XI MPD Collaboration Meeting

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In-Medium Modifications of Meson States and Higher-Order Cumulants

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Outline

- ***QCD-like effective model, Polyakov linear-sigma model:***
  - ***A novel estimation of the finite Isospin asymmetry***
  - ***The inclusion of finite magnetic field***
  - ***In-medium modifications of various meson states***
- ***Statistical thermal model, Hadron resonance gas model:***
  - ***In-medium modifications of multiplicity and particle ratios***
- ***Higher-order moments compared with experimental results***
- ***Conclusions***

Polyakov Linear-Sigma Model

In the mean-field approximation (MFA), the PLSM thermodynamic potential can be related the to grand-canonical function $\mathcal{Z}$, which is given in dependence of the temperatures T and the chemical potentials of $f-$th quark flavor μ_f,

$$\Omega(T,\mu_f) = \frac{-T \cdot \ln[\mathcal{Z}]}{V} = U(\sigma_u, \sigma_d, \sigma_s) + \mathcal{U}_{\text{Fuku}}(\phi, \bar{\phi}, T) + \Omega_{\bar{\psi}\psi}(T, \mu_f). \tag{17}$$

The chemical potentials μ_f are related to conserved quantum numbers of - for instance - baryon number (B), strangeness (S), electric charge (Q), and isospin (I) of each quark flavors,

$$\mu_u = \frac{\mu_B}{3} + \frac{2\mu_Q}{3} + \frac{\mu_I}{2}, \tag{18}$$

$$\mu_d = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \frac{\mu_I}{2}, \tag{19}$$

$$\mu_s = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \mu_S. \tag{20}$$

Polyakov Linear-Sigma Model

In expression (17), the first term $U(\sigma_u, \sigma_d, \sigma_s)$; the potential of the pure mesonic contributions, was given Eq. (16), while the second term $\mathcal{U}_{\text{Fuku}}(\phi, \bar{\phi}, T)$, the potential of Polyakov loop variables, was elaborate[...] The last term refers to the quarks and antiquarks contributions to the PLSM potential

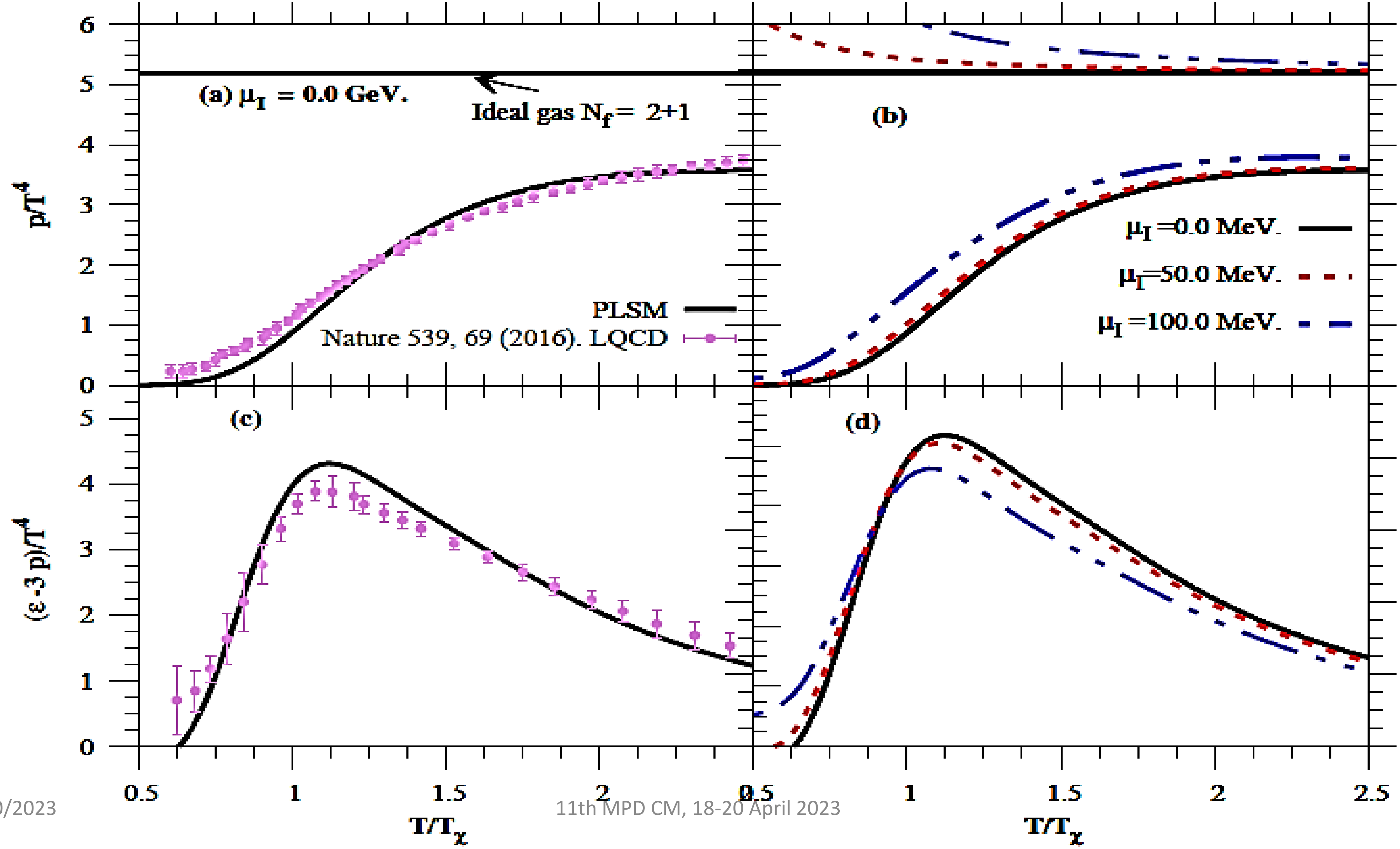
$$\Omega_{\bar{\psi}\psi}(T, \mu_f) = -2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3\vec{P}}{(2\pi)^3} \ln\left[1 + n_{q,f}(T,\ \mu_f)\right] + \ln\left[1 + n_{\bar{q},f}(T,\ \mu_f)\right], \tag{21}$$

where the number density distribution for particle is given as

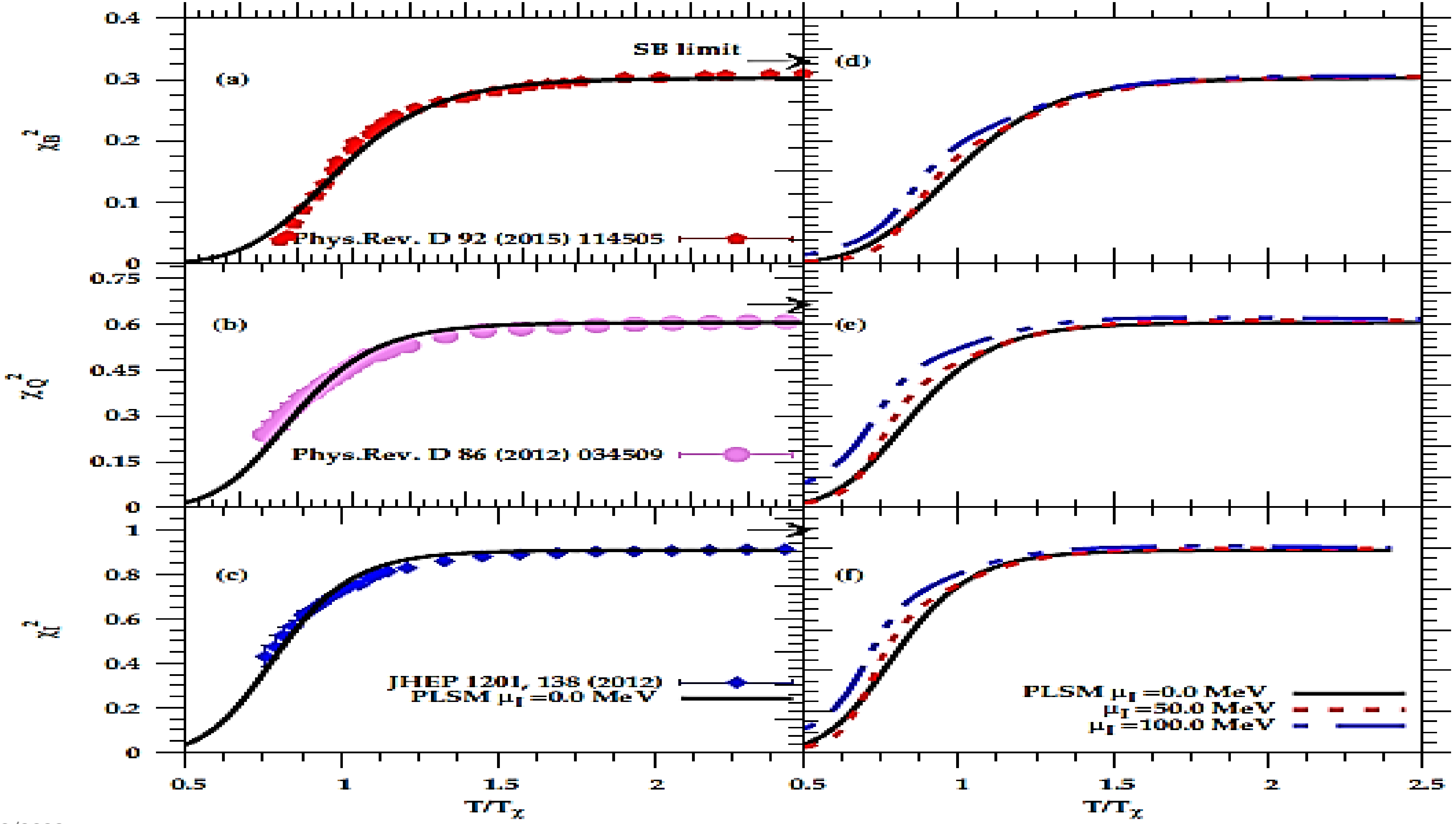
$$n_{q,f}(T,\ \mu_f) = 3\left(\phi + \bar{\phi} e^{-\frac{E_f - \mu_f}{T}}\right) \times e^{-\frac{E_f - \mu_f}{T}} + e^{-3\frac{E_f - \mu_f}{T}}, \tag{22}$$

which is identical to that of anti-particle $n_{q,f}(T,\ \mu_f)$ with $-\mu_f$ replacing $+\mu_f$ and the order parameter ϕ by its conjugate $\bar{\phi}$ or vice versa. $E_f = (\vec{P}^2 + m_f^2)^{1/2}$ is the energy-momentum dispersion relation with m_f being the mass of f^{th} quark flavor.

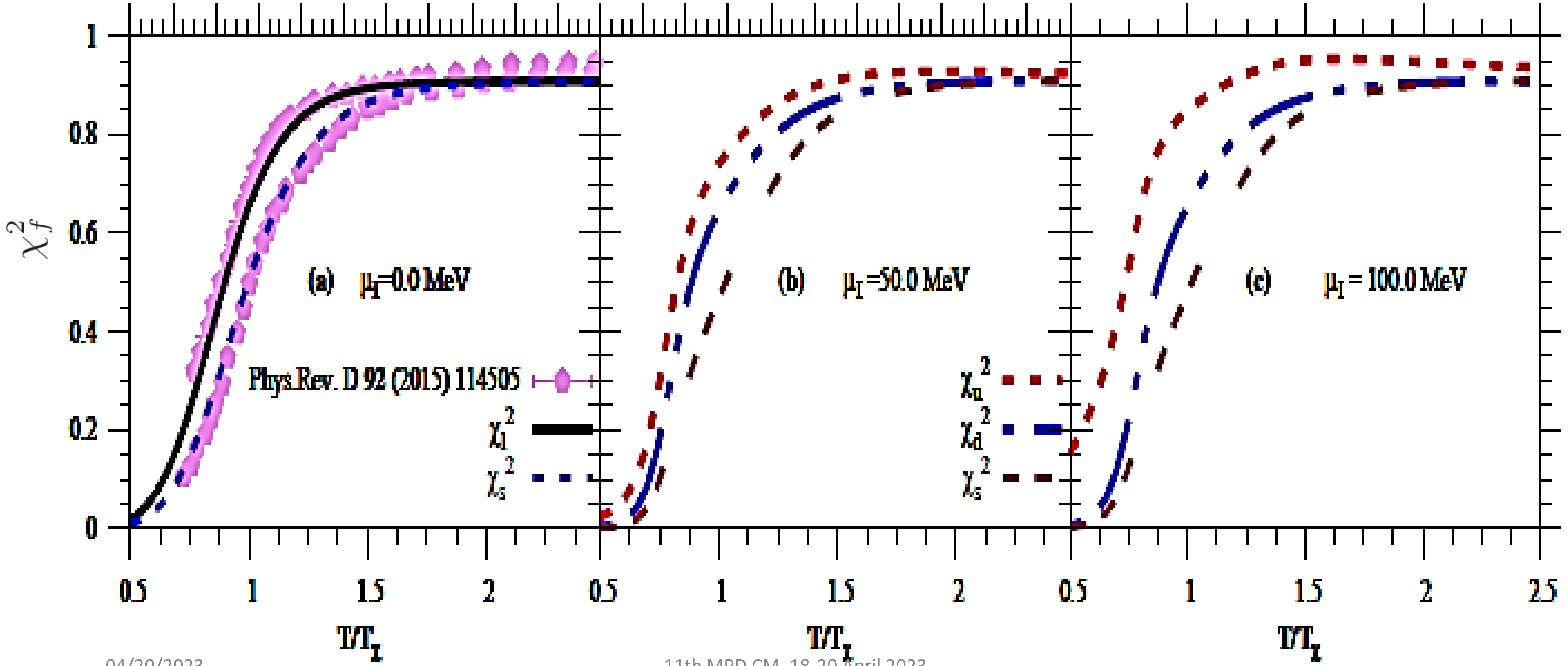
Polyakov Linear-Sigma Model



Polyakov Linear-Sigma Model



Polyakov Linear-Sigma Model



PLSM in Vanishing Magnetic Field

At vanishing magnetic field ($eB = 0$) but finite temperature (T) and baryon chemical potential (μ_f),

$$\Omega_{\bar{q}q}(T,\mu_f) = -2T \sum_{f=l,s} \int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3} \times$$

$$\left\{ \ln\left[1 + 3\left(\phi + \phi^* e^{-\frac{E_f-\mu_f}{T}}\right) e^{-\frac{E_f-\mu_f}{T}} + e^{-3\frac{E_f-\mu_f}{T}}\right] + \ln\left[1 + 3\left(\phi^* + \phi e^{-\frac{E_f+\mu_f}{T}}\right) e^{-\frac{E_f+\mu_f}{T}} + e^{-3\frac{E_f+\mu_f}{T}}\right]\right\}$$

PLSM in Finite Magnetic Field

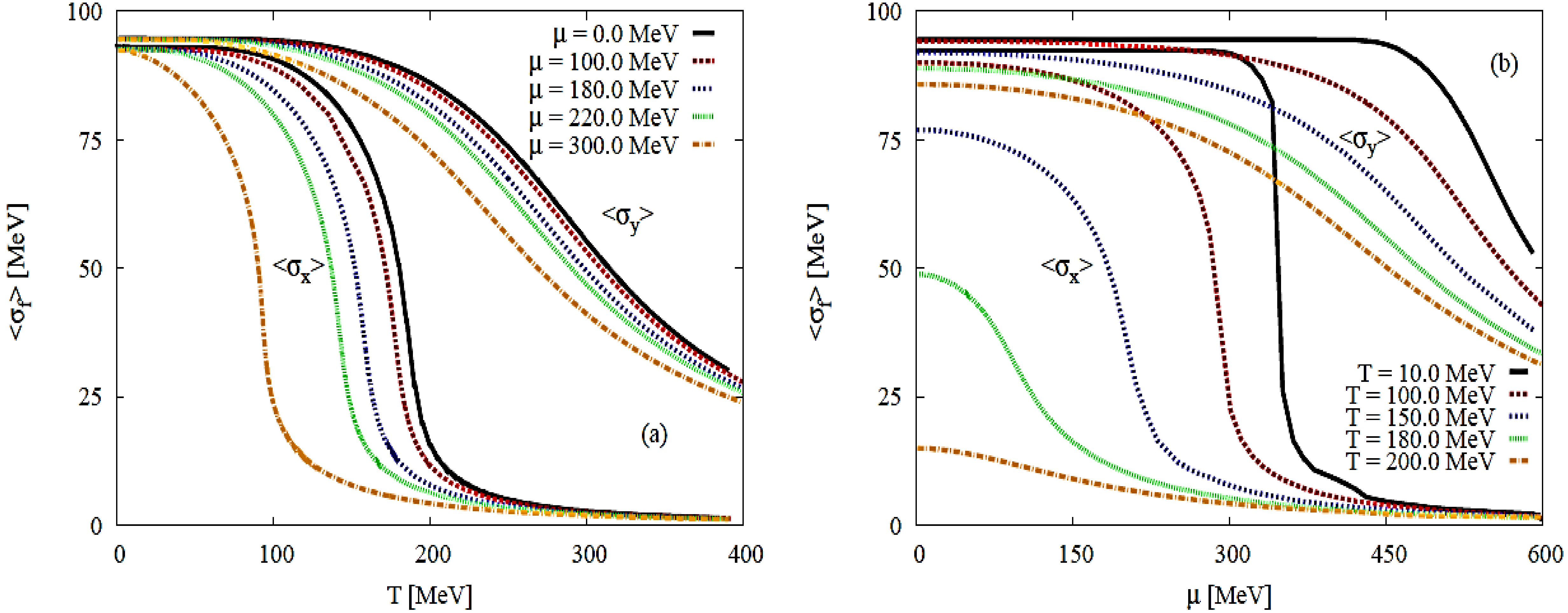
At finite magnetic background ($eB \neq 0$) where the magnetic field $\vec{B} = B\hat{e_z}$, all the spin directions should

$$\Omega_{\bar{q}q}(T, \mu_f, B) = -\sum_{f=l,s} \frac{|q_f| B T}{(2\pi)^2} \sum_{\nu=0}^{\nu_{max_f}} (2 - \delta_{0\nu}) \int_0^\infty dp_z \times$$

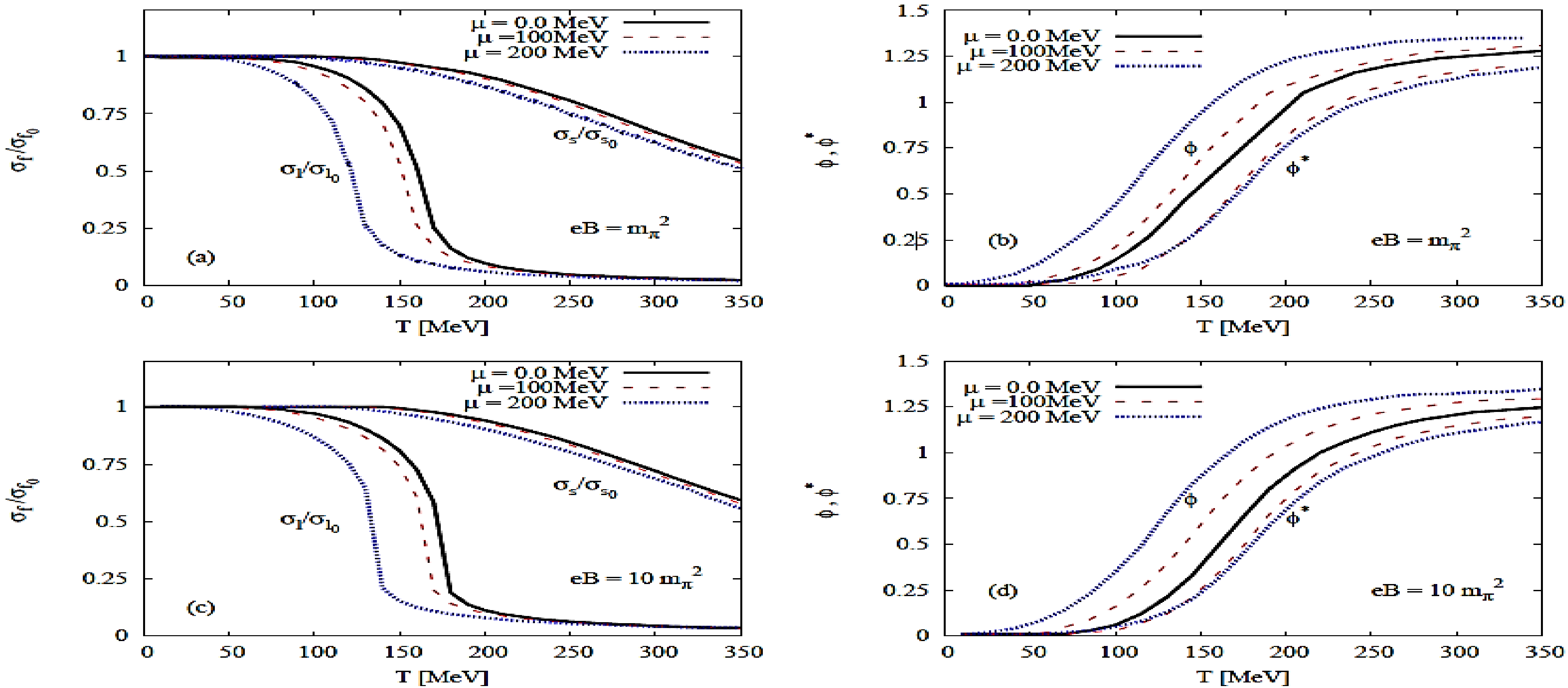
$$\left\{ \ln \left[1 + 3\left(\phi + \phi^* e^{-\frac{E_{B,f} - \mu_f}{T}}\right) e^{-\frac{E_{B,f} - \mu_f}{T}} + e^{-3\frac{E_{B,f} - \mu_f}{T}} \right] \right.$$

$$\left. + \ln \left[1 + 3\left(\phi^* + \phi e^{-\frac{E_{B,f} + \mu_f}{T}}\right) e^{-\frac{E_{B,f} + \mu_f}{T}} + e^{-3\frac{E_{B,f} + \mu_f}{T}} \right] \right\}$$

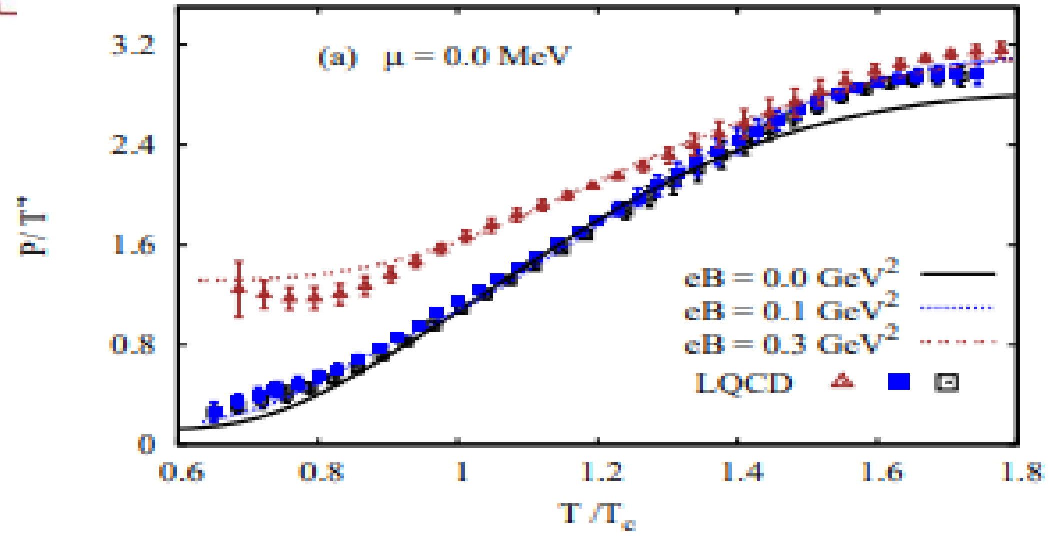
PLSM: Quark Condensates at vanishing eB

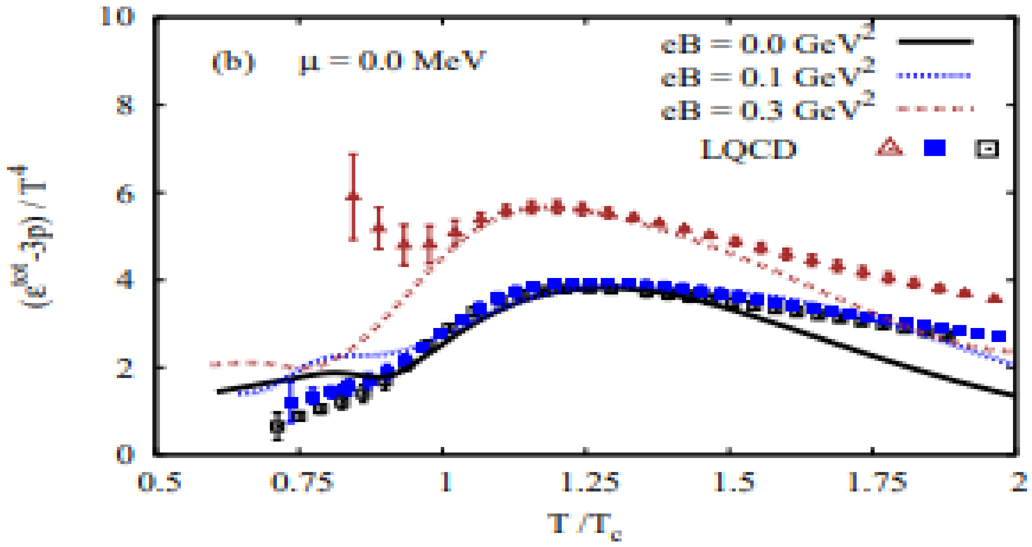


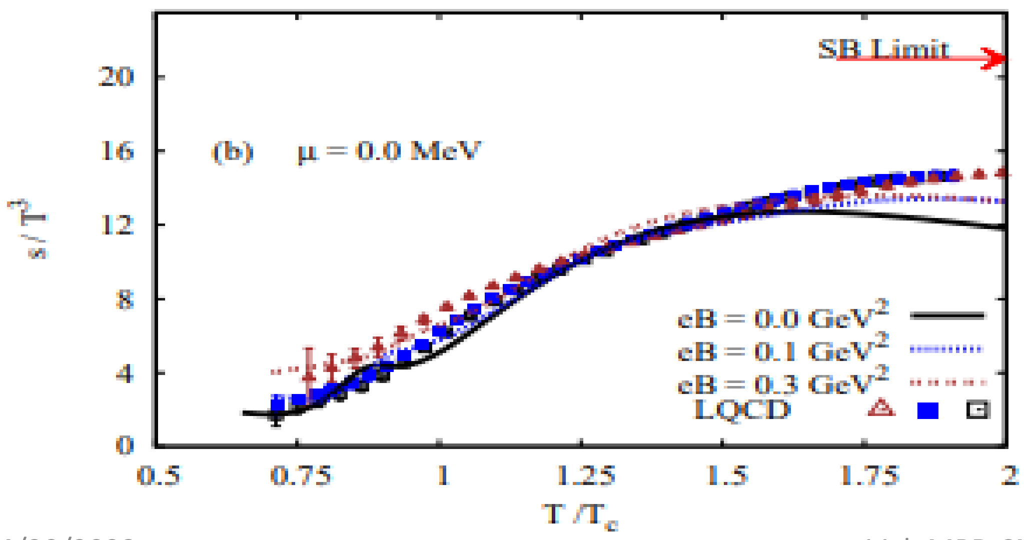
PLSM: Quark Condensates at finite eB

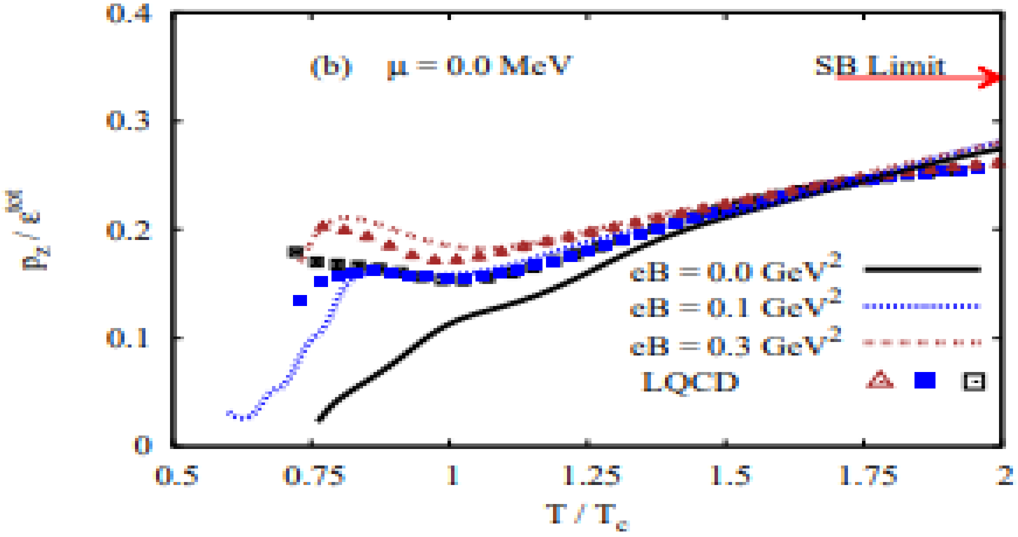


PLSM: Thermodynamics at finite eB

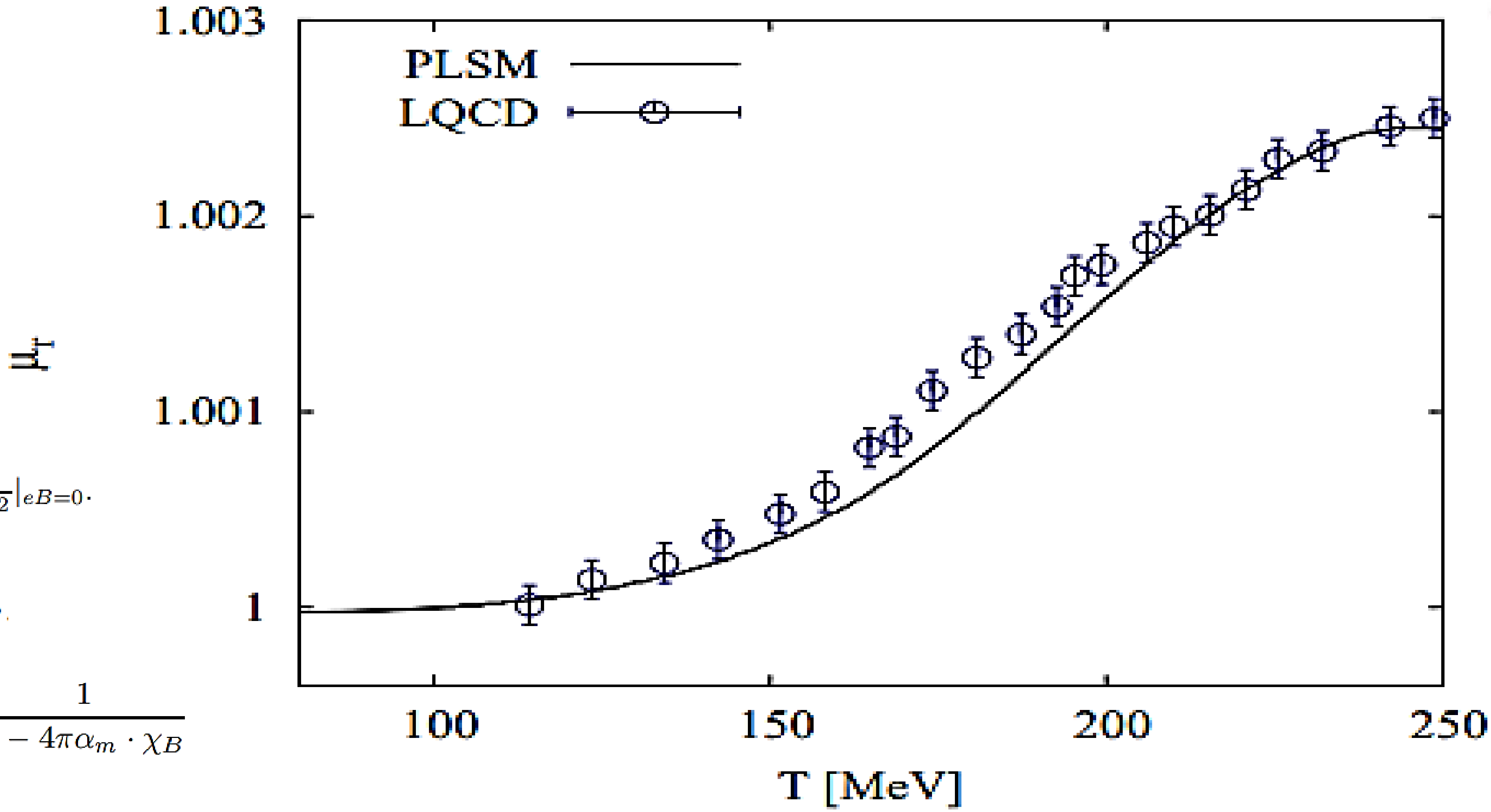








PLSM: Magnetization

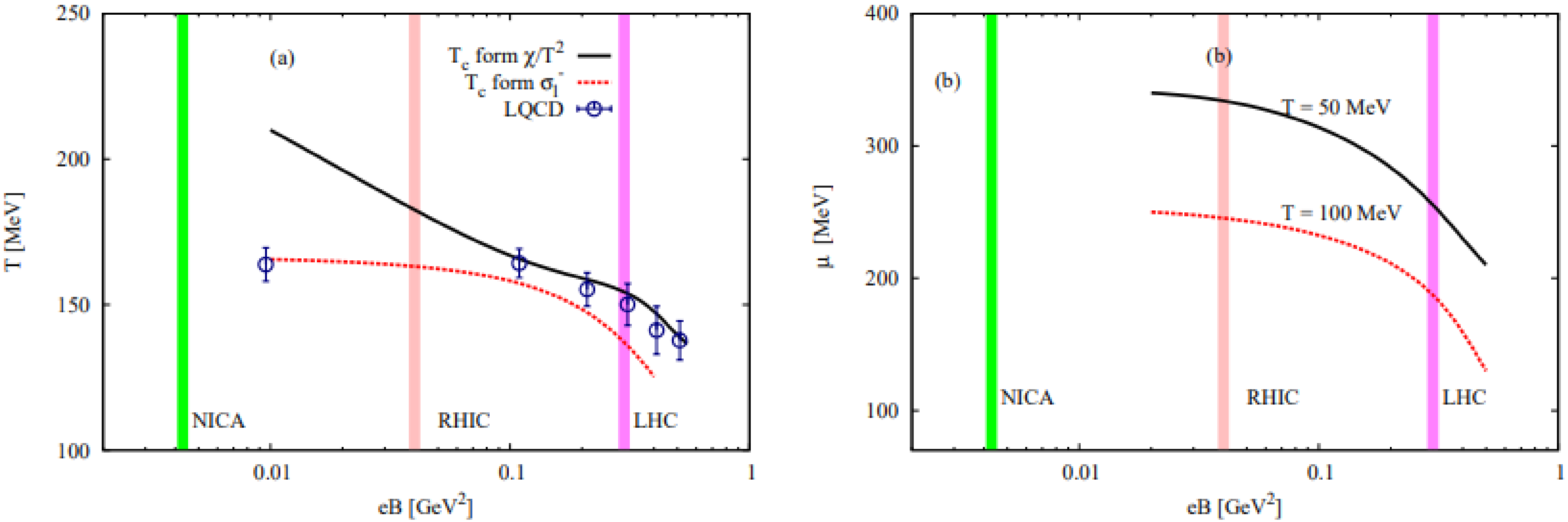


$$\chi_B = -\frac{1}{V}\frac{\partial^2 \mathcal{F}}{\partial (eB)^2}\Big|_{eB=0}.$$

$$\mu_r = 1 + \chi_B$$

$$\mu_B \equiv \frac{B^{ind}}{B^{ext}} = \frac{1}{1 - 4\pi\alpha_m \cdot \chi_B}$$

PLSM: QCD Phase Diagram at finite eB



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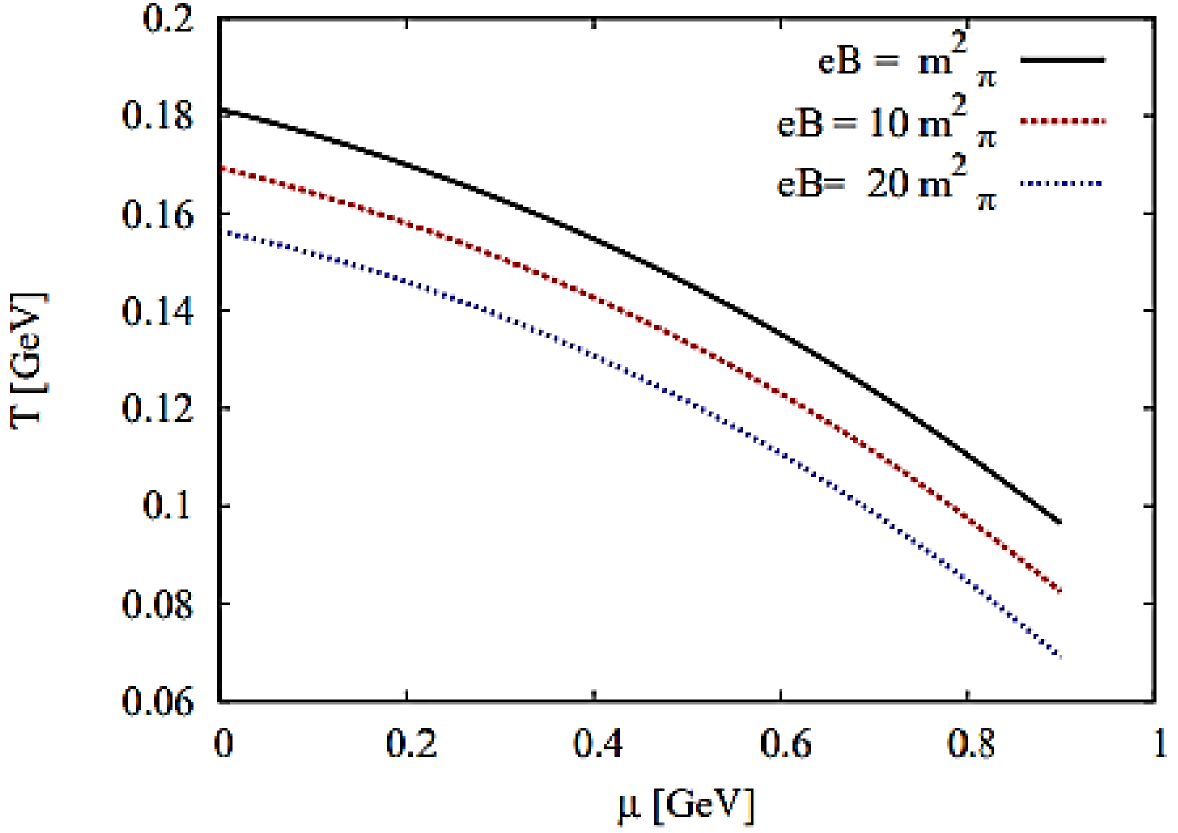
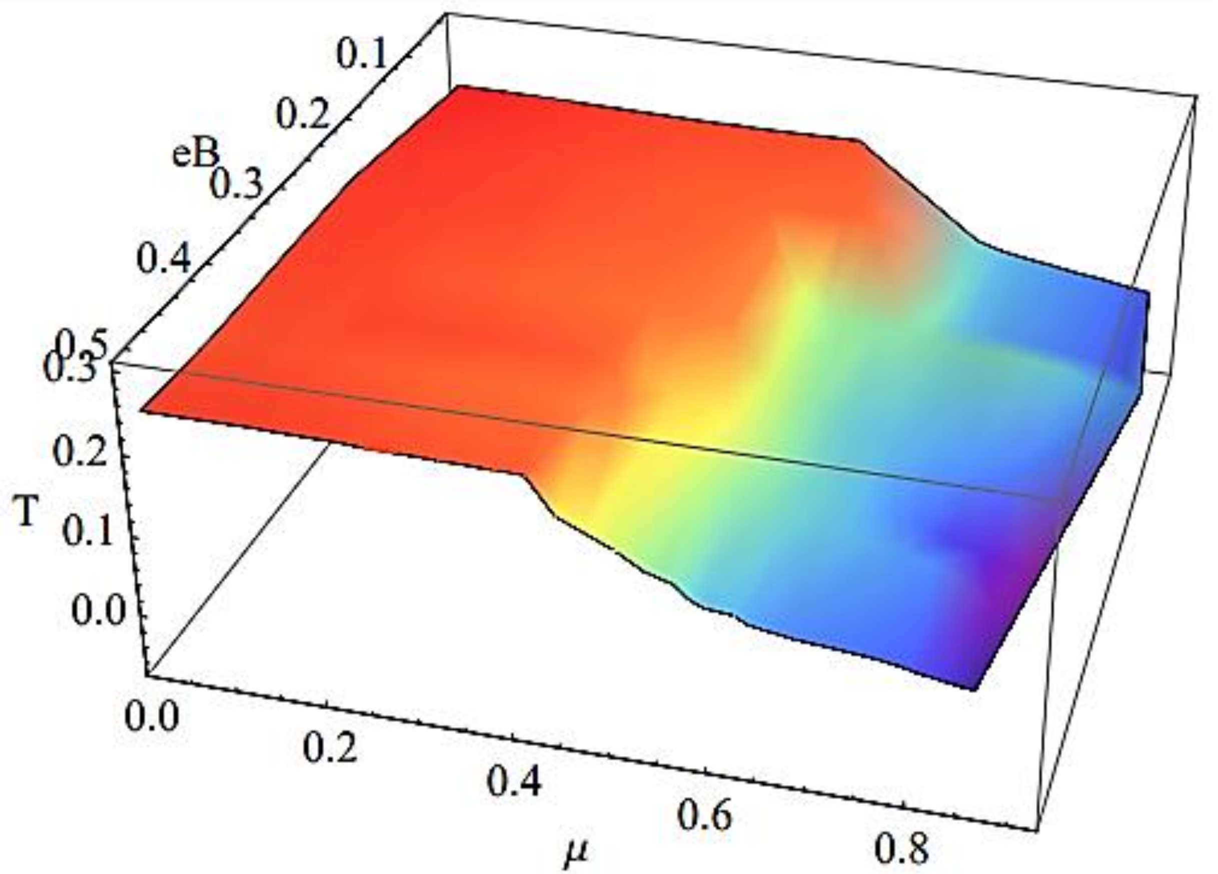
PLSM: QCD Phase Diagram at finite eB





PLSM: Chiral Phase Structure

The masses are defined by the second derivative of the grand potential

$$\Omega(T,\mu) = \frac{-T \ln \mathcal{Z}}{V} = U(\sigma_x,\sigma_y) + \mathcal{U}(\phi,\phi^*,T) + \Omega_{\bar{q}q}(T,\mu_f).$$

evaluated at its minimum with respect to the corresponding fields.

- In the present calculations, the minima are estimated by vanishing expectation values of all scalar, pseudoscalar, vector and axial-vector fields.
- The pure strange and non-strange condensates are finite.

$$m^2_{i,ab} = \left.\frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\right|_{\min}$$

where i stands for scalar, pseudoscalar, vector and axial-vector mesons and a and b range from 0, . . . , 8.

- In vacuum, mesonic sectors are formulated in non-strange and strange basis.

Polyakov Linear-Sigma Model

The Lagrangian of LSM with $N_f = 3$ quark flavors and $N_c = 3$ color degrees of freedom, where the quarks couple to the Polyakov-loop dynamics Φ-field represents a complex (3×3)-matrix for the $SU(3)_L \times SU(3)_R$ symmetric LSM Lagrangian $\mathcal{L}_{chiral} = \mathcal{L}_q + \mathcal{L}_m$, where the fermionic part reads

$$\mathcal{L}_q = \bar{q}\left[i\partial\!\!\!/ - g\, T_a\, (\sigma_a + i\,\gamma_5\,\pi_a + \gamma_\mu V_a^\mu + \gamma_\mu\gamma_5 A_a^\mu)\right] q,$$

with μ is an additional Lorentz index [ ], g is the flavor-blind Yukawa coupling of the quarks to the mesonic contribution $\mathcal{L}_m = \mathcal{L}_{SP} + \mathcal{L}_{VA} + \mathcal{L}_{Int} + \mathcal{L}_{U(1)_A}$ represented to $\mathcal{L}_{SP}$ scalars $(J^{PC} = 0^{++})$ and pseudoscalars $(J^{PC} = 0^{-+})$, $\mathcal{L}_{VA}$ to vectors $(J^{PC} = 1^{-})$ and axial-vectors $(J^{PC} = 1^{++})$ mesons and $\mathcal{L}_{Int}$ being the interaction between them. Finally the Lagrangian of the anomaly term is given by $\mathcal{L}_{U(1)_A}$

Polyakov Linear-Sigma Model

$$
\begin{aligned}
\mathcal{L}_{SP} &= \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + \mathrm{Tr}[H(\Phi + \Phi^\dagger)], \\
\mathcal{L}_{AV} &= -\frac{1}{4} \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] \\
&+ i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] \\
&+ g_5\,\mathrm{Tr}\,(L_\mu L^\mu)\,\mathrm{Tr}\,(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\,\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\,\mathrm{Tr}(R_\nu R^\nu)], \\
\mathcal{L}_{Int} &= \frac{h_1}{2} \mathrm{Tr}(\Phi^\dagger \Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[|L_\mu \Phi|^2 + |\Phi R_\mu|^2] + 2h_3\,\mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger), \\
\mathcal{L}_{U(1)_A} &= c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] + c_0[\mathrm{Det}(\Phi) - \mathrm{Det}(\Phi^\dagger)]^2 + c_1[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)]\,\mathrm{Tr}[\Phi\Phi^\dagger].
\end{aligned}
$$

PLSM: Chiral Phase Structure

- Scalar meson masses are given as

$$m_{i,ab}^2 = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Bigg|_{\min}$$

$$\begin{aligned}
m_{a_0}^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{3\lambda_2}{2}\bar{\sigma}_x^2 + \frac{\sqrt{2}c}{2}\bar{\sigma}_y, \\
m_{\kappa}^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\left(\bar{\sigma}_x^2 + \sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 2\bar{\sigma}_y^2\right) + \frac{c}{2}\bar{\sigma}_x, \\
m_{\sigma}^2 &= m_{s,00}^2 \cos^2\theta_s + m_{s,88}^2 \sin^2\theta_s + 2m_{s,08}^2 \sin\theta_s \cos\theta_s, \\
m_{f_0}^2 &= m_{s,00}^2 \sin^2\theta_s + m_{s,88}^2 \cos^2\theta_s - 2m_{s,08}^2 \sin\theta_s \cos\theta_s,
\end{aligned}$$

with

s (scalar) refers to i in

$$\begin{aligned}
m_{s,00}^2 &= m^2 + \frac{\lambda_1}{3}\left(7\bar{\sigma}_x^2 + 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 5\bar{\sigma}_y^2\right) + \lambda_2\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) - \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x + \bar{\sigma}_y\right), \\
m_{s,88}^2 &= m^2 + \frac{\lambda_1}{3}\left(5\bar{\sigma}_x^2 - 4\sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 7\bar{\sigma}_y^2\right) + \lambda_2\left(\frac{\bar{\sigma}_x^2}{2} + 2\bar{\sigma}_y^2\right) + \frac{\sqrt{2}c}{3}\left(\sqrt{2}\bar{\sigma}_x - \frac{\bar{\sigma}_y}{2}\right), \\
m_{s,08}^2 &= \frac{2\lambda_1}{3}\left(\sqrt{2}\bar{\sigma}_x^2 - \bar{\sigma}_x\bar{\sigma}_y - \sqrt{2}\bar{\sigma}_y^2\right) + \sqrt{2}\lambda_2\left(\frac{\bar{\sigma}_x^2}{2} - \bar{\sigma}_y^2\right) + \frac{c}{3\sqrt{2}}\left(\bar{\sigma}_x - \sqrt{2}\bar{\sigma}_y\right).
\end{aligned}$$

PLSM: Chiral Phase Structure

- Pseudoscalar meson masses read

$$m_{i,ab}^2 = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \zeta_{i,a} \partial \zeta_{i,b}}\Bigg|_{\min}$$

$$\begin{aligned}
m_\pi^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\bar{\sigma}_x^2 - \frac{\sqrt{2}c}{2}\bar{\sigma}_y, \\
m_K^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{2}\left(\bar{\sigma}_x^2 - \sqrt{2}\bar{\sigma}_x\bar{\sigma}_y + 2\bar{\sigma}_y^2\right) - \frac{c}{2}\bar{\sigma}_x, \\
m_{\eta'}^2 &= m_{p,00}^2 \cos^2\theta_p + m_{p,88}^2 \sin^2\theta_p + 2m_{p,08}^2 \sin\theta_p \cos\theta_p, \\
m_\eta^2 &= m_{p,00}^2 \sin^2\theta_p + m_{p,88}^2 \cos^2\theta_p - 2m_{p,08}^2 \sin\theta_p \cos\theta_p,
\end{aligned}$$

with $\tan 2\theta_i = \frac{2m_{i,08}^2}{m_{i,00}^2 - m_{i,88}^2}\ , \ i = s, p$

p (pseudoscalar) refers to i in

$$\begin{aligned}
m_{p,00}^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{3}\left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{c}{3}\left(2\bar{\sigma}_x + \sqrt{2}\bar{\sigma}_y\right), \\
m_{p,88}^2 &= m^2 + \lambda_1 \left(\bar{\sigma}_x^2 + \bar{\sigma}_y^2\right) + \frac{\lambda_2}{6}\left(\bar{\sigma}_x^2 + 4\bar{\sigma}_y^2\right) - \frac{c}{6}\left(4\bar{\sigma}_x - \sqrt{2}\bar{\sigma}_y\right), \\
m_{p,08}^2 &= \frac{\sqrt{2}\lambda_2}{6}\left(\bar{\sigma}_x^2 - 2\bar{\sigma}_y^2\right) - \frac{c}{6}\left(\sqrt{2}\bar{\sigma}_x - 2\bar{\sigma}_y\right),
\end{aligned}$$

PLSM: Chiral Phase Structure

- Vector meson masses are given as

$$
\begin{aligned}
m_\rho^2 &= m_1^2 + \frac{1}{2}(h_1+h_2+h_3)\,\bar{\sigma}_x^2 + \frac{h_1}{2}\bar{\sigma}_y^2 + 2\delta_x\,, \\
m_{K^\star}^2 &= m_1^2 + \frac{\bar{\sigma}_x^2}{4}(g_1^2+2h_1+h_2) + \frac{\bar{\sigma}_x\bar{\sigma}_y}{\sqrt{2}}(h_3-g_1^2) + \frac{\bar{\sigma}_y^2}{2}(g_1^2+h_1+h_2) + \delta_x + \delta_y\,, \\
m_{\omega_x}^2 &= m_\rho^2\,, \\
m_{\omega_y}^2 &= m_1^2 + \frac{h_1}{2}\bar{\sigma}_x^2 + \left(\frac{h_1}{2}+h_2+h_3\right)\bar{\sigma}_y^2 + 2\delta_y\,,
\end{aligned}
$$

and vectors V^μ refer to i

$$
m_{i,ab}^2 = \frac{\partial^2\Omega(T,\mu_f)}{\partial\zeta_{i,a}\partial\zeta_{i,b}}\bigg|_{\min}
$$

PLSM: Chiral Phase Structure

- And finally the axial-vectors masses are

$$
\begin{aligned}
m_{a_1}^2 &= m_1^2 + \frac{1}{2}\left(2g_1^2 + h_1 + h_2 - h_3\right)\bar{\sigma}_x^2 + \frac{h_1}{2}\bar{\sigma}_y^2 + 2\delta_x, \\
m_{K_1}^2 &= m_1^2 + \frac{1}{4}\left(g_1^2 + 2h_1 + h_2\right)\bar{\sigma}_x^2 - \frac{1}{\sqrt{2}}\bar{\sigma}_x\bar{\sigma}_y\left(h_3 - g_1^2\right) + \frac{1}{2}\left(g_1^2 + h_1 + h_2\right)\bar{\sigma}_y^2 + \delta_x + \delta_y, \\
m_{f_{1x}}^2 &= m_{a_1}^2, \\
m_{f_{1y}}^2 &= m_1^2 + \frac{\bar{\sigma}_x^2}{2}h_1 + \left(2g_1^2 + \frac{h_1}{2} + h_2 - h_3\right)\bar{\sigma}_y^2 + 2\delta_y.
\end{aligned}
$$

and axialvector A^μ refer to i

$$
m_{i,ab}^2 = \left.\frac{\partial^2\Omega(T,\mu_f)}{\partial\zeta_{i,a}\partial\zeta_{i,b}}\right|_{\min}
$$

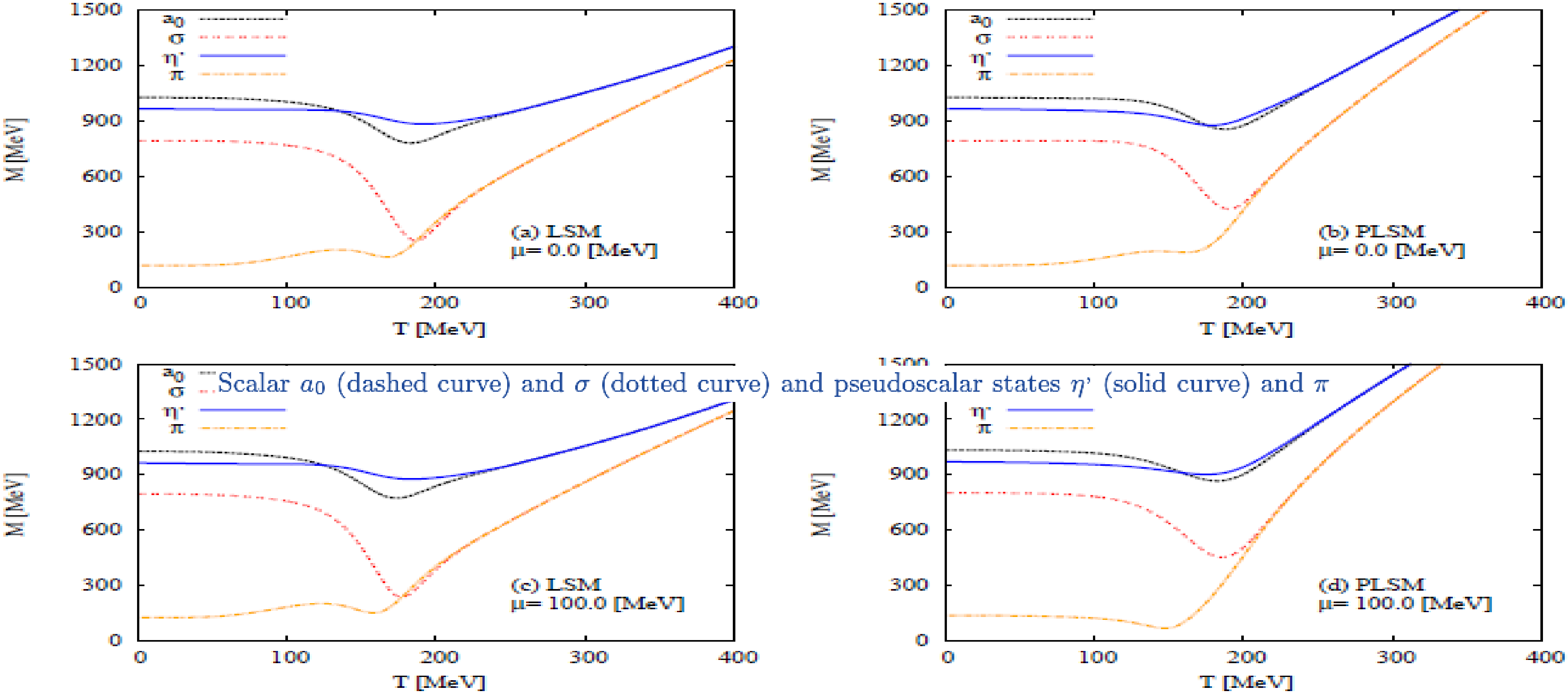
PLSM: meson masses at T=0

Sector	Symbol	PDG	PLSM	PNJL	Lattice QCD	
					Hot QCD	PACS-CS
Scalar $J^{PC}=0^{++}$	a_0	$a_0(980^{\pm 20})$	1026	837		
	κ	$K_0^*(1425^{\pm 50})$	1115	1013		
	σ	$\sigma(400-1200)$	800	700		
	f_0	$f_0(1200-1500)$	1284	1169		
Pseudoscalar $J^{PC}=0^{-+}$	π	$\pi^0(134.97^{\pm 6.9})$	120	126	$134^{\pm 6}$	$135.4^{\pm 6.2}$
	K	$K^0\ (497.614^{\pm 24.8})$	509	490	$422.6^{\pm 11.3}$	$498^{\pm 22}$
	η	$\eta(547.853^{\pm 27.4})$	553	505	$579^{\pm 7.3}$	$688^{\pm 32}$
	η'	$\eta'(957.78^{\pm 60})$	965	949	—	—
Vector $J^{PC}=1^{-}$	ρ	$\rho(775.49^{\pm 38.8})$	745	—	$756.2^{\pm 36}$	$597^{\pm 86}$
	ω_X	$\omega(782.65^{\pm 44.7})$	745	—	$884^{\pm 18}$	$861^{\pm 23}$
	K^*	$K^*(891.66^{\pm 26})$	894	—	$1005^{\pm 93}$	$1010.2^{\pm 77}$
	ω_y	$\phi(1019.455^{\pm 51})$	1005	—	—	—
Axial-Vector $J^{PC}=1^{++}$	a_1	$a_1(1030-1260)$	980	—		
	f_{1x}	$f_1(1281^{\pm 60})$	980	—		
	K_1^*	$K_1^*(1270^{\pm 7})$	1135	—		
	f_{1y}	$f_1(1420^{\pm 71.3})$	1315	—		

A comparison between (pseudo)-scalar and (axial)-vector meson sectors in PLSM (present work) and corresponding results from PNJL. Both are compared with the experimental measurements, PDG and the lattice QCD simulations

PLSM: In-medium meson masses

Left-hand panel (LSM) and right-hand panel (PLSM):



Scalar a_0 (dashed curve) and σ (dotted curve) and pseudoscalar states η' (solid curve) and π

PLSM: In-medium meson masses

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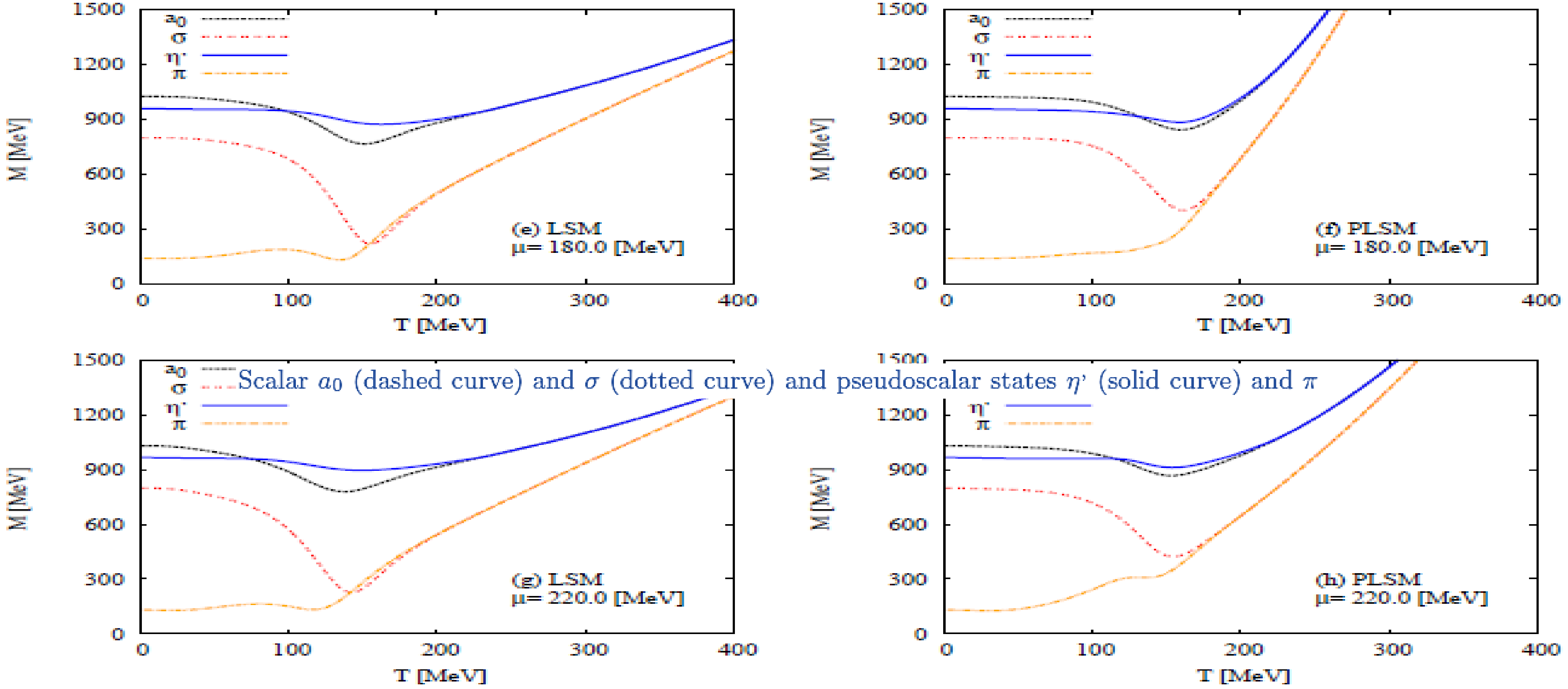

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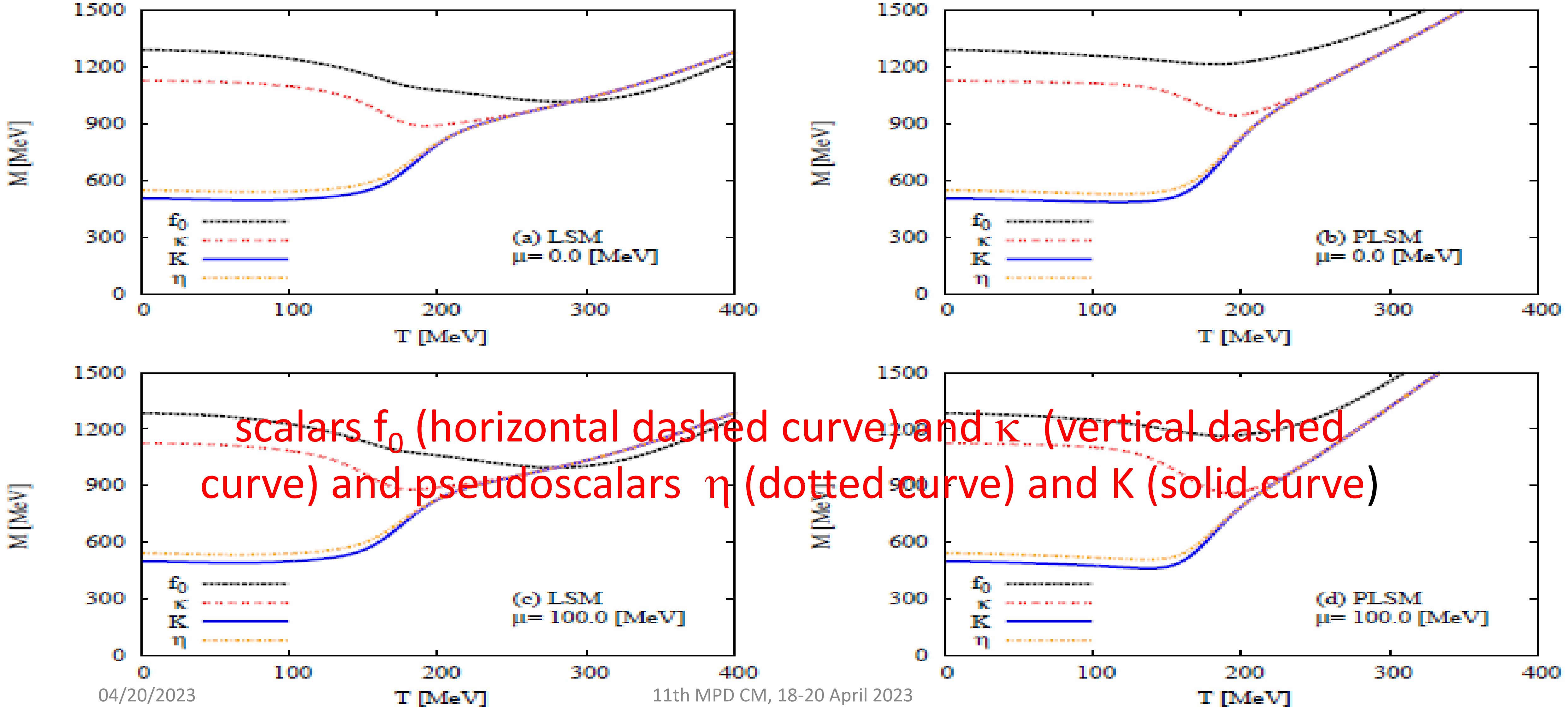
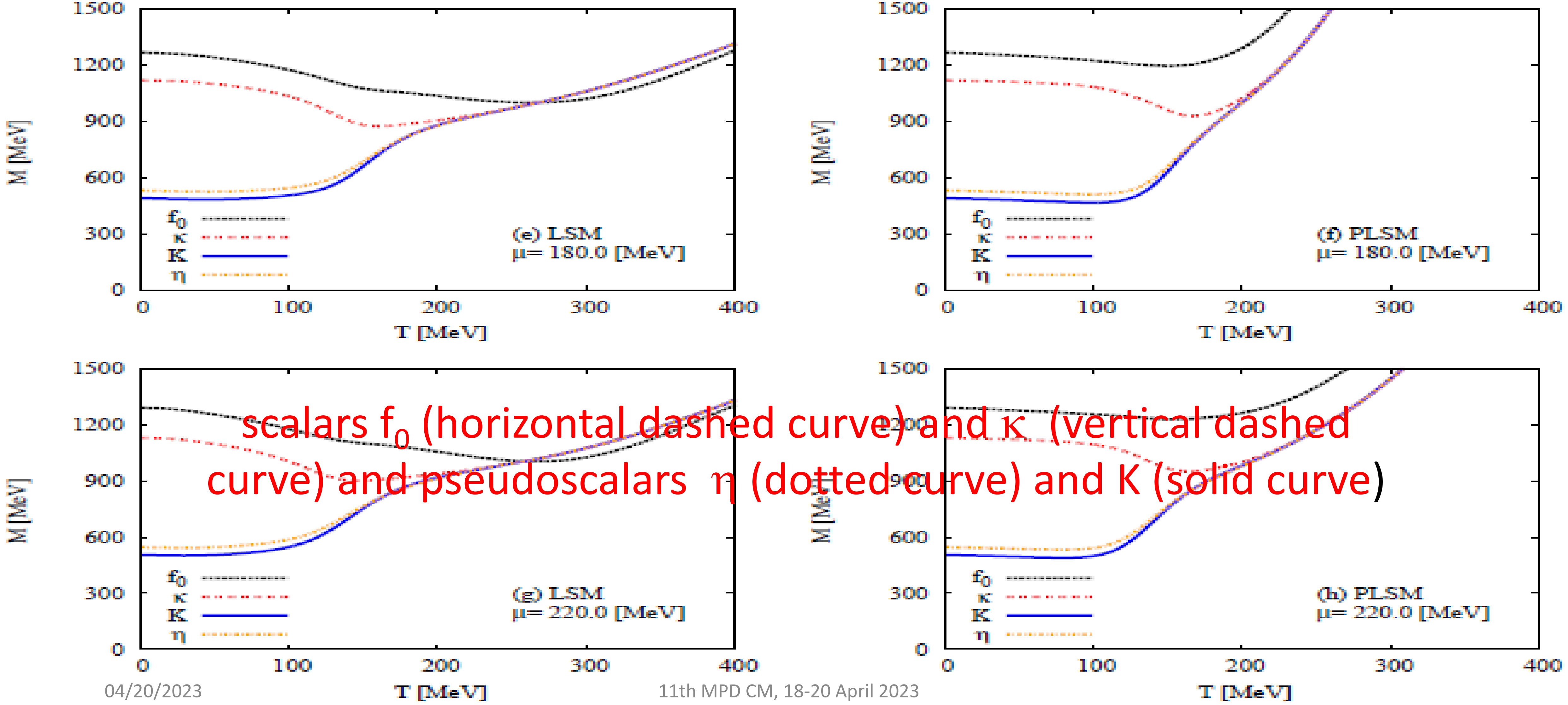


scalars f_0 (horizontal dashed curve) and κ (vertical dashed curve) and pseudoscalars η (dotted curve) and K (solid curve)

PLSM: In-medium meson masses

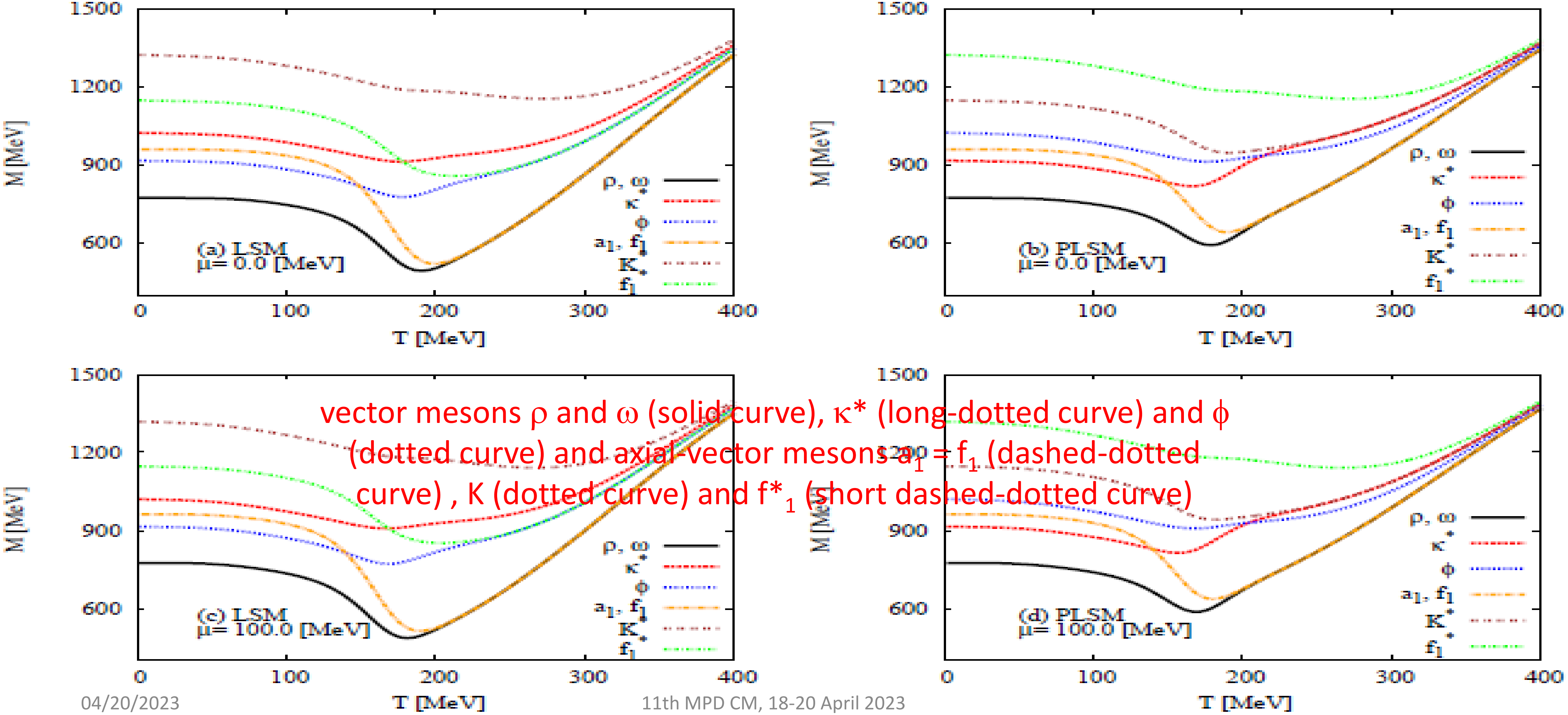
Left-hand panel (LSM) and right-hand panel (PLSM):



scalars f_0 (horizontal dashed curve) and κ (vertical dashed curve) and pseudoscalars η (dotted curve) and K (solid curve)

PLSM: In-medium meson masses

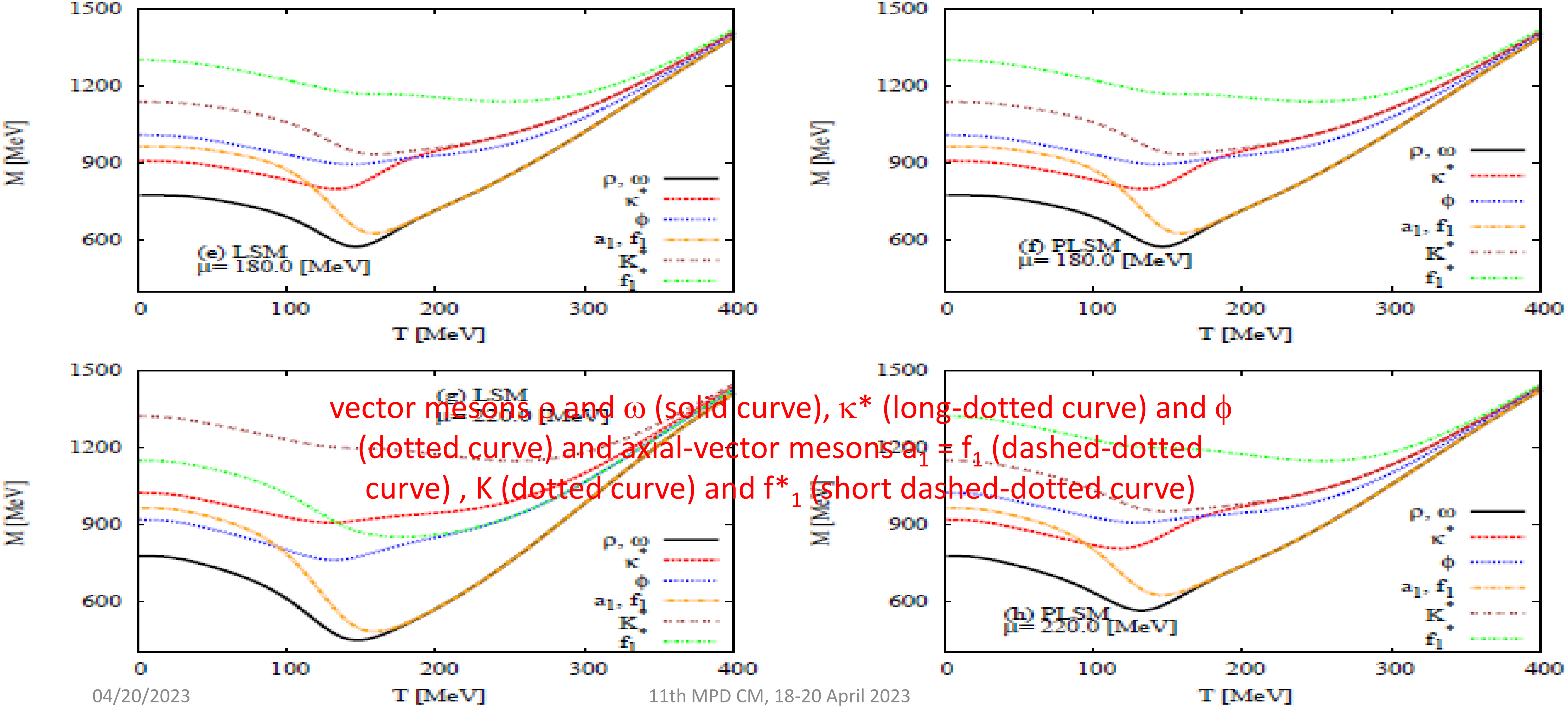
Left-hand panel (LSM) and right-hand panel (PLSM):



vector mesons ρ and ω (solid curve), κ^* (long-dotted curve) and ϕ (dotted curve) and axial-vector mesons $a_1 = f_1$ (dashed-dotted curve) , K (dotted curve) and f^*_1 (short dashed-dotted curve)

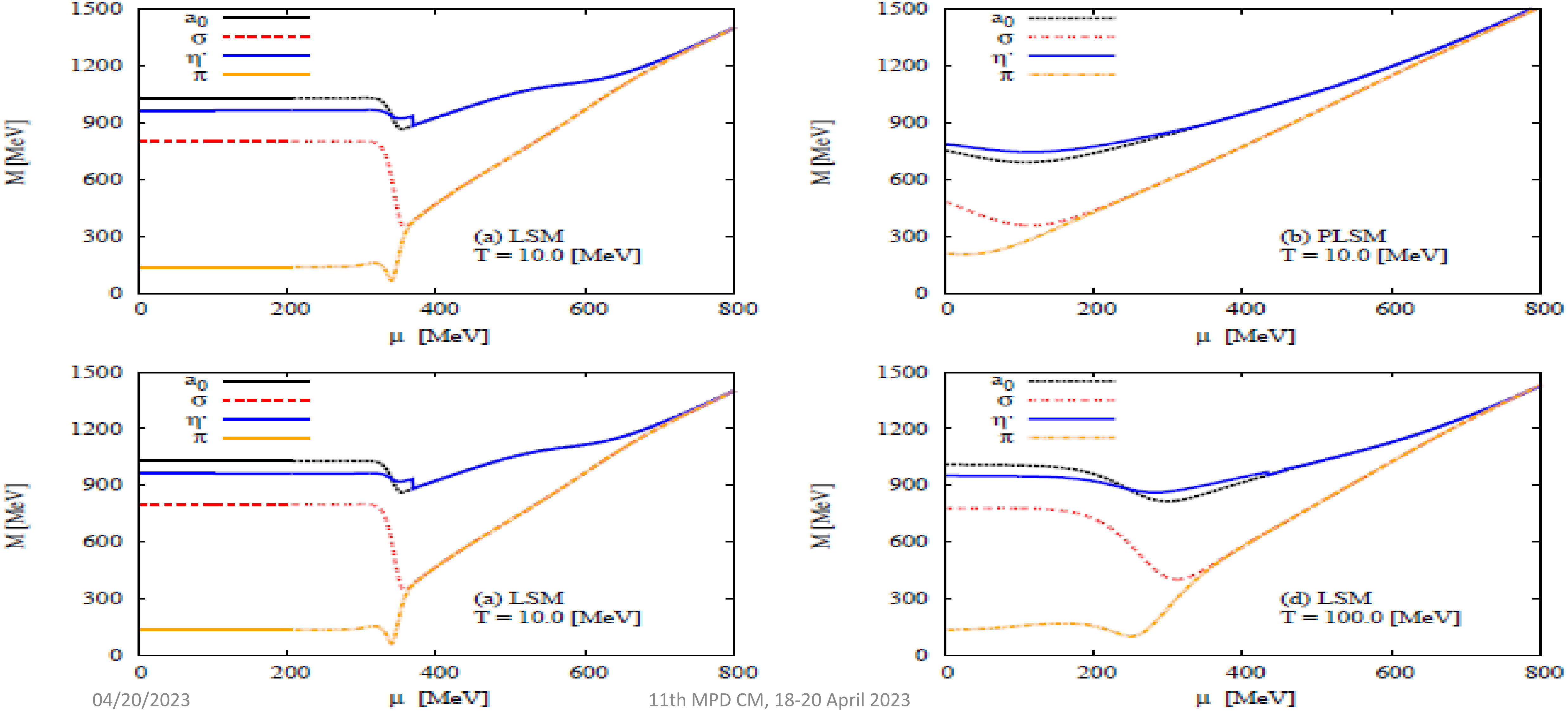
PLSM: In-medium meson masses

Left-hand panel (LSM) and right-hand panel (PLSM):

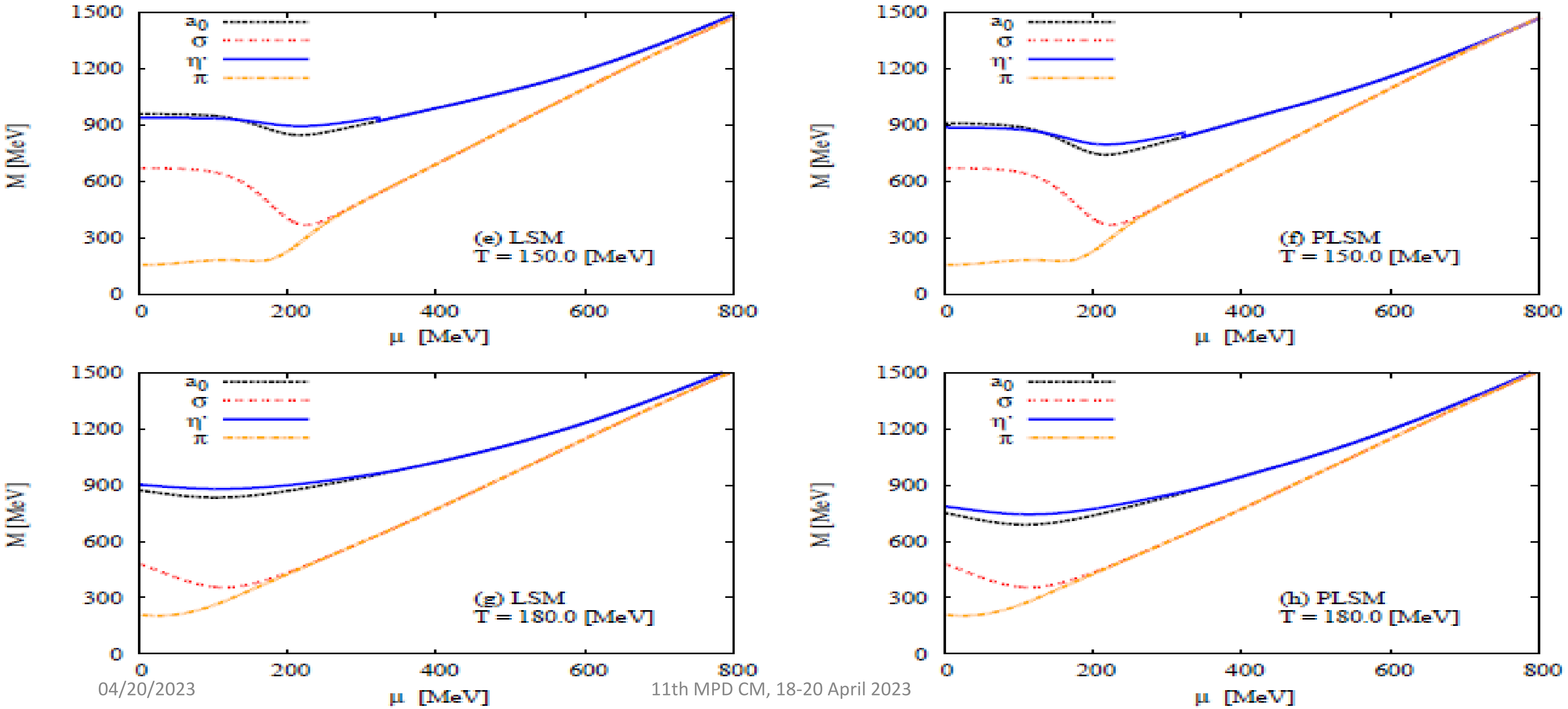


vector mesons ρ and ω (solid curve), κ* (long-dotted curve) and ϕ (dotted curve) and axial-vector mesons $a_1 = f_1$ (dashed-dotted curve) , K (dotted curve) and f^*_1 (short dashed-dotted curve)

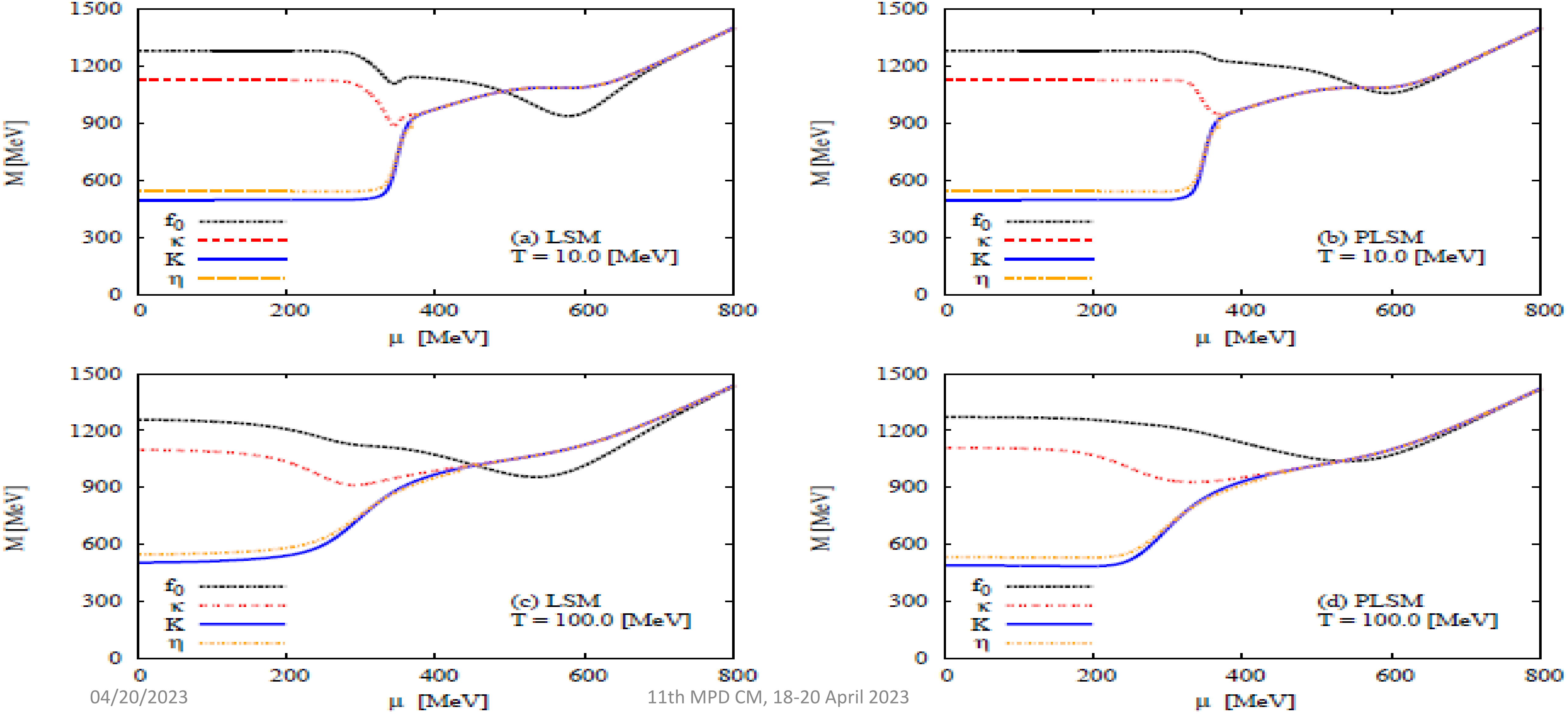
PLSM: In-medium meson masses



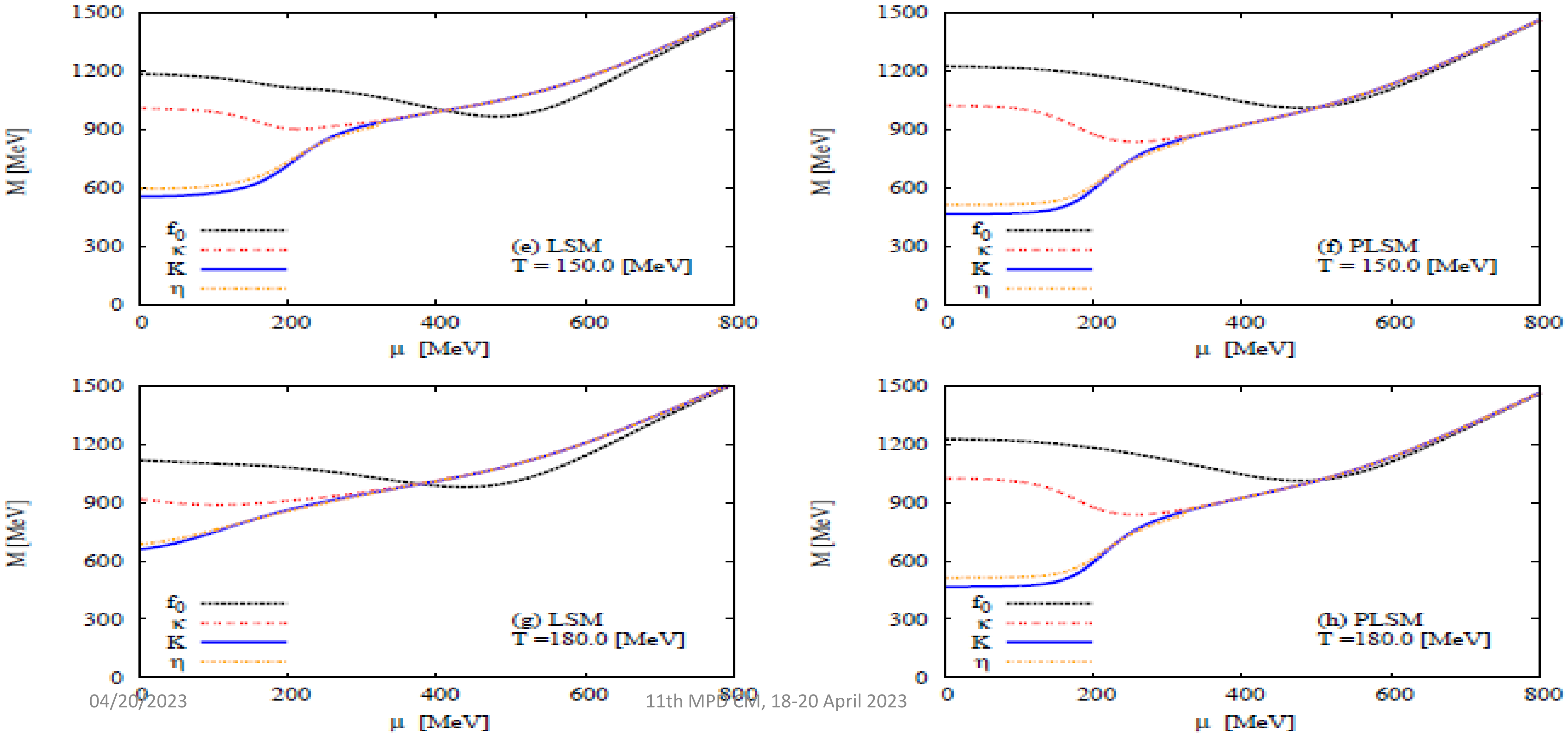
PLSM: In-medium meson masses



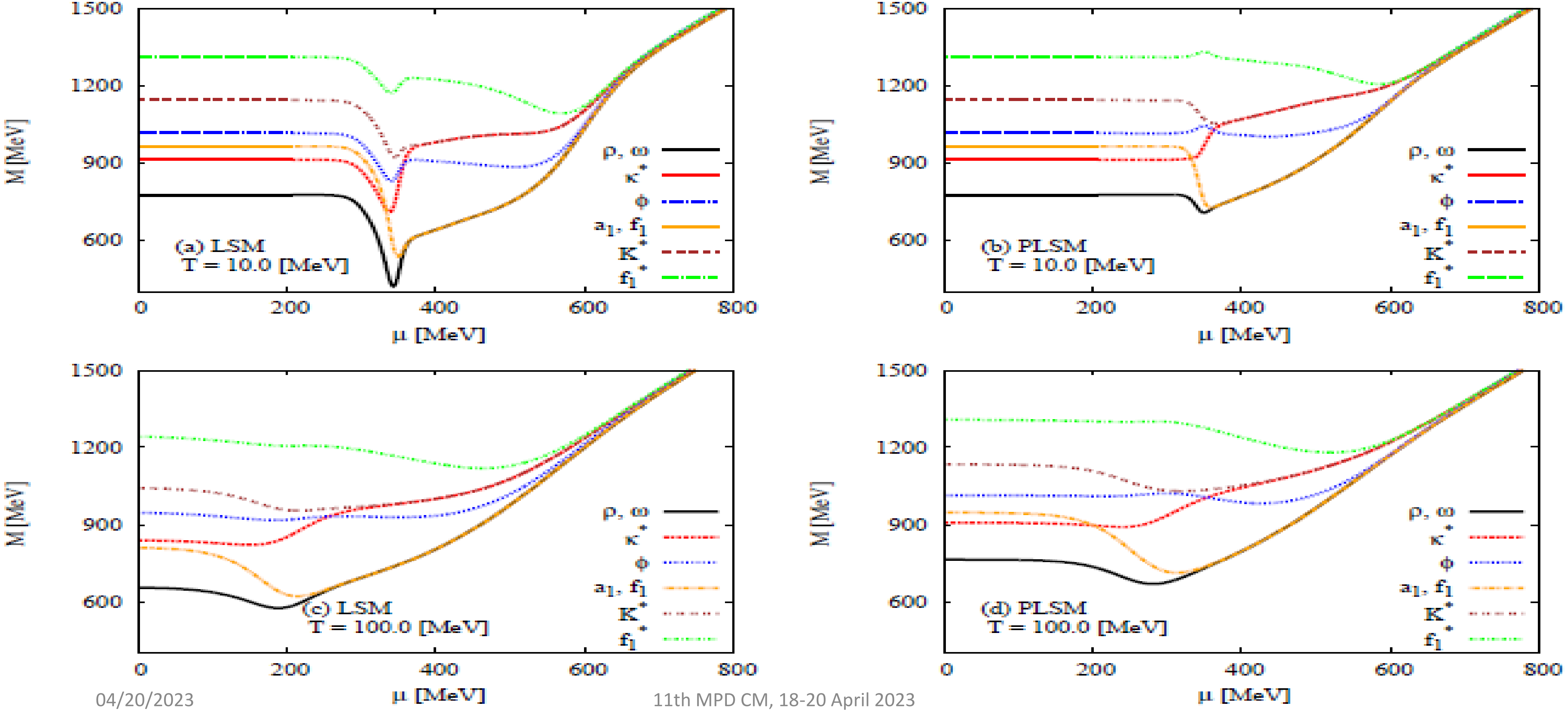
PLSM: In-medium meson masses



PLSM: In-medium meson masses



PLSM: In-medium meson masses



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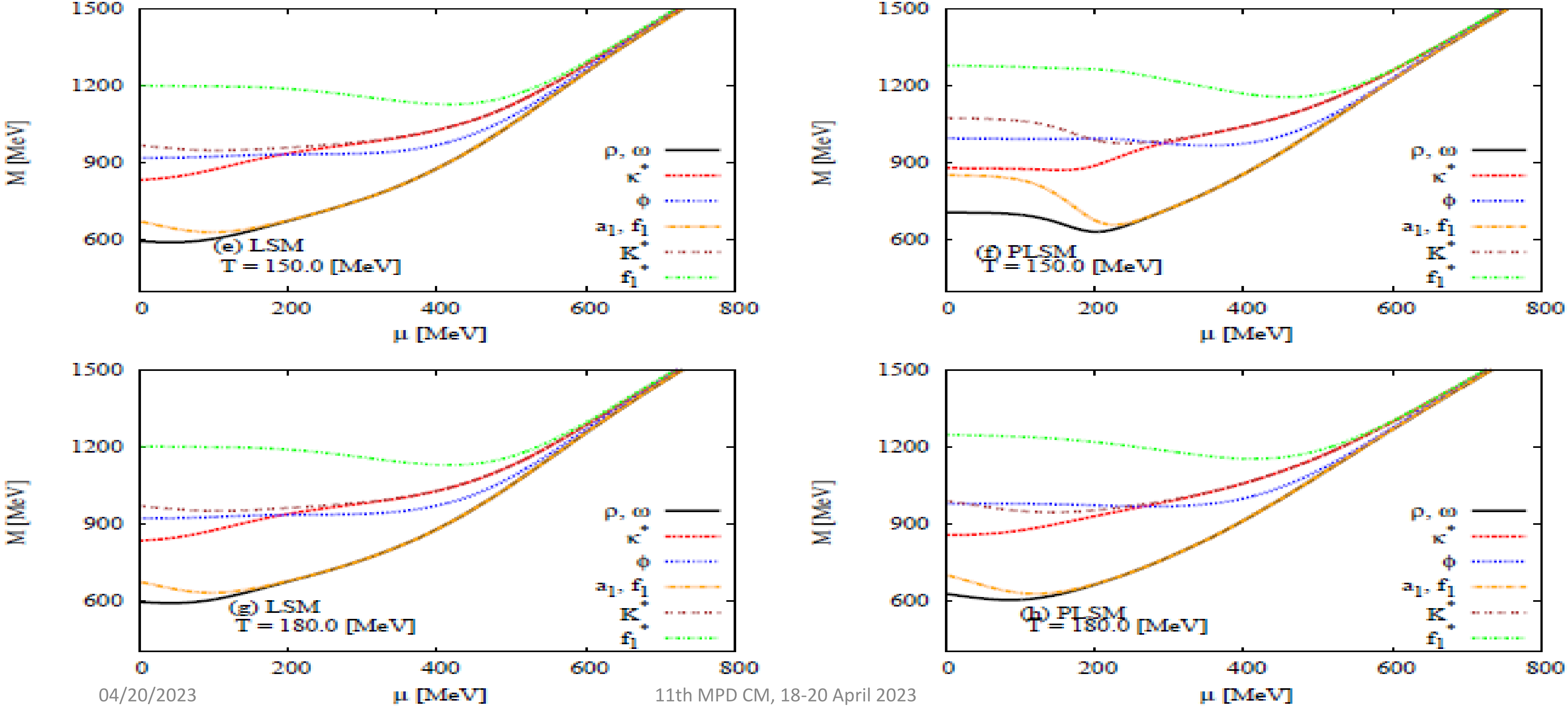
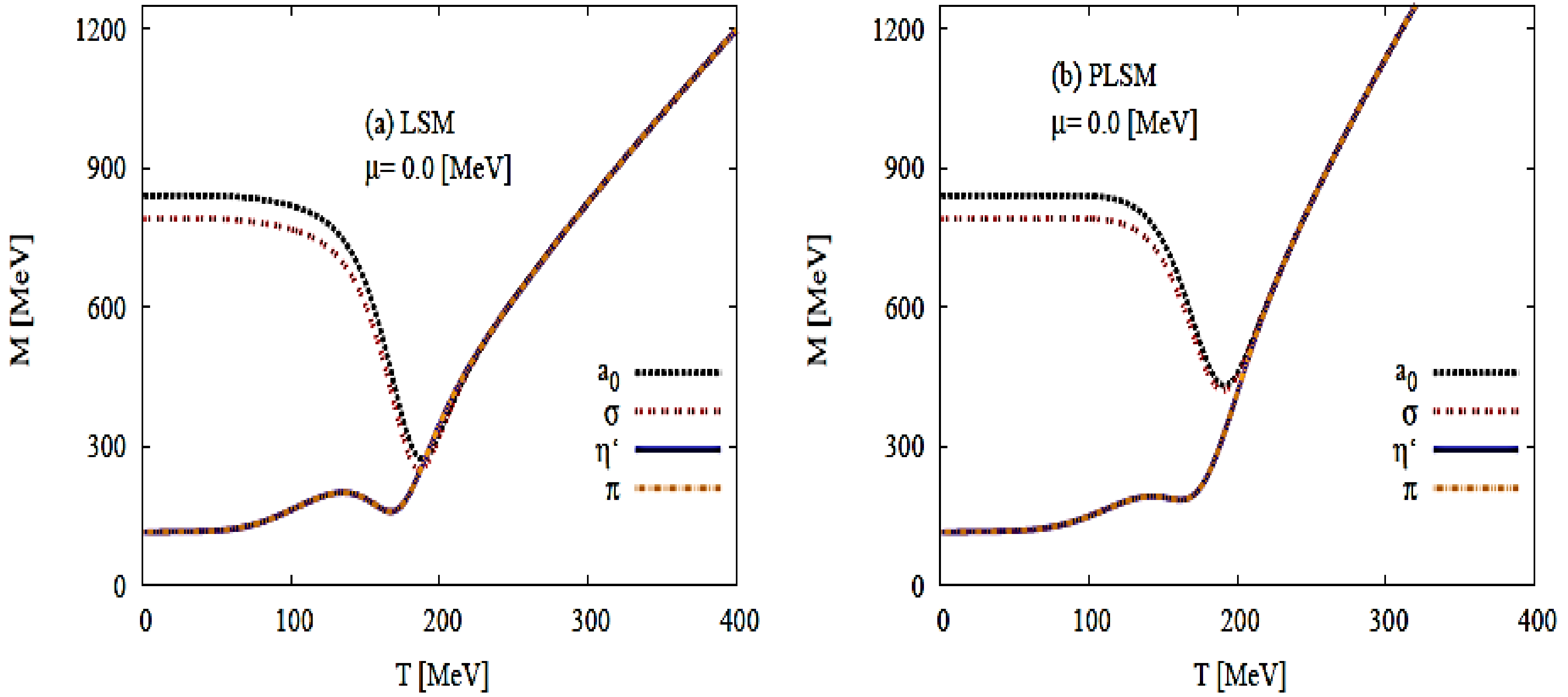
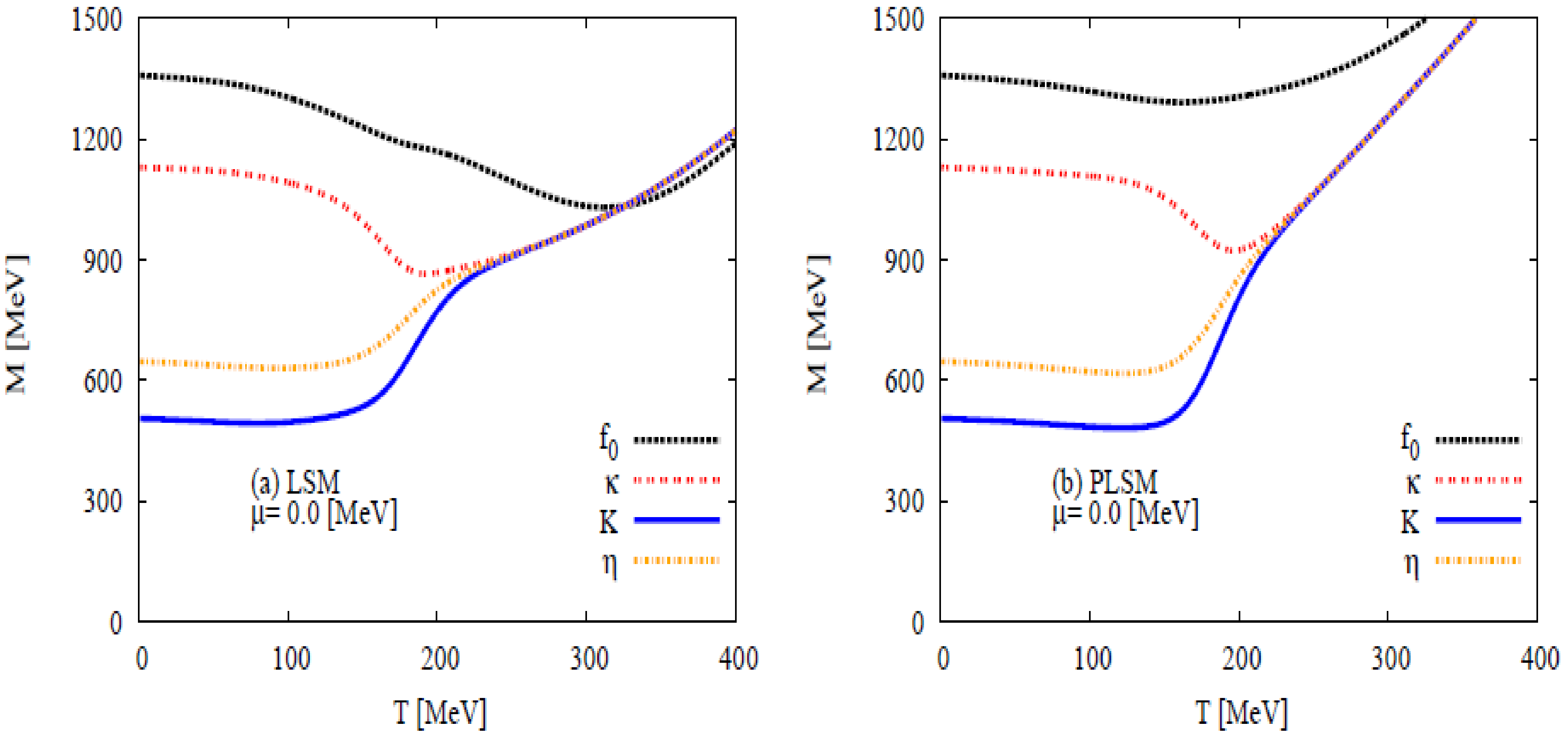


PLSM: In-medium meson masses



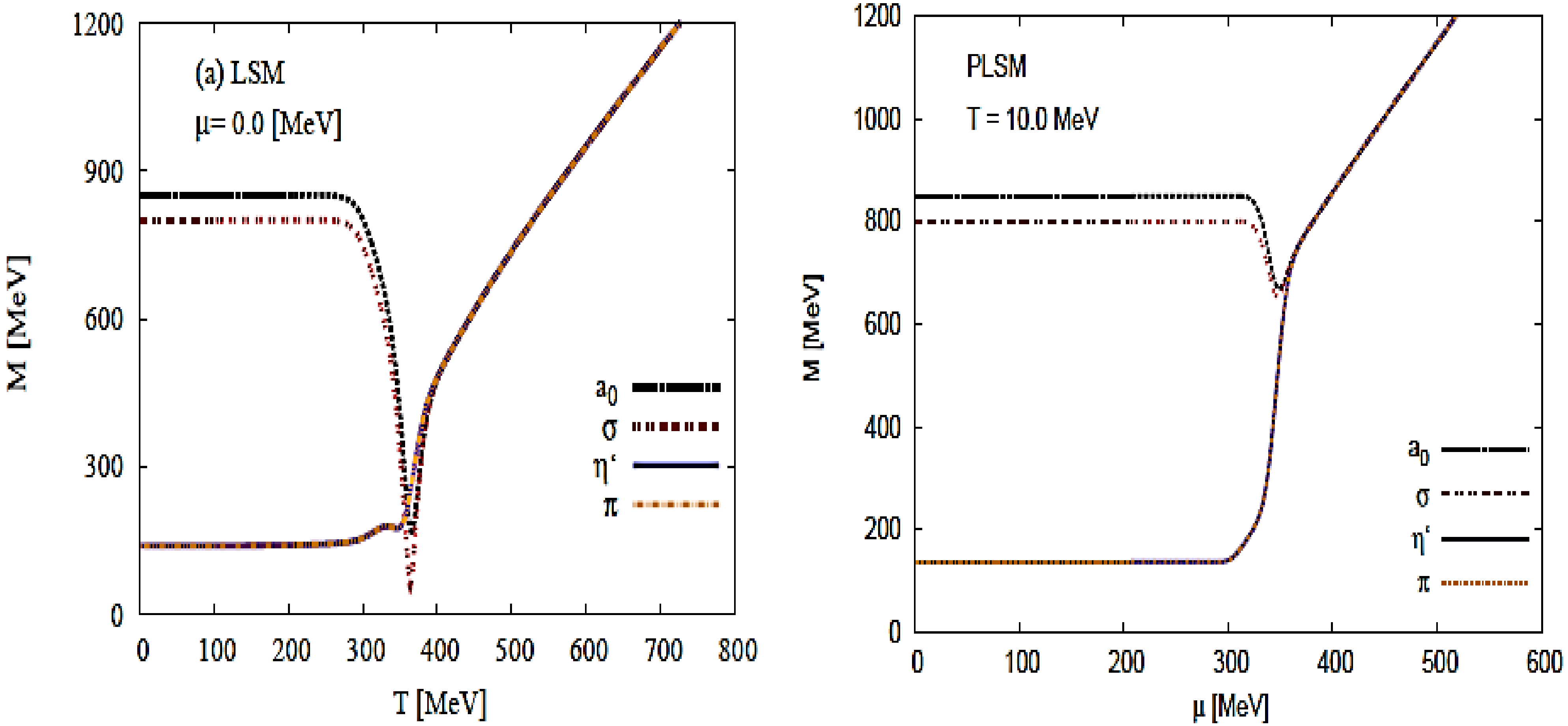
Left- (LSM) and right-hand panel (PLSM) show scalar a_0 (dashed curve) and σ (dotted curve) and pseudoscalar states η' (solid curve) and π (dashed-dotted curve) as function of temperature at vanishing baryon-chemical potentials $\mu = 0.0$ MeV.

PLSM: In-medium meson masses



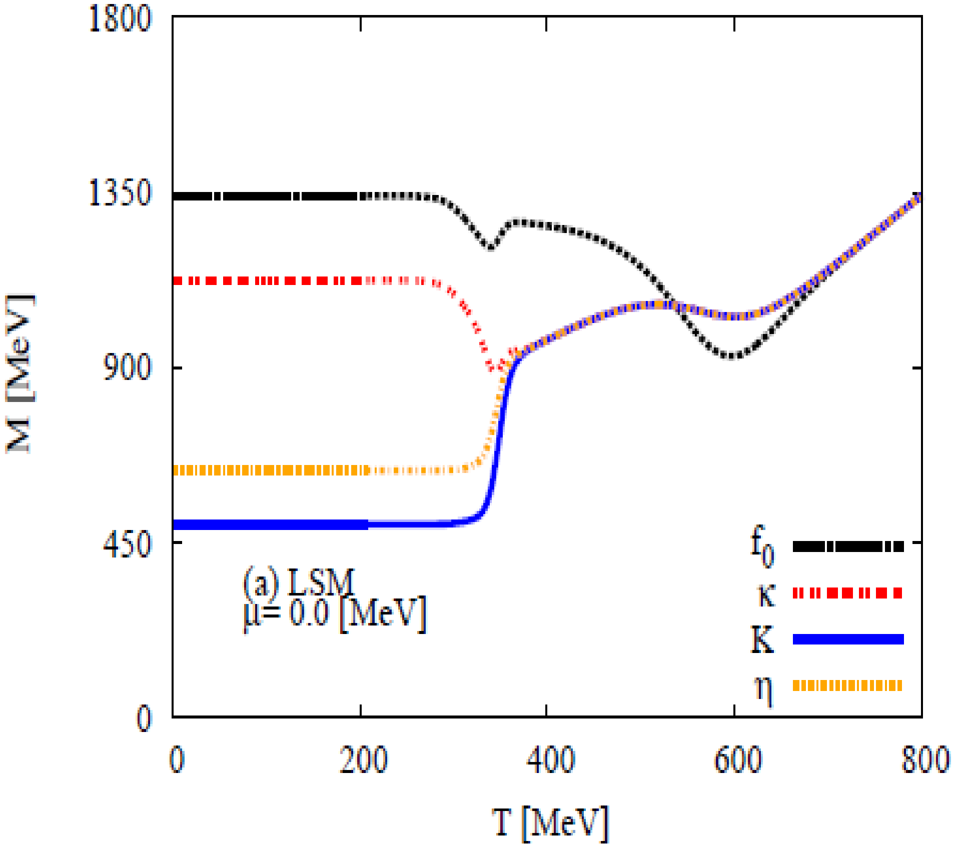
) Left- (LSM) and right-hand panel (PLSM) show scalars f_0 (horizontal dashed curve) and κ (vertical dashed curve) and pseudoscalars η (dotted curve) and K (solid curve) as function of temperature at vanishing baryon-chemical potentials $\mu = 0.0$ MeV.

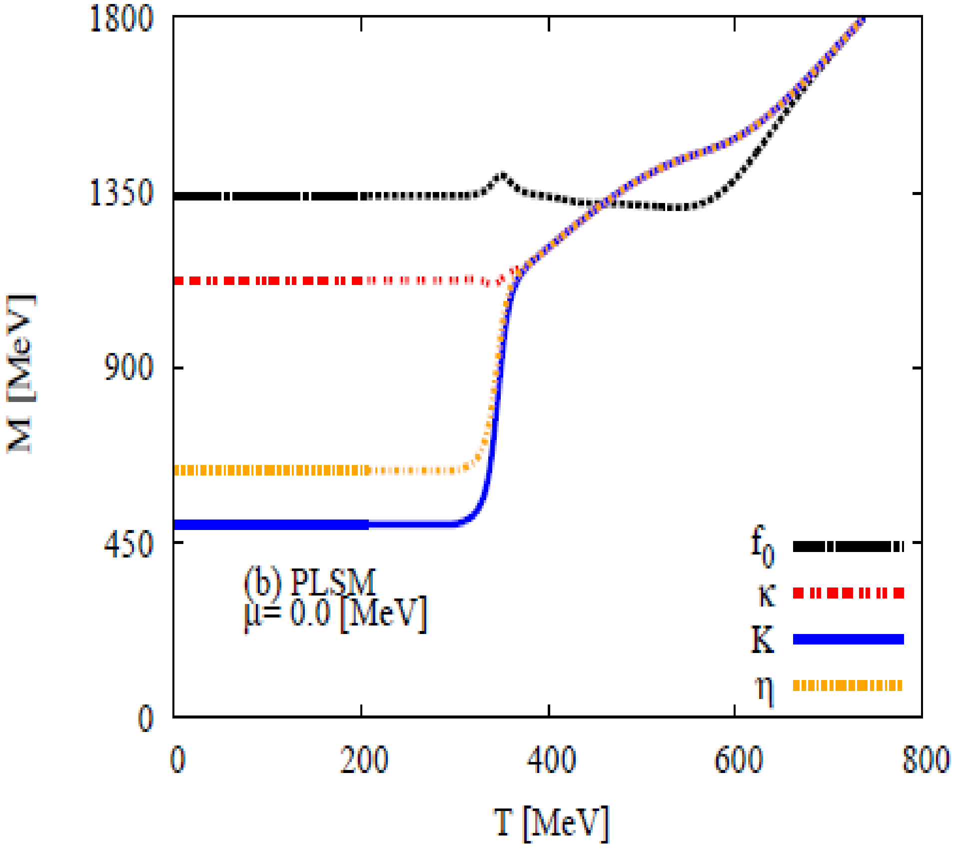
PLSM: In-medium meson masses



Left- (LSM) and right-hand panel (PLSM) present scalars a_0 (dashed curve) and σ (dotted curve) and pseudoscalars η' (solid curve) and π (dashed–dotted curve) in dense medium at fixed temperature $T = 10$ MeV.

PLSM: In-medium meson masses





Left- (LSM) and right-hand panel (PLSM) show scalars f_0 (horizontal dashed curve) and κ (vertical dashed curve) and pseudoscalars η (dotted curve) and K (solid curve) in dense medium at fixed temperature $T = 10$ MeV.

PLSM: Phase Structure Conclusions

- *We have studies the thermal, dense and magnetic in-medium modifications of* $a_0, \sigma, \eta', \pi, f_0, \kappa, \eta, K, \rho, \omega, \kappa^*, \phi, a_1, f_1, K^*$ *and* f_1^* *meson states*
- *This study shall be extended to charged meson states, e.g.,* K^+, K^-, π^+, π^-, *etc.*
- *The thermal and dense dependence of* K^+/π^+, K^-/π^-, *could be predicted, as well.*
- *Extending PLSM to include baryons is planned in near future.*

Hadron Resonance Gas Model

For an ideal gas of hadron resonances

$$Z(T,\mu,V) \;=\; \mathrm{Tr}\left[\exp^{\frac{\mu \mathbf{N}-\mathbf{H}}{\mathbf{T}}}\right]$$

$$\ln Z(T,\mu_i,V) \;=\; \sum_i \ln Z_i^1(T,V) = \sum_i \pm \frac{V g_i}{2\pi^2}\int_0^\infty k^2 dk \ln\left\{1 \pm \exp[(\mu_i - \varepsilon_i)/T]\right\}$$

where $\epsilon_i(k) = (k^2 + m_i^2)^{1/2}$ is the $i-$th particle dispersion relation, g_i is spin-isospin degeneracy factor and $\pm$ stands for bosons and fermions, respectively.

At finite temperature T and baryon chemical potential μ_i, the pressure of the i-th hadron resonance reads

$$p(T,\mu_i) \;=\; \pm\frac{g_i}{2\pi^2} T \int_0^\infty k^2 dk \ln\left\{1 \pm \exp[(\mu_i - \varepsilon_i)/T]\right\}$$

Hadron Resonance Gas Model

Switching between hadron and quark chemistry is possible from the analytic relations between the hadronic chemical potentials and the quark constituents;

$$\mu_i = 3 n_b \mu_q + n_s \mu_S,$$

- ***The chemical potential assigned to the light quarks is $\mu_q = (\mu_u + \mu_d)/2$ and the one assigned to strange quark reads $\mu_s = \mu_q - \mu_s$.***
- ***The strangeness chemical potential μ_S is calculated as a function of T and μ_n under the assumption that the overall strange quantum number must remain conserved in heavy-ion collisions.***
- ***The number density or particle multiplicity can be estimated as***

$$m_1(T, \mu_i) = \pm \frac{g_i}{2\pi^2} T \int_0^\infty \frac{e^{\frac{\mu_i - \varepsilon_i}{T}} \, k^2 \, dk}{1 \pm e^{\frac{\mu_i - \varepsilon_i}{T}}}.$$

Hadron Resonance Gas Model

- ***The "second" order moment is known as the variance reads***

$$m_2(T,\mu_i) = \pm\frac{g_i}{2\pi^2}T\int_0^\infty \frac{e^{\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}} - \frac{g_i}{2\pi^2}T\int_0^\infty \frac{e^{2\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^2}$$

- ***The "third" order moment measures of the lopsidedness of the distribution***

$$m_3(T,\mu_i) = \pm\frac{g_i}{2\pi^2}T\int_0^\infty \frac{e^{\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}} - \frac{g_i}{2\pi^2}3T\int_0^\infty \frac{e^{2\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^2} \pm \frac{g_i}{2\pi^2}2T\int_0^\infty \frac{e^{3\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^3}$$

- ***The "fourth" order moment compares the tallness and skinny or shortness and squatness, i.e., it defined the shape of the distribution***

$$m_4(T,\mu_i) = \pm\frac{g_i}{2\pi^2}T\int_0^\infty \frac{e^{\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}} - \frac{g_i}{2\pi^2}7T\int_0^\infty \frac{e^{2\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^2} \pm \frac{g_i}{2\pi^2}12T\int_0^\infty \frac{e^{3\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^3} - \frac{g_i}{2\pi^2}6T\int_0^\infty \frac{e^{4\frac{\mu_i-\varepsilon_i}{T}}\, k^2\, dk}{\left(1\pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^4}$$

Hadron Resonance Gas Model

- ***Now a general expression for the r-th order moment can be deduced***

$$m_r(T,\mu_i) = \frac{g_i}{2\pi^2} T \sum_{l=1}^{r} a_{r,l} \int_0^\infty \frac{e^{l\frac{\mu_i-\varepsilon_i}{T}}}{\left(1 \pm e^{\frac{\mu_i-\varepsilon_i}{T}}\right)^l} k^2\, dk,$$

where the coefficients read

$$a_{r,l} = (\pm 1)^l (-1)^{l+1} \left[l\, a_{r-1,l} + (l-1)\, a_{r-1,l-1}\right],$$

$$m_2 = \langle (\delta N)^2 \rangle \approx 2\langle N \rangle,$$

$$m_3 = \langle (\delta N)^3 \rangle \approx 4\langle N \rangle + C_3,$$

$$m_4 \simeq \langle (\delta N)^4 \rangle = 6m_3 + 3m_2^2 - 8m_2 + 8C_4,$$

$$\delta N = N - \langle N \rangle$$

$$C_3(p_1,p_2,p_3) = \sum_{p_1} \langle p_1p_1p_1 \rangle + 3 \sum_{p_1<p_2} \langle p_1p_2p_2 \rangle + 6 \sum_{p_1<p_2<p_3} \langle p_1p_2p_3 \rangle,$$

$$C_4(p_1,p_2,p_3,p_4) = \sum_{p_1<p_2} \langle p_1p_2p_2p_2 \rangle + \sum_{p_1<p_2<p_3} \langle p_1p_2p_3p_3 \rangle + 8 \sum_{p_1<p_2<p_3<p_4} \langle p_1p_2p_3p_4 \rangle,$$

Hadron Resonance Gas Model

- ***By considering the particle multiplicities,***

$$m_2 = \langle(\delta N)^2\rangle \simeq \langle N^2\rangle - \langle N\rangle^2,$$

$$m_3 = \langle(\delta N)^3\rangle \simeq \langle N^3\rangle - \langle N^2\rangle\langle N\rangle + 2\langle N^3\rangle^3,$$

$$m_4 = \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \simeq \langle\langle N\rangle^4\rangle - 2\langle\langle N^2\rangle^2\rangle - 5\langle\langle N^2\rangle\rangle^2 + 6\langle\langle N\rangle^2\rangle\,\langle\langle N^2\rangle\rangle$$

$$P(T,\mu_i) = \frac{g_i}{2\pi^2} T^4 \sum_{n=1}^{\infty} (\pm)^{n+1} e^{n\frac{\mu_i}{T}} \left(n\frac{m_i}{T}\right)^2 n^{-4} \mathrm{K}_2\left(n\frac{m_i}{T}\right)$$

$$m_r(T,\mu_i) = \frac{g_i}{2\pi^2} T^4 \sum_{n=1}^{\infty} (\pm)^{n+1} e^{n\frac{\mu_i}{T}} \left(n\frac{m_i}{T}\right)^2 n^{r-4} \mathrm{K}_2\left(n\frac{m_i}{T}\right)$$

Hadron Resonance Gas Model

- ***Chemical Freezeout***

$$\frac{\sigma^2}{\langle N\rangle} = \frac{\sum_{n=1}^{\infty}(\pm)^{n+1}\, e^{n\frac{\mu_i}{T}}\, \left(n\frac{m_i}{T}\right)^2 n^{-2} K_2\left(n\frac{m_i}{T}\right)}{\sum_{n=1}^{\infty}(\pm)^{n+1} e^{n\frac{\mu_i}{T}}\, \left(n\frac{m_i}{T}\right)^2 n^{-3} K_2\left(n\frac{m_i}{T}\right)},$$

$$\kappa\,\sigma^2 = \frac{\sum_{n=1}^{\infty}(\pm)^{n+1} e^{n\frac{\mu_i}{T}}\, \left(n\frac{m_i}{T}\right)^2 K_2\left(n\frac{m_i}{T}\right)}{\sum_{n=1}^{\infty}(\pm)^{n+1} e^{n\frac{\mu_i}{T}}\, \left(n\frac{m_i}{T}\right)^2 n^{-2} K_2\left(n\frac{m_i}{T}\right)} - 3\frac{g_i}{2\pi^2} T^4 \sum_{n=1}^{\infty}(\pm)^{n+1}\, e^{n\frac{\mu_i}{T}}\, \left(n\frac{m_i}{T}\right)^2 n^{-2} K_2\left(n\frac{m_i}{T}\right).$$

$$\left(\kappa\,\sigma^2\right)_b = -\frac{1}{4}\frac{\int_0^{\infty}\left\{\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]+2\right\}\ \mathrm{csch}\left[\frac{\varepsilon_i-\mu_i}{2T}\right]^4\ k^2\, dk}{\int_0^{\infty}\left(1-\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]\right)^{-1}\ k^2\, dk} + \frac{3\,g_i}{4\,\pi^2}\frac{1}{T^3}\int_0^{\infty}\left(1-\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]\right)^{-1}\ k^2\, dk,$$

$$\left(\kappa\,\sigma^2\right)_f = \frac{1}{4}\frac{\int_0^{\infty}\left\{\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]-2\right\}\ \mathrm{Sech}\left[\frac{\varepsilon_i-\mu_i}{2T}\right]^4\ k^2\, dk}{\int_0^{\infty}\left(\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]+1\right)^{-1}\ k^2\, dk} - \frac{3\,g_i}{4\,\pi^2}\frac{1}{T^3}\int_0^{\infty}\left(\cosh\left[\frac{\varepsilon_i-\mu_i}{T}\right]+1\right)^{-1}\ k^2\, dk.$$

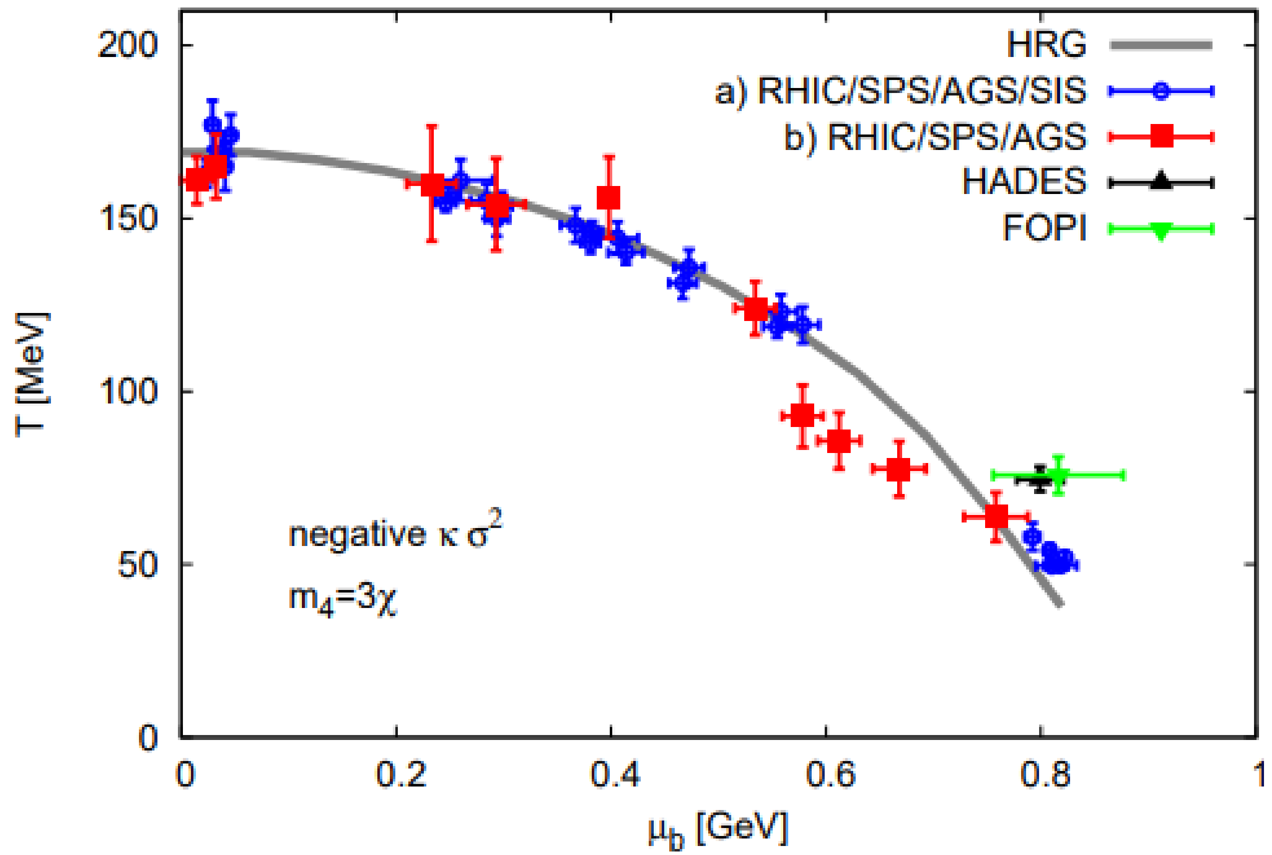
$$\frac{\sigma^2}{\langle N\rangle} = S\,\sigma$$

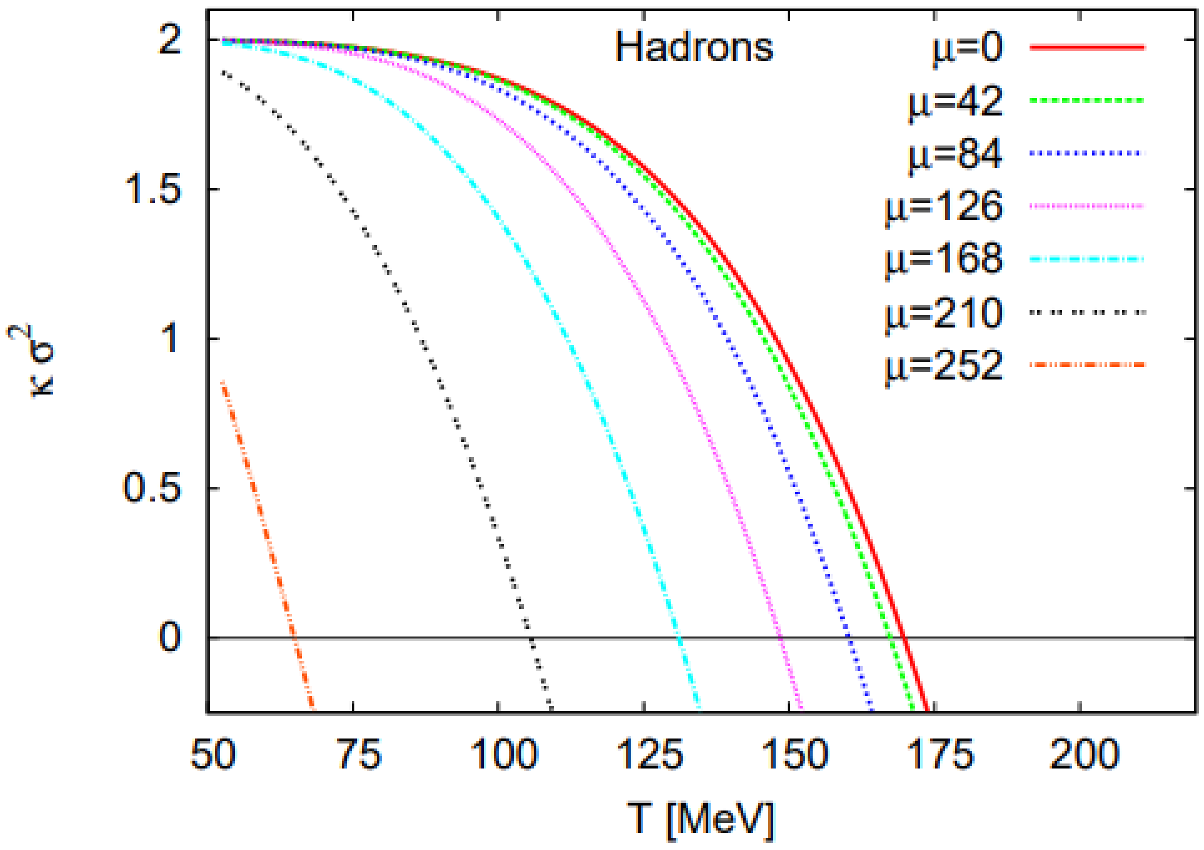
$$\kappa\,\sigma^2 \simeq 1 - 3\frac{g_i}{2\pi^2} T^4 \exp\left[\frac{\mu_i}{T}\right]\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right)$$

$$\frac{\sigma^2}{\langle N\rangle} = \frac{1}{2}\frac{\int_0^{\infty}\left(1\pm\mathrm{csch}\left[\frac{\varepsilon_i-\mu_i}{T}\right]\right)^{-1}\ k^2\, dk}{\int_0^{\infty}\left(1\pm e^{\frac{\varepsilon_i-\mu_i}{T}}\right)^{-1}\ k^2\, dk},$$

Hadron Resonance Gas Model

- ***Chemical Freezeout*** Physics of vanishing $\kappa\,\sigma^2$ or equivalently $m_4 = 3\chi^2$





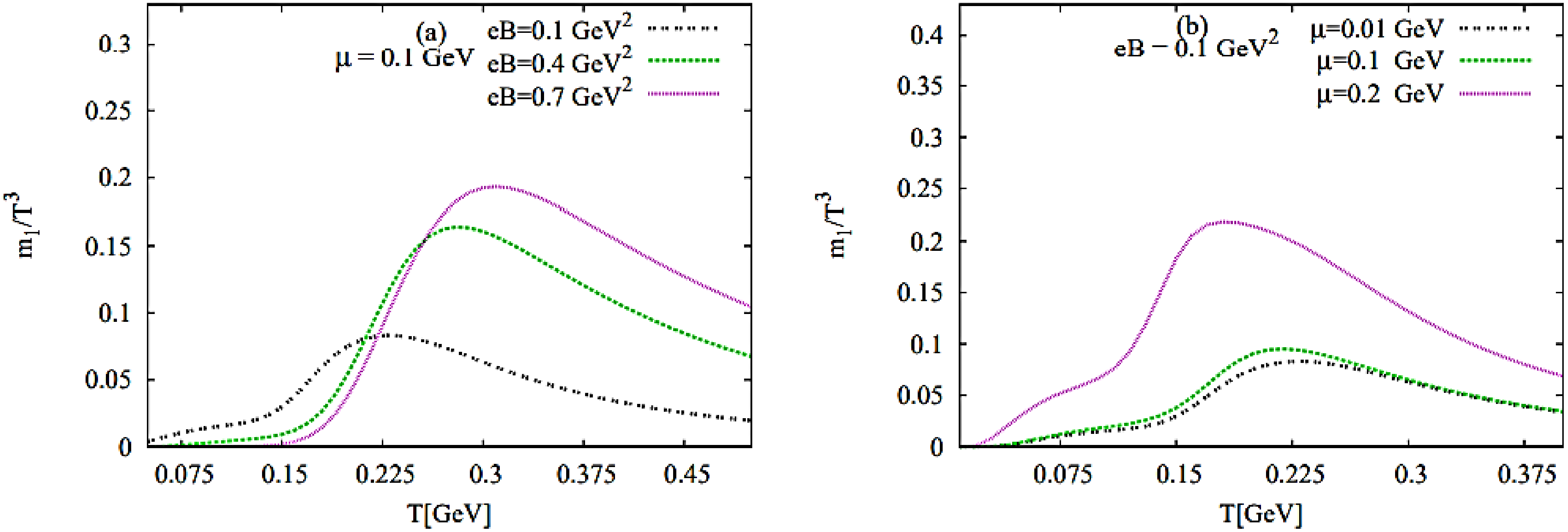
Higher-Order Moments in PLSM

The higher-order moment of the particle multiplicity is defined is

$$m_i = \frac{\partial^i}{\partial \mu^i} \frac{p(T, \mu, B)}{T^4},$$

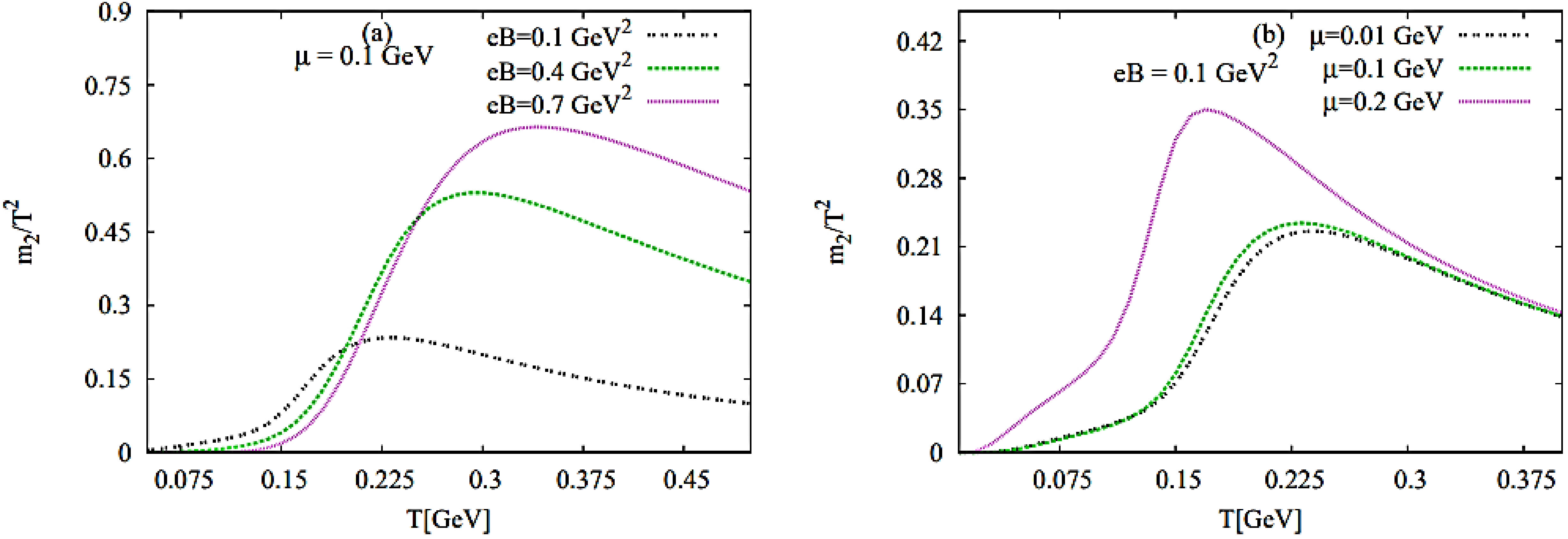
$$p(T, \mu, B) = -T \partial \ln \mathcal{Z}(T, \mu, B)/\partial V$$

$$\ln \mathcal{Z}(T, \mu, B) = -V\, \Omega(T, \mu, B)/T.$$



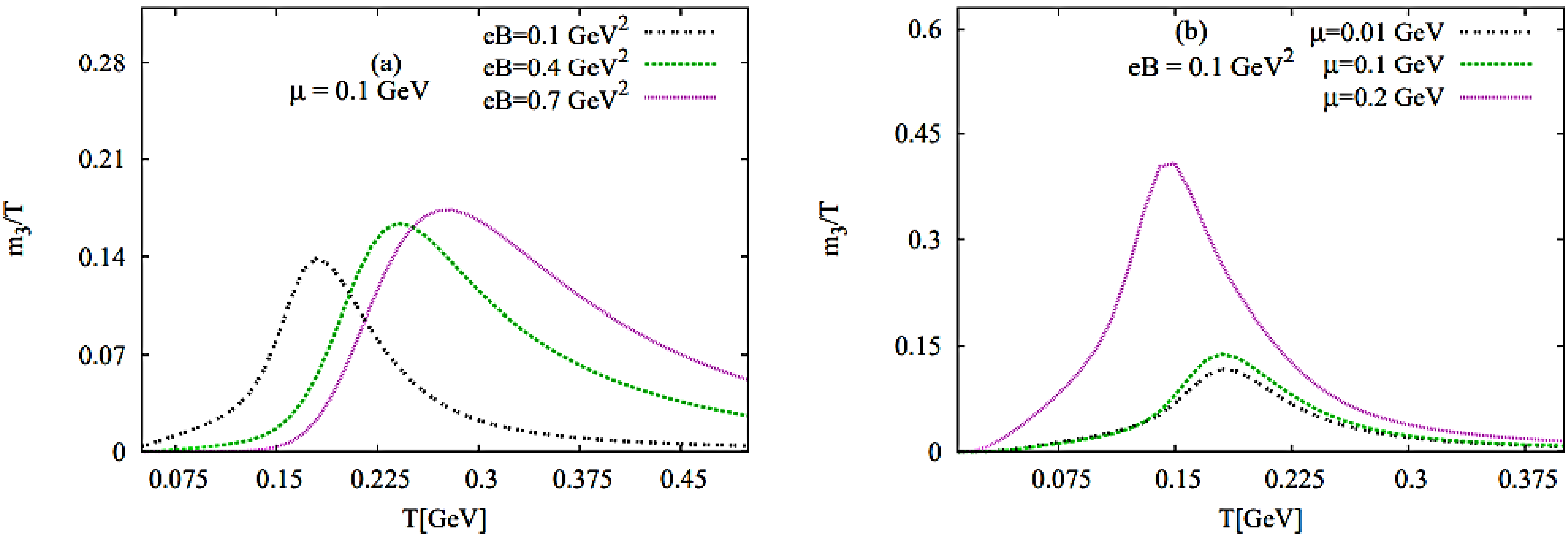
Higher-Order Moments in PLSM

- ***T-dependence of the second-order moment, at finite eB and μ***

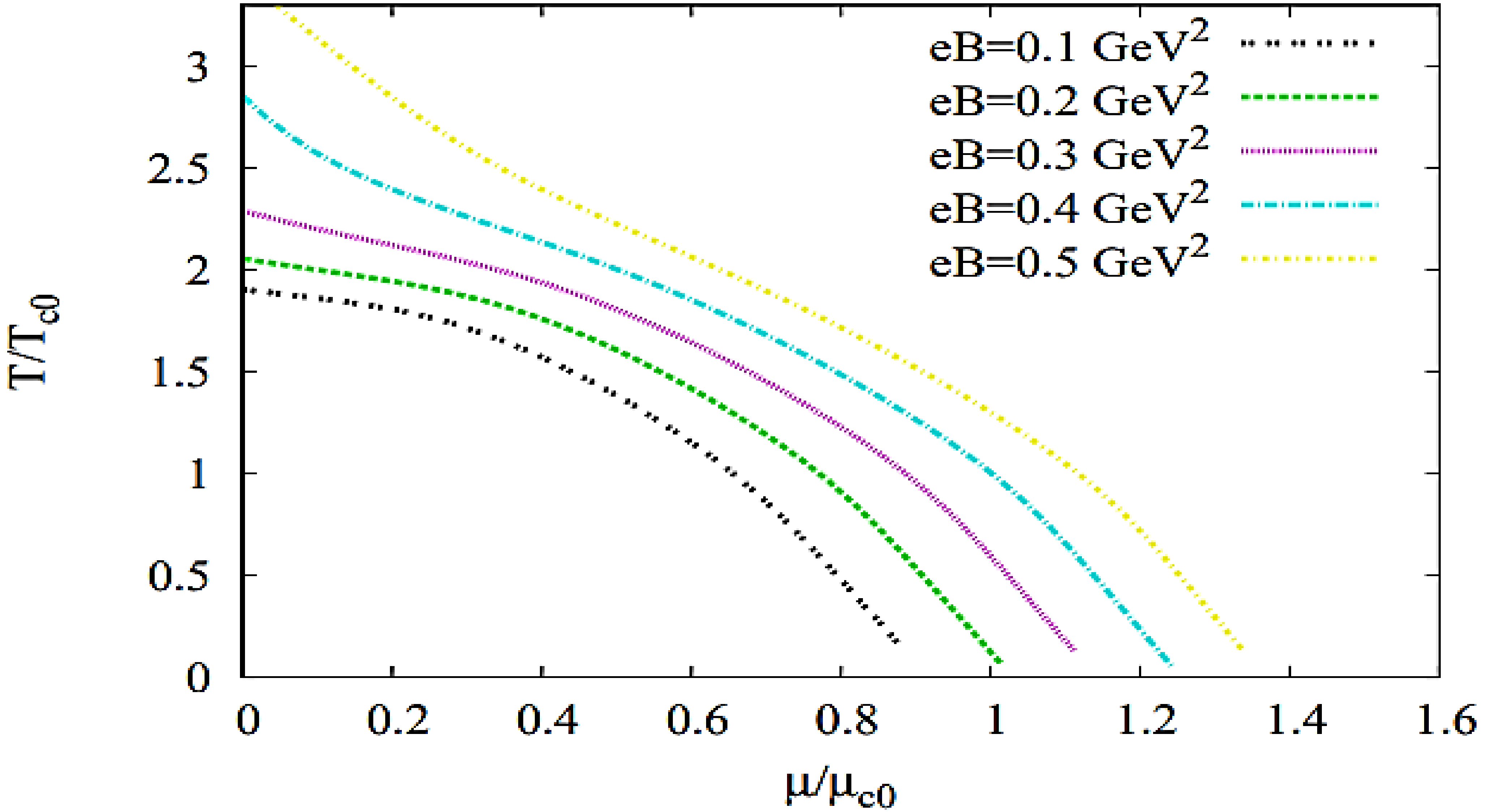


Higher-Order Moments in PLSM

- ***T-dependence of the third-order moment, at finite eB and μ***

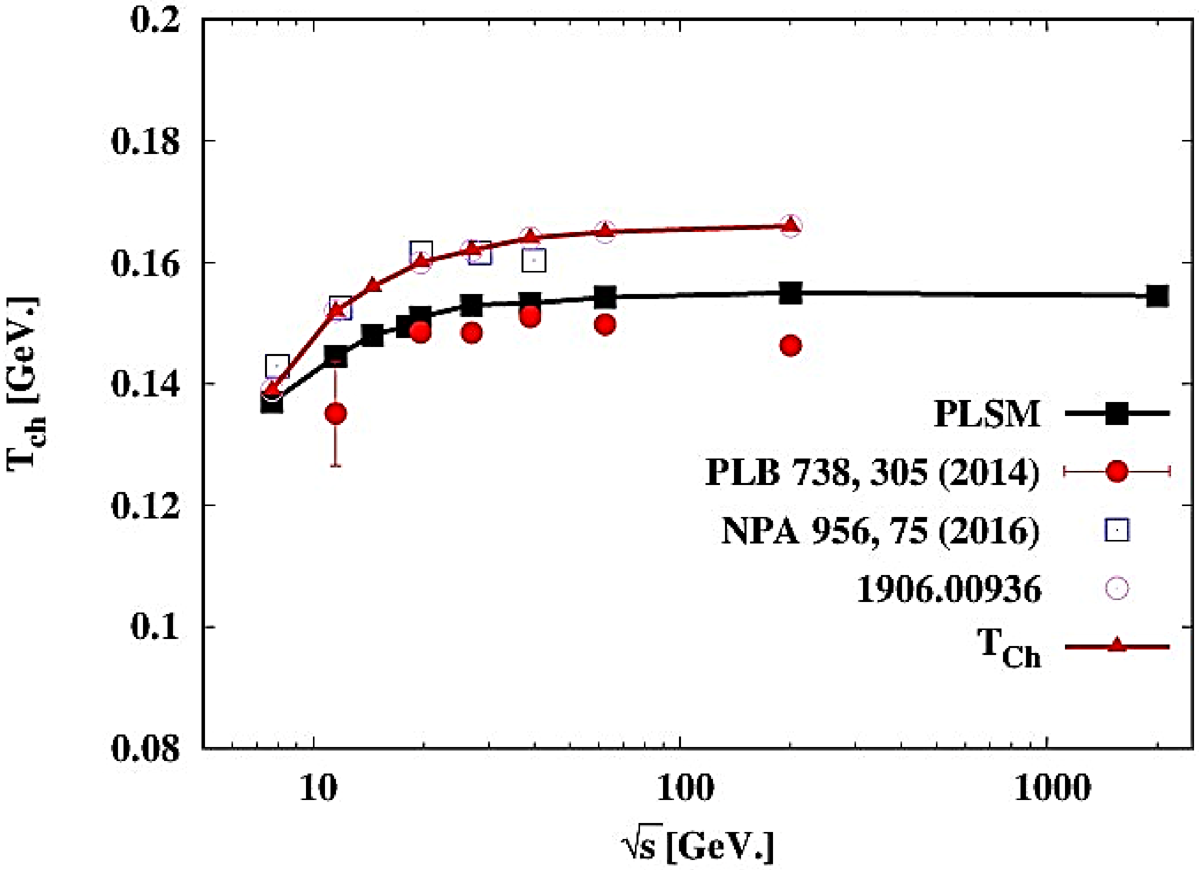


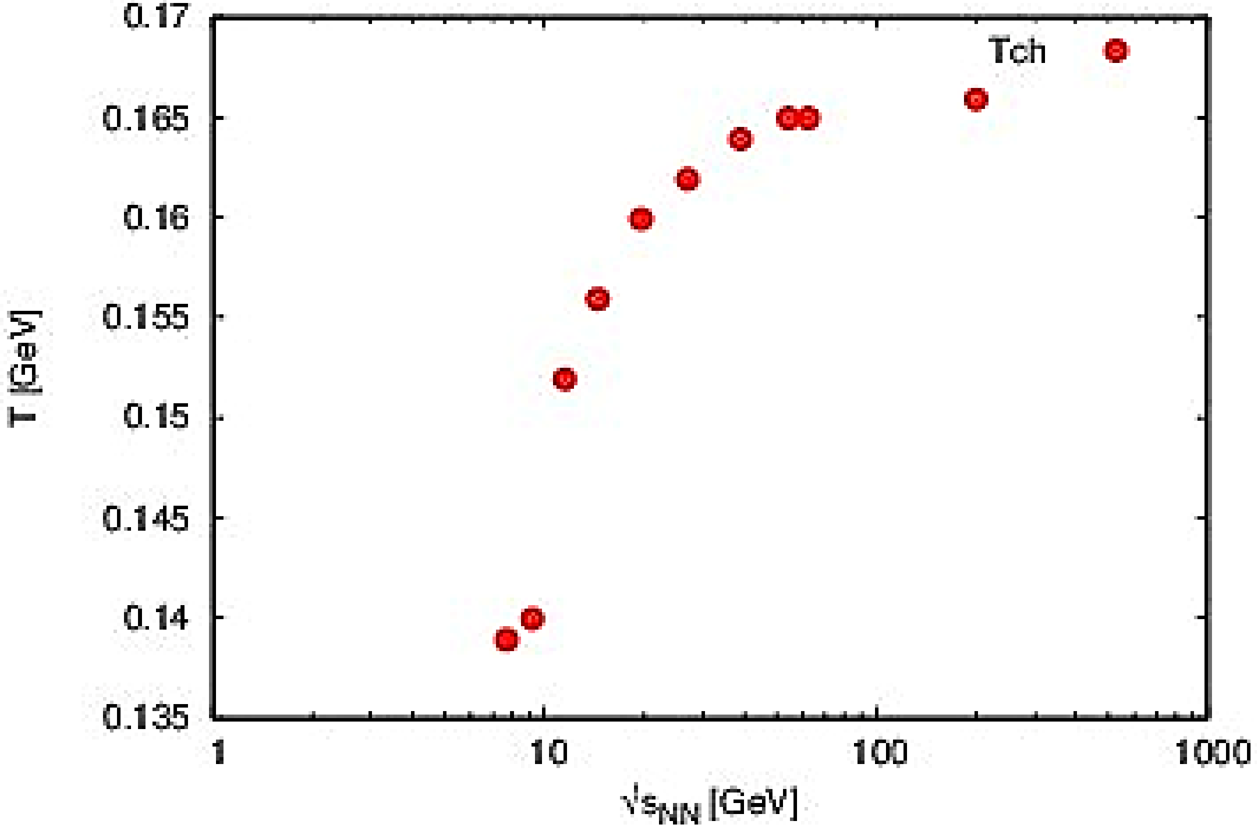
Higher-Order Moments in PLSM



Higher-Order Moments in PLSM & Experiment

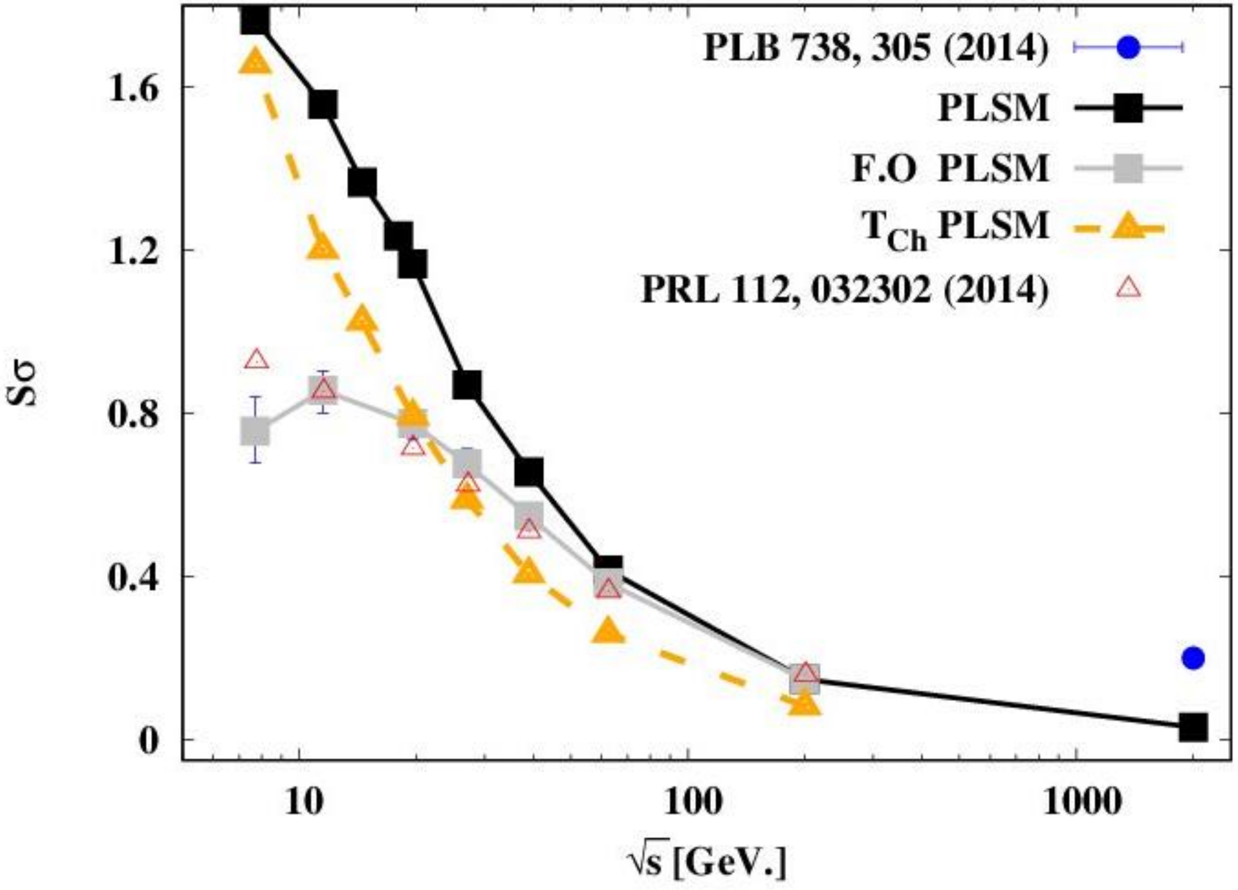
Freezeout Temperatures in PLSM

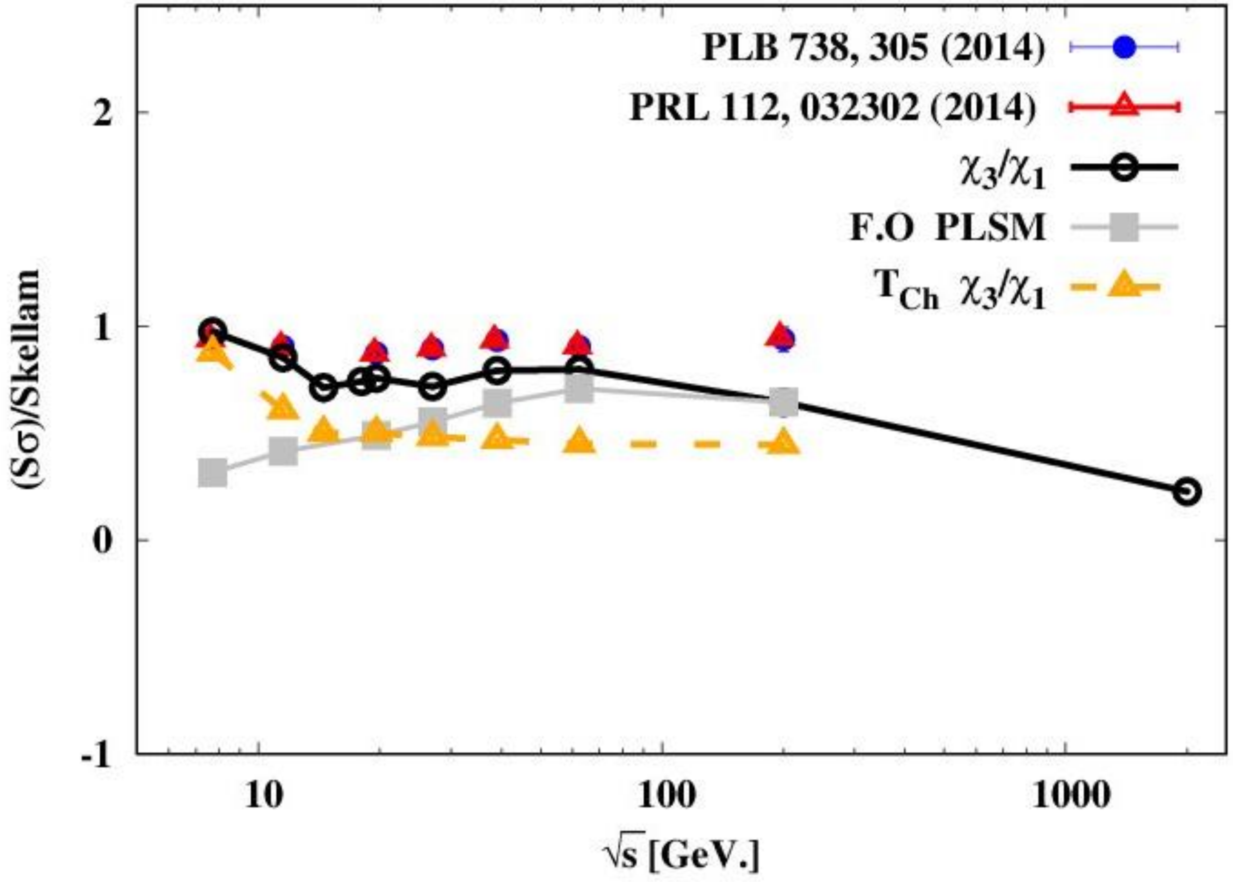




Higher-Order Moments in PLSM & Experiment

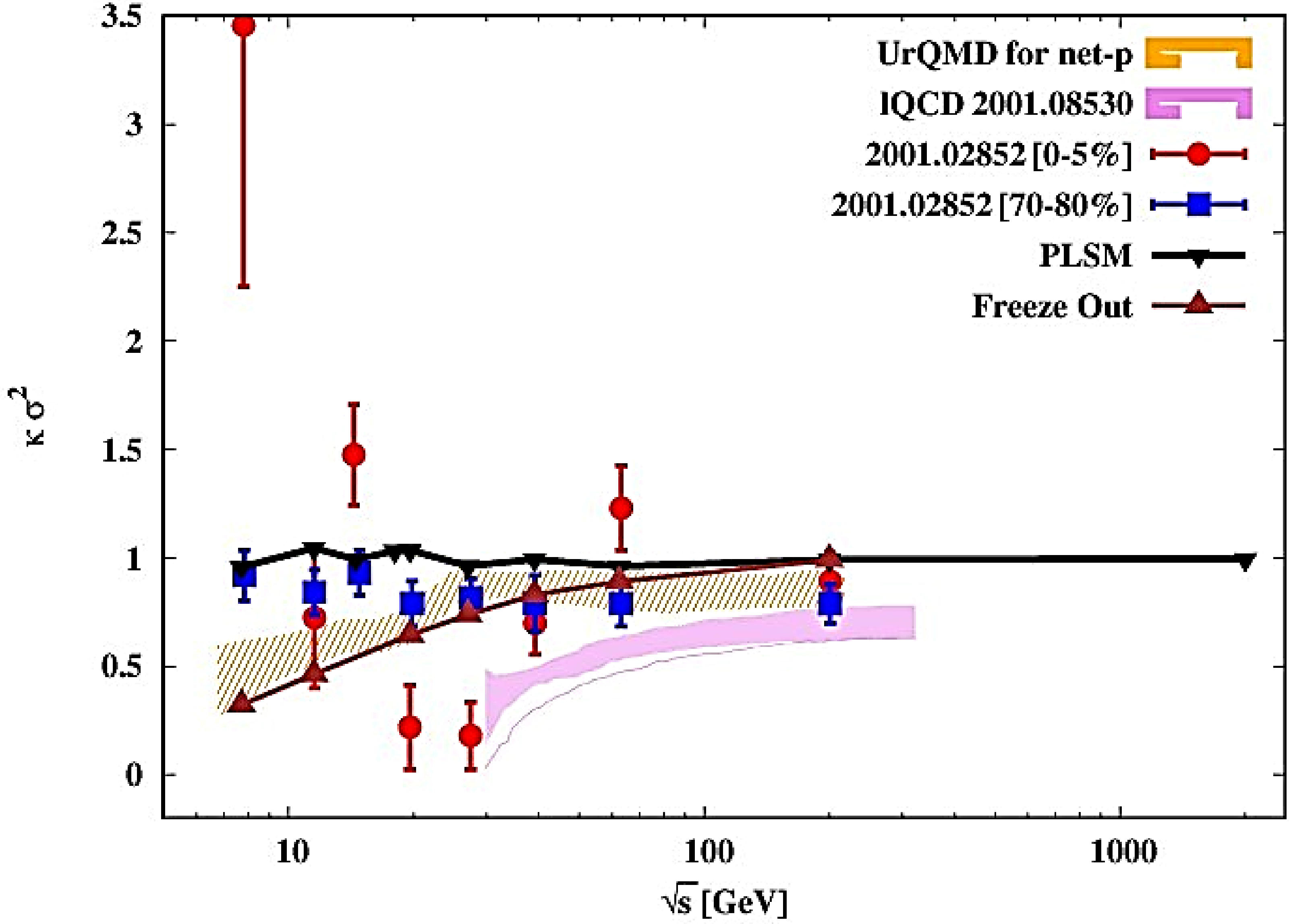
Freezeout Temperatures in PLSM





Higher-Order Moments in PLSM & Experiment

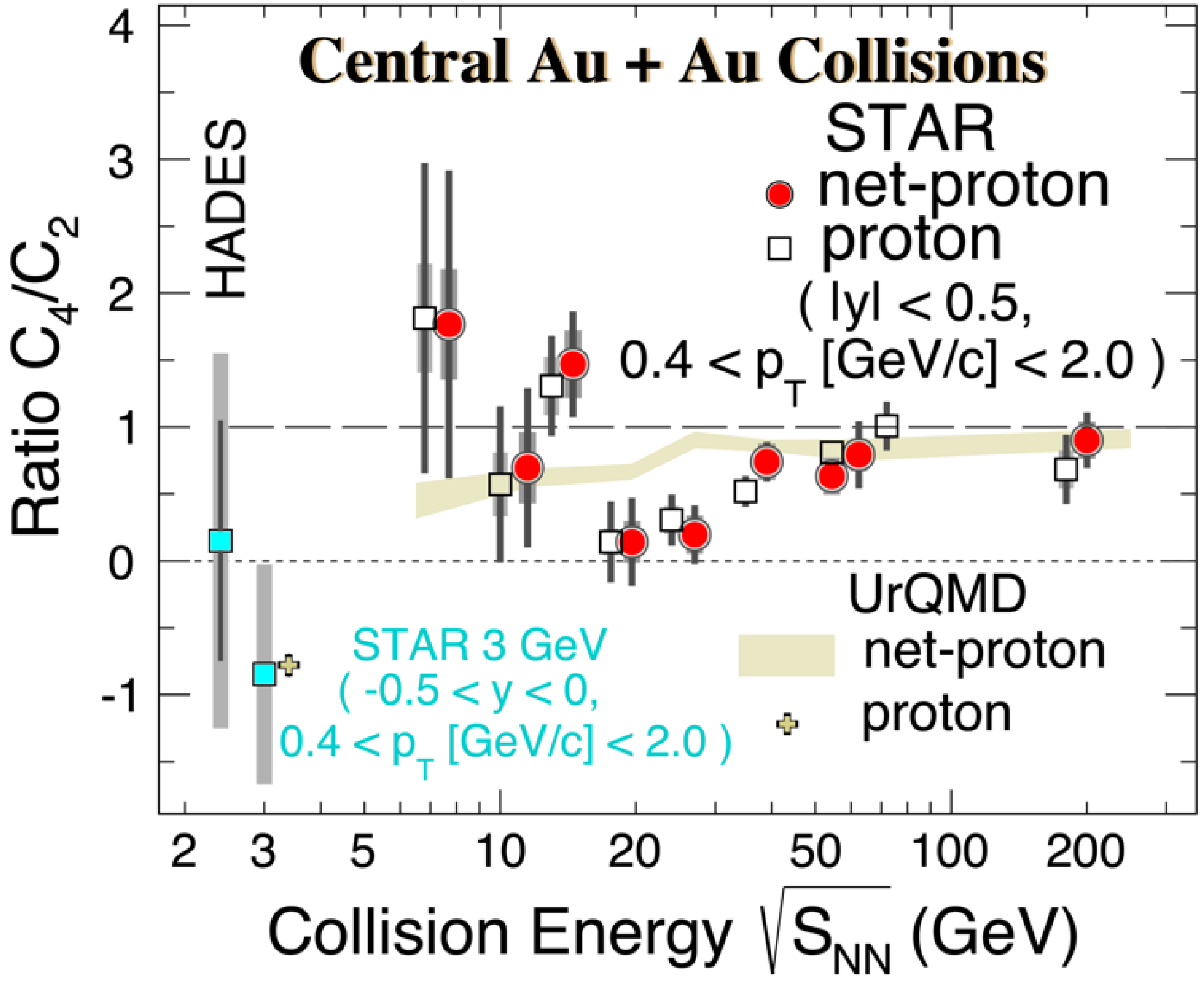
2020
STAR
Results

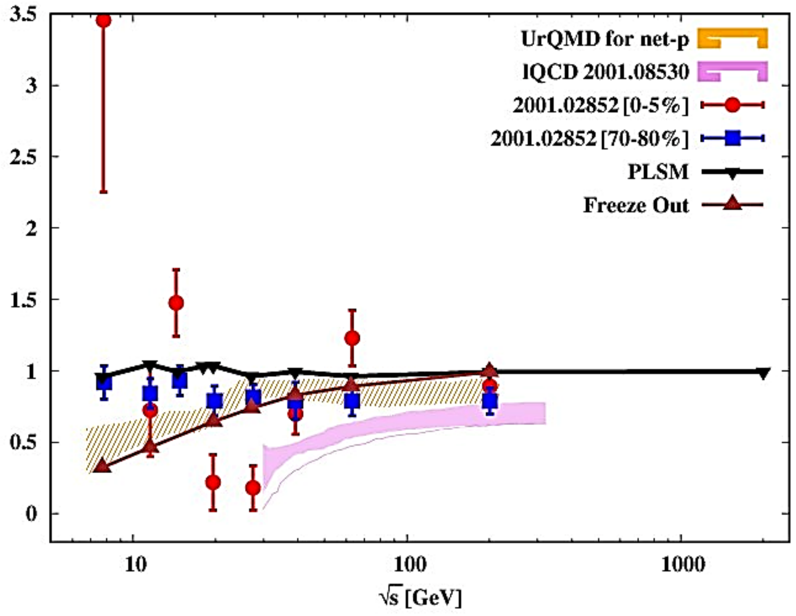


Higher-Order Moments in PLSM & Experiment

2022
STAR
Results

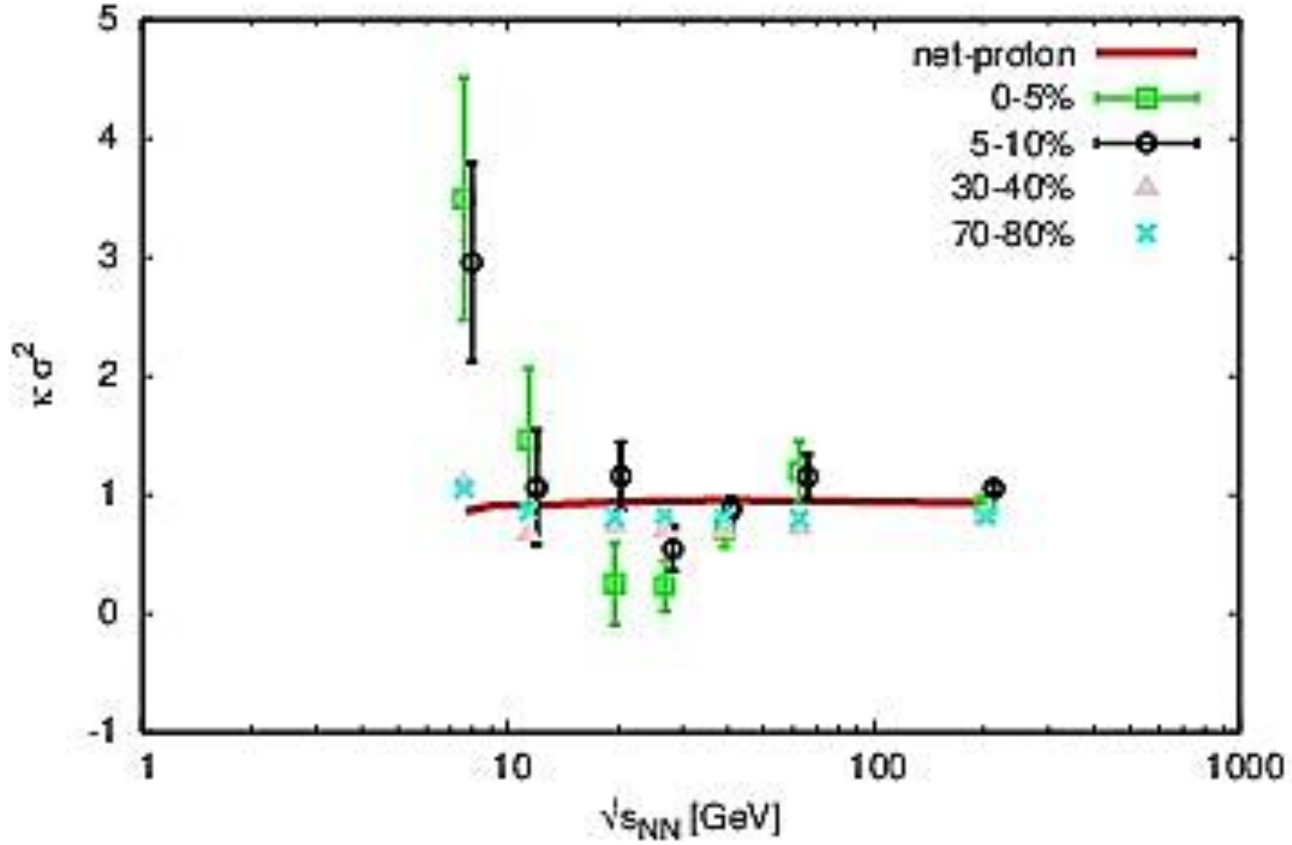
[PRL **128**, 202303](#)

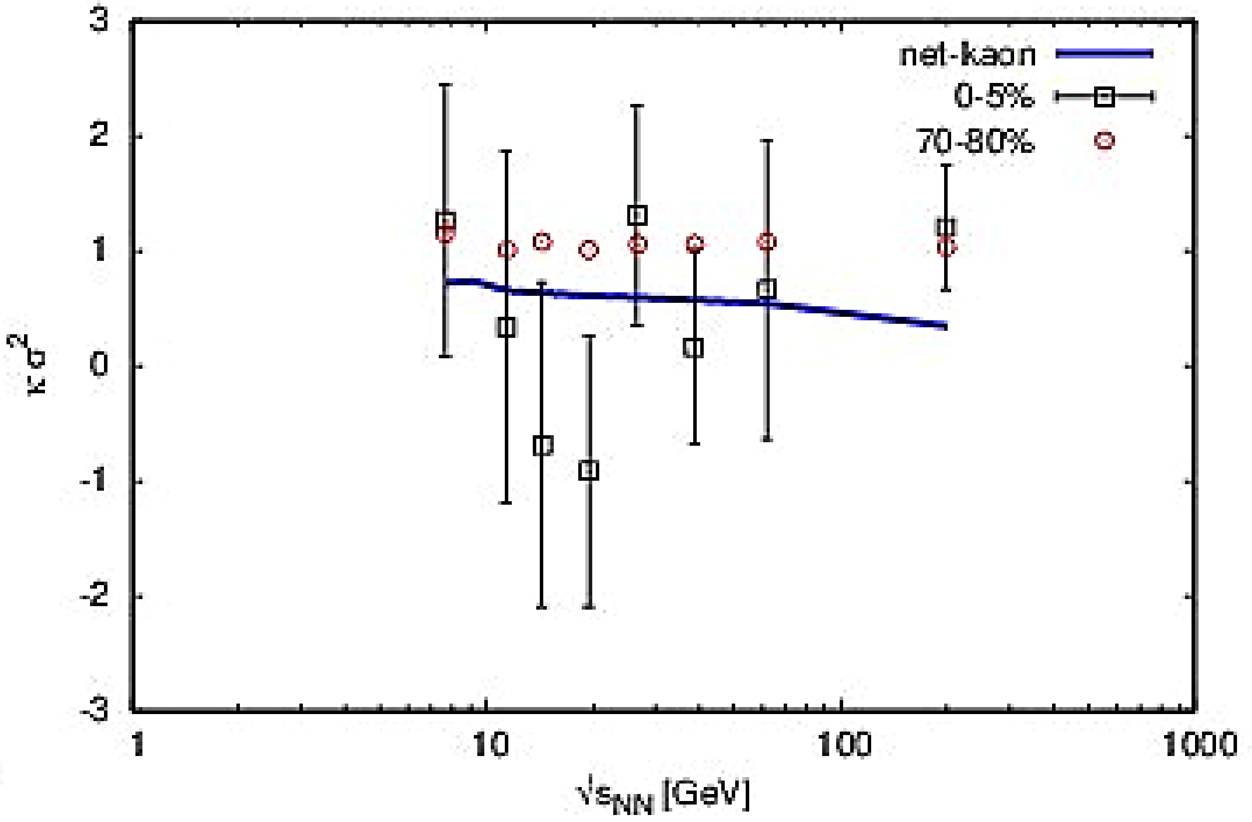




Higher-Order Moments in PLSM & Experiment

With Non-Extensive Statistics (not necessarily of Tsallis-type)





Higher-Order Moments in PLSM & Experiment

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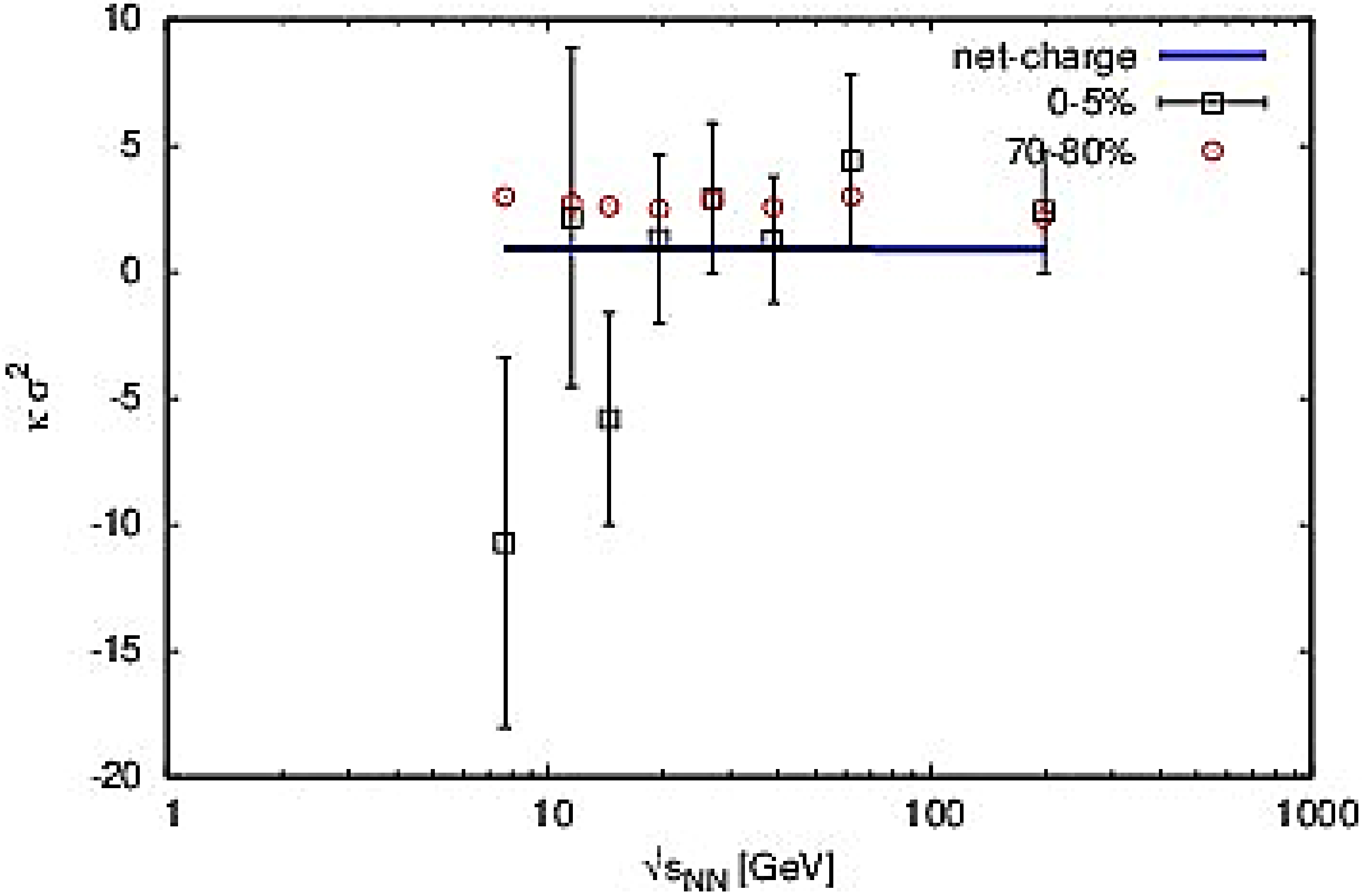

Conclusion: Higher-Order Moments

- We observe that HRGM excellently reproduces experimental multiplicity and particle ratios, i.e., first-order, why not the higher-order cumulants!

- The statistical nature seems to play an essential role
  - Microcanonical (fixed energy), canonical (fixed temperature) or grand canonical ensemble (fixed T and μ)
  - Extensive or nonextensive or generic (super) statistics
  - Ideal static equilibrium or interacting dynamic nonequilibrium

- Indeed, if statistical nature is essential for higher-order cumulants, it is as well essential for multiplicity, as well!

Thank you!

Спасибо!