

Особенности электронного строения 8-го периодов

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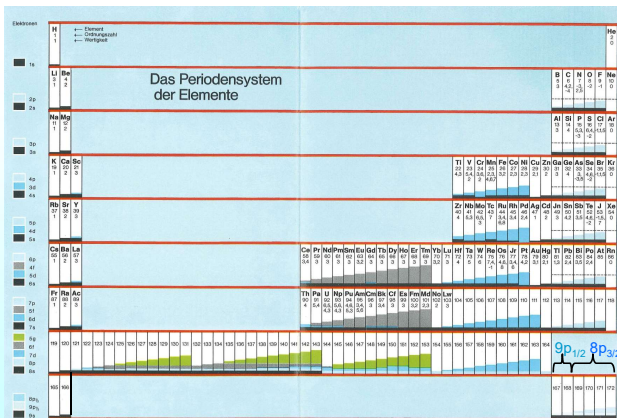
18 December, 2022

- Introduction
- Ground state electron configurations of superheavy elements (SHE)
- Relativistic effects in the electronic structure of SHE
- Orbital collapse of the 5g-states
- Summary and Outlook

Future Periodic Table of the Elements

1																	18		
1 H																	2 He		
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne		
11 Na	12 Mg	3	4	5	6	7	8	9	10	11	12	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar		
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr		
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe		
55 Cs	56 Ba	La →	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn		
87 Fr	88 Ra	89 Ac →	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111	112 Cn	113	114 Fl	115	116 Lv	117	118		
		;(119);(120);(121);		?												Z=???			
		Lanthanides →	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu			
		Actinides →	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr			
		Superactinides →	(122 - 155)																

Periodic Table of Fricke, Greiner and Waber



Introduction. Pyykkö's periodic table

Periodic Table 1-172

Period	1																18 Orbitals									
1	1 H	2														13	14	15	16	17	2 He	1s				
2	3 Li	4 Be														5 B	6 C	7 N	8 O	9 F	10 Ne	2s2p				
3	11 Na	12 Mg	3	4	5	6	7	8	9	10	11	12	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	3s3p							
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	4s3d4p							
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	5s4d5p							
6	55 Cs	56 Ba	57-71 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	6s5d6p							
7	87 Fr	88 Ra	89-103 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113	114	115	116	117	118	7s6d7p							
8	119	120	121-	156	157	158	159	160	161	162	163	164	139	140	169	170	171	172	8s7d8p							
9	165	166											167	168				9s9p								
6	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu					4f						
7	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr					5f						
8	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155					6f						
8	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	5g							

The table is taken from P. Pyykkö, Phys. Chem. Chem. Phys., 13, 161 (2011)

In this work, the results of the electronic-structure calculations for a number of super-heavy elements (SHE) of the 7th and 8th periods with atomic numbers $Z=111-170$ and their lighter homologous are performed [1-3].

- Ground state electron configurations of superheavy elements (SHE)
- Ionization potentials
- Electron affinities
- One-particle electron density
- Root-mean-square radii of the valence states
- Widths of the electron-density distribution of valence shells
- Electron localization functions (ELF)
- Shannon entropy
- Quantum electrodynamics (QED) corrections

1. M. Y. Kaygorodov et al., *Phys. Rev. A* 104, 012819 (2021)
2. I. I. Tupitsyn et al., *Opt. Spectr.* 129, 1038 (2021)
3. M. Y. Kaygorodov et al., *Phys. Rev. A* 105, 062805 (2022)

Introduction. Specific features of electronic structure and chemical properties of super-heavy elements of the 7th and 8th periods

The electronic structure of SHEs is unique in several aspects:

- Strong relativistic effects cause contraction of the s- and p- orbitals
- Spin-orbital splitting of valence p-shells reaches up about 10 eV in Og ($Z=118$) and about 420 eV for the 7p-orbital and about 75 eV for the 8p-orbital in element with atomic number $Z=165$.
- As a result, due to the strong relativistic contraction, the radial distribution of the electron density of the valence $7p_{1/2}$ -shell of the Og atom starts to overlap with the outer core shells and ELF is close to 0.5 in the valence region. In Ref. [1], this effect in Og was interpreted as smearing out the valence electron density distribution and its approaching to the case of the homogeneous electron gas.
- Starting from the $Z = 125$ element, the 5g-shell with the large angular momentum ($l = 4$) is occupied with electrons.
- The effective radial potential for the 5g-electron, which includes a large centrifugal repulsive term, has two potential wells which leads to the so-called *orbital collapse*.

[1] P. Jerabek, B. Schuetrumpf, P. Schwerdtfeger, and W. Nazarewicz, *Phys. Rev. Lett.* **120**, 053001 (2018).

In our work, we used two independent theoretical calculation methods.

- **Configuration Interaction Dirac-Fock-Sturm method (CI-DFS)**

At the first step, to obtain the one-electron wave functions for the occupied atomic shells, we use the Dirac-Fock method. Then the DFS orbitals are obtained by solving the DFS equations for the vacant shells. At the last step, the relativistic CI+MBPT method is used to obtain the many-electron wave functions and the total energies.

- **Fock Space Coupled-Cluster method (FS-CC)**

DIRAC, a relativistic ab initio electronic structure program, Release DIRAC21 (2021), <http://www.diracprogram.org>

FS-CC method, in contrast to the one-configuration coupled-cluster method is capable of providing not only the ground-state energy of an N-electron system, but also an important fraction of system's excitation spectrum, including ionization potentials, electron affinities, etc.

- **To evaluate the QED correction we use the model QED operator approach[1]**

[1] V.M. Shabaev, I.I. Tupitsyn, V.A. Yerokhin, *Comput.Phys.Commun.* 189, 175 (2015)

Ground state configurations

For $Z = 126$ the configurations with the *lowest Dirac-Fock (DF) energy* within the configuration average approximation are

Configuration	E_{DF}^{av} [a.u.]
$8p^1 7d^1 6f^2 5g^2$	-66298.183666
$8p^1 6f^3 5g^2$.183121
$8p^1 6f^4 5g^1$.168137
$8p^2 6f^2 5g^2$.146963
$8p^1 7d^2 6f^1 5g^2$.114149

- $8p^2 6f^2 5g^2$ Mann et al., 1970 (DF)
- $8p^1 7d^1 6f^2 5g^2$ Fricke et al., 1977 (DFS)
- $8p^1 6f^4 5g^1$ Umemoto and Saito, 1997 (DF+PZ SIC)
- $8p^1 6f^3 5g^2$ (0.98) Nefedov et al., 2006 (MCDF)
- $8p^1 7d^1 6f^2 5g^2$ Zhou et al., 2017 (DF)

Ground electron configurations

Таблица: Ground state electron configurations

Core: [Og] 8s ²				
Z	Conf.	J	Ref[1]	Ref[2]
125	5g ¹ 6f ² 7d ¹ 8p ¹	8.5	5g ¹ 6f ³ 8p ¹	5g ¹ 6f ² 8p ²
126	5g ² 6f ² 7d ¹ 8p ¹	10	5g ² 6f ² 7d ¹ 8p ¹	5g ² 6f ³ 8p ¹
127	5g ³ 6f ² 7d ¹ 8p ¹	13.5	5g ³ 6f ² 8p ²	5g ³ 6f ² 8p ²
Core: [Og] 8s ² 8p _{1/2} ²				
144	5g ¹⁸ 6f ¹ 7d ³	4.0	5g ¹⁸ 6f ¹ 7d ³	5g ¹⁷ 6f ¹ 7d ³
145	5g ¹⁸ 6f ³ 7d ²	6.5	5g ¹⁸ 6f ³ 7d ²	5g ¹⁸ 6f ³ 7d ²
162	5g ¹⁸ 6f ¹⁴ 7d ⁸	4.0	5g ¹⁸ 6f ¹⁴ 7d ⁸	5g ¹⁸ 6f ¹⁴ 7d ⁷ 9s ¹
163	5g ¹⁸ 6f ¹⁴ 7d ⁹	2.5	5g ¹⁸ 6f ¹⁴ 7d ⁹	5g ¹⁸ 6f ¹⁴ 7d ⁸ 9s ¹
164	5g ¹⁸ 6f ¹⁴ 7d ¹⁰	0.0	5g ¹⁸ 6f ¹⁴ 7d ¹⁰	5g ¹⁸ 6f ¹⁴ 7d ⁹ 9s ¹
165	5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 9s ¹	0.5	5g ¹⁸ 6f ¹⁴ 5g ¹⁸ 7d ¹⁰ 9s ¹	—
166	5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 9s ²	0.0	5g ¹⁸ 6f ¹⁴ 5g ¹⁸ 7d ¹⁰ 9s ²	—

[1] B. Fricke and G. Soff, *Atomic Data and Nuclear Data Tables* 19, 83 (1977).

[2] V.I. Nefedov, M. Trzhaskovskaya, *Dokl. Phys. Chem.* 408, 149 (2006).

Electronic Configuration of E140

Method	Ground state	Ref.	Year
high-sectors FSC	$5g^{18}8p^2$ –not!	Eliav	2019
MCDF (OL) + QED	$5g^{15}8p^46f$ (?)	Indelicato	2011
MCDF (AL)	$5g^{14}6f^37d8p^2$	Nefedov	2006
MCDF (AL)	$5g^{16}8p^2$ (2+)	Pyykkö	2011
DF	$5g^{14}6f^37d8p^2$	Fricke	1971
DF	$5g^{18}7d^3$	Nefedov	2004
Madelung	$5g^{18}8p^2$	-	-

QED ~ 10%

Presently impossible to accurately predict the ground state!

IH-RFSCC electronic configurations and properties of SHE with $164 \leq Z \leq 174$

Z	Configuration E. Eliav	Possible group	Period	Properties (eV) IP EA	
164	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ²	12, 14, 18	8	7.513	0.071
165	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s	1	9	5.652	0.987
166	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ²	2	9	7.345	0.518
167	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 8p _{3/2}	15	8	6.639	1.290
168	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 8p _{3/2} ²	16	8	8.055	
169	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ² 8p _{3/2}	15	8	8.968	2.518
170	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ² 8p _{3/2} ²	16	8	9.975	
171	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ² 8p _{3/2} ³	17	8	10.917	3.922
172	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ²	18	8	11.967	<0
173	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ² 10s	1	10	3.070	0.478
174	[Og]8s ² 5g ¹⁸ 6f ¹⁴ 7d ¹⁰ 8p _{1/2} ² 9s ² 9p _{1/2} ² 10s8d	4	9	3.614	

Z125 Ground configuration

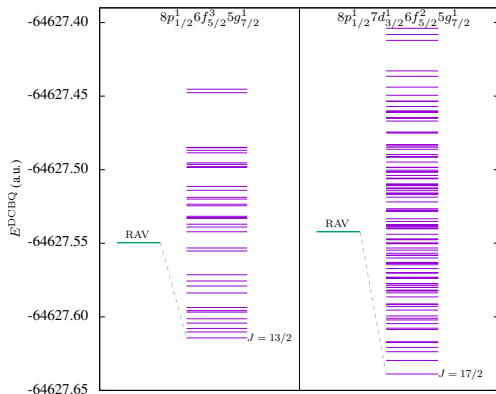


Рис.: Relativistic-configuration-average (RAV) energies calculated for the configurations $8p^1 6f^3 5g^1$ (left) and $8p^1 7d^1 6f^2 5g^1$ (right) of the SHE with $Z = 125$ and all the possible levels which contribute to these average energies.

Таблица: Spin-orbit splitting Δ_{SO} [eV]

Z	$7p_{1/2}$	$7p_{3/2}$	Δ_{SO}
114	10.4	4.5	5.9
116	14.3	6.1	8.2
118	20.1	8.3	11.8
	$8p_{1/2}$	$8p_{3/2}$	
125	5.3	2.4	2.9
144	13.8	2.7	11.1
145	16.2	2.7	13.5
164	63.2	3.5	59.7
165	79.7	5.0	74.7
166	90.1	6.6	83.5

Z=164. Electron configuration: [Og]5g¹⁸8s²8p²6f¹⁴7d¹⁰.

Таблица: One-electron energies ε and mean radii of the **core** electrons

	ε [keV]	$\langle r \rangle$ [a.u.]
1s 1/2	-770.6	0.0028
2p 1/2	-343.0	0.0038
2s 1/2	-196.3	0.0130
2p 3/2	-66.2	0.0279

$mc^2 = 510.7$ keV

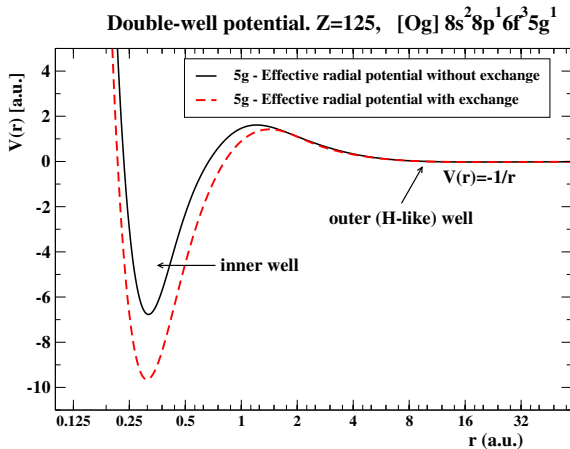
Таблица: One-electron energies ε and mean radii of the **valence** electrons

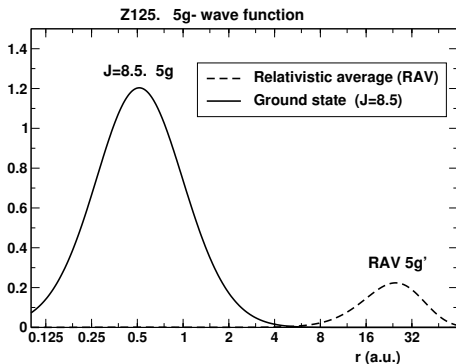
	ε [eV]	$\langle r \rangle$ [a.u.]
7p 1/2	-469.73	0.59
7p 3/2	-65.85	1.28
8s 1/2	-64.38	1.37
8p 1/2	-63.18	1.30
7d 3/2	-15.96	1.68
7d 5/2	-7.17	2.15
8p 3/2	-3.54	5.14

Double-well potential

Effective radial equation:

$$V(r) = -\frac{Z}{r} + V_H(r) + V_{\text{ex}}(r) + \frac{1}{2} \frac{l(l+1)}{r^2} \quad (1)$$





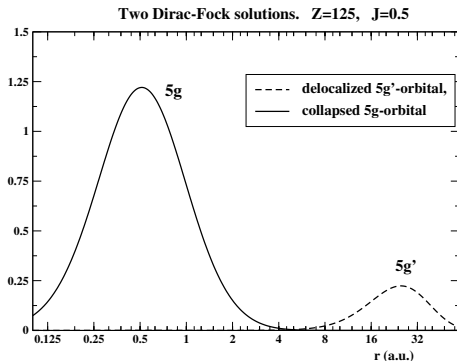
Nonrelativistic: $\epsilon_{5g}^H = -\frac{Z^2}{2n^2} = -1/(2 \cdot 5^2) = -0.02 \text{ a.u.}$ (2)

ϵ_{5g}	$= -0.53677445 \text{ a.u.}$	$\langle r \rangle_{5g}$	$= 0.71289 \text{ a.u.}$
ϵ_{5g}'	$= -0.02000182 \text{ a.u.}$	$\langle r \rangle_{5g}'$	$= 27.4929 \text{ a.u.}$
ϵ_{5g}^H	$= -0.02000001 \text{ a.u.}$	$\langle r \rangle_{5g}^H$	$= 27.5000 \text{ a.u.}$

Total Energies

$J = 8.5 :$	E_{inner}	$=$	-64846.3675 a.u.
RAV :	E_{outer}	$=$	-64846.0967 a.u.

Orbital collapse. Two solutions

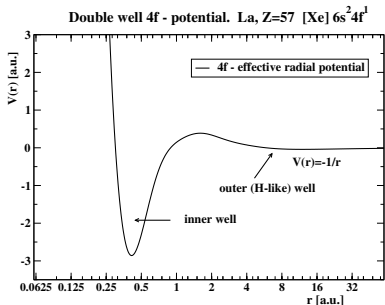


$$\begin{array}{ll}
 \varepsilon_{5g} & = -0.51464169 \text{ a.u.} & \langle r \rangle_{5g} & = 0.71289 \text{ a.u.} \\
 \varepsilon_{5g'} & = -0.02000147 \text{ a.u.} & \langle r \rangle_{5g'} & = 27.4943 \text{ a.u.} \\
 \varepsilon_{5g}^H & = -0.02000001 \text{ a.u.} & \langle r \rangle_{5g}^H & = 27.5000 \text{ a.u.}
 \end{array} \quad (5)$$

Total Energies($J = 0.5$) :

$$\begin{array}{ll}
 E_{\text{inner}} & = -64846.2788 \text{ a.u.} \\
 E_{\text{outer}} & = -64846.0878 \text{ a.u.}
 \end{array} \quad (6)$$

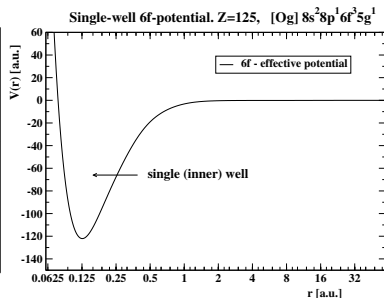
Orbital collapse. Two solutions. La ($Z=57$)



$$\begin{aligned} \epsilon_{4f} &= -0.2304 \text{ a.u.} \\ \epsilon_{4f'} &= -0.03176 \text{ a.u.} \\ \epsilon_{4f'}[1] &= -0.0316 \text{ a.u.} \\ \epsilon_{4f}^H &= -0.03125 \text{ a.u.} \end{aligned}$$

Total Energies ($J=0.5$):

$$\begin{aligned} E_{\text{inner}} &= -8493.5483 \text{ a.u.} \\ E_{\text{outer}} &= -8493.4767 \text{ a.u.} \end{aligned}$$



$$\begin{aligned} \langle r \rangle_{4f} &= 1.27452 \text{ a.u.} \\ \langle r \rangle_{4f'} &= 17.1653 \text{ a.u.} \\ &= -- \\ \langle r \rangle_{4f}^H &= 17.9999 \text{ a.u.} \end{aligned}$$

(7)

$$\begin{aligned} E_{\text{inner}}[3] &= -8493.6247 \text{ a.u.} \\ E_{\text{outer}}[3] &= -8493.5512 \text{ a.u.} \end{aligned}$$

(8)

1. I.M. Band and V.I. Fomichev, *Phys.Letters A*, **75**, 178 (1980)

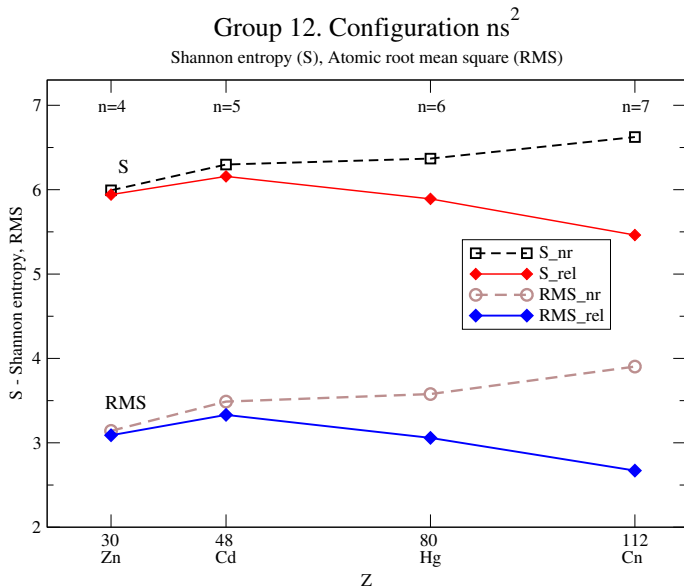
2. J.-R Connerade and R.C. Kamatak, *Handbook on the Physics and Chemistry of Rare Earths*, v. 28, p. 1 (2000)

Thank You for Attention.

Ground state configurations

Таблица: Ground state electron configurations

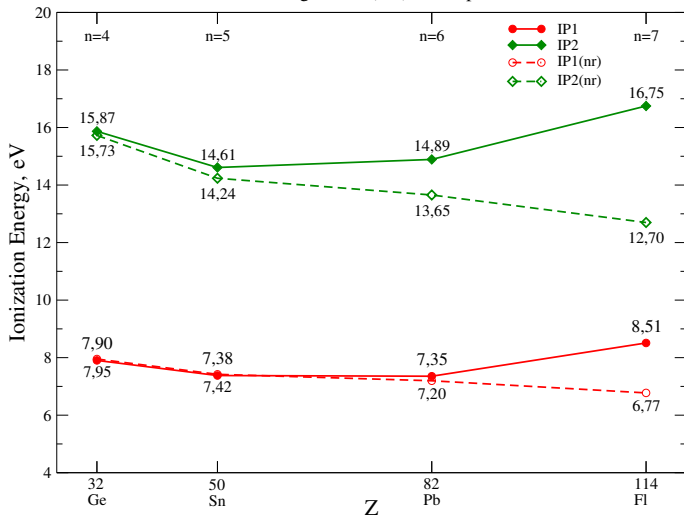
Core: [Rn] 5f ¹⁴		
Z	Conf.	J
111	6d ⁹ 7s ²	2.5
112	6d ¹⁰ 7s ²	0.0
113	6d ¹⁰ 7s ² 7p ¹	0.5
114	6d ¹⁰ 7s ² 7p ²	0.0
118	6d ¹⁰ 7s ² 7p ⁶	0.0
119	6d ¹⁰ 7s ² 7p ⁶ 8s ¹	0.5
120	6d ¹⁰ 7s ² 7p ⁶ 8s ²	0.0



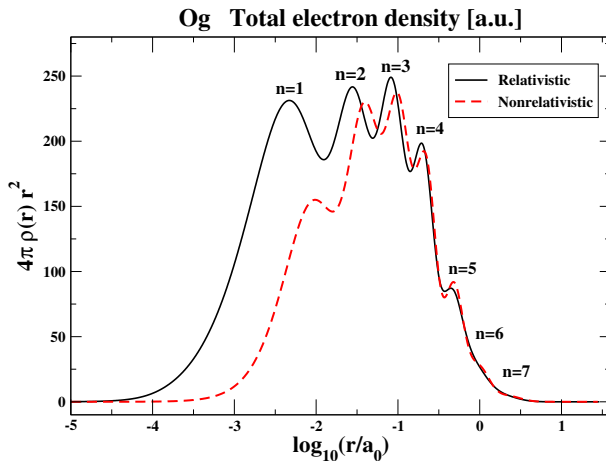
14th group of elements

Group 14: Ionization Potentials

configuration $(n-1)d^{10}ns^2np^2$



Electron density distribution



Radial electron density $4\pi\rho(r)r^2$ plot for Og in both relativistic and non-relativistic approximations.

There is no visible peak in the valence shell region. Electron density alone does not show any valence shell structure

Nonrelativistic Electron Localization Function (ELF)

The electron localization function is defined by [4]

$$\text{ELF}(\mathbf{r}) = \left(1 + \left[\frac{D(\mathbf{r})}{D_0(\mathbf{r})} \right]^2 \right)^{-1}, \quad (9)$$

where

$$D(\mathbf{r}) = \frac{1}{2} \left[\tau - \frac{1}{4} \frac{|\nabla \rho(\mathbf{r})|^2}{\rho(\mathbf{r})} \right], \quad \text{and} \quad \tau = \sum_{i,\sigma} |\nabla \varphi_{i\sigma}(\mathbf{r})|^2. \quad (10)$$

Here ρ is total density and τ the kinetic energy density.

$D_0(\mathbf{r})$ corresponds to a uniform electron gas (Thomas-Fermi) kinetic energy density

$$D_0(\mathbf{r}) = \frac{3}{10} (3\pi^2)^{2/3} \rho^{5/3}(\mathbf{r}). \quad (11)$$

The ELF values lie by definition between zero and one. Small values are typical for the region between two electron shells. In a homogeneous electron gas $\text{ELF} = 0.5$.

[4] A. D. Becke and K. E. Edgecombe, *J. Chem. Phys.* v.92, 5397 (1990).

Relativistic Electron Localization Function (ELF)

$$D(\mathbf{r}) = \sum_{\lambda=1,2} \left[W^\lambda(r) T^\lambda(\mathbf{r}) - \frac{1}{8} \frac{|\nabla \rho^\lambda(r)|^2}{\rho(r)} \right], \quad (12)$$

where $\rho(r)$ – total electron density

$$\rho(r) = \sum_{\lambda=1,2} \rho^\lambda(r), \quad \rho^\lambda(r) = \frac{4\pi}{r^2} \sum_a q_a \begin{cases} P_a^2(r), & \lambda = 1, \\ Q_a^2(r), & \lambda = 2. \end{cases} \quad (13)$$

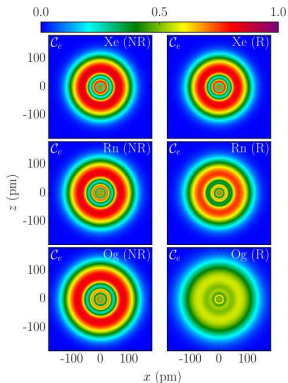
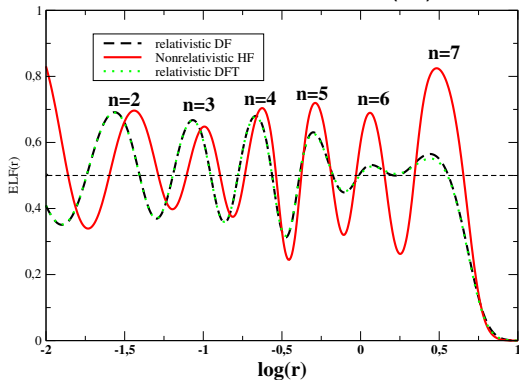
$T^\lambda(\mathbf{r})$ in formula (12) is the relativistic analogue of the non-relativistic kinetic energy density

$$T^\lambda(\mathbf{r}) = \sum_a q_a t_a^\lambda(\mathbf{r}), \quad t_a^\lambda(\mathbf{r}) = \frac{1}{2} \frac{1}{2j_a + 1} \sum_{\mu_a, \sigma} |\nabla \phi_{a\mu_a}^\lambda(\mathbf{r}, \sigma)|^2 \quad (14)$$

and $W^\lambda(r)$ is a weight function that has the form

$$W^\lambda(r) = \frac{\rho^\lambda(r)}{\rho(r)}. \quad (15)$$

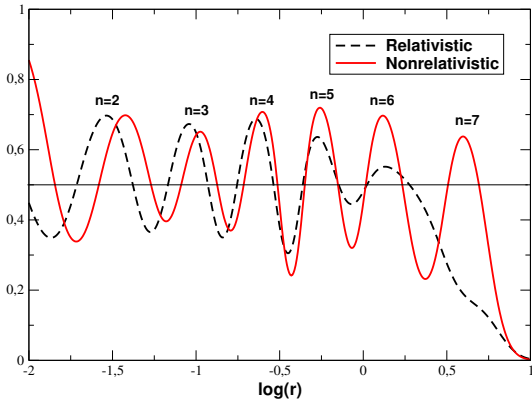
Og (Z=118). Configuration $7s^2 7p^6$
Electronic Localization Function (a.u.)



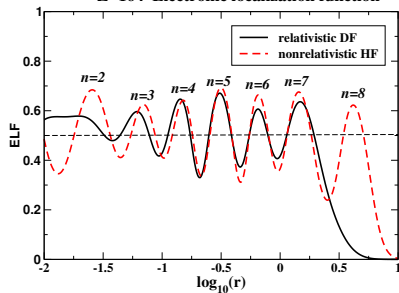
“Spin-orbit splitting in the 7p electronic shell becomes so large (~ 10 eV) that Og is expected to show uniform-gas-like behavior in the valence region” [5].

[5] *Jerabek et al., PRL 120, 053001 (2018).*

Fl (Z=114). Configuration $7s^2 7p^2$
Electronic Localization Function (a.u.)



Z=164 Electronic localization function



Z=164 Electronic localization function

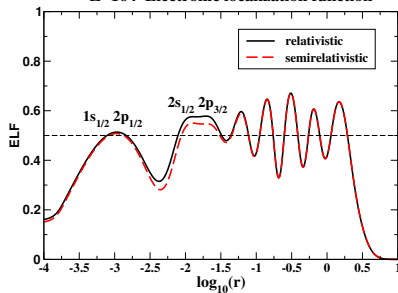


Таблица: Comparison of contributions to the Electron Affinities (EA) of Og ($Z=118$) atom ($[\text{Rn}]5f^{14}6d^{10}7s^27p^6$) calculated with the CI-DFS and FSCC methods in the present work [1] with results of Refs. [2-5] (eV). In Ref. [4] a combination of CI with the many-body perturbation theory is used.

Reference	$EA_{\text{CI-DFS}}$	$EA_{\text{FSCC-SD}}$	δEA^{T}	δEA^{QED}	EA^{Total}
Kaygorodov <i>et al.</i> [1]	0.070(10)	0.070(2)	0.008(3)	-0.002(1)	0.076(4)
Eliav <i>et al.</i> [2]					0.056(10)
Goidenko <i>et al.</i> [3]		0.064(2)		-0.0059(5)	0.058(3)
Lackenby <i>et al.</i> [4]					0.096
Guo <i>et al.</i> [5]					0.080(6)

1. M. Y. Kaygorodov *et al.*, *PRA*, 104, 012819, (2021).
2. E. Eliav, U. Kaldor, Y. Ishikawa, and P. Pyykkö, *PRL* 77, 5350 (1996).
3. I. Goidenko, L. Labzowsky, E. Eliav, U. Kaldor, and P. Pyykkö, *PRA* 67, 020102 (2003).
4. B. G. C. Lackenby, V. A. Dzuba, and V. V. Flambaum, *PRA* 98, 042512 (2018).
5. Y Guo, *et al.*, *arXiv:2107.02164 [physics]*, (2021)

Shannon entropy

According to the formula of K. Shannon [1], the amount of information is defined as:

$$S = - \sum_{i=1}^N p_i \ln p_i \quad (16)$$

where N is the number of random events, p_i is the probability of the i -th event and

$$\sum_{i=1}^N p_i = 1. \quad (17)$$

The minimum value of S is reached for the deterministic event when one of the probabilities of p_i is 1, and the rest are zero. In this case, $S = S_{\min} = 0$.

The maximum value of S is reached for an equally probable distribution $p_i = 1/N$. Then, $S_{\max} = \ln(N)$

$$0 \leq S \leq \ln(N) \quad (18)$$

For continuous distribution

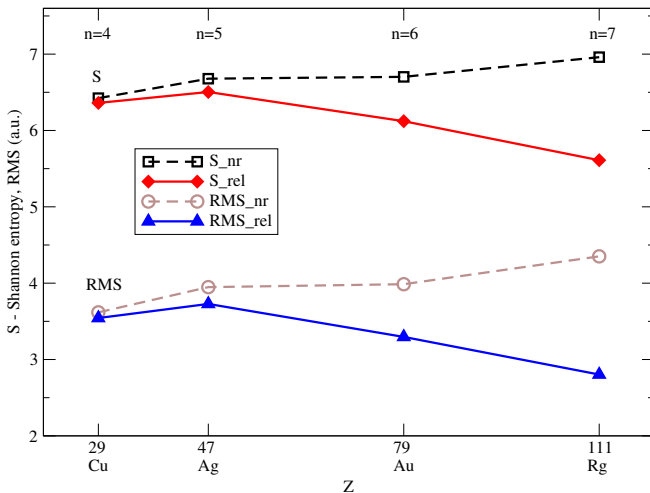
$$S = - \int d\mathbf{r} \ln(\rho(\mathbf{r})) \rho(\mathbf{r}), \quad \int d\mathbf{r} \rho(\mathbf{r}) = 1. \quad (19)$$

[1] C.E. Shannon, *Bell Syst. Tech. J.* 27, 379; 623 (1948).

11th group of elements

Group 11. Shannon entropy (S). Atomic root mean square (RMS)

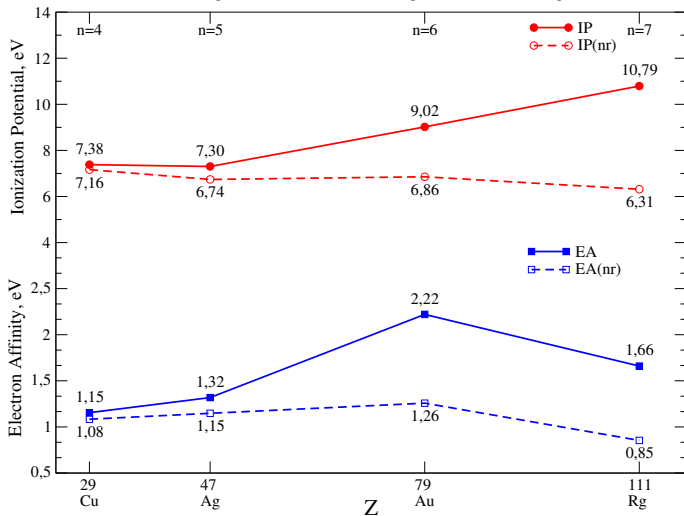
Configuration $(n-1)d^{10}ns^1$ for Cu, Ag, Au and $6d^97s^2$ for Rg



11th group of elements

Group 11: Ionization Potential & Electron Affinity

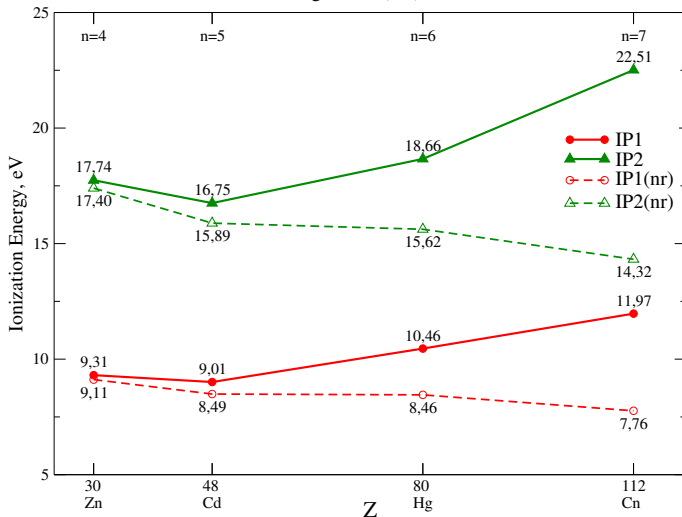
Configuration $(n-1)d^{10}ns^1$ for Cu, Ag, Au and $6d^97s^2$ for Rg



12th group of elements

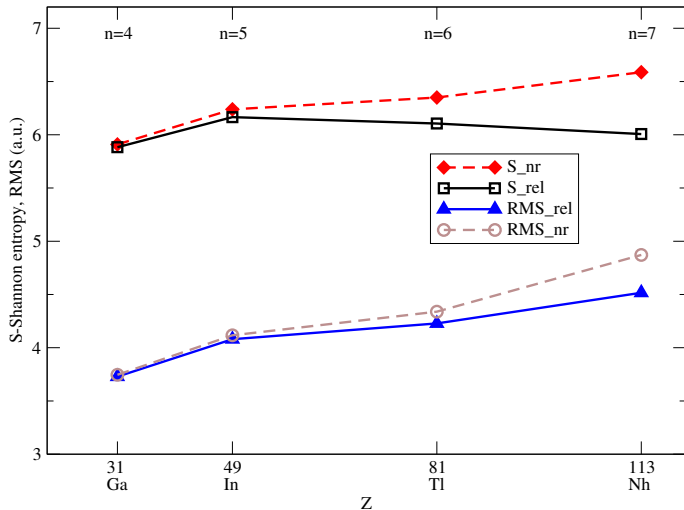
Group 12: Ionization Potentials

configuration $(n-1)d^{10}ns^2$



Group 13. Configuration $7s^2 p^1$

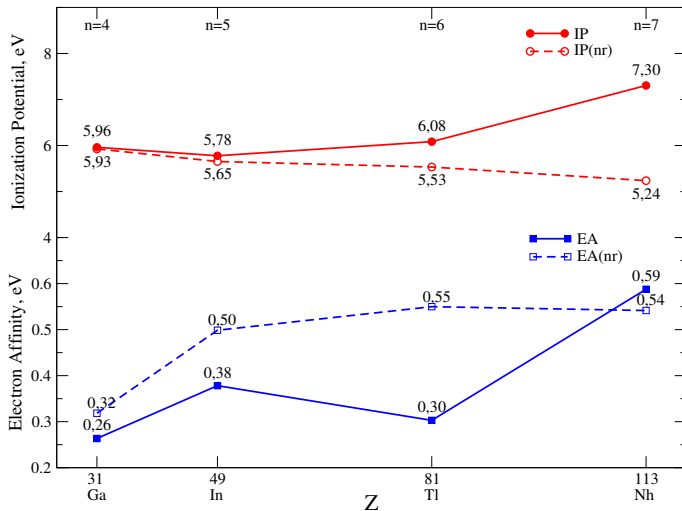
Shannon entropy (S), Atomic root mean square radius (RMS)



13th group of elements

Group 13: Ionization Potential & Electron Affinity

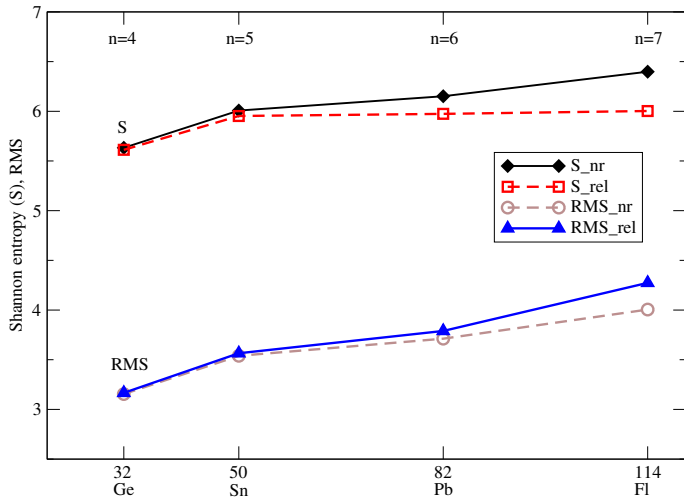
configuration $(n-1)d^{10}ns^2np^1$



14th group of elements

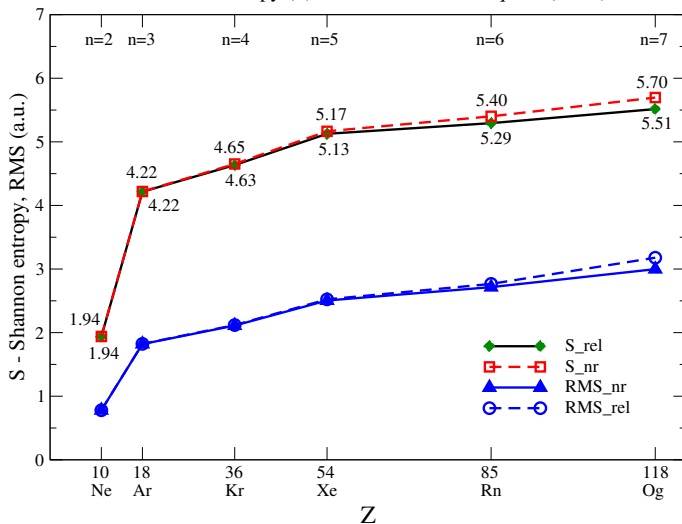
Group 14. Configuration $ns^2 p^2$

Shannon entropy (S), Atomic root mean square (RMS)



18th group of elements

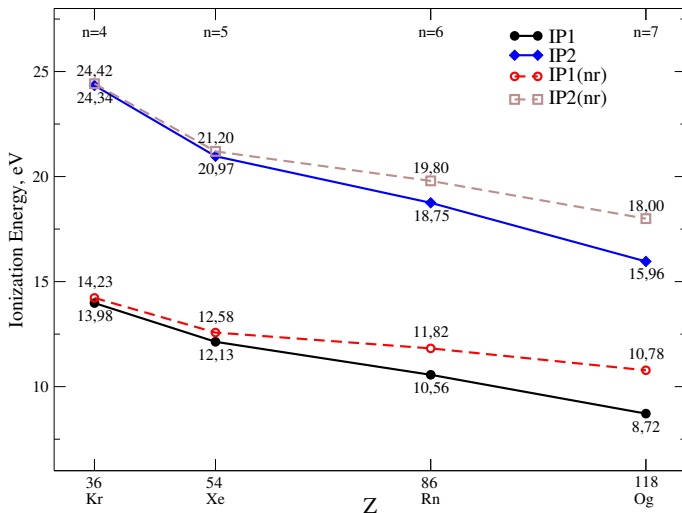
Group 18. Configuration $ns^2 np^6$
 Shannon entropy (S), Atomic root mean square (RMS)



18th group of elements

Group 18: Ionization Potentials

configuration $(n-1)d^{10}ns^2np^6$



Group 1. Ionization Potentials and RMS, ns¹

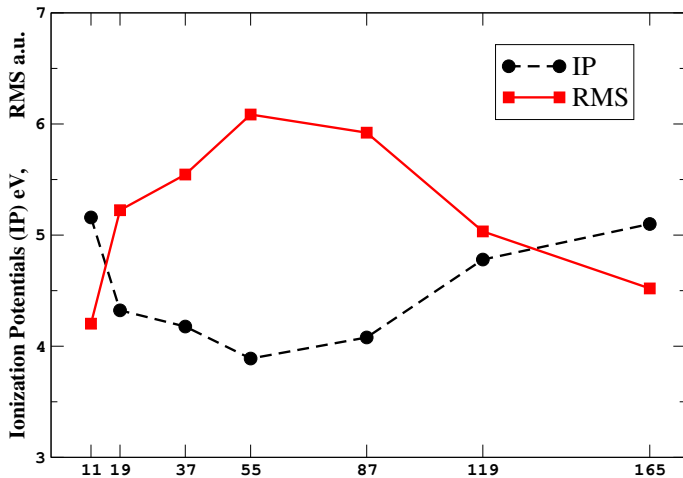


Таблица: One-electron energies and RMS valence shell radii of atoms with a serial number $Z=163$ and $Z=164$ and its homologous

n	Z	$\epsilon(d_{3/2})$	$\epsilon(d_{5/2})$	$\epsilon(ns)$	RMS($d_{5/2}$)	RMS($d_{5/2}$)	RMS(ns)
4	29	-13.276	-12.906	-6.661	0.991	1.002	3.261
5	47	-14.312	-13.646	-6.453	1.371	1.396	3.451
6	79	-13.430	-11.667	-7.937	1.536	1.619	3.061
7	111	-14.082	-11.047	-11.593	1.708	1.868	2.588
8	163	-14.396	-6.310	-60.791	1.745	2.282	1.401
4	30	-20.977	-20.537	-8.126	0.875	0.883	2.848
5	48	-20.089	-19.290	-7.658	1.254	1.274	3.089
6	80	-17.689	-15.637	-8.926	1.431	1.499	2.843
7	112	-15.313	-12.030	-12.275	1.643	1.795	2.499
8	164	-15.875	-7.138	-65.815	1.680	2.161	1.359

For atom $Z=164$ $\text{RMS}(5g) \simeq 0.32$ a.u.

Dirac-Fock-Sturm method (DFS)

Within the DCB approximation, the many-electron wave function $\Psi(JM)$ can be obtained in the form of an expansion in terms of the configuration-state functions (CSF) $\Phi_I(JM)$:

$$\Psi(JM) = \sum_I C_I^{JM} \Phi_I(JM). \quad (20)$$

Each function $\Phi_I(JM)$ is an eigenfunction of the operators \hat{J}^2 and \hat{J}_z and there is a linear combination of Slater determinants

$$\sum_K H_{KI} C_K^{JM} = E_I(J) C_I^{JM}, \quad (21)$$

where $H_{KI} = \langle \Phi_K | \hat{H}_{\text{DCB}} | \Phi_I \rangle$.

The Slater determinants are constructed from the one-electron wave functions ψ_i obtained by the Dirac-Fock method in the basis of Dirac-Fock-Sturm (DFS) orbitals φ_k :

$$\psi_i = \sum_k u_{ki} \varphi_k. \quad (22)$$

The one-electron DFS functions, obtained by numerically solving the Dirac-Fock (DF) integro-differential equations for the occupied in the ground and low-lying excited states.

Dirac-Fock-Sturm method (DFS)

For virtual (high-lying vacant) one-electron states, the functions φ_k are obtained by numerically solving the Dirac-Fock-Sturm equations

$$\left[\hat{h}_{\text{DF}} - \varepsilon_0 \right] \varphi_k = \mu_k W(r) \varphi_k, \quad (23)$$

where \hat{h}_{DF} — is the Dirac-Fock operator, ε_0 — is the reference one-electron energy, and $W(r)$ is a positive weight function tending to zero at infinity.

We note that all the DFS orbitals have approximately the same characteristic size and the same asymptotic at infinity, determined by the reference energy ε_0 :

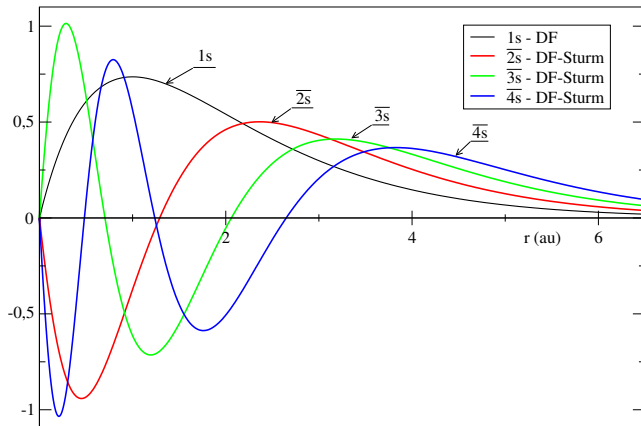
$$\varphi_k(r) \xrightarrow[r \rightarrow \infty]{} C_k \exp(-\sqrt{2\varepsilon_0} r). \quad (24)$$

The function tending to a constant for $r \rightarrow 0$ was chosen as the weight function $W(r)$:

$$W(r) = \frac{1 - \exp(-(\alpha r)^2)}{(\alpha r)^2}. \quad (25)$$

Hydrogen-like (Coulomb) Sturmians

Sturmian orbitals. Hydrogen



One-electron model QED potential

The one-electron QED potential V^{QED} can be considered as the sum of *Vacuum Polarization* (VP) potential V^{VP} and *Self-Energy* (SE) potential V^{SE}

$$V^{\text{QED}} = V^{\text{VP}} + V^{\text{SE}} \quad (26)$$

With a good accuracy VP contribution can be presented as the sum of the local Uehling and Wichmann-Kroll potentials

$$V^{\text{VP}} = V_{\text{Uehl}} + V_{\text{WK}} \quad (27)$$

The calculation of the self-energy is a complicated and time-consuming part of the QED corrections.

We suppose that there exist one-electron self-energy (SE) operator $\hat{\Sigma}$ which can be directly included into the Dirac-Coulomb-Breit (DCB) many-electron calculations. The approximation to the self-energy operator $\hat{\Sigma}$ based on the expansion (1) is given by

$$V^{\text{SE}} = \sum_{i,k=1}^n |\psi_i^{(0)}\rangle \Sigma_{ik} \langle \psi_k^{(0)}|, \quad \text{where} \quad \Sigma_{ij} = \langle \psi_i^{(0)} | \hat{\Sigma} | \psi_j^{(0)} \rangle. \quad (28)$$

The SE operator has to be localized in a small region whose size is on the order of Compton wavelength (1/137 a.u.)

One-electron model QED potential

The hydrogen-like wave functions should be replaced by the properly localized so-called projected functions ϕ_i . Then

$$V^{\text{SE}} = \sum_{i,k=1}^n |\phi_i\rangle B_{ik} \langle \phi_k|, \quad (29)$$

The matrix elements B_{ik} are chosen so that the matrix elements of the model operator V_{ik}^{SE} calculated with hydrogen like wave functions $\psi_i^{(0)}$ have to be equal to matrix elements Q_{ik} of the symmetrized exact one-loop energy-dependent SE operator $\Sigma(\varepsilon)$

$$\langle i | V^{\text{SE}} | k \rangle = Q_{ik} = \frac{1}{2} [\Sigma_{ik} + \Sigma_{ki}] \quad (30)$$

Then

$$B_{ik} = \sum_{j,l=1}^n (D^{-1})_{ji} Q_{jl} (D^{-1})_{lk}, \quad D_{ik} = \langle \phi_i | \psi_k^{(0)} \rangle \quad (31)$$

This approximation to the SE operator is also not very successful. If any strongly localized function is orthogonal to the finite set of projection functions ϕ_i then the corresponding SE contribution will be equal to zero.

One-electron model QED potential

To overcome this problem we introduce the local potential $V_{\text{loc}}^{\text{SE}}$ in (4) by the following way

$$V^{\text{SE}} = V_{\text{loc}}^{\text{SE}} + \sum_{i,k=1}^n |\phi_i\rangle \Delta B_{ik} \langle \phi_k|, \quad (32)$$

where

$$\Delta B_{ik} = \sum_{j,l=1}^n (D^{-1})_{ji} \Delta \Sigma_{jl} (D^{-1})_{lk}, \quad (33)$$

and $\Delta \Sigma_{ik} = \Sigma_{ik} - \langle \psi_i^{(0)} | V_{\text{loc}}^{\text{SE}} | \psi_k^{(0)} \rangle$.

At the present time, the matrix Σ is constructed in the interval $3 \leq Z \leq 125$. However, A. Malyshev is finishing work on expanding the region of Z up to $Z=170$.

This approximation to the SE operator was used in our papers [1-3]:

1. V.M. Shabaev, I.I. Tupitsyn, V.A. Yerokhin, *Phys. Rev. A*, **88**, 012513 (2013)
2. V.M. Shabaev, I.I. Tupitsyn, V.A. Yerokhin, *Computer Phys. Comm.*, **189**, 175 (2015)
3. I.I. Tupitsyn, M.G. Kozlov, M.S. Safronova, V.M. Shabaev, and V.A. Dzuba, *PRL*, **117**, 253001 (2016)

Таблица: Contributions to the first ionization potential I_1 and electron affinity EA of Rg ($Z=111$) atom (eV). Configuration: $6d^9 7s^2$

Contribution	I_1	EA
FSCC-SD	10.94(6)	1.92(6)
FSCC-T	-0.585	-0.076
Gaunt (FSCC)	0.026(1)	0.024(2)
Retard (CI)	-0.002(1)	-0.002(1)
Freq (CI)	-0.0006(3)	0.0001(3)
QED (CI)	0.021(3)	0.019(3)
Total	10.40(6)	1.89(6)
Eliav <i>et al.</i>	10.60	1.565
Lackenby <i>et al.</i>	11.175	
Hangele <i>et al.</i>	11.871	2.5077

1. E. Eliav et al., PRL, 73, 3203, (1994).
2. B. G. C. Lackenby et al., PRA, 101, 012514 (2020).
3. T. Hangele et al, J. Chem. Phys., 136, 214105 (2012).

Таблица: Contributions to the ionization potentials (IP) of Cn atom ($Z = 112$ element), in eV.

Contribution	$I_2^{J=0}$	$I_2^{J=2}$	$I_2^{J=3}$	$I_2^{J=4}$	I_1
FSCC-SD	23.32(4)	22.53(4)	22.69(4)	22.46(4)	11.93(4)
FSCC-T	-0.143(2)	-0.238(2)	-0.159(2)	-0.302(2)	-0.006(2)
Gaunt (FSCC)	-0.100(3)	-0.031(3)	-0.074(3)	0.025(2)	0.027(2)
Gaunt (CI)				0.029(2)	0.031(2)
Retard (CI)				-0.001(1)	-0.002(1)
Freq (CI)				-0.001(1)	-0.0006(2)
QED (CI)				0.025(3)	0.022(3)
Total	23.07(4)	22.26(4)	22.45(4)	22.20(4)	11.97(4)
Eliav <i>et al.</i>	23.28	22.54	22.68	22.49	11.97
Nash					11.675
Yu <i>et al.</i>				21.98	11.73
Lackenby <i>et al.</i>				22.84	12.14
Hangele <i>et al.</i>				21.989	11.353

1. E. Eliav *et al.*, PRA, 52,2765, (1995).
2. T. Hangele *et al.*, J. Chem. Phys., 136, 214105 (2012).
3. B. G. C. Lackenby *et al.*, PRA, 101, 012514 (2020).
4. C. S. Nash *et al.*, J. Phys. Chem. A, 109, 3493, (2005).
5. Y. J. Yu *et al.*, EPJ D, 44, 51, (2007).

Таблица: Contributions to the ionization potentials (IP) and electron affinity (EA) of Nh atom ($Z = 113$ element), in eV.

Contribution	I_3	I_2	I_1	EA
FSCC-SD	33.42(3)	23.91(3)	7.38(1)	0.66(1)
FSCC-T	-0.060(24)	0.008(3)	0.049(19)	0.023(9)
Gaunt (CI)	-0.060(6)	-0.049(6)	-0.055(6)	-0.040(10)
Retard (CI)	0.004(1)	0.003(1)	0.004(1)	0.003(1)
Freq (CI)	-0.005(1)	-0.005(1)	-0.003(1)	-0.002(1)
QED (CI)	-0.088(6)	-0.078(8)	-0.001(1)	0.007(4)
Total	33.21(4)	23.79(3)	7.37(2)	0.65(1)
Dzuba <i>et al.</i>	33.5	23.6	7.37	
Eliav <i>et al.</i>	33.47	23.96	7.306	0.68(5)
Pershina <i>et al.</i>			7.420	
Hangele <i>et al.</i>		23.6271	7.2779	

1. V. A. Dzuba, PRA, 93, 032519 (2016).
2. E. Eliav et al., PRA, 53, (1996).
3. V. Pershina et al., J. Chem. Phys., 131, 084713, (2009).
4. T. Hangele et.al., J. Chem. Phys., 136, 214105, (2012).

Таблица: Contributions to the ionization potentials (IP) of FI atom $Z = 114$ element, in eV.

Contribution	I_4	I_3	I_2	I_1
SD	46.19(4)	35.73(4)	16.92(4)	8.66(4)
T	-0.074(25)	-0.019(6)	0.054(18)	0.036(12)
Gaunt	-0.076(2)	-0.069(2)	-0.068(2)	-0.044(2)
Retard	0.006(2)	0.004(2)	0.006(2)	0.005(2)
Freq	-0.006(1)	-0.006(1)	-0.004(1)	-0.003(1)
QED	-0.105(5)	-0.093(5)	-0.004(2)	-0.003(3)
Total	45.94(5)	35.55(4)	16.90(4)	8.65(4)
Dzuba <i>et al.</i>			17.00	8.37
Landau <i>et al.</i>	46.272	35.739	16.871	8.539
Yu <i>et al.</i>	46.57	35.82	17.22	8.28
Nash				8.529
Hangele <i>et al.</i>		35.3826	16.1105	7.2601

1. V. A. Dzuba, *PRA*, 93, 032519 (2016).
2. A. Landau *et al.*, *J.Chem. Phys.*, 114, (2001).
3. Y. J. Yu *et al.*, *J. Chem. Phys.*, 128, 124316, (2008).
4. C. S. Nash *et al.*, *J. Phys. Chem. A*, 109, 3493, (2005).
5. T. Hangele, *et al.*, *J. Chem. Phys.*, 136, p. 214105, (2012)

PERIODIC TABLE

Atomic Properties of the Elements

Group
1
IA

1	¹ H Hydrogen 1.008 1.00784 13.5964
2	³ Li Lithium 6.94 6.938 5.20917
3	⁴ Be Beryllium 9.0122 9.012182 9.3227
11	¹¹ Na Sodium 22.990 22.98976928 5.1391
12	¹² Mg Magnesium 24.305 24.304094 5.6462
19	¹⁹ K Potassium 39.098 39.0983 4.3407
37	³⁷ Rb Rubidium 85.468 85.4678 10.771
55	⁵⁵ Cs Cesium 132.91 132.90545 3.8930
87	⁸⁷ Fr Francium (223) ([Rn]7s ¹) 4.0727

FREQUENTLY USED FUNDAMENTAL PHYSICAL CONSTANTS¹

¹ second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ¹³³Cs

speed of light in vacuum	<i>c</i>	299 792 458 m s ⁻¹	(exact)
Planck constant	<i>h</i>	6.626 070 15 × 10 ⁻³⁴ J Hz ⁻¹	(exact)
elementary charge	<i>e</i>	1.602 176 634 × 10 ⁻¹⁹ C	(exact)
Avogadro constant	<i>N_A</i>	6.022 140 76 × 10 ²³ mol ⁻¹	(exact)
Boltzmann constant	<i>k</i>	1.380 649 × 10 ⁻²³ J K ⁻¹	(exact)
electron volt	eV	1.602 176 634 × 10 ⁻¹⁹ J	(exact)
electron mass	<i>m_e</i>	9.109 383 70 × 10 ⁻³¹ kg	(exact)
energy equivalent	<i>m_ec²</i>	0.510 998 950 MeV	(exact)
proton mass	<i>m_p</i>	1.672 621 923 4 × 10 ⁻²⁷ kg	(exact)
energy equivalent	<i>m_pc²</i>	938.272 088 MeV	(exact)
fine-structure constant	<i>α</i>	1/137.035 999	(exact)
Rydberg energy	<i>R_∞</i>	13.605 693 1230 eV	(exact)
Newtonian constant of gravitation	<i>G</i>	6.674 × 10 ⁻¹¹ m ³ kg ⁻¹ s ⁻²	(exact)

¹For the most accurate values of these and other constants, visit nist.gov/constants.

- Solids
- Liquids
- Gases
- Artificially Prepared

Physical Measurement Laboratory www.nist.gov/pml
Standard Reference Data www.nist.gov/srd

13	¹³ B Boron 10.81 10.8107 8.2980	14	¹⁴ C Carbon 12.011 12.0107 16.2982	15	¹⁵ N Nitrogen 14.007 14.00643 14.5441	16	¹⁶ O Oxygen 15.999 15.99903 13.8181	17	¹⁷ Cl Chlorine 35.45 35.453 7.4225	18	¹⁸ Ar Argon 39.948 39.948 13.5969
13	¹³ Al Aluminum 26.982 26.9815386 10.7639	14	¹⁴ Si Silicon 28.086 28.0858309 9.1517	15	¹⁵ P Phosphorus 30.974 30.973762 10.4867	16	¹⁶ S Sulfur 32.06 32.065 10.8662	17	¹⁷ Cl Chlorine 35.45 35.453 7.4225	18	¹⁸ Ar Argon 39.948 39.948 13.5969
31	³¹ Ga Gallium 69.723 69.7231 5.9903	32	³² Ge Germanium 72.630 72.6305 7.8904	33	³³ As Arsenic 74.922 74.9216 9.7886	34	³⁴ Se Selenium 78.971 78.9718 10.4513	35	³⁵ Br Bromine 79.904 79.904 11.8138	36	³⁶ Kr Krypton 83.798 83.798 13.0906
49	⁴⁹ In Indium 114.82 114.818 12.1141	50	⁵⁰ Sn Tin 118.710 118.710 7.3430	51	⁵¹ Sb Antimony 121.76 121.757 10.0097	52	⁵² Te Tellurium 127.60 127.603 10.0097	53	⁵³ I Iodine 126.905 126.90545 10.4513	54	⁵⁴ Xe Xenon 131.29 131.294 10.7485
81	⁸¹ Tl Thallium 204.38 204.383 6.1083	82	⁸² Pb Lead 207.2 207.2 7.4167	83	⁸³ Bi Bismuth 208.98 208.9804 7.2855	84	⁸⁴ Po Polonium (209) ([Po]) 8.414	85	⁸⁵ At Astatine (210) ([At]) 9.3175	86	⁸⁶ Rn Radon (222) ([Rn]) 10.7485
113	¹¹³ Nh Nihonium (286) ([Nh]) 8.285	114	¹¹⁴ Fl Flerovium (289) ([Fl]) 8.285	115	¹¹⁵ Mc Moscovium (289) ([Mc]) 8.285	116	¹¹⁶ Lv Livermorium (293) ([Lv]) 8.285	117	¹¹⁷ Ts Tennessine (294) ([Ts]) 8.285	118	¹¹⁸ Og Oganesson (294) ([Og]) 8.285

Atomic Number	58
Ground State	¹³⁶ Gd
Symbol	Ce
Name	Cerium
Standard Atomic Weight (<i>A_r</i>)	140.12 ([Ce]4f5d6s ²) 5.5396
Ground-state Configuration	[Xe]4f5d6s ²
Ionization Energy (eV)	5.5396

57	⁵⁷ La Lanthanum 138.91 138.90547 5.5769	58	⁵⁸ Ce Cerium 140.12 140.12 5.5396	59	⁵⁹ Pr Praseodymium 140.91 140.90765 5.4702	60	⁶⁰ Nd Neodymium 144.24 144.242 5.5250	61	⁶¹ Pm Promethium (145) ([Pm]) 5.577	62	⁶² Sm Samarium 150.36 150.36 5.6437	63	⁶³ Eu Europium 151.96 151.964 5.6704	64	⁶⁴ Gd Gadolinium 157.25 157.25 6.1498	65	⁶⁵ Tb Terbium 158.93 158.92535 5.8388	66	⁶⁶ Dy Dysprosium 162.50 162.50085 5.9391	67	⁶⁷ Ho Holmium 164.93 164.93032 6.0215	68	⁶⁸ Er Erbium 167.26 167.259 6.1077	69	⁶⁹ Tm Thulium 168.93 168.934 6.1843	70	⁷⁰ Yb Ytterbium 173.05 173.0546 6.2542	71	⁷¹ Lu Lutetium 174.97 174.96706 6.4259
89	⁸⁹ Ac Actinium (227) ([Ac]) 5.3802	90	⁹⁰ Th Thorium (232) ([Th]) 6.3067	91	⁹¹ Pa Protactinium (231) ([Pa]) 5.89	92	⁹² U Uranium (238.03) ([U]) 6.1941	93	⁹³ Np Neptunium (237) ([Np]) 6.2655	94	⁹⁴ Pu Plutonium (244) ([Pu]) 6.0258	95	⁹⁵ Am Americium (243) ([Am]) 5.9738	96	⁹⁶ Cm Curium (247) ([Cm]) 5.9914	97	⁹⁷ Bk Berkelium (247) ([Bk]) 6.1078	98	⁹⁸ Cf Californium (251) ([Cf]) 6.2817	99	⁹⁹ Es Einsteinium (252) ([Es]) 6.3676	100	¹⁰⁰ Fm Fermium (257) ([Fm]) 6.50	101	¹⁰¹ Md Mendelevium (258) ([Md]) 6.58	102	¹⁰² No Nobelium (259) ([No]) 6.66	103	¹⁰³ Lr Lawrencium (260) ([Lr]) 4.96

¹Based upon ¹²C. () indicates the mass number of the longest-lived isotope.

For the most precise values and uncertainties visit www.nist.gov/pml and www.nist.gov/srd.