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Hadrons as Coherent states of partons

The Actual Problems of Microworld Physics **2023**

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Plan

- Introduction. Some results of multi particle production on LHC.
- Kinematic of the of the $pp-$ collisions in the quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space.
- Conventional coherent states on horosphere of Lobachevsky momentum space. Multiplicity distribution. Coordinate and momentum representation of the coherent states and multiplicity distribution.
- Theory Field point of view. Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles.
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Plan

- Scalar invariance.
- Scalar non invariance.
- Violation of scale invariance. Kinematic region of born particles. Strings.
- Conclusion
- .

Typical sizes

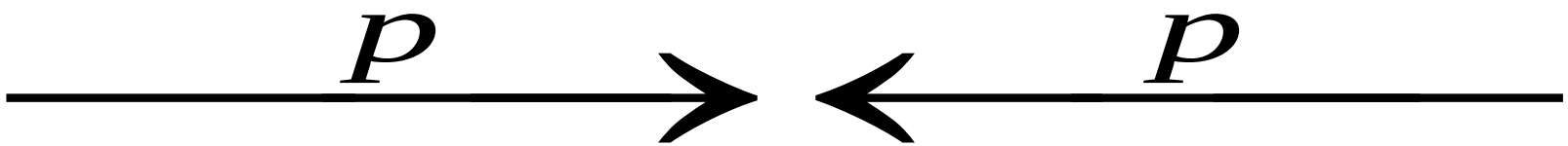
$$r_{0s} = 1,46 \cdot 10^{-15}, r_0 \square 2,3 \cdot 10^{-15}, r = \frac{1}{\sqrt{S}} \square 10^{-18}, \sqrt{S} = 7TeV$$

The ATLAS Collaboration. Two-particle Bose–Einstein correlations in *pp* collisions at $\sqrt{S}$ 0.9 and 13 TeV measured with the ATLAS detector. CERN-PH-EP-2014-264. Submitted to: EPJC.

$$r_{эф} \leq \frac{\ln P(S)}{m_\pi}$$

A.A.Logunov, M.A.Mestvirishvili, V.A. Petrov. In book General principles of the quantum fields theory and hadrons interactions under high energy M. : Nauka 1977

Kinematics of the collisions of hadrons



$$p_1 = (ip_{01}, \vec{p}_1) \qquad p_2 = (ip_{02}, \vec{p}_2)$$

$$p_1^{\,2} = \vec{p}_1^{\,2} - p_{01}^{\,2} = \vec{p}_2^{\,2} - p_{02}^{\,2} = -m_p^{\,2} \qquad (1.1)$$

- Here m_p - proton mass, we use the system of the physical units with $\hbar = c = 1$, $\sqrt{s}$ - system mass energy of protons.

Kinematics of the collisions of hadrons

$$S = -(p_1 + p_2)^2 = -P^2 =$$

$$-(p_{x1} + p_{x2})^2 - (p_{y1} + p_{y2})^2 - (p_{z1} + p_{z2})^2 + (p_{01} + p_{02})^2, (1.2)$$

$$P = (\vec{P}, iP_0) = [p_{x1} + p_{x2}, p_{y1} + p_{y2}, p_{z1} + p_{z2}, +i(p_{01} + p_{02})]$$

In the laboratory systems, where second proton is in the rest

$$P = (\vec{P}, iP_0) = [p_x, p_y, p_z, i(p_0 + m_p)] \quad (1.3)$$

and four dimensional momentum of the falling proton denote as

$$p = (p_x, p_y, p_z, ip_0) = (\vec{p}, ip_0)$$

Quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space

$$P_z = \frac{\sqrt{S}}{2}[e^{q_z/\sqrt{S}} + (\frac{q_x{}^2 + q_y{}^2}{S} + 1)e^{-q_z/\sqrt{S}}], \qquad P_x = q_x e^{-q_z/\sqrt{S}},$$

(1.4)

$$P_y = q_y e^{-q_z/\sqrt{S}}, \qquad P_0 = \frac{\sqrt{S}}{2}[e^{q_z/\sqrt{S}} + (\frac{q_x{}^2 + q_y{}^2}{S} - 1)e^{-q_z/\sqrt{S}}].$$

Inverse formulas

$$q_x = \frac{P_x\sqrt{S}}{P_z - P_0}, q_y = \frac{P_y\sqrt{S}}{P_z - P_0}, q_z = \sqrt{S}\ln\frac{\sqrt{S}}{P_z - P_0}. \qquad (1.5)$$

Quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space

Metric element in these coordinates expressed as follows

$$dl_m{}^2 = e^{-2q_z/\sqrt{S}}(dq_x{}^2 + dq_y{}^2) + dq_z{}^2 \quad . \qquad (1.6)$$

The element of the volume in the momentum space in the horospherical (quasi Cortesian) coordinates

$$dV_m = \sqrt{g}\,dq_x dq_y dq_z = e^{-2q_z/\sqrt{S}} dq_x dq_y dq_z \quad . \qquad (1.7)$$

Quantum mechanical variables (momenta, coordinates) on horosphere

$$q_x, q_y, \quad x = -i\hbar \frac{\partial}{\partial q_x}, \qquad y = -i\hbar \frac{\partial}{\partial q_y}. \tag{1.8}$$

Heisenberg-Weyl algebra is the base for definition of the coherent states

$$\begin{gathered} [x, q_x] = [y, q_y] = -i\hbar I, \quad [x, y] = [q_x, q_y] = 0, \\ [x, I] = [I, y] = [q_x, I] = [q_y, I,] = 0, \end{gathered} \tag{1.9}$$

Creation and annihilation operators connected with our problem

$$a_x = \frac{Rq_x + i\frac{x}{R}}{\sqrt{2}}, a_x^+ = \frac{Rq_x - i\frac{x}{R}}{\sqrt{2}}, a_y = \frac{Rq_y + i\frac{y}{R}}{\sqrt{2}}, a_y = \frac{Rq_y - i\frac{y}{R}}{\sqrt{2}}; \tag{1.10}$$

Yu. A. Kurochkin, I. Rybak, Dz. V. Shoukovy *Coherent states on the horosphere of the three dimensional Lobachevsky space. Doklady NAS Belarus 2014, J.Math.Phys. 57(8):082111 (2016)*

Heisenberg-Weyl algebra in terms of the creation and annihilation operators

$$\left[a_k, a_l^+\right] = \delta_{kl} I, \left[a_k^+, a_l^+\right] = \left[a_k, a_l\right] = \left[a_k, I\right] = \left[a_k^+, I\right] = 0. \qquad (1.11)$$

- where $k, l = 1, 2$ corresponds to x and y
- Definition of the coherent states and some
- properties

$$a_x \left|z_1\right\rangle = z_1 \left|z_1\right\rangle, \; a_y \left|z_2\right\rangle = z_2 \left|z_2\right\rangle \qquad (1.12)$$

$$\left\langle z_1 \middle| z_1 \right\rangle = \exp\left|z_1\right|^2, \qquad \left\langle z_2 \middle| z_2 \right\rangle = \exp\left|z_2\right|^2$$

Definition of the coherent states and some properties

$$\left|z_1, z_2\right\rangle = \exp\left(z_1 a_x^+\right)\exp\left(z_2 a_y^+\right)\left|0,0\right\rangle \qquad (1.13)$$

- $\left|0,0\right\rangle$-vacuum state, $\quad a_x\left|0,0\right\rangle = a_y\left|0,0\right\rangle = 0 \qquad (1.14)$

$$\int \left|z_1, z_2\right\rangle\left\langle z_1, z_2\right| d\mu(z_1, z_2) = \int \left|z_1\right\rangle\left\langle z_1\right| d\mu(z_1) \int \left|z_2\right\rangle\left\langle z_2\right| d\mu(z_2) = I$$

$$\Delta x \Delta q_x = \frac{\hbar}{2}, \quad \Delta y \Delta q_y = \frac{\hbar}{2} \qquad (1.15)$$

Average number of the excited quanta

$$\bar{n}_1 = \exp(-|z_1|^2)\langle z_1|a^+{}_x a_x|z_1\rangle = |z_1|^2, \quad \bar{n}_2 = \exp(-|z_2|^2)\langle z_2|a^+{}_y a_y|z_2\rangle = |z_2|^2$$

- $$\bar{n} = \bar{n}_1 + \bar{n}_2 = |z_1|^2 + |z_2|^2 \qquad (1.16)$$

Multiplicity distribution

$$P(n) = \frac{\exp(-\bar{n})\bar{n}^{\,n}}{n!} \qquad (1.17)$$

The main hypothesis:
The partons are the coherent states which are built from variables of horosphere Lobachevsky raletivistic momentum space.

Coordinate representation of coherent states

$$\langle x, y | z_1, z_2 \rangle \propto e^{i\frac{\sqrt{2}}{R}(\beta_1 x + \beta_2 y)} \times e^{-\frac{1}{2R^2}[(x-\sqrt{2}R\alpha_1)^2 + (y-\sqrt{2}R\alpha_2)^2]}$$

$$\left|\langle x, y | z_1, z_2 \rangle\right|^2 \propto e^{-\frac{1}{2R^2}[(x-\sqrt{2}R\alpha_1)^2 + (y-\sqrt{2}R\alpha_2)^2]} \qquad (1.18)$$

$$z_1 = \alpha_1 + i\beta_1, \alpha_1 = |z_1|\cos\vartheta_1, \beta_1 = |z_1|\sin\vartheta_1, \; z_2 = \alpha_2 + i\beta_2, \alpha_2 = |z_2|\cos\vartheta_2, \beta_2 = |z_2|\sin\vartheta_2$$

$$\sqrt{2}R\alpha_1 = \sqrt{2}R|z_1|\cos\vartheta_1 = \sqrt{2n}R\cos\vartheta_1 = r_0, R = 0{,}84 \times 10^{-15}$$

Dependence of the radius of correlations on the average multiplicity

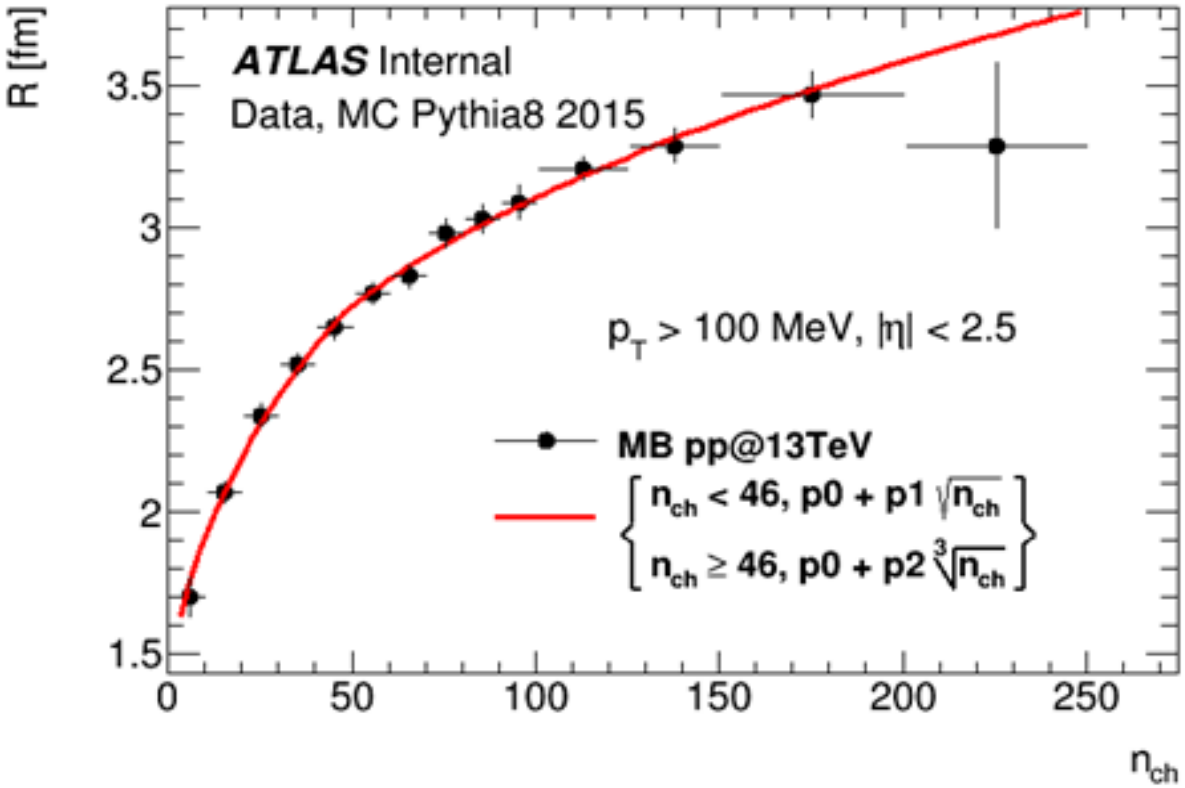


Figure 1. Approximation of the experimental data by the theoretical dependence, where p0 = 1.26 0.15, p1 = 0.21 0.03, p2 = 0.40 0.03. Black dots indicate experimental data.

Dependence of the radius of correlations on the average multiplicity stages 1 and 2

doi:10.17182/hepdata.132012.v1/t79

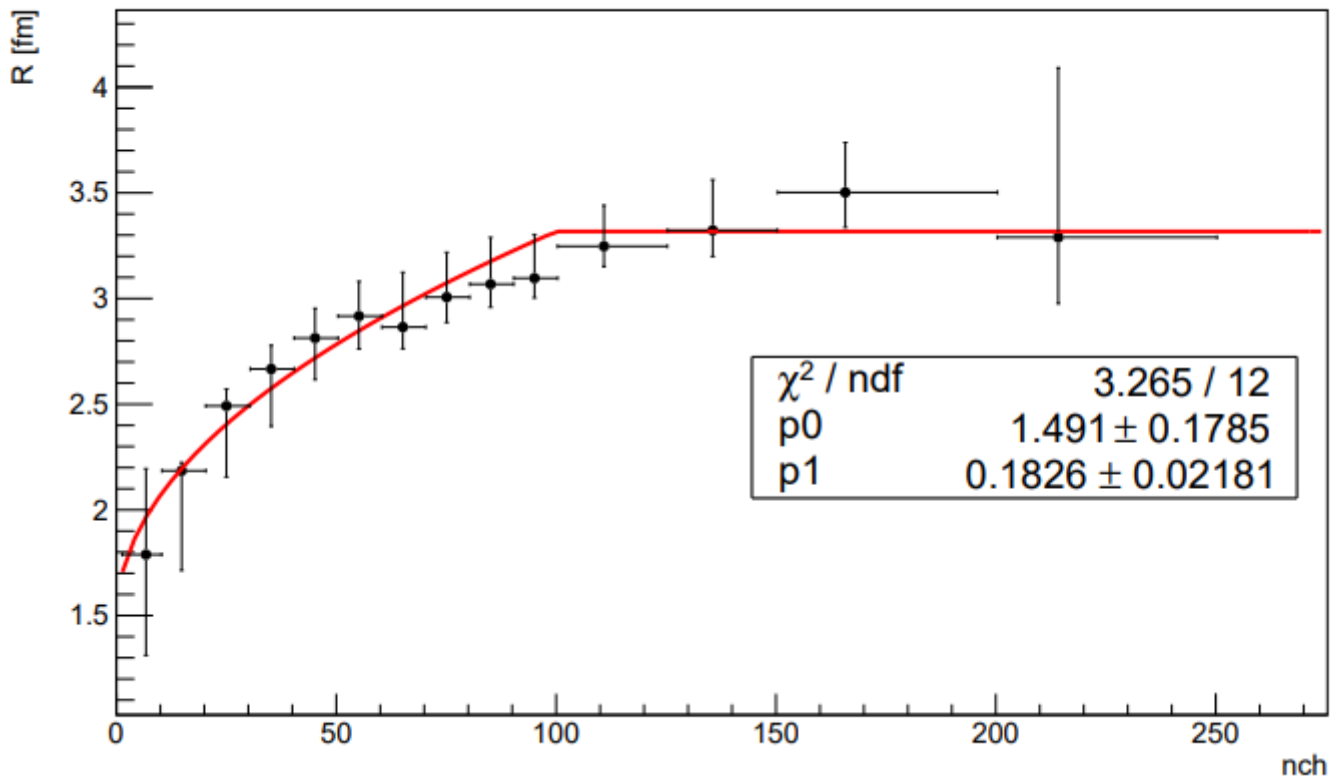


Figure 2. Approximation of the experimental data by the theoretical dependence $p_0 + p_1 \sqrt{n_a}$

Momentum representation of coherent states

$$\left\langle q_x, q_y \middle| z_1, z_2 \right\rangle \square\ e^{iR\sqrt{2}(\alpha_1 q_x + \alpha_2 q_y)} \times e^{-R^2/2\left[\left(q_x - \sqrt{2}/R\,\beta_1\right)^2 + \left(q_y - \sqrt{2}/R\,\beta_2\right)^2\right]}$$

$$\left|\left\langle q_x, q_y \middle| z_1, z_2 \right\rangle\right|^2 \square\ e^{-R^2/2\left[\left(q_x - \sqrt{2}/R\,\beta_1\right)^2 + \left(q_y - \sqrt{2}/R\,\beta_2\right)^2\right]} \qquad (1.19)$$

$$z_1 = \alpha_1 + i\beta_1, \alpha_1 = |z_1|\cos\vartheta_1, \beta_1 = |z_1|\sin\vartheta_1, \qquad z_2 = \alpha_2 + i\beta_2, \alpha_2 = |z_2|\cos\vartheta_2, \beta_2 = |z_2|\sin\vartheta_2$$

$$\sqrt{2}R\alpha_1 = \sqrt{2}R|z_1|\cos\vartheta_1 = \sqrt{2n}R\cos\vartheta_1 = r_0, R = 0{,}84\times 10^{-15}$$

Theory Field point of view

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

Y.Kurochkin, Y. Kulchitsky., S.Harkusha, and N.Russakovich / Solutions of the Klein-Fock-Gordon equations and Coherent states on Horosphere of the Lobachevsky Momentum space// Physics of elementary Particles and Atomic Nuclei. Theory 2021, T.18, N 7, pp716-720.

Theoretical Field picture

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

We model an incident hadron in the laboratory reference frame as scalar particle described by the equation Klein-Gordon-Fock

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial {x_0}^2}-S\right)\Psi(x,y,z,x_0)=0 \qquad (2.1)$$

Y.Kurochkin, Y. Kulchitsky., S.Harkusha, and N.Russakovich / Solutions of the Klein-Fock-Gordon equations and Coherent states on Horosphere of the Lobachevsky Momentum space// Physics of elementary Particles and Atomic Nuclei. 2021, T.18, N 7, pp716-720.

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

$$-P_x^{\,2} - P_y^{\,2} - P_z^{\,2} + P_0^{\,2} + S \rightarrow \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x_0^{\,2}} - S \qquad (2.2)$$

It is taken into account that the momentum of the system in the laboratory frame of reference is given by the expression

$$P = (\vec{P}, iP_0) = [p_x, p_y, p_z, i(p_0 + m_p)]$$

in the laboratory reference frame (rest frame, for example, of the second proton)

$$p = (p_x, p_y, p_z, ip_0) = (\vec{p}, ip_0)$$

-4 momentum of the incident proton in the reference frame where the second proton is at rest. The mass term is removed by multiplying the wave function by the factor $\exp imx_0$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

As is well known, the solution of Eq. (2.1) can be represented as the Fourier integral

$$\Psi^{\rightarrow\pm}(x, y, z, x_0) =$$

$$= (2\pi)^{-3/2} \int \delta(P^2 + S)\Phi^{\pm}(P_x, P_y, P_z, P_0) \exp[\pm i(xP_x + yP_y + zP_z - x_0 P_0)] d^4 P$$

This integral is defined on the impulse hyperboloid, as evidenced by the -function, and is invariant under transformations of the group of motions of this hyperboloid on which the geometry of the three-dimensional Lobachevsky space is realized. Transit from coordinates (1.3 to coordinates (1,4)

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar invariance

$$\Psi^{\to\pm}(x,y,z,x_0)=(2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}dq_z dq_x dq_y \varphi^{\pm}(q_x,q_y,q_z)\exp\pm i\{xq_x e^{-q_z/\sqrt{s}}+yq_y e^{-q_z/\sqrt{s}}+$$

$$+z\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}}+(\frac{q_x^{\ 2}+q_y^{\ 2}}{S}-1)e^{-q_z/\sqrt{s}}]-x_0\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}}+(\frac{q_x^{\ 2}+q_y^{\ 2}}{S}+1)e^{-q_z/\sqrt{s}}]\}. \qquad (2.3)$$

.

Let's make the following large-scale transformations in the above expression

$$q_x = kn_x \qquad q_y = kn_y \qquad (2.4)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar invariance

$$\Psi^{\pm}(x,y,z,x_0) =$$

$$(2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}dq_z d(kn_x)d(kn_y)\varphi^{\pm}(kn_x,kn_y,q_z)\exp\pm i\{xkn_x e^{-q_z/\sqrt{s}} + ykn_y e^{-q_z/\sqrt{s}} +$$

$$+z\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}} + (k^2\frac{n_x^{\ 2}+n_y^{\ 2}}{S}-1)e^{-q_z/\sqrt{s}}] - x_0\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}} + (k^2\frac{n_x^{\ 2}+n_y^{\ 2}}{S}+1)e^{-q_z/\sqrt{s}}]\}. \tag{2.5}$$

Let us now consider some obvious approximations, namely

$$k^2\frac{n_x^{\ 2}+n_y^{\ 2}}{S} \ll 1. \tag{2.6}$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

Then

$$\psi^{\pm}(x,y,z,x_0) =$$

$$(2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}dq_z d(kn_x)d(kn_y)\varphi^{\pm}(kn_x,kn_y,q_z)\exp\pm i\{xkn_x e^{-q_z/\sqrt{s}} + ykn_y e^{-q_z/\sqrt{s}} -$$

$$-z\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}} - e^{-q_z/\sqrt{s}}] - x_0\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}} + e^{-q_z/\sqrt{s}}]\}, \qquad (2.7)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

Developing the main hypothesis adopted according to which it is assumed that partons (constituents of the incident particle) are excitations on the horosphere, it is natural to consider *k* as a fraction of the momentum of one parton in the momenta q_x and q_y (By virtue of the Euclidean, and hence the isotropy, it is natural to consider the same in both directions). Since before the interaction the number of components of a scalar hadron is not determined, we require the invariance of the function under transformations

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar invariance.

Scale invariance will be observed

$$\psi^{\pm}(x,y,z,x_0)=\psi^{\pm}(kx,ky,z,x_0)=$$

$$(2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}dq_z d(kn_x)d(kn_y)\varphi^{\pm}(kn_x,kn_y,q_z)\exp\pm i\{xkn_x e^{-q_z/\sqrt{s}}+ykn_y e^{-q_z/\sqrt{s}}-$$

$$-z\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}}-e^{-q_z/\sqrt{s}}]-x_0\frac{\sqrt{S}}{2}[e^{q_z/\sqrt{s}}+e^{-q_z/\sqrt{s}}]\},\qquad(2.8)$$

if there $\varphi^{\pm}$ is a homogeneous function in variables q_x , q_y the degree of homogeneity -1, i.e.

$$\varphi^{\pm}(kn_x,kn_y,q_z)=k^{-1}k^{-1}\varphi^{\pm}(n_x,n_y,q_z)=k^{-2}\varphi^{\pm}(n_x,n_y,q_z)=\varphi^{\pm}(q_x,q_y,q_z)\qquad(2.9)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar invariance

The expression, as is easy to see, satisfies the two-dimensional wave equation. To establish this fact, it is convenient to represent expression () in the variables of the light cone (wave front), namely

$$\psi^{\pm}(x,y,z,x_0) = (2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}\, dq_z dn_x dn_y \varphi^{\pm}(n_x,n_y,q_z)\exp\pm i[xn_x e^{-q_z/\sqrt{s}} + yn_y e^{-q_z/\sqrt{s}} -$$

$$-(z+x_0)\frac{\sqrt{S}}{2}e^{q_z/\sqrt{s}} + (z-x_0)\frac{\sqrt{S}}{2}e^{-q_z/\sqrt{s}}]. \qquad (2.10)$$

.

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar invariance

and therefore

$$-\frac{\partial^2\psi}{\partial w\partial\overline{w}}=\frac{\partial^2\psi}{\partial z^2}-\frac{\partial^2\psi}{\partial x_0{}^2}=S\psi \qquad (2.11)$$

Here the variables of light cones are introduced

$$w=(z+x_0),\overline{w}=(z-x_0),\quad \frac{\partial}{\partial w}=\frac{1}{2}(\frac{\partial}{\partial z}+\frac{\partial}{\partial x_0}),\qquad \frac{\partial}{\partial\overline{w}}=\frac{1}{2}(\frac{\partial}{\partial z}-\frac{\partial}{\partial x_0})$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles

We will interpret the inverse value k as a value proportional to the number of partons. By virtue of the scale invariance described above, function (2.9) the wave function of a quantum system propagating along an axis is actually independent of the number of components. The significance of the number of components will become apparent during the period of pre-hadronization. In this case, the resulting hadrons will be described by functions for which the scale invariance is not satisfied. Thus, hadronization can be considered as a second-order “phase” transition occurring near the critical point, accompanied by violation of the scale invariance (2.5) – (2.9) in the plane perpendicular to the propagation direction. Symmetry breaking is known to be decisive for the second-order “phase” transition

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles. Strings.

Now consider case when

$$k^2 \frac{{n_x}^2 + {n_y}^2}{S} \gg 1, \qquad (2.12)$$

$$\Psi^{\pm}(x, y, z, x_0) =$$

$$(2\pi)^{-3/2} \int e^{-2q_z/\sqrt{S}} dq_z d(k'n_x) d(k'n_y) \varphi^{\pm}(k'n_x, k'n_y, q_z) \exp \pm i[xk'n_x e^{-q_z/\sqrt{s}} + yk'n_y e^{-q_z/\sqrt{s}} +$$

$$+ z \frac{\sqrt{S}}{2} (e^{q_z/\sqrt{s}} + k'^2 \frac{{n_x}^2 + {n_y}^2}{S} e^{-q_z/\sqrt{s}}) - x_0 \frac{\sqrt{S}}{2} (e^{q_z/\sqrt{s}} + k'^2 \frac{{n_x}^2 + {n_y}^2}{S} e^{-q_z/\sqrt{s}})], \qquad (2.13)$$

Since conditions (2.7) and (3.1) are two different states of the system, two different modes of behavior before the process of multiple birth, in what follows for the conditions following from (3.1) we will use k' instead k of .

Violation of scale invariance. Kinematic region of born particles. Strings.

or $\Psi^{\pm}(x, y, z, x_0) =$

$$(2\pi)^{-3/2}\int e^{-2q_z/\sqrt{S}}dq_z d(k'n_x)d(k'n_y)\varphi^{\pm}(k'n_x, k'n_y, q_z)\exp\pm i[xk'n_x e^{-q_z/\sqrt{s}} + yk'n_y e^{-q_z/\sqrt{s}} +$$

$$+(z-x_0)\frac{\sqrt{S}}{2}(e^{q_z/\sqrt{s}} + k'^2\frac{n_x^{\ 2}+n_y^{\ 2}}{S}e^{-q_z/\sqrt{s}})]. \qquad (2.14)$$

Obviously, in this case, the resulting wave is a retarded solution of the two-dimensional wave equation as follow

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles. Strings.

$$-\frac{\partial^2\psi}{\partial w\partial\bar{w}}=\frac{\partial^2\psi}{\partial z^2}-\frac{\partial^2\psi}{\partial x_0{}^2}=0 \qquad \frac{\partial\psi}{\partial w}=\frac{\partial\psi}{\partial z}+\frac{\partial\psi}{\partial x_0}=0 \qquad (2.15)$$

$$\frac{\partial\psi}{\partial\bar{w}}=\frac{\partial\psi}{\partial z}-\frac{\partial\psi}{\partial x_0}=\frac{\sqrt{S}}{2}(e^{q_z/\sqrt{s}}+k'^2\frac{n_x{}^2+n_y{}^2}{S}e^{-q_z/\sqrt{s}})\psi \qquad (2.16)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles. Strings.

Let us examine conditions (2.6) and (2.12) for consistency. To do this, in these expressions, we pass to the physical components of the 4-momentum (1.3) in the laboratory system of reference. In this case, condition (2.12) can be represented as

$$k'^2 \approx \frac{S}{n_x^{\,2} + n_y^{\,2}} = \frac{S}{p_x^{\,2} + p_y^{\,2}} e^{-2q_z/\sqrt{S}} = \frac{S}{p_x^{\,2} + p_y^{\,2}} \left(\frac{p_0 + m - p_z}{\sqrt{S}} \right)^2 \qquad (2.17)$$

Here it is taken into account that in the laboratory system of reference

$$p_0 \approx m, \;\; S = 2mp_0, \;\; p_x^{\,2} + p_y^{\,2} = p_0^{\,2} - p_z^{\,2} - S. \qquad (2.18)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles. Strings.

$$k'^2 \square \frac{S}{p_x^{\ 2} + p_y^{\ 2}} \left(\frac{p_0 + m - p_z}{\sqrt{S}} \right)^2 = \frac{\left(\frac{S}{2m} - p_z \right)^2}{\left(\frac{S}{2m} \right)^2 - p_z^{\ 2} - S} \qquad (2.19)$$

when we take into account $\left(\frac{S}{2m} \right)^2 \gg S,$ then

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles

$$k'^2 \gg \frac{\left(\frac{S}{2m} - p_z\right)^2}{\left(\frac{S}{2m}\right)^2 - p_z{}^2} = \frac{\frac{S}{2m} - p_z}{\frac{S}{2m} + p_z} \qquad (2.20)$$

$$\frac{1}{k'} \ll \left(\frac{\frac{S}{2m} + p_z}{\frac{S}{2m} - p_z}\right)^{\frac{1}{2}} = \left(\frac{p_0 + p_z}{p_0 - p_z}\right)^{\frac{1}{2}} \qquad (2.21)$$

Obviously, a similar condition for (2.6) in this case can be represented as

$$\frac{1}{k} \gg \left(\frac{\frac{S}{2m} + p_z}{\frac{S}{2m} - p_z}\right)^{\frac{1}{2}} = \left(\frac{p_0 + p_z}{p_0 - p_z}\right)^{\frac{1}{2}} \qquad (2.22)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles

Let us take the expressions (2.21) and (2.22) and take into account that while maintaining the plus sign on the right side, the directions of the inequalities will not change, as a result from expression (2.22) we will have

$$ln\frac{1}{k} \gg \frac{1}{2}ln\left(\frac{\frac{S}{2m}+p_z}{\frac{S}{2m}-p_z}\right) = \frac{1}{2}ln\left(\frac{p_0+p_z}{p_0-p_z}\right) \qquad (2.23)$$

The state described by inequalities (2.20)–(2.22) corresponds to the inequality

$$ln\frac{1}{k'} \ll \frac{1}{2}ln\left(\frac{\frac{S}{2m}+p'_z}{\frac{S}{2m}-p'_z}\right) = \frac{1}{2}ln\left(\frac{p_0+p'_z}{p_0-p'_z}\right) \qquad (2.24)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Let us consider how the transition from the regime corresponding to scale invariance, assuming that the incident particle potentially consists of composite partons unobservable before the interaction (the state of pre-hadronization) to the multiple production of particles, can be carried out. We will assume such a transition from the state corresponding to inequality (2.24) to the state corresponding to inequality (2.23) as a continuous or “phase” transition, for which the scale invariance violation is the determining factor and, in particular, when the wave function (2.8) – (2.10) satisfying the two-dimensional Klein-Fock equation (2.11) transforms into function (2.13), (2.14) that is non-invariant under scaling transformations (2.4). This function already satisfies the two-dimensional wave equation (2.15), which describes the strings, and equation (2.16).

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Violation of scale invariance. Kinematic region of born particles

Inequalities (2.23), (2.24) are not violated up to $p_z = 0$, and $p_z = 0$ when equality is possible.
The expressions on the right side of (2.23), (2.24) are variables which are called speed. They are usually referred to as . Those.

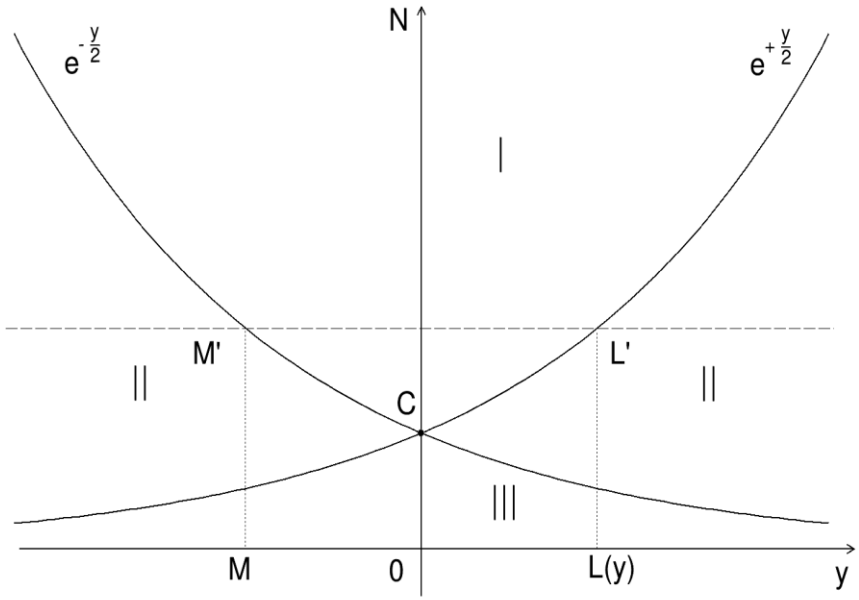
$$y = \frac{1}{2}\ln\frac{p_0 + p_z}{p_0 - p_z}, \qquad -y = -\frac{1}{2}\ln\frac{p_0 + p_z}{p_0 - p_z} = \frac{1}{2}\ln\frac{p_0 - p_z}{p_0 + p_z} \qquad (2.25)$$

Resolving logarithmic inequalities (2.23), (2.24) we obtain

$$\frac{1}{k} = N \gg Ce^{y} \qquad (2.26) \qquad\qquad \frac{1}{k'} = N' \ll Ce^{y'} \qquad (2.27)$$

Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

Figure 3 Phase diagram of the transition hadron - coherent state of partons, defined as excitations on the horosphere of the Lobachevsky momentum space into a set of born particles. . Kinematic phase diagram demonstrating the regions defined by inequalities (3.16), (3.17).



Hadron as a scalar particle on a hyperboloid of momentum space, with curvature determined by the energy of colliding particles. Scalar non invariance

On the lines separating the phases, which are exponential functions of the speed, inequalities (3.16), (3.17) are transformed into equalities. In this case, we will take as a condition at the phase boundaries and , respectively.

In inequalities (), () and in the diagram (Figure 3), the value S is assumed to be fixed. In this case, the change in speed is completely determined by p_z, which is a consequence of condition (1.2). It should be borne in mind the kinematic restrictions on the admissible values of the speeds and, accordingly, the restrictions on the regions I, II, III. Kinematic restrictions single out a finite region on the plane (*N,y*).

Conclusion

- Thus, in our approach, we distinguish two stages of the process.
- Stage 1 with dependence of the radius of correlations on the average multiplicity $p_0 + p_1 \sqrt{n_a}$, , $n_a < 110$ corresponds to II domain on the phath diagram
- And stage 2 $n_a > 110$ corresponds to III domain on the phath diagram Figure 3,
-

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Thank you for your attention