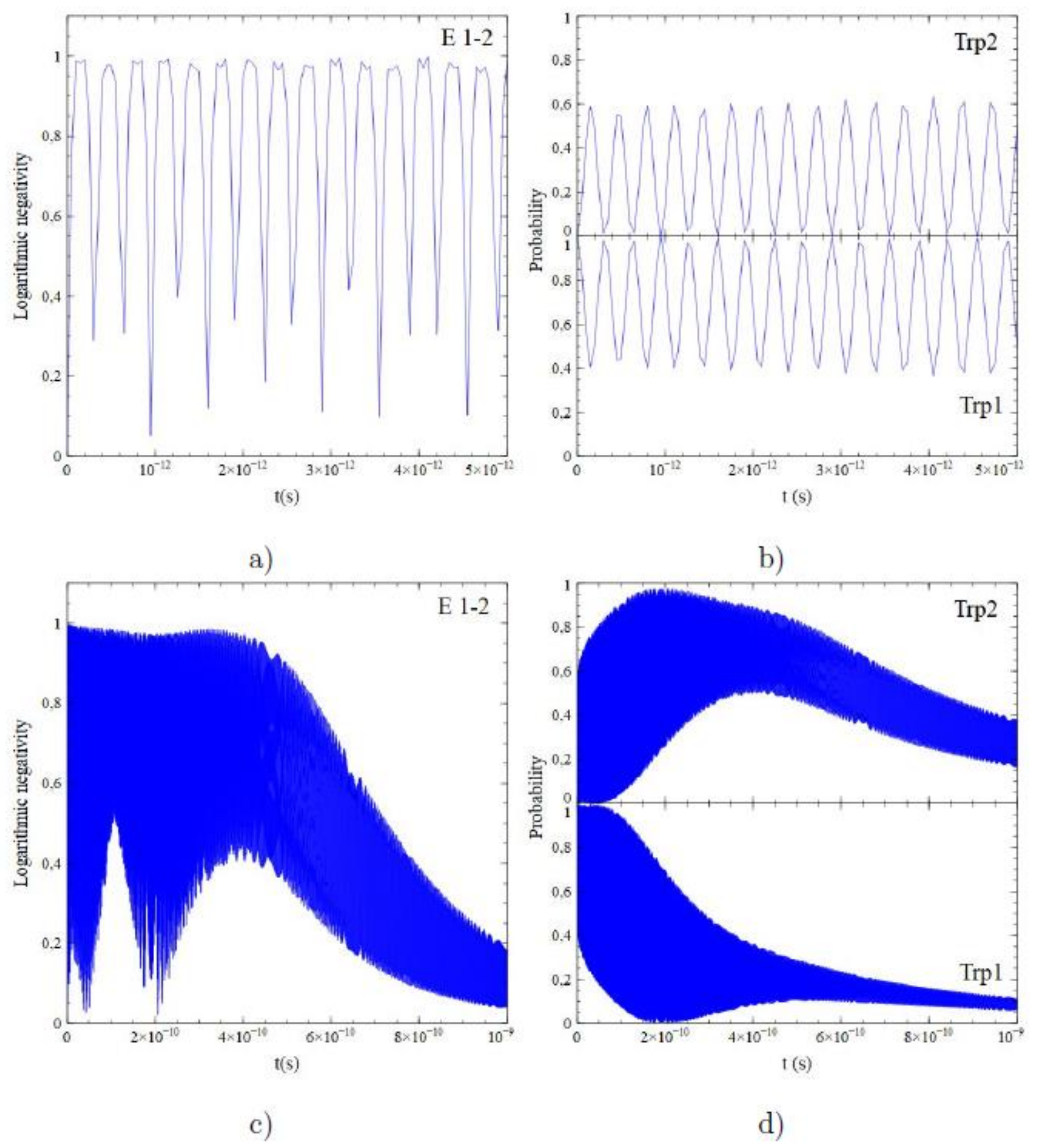


Figure 7: Logarithmic negativity a), c); time-depending probable excitation location on the tryptophans b), d). $\kappa_n = 0.04$; $k = 4.5\ N/m$.

The states are correlated, that is, they fluctuate synchronously-entangled.

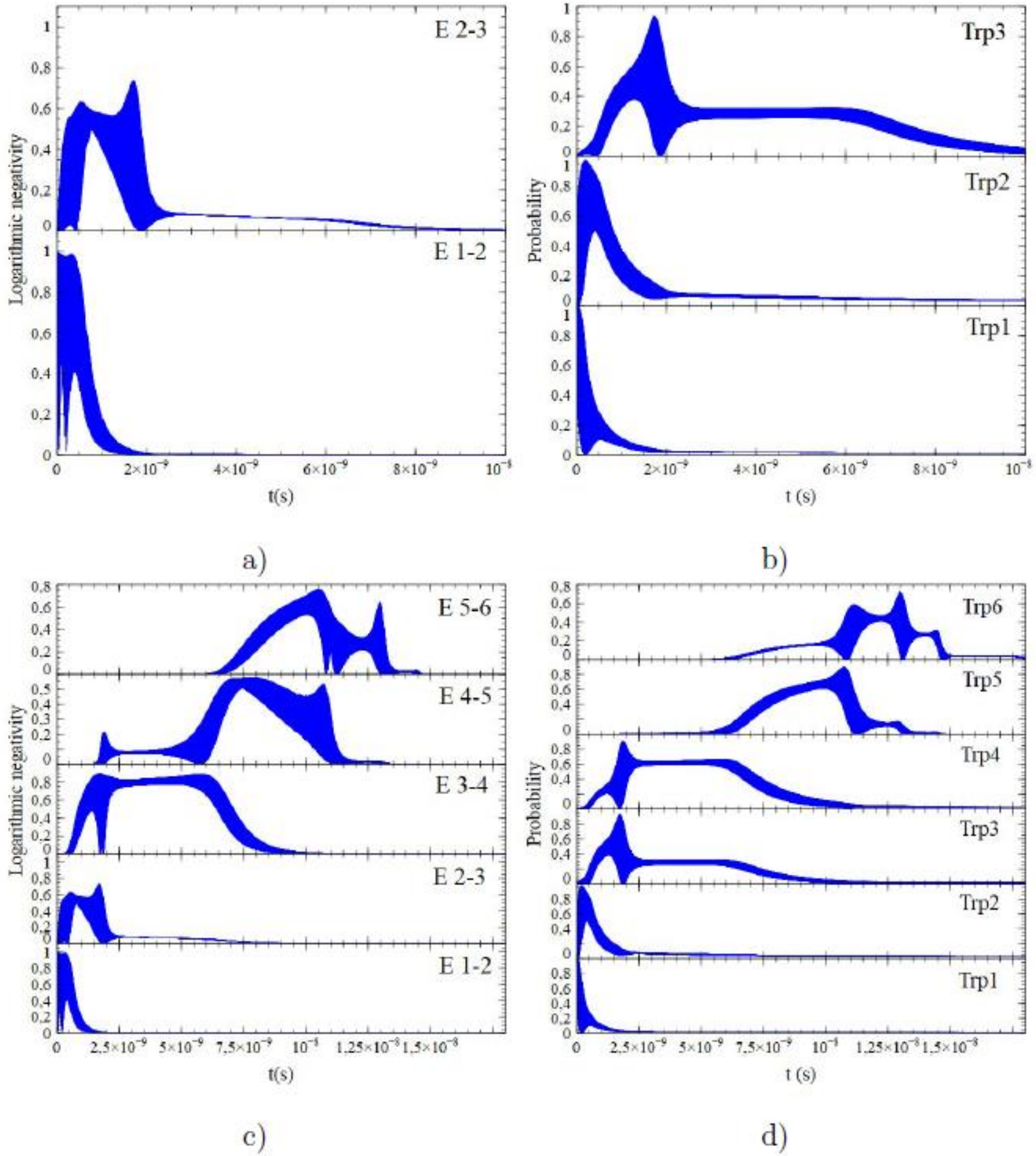
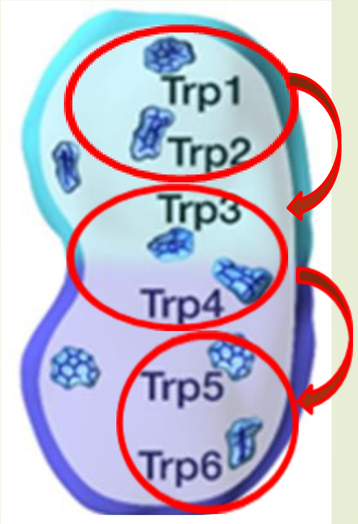


Figure 8: Logarithmic negativity a), c); time-depending probable excitation location on the tryptophans b), d). $\kappa_n = 0.04$; $k = 4.5\ N/m$.



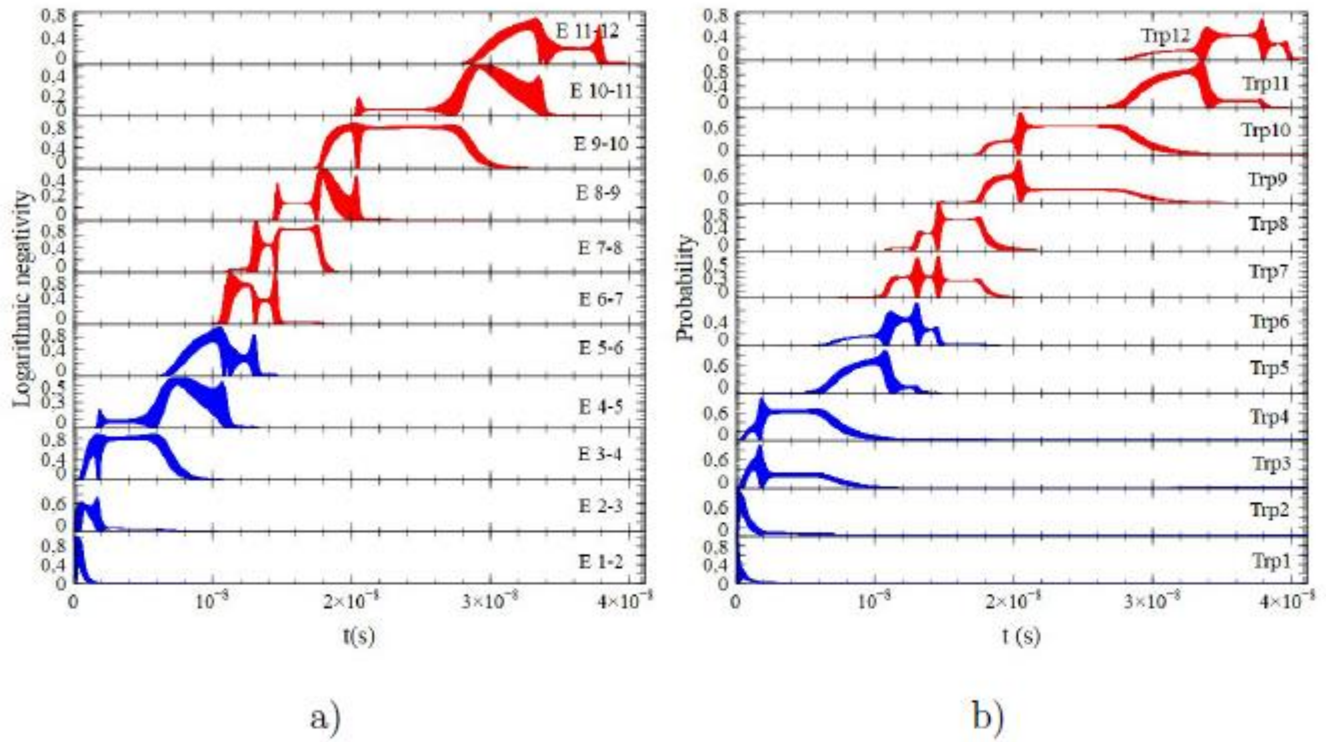
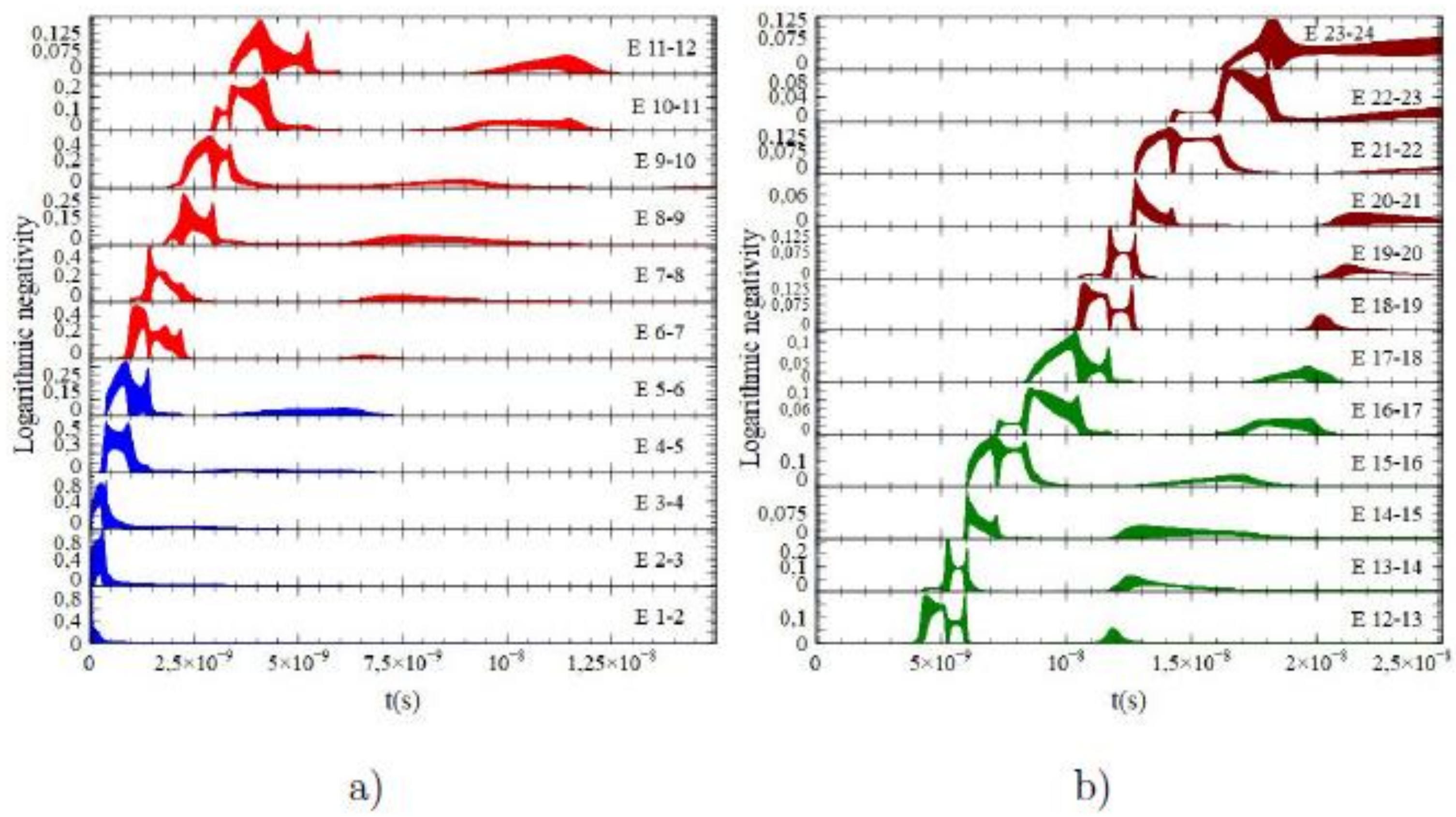


Figure 10: Time-depending probable excitation location on the tryptophans a), logarithmic negativity b). $\kappa_n = 0.04$; $k = 4.5\ N/m$.

The process of quantum entangled states migration between tryptophans has been considered.



The results of the work allow us to talk about the signal function of microtubule tryptophans working as a quantum repeater that transmits quantum entangled states by relaying through intermediate tryptophans.

E 1-2 → E 2-3 → E 3-4 → E 4-5 →...E 23-24

Conclusion

1. Energy transfer mechanisms in microtubules tryptophan system have been determined.

2. Taking into account the spacing between adjacent tryptophan groups and exited state migration time, the rate of propagation of the excited states falls within the range of nerve impulse velocity.

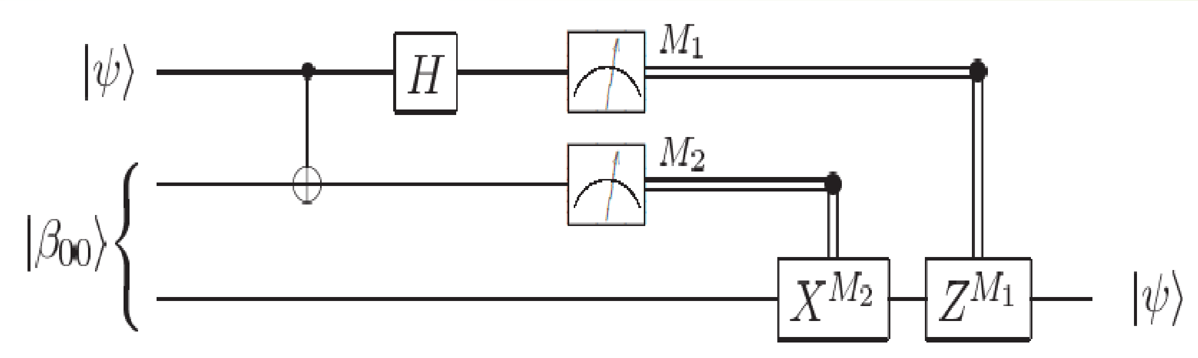
3. The process of quantum entangled states migration between tryptophans has been considered.

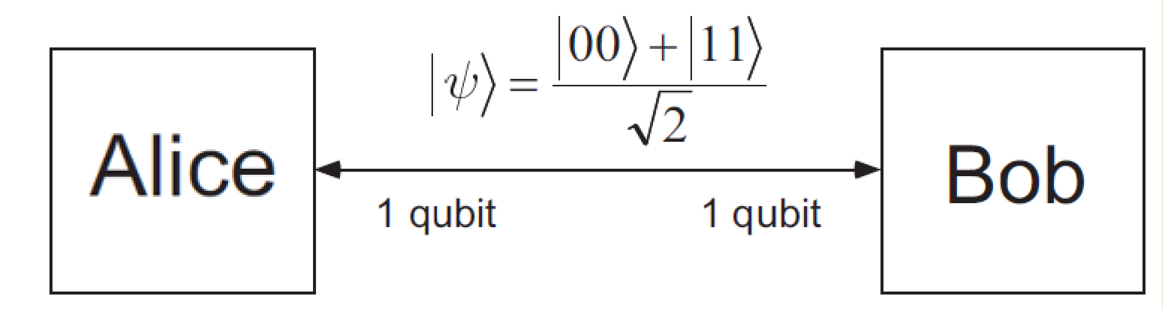
4. The results of the work allow us to talk about the signal function of microtubule tryptophans working as a quantum repeater that transmits quantum entangled states by relaying through intermediate tryptophans.

S. Eh. Shirmovsky, A. V. Chizhov 2023. Modeling of the entangled states transfer processes in microtubule tryptophan system. BioSystems has been accepted.

Development prospects:

1. Quantum teleportation.
2. Superdense coding in microtubules.





Michael A. Nielsen & Isaac L. Chuang, Quantum Computation and Quantum Information

Appendixes

Microtubules

Logarithmic negativity

$$E_{\mathcal{N}}(\hat{\varrho}) \equiv \log_2 ||\hat{\varrho}^{PT}||$$

$$|\psi\rangle = \sum_{n}^{N} A_n(t)|i\rangle \equiv \sum_{n}^{N} A_n(t)|0_1,\ldots,1_n,\ldots,0_N\rangle \quad \hat{\varrho}_{sys}(t) = |\psi\rangle\langle\psi|$$

$$\hat{\varrho}_{\{ij\}}(t) = \mathrm{Tr}_{k\neq i,j}\,\hat{\varrho}_{sys}(t)$$

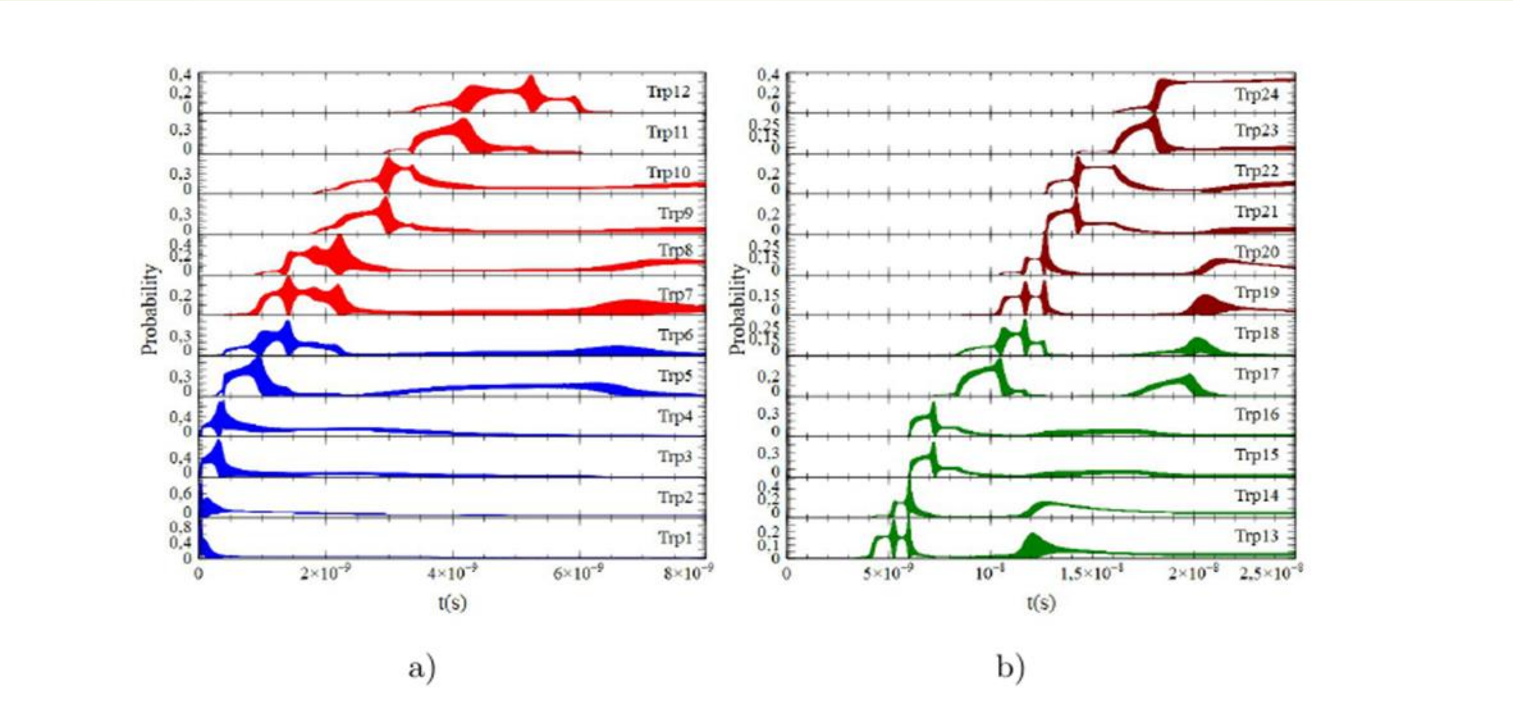
$$\psi = \begin{pmatrix} |1_i,0_j\rangle \\ |0_i,1_j\rangle \\ |0_i,0_j\rangle \\ |1_i,1_j\rangle \end{pmatrix} \qquad \psi^{\dagger} = (\langle 1_i,0_j|, \langle 0_i,1_j|, \langle 0_i,0_j|, \langle |1_i,1_j|)$$

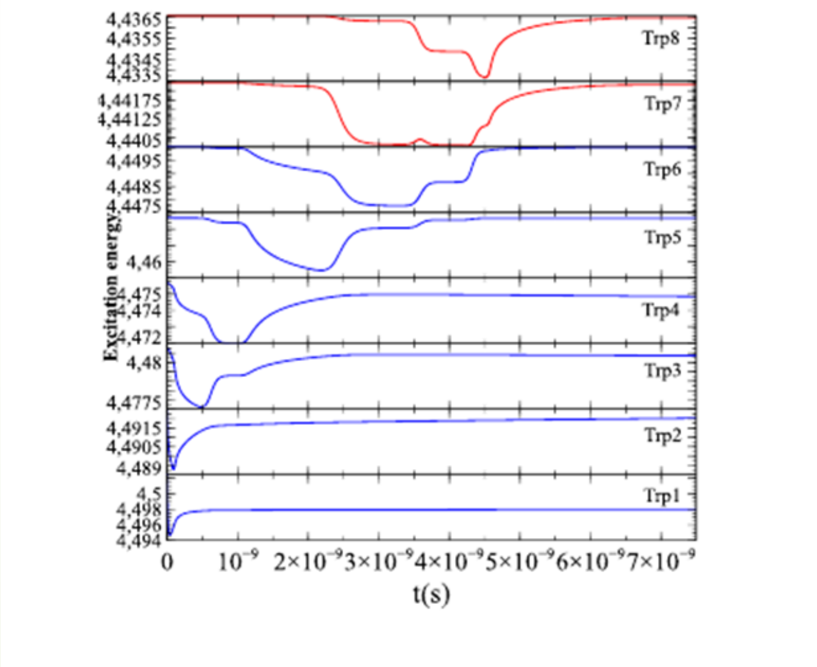
$$\hat{\varrho}_{\{ij\}}(t) = \psi^{\dagger}\mathbf{V}_{\{ij\}}(t)\psi \qquad \hat{\varrho}_{\{ij\}}^{PT}(t) = \psi^{\dagger}\mathbf{V}_{\{ij\}}^{PT}(t)\psi$$

$$\mathbf{V}_{\{ij\}}(t) = \begin{pmatrix} |A_i(t)|^2 & A_i^*(t)A_j(t) & 0 & 0 \\ A_j^*(t)A_i(t) & |A_j(t)|^2 & 0 & 0 \\ 0 & 0 & 1-|A_i(t)|^2-|A_j(t)|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{V}_{\{ij\}}^{PT}(t) = \begin{pmatrix} |A_i(t)|^2 & 0 & 0 & 0 \\ 0 & |A_j(t)|^2 & 0 & 0 \\ 0 & 0 & 1-|A_i(t)|^2-|A_j(t)|^2 & A_j^*(t)A_i(t) \\ 0 & 0 & A_i^*(t)A_j(t) & 0 \end{pmatrix}$$

$$\lambda_N = \frac{1}{2}\left[1-|A_i(t)|^2-|A_j(t)|^2-\sqrt{(1-|A_i(t)|^2-|A_j(t)|^2)^2+4|A_i(t)|^2|A_j(t)|^2}\right]$$

$$E_{\mathcal{N}} = \log_2[1+2|\lambda_N|] = \log_2\left[|A_i(t)|^2+|A_j(t)|^2+\sqrt{(1-|A_i(t)|^2-|A_j(t)|^2)^2+4|A_i(t)|^2|A_j(t)|^2}\right]$$


Trp12
Trp11
Trp10
Trp9
Trp8
Trp7
Trp6
Trp5
Trp4
Trp3
Trp2
Trp1
Probability
t(s)
a)
Trp24
Trp23
Trp22
Trp21
Trp20
Trp19
Trp18
Trp17
Trp16
Trp15
Trp14
Trp13
Probability
t(s)
b)


Trp8
Trp7
Trp6
Trp5
Trp4
Trp3
Trp2
Trp1
Excitation energy
t(s)

Pure separable states

$$|\psi\rangle_A = \sum_i c_i^A |i\rangle_A \text{ and } |\phi\rangle_B = \sum_j c_j^B |j\rangle_B.$$

$$|\psi\rangle_{AB} = \sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_B.$$

$$c_{ij} = c_i^A c_j^B, \quad |\psi\rangle_{AB} = |\psi\rangle_A \otimes |\phi\rangle_B.$$

Inseparable, an entangled state

$$c_{ij} \neq c_i^A c_j^B, \quad |\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\phi\rangle_B.$$

Mixed separable states

$$\rho = \sum_i w_i \rho_i^A \otimes \rho_i^B,$$

A state is entangled if it is not separable.

$$\rho \neq \sum_i w_i \rho_i^A \otimes \rho_i^B.$$