

# Relativistic Boltzmann solver with GPGPU for astrophysical applications

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# Radiation sources

## Stationary:

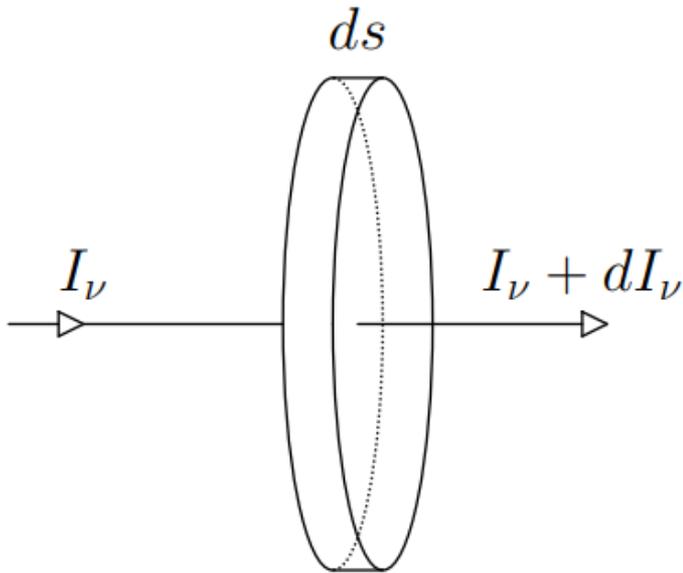
- Quasar / Blazar
- Radiopulsar
- X-ray pulsar

## Non-stationary (flashes):

- Supernova explosion
- GRB (Gamma-ray burst)
- FRB (Fast radio burst)

All are non-thermal !!!

# Radiation transfer equation and kinetic equation



$$dI_\nu = -\kappa_\nu I_\nu ds$$

$\kappa_\nu$  : absorption coefficient [ $\text{cm}^{-1}$ ]

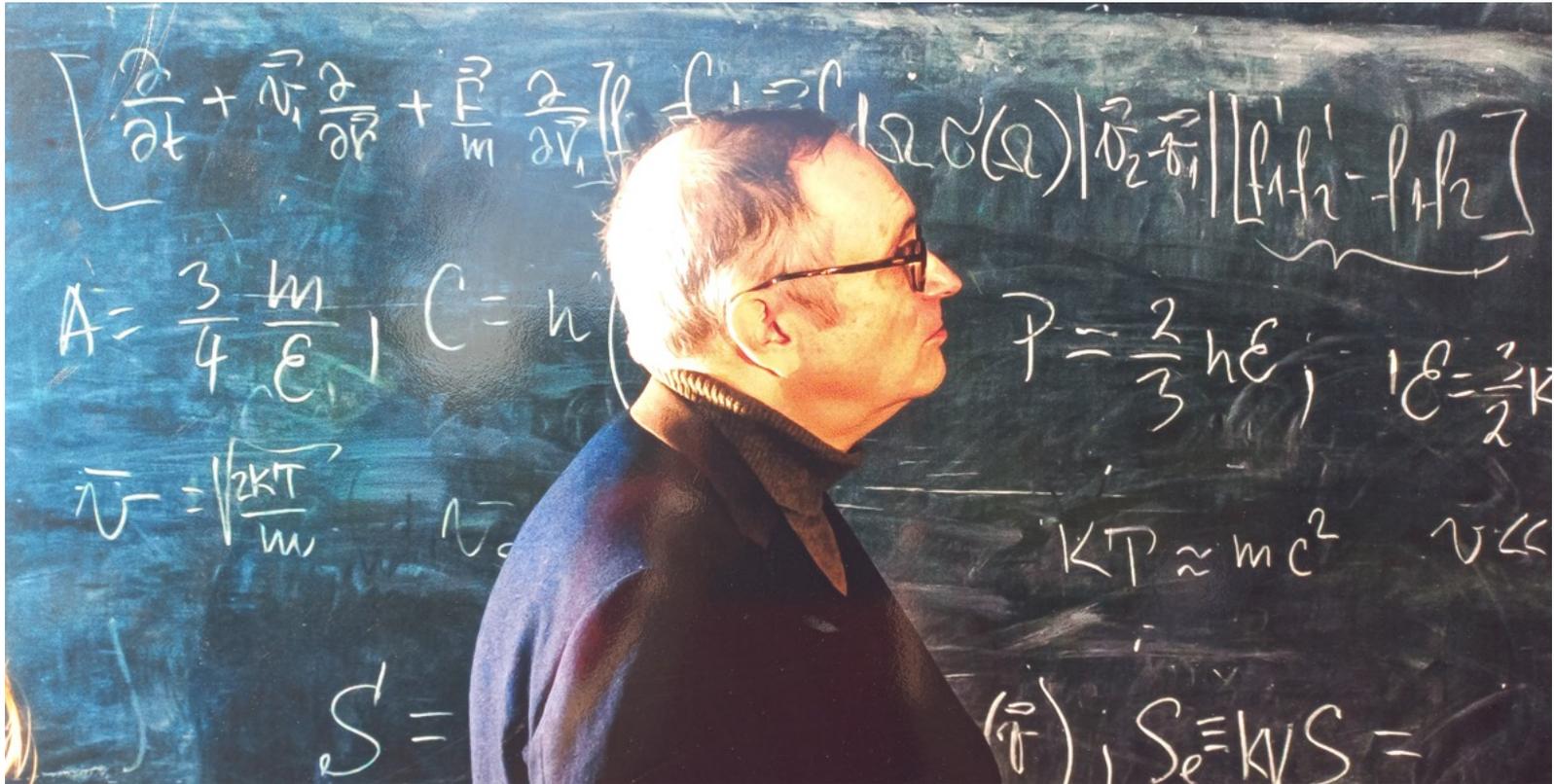
$j_\nu$  : emission coefficient

$$\frac{dI_\nu}{ds} = j_\nu - \kappa_\nu I_\nu$$

$$I_\nu = \frac{h^4}{c^2} \nu^3 f$$

$f(t, \vec{r}, \vec{p})$  – time dependent one-particle distribution function

$$\frac{df}{ds} = \text{St} f$$



credit: JINR Bogoliubov Laboratory of Theoretical Physics

# Basic QED processes

*Binary processes*

$e^+e^- \leftrightarrow \gamma\gamma$
$e\gamma \leftrightarrow e\gamma$
$e^+e^- \leftrightarrow e^+e^-$
$ee \leftrightarrow ee$

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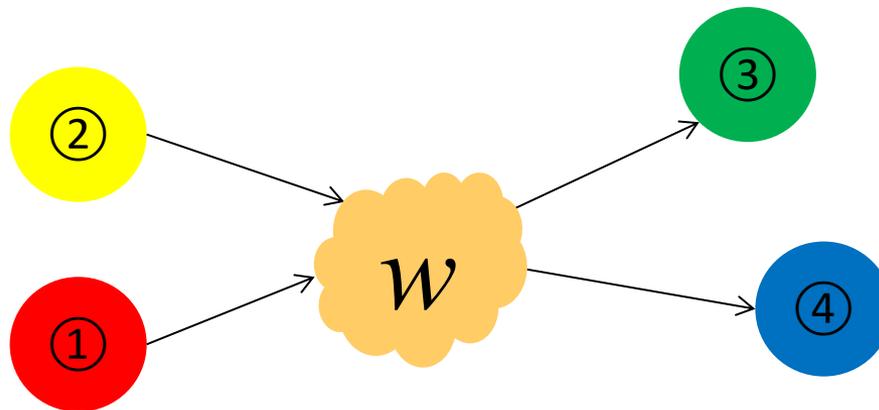
## *Triple processes*

$\gamma\gamma \leftrightarrow \gamma e^+e^-$
$\gamma e \leftrightarrow ee^+e^-$
$e\gamma \leftrightarrow e\gamma\gamma$
$ee \leftrightarrow ee\gamma$
$e^+e^- \leftrightarrow e^+e^-\gamma$
$e^+e^- \leftrightarrow \gamma\gamma\gamma$

# Collision integral for binary processes

$$\int d^3 p_2 d^3 p_3 d^3 p_4 [ w_{(3,4;1,2)} f_{III} f_{IV} - w_{(1,2;3,4)} f_I f_{II} ]$$

5D integral



$$w = \frac{\hbar^2 c^6}{4\pi^2} \frac{|M_{if}|^2}{16\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)$$

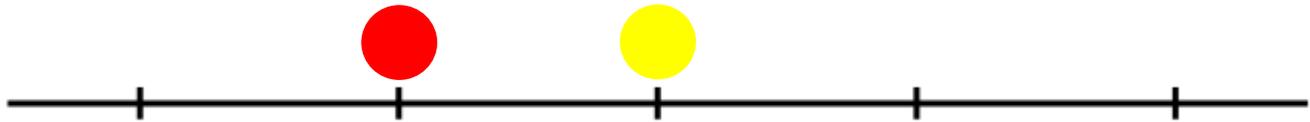
# Particle splitting (between cell interpolation)

$$\varepsilon_3 = \varepsilon_3(\vec{p}_1, \vec{p}_2, \vec{p}_4)$$

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$$\varepsilon_4 = \varepsilon_4(\vec{p}_1, \vec{p}_2, \vec{p}_4)$$

*Income:*



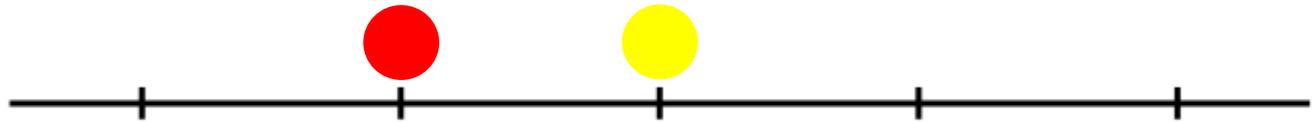
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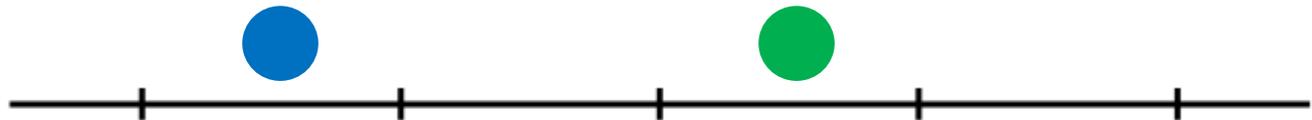
$$\vec{p}_3 = \vec{p}_3(\vec{p}_1, \vec{p}_2, \vec{p}_4)$$

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*Income:*



*Outcome:*



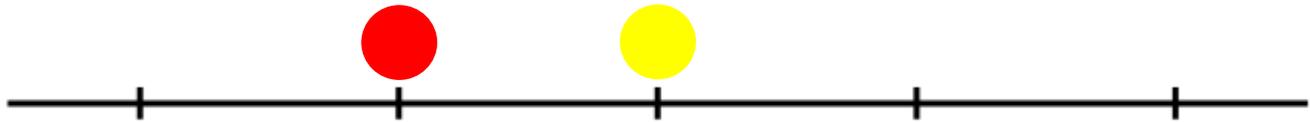
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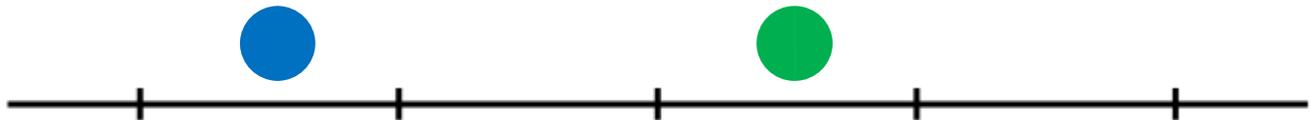
$$\vec{p}_3 = \vec{p}_3(\vec{p}_1, \vec{p}_2, \vec{p}_4)$$

$$\varepsilon_4 = \varepsilon_4(\vec{p}_1, \vec{p}_2, \vec{p}_4)$$

*Income:*



*Outcome:*



# Timing for binary processes

$$\vec{p} = (\varepsilon, \mu, \varphi)$$

grid nodes:  $(n_\varepsilon, n_\mu, n_\varphi = 2n_\mu)$

CPU time (in seconds) of each reaction angular integration

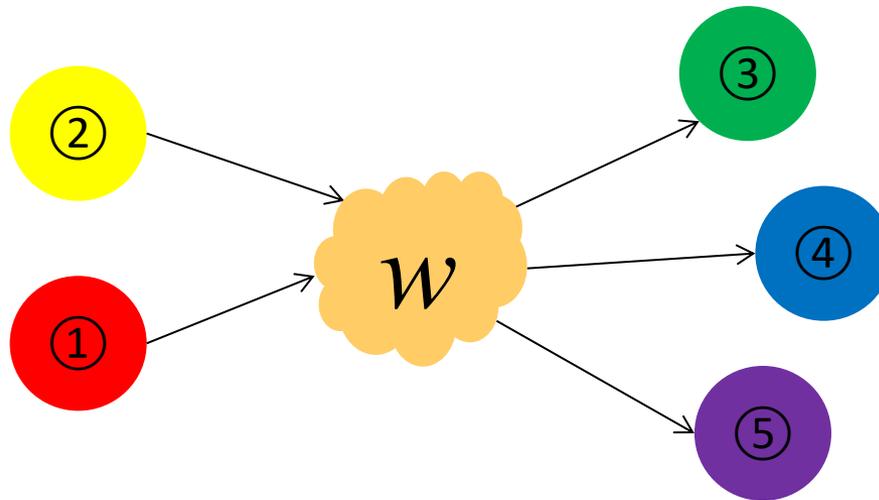
$n_\mu$	16	32	64	128	n
$e + \gamma \leftrightarrow e + \gamma$	2.215	14.48	113.2	590.1	2.7
$\gamma + \gamma \rightarrow e^+ + e^-$	2.106	14.73	100.2	543.1	2.7
$e^+ + e^- \rightarrow \gamma + \gamma$	0.531	3.619	28.82	223.2	2.9
$e^\pm + e^\pm \rightarrow e^\pm + e^\pm$	2.418	16.87	130.5	1030	2.9
$e^\pm + e^\mp \rightarrow e^\pm + e^\mp$	3.354	22.74	178.6	1113	2.8

\*M.A. Prakapenia et al., Journal of Computational Physics. 2018. vol. 373. p. 533.

# Collision integral for triple processes

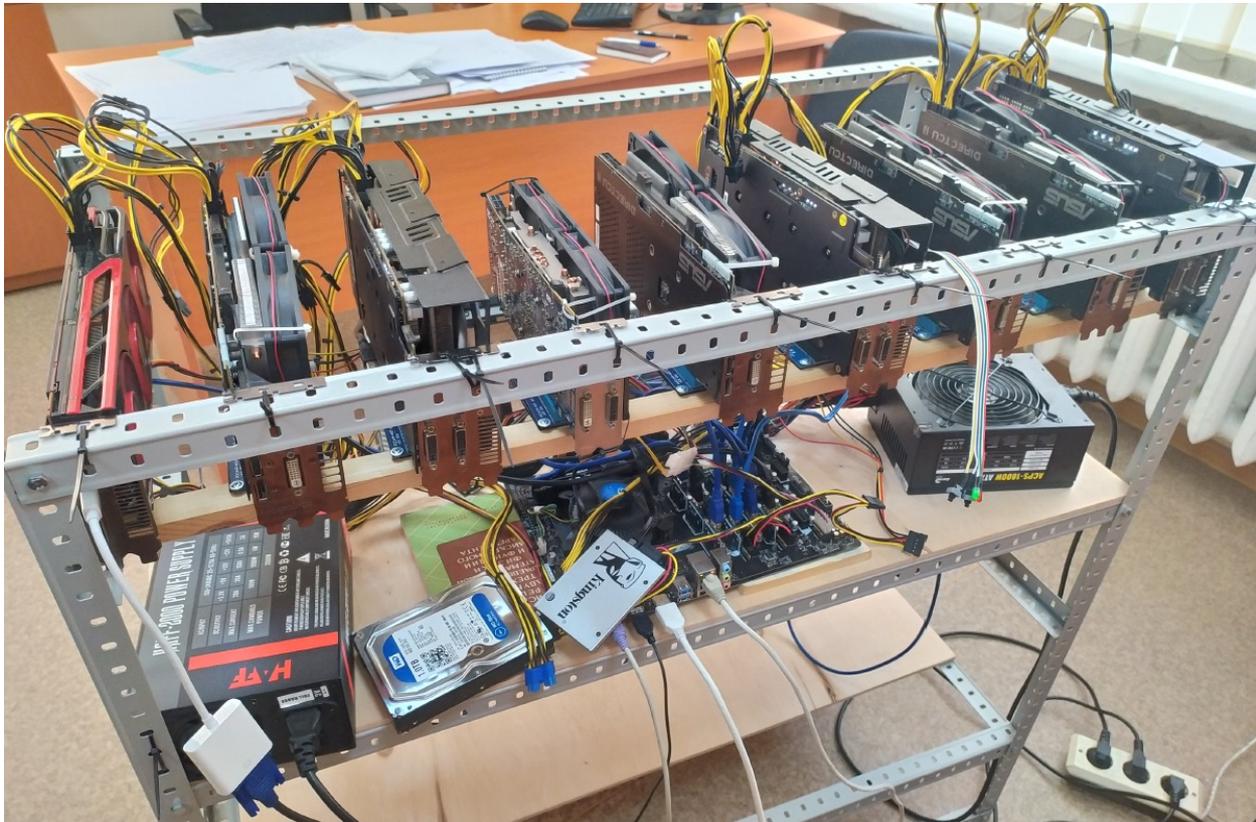
$$\text{St } f_I = \int d^3 p_2 d^3 p_3 d^3 p_4 d^3 p_5 [ w_{(3,4,5|1,2)} f_{III} f_{IV} f_V - w_{(1,2|3,4,5)} f_I f_{II} ]$$

8D integral



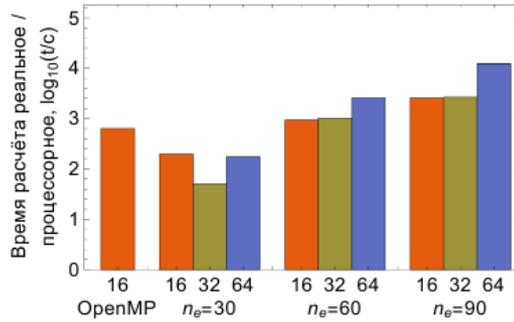
# “Our machine”

Основная рабочая станция, 14 Tahiti @ GPU 1000 МГц, память 1500 МГц,  
Intel(R) Core(TM) i3-7100 CPU @ 3.90GHz, 16GB DDR4-2666 Kingston HyperX KHX2666C16/16G,  
Windows 10 x64 v2004, Microsoft Visual Studio 2019 v16.6.3, AMD Adrenalin Driver 20.1.1

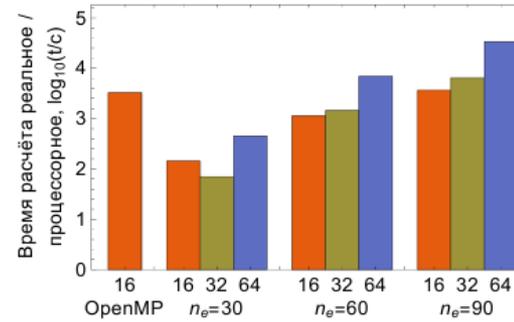


# OpenMP vs GPGPU

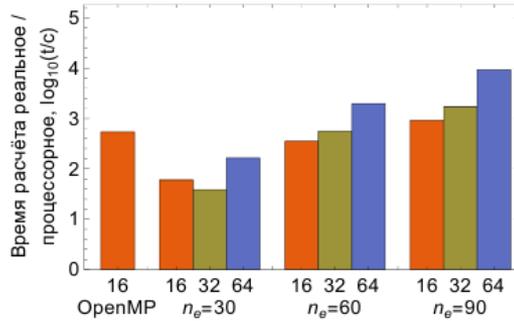
$$e^\pm \gamma \rightarrow e^\pm \gamma_1 \gamma_2$$



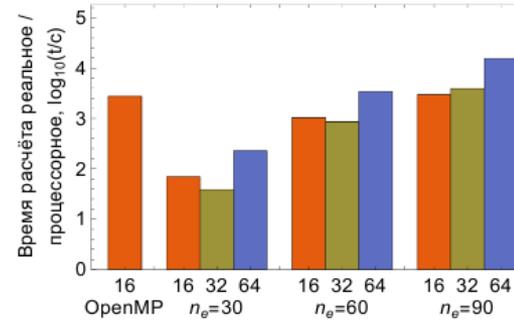
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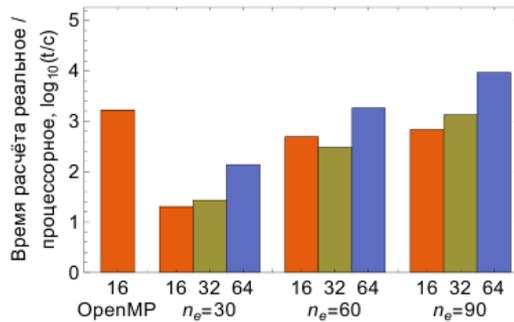
$$\gamma_1 \gamma_2 \rightarrow e^+ e^- \gamma$$



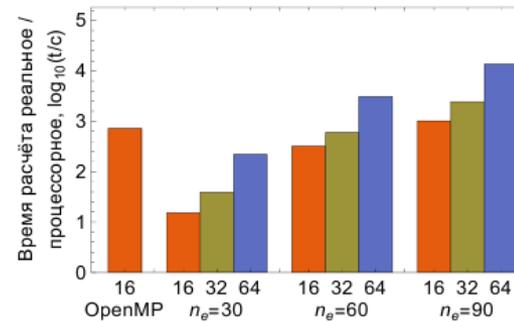
$$\gamma_1 \gamma_2 \leftarrow e^+ e^- \gamma$$



$$e^+ e^- \rightarrow \gamma_1 \gamma_2 \gamma_3$$

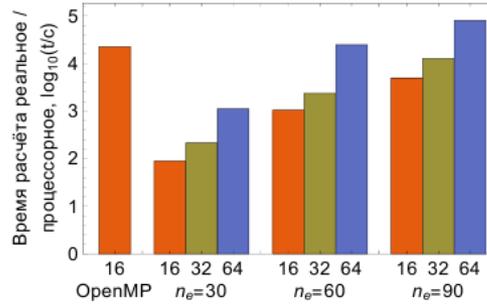


$$e^+ e^- \leftarrow \gamma_1 \gamma_2 \gamma_3$$

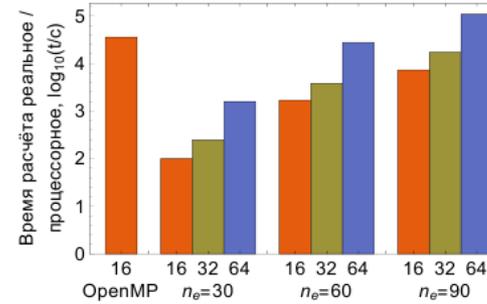


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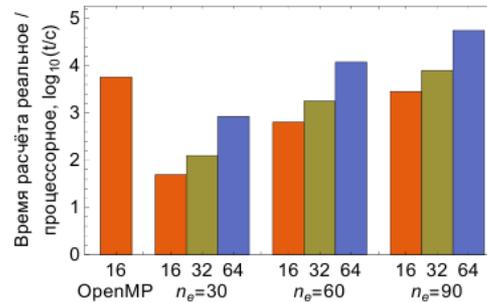
$$e^+e^- \rightarrow e^+e^-\gamma$$



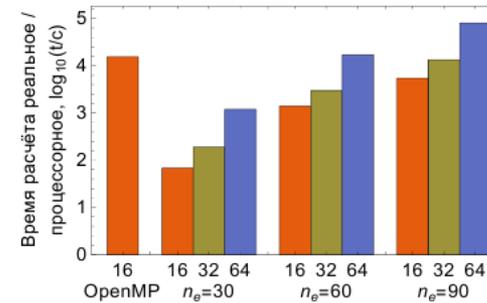
$$e^+e^- \leftarrow e^+e^-\gamma$$



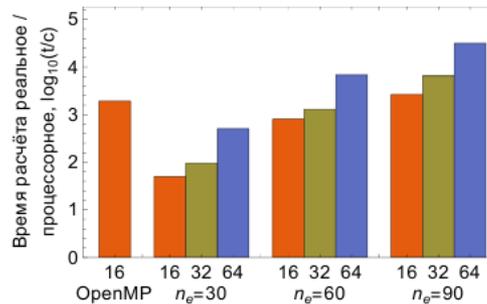
$$e_1^+e_2^+ \rightarrow e_1^+e_2^+\gamma$$



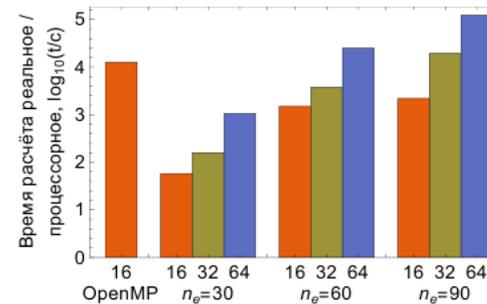
$$e_1^+e_2^+ \leftarrow e_1^+e_2^+\gamma$$



$$e^+\gamma \rightarrow e^+e^+e^-$$

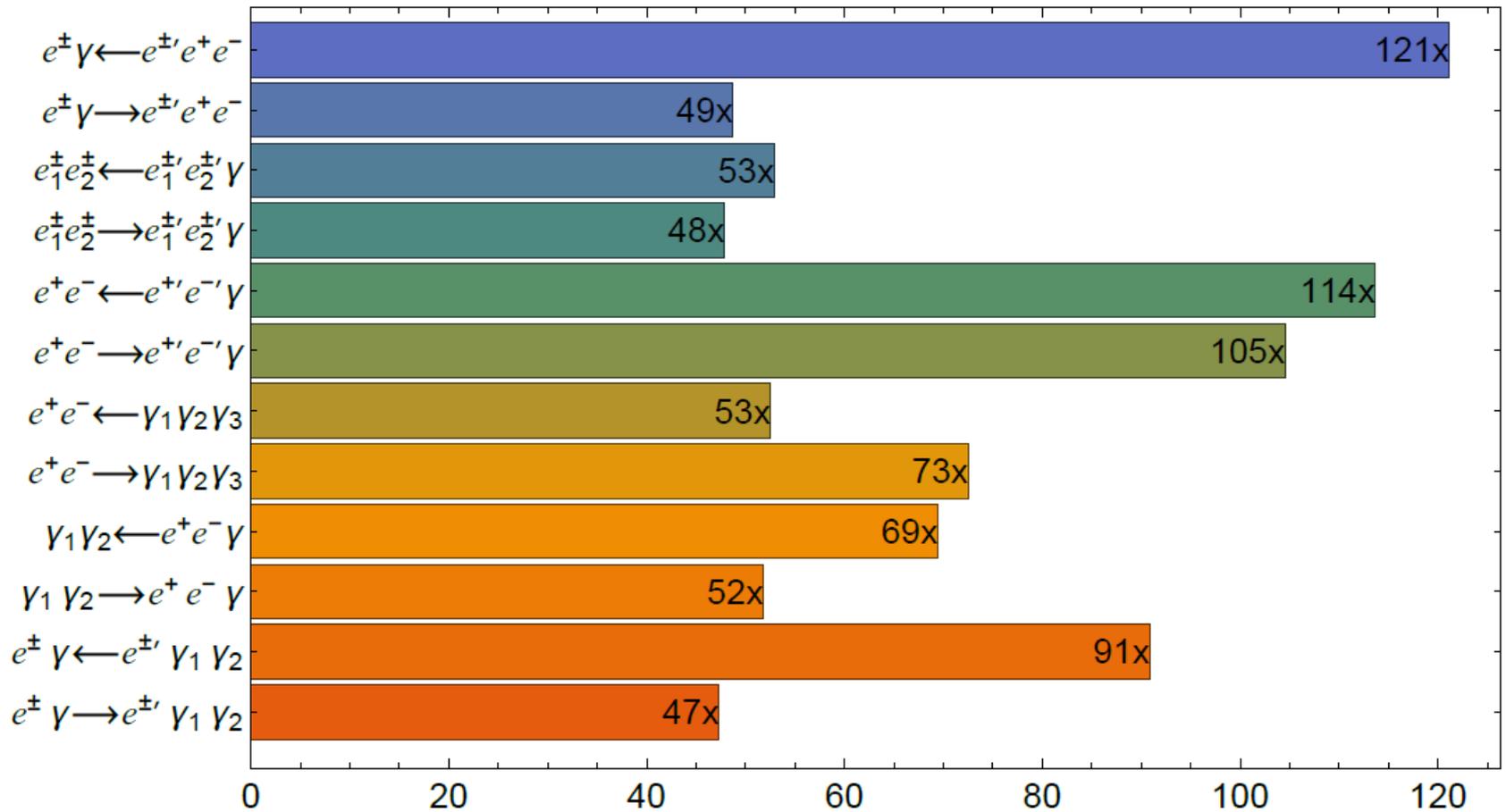


$$e^+\gamma \leftarrow e^+e^+e^-$$



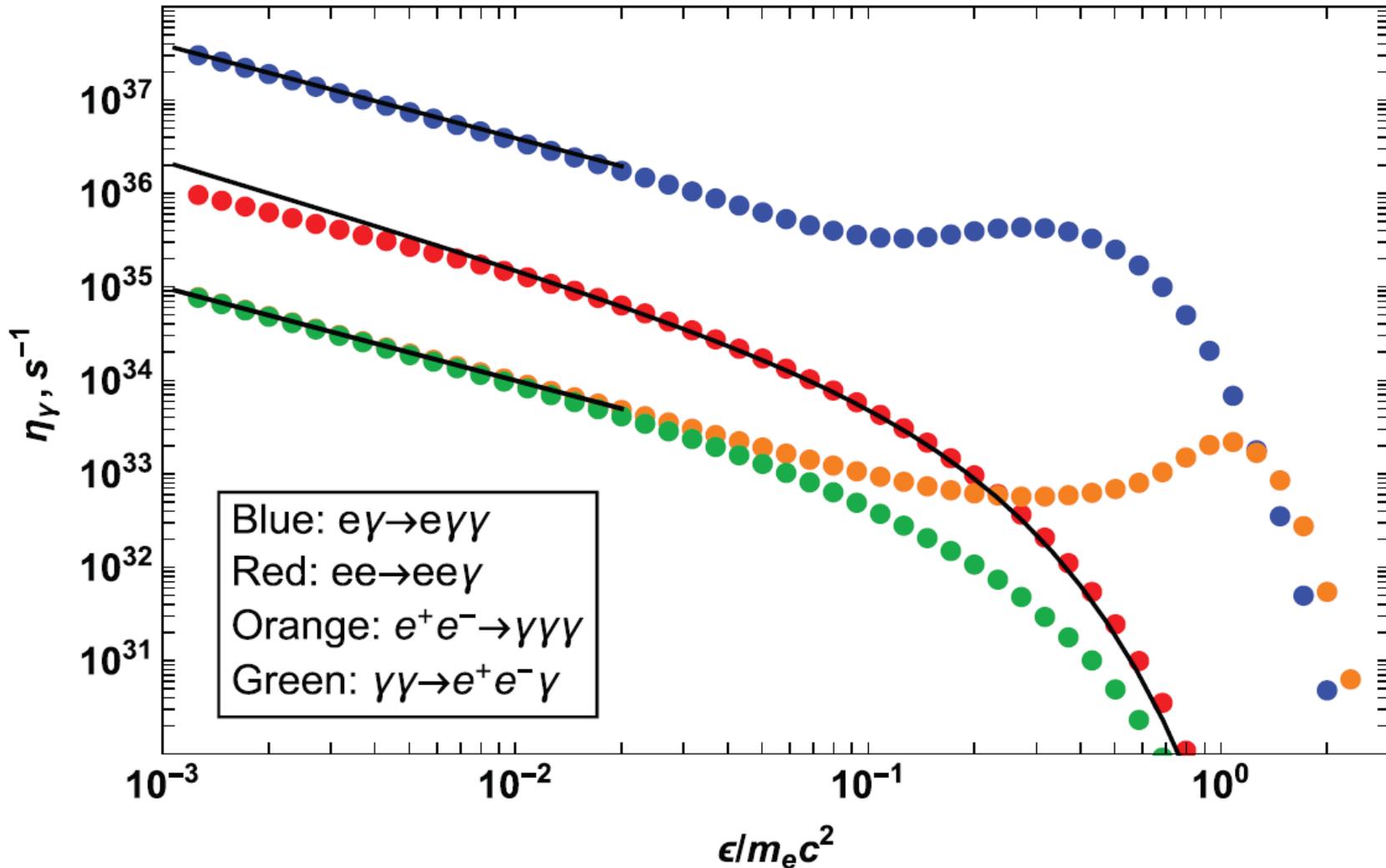
# Speed-up compared to 1 core CPU

$$n_e = 20, n_\mu = 64$$



# Comparison with analytical formulas

Thermal photon emissivity as function of photon energy



***Thank you***