

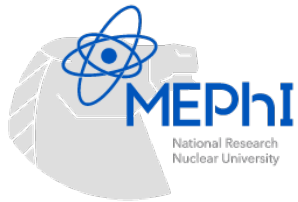
Review of existing models and results on flow at the BM@N energy range

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14 – 19 May 2023

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EOS for high baryon density matter

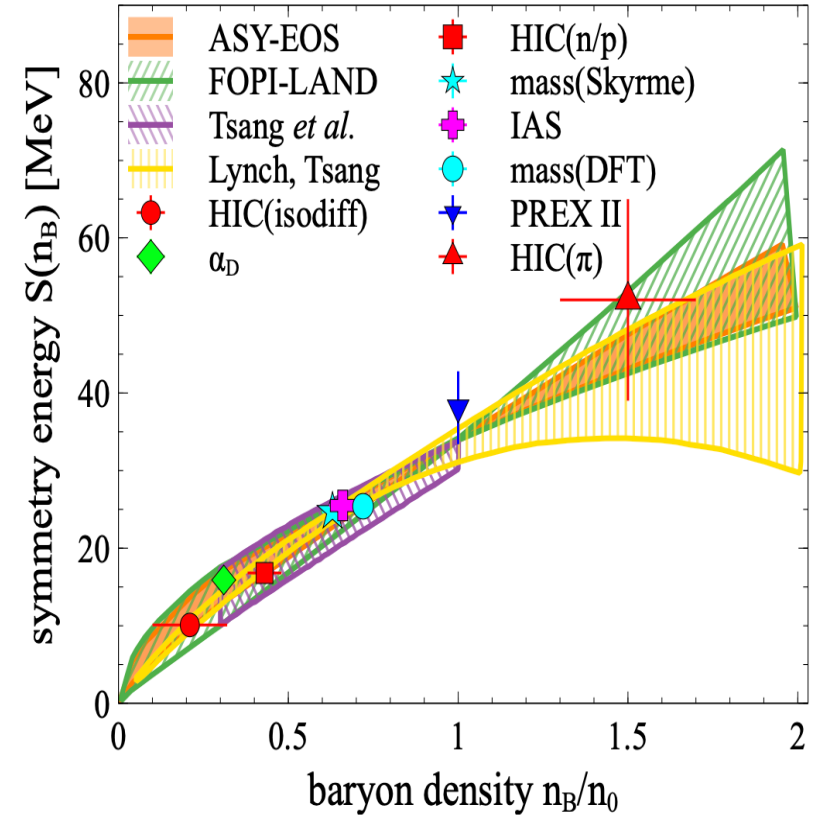
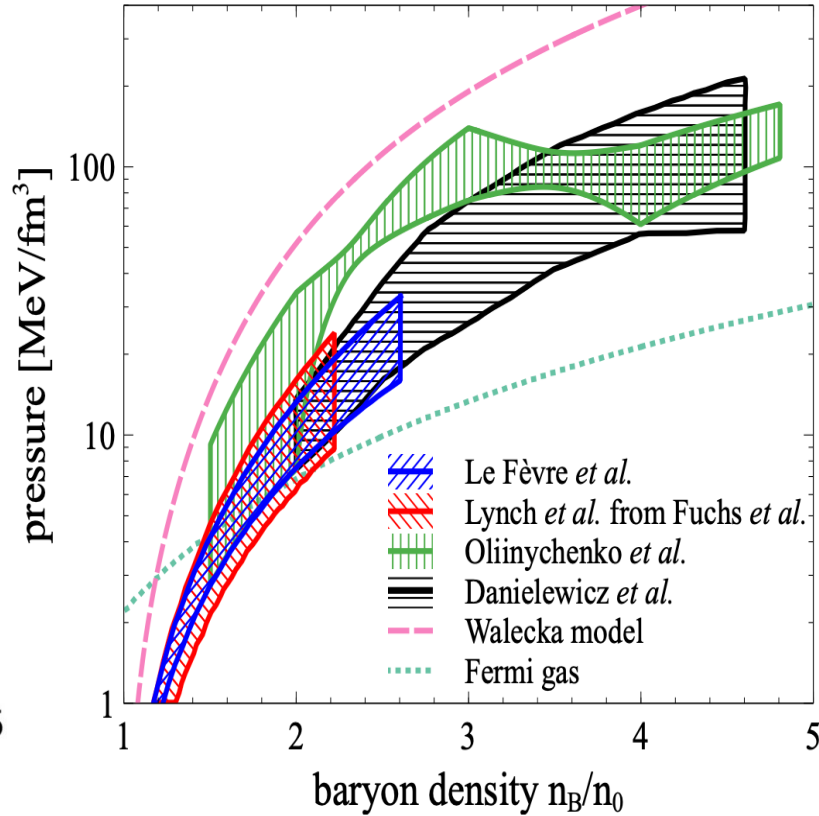
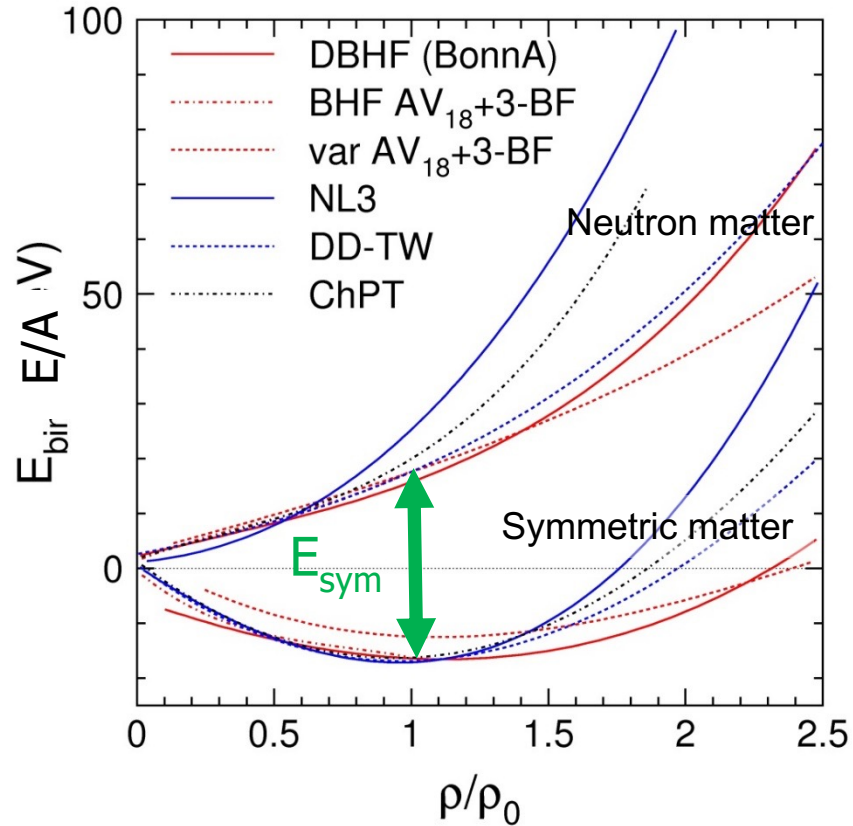
The binding energy per nucleon: $E_A(\rho, \delta) = E_A(\rho, 0) + E_{sym}(\rho)\delta^2 + O(\delta^4)$

Isospin asymmetry:

$$\delta = (\rho_n - \rho_p) / \rho$$

Symmetric matter

Symmetry energy

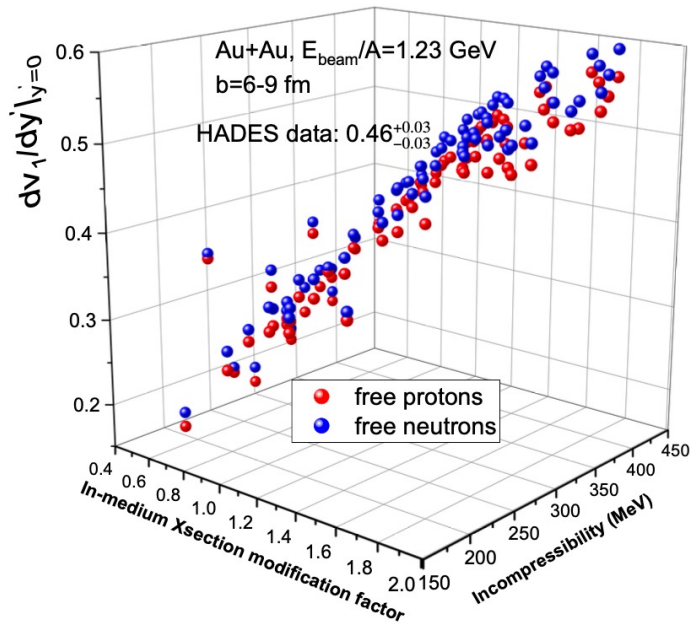


Ch. Fuchs and H.H. Wolter, EPJA 30 (2006) 5

A. Sorensen et. al., arXiv:2301.13253 [nucl-th] (2023)

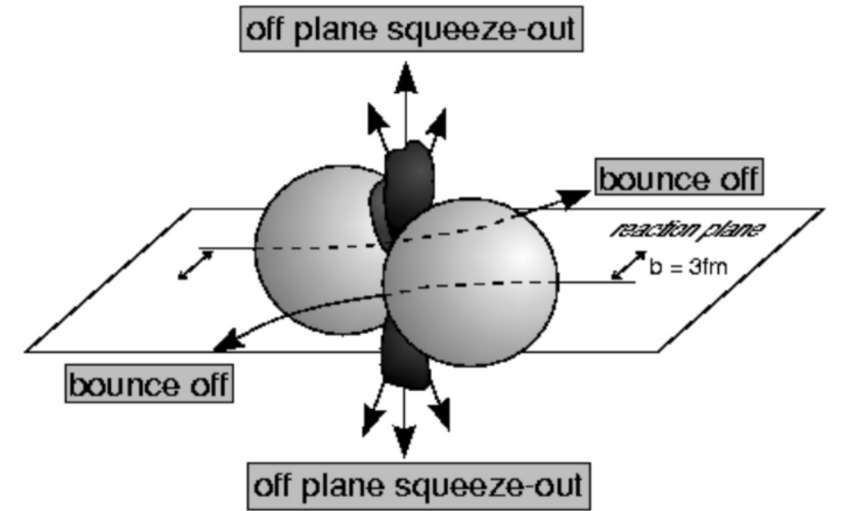
New data is needed to further constrain transport models with hadronic d.o.f.

Sensitivity of the collective flow to the EOS



Incompressibility K_0 :
 parameter which specifies the behavior of EOS in the given baryon densities $K_0 = K_0(\rho)$

Models with flexible EOS for different (K_0, ρ) are required

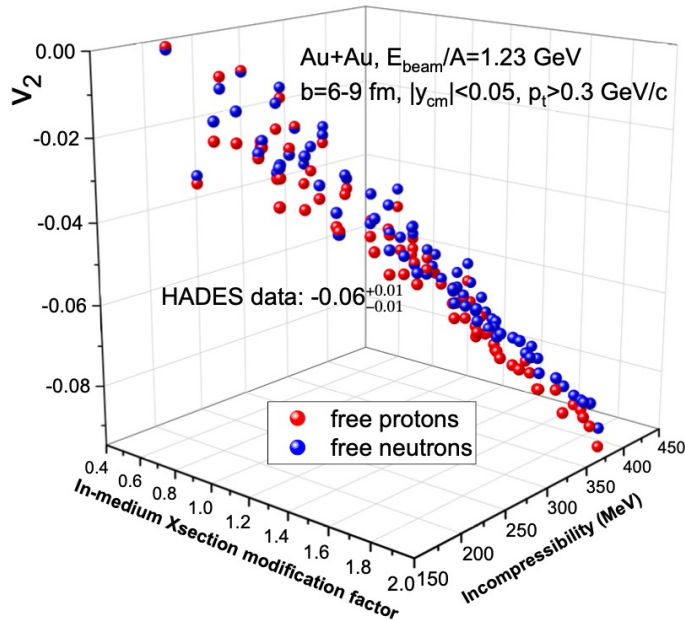


$$\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1} v_n \cos[n(\phi - \Psi_{RP})], \quad v_n = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

v_1 is called directed and v_2 is called elliptic flow

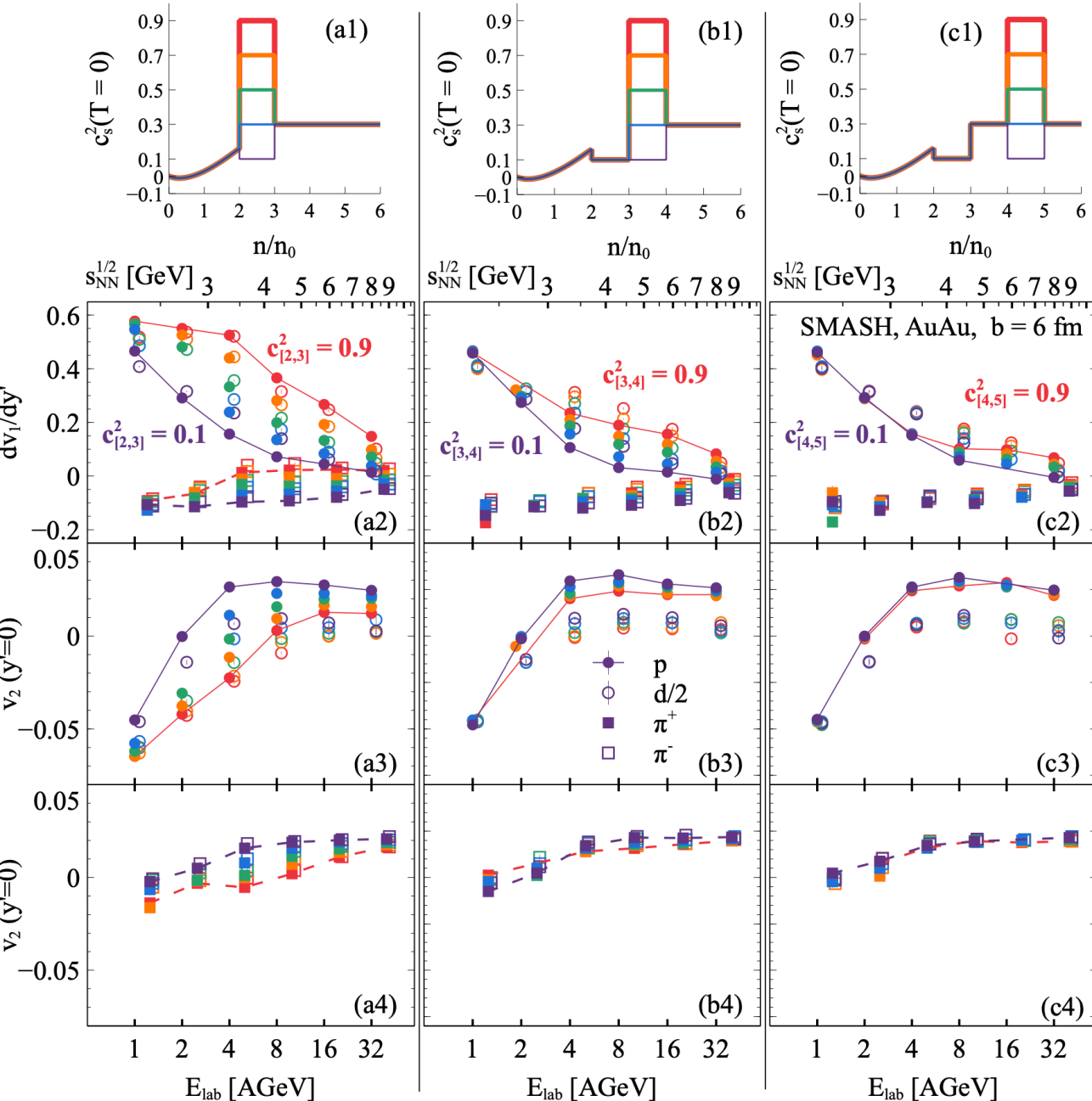
Collective flow is sensitive to:

- **Compressibility of the created in the collision matter**
- **Time of the interaction between the matter within the overlap region and spectators**



Sensitivity of the collective flow to the EOS

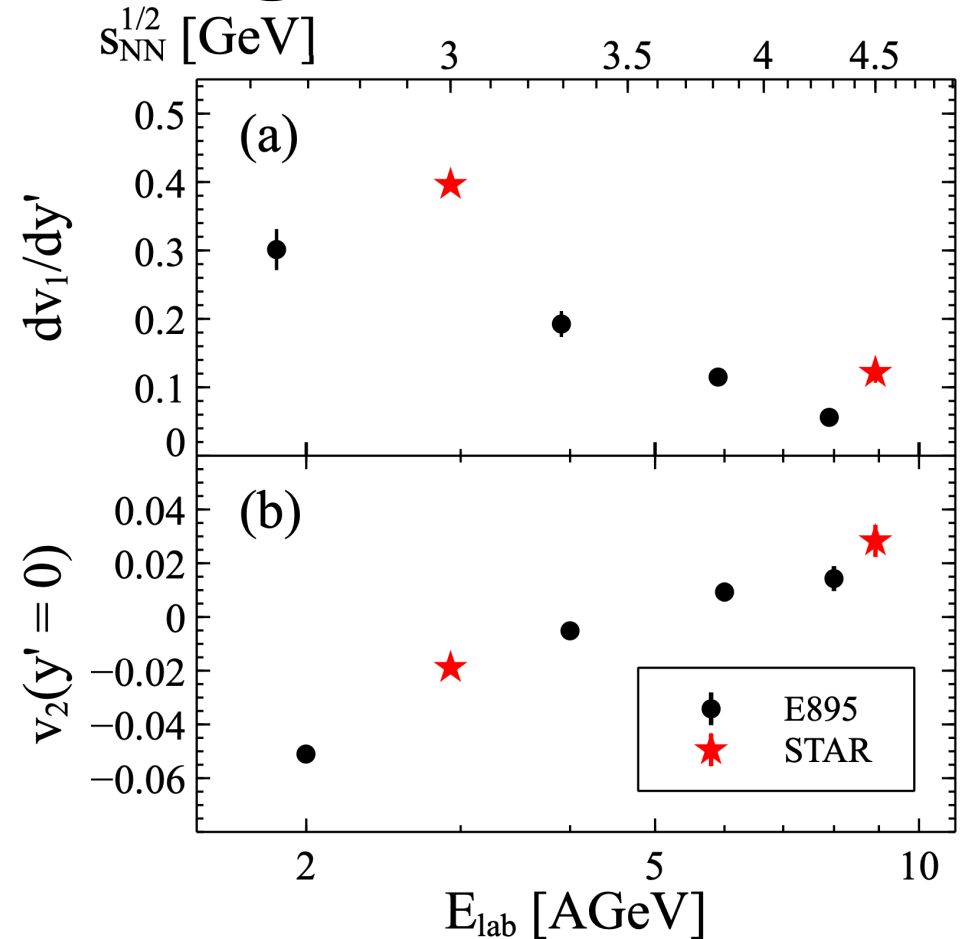
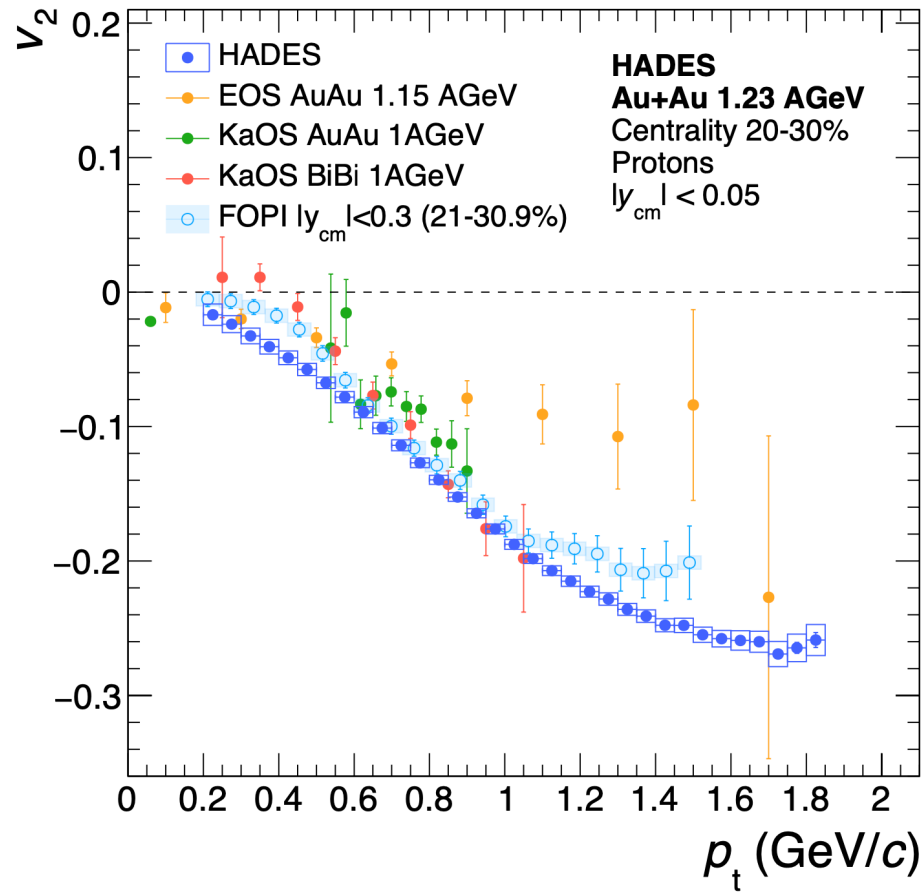
D. Oliinychenko et. al., arXiv:2208.11996 [nucl-th] (2023)



- SMASH model with flexible EOS was used to test the sensitivity of the v_n to changes of EOS in a specific density range n/n_0 :
 - $2 < n_B < 3$: dv_1/dy' and v_2 of pions, protons and deuterons are very sensitive to the EOS
 - $3 < n_B < 4$: dv_1/dy' and v_2 of protons and deuterons are sensitive to the EOS
 - $3 < n_B < 4$: weak sensitivity to the EOS

The most precise constraints can be achieved from the flow of identified hadrons (pions, protons and deuterons)

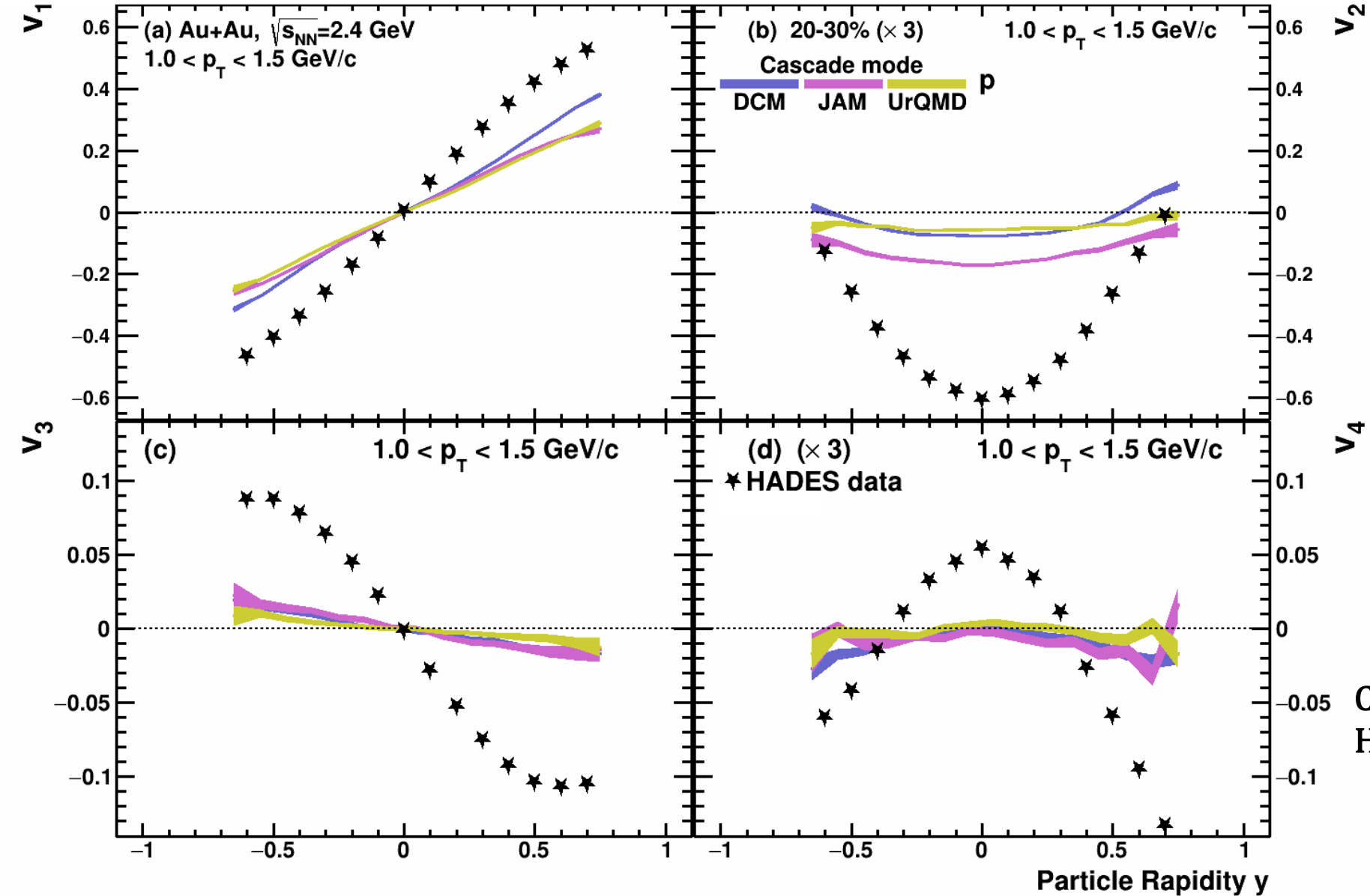
Why do we need new measurements at BM@N and MPD?



- The main source of existing systematic errors in v_n measurements is the difference between results from different experiments (for example, FOPI and HADES, E895 and STAR)
- New data from the future BM@N ($\sqrt{s_{NN}}=2.3-3.3$ GeV) and MPD ($\sqrt{s_{NN}}=4-11$ GeV) experiments will provide more detailed and robust v_n measurements

$v_n(y)$ in Au+Au $\sqrt{s_{NN}}=2.4$ GeV: cascade models

P. Parfenov, Particles 5, no.4, 561-579 (2022)



Kinematic cuts:

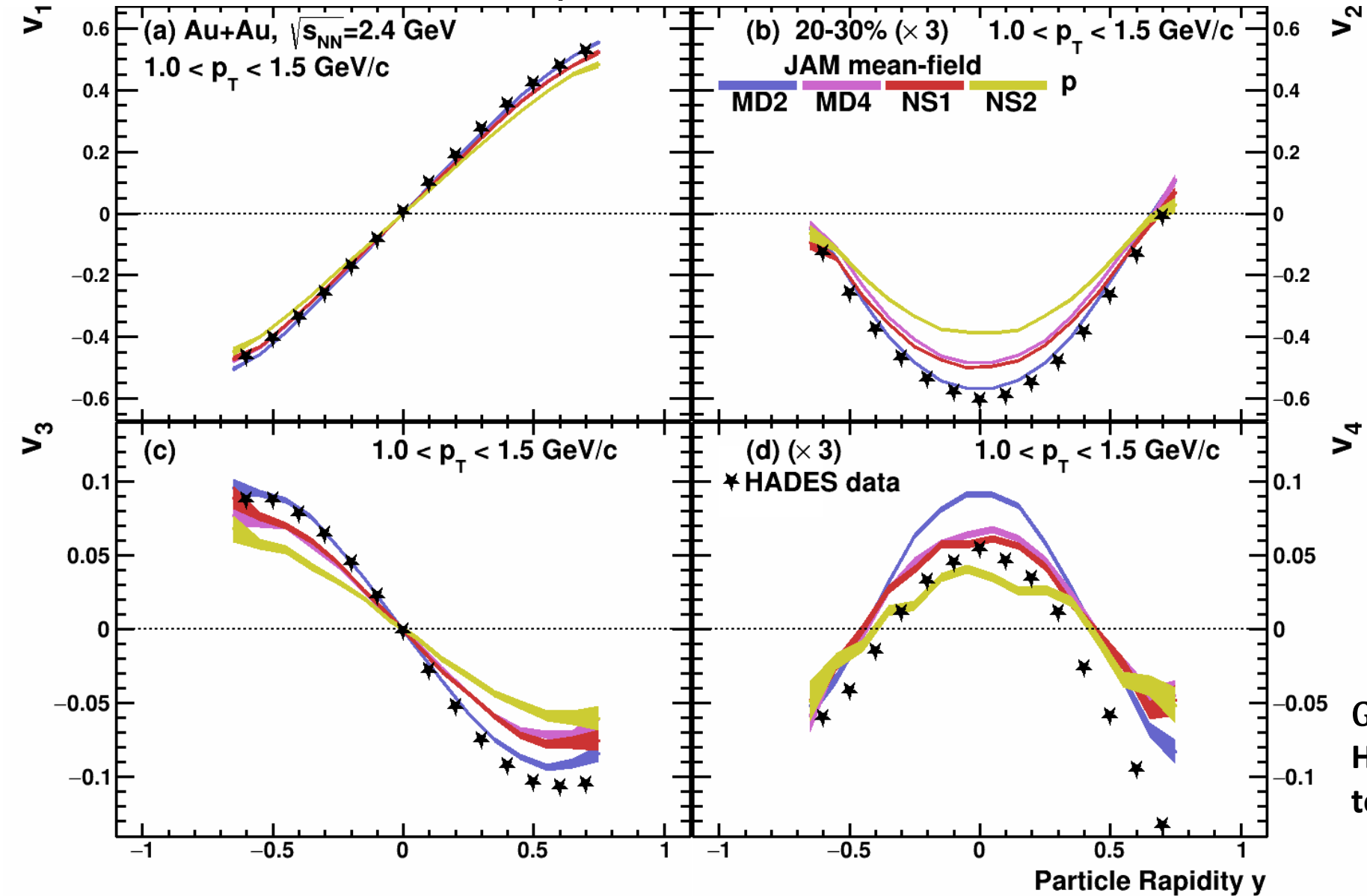
$V_{1,3}(y): 1.0 < p_T < 1.5$ GeV/c

$V_{2,4}(y): 1.0 < p_T < 1.5$ GeV/c

Cascade models fail to reproduce HADES experimental data

$v_n(y)$ in Au+Au $\sqrt{s_{NN}}=2.4$ GeV: model vs. HADES data

P. Parfenov, Particles 5, no.4, 561-579 (2022)



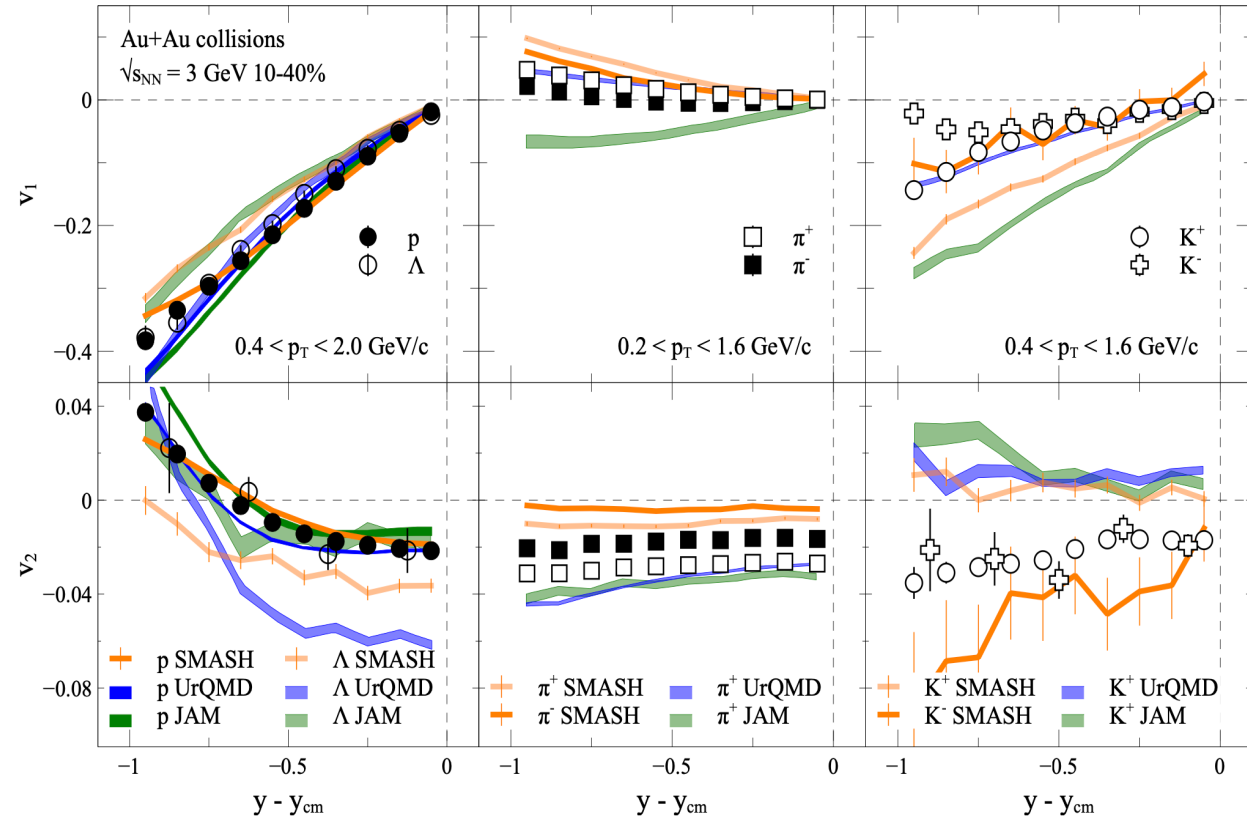
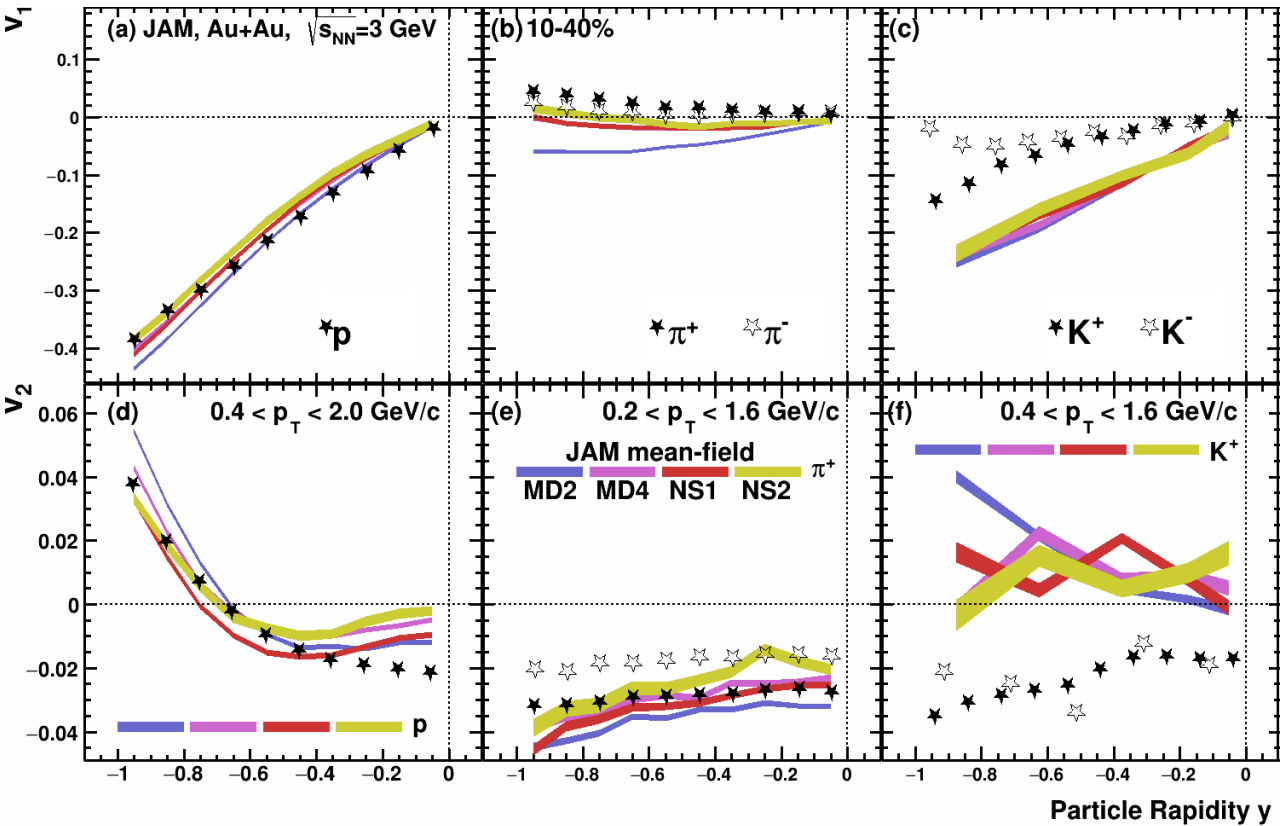
Kinematic cuts:

$V_{1,3}(y)$: $1.0 < p_T < 1.5$ GeV/c

$V_{2,4}(y)$: $1.0 < p_T < 1.5$ GeV/c

Good agreement for $v_n(y)$
 Higher harmonics are more sensitive
 to different EOS than v_1

$v_{1,2}(y)$ in Au+Au $\sqrt{s_{NN}}=3$ GeV: model vs. STAR data



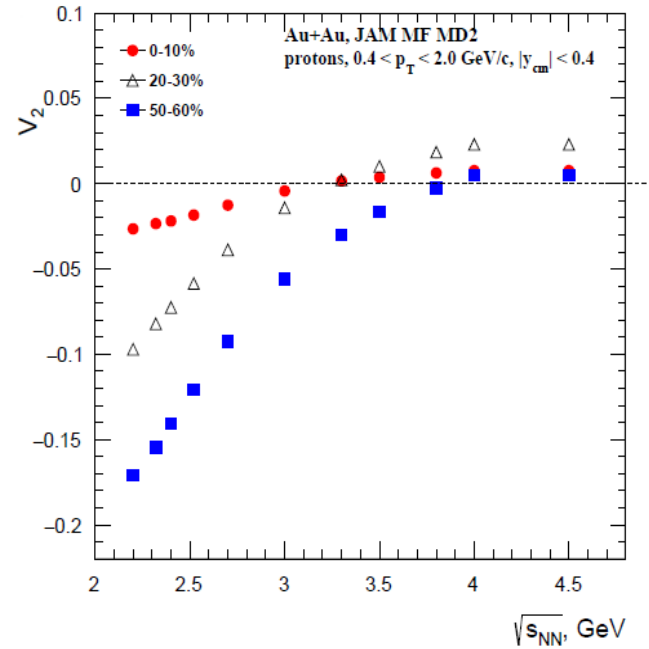
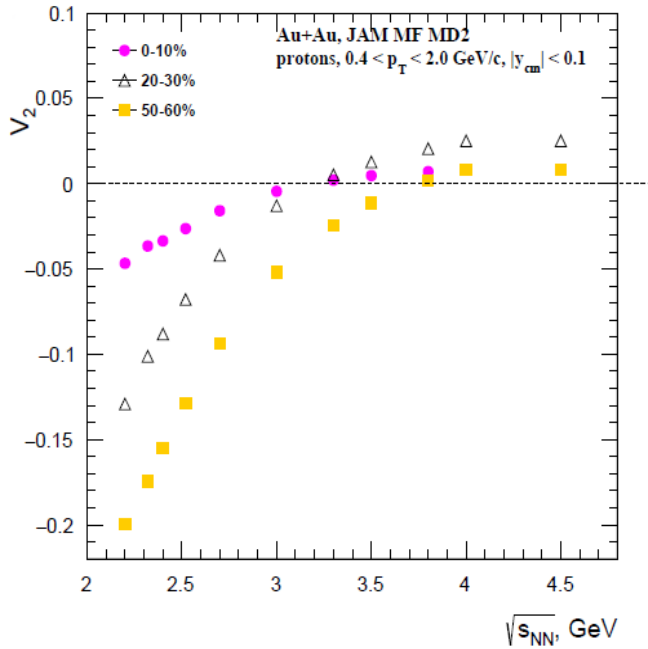
A. Sorensen et. al., arXiv:2301.13253 [nucl-th] (2023)

Models do not describe all particle species equally well

v_1, v_2 of protons are described by JAM, UrQMD (hard EOS) and SMASH (hard EOS with softening at higher densities)

$v_2(y)$ transition from out-of-plane to in-plane

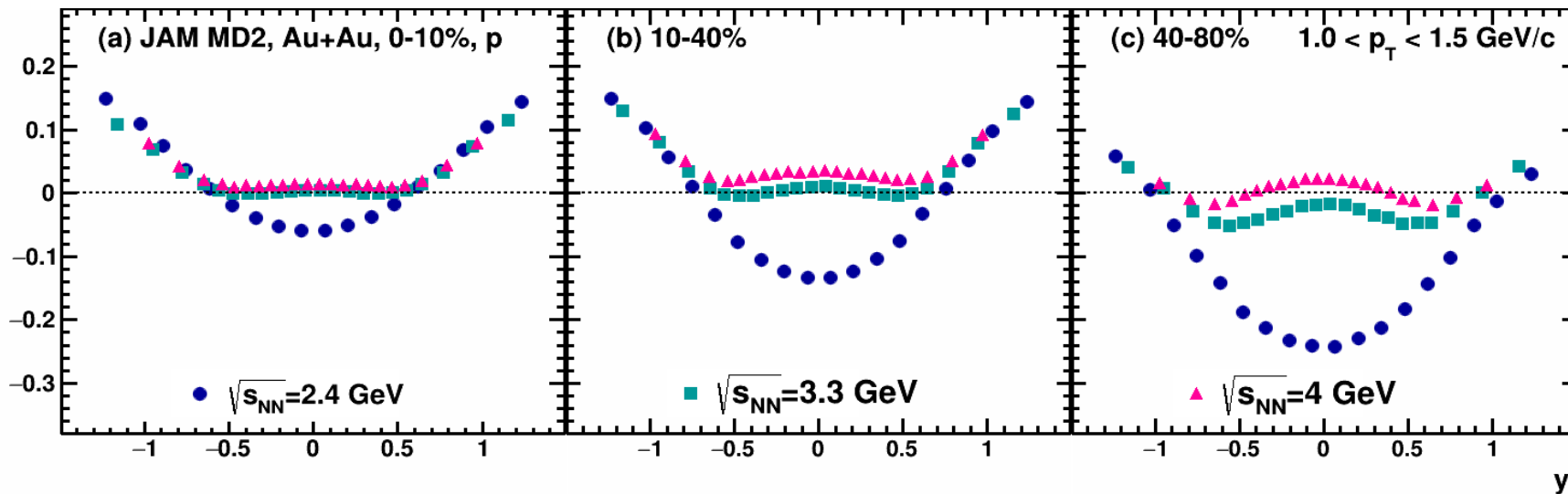
P. Parfenov, Particles 5, no.4, 561-579 (2022)



Transition of v_2 from out-of-plane to in-plane can be a good tool to constrain models and extract information about EOS

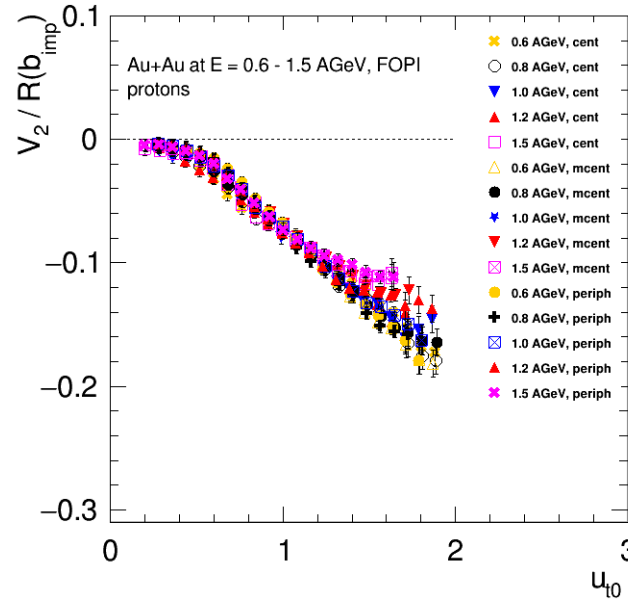
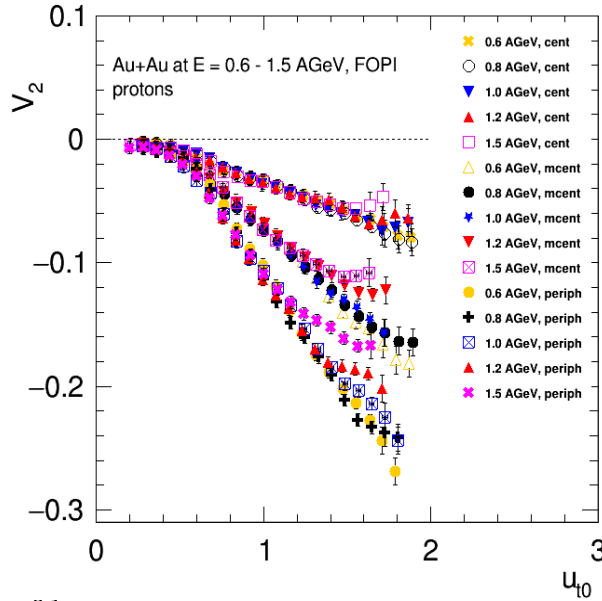
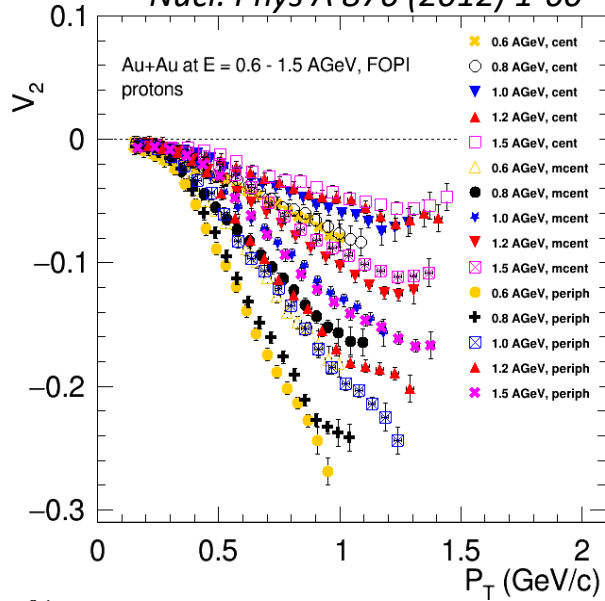
- $v_2 \approx 0$ in midrapidity at $\sqrt{s_{NN}}=3.3$ GeV for central and mid-central collisions
- $v_2 < 0$ for peripheral collisions

Transition from out-of-plane to in-plane depends on centrality and rapidity



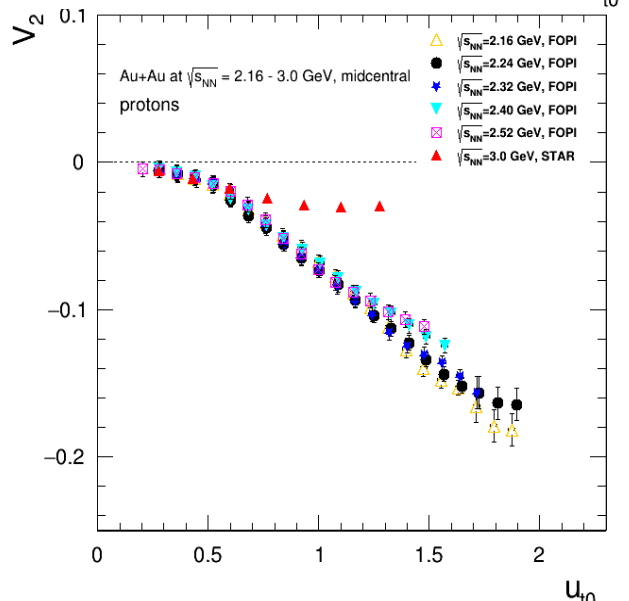
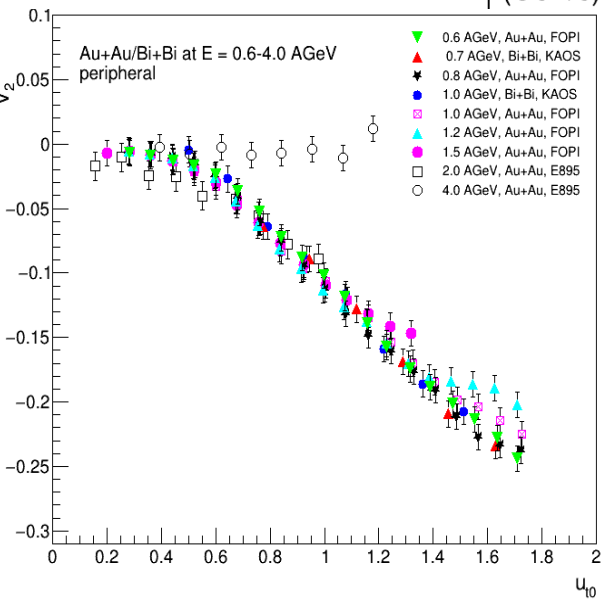
Scaling relations at SIS – scaling with passage time

Nucl. Phys A 876 (2012) 1-60



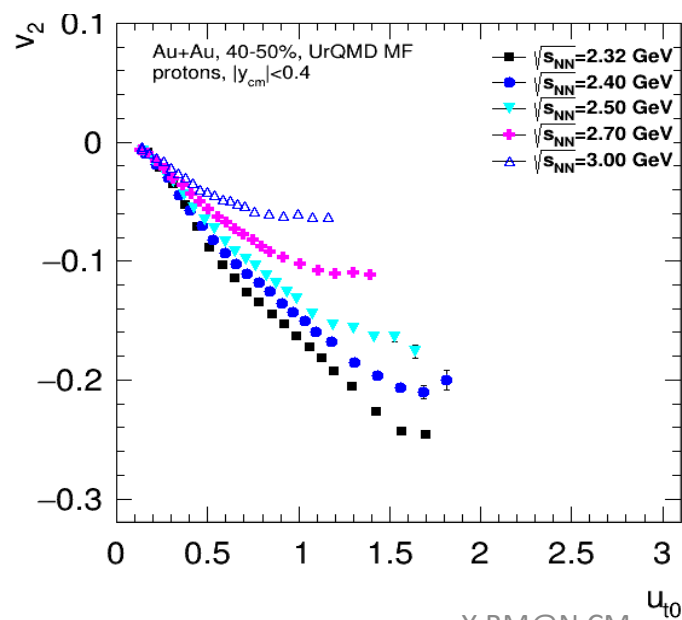
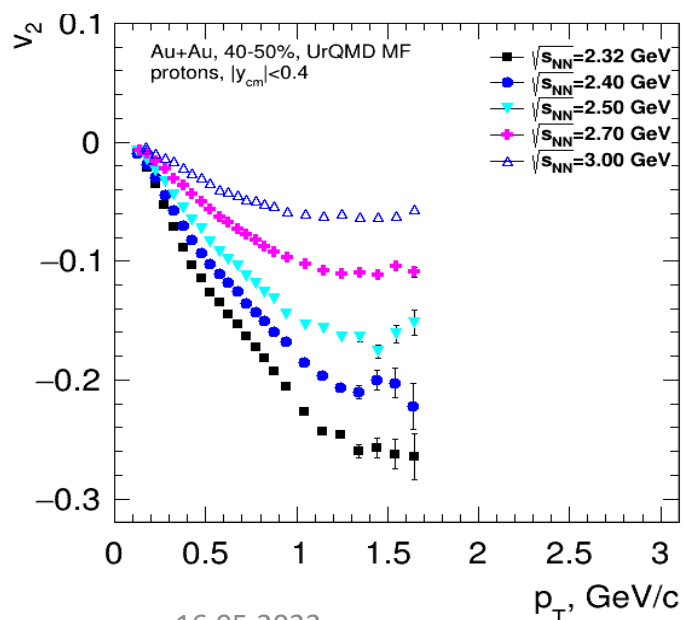
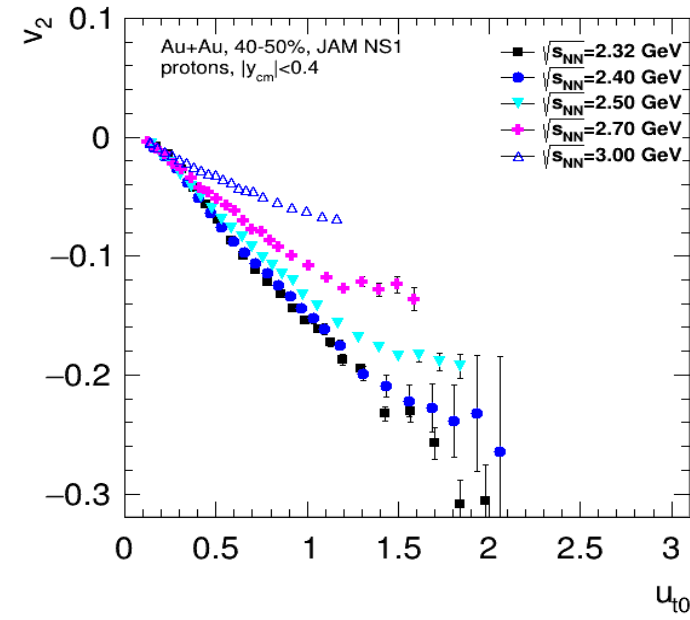
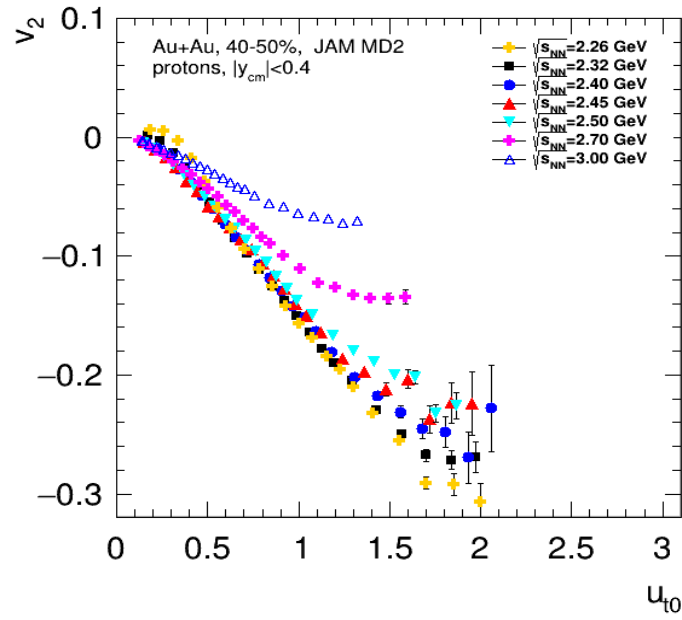
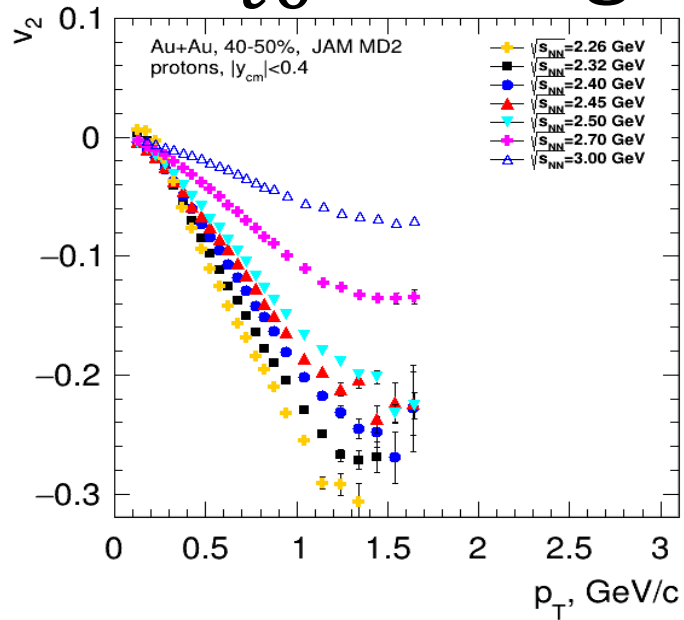
$$u_{t0} = \frac{p_T}{m_0 \beta_{CM} \gamma_{CM}} \equiv \frac{p_T t_{pass}}{2R m_0}$$

$$t_{pass} = \frac{2R}{\beta_{CM} \gamma_{CM}}$$



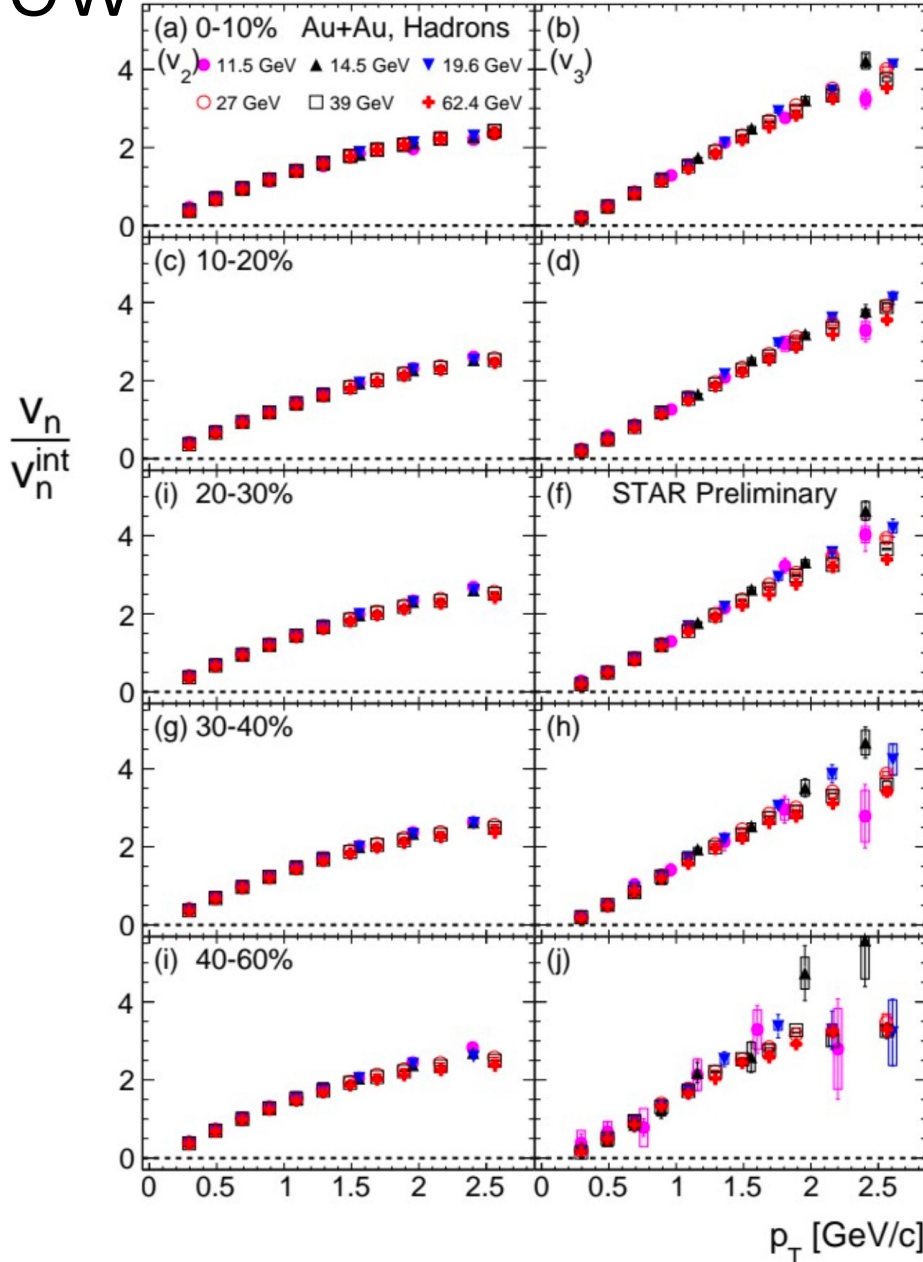
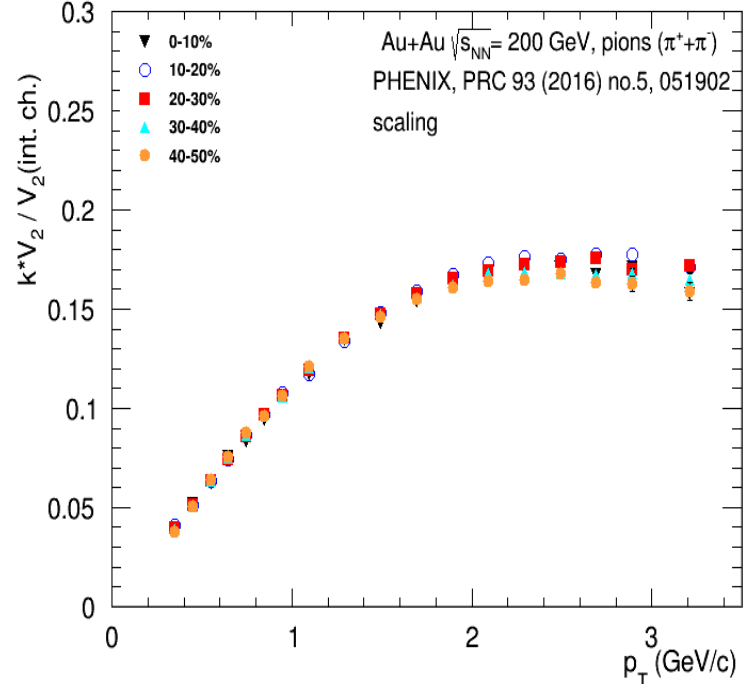
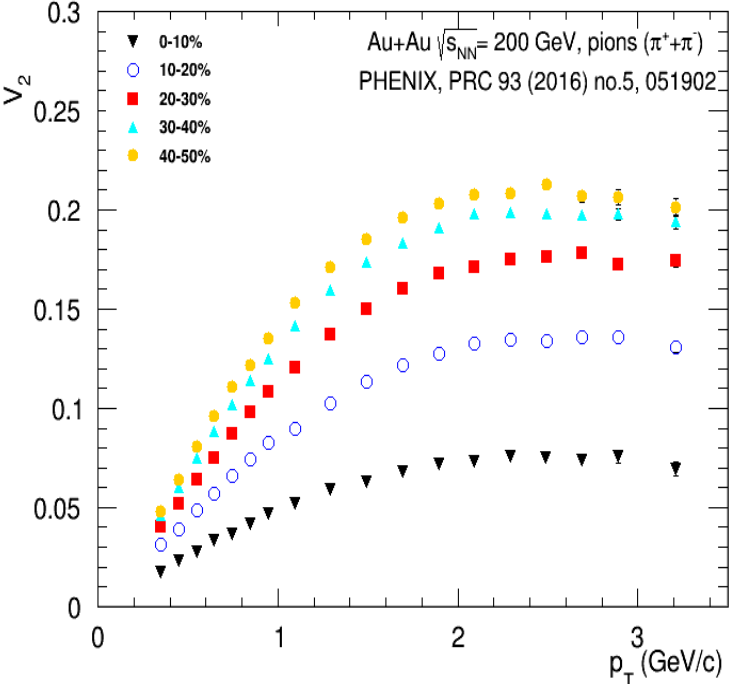
- The rather good scaling observed suggests that c_s does not change significantly over beam energy range $E_{kin} = 0.4 - 2$ AGeV ($\sqrt{s_{NN}} = 2 - 2.7$ GeV)
- Scaling breaks at $E_{kin} = 2.9$ AGeV ($\sqrt{s_{NN}} = 3$ GeV)

u_{t0} scaling: mean-field models



- Scaling holds for both JAM and UrQMD models with mean-field potentials for all EOS
- Similar trend with experimental data: scaling breaks at around $\sqrt{s_{NN}} \geq 2.7$ GeV
- Scaling can provide additional constraints for models

Scaling with integral anisotropic flow

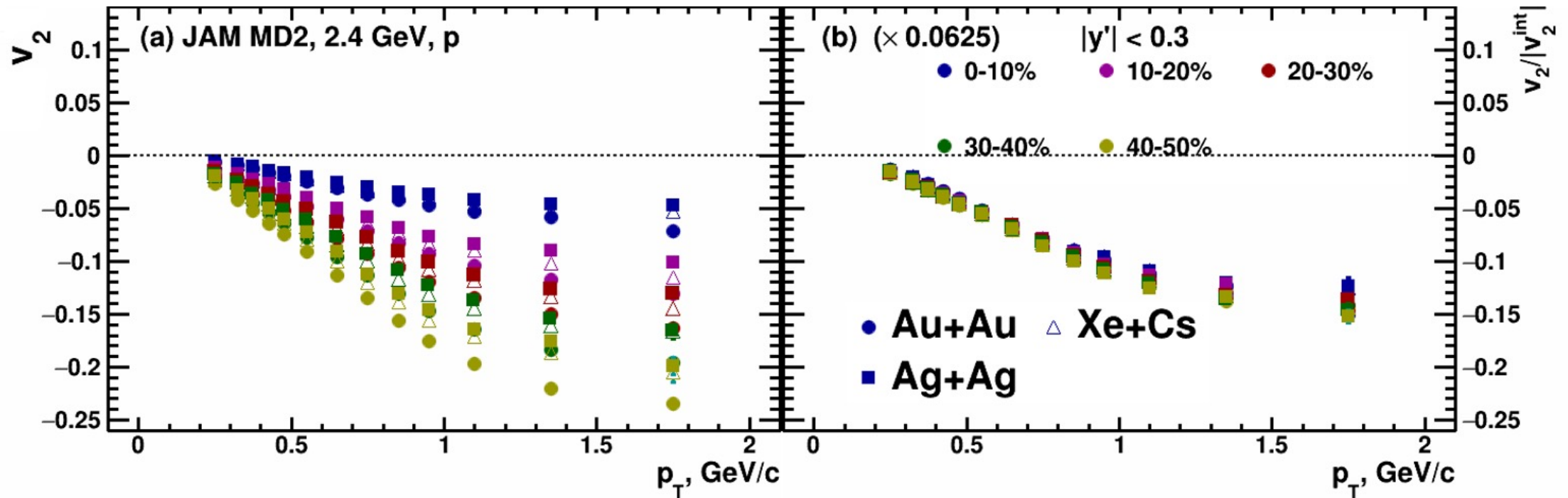


$$v_n(int.) \equiv |v_n^{int}| = |\langle v_n(p_T, y, centrality, PID) \rangle_{p_T, y}|$$

- Scaling works at top RHIC and BES energy range
- Similar trend for pions, kaons and protons

$|v_n^{int}|$ scaling: JAM MD2 model – Nuclotron energies

$$|v_n^{int}| = |\langle v_n(p_T, y, \text{centrality, PID}) \rangle_{p_T, y}|$$

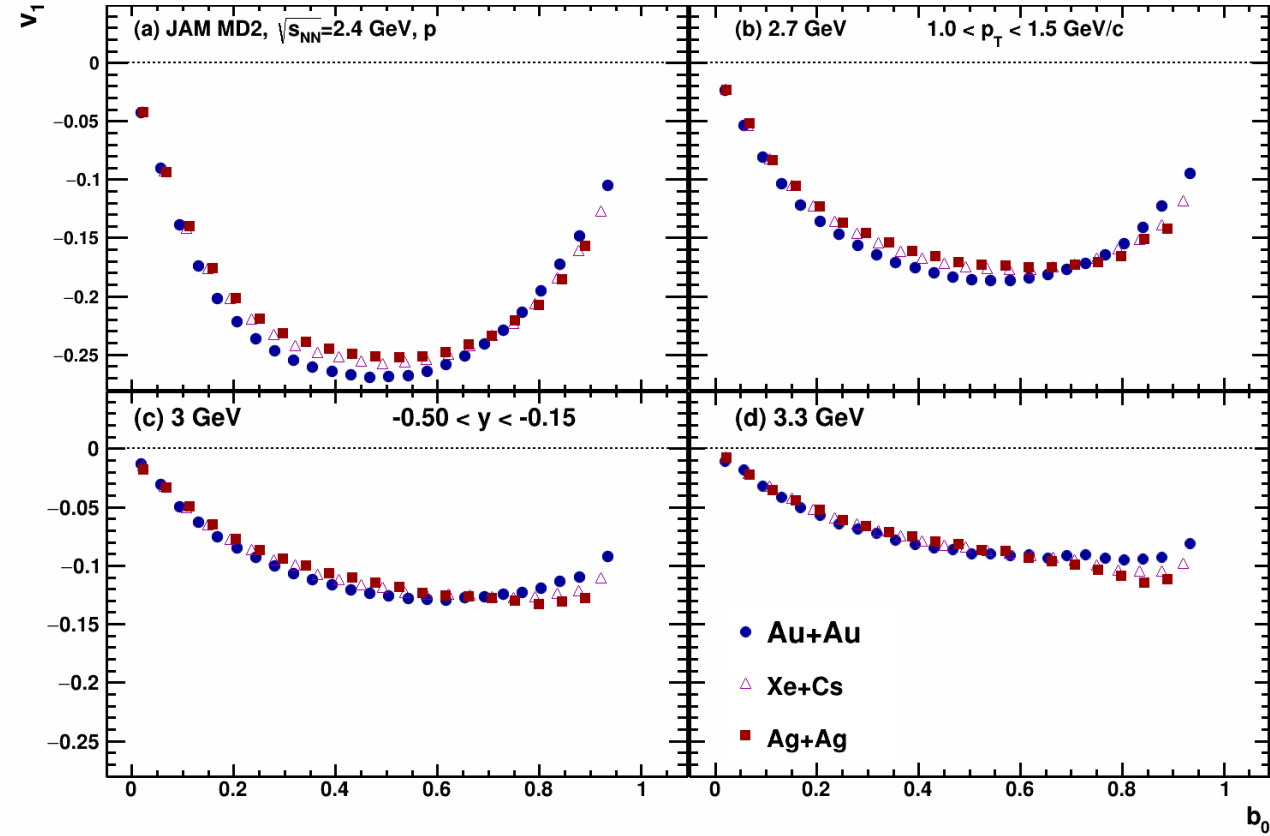
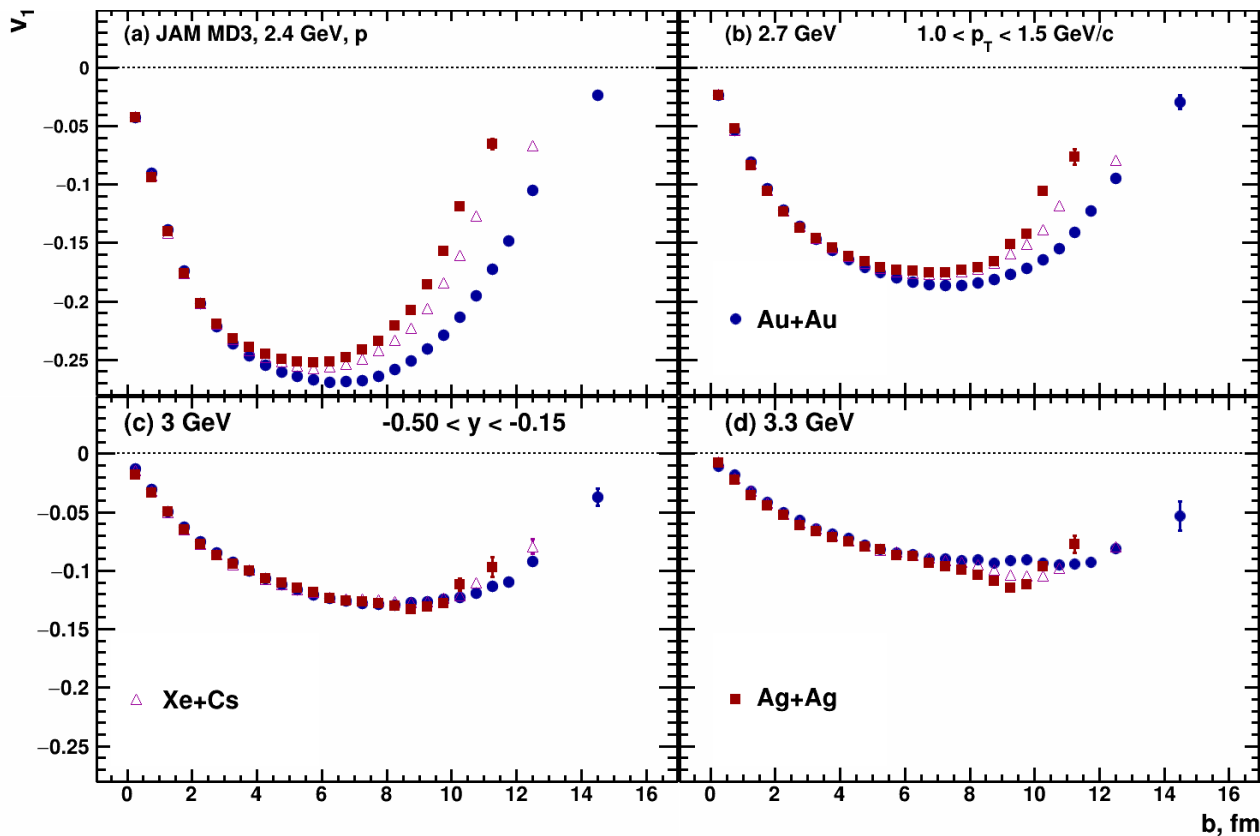


Scaling works for JAM model at $\sqrt{s_{NN}} = 2.4$ GeV for Au+Au, Xe+Cs and Ag+Ag collisions
 Provides a useful tool to make comparison of v_n results from different colliding systems

Scaling with system size

$$b_0 = b/b_{max}$$

$$b_{max} = 1.15 \left(A_{targ}^{1/3} + A_{proj}^{1/3} \right)$$



- Scaling with b_0 can be useful for comparison of the v_n results for different colliding systems
- Difference between v_n for Au+Au, Xe+Cs and Ag+Ag decreases with increasing $\sqrt{s_{NN}}$

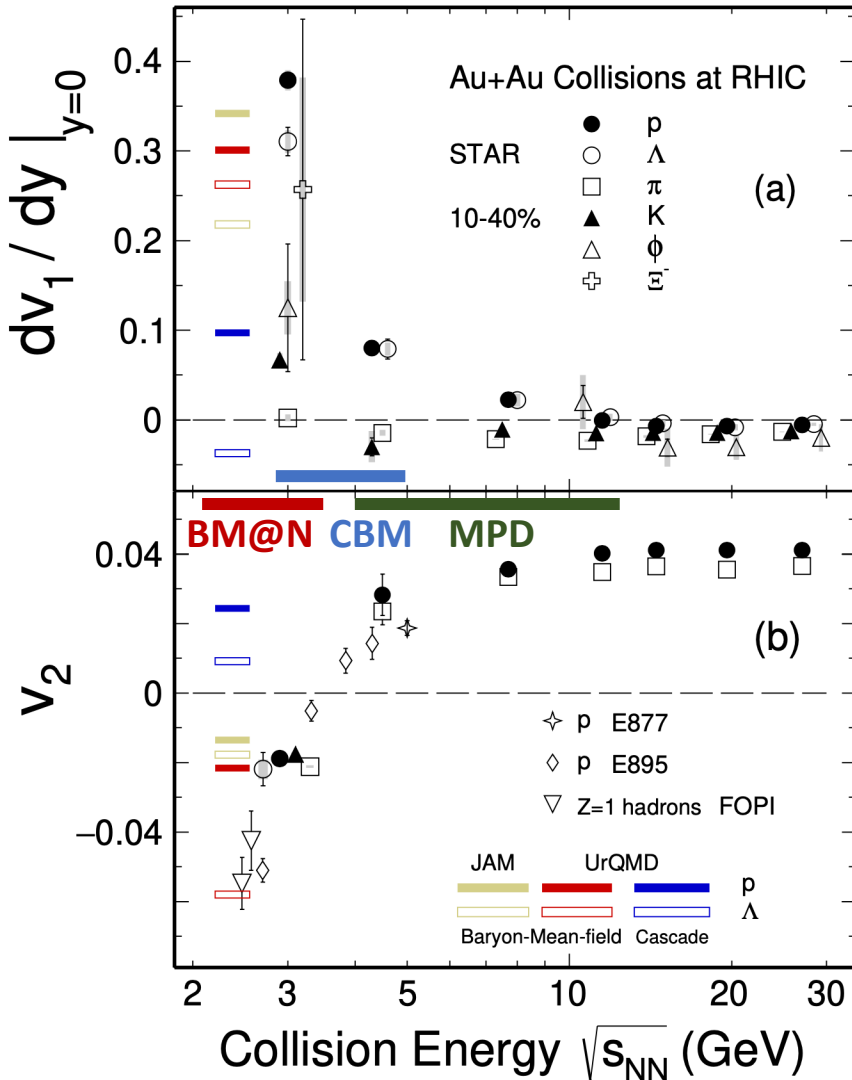
Summary

- **Comparison with STAR BES at $\sqrt{s_{NN}}=3$ GeV and HADES at $\sqrt{s_{NN}}=2.4$ GeV:**
 - Good overall agreement with experimental data for protons for v_n for JAM, UrQMD, SMASH with hard EOS
 - Models do not describe all particle species equally well (mesons, Λ)
- **Study of collision energy dependence of v_n :**
 - $|v_{1,3}|$ decreases with increasing collision energy
 - $v_2 \approx 0$ in midrapidity at $\sqrt{s_{NN}}=3.3$ GeV for central and mid-central collisions
 - Out-of-plane to in-plane transition of v_2 also depends on centrality and rapidity range
- **Scaling relations can be used to compare results from BM@N with the existing experimental data for $\sqrt{s_{NN}} \leq 3$ GeV and further constrain models:**
 - Scaling with passage time holds up for energies $\sqrt{s_{NN}} = 2 - 2.7$ GeV and breaks at $\sqrt{s_{NN}} \geq 3$ GeV
 - Scaling with integral anisotropic flow holds up for a wide energy range and breaks in the energy range where v_2 changes sign (near $\sqrt{s_{NN}}=3.3$ GeV)
 - Scaling with system size provides a useful tool to make comparison of v_n results from different colliding systems
- New data from the future BM@N ($\sqrt{s_{NN}}=2.3-3.3$ GeV) and MPD ($\sqrt{s_{NN}}=4-11$ GeV) experiments will provide more detailed and robust v_n measurements
- To perform more detailed study, different colliding systems, models and EOS are needed

Backup slides

Anisotropic flow in Au+Au collisions at FAIR/NICA energies

M. Abdallah et al. [STAR Collaboration] 2108.00908 [nucl-ex]



$$\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1} v_n \cos[n(\phi - \Psi_{RP})], \quad v_n = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

Strong energy dependence of dv_1/dy and v_2 at $\sqrt{s_{NN}}=2-11$ GeV

Anisotropic flow at FAIR/NICA energies is a delicate balance between:

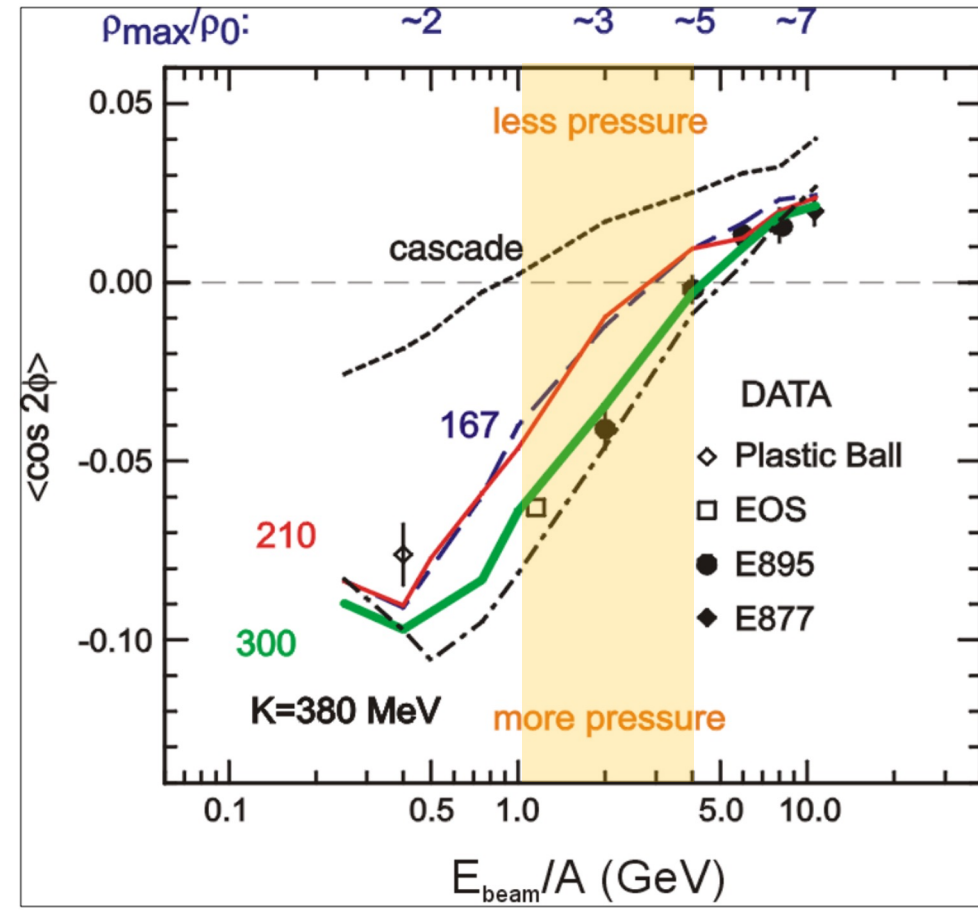
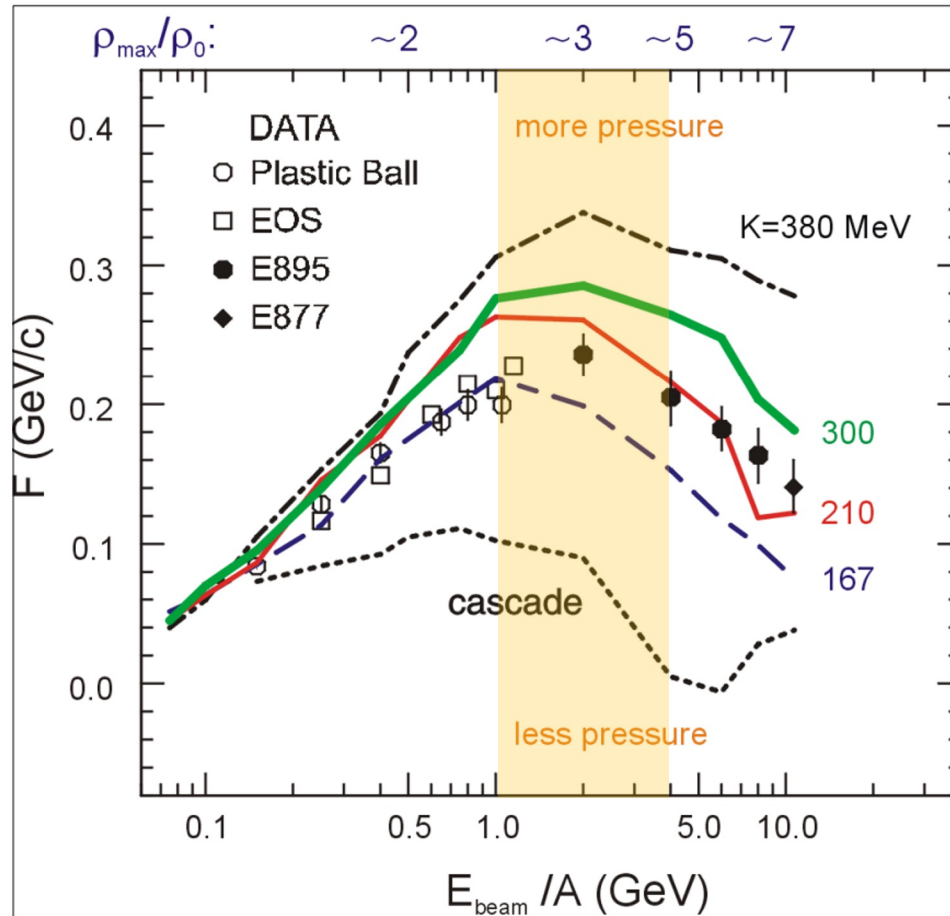
- I. The ability of pressure developed early in the reaction zone ($t_{exp} = R/c_s$) and
- II. The passage time for removal of the shadowing by spectators ($t_{pass} = 2R/\gamma_{CM}\beta_{CM}$)

Goal of this work:

- Perform simulation with different models and make comparison with STAR BES (3, 4.5, 7.7, 11.5 GeV) and HADES (2.4 GeV) published experimental data
- Make predictions for the anisotropic flow measurements $v_n(p_T, y)$ at BM@N ($\sqrt{s_{NN}}=2.3-3.3$ GeV) and MPD ($\sqrt{s_{NN}}=4-11$ GeV) energies

Interpretation of the previous flow data

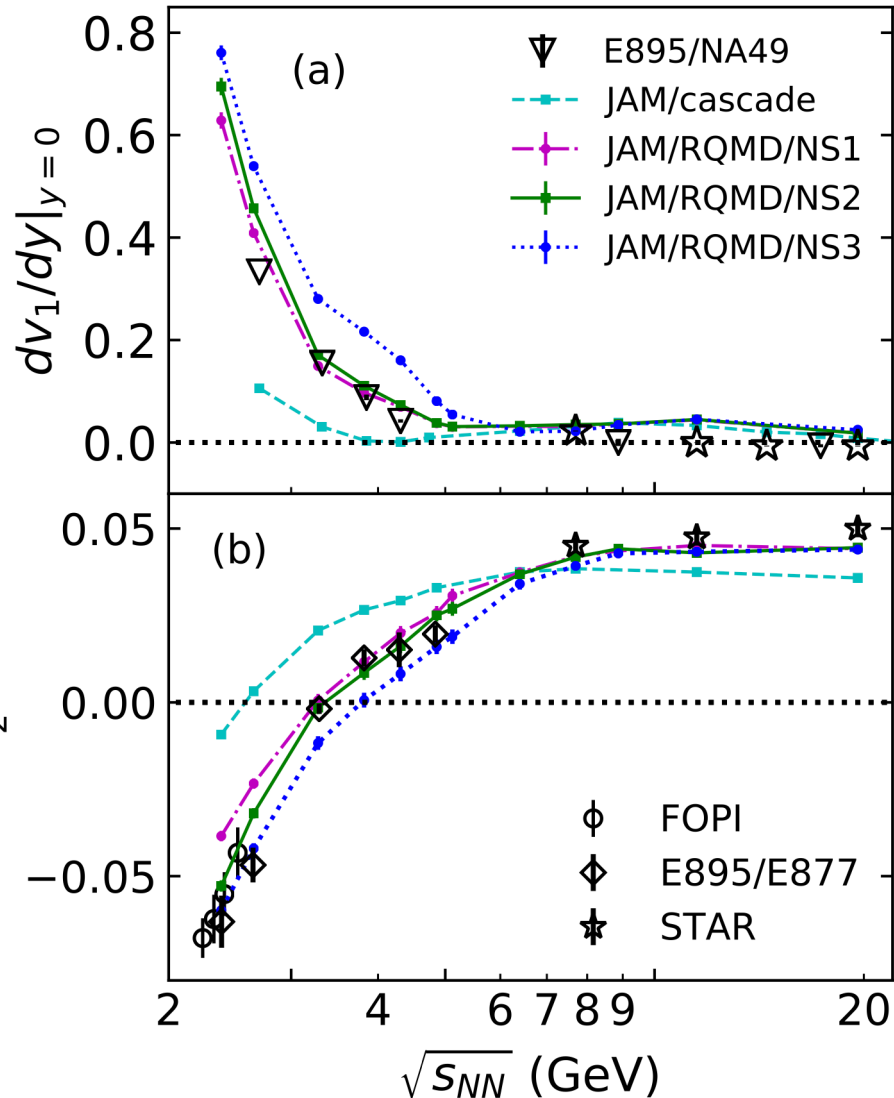
P. DANIELEWICZ, R. LACEY, W. LYNCH
[10.1126/science.1078070](https://doi.org/10.1126/science.1078070)



- The flow data from E895 experiment have ambiguous interpretation: v_1 suggests soft EOS while v_2 corresponds to hard EOS
- Additional measurements are essential to clarify the previous measurements

Anisotropic flow study at $\sqrt{s_{NN}}=2-4$ GeV with JAM model

Y.Nara, et al., Phys. Rev. C 100, 054902 (2019)



To study energy dependence of v_n , JAM microscopic model was selected (ver. 1.90597)

NN collisions are simulated by:

- $\sqrt{s_{NN}} < 4$ GeV: resonance production
- $4 < \sqrt{s_{NN}} < 50$ GeV: soft string excitations
- $\sqrt{s_{NN}} > 10$ GeV: minijet production

We use RQMD with relativistic mean-field theory (non-linear σ - ω model) implemented in JAM model

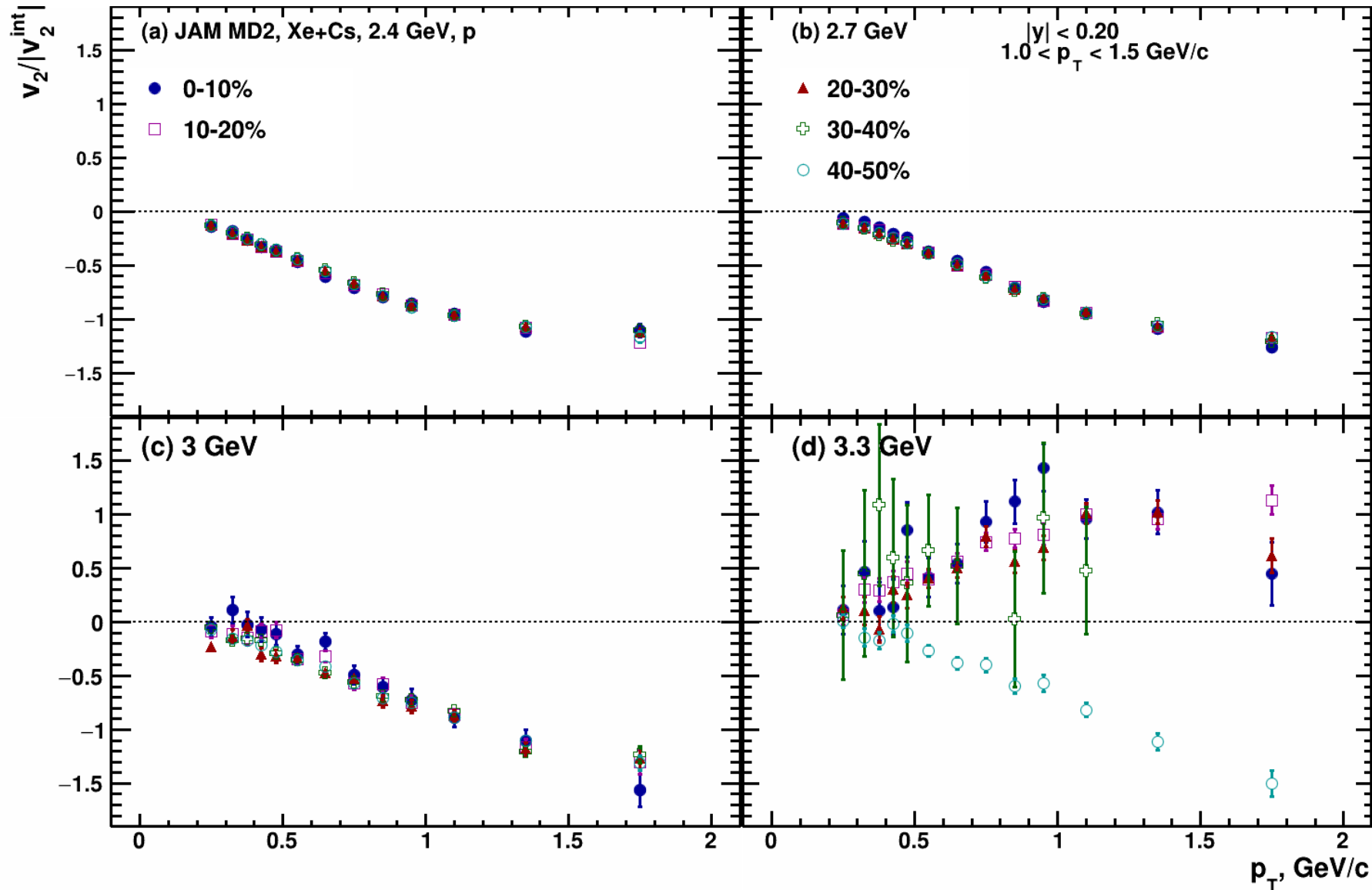
Different EOS were used:

- **MD2** (momentum-dependent potential): $K=380$ MeV, $m^*/m=0.65$, $U_{opt}(\infty)=30$
- **MD4** (momentum-dependent potential): $K=210$ MeV, $m^*/m=0.83$, $U_{opt}(\infty)=67$
- **NS1**: $K=380$ MeV, $m^*/m=0.83$, $U_{opt}(\infty)=95$
- **NS2**: $K=210$ MeV, $m^*/m=0.83$, $U_{opt}(\infty)=98$

Y.Nara, T.Maruyama, H.Stoecker Phys. Rev. C 102, 024913 (2020)

Y.Nara, H.Stoecker Phys. Rev. C 100, 054902 (2019)

$|v_2^{int}|$ scaling: JAM MD2 model – Nuclotron energies



Scaling works for energy range $\sqrt{s_{NN}} = 2.4 - 3$ GeV and breaks at $\sqrt{s_{NN}} = 3.3$ GeV where v_2 changes sign

EOS for high baryon density matter

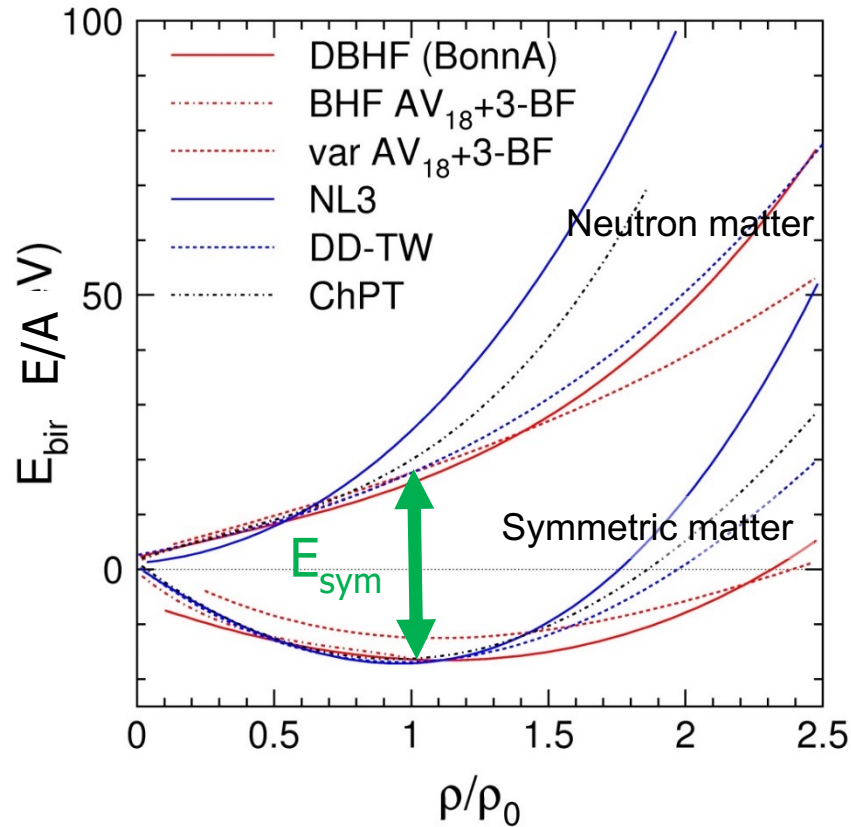
The binding energy per nucleon: $E_A(\rho, \delta) = E_A(\rho, 0) + E_{sym}(\rho)\delta^2 + O(\delta^4)$

Isospin asymmetry:

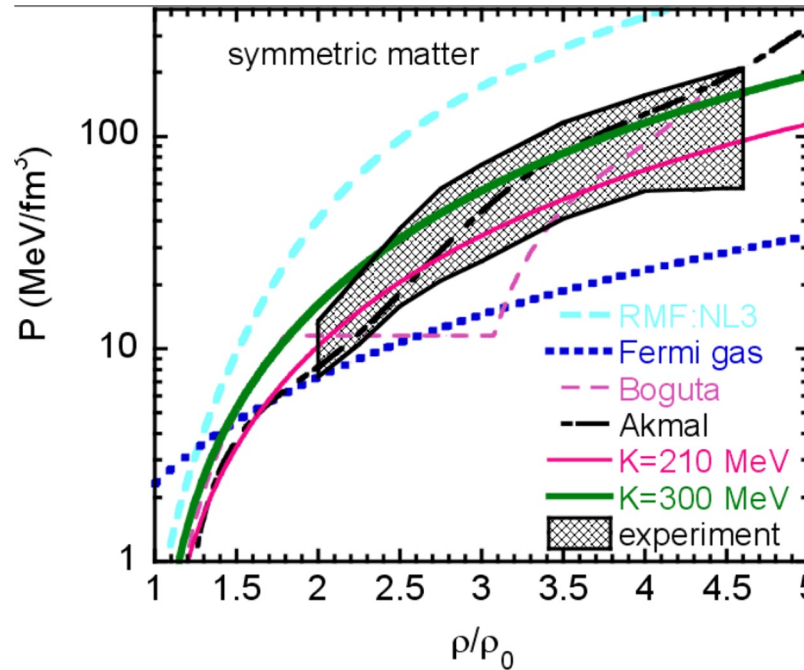
$$\delta = (\rho_n - \rho_p) / \rho$$

Symmetric matter

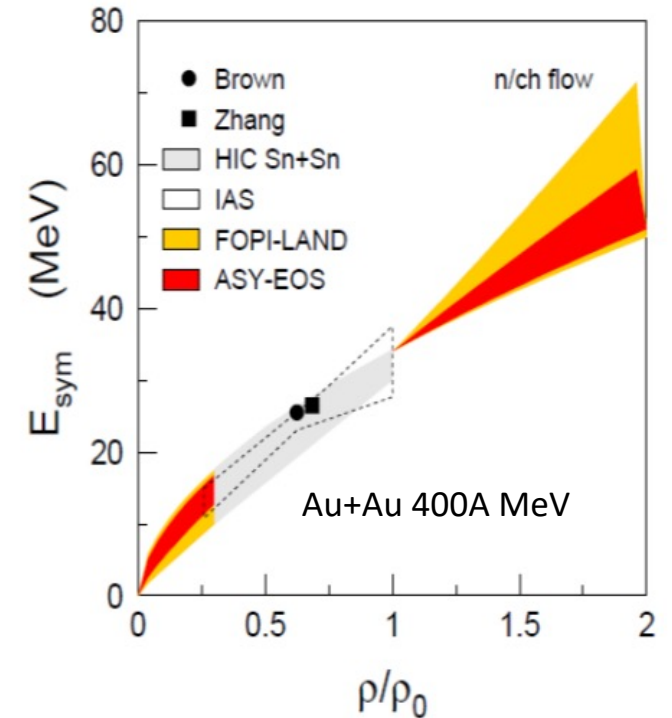
Symmetry energy



Ch. Fuchs and H.H. Wolter, EPJA 30 (2006) 5



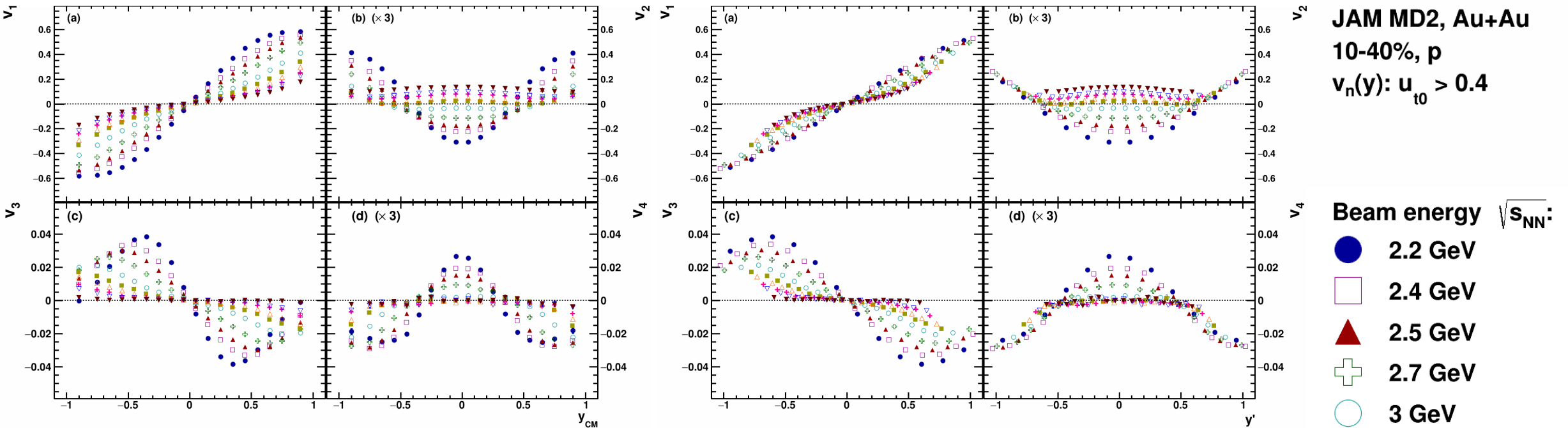
E895: elliptic flow of protons
P. Danielewicz, R. Lacey, W.G. Lynch,
Science 298 (2002) 1592



ASY-EOS: Elliptic flow of
neutrons/charged particles

P. Russotto et al.,
Phys. Rev. C 94 (2016) 034608

y' scaling: mean-field models

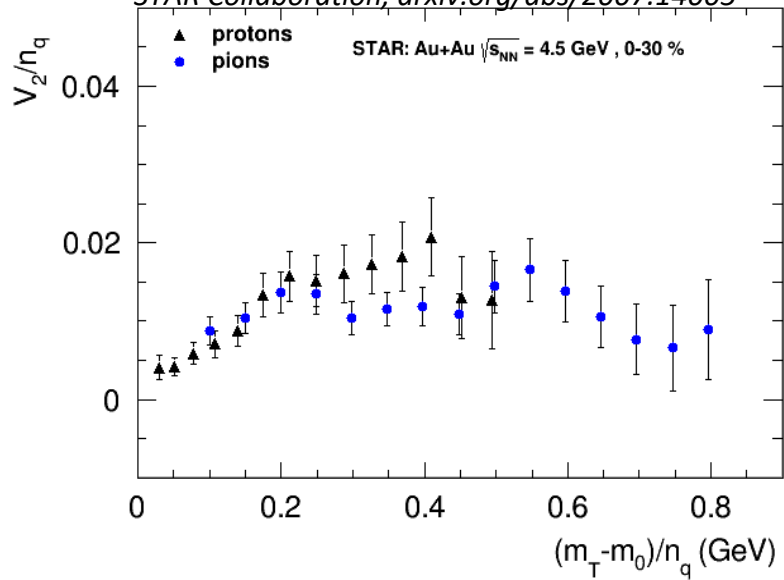


$$y' = y/y_{beam}, \quad t_{pass} = \frac{2R}{\gamma_{CM}\beta_{CM}} \equiv \frac{2R}{\sinh y_{beam}}$$

- Scaled rapidity $y' = y_{CM}/y_{beam}$ dependence simplifies the energy dependence of $v_n(y)$ and may reflect the partial scaling of v_n with t_{pass}

NCQ scaling: hybrid and cascade models

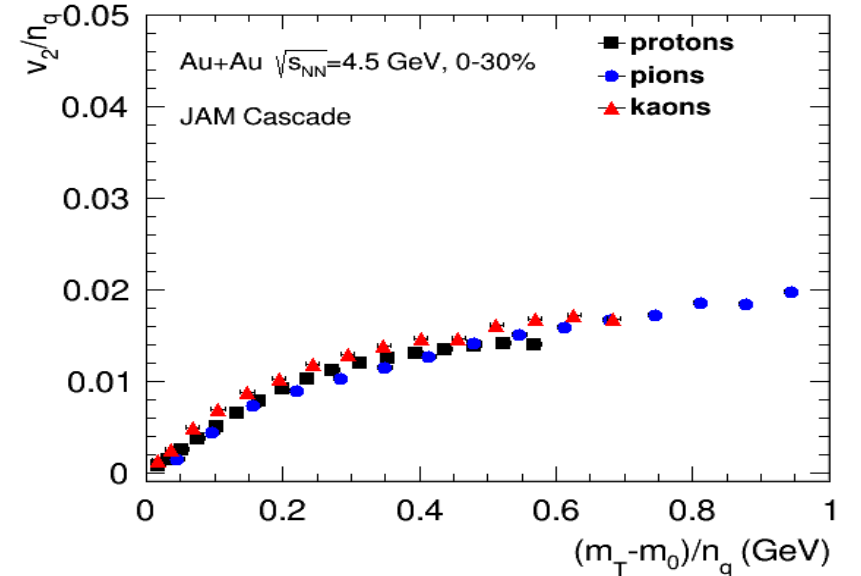
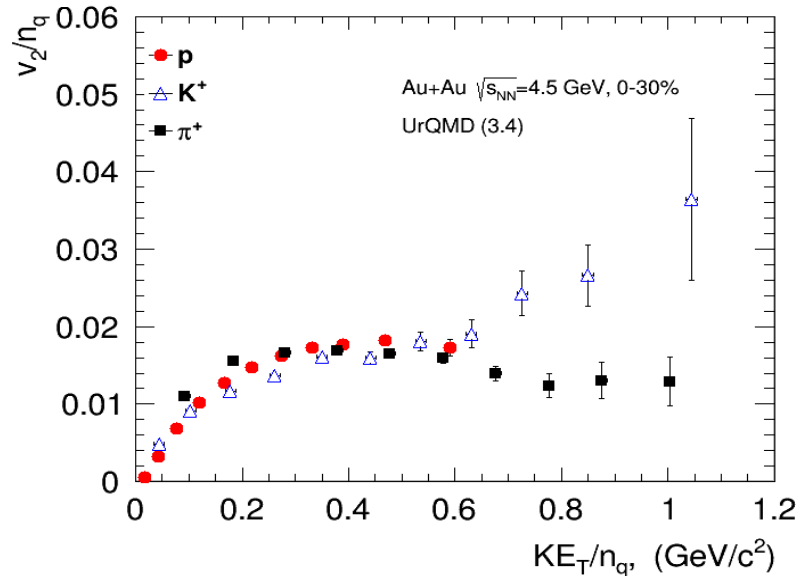
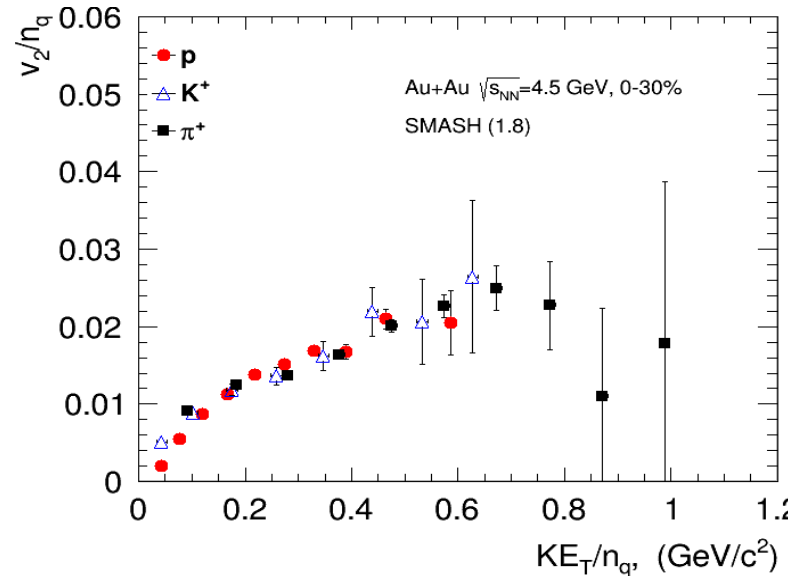
STAR Collaboration, arxiv.org/abs/2007.14005



$$\text{NCQ scaling: } v_n(\mathbf{p}_T) \rightarrow v_n/n_q^{n/2} \left(\frac{KE_T}{n_q} \right) \quad n_q = \begin{cases} 2 \text{ for mesons} \\ 3 \text{ for baryons} \end{cases} \quad KE_T = \sqrt{m^2 + p_T^2} - m$$

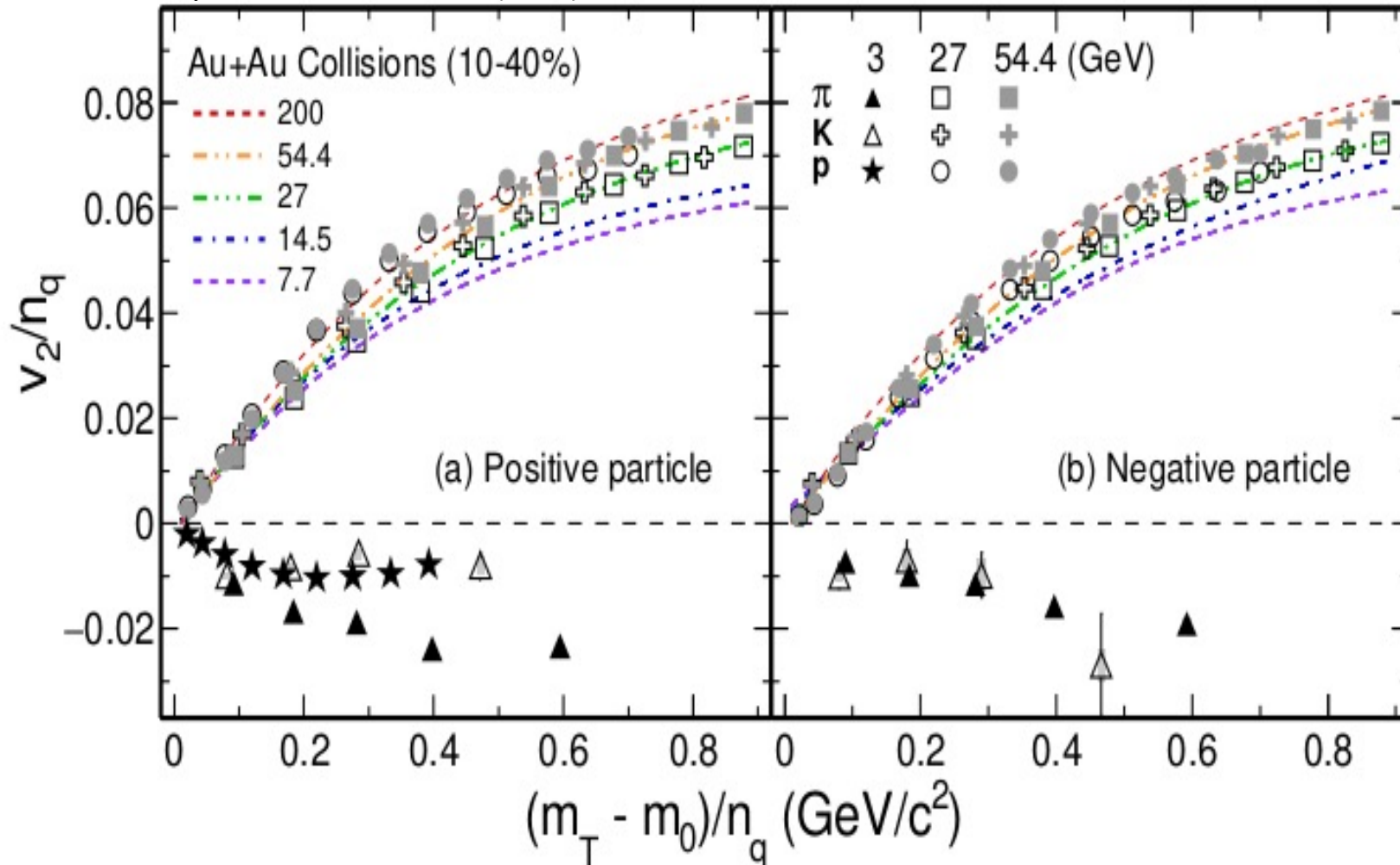
- Scaling holds up at 4.5 GeV in STAR data and pure string/hadronic cascade models (without partonic d.o.f.)

KE_T/n_q scaling at 4.5 GeV might be accidental – more careful studies should be performed



Dissappearance of partonic collectivity in $\sqrt{s_{NN}} = 3$ GeV Au+Au collisions at RHIC

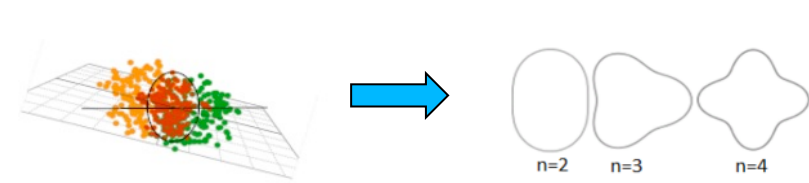
Phys. Lett. B 827, 137003 (2022)



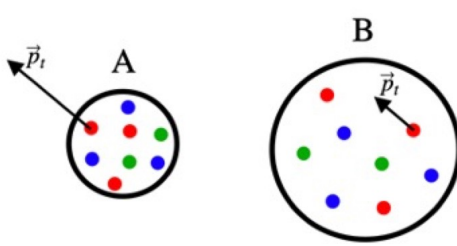
Breaking of NCQ scaling at 3 GeV

“imply the vanishing of partonic collectivity and a new EOS, likely dominated by baryonic interactions in the high baryon density region”

$v_n - [p_T]$ correlation measurements



small R, large $\langle p_T \rangle$ large R, small $\langle p_T \rangle$



$$\langle p_T \rangle \sim 1/R$$

- v_n is sensitive to the initial shape of the collision geometry (but also thermalization, etc.)
- $[p_T]$ is sensitive to the initial size of the overlap region (but also thermalization, etc.)

$$\frac{dN}{d\varphi} \sim 1 + \sum_{n=1} v_n \cos[n(\varphi - \Psi_n)],$$

$$v_n \propto \varepsilon_n, \quad n = 1, 2$$

$$\rho(v_2^2, [p_T]) = \frac{\text{cov}(v_2^2, [p_T])}{\sqrt{\text{var}(v_2^2)_{dyn}} \sqrt{c_k}}$$

$$\text{cov}(v_2^2, [p_T]) = \left\langle \frac{\sum_{A,C} e^{i2(\varphi_1^A - \varphi_2^C)} \sum_B (p_{T,B} - \langle [p_T] \rangle)}{M_A M_C M_B} \right\rangle$$

➤ The $\rho(v_2^2, [p_T])$ is sensitive to initial state and its entropy density profile

➤ The $\text{cov}(v_2^2, [p_T])$ is sensitive to η/s

$$\text{var}(v_2^2)_{dyn} = \langle v_2^4 \rangle - \langle v_2^2 \rangle^2 \quad c_k = \left\langle \frac{1}{M_B(M_B - 1)} \sum_B \sum_{B' \neq B} (p_{T,B} - \langle [p_T] \rangle)(p_{T,B'} - \langle [p_T] \rangle) \right\rangle$$

The precise set of measurements for $\text{var}([p_T])$, $\text{var}(v_2^2)$, $\text{cov}(v_2^2, [p_T])$ and $\rho(v_2^2, [p_T])$ as a function of beam-energy and centrality could help precision extraction of the temperature and baryon chemical-potential dependence of η/s