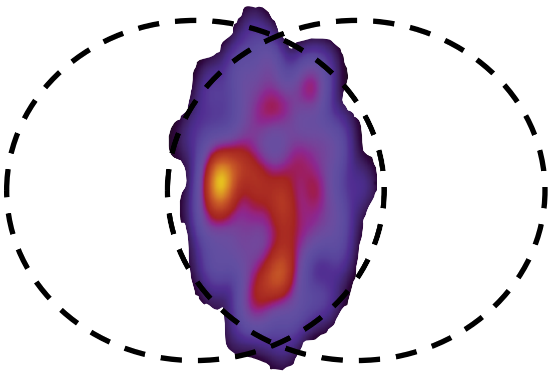
Skewness of the elliptic flow distribution from the MPD

Jovan Milošević
University of Belgrade
Vinča Institute of Nuclear Sciences, Serbia

on behalf of the Vinca MPD group and SPiRL Huzhou University

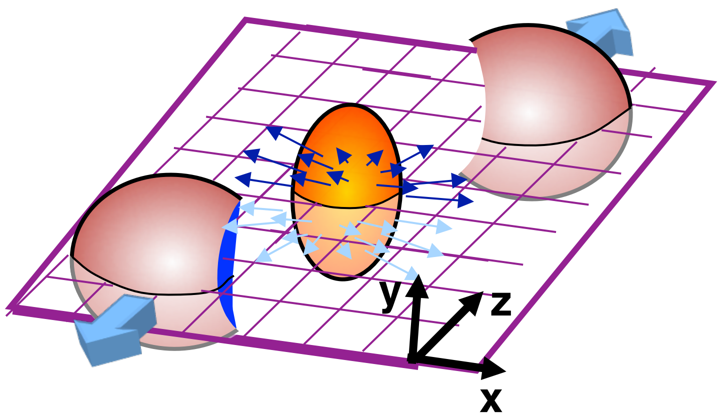
Outline

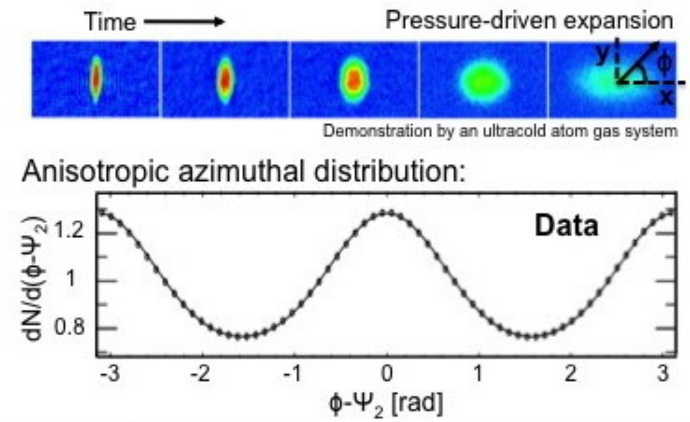


- Motivation
- Collectivity in nuclear collisions – studied by Q-cumulants
- Centrality dependence of the hydrodynamic probes
- Central moments of the $\mathbf{v}_2$ distribution
- Do central moments change with incident energies?
- Conclusions

Motivation

Azimuthal anisotropy





Ψ_n (angle of n-th-order flow symmetry plane)

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_n)]$$

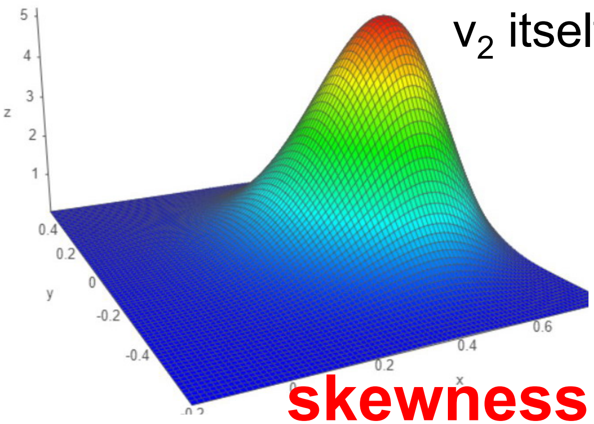
v_n – *Fourier harmonics* depend on

- initial state geometry
- initial state fluctuations
- medium transport properties (e.g. η/s)

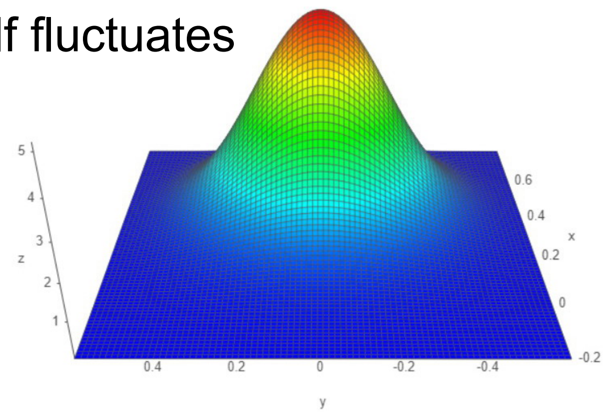
$$v_n \equiv \langle \cos[n(\phi - \Psi_n)] \rangle$$

Event-by-event v_n distribution is not Gaussian-like

Shape of the v_2 distribution

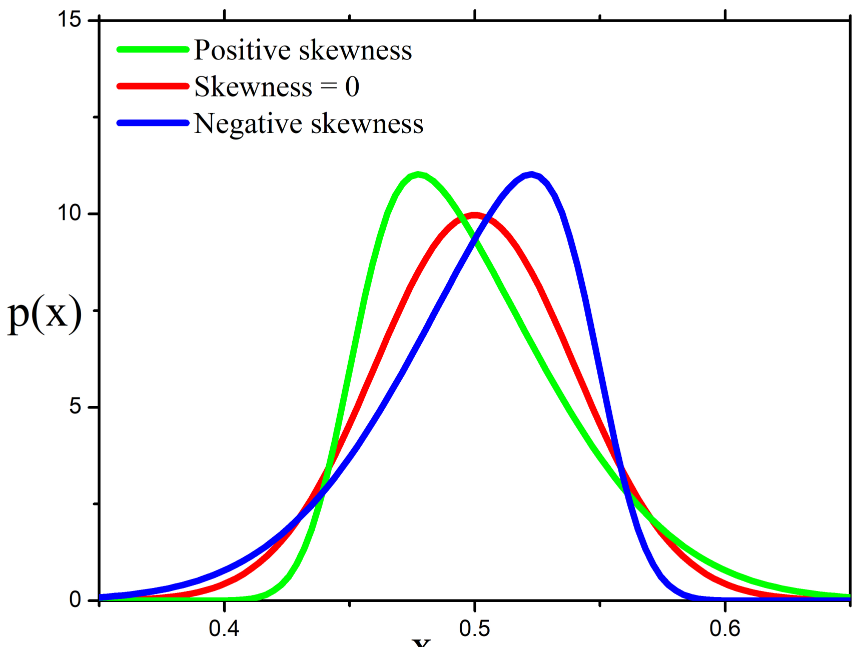


v_2 itself fluctuates

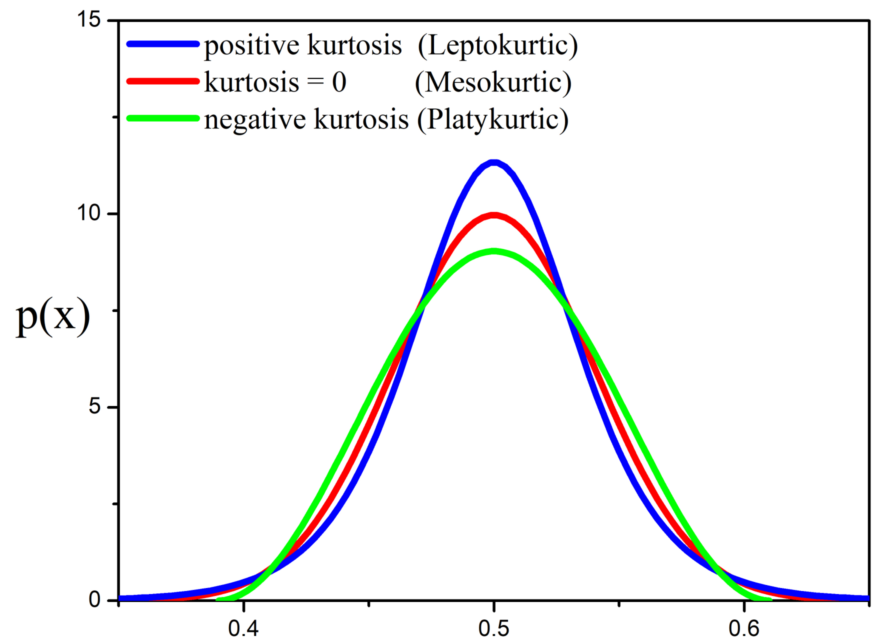


- Non-Gaussianities are present in the early stage
- Partially washed out during hydro expansion
- Can make constraints on the theoretical models

skewness



kurtosis



x

Q-cumulants

v_2 shape studied via Q-cumulant method

Multi-particle correlations

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1-\phi_2)} \right\rangle \qquad \langle 4 \rangle \equiv \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle$$

Q-vector

$$Q_n = \sum_{i=1}^{M} e^{in\varphi_i}$$

Ideal detector case

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2\,\mathrm{Re}\left[Q_{2n} Q_n^* Q_n^*\right]}{M(M-1)(M-2)(M-3)} - 2\frac{2(M-2)|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}$$

Averaging over all events

$$\langle\langle 2 \rangle\rangle \equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle \qquad \langle\langle 4 \rangle\rangle \equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle$$

Formulas become more and more larger with an increase of the cumulant order

Q-cumulants and v_n

[Phys. Rev. C 104 (2021) 034906]:

$$c_n\{2k\} = \langle\langle 2k \rangle\rangle - \sum_{m=1}^{k-1} \binom{k}{m}\binom{k-1}{m} \langle\langle 2m \rangle\rangle c_n\{2k-2m\}$$

$$c_n\{2\} = \langle\langle 2 \rangle\rangle$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2$$

General formulas for any order

$$v_n\{2k\} = \sqrt[2k]{\frac{(2k)!}{2^{2k}(k!)^2}\left[\frac{d^{2k}}{dl^{2k}} \ln I_0(l)\Big|_{l=0}\right]^{-1} c_n\{2k\}}$$

$$v_n\{2\} = \sqrt{c_n\{2\}}$$

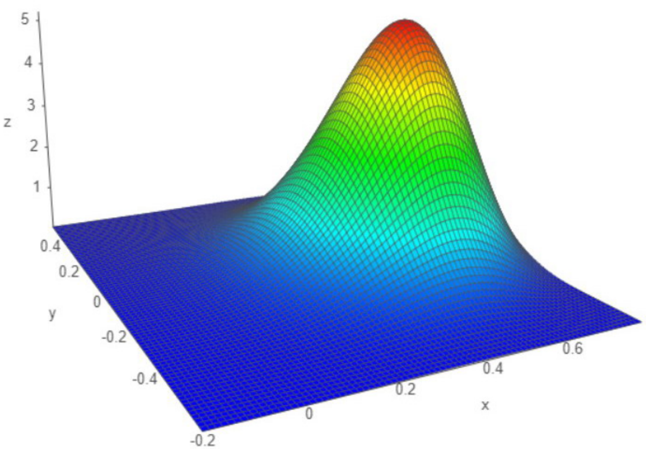
$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$

Expansion of hydro probes in central moments

$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} - \frac{4\kappa_{40} + \dfrac{8(p_{50} + p_{32})}{\bar{v}_2}}{11\left[2\bar{v}_2 s_{30} + 3(\kappa_{40} + \kappa_{22}) + \dfrac{3(p_{50} + 2p_{32} + p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{12})}{2\bar{v}_2}\right]}$$

$$h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{88p_{50}}{95\left[4\bar{v}_2^2 s_{30} - 2\bar{v}_2(\kappa_{40} - 3\kappa_{22}) - 13(p_{50} + 10p_{32} - 3p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{32})\right]}$$

If higher moments κ, p, … negligible
➔ h_i are centrality independent



Taylor expansions

$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} - \frac{1}{11}\frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2 + \dfrac{(\sigma_y^2 - \sigma_x^2)s_{30}}{3\bar{v}_2^3}}$$

negligible

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{1}{19}\frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2 + \dfrac{(\sigma_y^2 - \sigma_x^2)s_{30}}{33\bar{v}_2^3}}$$

Hydrodynamics probes and central moments NICA MPD case JAM

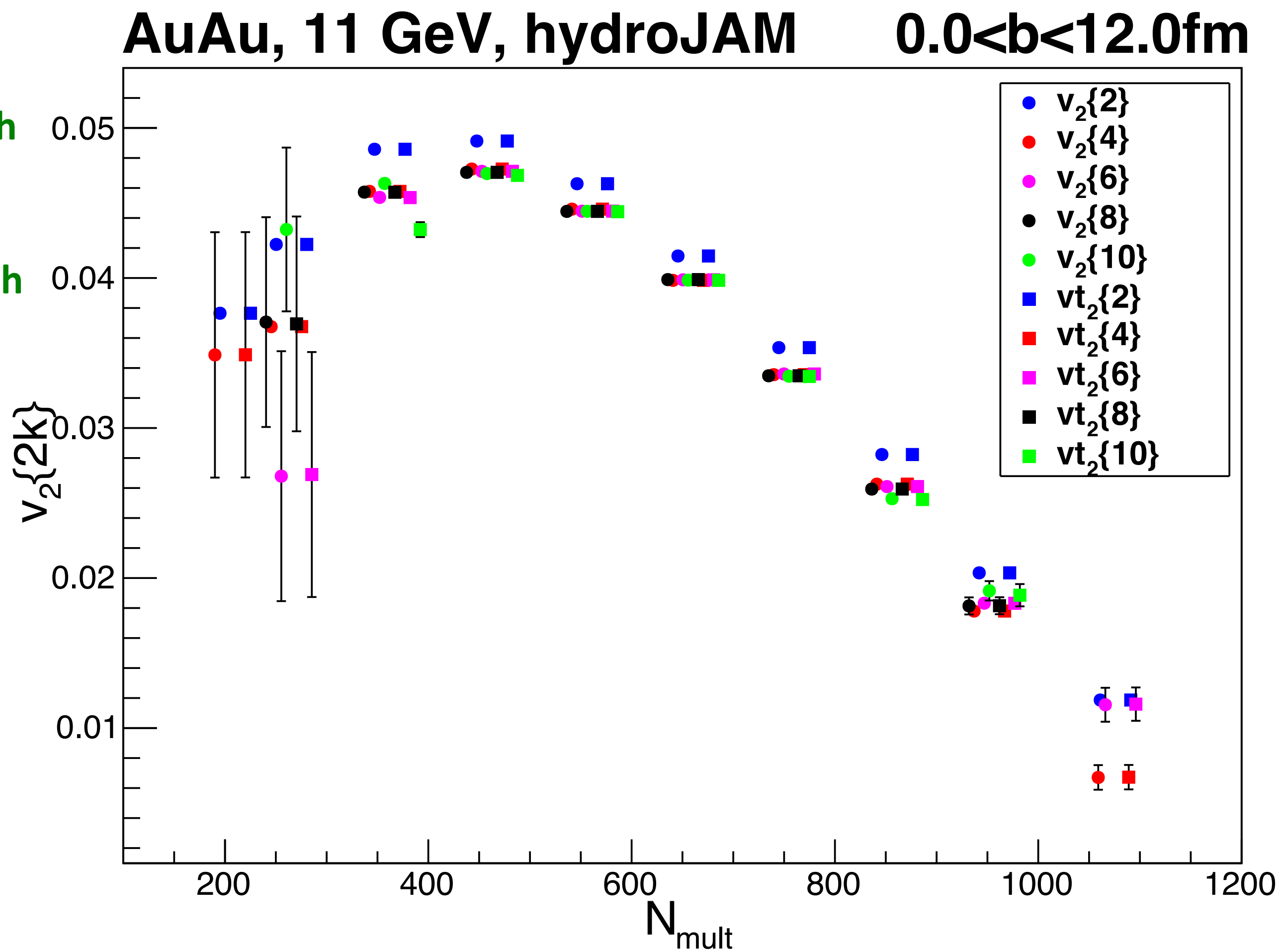
$v_2\{2k\}$ from Q-cumulants - NICA

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c

- 0.0 < b < 12.0 fm
- stat.: 1.068 B events

- $v_2\{2k\}$ are well measured in semicentral collisions
- $v_2\{2k\}$ are not well enough ordered. It could be a problem with JAM itself.
- Codes for calculations with and without efficiency corrections.
- closed circles (squares): results without (with) efficiency corrections (efficiency randomly distributed between 95 and 100%)
- With real data and real efficiencies the two results will differ.



hydro check 1 - NICA

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c

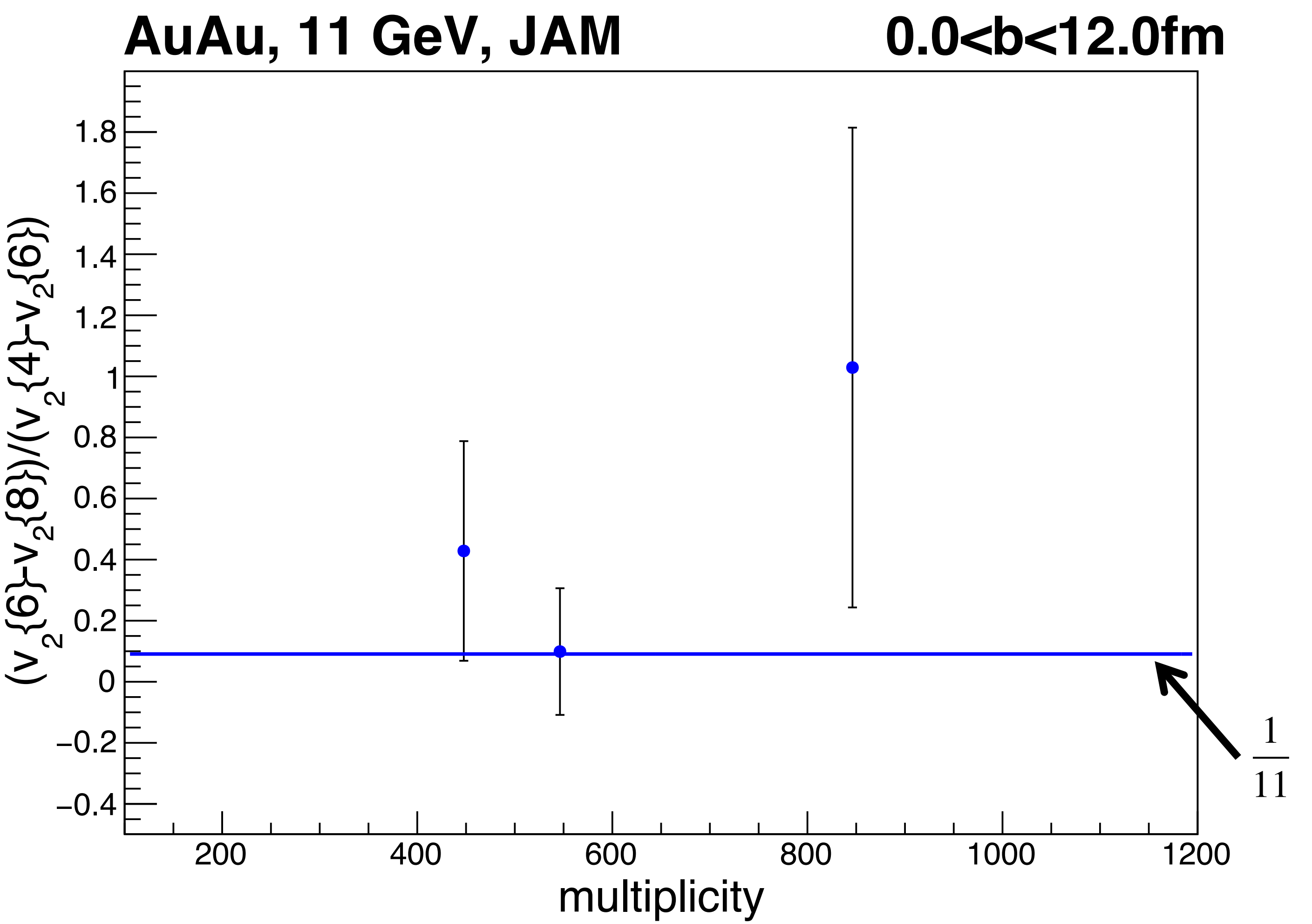
- $0.0 < b < 12.0$ fm
- stat.: 1.068 B events

With condition: $v_2\{4\} > v_2\{6\} > v_2\{8\} > v_2\{10\}$

If centrality independent, this ratio should be 1/11 = 0.090909….

Points are reconstructed by chance. Further increase of stat. will not help

Note: If a proper cumulant ordering exists in the real data, that would significantly decrease the stat. uncertainties



hydro check 2 - NICA

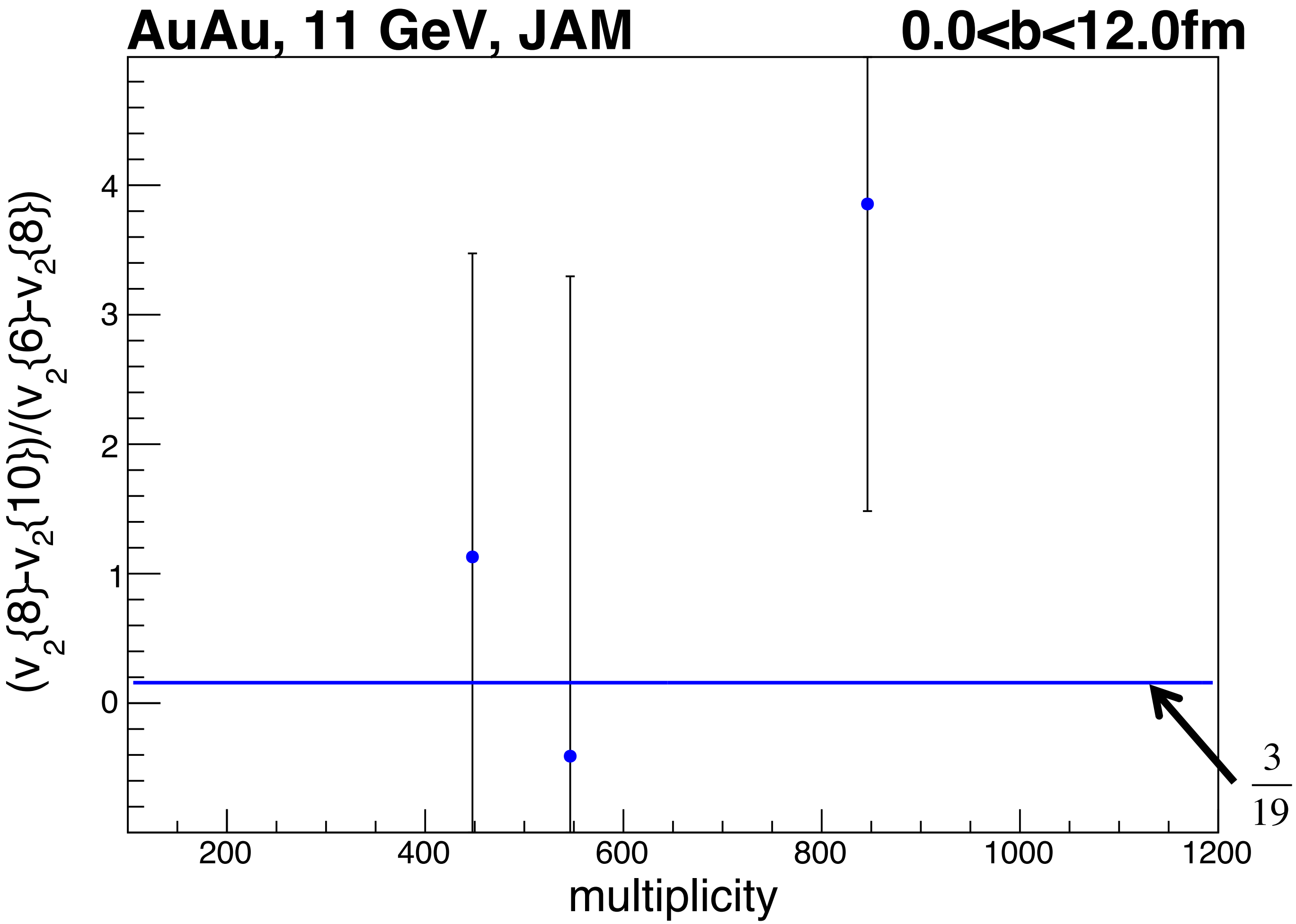
✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c
- 0.0 < b < 12.0 fm
- stat.: 1.068 B events

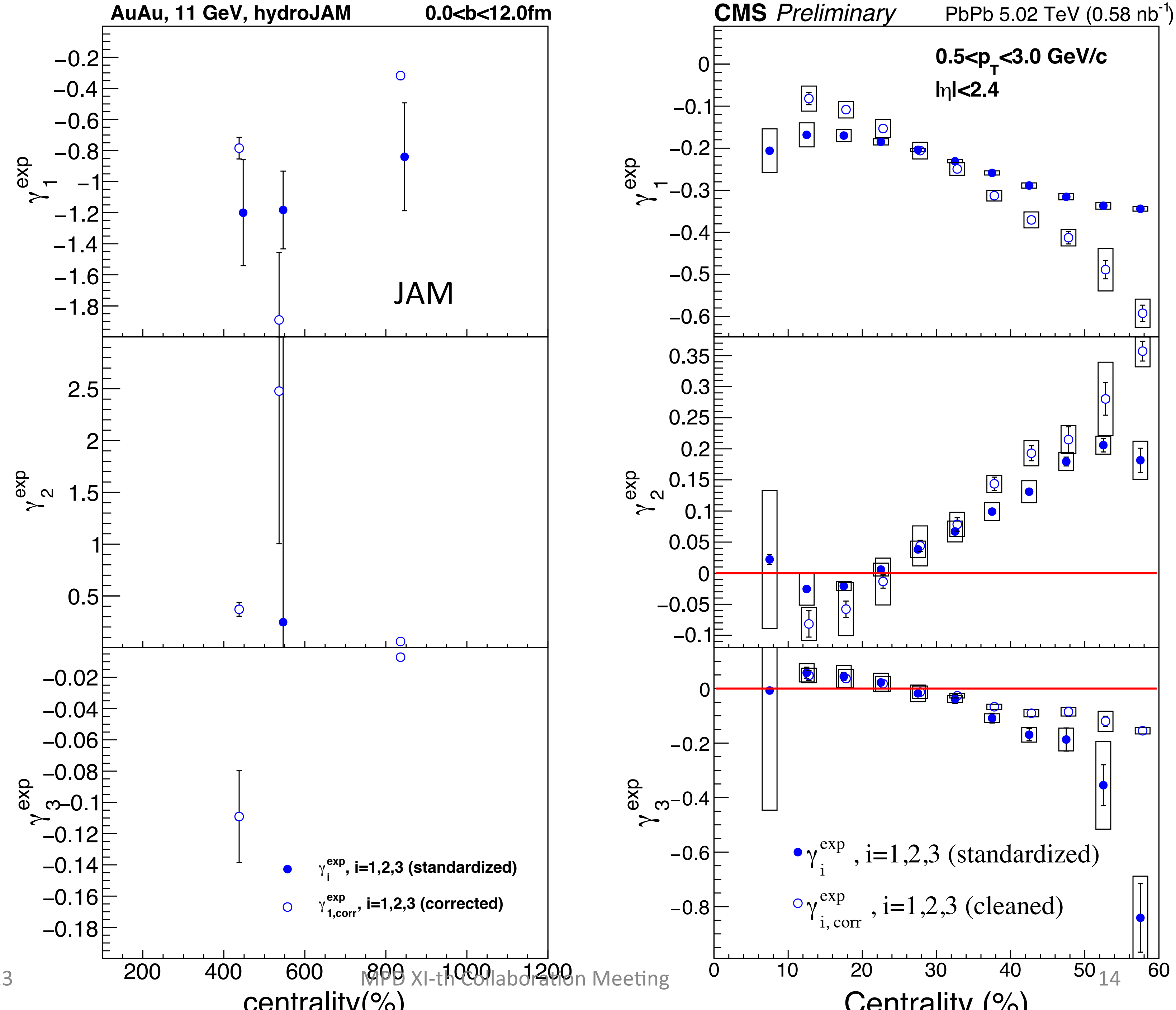
With condition: $v_2\{4\} > v_2\{6\} > v_2\{8\} > v_2\{10\}$

If centrality independent, this ratio should be 3/19 = 0.1578947….

Note: Again, if cumulants would be well ordered than the stat. uncertainties would become smaller



Skewness, kurtosis and superskewness



CMS-PAS-HIN-21-010

NICA MPD case
vHLLE+UrQMD

$v_2\{2k\}$ from Q-cumulants – NICA MPD

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV

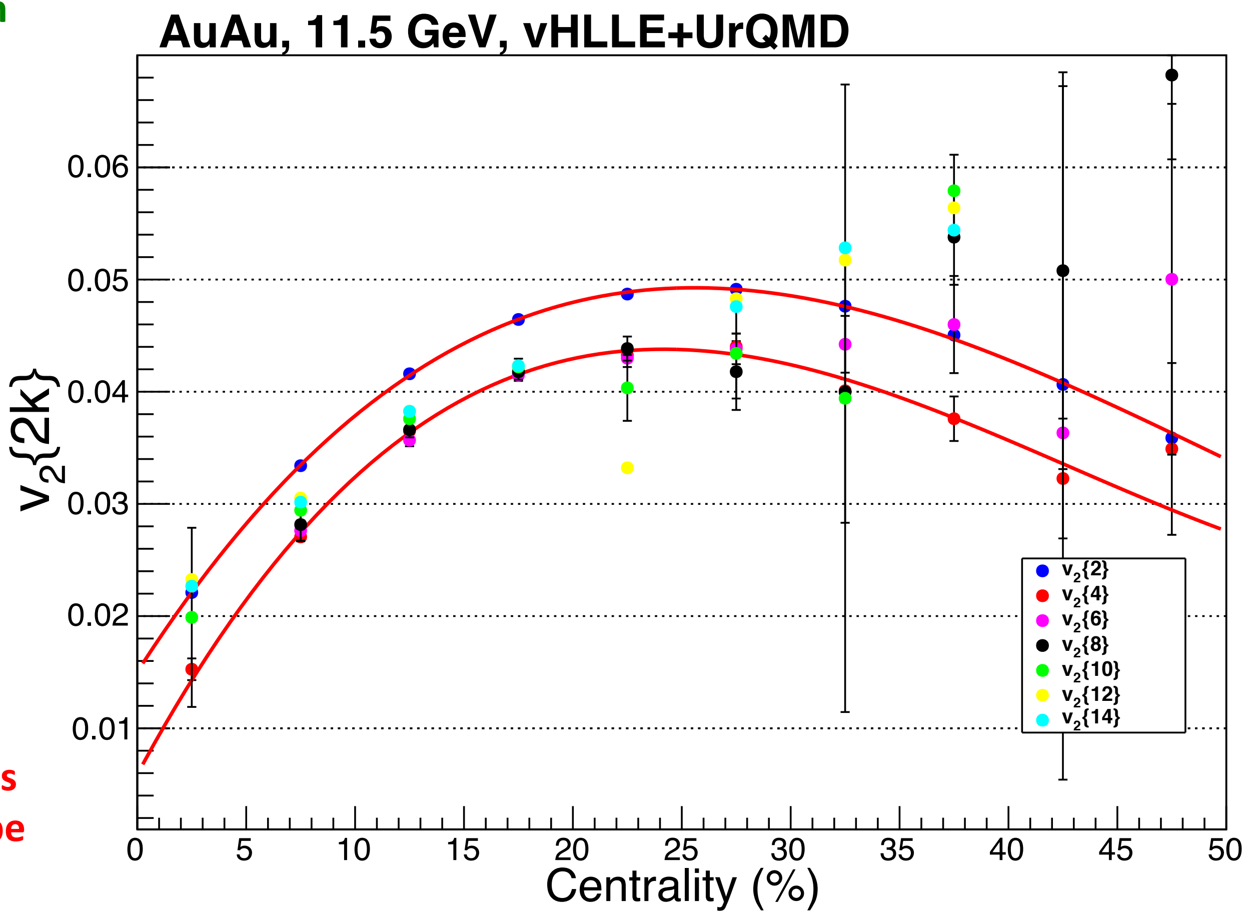
- In 10 centrality classes 0 – 50%
- PID: π^+, π^- , $|\eta| < 1.5$, $0.2 < p_T < 2.0$ GeV/c

- Minimum bias
- stat.: 34 M events

- $v_2\{2k\}$ are well measured in semicentral collisions
- As hydro model, $v_2\{2k\}$ should be well enough ordered
- Statistics is about 60 times smaller wrt CMS data.
- With real data (1 B MB) $v_2\{2k\}$ could be well measured
- With the current statistics, ONLY the condition

$$v_2\{4\} > v_2\{6\}$$

is satisfied ONLY in 10-30% centrality bin. So, hydro probes and central moments cannot be measured



Hydro probes vHLLE

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV

- In 10 centrality classes 0 up to 50%
- PID: π^+, π^- , $|\eta| < 1.5$, $0.2 < p_T < 2.0$ MeV/c

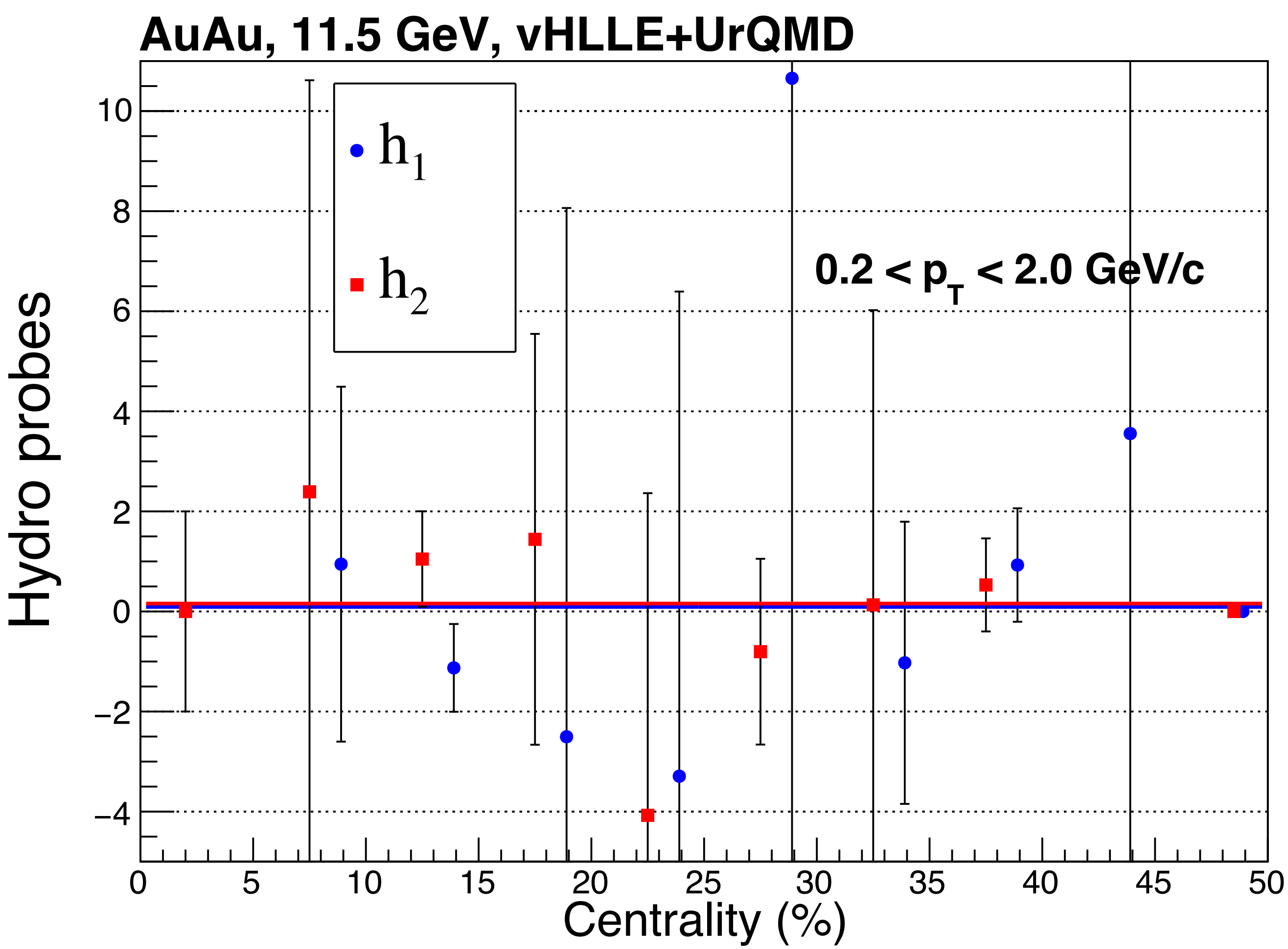
- $0.0 < b < 12.0$ fm
- stat.: 34 M events

Without condition: $v_2\{4\} > v_2\{6\} > v_2\{8\} > v_2\{10\}$

If centrality independent, this ratio should be 1/11 = 0.090909….

Points are reconstructed by chance. Statistics is to small

Note: If a proper cumulant ordering exists in the real data, that would significantly decrease the stat. uncertainties



vHLLE v_2 central moments

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV

- In 10 centrality classes 0 up to 50%
- PID: π^+, π^- , $|\eta| < 1.5$, $0.2 < p_T < 2.0$ MeV/c
- $0.0 < b < 12.0$ fm
- stat.: 34 M events

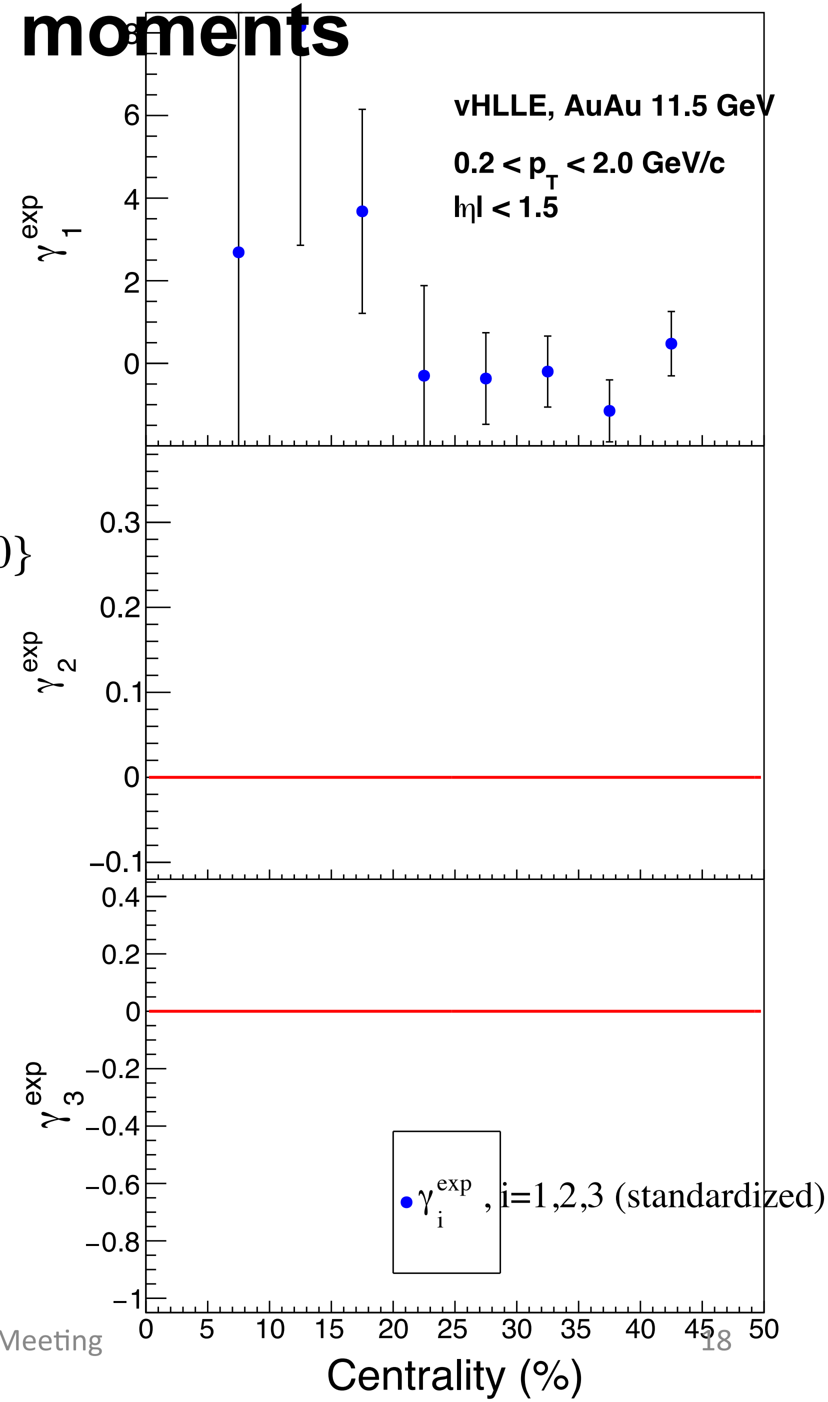
Due to improper ordering, central moments could be reconstructed with a wrong sign

Without condition:

$$v_2\{4\} > v_2\{6\} > v_2\{8\} > v_2\{10\}$$

Points are mainly reconstructed by chance. Statistics is to small

Note: If a proper cumulant ordering exists in the real data, that would significantly decrease the stat. uncertainties



Elliptic power distributions

- In order to increase the statistics at NICA top energy, we used **elliptic power distribution (EPD)**

$$\frac{dN}{d\varepsilon_2} = 2\alpha(1-\varepsilon_0^2)^{\alpha+1/2}\varepsilon_2 \frac{(1-\varepsilon_2^2)^{\alpha-1}}{(1+\varepsilon_0\varepsilon_2)^{2\alpha+1}}\ {}_2F_1\left(\frac{1}{2}, 2\alpha+1; 1; \frac{2\varepsilon_0\varepsilon_2}{1+\varepsilon_0\varepsilon_2}\right)$$

where α and ε_0 are power and ellipticity parameters obtained by `Trento`. The scaling factor κ_2 between the v_2 and initial eccentricity ε_2, $v_2 = \kappa_2\varepsilon_2$, is chosen to imitate the MPD v_2 centrality distribution

- The scaling factor κ_2 is obtained by fitting $v_2\{4\}$ vHLLE with pol3
- Small vHLLE statistics could cause a poor $v_2\{4\}$ reconstruction. As a consequence, this can make a wrong positioning of $v_2\{2\}$ and higher $v_2\{2k\}$, k=3,4,…
- Expectation: NICA MPD will collect about 1B MB events per year. That should be enough to perform precise enough measurements of the hydro probes and central moments

Hydro probes vHLLE

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV

- In 9 centrality classes 5 up to 50%
- PID: π^+, π^- , $|\eta| < 1.5$, $0.2 < p_T < 2.0$ GeV/c

✧ PbPb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV

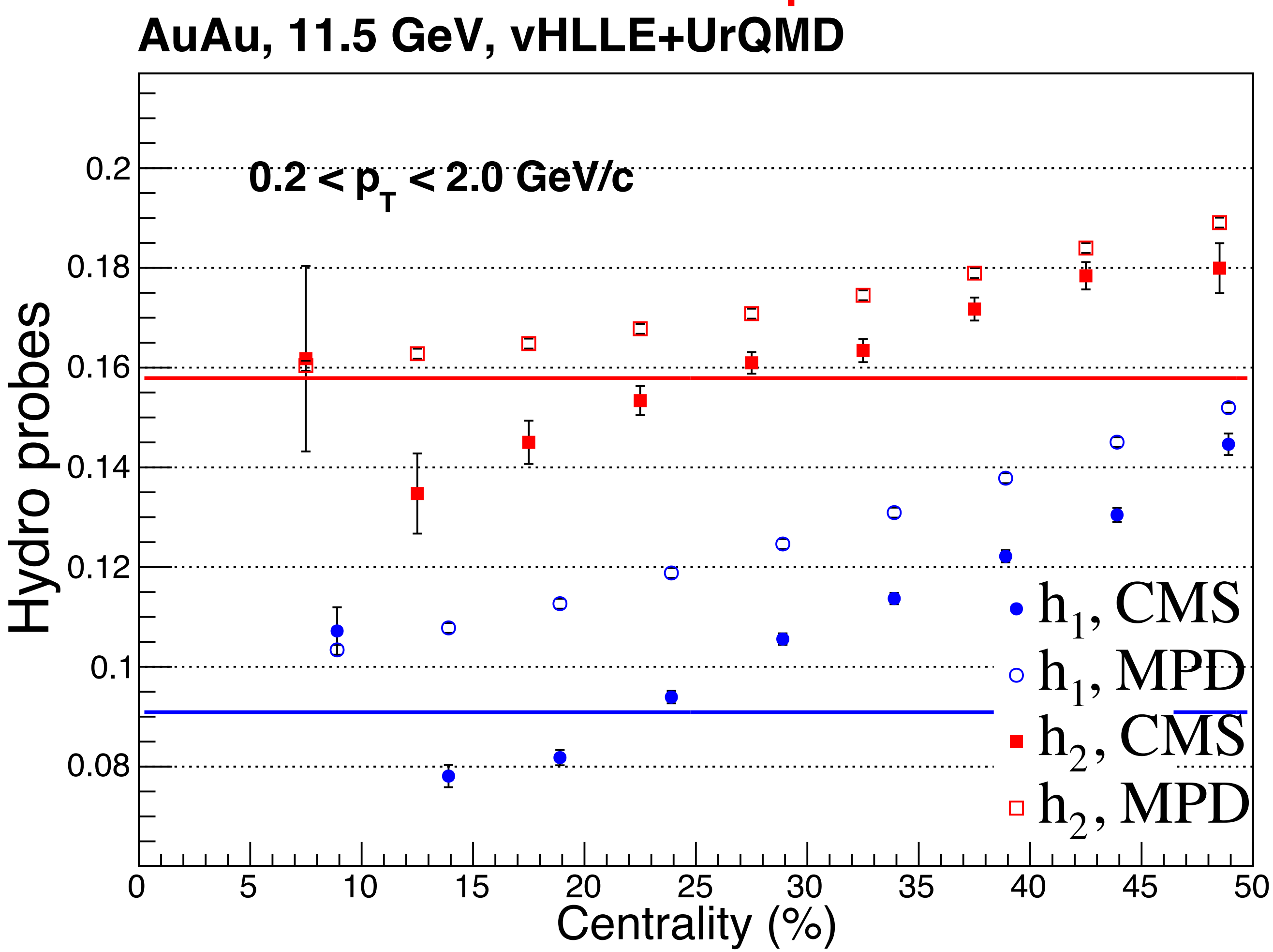
- PID: charged, $|\eta| < 2.4$, $0.5 < p_T < 3.0$ GeV/c

Trento prediction gives values and shape similar to the CMS experimental measurements

It could be the case, if α and ε_0 does not depend on incident energy.

A long time of collision at the NICA energy can wash out it and change initial conditions significantly wrt LHC energies

Trento **prediction:**



Skewness from the EPD

✧ **AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV**

- **In 9 centrality classes 5 up to 50%**
- **PID: π^+, π^-, $|\eta| < 1.5$, $0.2 < p_T < 2.0$ GeV/c**

✧ **PbPb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV**

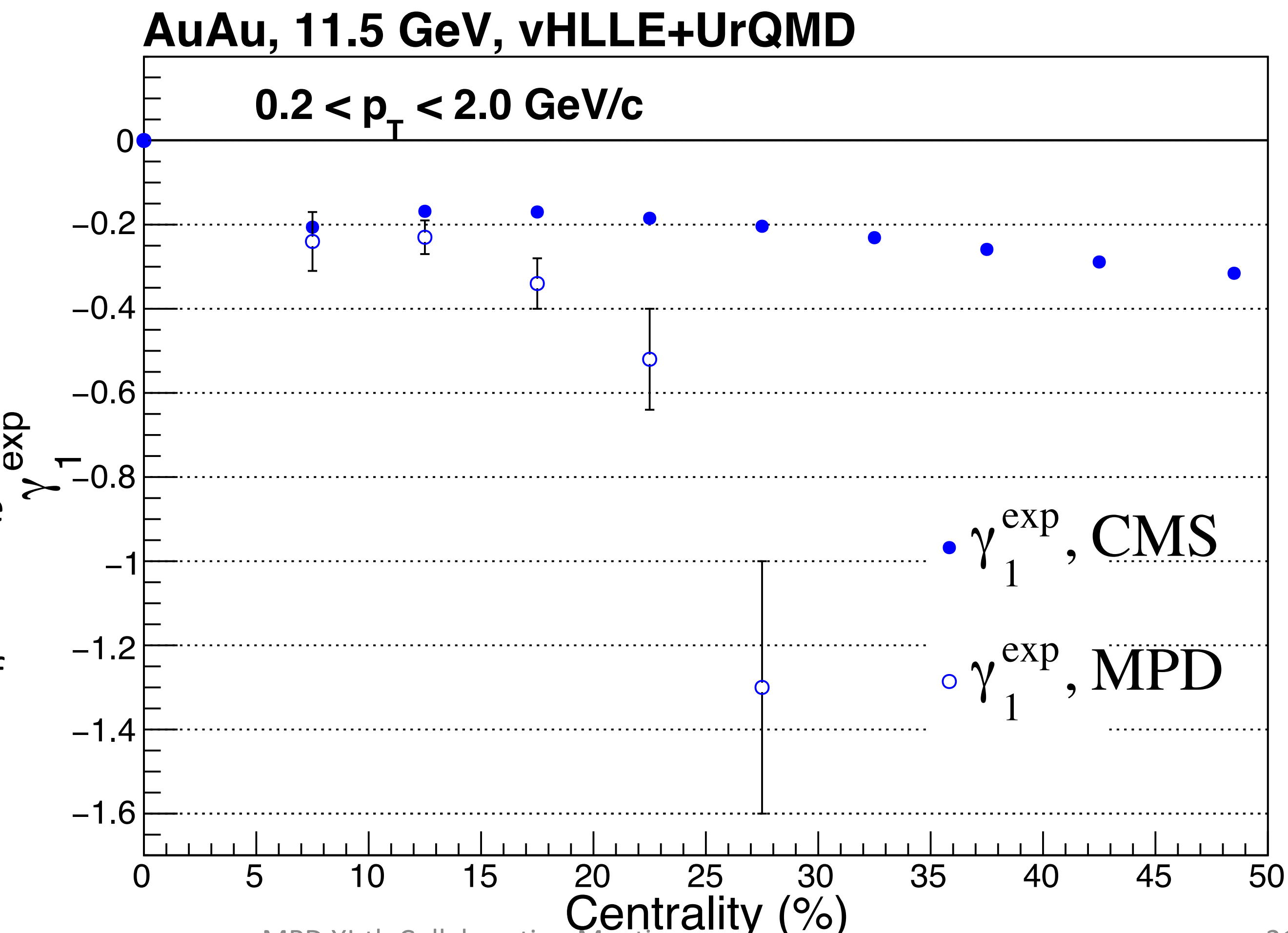
- **PID: charged, $|\eta| < 2.4$, $0.5 < p_T < 3.0$ GeV/c**

Skewness measured from $v_2\{2k\}$ cumulants obtained from the EPD simulations

The event statistics is comparable with the CMS statistics

As we do not know the real values of the EPD parameters, this just shows the feasibility of the measurements of the skewness **only**

EPD **prediction:**



Machine learning & cumulant splitting

- **The most simple case**
- **Fixed v_2=0.02**
- **Fixed and high number of tracks**
- **Training 90k events, test 10k events**

A simple toy model to simulate v_2 only

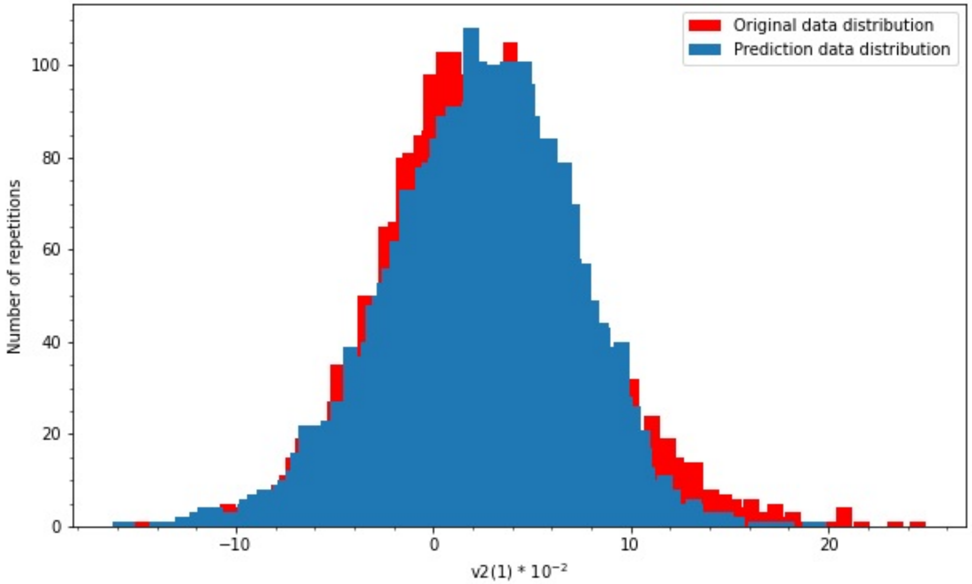
Stat.: 100 k events with 500 tracks

XGBoost Machine Learning model used to reconstruct v_2 (with 1M estimators)

ML learns from v_2=<cos(2ϕ)> values calculated in each event (training)

ML nicely reproduces the mean value and its statistical uncertainty

ML **prediction:**



Machine learning & cumulant splitting

- **In 4 the most central classes 0 up to 20%**
- **Realistic multiplicity dependence (from vHLLE)**
- **PID: π^+, π^- , |η| < 1.5, 0.2 < p_T < 2.0 GeV/c**

ML **prediction:**

Again a simple toy model to simulate v_2 with realistic values

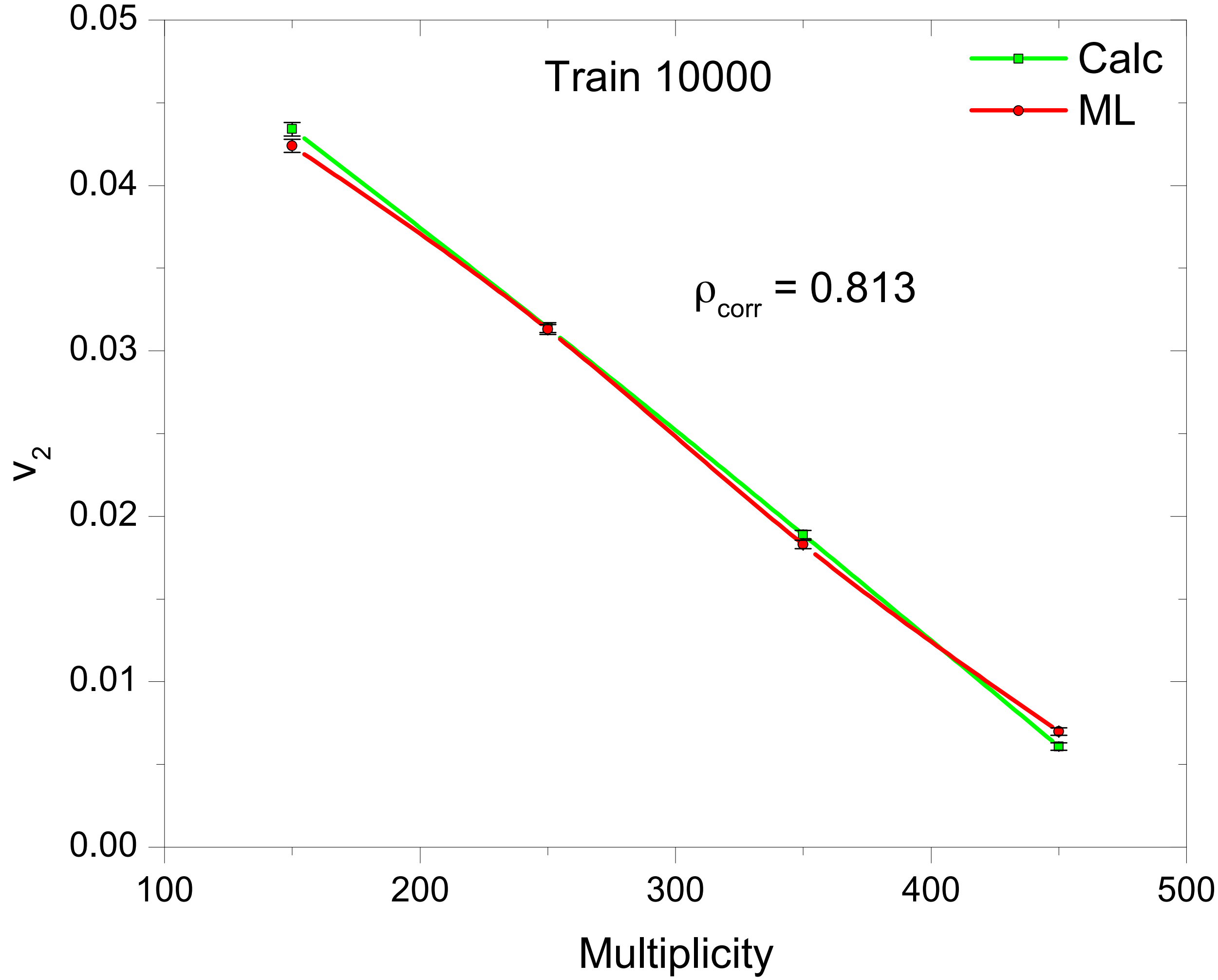
Stat.: 100 k events in each centrality class

XGBoost Machine Learning model used to reconstruct v_2

ML learns from $v_2 = \langle\cos(2\phi)\rangle$ values calculated in each event (training: 10/100k)

ML nicely reproduces the mean value and its statistical uncertainty

High correlation between calculated and ML predicted values



Machine learning & cumulant splitting

- **In 4 the most central classes 0 up to 20%**
- **Realistic multiplicity dependence (from vHLLE)**
- **PID: π^+, π^- , $|\eta| < 1.5$, $0.2 < p_T < 2.0$ GeV/c**

ML **prediction:**

Again a simple toy model to simulate v_2 with realistic values

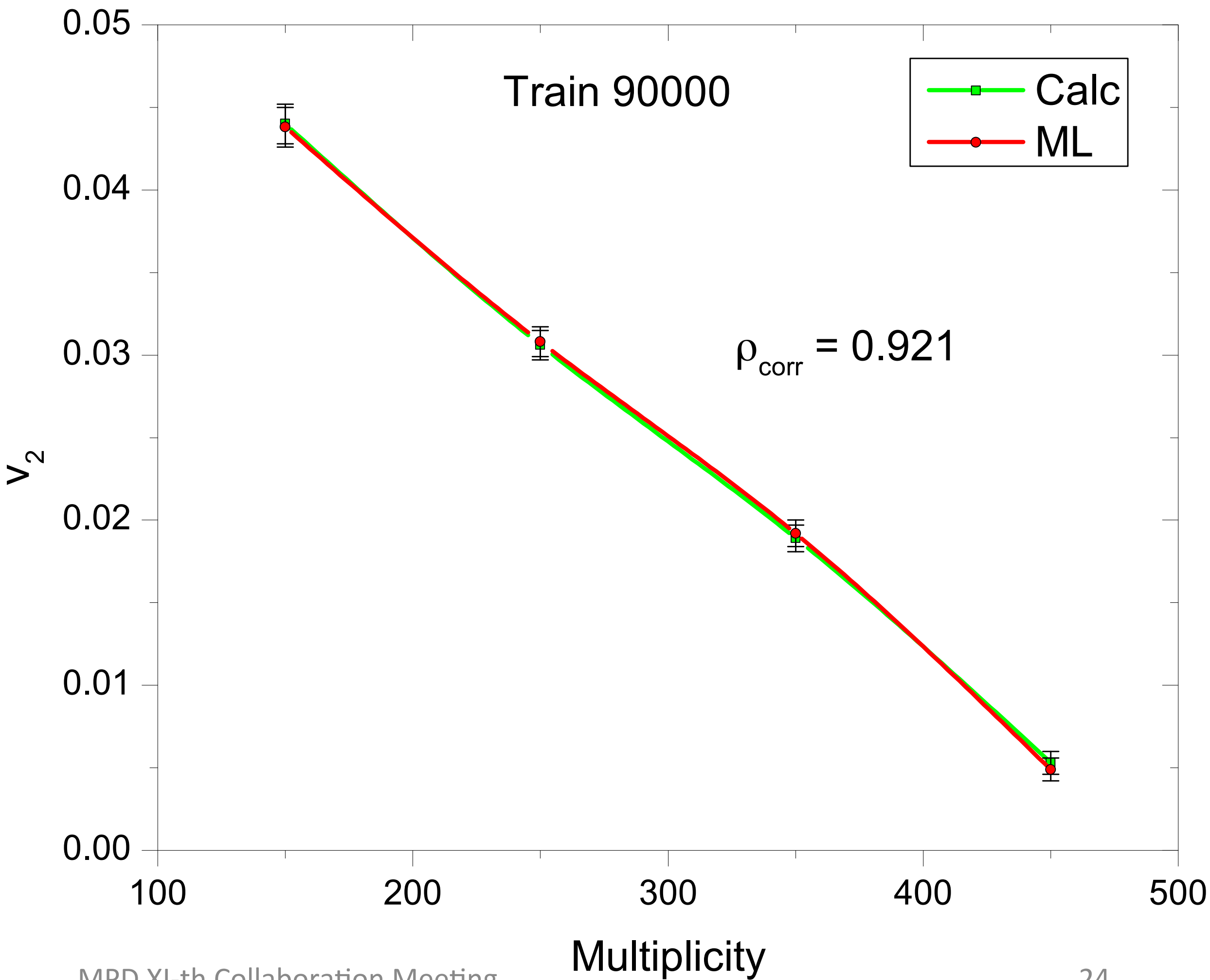
Stat.: 100 k events in each centrality class

XGBoost Machine Learning model used to reconstruct v_2

ML learns from $v_2 = \langle \cos(2\phi) \rangle$ values calculated in each event (training: 90/100k)

ML nicely reproduces the mean value and its stat. uncertainty

Higher correlation between calculated and ML predicted values with a larger number of the training events



Conclusions

- At NICA energy huge statistics is needed to perform measurements of the hydrodynamics probes and central moments
- We ran JAM and vHLLE+UrQMD model
- JAM analysis performed with a high statistics, but seems there is no splitting between cumulants
- vHLLE+UrQMD should have splitting, but the current statistics is too small
- We used a toy EPD model with parameters obtained from `Trento` and vHLLE data to increase the statistics
- We started to use ML in order to see is it able to recognize the cumulant splitting, and to properly measure hydro probes and central v_2 moments
- Expectation: NICA MPD will collect about 1B MB events per year. That should be enough to perform precise enough measurements of the hydro probes and central moments

Backup

Hydrodynamics probes
and
central moments
LHC CMS case

v_2 from Q-cumulants at LHC energy

- Flow fluctuations, σ_v, -> a gap between $v_2\{2\}$ and higher-order cumulants based $v_2\{2k\}$: $v_2\{2\}^2 = v_2\{2k\}^2 + 2\sigma_v^2$, for (k>1)

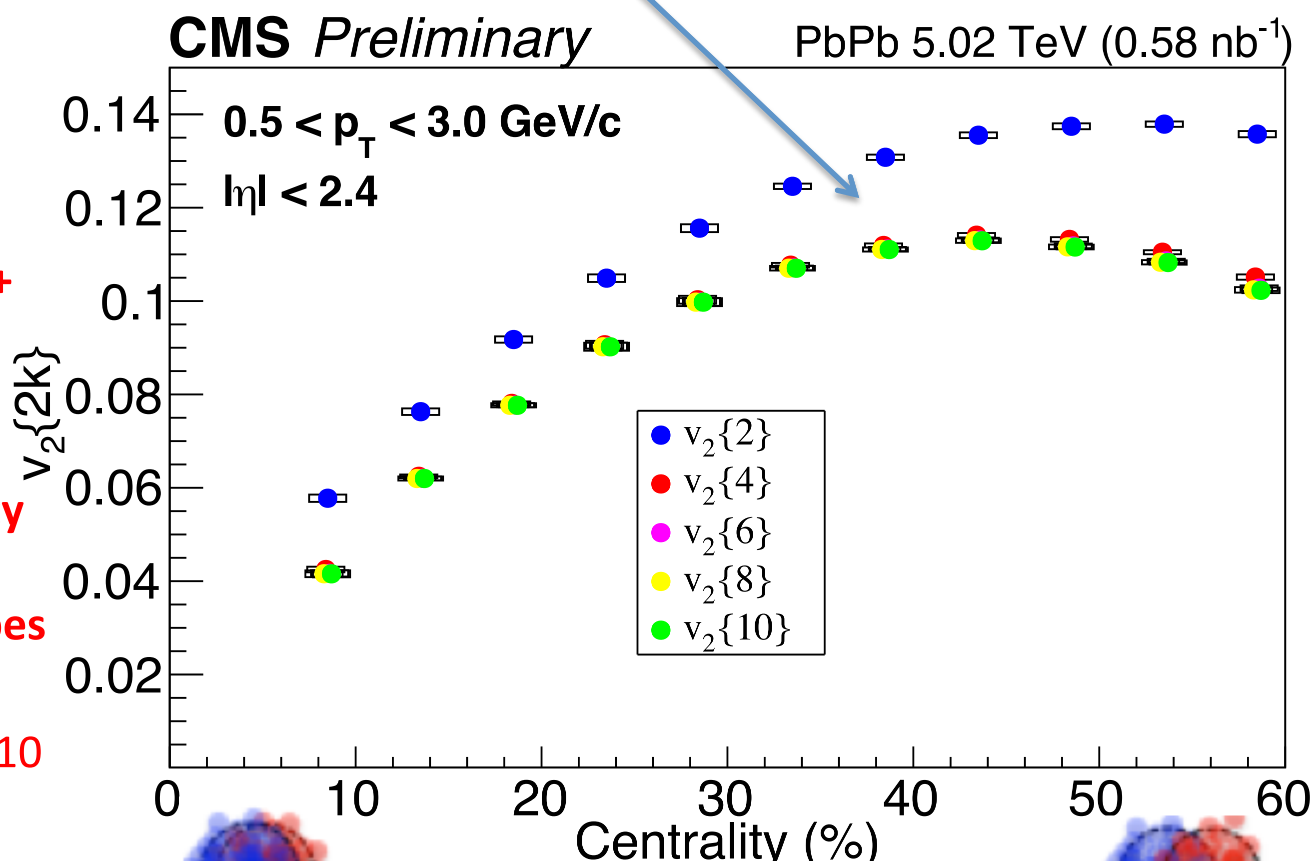
fine splitting

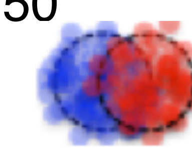
$$v_2\{4\} >\approx v_2\{6\} >\approx v_2\{8\} >\approx v_2\{10\}$$

- Syst. uncertainties ~ 2 orders of magnitude greater wrt stat. ones

Dominant source syst. uncertainties: variation on criteria for tracks

- Large v_2 magnitude + large multiplicity ➔ high Q-cumulant feasibility + small statistical uncertainty
- Allows to perform hydrodynamics probes





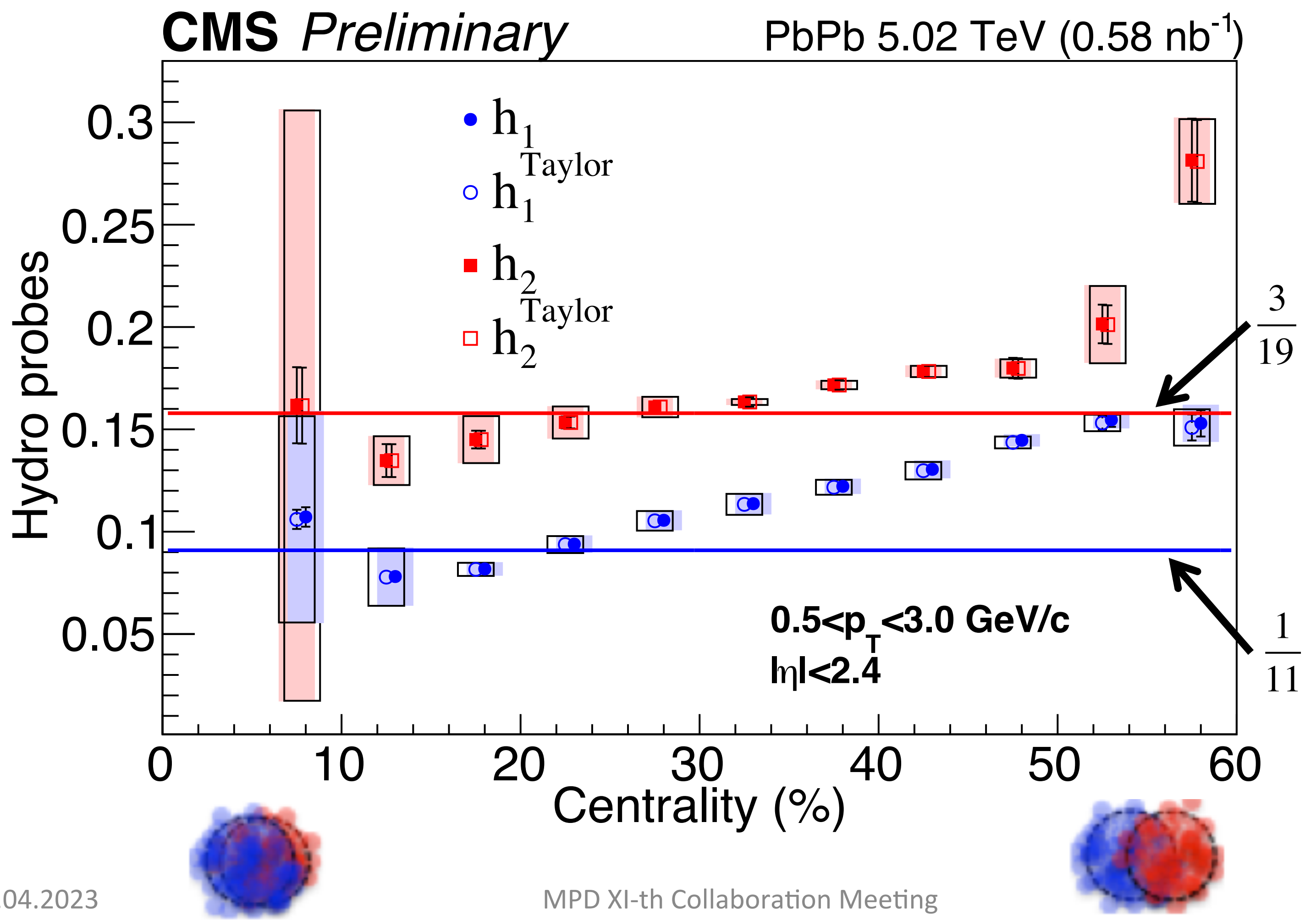
CMS-PAS-HIN-21-010

Quark Matter 2022

Hydrodynamic probes

$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx h_1^{Taylor} = \frac{1}{11} - \frac{1}{11}\frac{v_2^2\{4\} - 12v_2^2\{6\} + 11v_2^2\{8\}}{v_2\{4\}^2 - v_2\{6\}^2}$$

$$h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx h_2^{Taylor} = \frac{3}{19} - \frac{1}{19}\frac{3v_2^2\{6\} - 22v_2^2\{8\} + 19v_2^2\{10\}}{v_2\{6\}^2 - v_2\{8\}^2}$$



CMS-PAS-HIN-21-010

Higher-order moments necessary to describe data

Standardized & Corrected moments

PRC 99 (2019) 014907

$$\gamma_1^{\exp} = -2^{3/2} \frac{v_2\{4\}^3 - v_2\{6\}^3}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^{3/2}} \approx -2^{3/2} \frac{-s_{30} - O_N}{\left[2\sigma_x^2 + O_D\right]^{3/2}} \approx \frac{s_{30}}{\sigma_x^3} \equiv \gamma_1$$

$$\gamma_2^{\exp} = -\frac{3}{2} \frac{v_2\{4\}^4 - 12 v_2\{6\}^4 + 11 v_2\{8\}^4}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} \equiv \gamma_2$$

Kurtosis

$$\gamma_3^{\exp} = 6\sqrt{2} \frac{3 v_2\{6\}^5 - 22 v_2\{8\}^5 + 19 v_2\{10\}^5}{\left[v_2\{2\}^2 - v_2\{4\}^2\right]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} \equiv \gamma_3$$

Superskewness

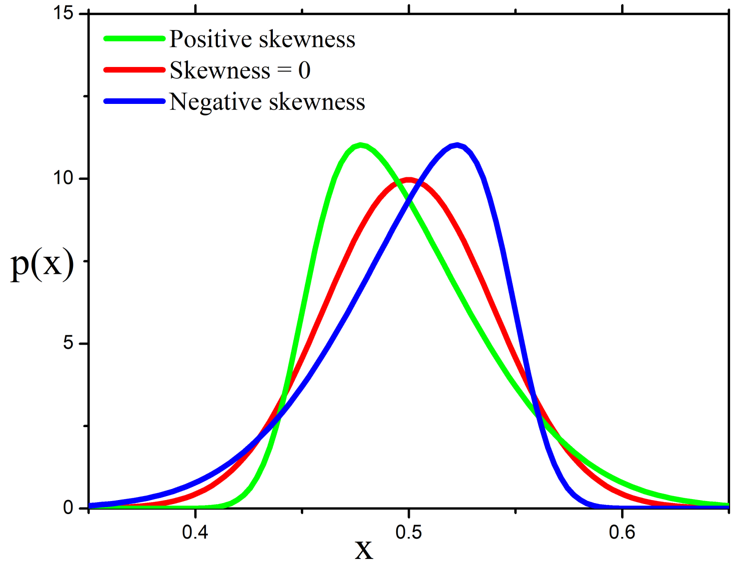
Conditions: $s_{12} \approx \frac{s_{30}}{3}$ $\kappa_{22} \approx \frac{\kappa_{40}}{3}$ $p_{32} \approx p_{14} \approx \frac{p_{50}}{5}$ Ell. pow. distr. param. $\varepsilon_0 \leq 0.15$

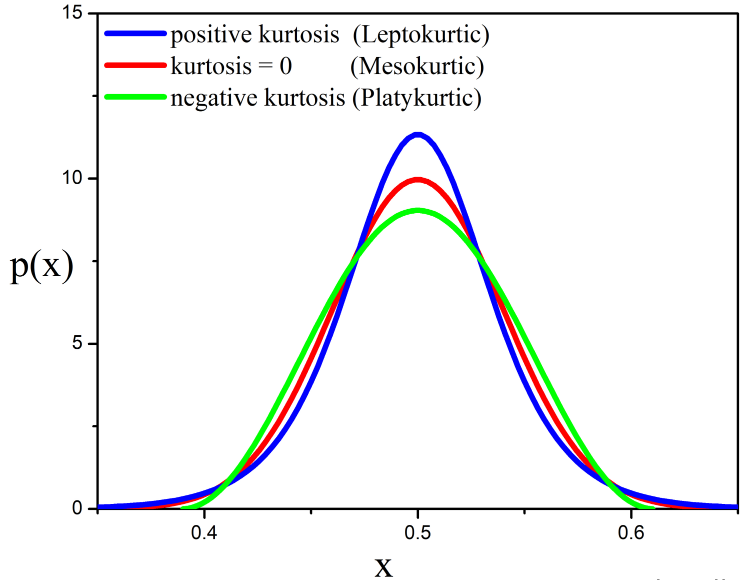
$$\gamma_{1,corr}^{\exp} = -2^{3/2} \frac{187 v_2\{8\}^3 - 16 v_2\{6\}^3 - 171 v_2\{10\}^3}{\left[v_2\{2\}^2 - 40 v_2\{6\}^2 + 495 v_2\{8\}^2 - 456 v_2\{10\}^2\right]^{3/2}}$$

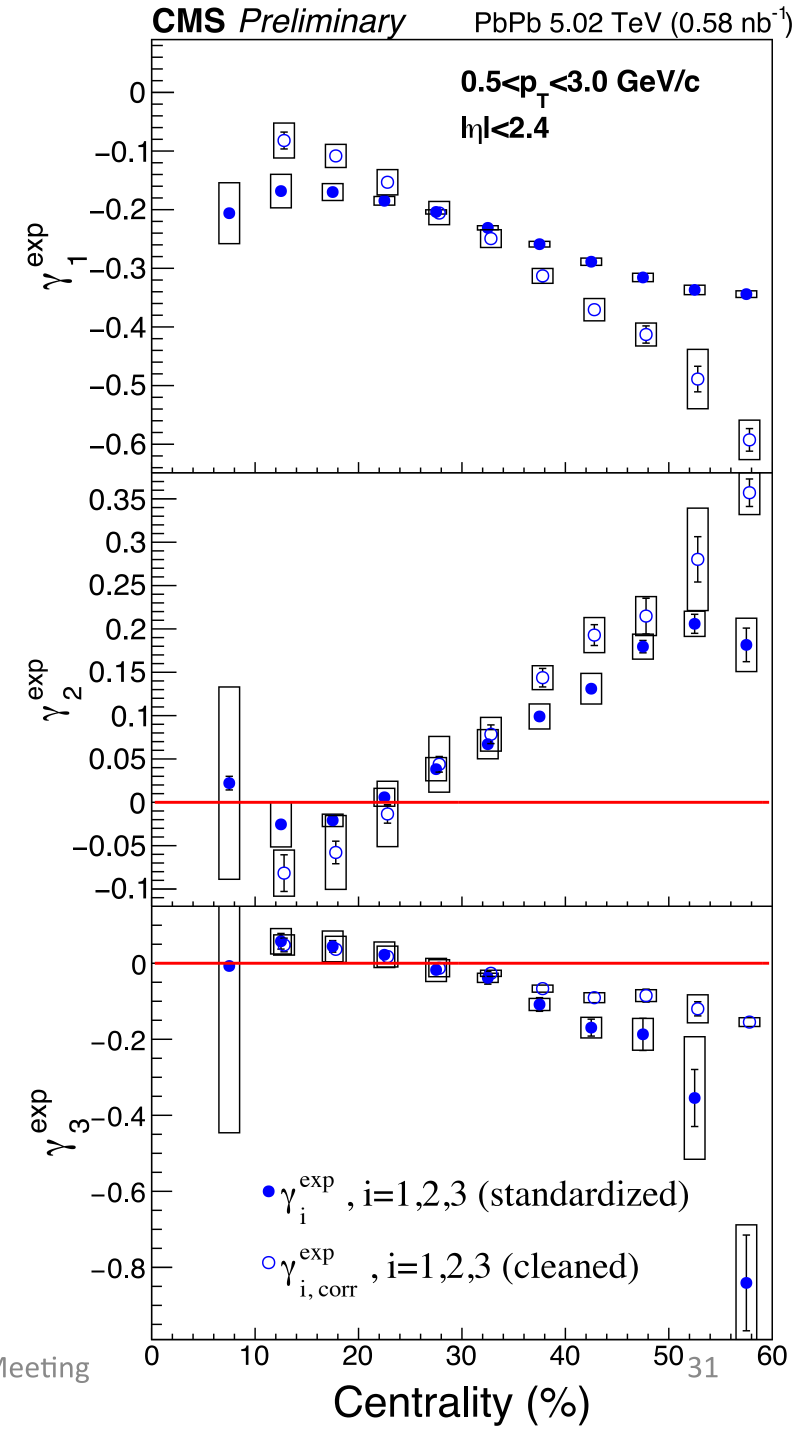
Skewness, kurtosis and superskewness

Important for the initial-stats ↑

Important for a proper hydro description ↑







CMS-PAS-HIN-21-010

$v_2\{2k\}$ from Q-cumulants - NICA

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.5 GeV

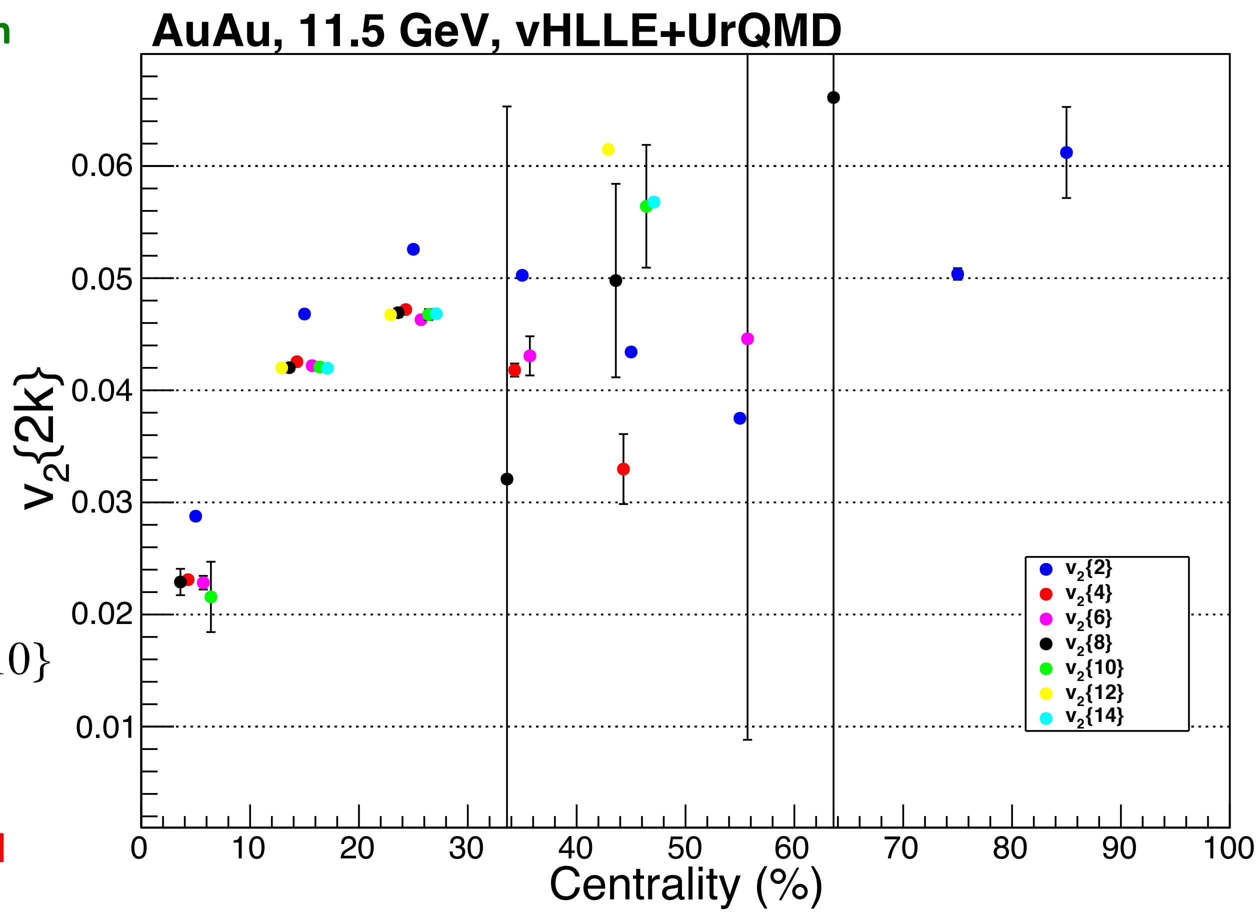
- Minimum bias
- stat.: 34 M events

- In 10 centrality classes
- PID: p, π^+, π^-, p_T > 300 MeV/c

- $v_2\{2k\}$ are well measured in semicentral collisions
- As hydro model, $v_2\{2k\}$ should be well enough ordered.
- Statistics is about 60 times smaller wrt CMS data.
- With real data (1 B MB) $v_2\{2k\}$ could be well measured
- With the current statistics, the condition

$$v_2\{4\} > v_2\{6\} > v_2\{8\} > v_2\{10\}$$

is not satisfied in any bin. So, hydro probes and central moments cannot be measured



$v_n\{10\}$ from Q-cumulants

10-th order Q-cumulant $\langle\langle 10\rangle\rangle = \langle\langle e^{in(\phi_1+\phi_2+\phi_3+\phi_4+\phi_5-\phi_6-\phi_7-\phi_8-\phi_9-\phi_{10})}\rangle\rangle$

$$c_n\{10\} = \langle\langle 10\rangle\rangle - 25\cdot\langle\langle 2\rangle\rangle\langle\langle 8\rangle\rangle - 100\cdot\langle\langle 4\rangle\rangle\langle\langle 6\rangle\rangle$$
$$+400\cdot\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle^2 + 900\cdot\langle\langle 2\rangle\rangle\langle\langle 4\rangle\rangle^2$$
$$-360\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^3 + 2880\cdot\langle\langle 2\rangle\rangle^5$$

- **For the first time $v_n\{10\}$** $\quad v_n\{10\} = \sqrt[10]{\frac{1}{456}c_n\{10\}}$

Statistical uncertainties of the $v_n\{2k\}$ (k=1,…,5) cumulants are calculated analytically using the data [Phys. Rev. C 104 (2021) 034906 arXiv:2104.00588 [nucl-th]]

$$s^2[v_n\{10\}]\cdot 4560^2(v_n\{10\})^{18} = A^2\sigma^2_{\langle\langle 2\rangle\rangle} + B^2\sigma^2_{\langle\langle 4\rangle\rangle}$$
$$+C^2\sigma^2_{\langle\langle 6\rangle\rangle} + D^2\sigma^2_{\langle\langle 8\rangle\rangle} + \sigma^2_{\langle\langle 10\rangle\rangle} + 2AB\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 4\rangle\rangle}$$
$$+2AC\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 6\rangle\rangle} + 2AD\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 8\rangle\rangle} + 2A\sigma_{\langle\langle 2\rangle\rangle,\langle\langle 10\rangle\rangle}$$
$$+2BC\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 6\rangle\rangle} + 2BD\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 8\rangle\rangle} + 2B\sigma_{\langle\langle 4\rangle\rangle,\langle\langle 10\rangle\rangle}$$
$$+2CD\sigma_{\langle\langle 6\rangle\rangle,\langle\langle 8\rangle\rangle} + 2C\sigma_{\langle\langle 6\rangle\rangle,\langle\langle 10\rangle\rangle} + 2D\sigma_{\langle\langle 8\rangle\rangle,\langle\langle 10\rangle\rangle}$$

$$A = 14400\langle\langle 2\rangle\rangle^4 - 10800\langle\langle 2\rangle\rangle^2\langle\langle 4\rangle\rangle$$
$$+800\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle + 900\langle\langle 4\rangle\rangle^2 - 25\langle\langle 8\rangle\rangle$$
$$B = 1800\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle - 3600\langle\langle 2\rangle\rangle^3 - 100\langle\langle 6\rangle\rangle$$
$$C = 400\langle\langle 2\rangle\rangle^2 - 100\langle\langle 4\rangle\rangle$$
$$D = -25\langle\langle 2\rangle\rangle$$

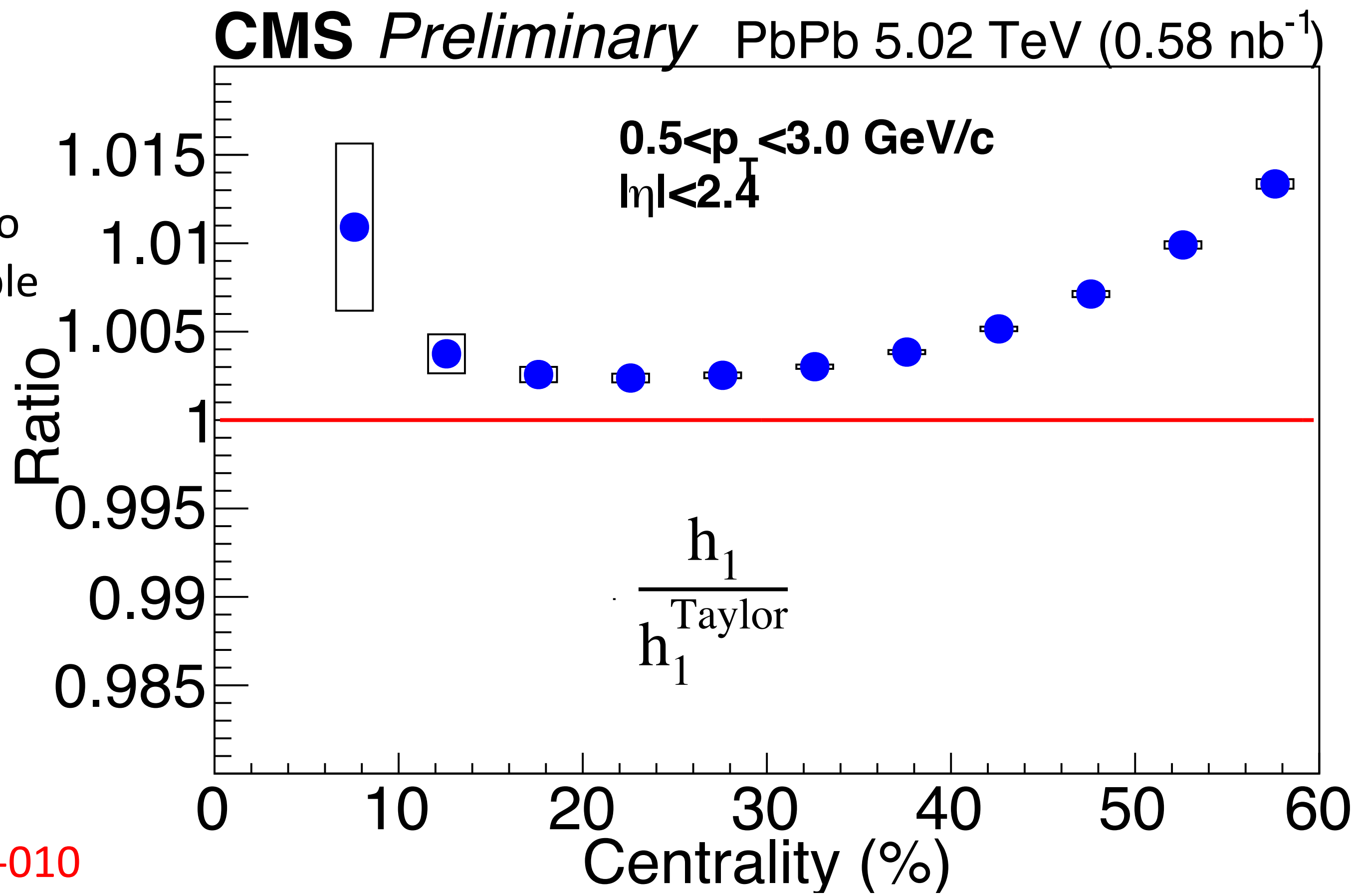
Ratio between probe and its Taylor expansion

$$\frac{h_1}{h_1^{Taylor}} \approx \frac{\dfrac{v_2\{6\}-v_2\{8\}}{v_2\{4\}-v_2\{6\}}}{\dfrac{1}{11}-\dfrac{1}{11}\dfrac{v_2^2\{4\}-12v_2^2\{6\}+11v_2^2\{8\}}{v_2\{4\}^2-v_2\{6\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated

-Syst. uncertainties dominates

-Term proportional to $(\sigma_y^2-\sigma_x^2)$ is negligible in accordance with *PRC* ***95*** *(2017) 014913*



CMS-PAS-HIN-21-010

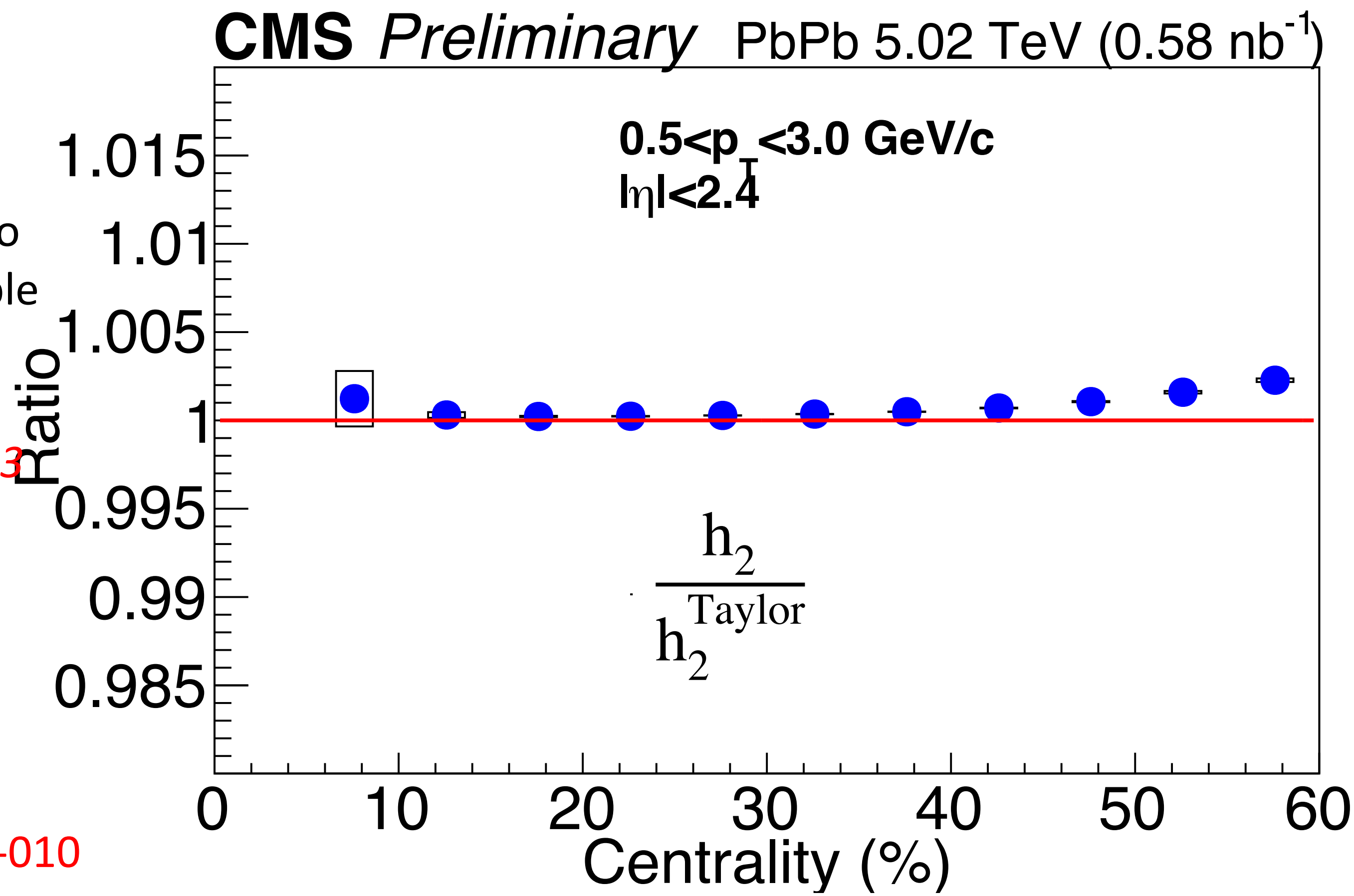
Ratio between the new probe and its Taylor expansion

$$\frac{h_2}{h_2^{Taylor}} \approx \frac{\dfrac{v_2\{8\}-v_2\{10\}}{v_2\{6\}-v_2\{8\}}}{\dfrac{3}{19}-\dfrac{1}{19}\dfrac{3v_2^2\{6\}-22v_2^2\{8\}+19v_2^2\{10\}}{v_2\{6\}^2-v_2\{8\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated

-Syst. uncertainties dominates

-Term proportional to $(\sigma_y^2-\sigma_x^2)$ is negligible in accordance with *PRC* ***95*** *(2017) 014913*

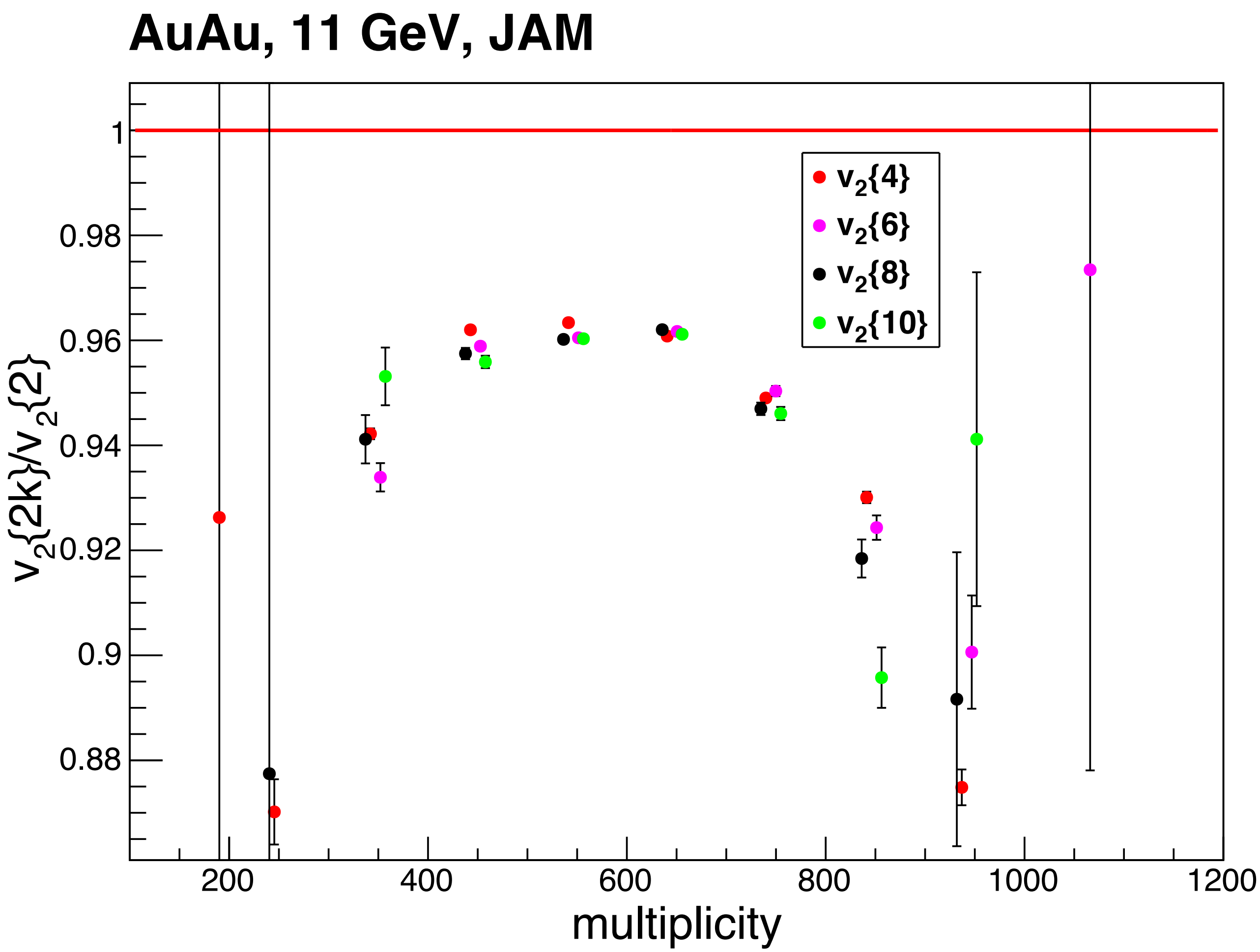


CMS-PAS-HIN-21-010

$v_2\{2k\}/v_2\{2\}$

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV
- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c
- $0.0 < b < 12.0$ fm, stat.: 1.068 B events

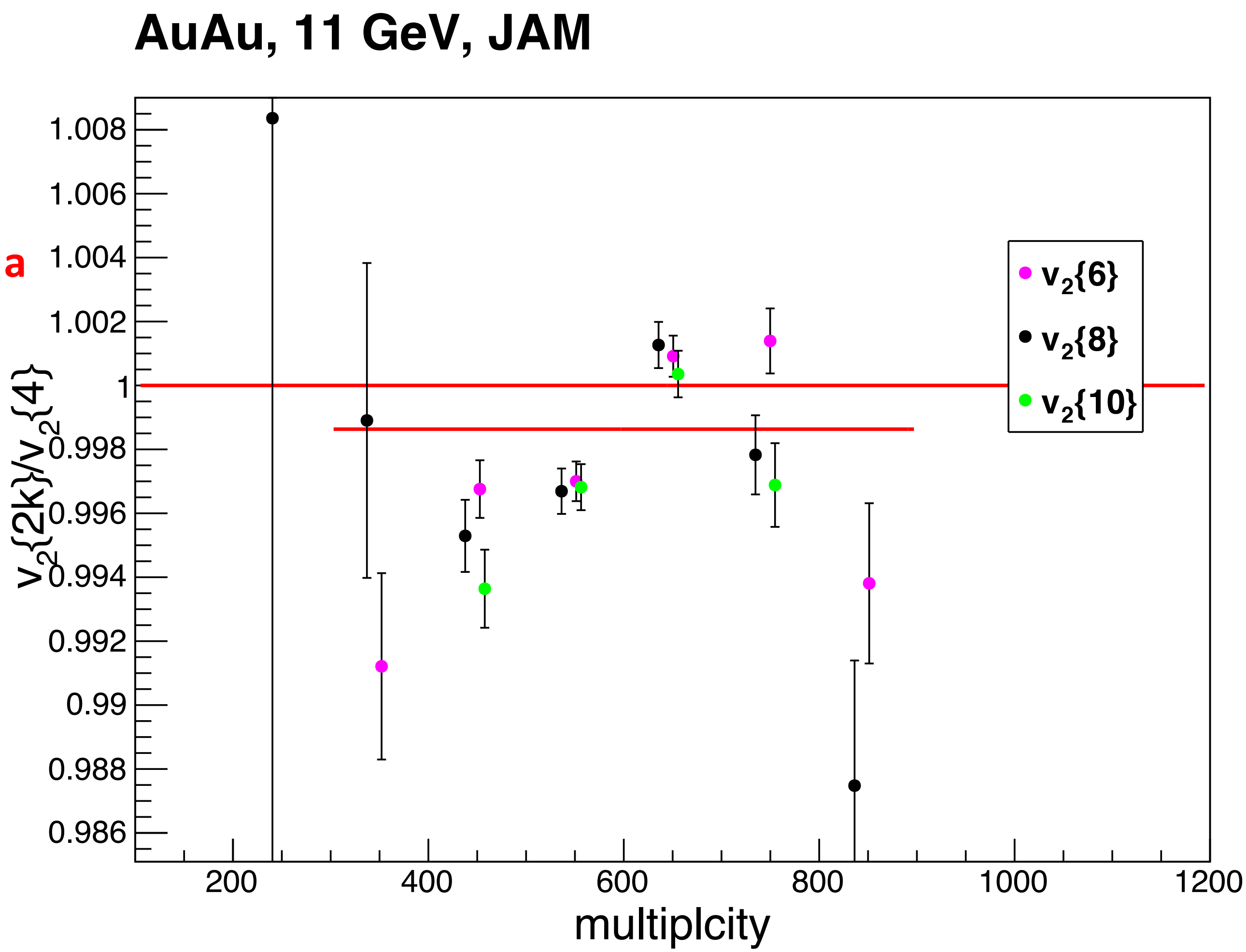
- A clear separation between $v_2\{2\}$ and $v_2\{2k\}$ (k>1) is seen
- It is purely due to the flow fluctuations: $v_2^2\{2\} = v_2^2\{2k\} + 2\sigma_v^2$ for (k>1)
- It seems that higher order cumulants $v_2\{2k\}$ are rarely well ordered
- One needs a good ordering in order to be able to perform hydro checks



$v_2\{2k\}/v_2\{4\}$

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c
- $0.0 < b < 12.0$ fm, stat.: 1.068 B events



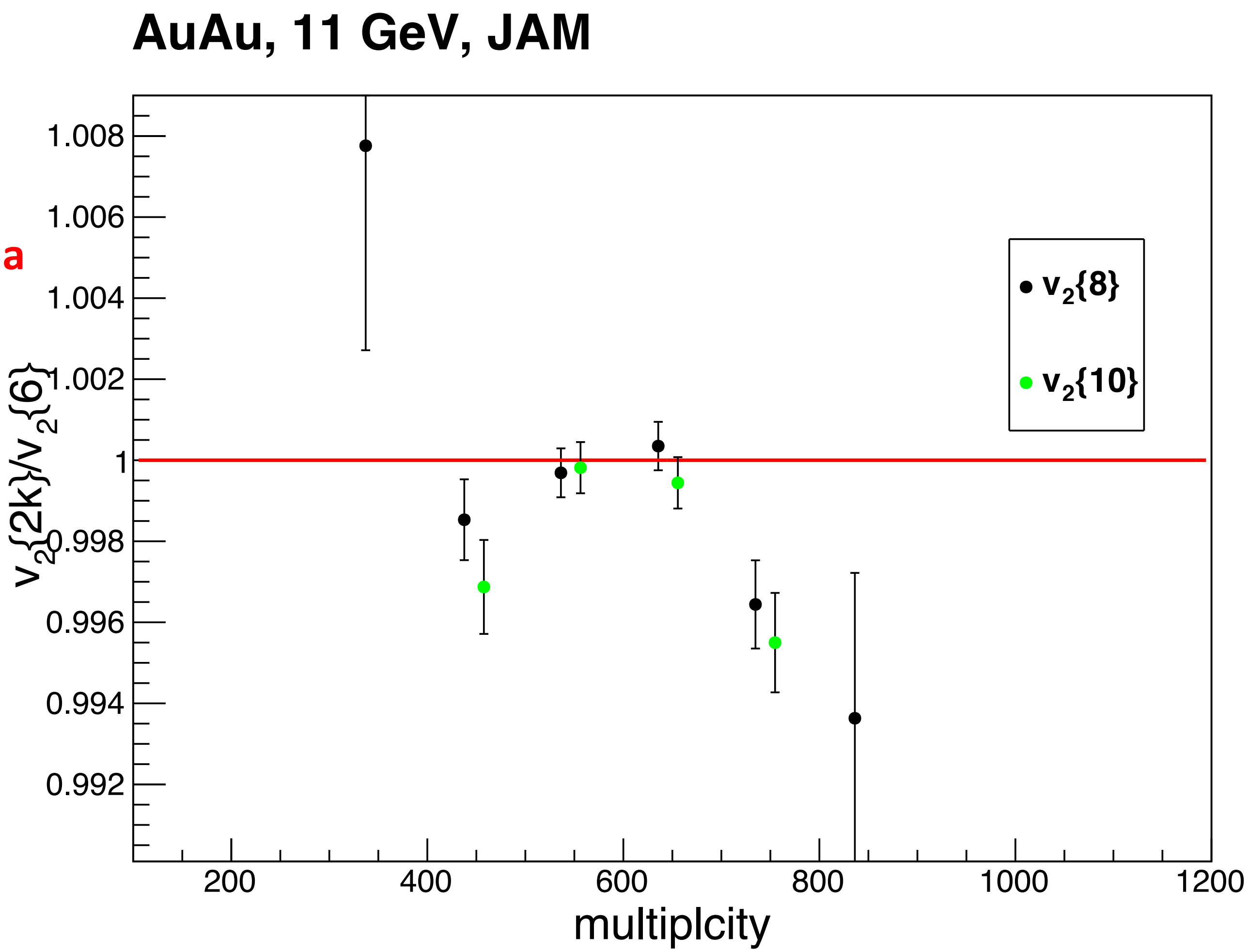
- Here, again, we see bins where there is no a proper ordering between higher order cumulants: $v_2\{4\} >\approx v_2\{6\} >\approx v_2\{8\} >\approx v_2\{10\}$

$v_2\{2k\}/v_2\{6\}$

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c
- $0.0 < b < 12.0$ fm, stat.: 1.068 B events

- Here, again, we see bins where there is no a proper ordering between higher order cumulants: $v_2\{6\} >\approx v_2\{8\} >\approx v_2\{10\}$



$v_2\{2k\}/v_2\{8\}$

✧ AuAu collisions at $\sqrt{s_{NN}}$ = 11.0 GeV

- In 10 multiplicity classes from 100 up to 1200
- PID: p, π^+, π^- , $|\eta| < 1.5$, $p_T > 200$ MeV/c
- $0.0 < b < 12.0$ fm, stat.: 1.068 B events

- Splitting for the range 400-800 is still visible: $v_2\{8\} \gtrsim v_2\{10\}$
- It seems that the splitting from JAM becomes more and more fine with an increase of k, which is correct.

Anyhow, we could expect splitting in the experimental MPD NICA data