QUANTUM COMPETITION

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Markov Andrei Andrevich

(1856- 1922)

Dynamics of probabilities

Markov chains

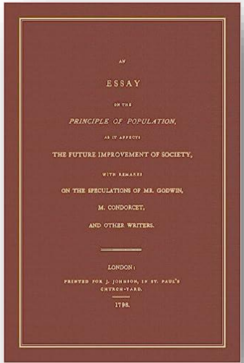
Markov processes

"future is independent of the past, given the present"

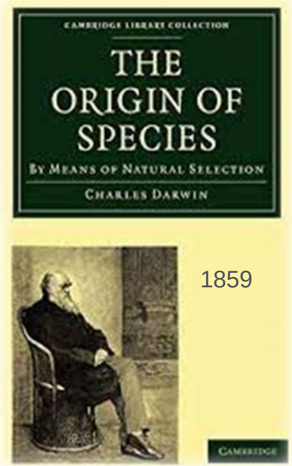
1906. A simple case—a system with just **two states**.

1913, paper on *Onegin poem.* Chain of two states, vowels and consonants in the first 20,000 letters of the poem. Letters are not independent …

Thomas Robert **Malthus**
(1766—1834)



Principle of population
(1809)



1859

Charles Robert Darwin
(1809-1882)

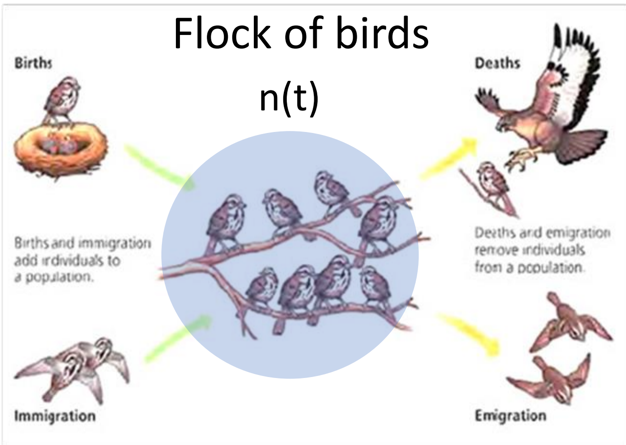
Natural selection

struggle for existence

Birth and Death Process
with immigration

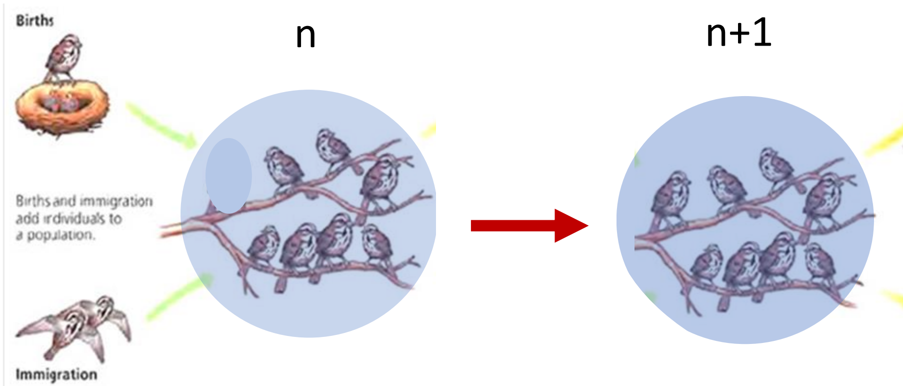
Flock of birds

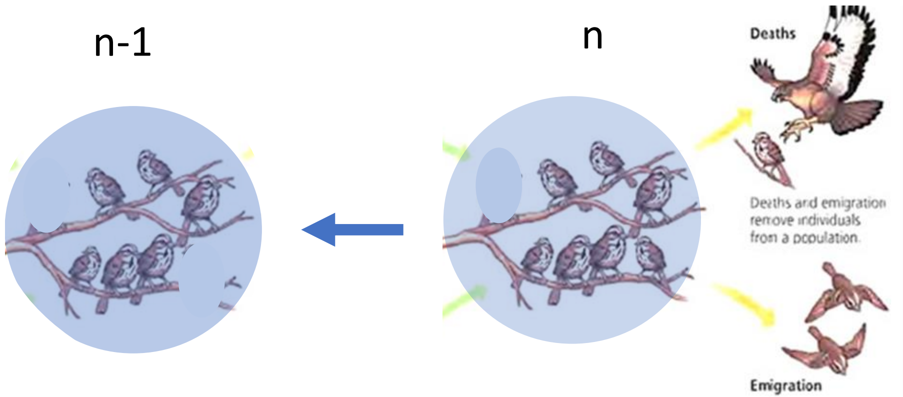
n(t)



Organisms produce more offspring than - given the limited amounts of resources - can ever survive, and organisms therefore **compete** for survival. Only the successful competitors will reproduce themselves. It was Charles Darwin who first discussed **this competition** and described it as the "struggle for existence". The **struggle for existence** takes place within a web of ecological relations.

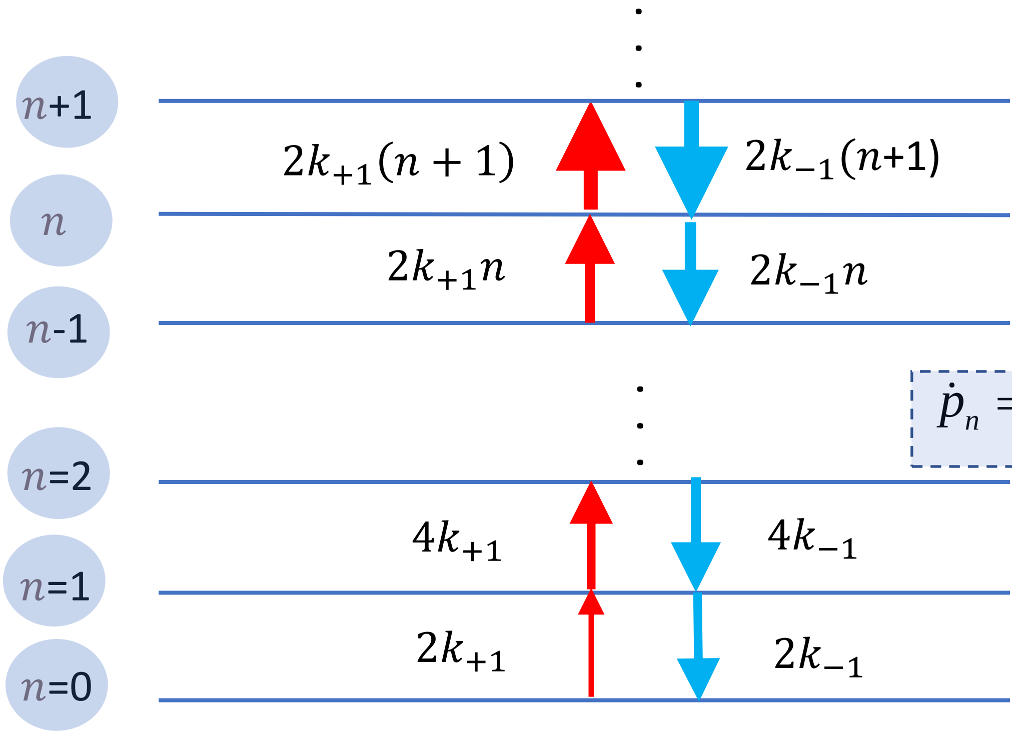
Birth


Births
Births and immigration add individuals to a population.
Immigration
n
n+1

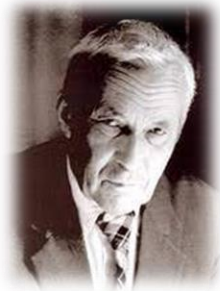

n-1
n
Deaths
Deaths and emigration remove individuals from a population.
Emigration

Death

The BD stochastic linear Markov jump process with immigration



The **state-dependent** up/down **transition rates**



Kolmogorov
Andrei Nikolaevich
(1903-1987)

Kolmogorov equations

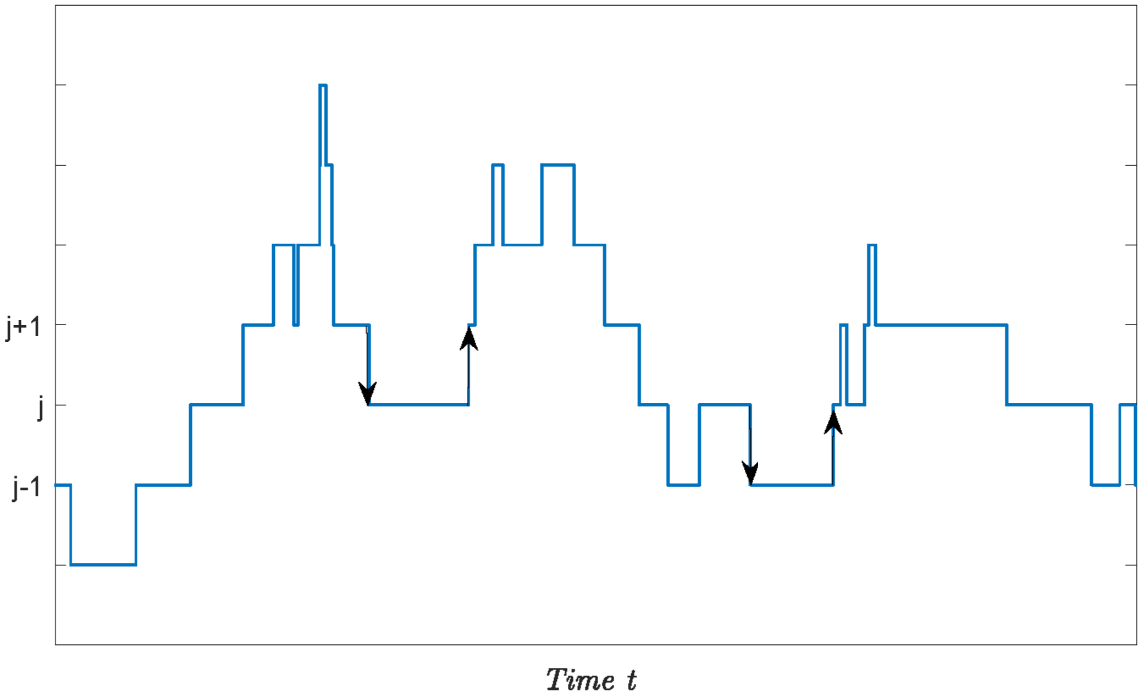
$$\dot{p}_n = -\gamma_n p_n + 2k_{+1} n p_{n-1} + 2k_{-1}(n+1) p_{n+1}$$

$$\gamma_n = 2(\Gamma n + k_{+1}), \quad \Gamma = k_{+1} + k_{-1}$$

Steady state solution

$$p_{n\,(t \to \infty)} \sim e^{-\frac{k_{+1}}{\Gamma} n}$$

Trajectories of the BD jump Markov process

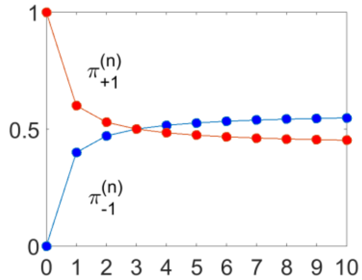


A particular realization of the classical BD process

Classical BD trajectories evolution/ simulation

Up or down ?

$$\pi_{+1}^{(n)} + \pi_{-1}^{(n)} = 1$$



The next jump conditional probability:

$$\pi_{\xi}^{(n)} = k_{\xi}(2n + \xi + 1) / \gamma_n ,$$

$$\gamma_n = 2\left(\Gamma n + k_{+1}\right), \quad \Gamma = k_{+1} + k_{-1}$$

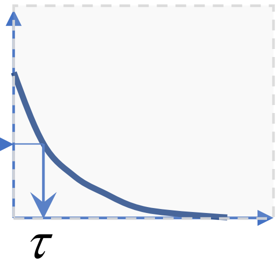
The waiting time distribution function (WTDF) of the next jump:

When?

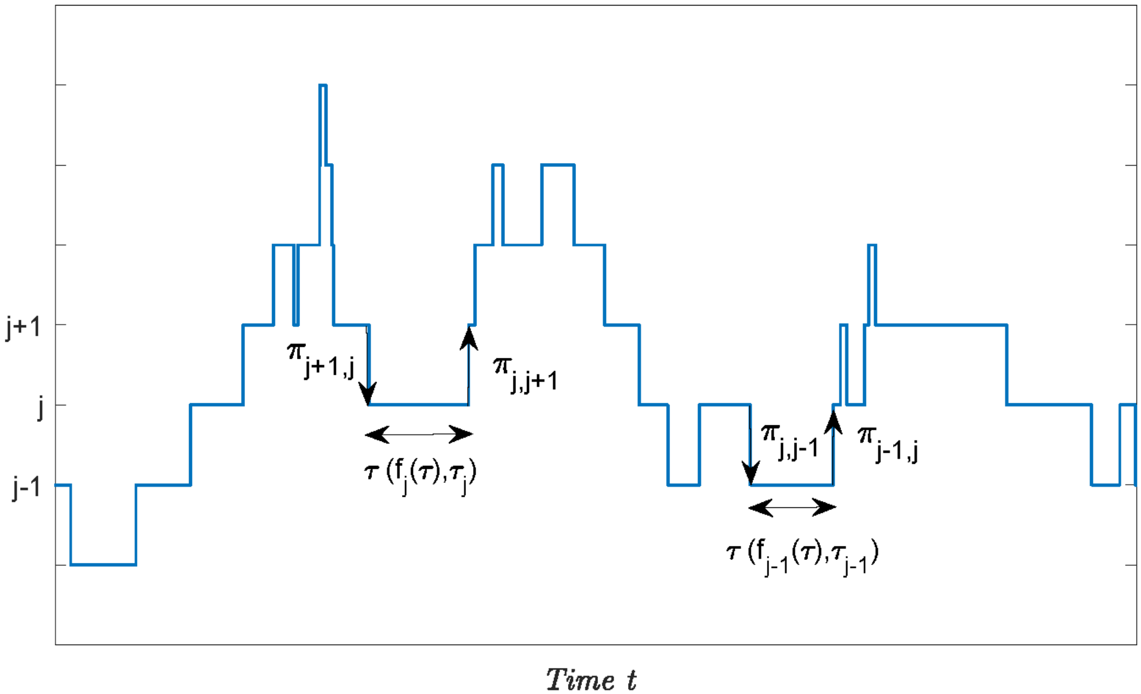
$$f_n(\tau) = e^{-\gamma_n \tau}$$

→ *Markov process*

→ $x \in [0,1]$ →



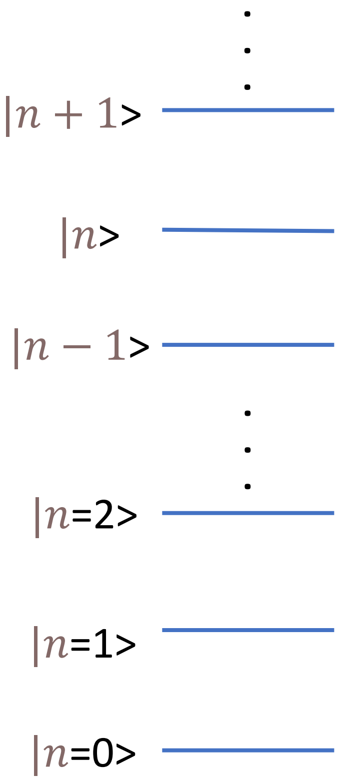
Trajectories of the BD jump Markov process



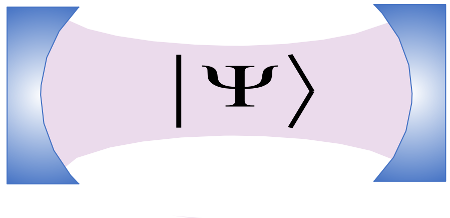
A particular realization of the classical BD process

QUANTUM

Quantum



Birds → Photons



$|\Psi\rangle$ —

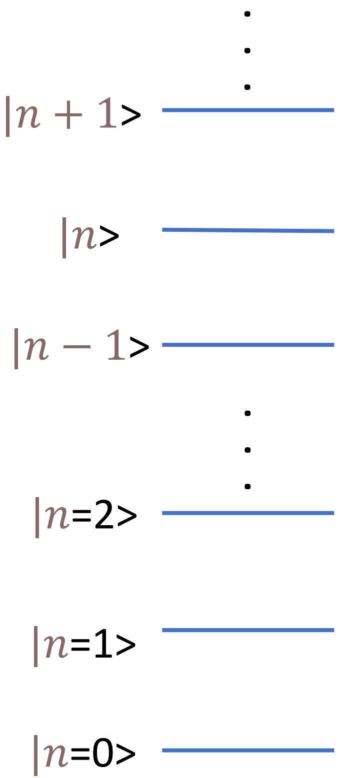
state of knowledge, or objective state of the system

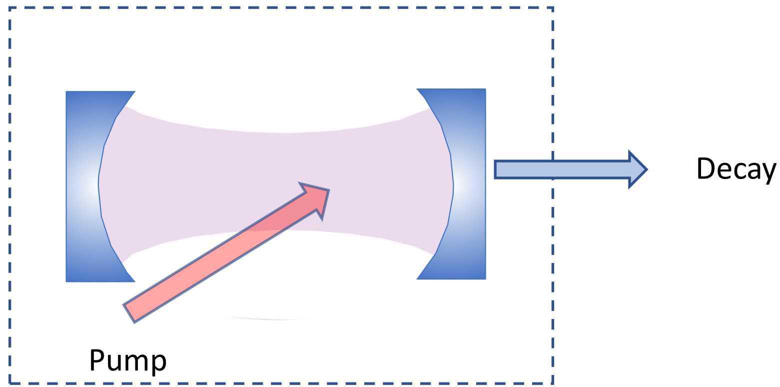
$$|\Psi\rangle = \sum_{n=0} C_n |n\rangle$$

Fock states

Vladimir Aleksandrovich Fock
(1898 –1974)

Quantum Open System

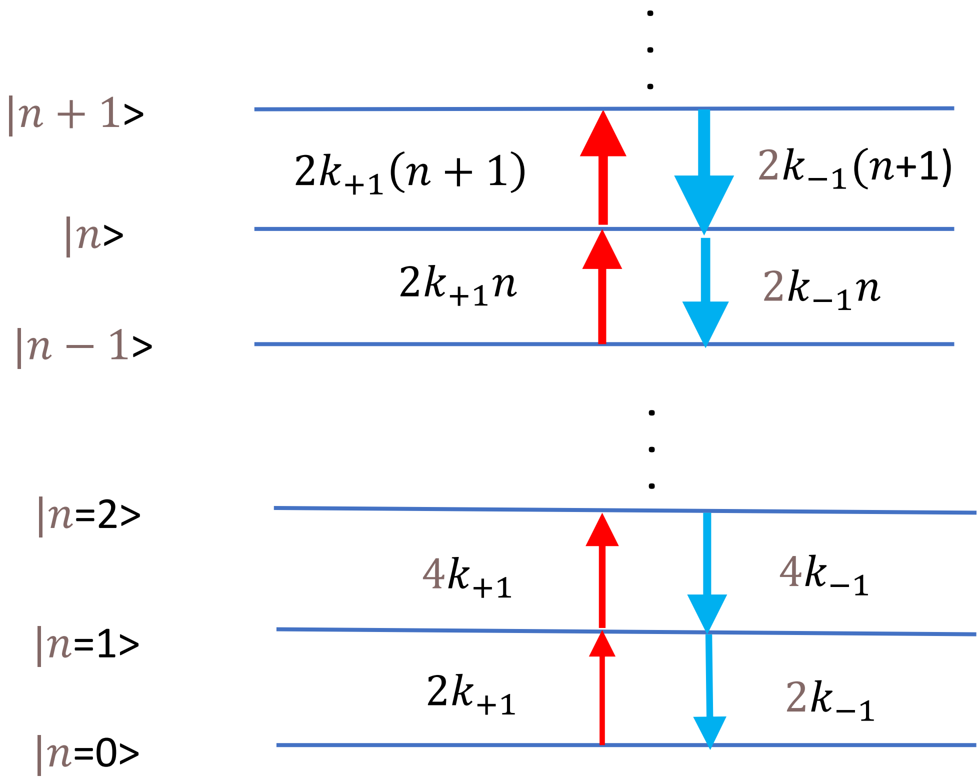




$$\dot{\rho} = \sum_{\xi=\pm1} k_{\xi} \left([a_{\xi}, \rho a_{\xi}^{\dagger}] + [a_{\xi}\rho, a_{\xi}^{\dagger}] \right),$$

$$a_{+1} = a^{\dagger},\ a_{-1} = a$$

Quantum Open System



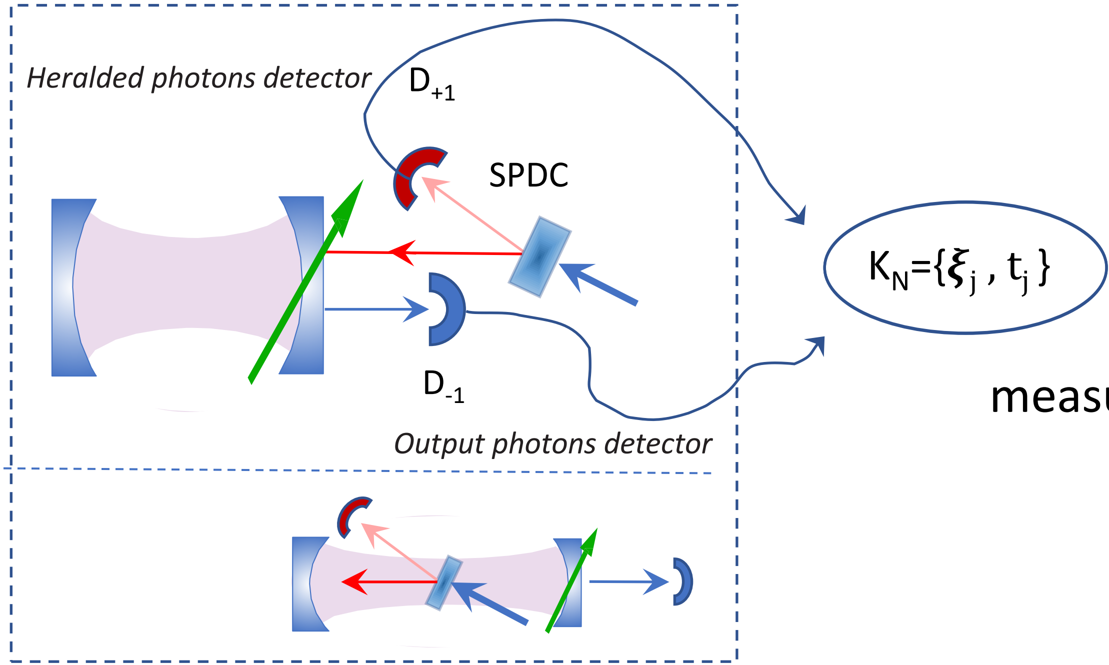
$$\dot{\rho} = \sum_{\xi=\pm1} k_\xi ([a_\xi, \rho a_\xi^\dagger] + [a_\xi \rho, a_\xi^\dagger])$$

Diagonal elements =>
Kolmogorov equations

$$\rho_{t\to\infty}^{(n)} \sim e^{-\frac{k_{+1}}{\Gamma} n}$$

Thermal state distribution

Continuous Measurement of the Cavity Energy Change in Quanta



The raw measurement record

Scheme of proof-of-principle experiment (I/O photons counting)

Measurement Record and Conditional States

$$\begin{array}{llllllll} \textit{Counts times} \rightarrow & t_1, & t_2, & t_3, & t_4, & \ldots, & t_{i-1}, & t_i, \quad \ldots \\ \textit{Counts types} \rightarrow & \xi_1, & \xi_2, & \xi_3, & \xi_4, & \ldots, & \xi_{i-1}, & \xi_i, \quad \ldots \end{array} \Bigg\} \textit{Measurement Record}$$

$$\textit{Aftercounts states} \rightarrow \rho^{(1)}, \quad \rho^{(2)}, \quad \rho^{(3)}, \quad \rho^{(4)}, \quad \ldots, \quad \rho^{(i-1)}, \quad \rho^{(i)}, \quad \ldots \Big\} \textit{Quantum Trajectory}$$

ρ_0	→	$\{\xi_k, t_k\}_N$	→	$\rho^{(N)}_{\{\xi_k, t_k\}}$
A priori state		*Record*		*A posteriori state*

Markov property: $\rho^{(i)} = \hat{M}\rho^{(i-1)}$

$$\xi_{i-1}, t_{i-1} = \xi', t' \qquad \xi_i, t_i = \xi, t$$

$$\rho^{(i-1)} = \rho(\xi', t') \qquad \rho^{(i)} = \rho(\xi, t)$$

ME Unraveling and Quantum Markov Chain

$$\rho(\xi,t) = \frac{J_\xi S^{(t-t')}\rho(\xi',t')}{Tr[J_\xi S^{(t-t')}\rho(\xi',t')]}$$

The **conditional** field state $\rho(\xi, t)$ at the moment t right after the ξ-count

Kilin (90), Barchielli, Belavkin (91), Carmichael (93) …

ξ-count (jump) → $J_\xi x = 2k_\xi a_\xi x a_\xi^\dagger$

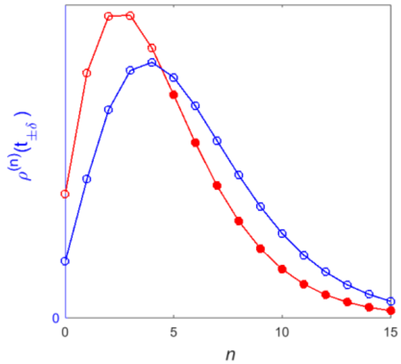
No counts → $S^{(\tau)} x = B^{(\tau)} x \left(B^{(\tau)}\right)^\dagger$

$$\begin{cases} B^{(\tau)} = e^{-iH_{eff}\tau/\hbar} \\ H_{eff} = -i\hbar \sum_{\xi=\pm1} k_\xi a_\xi^\dagger a_\xi \end{cases}$$

The same record presents subjectively different QTs

Opposite Conditional State Transformations

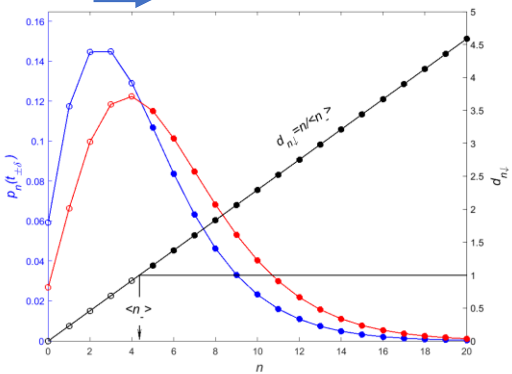
No-jump
state evolution →



$$\frac{\dot{\rho}_S^{(n)}}{\rho_S^{(n)}} = -2\Gamma\left(n - \langle n_\tau \rangle\right)$$

$$\langle n_\tau \rangle = \sum_n n \rho_S^{(n)}(t' + \tau)$$

After jump
state redistribution →



$$\frac{\rho^{(n+\xi)}(\xi,t)}{\rho_S^{(n)}(t)} = \frac{2n+\xi+1}{2\langle n_{t-t'} \rangle + \xi + 1} = d_{n\xi}$$

Quantum Trajectories Evolution/Simulation

The next jump conditional probability:

Up or down ?

$$\pi_{\xi\xi'} = Tr\int_{0}^{\infty} d\tau J_{\xi} S^{(\tau)} \rho(\xi') = \sum_{n} \pi_{\xi}^{(n)} \rho_{nn}(\xi')$$

$$\pi_{+1\xi'} + \pi_{-1\xi'} = 1$$

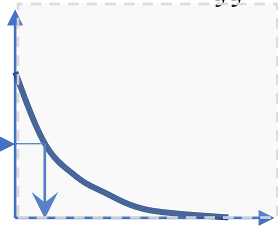
$$\pi_{\xi}^{(n)} = k_{\xi}(2n + \xi + 1) / \gamma_{n},$$

The waiting time distribution function (WTDF) of the next jump:

When?

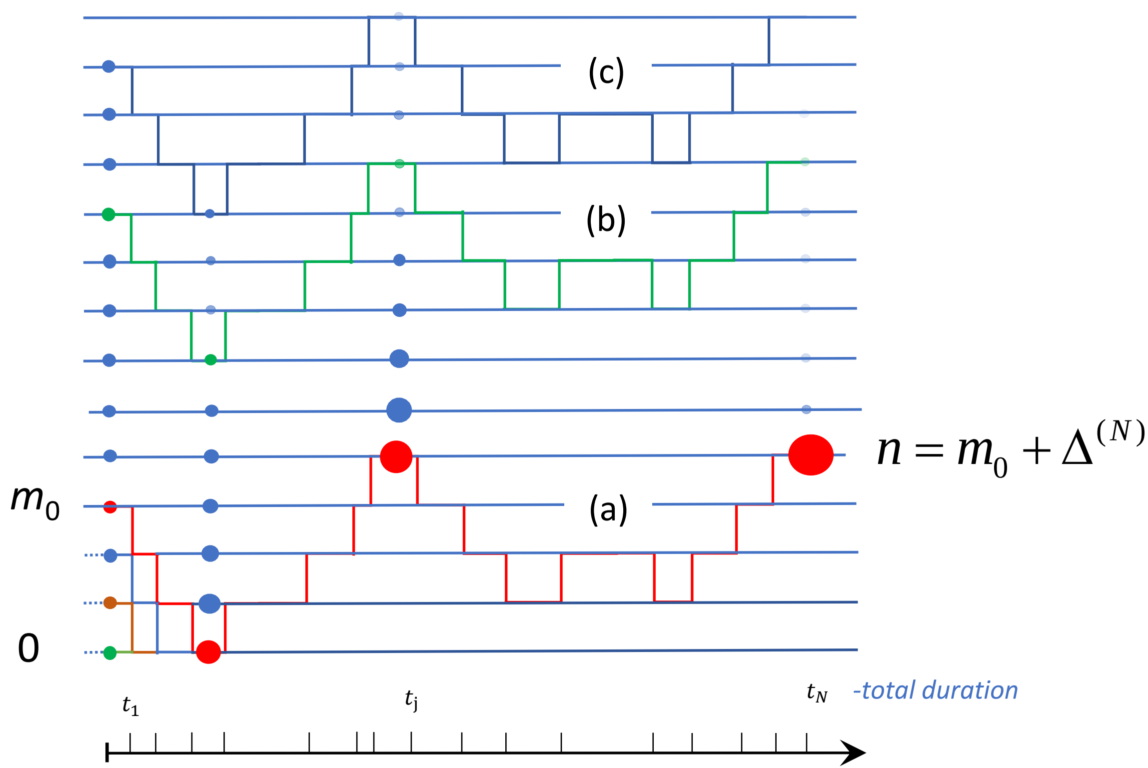
$$\omega_{\xi\xi'}(\tau) = \frac{1}{p_{\xi\xi'}} Tr\int_{\tau}^{\infty} dt J_{\xi} S^{(t)} \rho(\xi') = \sum_{n} p_{\xi\xi'}^{(n)} e^{-\gamma_{n}\tau}.$$

$\rightarrow x \in [0,1] \rightarrow$



Semi-Markov process

Quantum **Compound** Trajectories and Stochastic Processes



QUANTUM COMPETITION

TWO THEOREMS

Conditional State Evolution (two theorems)

Theorem 1: Independently of the *a priori* state of an open cavity field mode, the field state after **a long single run** becomes, in ergodic case, **a random Fock state**.

Theorem 2: If during a long single run with a record $\{\xi_k, t_k\}$ of N counts a Fock state has been created, this state **can be inferred** with unit asymptotic fidelity for ergodic regime using only the sequence $\{\xi_k\}$ of counts' types and its total duration t_N without referring to the initial state and times $\{t_k\}$ of intermediate clicks.

Proof of Theorems

The DM for a **sub-ensemble** conditioned by the record $\{\xi k, tk\}$ of N counts.

$$\langle n|\rho^{(N)}_{\{\xi_k,t_k\}}|n\rangle = \delta_{n,m+\Delta^{(N)}}\mathfrak{A}^{(N)}_{m,\{\xi_k\},t_N}\langle m|\rho_0|m\rangle, \quad (A)$$

m-trajectories

$\{\xi_k\}:|m\rangle \mapsto |n\rangle$

*A **partial** m-to-n correspondence*

$n-m=\Delta^{(N)}$

Energy conservation law

The measured change of the cavity energy in quanta

The conditional transition probability (CTP)

$$\mathfrak{A}^{(N)}_{m,\{\xi_k\},t_N} = e^{-2\Gamma m t_N}(m)_{N,f} / \langle e^{-2\Gamma m t_N}(m)_{N,f}\rangle_{\rho_0}, \quad (B)$$

no-jump maps

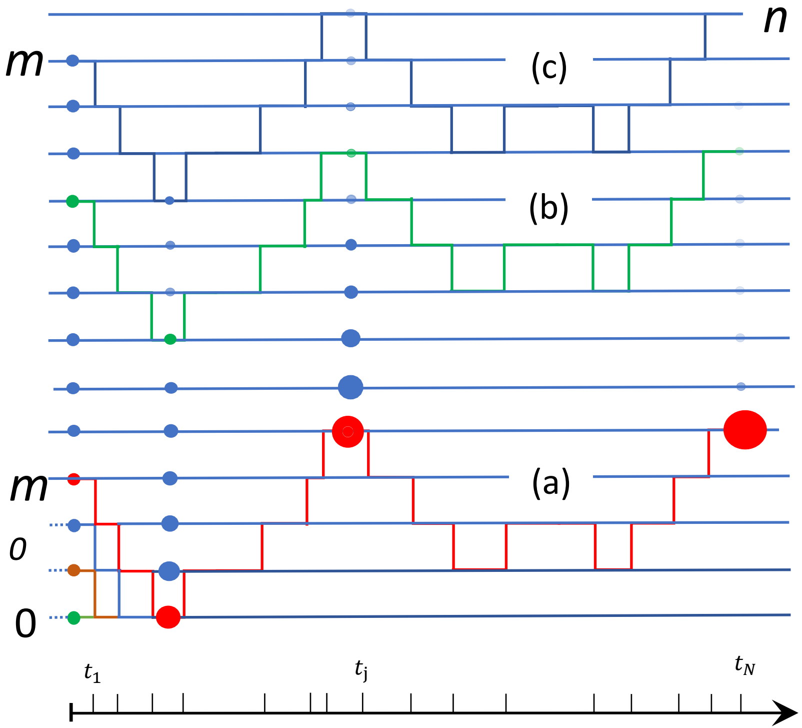
jump maps

$(m)_{N,f} = \prod_{k=1}^{N}\left(m+f^{(k)}\right),$ *Generalized Pochhammer symbol*

$f^{(k)} = \Delta^{(k)} + (1-\xi_k)/2$ *f-key*

Quantum **Compound** Trajectories

A sum of m-trajectories $\{\xi_k\} : |m\rangle \mapsto |n\rangle$

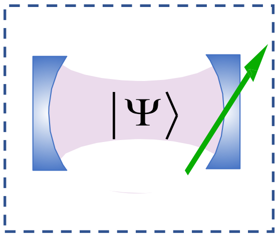


$n = m + \Delta^{(N)}$

- *low weight m-trajectories* $(m > m_0)$
- *dominant m_0-trajectory* $(m_0 = -\Delta_{min})$
- *truncated m-trajectories* $(m < m_0)$

FROM UNKNOWN TO DEFINETLY KNOWN

Quantum No Cloning & Quantum Transformation



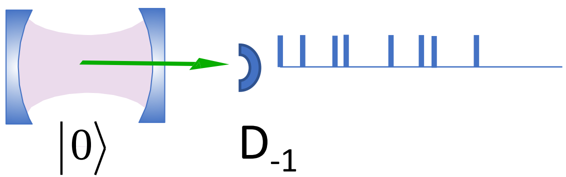
One copy of **unknown** state → No cloning theorem

(*Wootters , Zurek , 1982*)

How to use unknown state? → To transform to **a known nontrivial state**

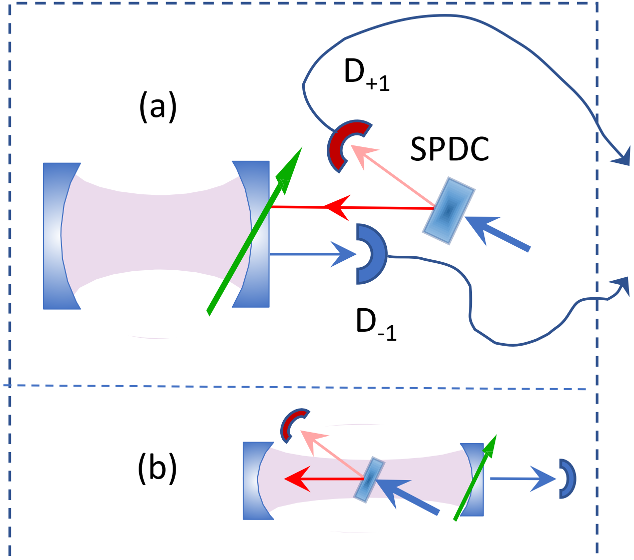
(But not, how to know ?)

QND → $|n\rangle$



Vacuum initialization

Fock State Inferring



Raw measurement record

$K_N=\{\xi_j, t_j\}$

$$\Delta^{(N)} = \sum_{j=1}^{N} \zeta_j$$

- total energy change in quanta

$\zeta_j \in \pm 1$

Vacuum initialization → CM → $|\Psi_N\rangle = |n\rangle, \quad n = \Delta^{(N)}$!

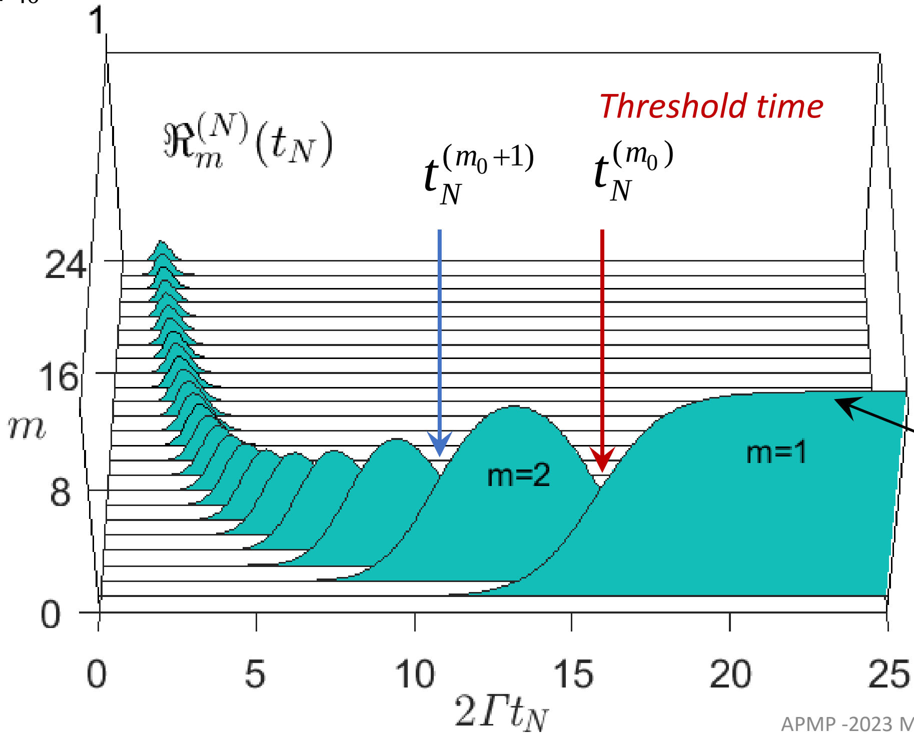
~~Vacuum initialization~~ → CM → $|\Psi_N\rangle = ?$

ENERGY-TO-TIME DECODING

Duration of m-trajectories for **given** *f*-key

(When m-trajectories end?)

$f_{40}=1100122112210000111234433443222111111100,\quad m_0=1\quad (m_0=-f_{min}+1\equiv-\Delta_{min})$



Time scaling

$$2\Gamma t_N^{(m)}=\log\left(\frac{(m+1)_{N,f}}{(m)_{N,f}}\right)$$

$$\Re_{m_0}^{(N)}(t_N)=\frac{1}{1+e^{-2\Gamma(t_N-t_N^{(m_0)})}}$$

Logistic function $\quad t_N>t_N^{(m_0)}$

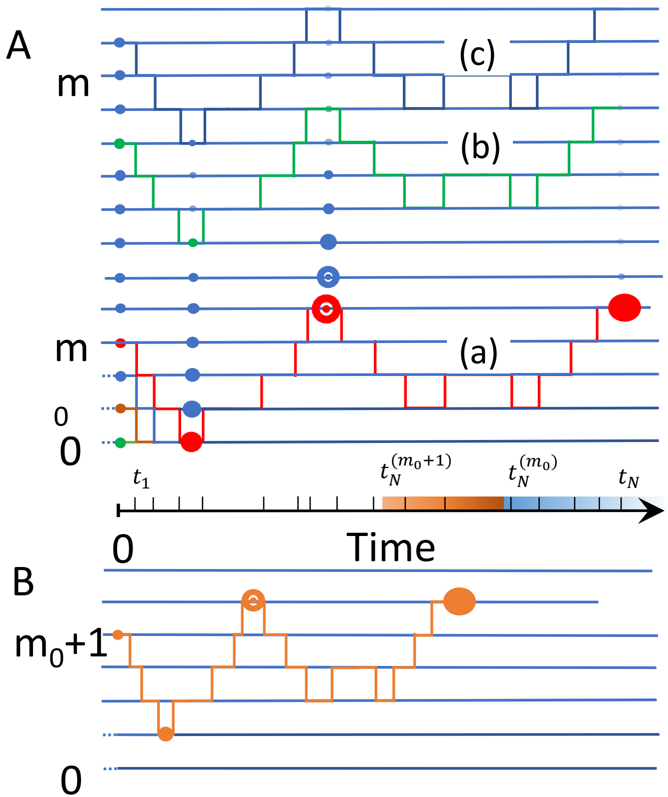
Energy-to-Time Decoding:

time scaling and Fock state inferring

$n \rightleftarrows n+1$

indistinguishable in frequency,

but can be discerned by transition rates $2k_\xi(n+1)$



Time windows $m > m_0$

$$\mathbf{T}_N^{(m)} = (t_N^{(m+1)}, t_N^{(m)})$$

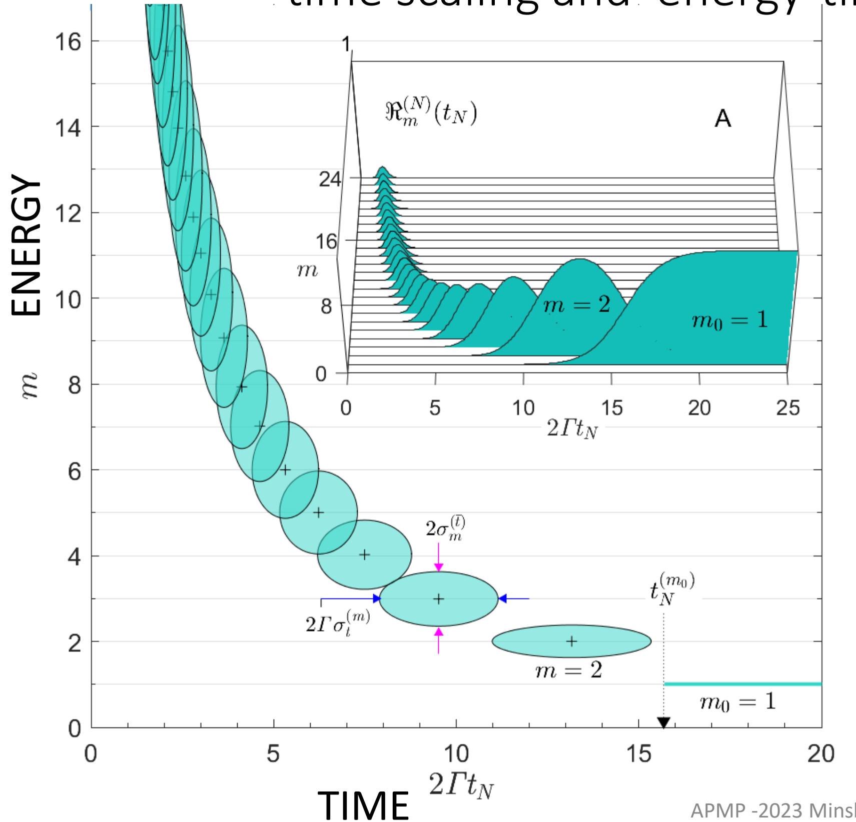
Condition of Inferring

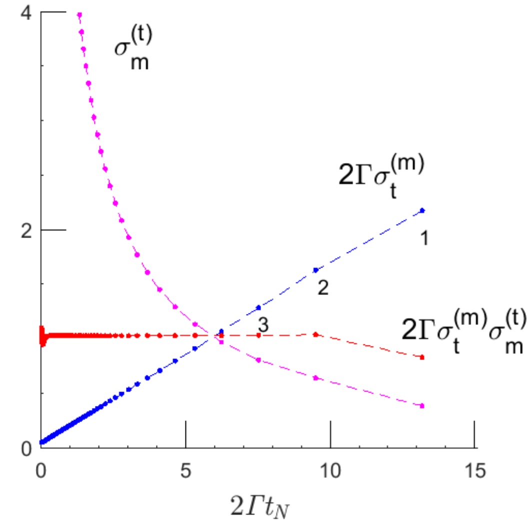
$$t_N > t_N^{(m_0)} > T_0$$

Mean First Passage Time from any state to vacuum

Energy-to-Time Decoding:

time scaling and energy-time uncertainty relation





$$\sigma_m^{(t)}\sigma_t^{(m)} = 1/2\Gamma$$

Energy-time uncertainty equality

Mandelstam, Tamm (1945)

K4HQS PROTOCOL

Key for a Hidden Quantum State

K4HQS Protocol

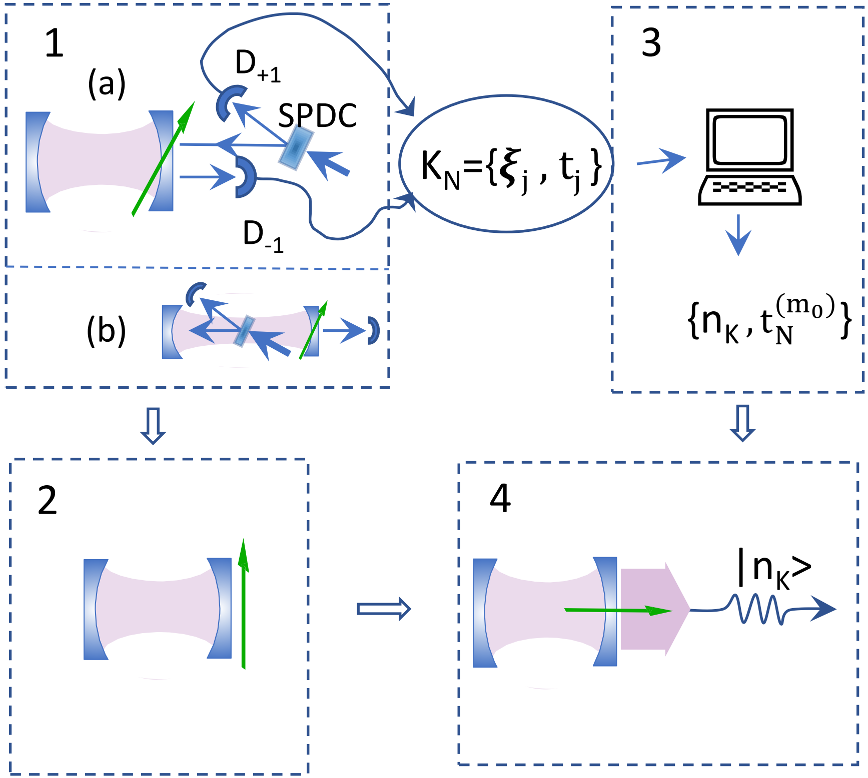
1. Register a random sequence $K_N = \{\xi_k, t_k\}$ of N clicks from ideal detectors monitoring photons leaving and entering the cavity.

2. ”Lock” the cavity from the environment to leave the In-cavity field unchanged.

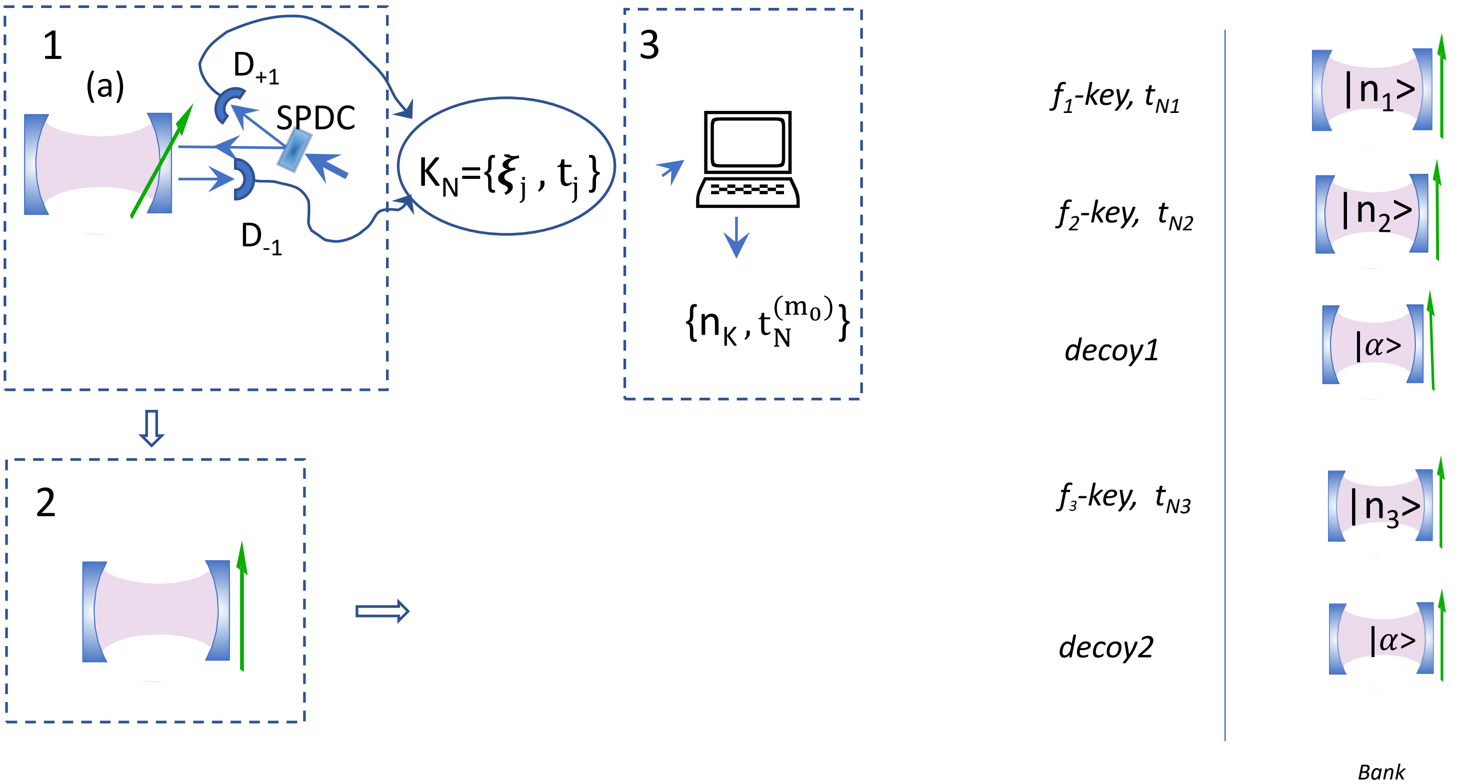
3. For the registered sequence $\{\xi_k\}$, find the minimum $\Delta_{\min}$ and the total $\Delta^{(N)}$ values of energy change and calculate the threshold $t_N^{(m_0)}$

4. Provided that , $t_N > t_N^{(m_0)}$ the intracavity field will be in the Fock state $\left| n_k = \Delta^{(N)} - \Delta_{\min} \right\rangle$. Unlock the cavity to use this state .

5. If $t_N < t_N^{(m_0)}$, one can start the protocol over, or find the $\mathbf{T}_N^{(m)}$-window wherein t_N is located, and determine m.



Created State as a Hidden Resource



Non-ideal Detection and Protocol Feasibility

Protocol feasibility

$$T_{e0} = -(2k_{-1}(1-q))^{-1}\log(1-q)$$ ← First Passage Time from equilibrium to |0>

$$\langle t_N \rangle > t_N^{(m_0)} > T_{e0}$$ → $N < 50, \quad q = \frac{k_{+1}}{k_{-1}} < 0.75$

Non-ideal detection

Detectors with error ϵ → $F = 1 - N\epsilon \simeq 0.8 - 0.9$

$N\epsilon$ events will not be registered

$N = 40 - 20$

Superconducting nanowire single-photon detectors with 99.5% detection efficiency *(J. Chang, et al, APL Photonics 6, 036114 (2021))*

Immunity to paired errors:

$Q_{0.05} = -0.88$, $Q_{0.1} = -0.76$

Q - Mandel parameter

POVM

The considered model of **continuous measurement of the cavity energy change**, which in fact goes back to the origins of Planck's quantum theory, shows that as time goes by, the *a posteriori* quantum state has progressively larger overlap with a **random energy eigenstate**.

For **the ergodic regime** in the limit $t \to \infty$, the realized measurement becomes an **orthogonal POVM measurement**.

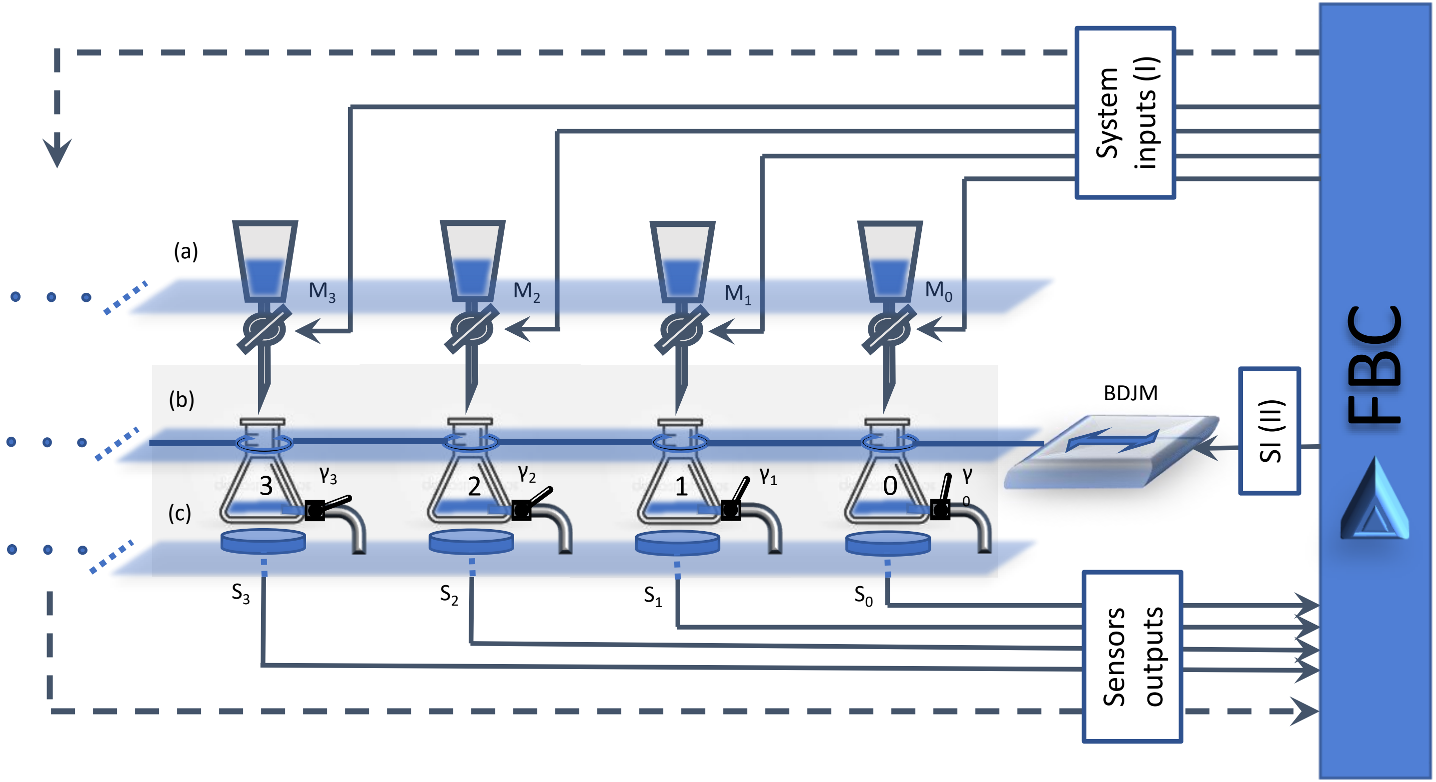
POVM measurement		
	The two-integers outcome	$(\Delta^{(N)}, m_0 = -\Delta_{min})$
	The measurement operators	$A^{(m_0)} = \vert m_0 + \Delta^{(N)}\rangle\langle m_0 \vert$
	The orthogonal set of the POVM elements	$E^{(m_0)} = \vert m_0\rangle\langle m_0 \vert, m_0 = 0,1,2,\ldots$ $\sum_{m_0} (A^{(m_0)})^\dagger A^{(m_0)} = \sum_{m_0} E^{(m_0)} = I$

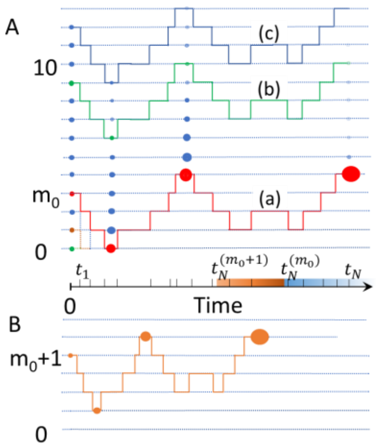
A posteriori state can be **asymptotically stable**, that is, independent of the a priori state. I.e. **two** initial states-of-knowledge (e.g., complete and limited) will converge together as data is obtained, iff both contain m_0

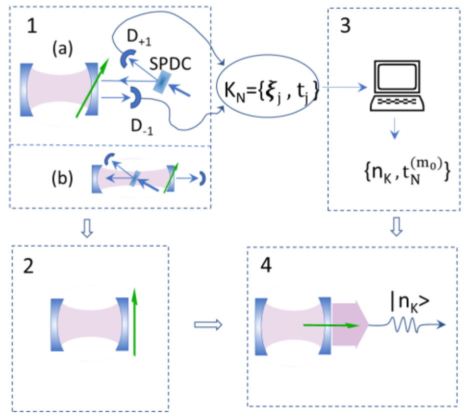
Handel (09), Jacobs (14)

The considered quantum stochastic BD process has the classical counterpart, **the collective BD process**, which has not yet been discussed in the literature.

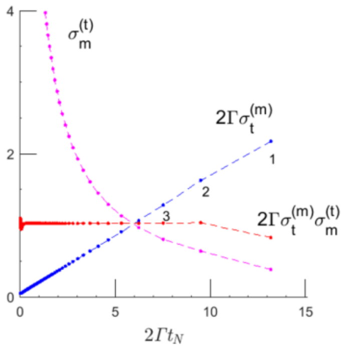
Collective BD Process

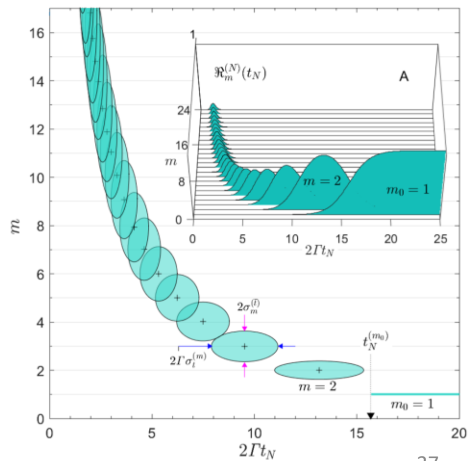



A
B
10
m0
0
m0+1
(a)
(b)
(c)
Time


1
(a)
(b)
SPDC
2
3
4
|nK>

Thank you!


$\sigma_m^{(t)}$
$2\Gamma\sigma_t^{(m)}$
$2\Gamma\sigma_t^{(m)}\sigma_m^{(t)}$
$2\Gamma t_N$


A
$m = 2$
$m_0 = 1$
$2\Gamma t_N$

Plan

- Classical Markov BD jump process
- Quantum compound trajectories and stochastic processes
- Scheme of proof-of-principle experiment
- Conditional state evolution (two theorems)
- Energy-to-time decoding: time scaling and energy-time uncertainty relation
- K4HQS Protocol
- The created state as a hidden resource
- Non-ideal detection and protocol feasibility
- Interpretation and discussion