Alignment with Tracks

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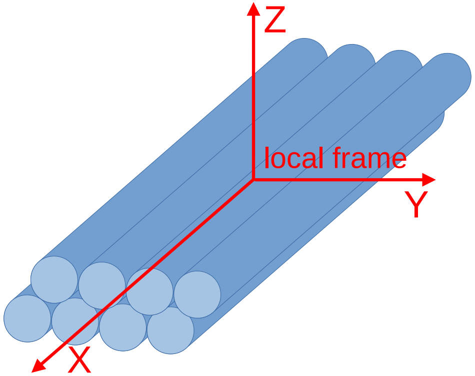
Alignment

Alignment goal: determination of (small) alignment corrections.

actual geometry = nominal geometry + alignment corrections

- alignment corrections are defined in the (nominal) local reference frame of the detector element which has to be aligned
- normally the are 6 alignment parameters: 3 shifts and 3 rotations



detector element

Y - precision coordinate; X - second coordinate
shift_X goes out for tube detectors
knowledge of second coordinate required for rot_Y and rot_Z parameters (no need to be precise)

example of a line with alignment constants:

A | ID_r | ID_eta | ID_phi | shift_X | shift_Y | shift_Z | rot_X | rot_Y | rot_Z

Alignment with Tracks. Least Square Minimization.

Least-square minimization of track hit residuals:

track hit residual: $\epsilon_k(\boldsymbol{a}) = y_k^{meas} - y_k^{track}$,

y_k^{meas} - measured hit position;
y_k^{track} - expectation of track position;
$\boldsymbol{a}$ - vector of alignment parameters;

Global χ^2: $\chi^2 = \sum_k \frac{\epsilon_k^2}{\sigma_k^2};$ index k runs over all hits on all tracks.

Minimization: $\frac{d\chi^2}{da_i} = 0;$ derivatives for each alignment parameter.

System of linear equations: $\boldsymbol{C} \cdot \boldsymbol{a} = \boldsymbol{b};$
Solution: $\boldsymbol{a} = \boldsymbol{C}^{-1} \cdot \boldsymbol{b};$ $\boldsymbol{C}^{-1}$ – covariance matrix

- iterations required (wrong initial geometry was used for track fit)

MILLEPEDE

Expected value of a measurement in a linear model: $y_k^{exp} = \boldsymbol{a}^T \boldsymbol{d}_k + \boldsymbol{\alpha}^T \boldsymbol{\delta}_k$

$\boldsymbol{a}$, $\boldsymbol{\alpha}$ - vectors of alignment and track parameters;

$\boldsymbol{d}_k$, $\boldsymbol{\delta}_k$ - vectors of derivatives for the k-th measurement.

Global χ^2:

$$\chi^2 = \sum_k \frac{(y_k^{meas} - y_k^{exp})^2}{\sigma_k^2} \ ,$$ index k runs over all hits on all tracks.

System of linear equations:

$$\left(\begin{array}{c|ccc} \sum \boldsymbol{C}_i & \dots & \boldsymbol{G}_i & \dots \\ \hline \vdots & \ddots & 0 & 0 \\ \boldsymbol{G}_i^T & 0 & \boldsymbol{\Gamma}_i & 0 \\ \vdots & 0 & 0 & \ddots \end{array}\right) \times \left(\begin{array}{c} \boldsymbol{a} \\ \hline \vdots \\ \boldsymbol{\alpha}_i \\ \vdots \end{array}\right) = \left(\begin{array}{c} \sum \boldsymbol{b}_i \\ \hline \vdots \\ \boldsymbol{\beta}_i \\ \vdots \end{array}\right)$$

Size_of_matrix_to_invert =
size_of_vector_$\boldsymbol{a}$ + size_of_vector_$\boldsymbol{\alpha}$ $\times$ number_of_tracks

Matrices $\boldsymbol{C}_i$, $\boldsymbol{\Gamma}_i$, $\boldsymbol{G}_i$ and vectors $\boldsymbol{b}_i$, $\boldsymbol{\beta}_i$ are contributions from the i-th track to the system.

Solution for alignment parameters:

$$\boldsymbol{a} = \boldsymbol{C}'^{-1}\boldsymbol{b}' \ , \quad \text{where} \quad \boldsymbol{C}' = \sum_i \boldsymbol{C}_i - \sum_i \boldsymbol{G}_i \boldsymbol{\Gamma}_i^{-1} \boldsymbol{G}_i^T \ , \qquad \boldsymbol{b}' = \sum_i \boldsymbol{b}_i - \sum_i \boldsymbol{G}_i \left(\boldsymbol{\Gamma}_i^{-1} \boldsymbol{\beta}_i\right) \ .$$

Advantages: Can deal with large number of correlated alignment parameters, unbiased, fast.

Constraints

Alignment is relative to global shift and global rotation of the detector

mathematically it means that matrix $\boldsymbol{C} \cdot \boldsymbol{a} = \boldsymbol{b}$; *can not be inverted*

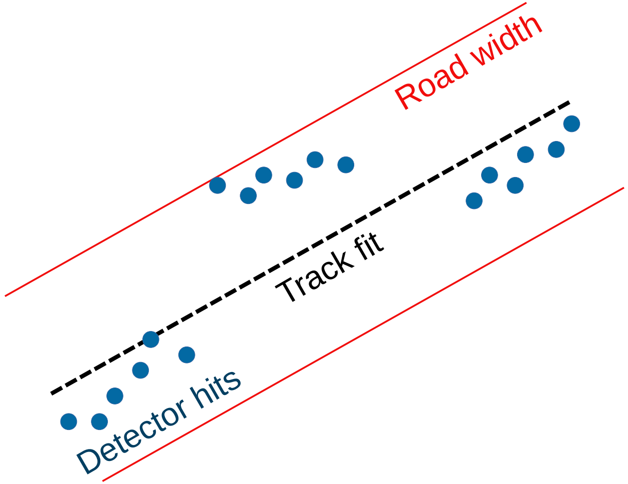
constraints needed: *number of constraints corresponds to the set of alignment parameters per one detector element*

- addition of constraints in a form of strict linear equations between any number of alignment parameters (nontrivial implementation); alignment can be done as:
  – relative to a particular detector element
  – relative to the center of mass of the detector
- addition of fictional measurements (with small weights) to the matrix

Outlier Suppression

most important part in terms of alignment precision!

even one ”bad” track can screw up your whole matrix!



outlier rejection

- rejection of hits/tracks outside the *”road width”*
- decrease the *”road width”* with next alignment iterations and make a new decision on each iteration

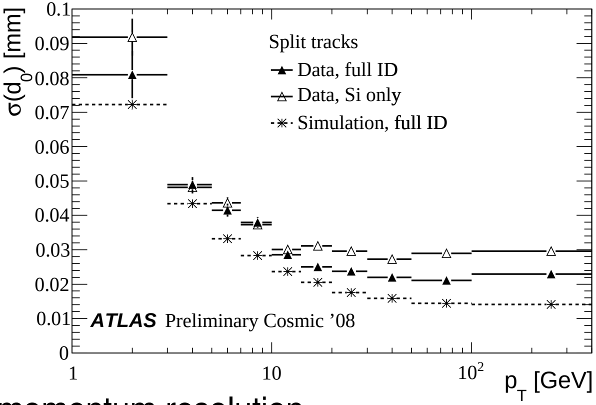
outlier downweighting

- introduce additional errors (decrease weights) to the hits according to the distance to track

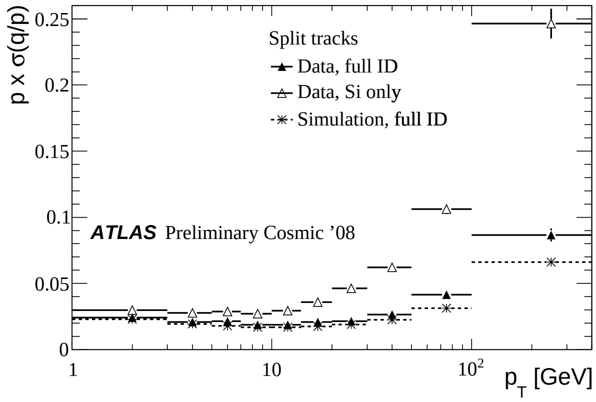
the reason why MILLEPEDE requires iterations

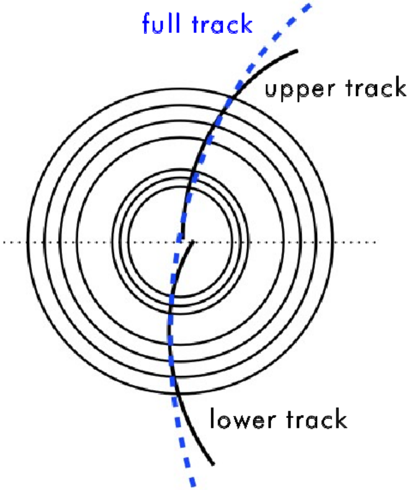
ATLAS Inner Detector. Track Parameter Resolution. Cosmic.

impact parameter resolution



momentum resolution





- split cosmic tracks into top and bottom parts to plot top-bottom distributions ($\sigma = \sigma_{tb}/\sqrt{2}$)
- resolution:
  - low p_t: dominated by multiple scattering
  - high p_t: dominated by intrinsic resolution and misalignment