Physics beyond the Standard model at future lepton colliders

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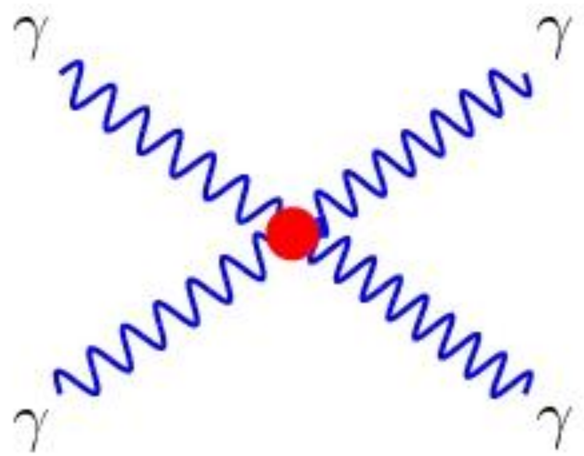
Plan of the talk

1. Probing anomalous $\gamma\gamma\gamma\gamma$ and $\gamma\gamma\gamma Z$ couplings in collision of Compton backscattered (CB) photons at the CLIC.
2. Study of invisible massive dark photon in γe scattering at future lepton colliders ILC, CLIC, and CEPC.
3. Search for axion-like particles (ALPs) via vector boson scattering at future muon collider.
4. Conclusions.

Anomalous gauge couplings

Light-by-light scattering (LBL) of CB photons at the CLIC

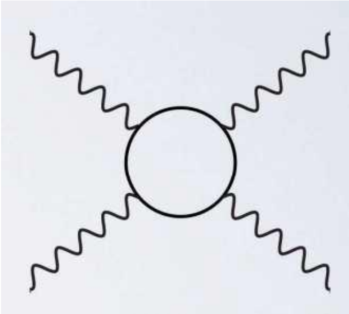


Anomalous diagram for process γγ → γγ

The goal is to examine anomalous quartic gauge couplings (QGCs) via collision γγ → γγ at the CLIC

New physics contributions to 4γ couplings

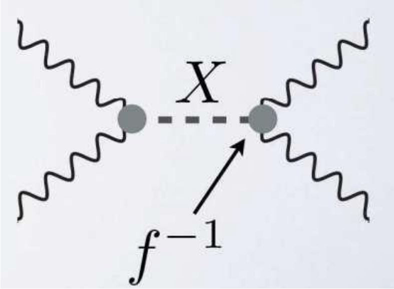
New charged particles via loops



$\zeta_i \sim Q^4\, m^{-4}$

Example: top partner

New neutral particles at tree level

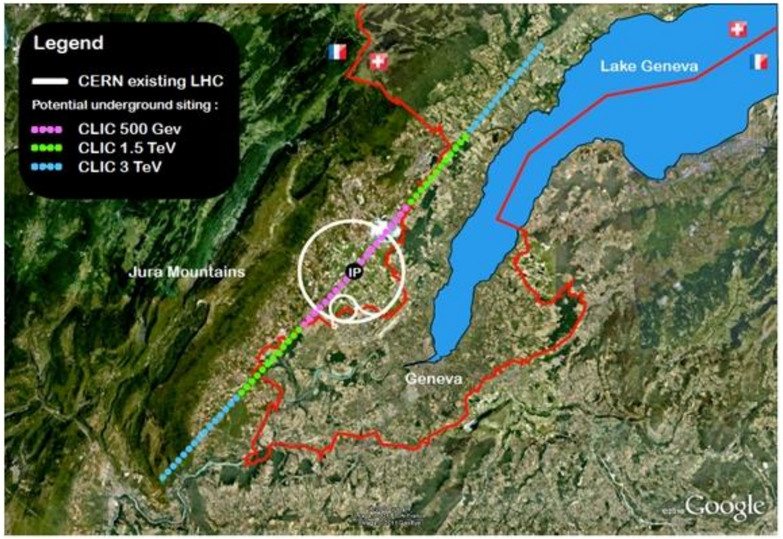


$\zeta_i \sim f^{-2}\, m^{-2}$

Example: KK gravitons, radion (warped extra dimension)

If f_{KK} ~ TeV and m_{KK} ~ few TeV, then $\zeta_i \sim 10^{-2} - 10^{-1}$ TeV^{-4}

Compact Linear Collider (CLIC)



	Beam energy	Integrated luminosity (unpolarized beams)
1st stage	190 GeV	1.0 ab^{-1}
2nd stage	750 GeV	2.5 ab^{-1}
3rd stage	1500 GeV	5.0 ab^{-1}

λ_0 is helicity of laser photon beam
λ_e is helicity of electron beam before
Compton backscattering

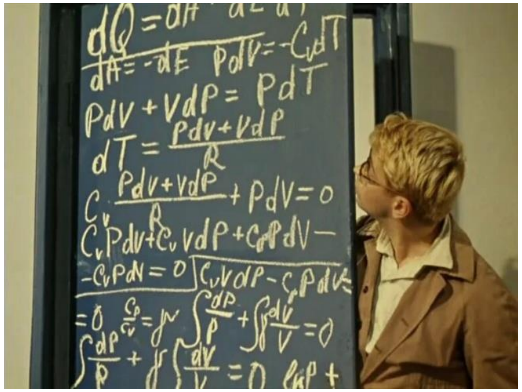
$$(\lambda_e^{(1)}, \lambda_0^{(1)}; \lambda_e^{(2)}, \lambda_0^{(2)}) = (0.8, 1; 0.8, 1) \ ,$$
$$(\lambda_e^{(1)}, \lambda_0^{(1)}; \lambda_e^{(2)}, \lambda_0^{(2)}) = (-0.8, 1; -0.8, 1)$$

CLIC energy stages and integrated luminosities for unpolarized and polarized electron beams

		L, fb^{-1}		
Stage	$\sqrt{s}$, GeV	$\lambda_e = 0$	$\lambda_e = -0.8$	$\lambda_e = 0.8$
2	1500	2500	2000	500
3	3000	5000	4000	1000

Differential cross section for LBL scattering

$$
\begin{aligned}
\frac{d\sigma}{d\cos\theta} = \frac{1}{128\pi s} \int\limits_{x_{1\,\min}}^{x_{\max}} \frac{dx_1}{x_1} f_{\gamma/e}(x_1) \int\limits_{x_{2\,\min}}^{x_{\max}} \frac{dx_2}{x_2} f_{\gamma/e}(x_2) \\
\times \Big\{ \Big[1 + \xi\left(E_\gamma^{(1)}, \lambda_0^{(1)}\right) \xi\left(E_\gamma^{(2)}, \lambda_0^{(2)}\right)\Big] \\
\times \left(|M_{++++}|^2 + |M_{++--}|^2\right) \\
+ \Big[1 - \xi\left(E_\gamma^{(1)}, \lambda_0^{(1)}\right) \xi\left(E_\gamma^{(2)}, \lambda_0^{(2)}\right)\Big] \\
\times \left(|M_{+-+-}|^2 + |M_{+--+}|^2\right) \Big\}
\end{aligned}
$$



$f_{\gamma/e}(x)$ - CB photon distribution in energy fraction $x = E_\gamma/E_e$

$\xi(E_\gamma,\lambda_0)$ - CB photon helicity

$x_{1,min} = (p_t/E_e)^2$, $x_{2,min} = (p_t/x_1E_e)^2$, $x_{max} = 0.83$

Effective Lagrangian

Effective Lagrangian of dimension-8 operators which contribute to neutral anomalous QGCs

$$\mathcal{L}_{\rm QNGC} = \frac{c_8}{\Lambda^4} B_{\rho\sigma} B^{\rho\sigma} B_{\mu\nu} B^{\mu\nu} + \frac{c_9}{\Lambda^4} W^a_{\rho\sigma} W^{a\rho\sigma} W^b_{\mu\nu} W^{b\mu\nu} + \frac{c_{10}}{\Lambda^4} W^a_{\rho\sigma} W^{b\rho\sigma} W^a_{\mu\nu} W^{b\mu\nu}$$
$$+ \frac{c_{11}}{\Lambda^4} B_{\rho\sigma} B^{\rho\sigma} W^a_{\mu\nu} W^{a\mu\nu} + \frac{c_{13}}{\Lambda^4} B_{\rho\sigma} B^{\sigma\nu} B_{\nu\mu} B^{\mu\rho} + \frac{c_{14}}{\Lambda^4} W^a_{\rho\sigma} W^{a\sigma\nu} W^b_{\nu\mu} W^{b\mu\rho}$$
$$+ \frac{c_{15}}{\Lambda^4} W^a_{\rho\sigma} W^{b\sigma\nu} W^a_{\nu\mu} W^{b\mu\rho} + \frac{c_{16}}{\Lambda^4} B_{\rho\sigma} B^{\sigma\nu} W^a_{\nu\mu} W^{a\mu\rho}$$

Effective Lagrangian for anomalous γγγγ couplings (in terms of physical fields)

$$L_{\gamma\gamma\gamma\gamma} = \varsigma_1 (F_{\mu\nu} F^{\mu\nu})(F_{\rho\sigma} F^{\rho\sigma}) + \varsigma_2 (F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu})$$

Couplings ς_1, ς_2 have dimension -4

Numerical analysis

Electron beam helicities:

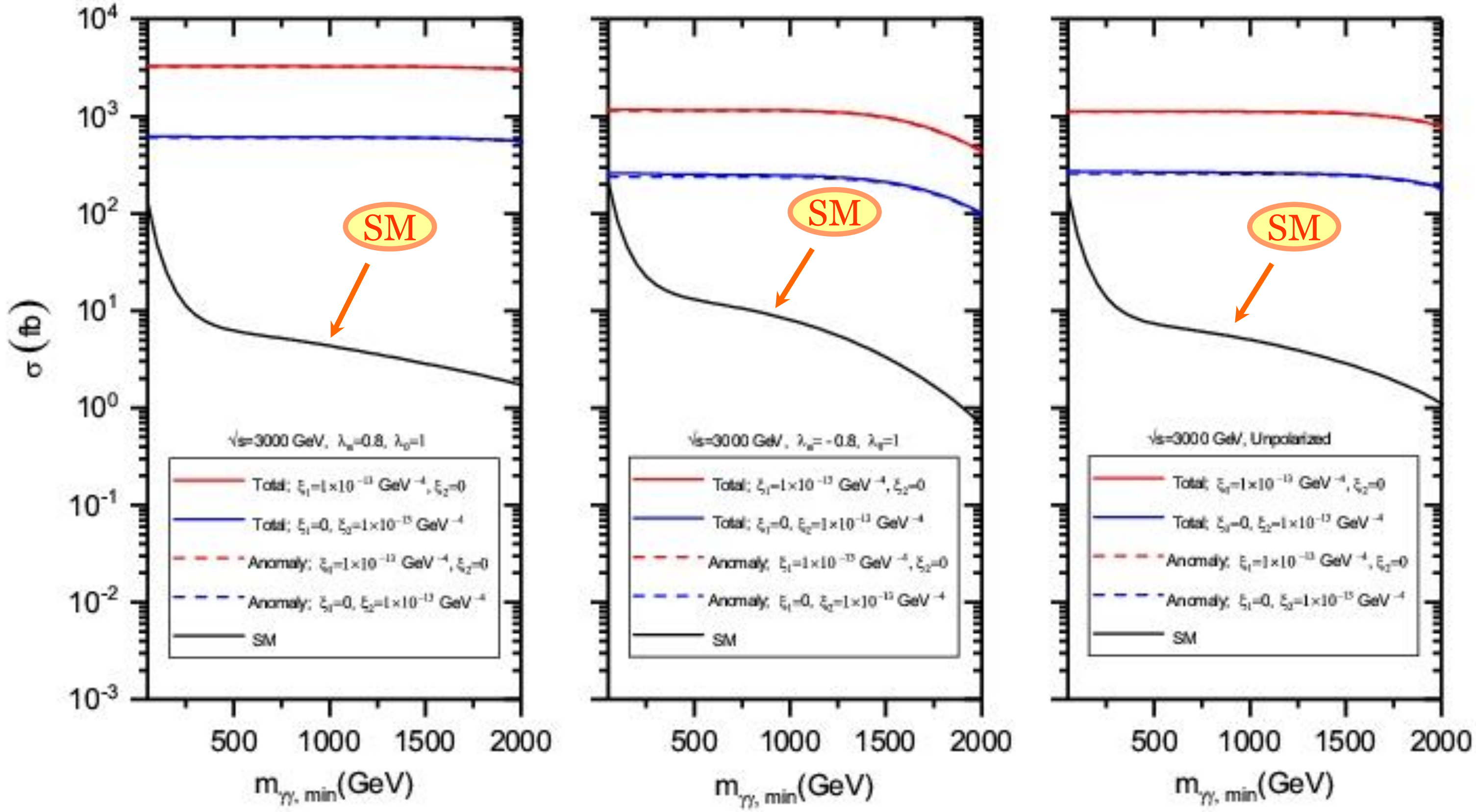
λ_e = 0.8, -0.8, and 0 (unpolarized case)

Cuts on the invariant mass and rapidities of final photons:

$m_{\gamma\gamma} > 200$ GeV, $|\eta| < 2.5$

SM amplitude: *(G.Gounaris et al., EPJC 9, 673, 1999)*

M_W (W-loop, dominates at $\sqrt{s} > 200$ GeV) and M_f (fermion-loop) contributions



Total cross sections for γγ → γγ scattering vs. minimal invariant mass of outgoing photons for √s = 3 TeV. The left, middle and right panels correspond to λ_e= 0.8, -0.8, 0. The solid curves (from top downwards): ($\varsigma_1 = 10^{-13}$ GeV^{-4}, $\varsigma_2 = 0$), ($\varsigma_1 = 0$, $\varsigma_2 = 10^{-13}$ GeV^{-4}), SM.

Exclusion significance (δ = percentage systematic error)

(Y.Zhang & J.Shen, EPJC 80, 811, 2020)

$$S_{\mathrm{excl}} = \sqrt{2}\left[s - b\ln\left(\frac{b+s+x}{2b}\right) - \frac{1}{\delta^2}\ln\left(\frac{b-s+x}{2b}\right) - (b+s-x)\left(1+\frac{1}{\delta^2 b}\right)\right]^{1/2}$$

s (b) = number of signal (background) events

$$x = \sqrt{(s+b)^2 - 4\delta^2 s b^2/(1+\delta^2 b)}$$

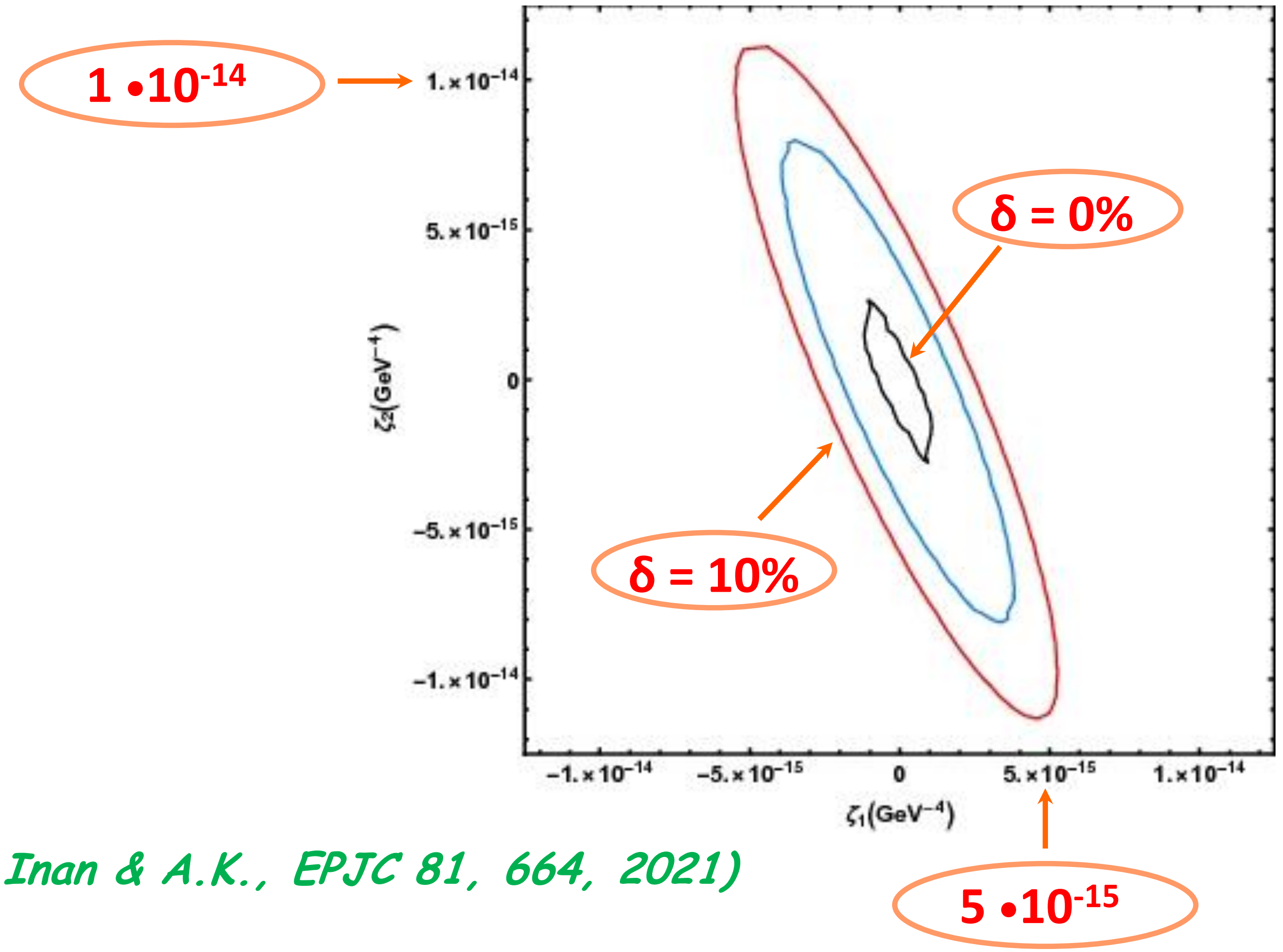
In the limit δ = 0

$$S_{\mathrm{excl}} = \sqrt{2\left[s - b\ln\left(1+\frac{s}{b}\right)\right]}$$

s << b →

$$S_{\mathrm{excl}} = \frac{s}{\sqrt{b}}$$

$S_{\mathrm{excl}} \geq 1.645$ is the region that can be excluded at 95% C.L.



$|\eta| < 2.5$

$m_{\gamma\gamma} > 1$ TeV

(S.Inan & A.K., EPJC 81, 664, 2021)

95% C.L. exclusion region for anomalous couplings ζ_1, ζ_2 for unpolarized LBL scattering at the CLIC. Systematic errors are 0%, 5%, and 10%. The collision energy is 3 TeV, integrated luminosity is 5 ab^{-1}. Couplings ζ_1, ζ_2 are in GeV^{-4}.

Exclusion limits on anomalous couplings for polarized LBL scattering at the CLIC with energy 3.0 TeV

Helicity		0	-0.8	0.8
Luminosity, fb^{-1}		5000	4000	1000
$\lvert\zeta_1\rvert$, GeV^{-4} $(\zeta_2 = 0)$	$\delta = 0\%$	6.85×10^{-16}	8.82×10^{-16}	8.73×10^{-16}
	$\delta = 5\%$	1.90×10^{-15}	2.48×10^{-15}	1.56×10^{-15}
	$\delta = 10\%$	2.63×10^{-15}	3.37×10^{-15}	2.12×10^{-15}
$\lvert\zeta_2\rvert$, GeV^{-4} $(\zeta_1 = 0)$	$\delta = 0\%$	1.43×10^{-15}	1.85×10^{-15}	1.82×10^{-15}
	$\delta = 5\%$	3.99×10^{-15}	5.12×10^{-15}	3.28×10^{-15}
	$\delta = 10\%$	5.53×10^{-15}	7.10×10^{-15}	4.46×10^{-15}

(S.Inan & A.K., EPJC 81, 664, 2021)

Bounds on anomalous couplings for $\gamma\gamma\gamma\gamma$ vertex at the LHC and HL-LHC

LHC, L=300 fb^{-1} :
$|\varsigma_1| < 1.5\cdot10^{-14}$ GeV^{-4}, $|\varsigma_2| < 3.0\cdot10^{-14}$ GeV^{-4}

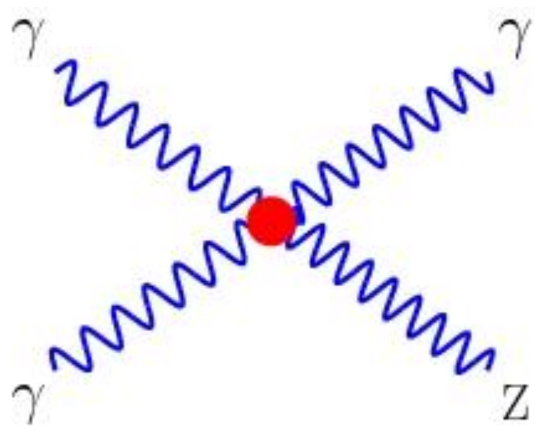
(S.Fichet et al., JHEP 02, 165, 2015)

HL-LHC, L=3000 fb^{-1} :
$|\varsigma_1| < 7.0\cdot10^{-15}$ GeV^{-4}, $|\varsigma_2| < 1.5\cdot10^{-14}$ GeV^{-4}

(S.Fichet et al., Phys. Rev. D 89, 114004, 2014)

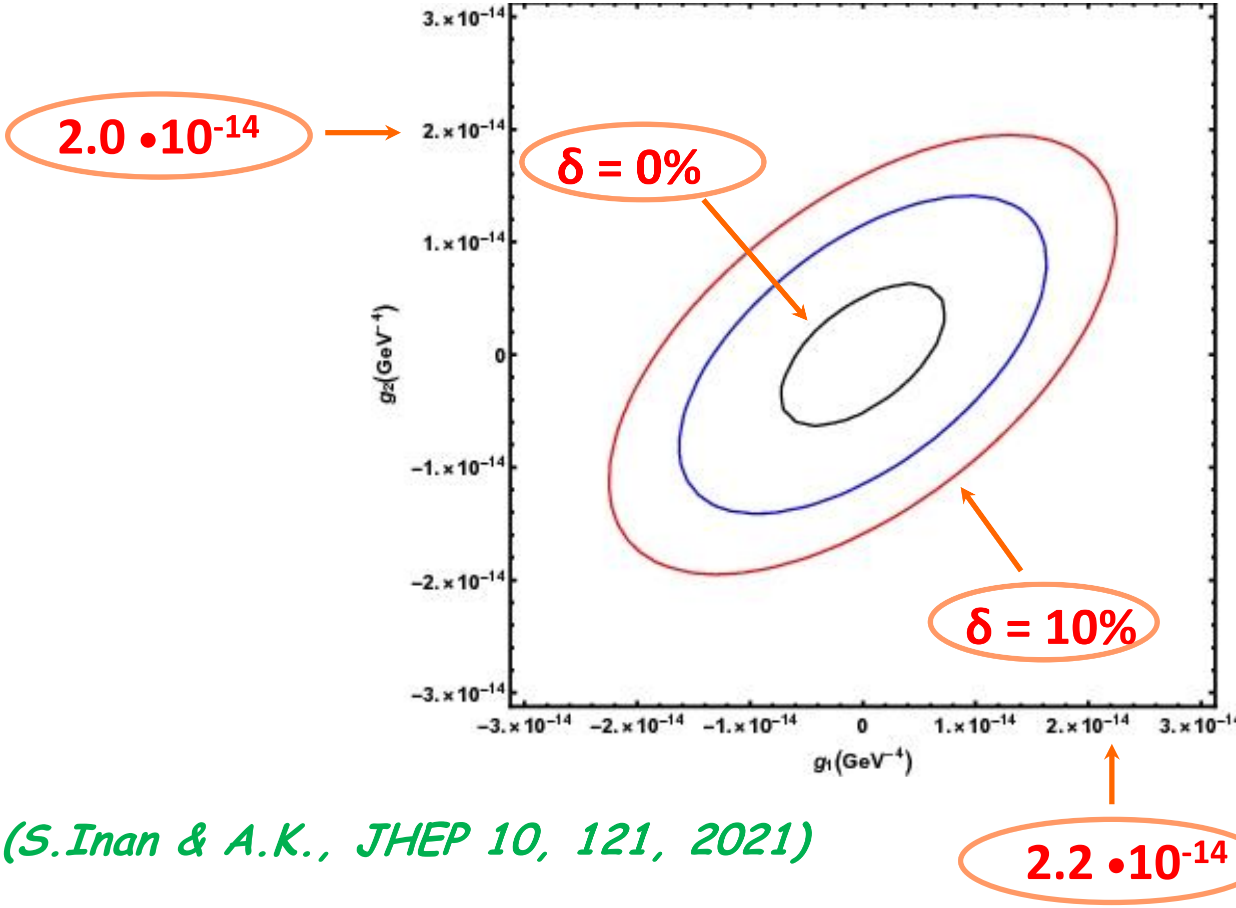
γZ production in photon-photon scattering of CB photons at the CLIC

Anomalous diagram for process γγ → γZ



Z → e, μ

$$L_{\gamma\gamma\gamma Z} = g_1(F_{\mu\nu}F^{\mu\nu})(F_{\rho\sigma}Z^{\rho\sigma}) + g_2(F_{\mu\nu}\tilde{F}^{\mu\nu}F_{\rho\sigma}\tilde{Z}^{\rho\sigma})$$



(S.Inan & A.K., JHEP 10, 121, 2021)

$|\eta| < 2.5$

$m_{\gamma Z} > 1$ TeV

95% C.L. exclusion region for anomalous couplings g_1, g_2 for unpolarized $\gamma\gamma \to \gamma Z$ scattering at the CLIC. Systematic errors are 0%, 5%, and 10%. The collision energy is 3 TeV, integrated luminosity is 5 ab^{-1}. Couplings g_1, g_2 are in GeV^{-4}.

Bounds on anomalous couplings for $\gamma\gamma\gamma Z$ vertex at the LHC and HL-LHC

SM expectation: $\mathcal{B}(Z \to \gamma\gamma\gamma) = 5.41 \cdot 10^{-10}$

$\mathcal{B}(Z \to \gamma\gamma\gamma) < 0.8 \cdot 10^{-5}$ (LEP)
$\mathcal{B}(Z \to \gamma\gamma\gamma) < 2.2 \cdot 10^{-6}$ (ATLAS)

→ **$|f_i| < 1.3 \cdot 10^{-9}$ GeV^{-4}** **where $f_i = g_i/8$**

From $\gamma\gamma \to \gamma Z$ at LHC:

LHC, L=300 fb^{-1} :
$|f_i| < 1.0 \cdot 10^{-13}$ GeV^{-4}

HL-LHC, L=3000 fb^{-1} :
$|f_i| < 7.8 \cdot 10^{-14}$ GeV^{-4}

(C.Baldenegro et al., JHEP 06, 142, 2017)

Dark photons

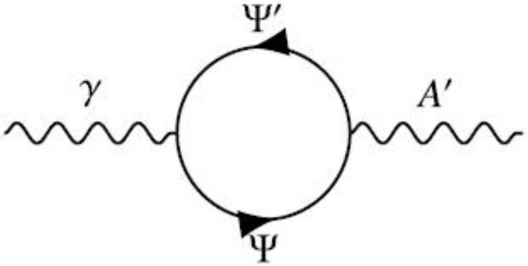
Invisible dark photons in γe scattering at future lepton colliders ILC, CLIC, CEPC

We consider dark matter (DM) scenario in which **no** DM are charged under SM gauge group

Lightest stable DM particles can only interact with SM through exchange of massive vector mediator, dark photon (DP) A’

DP kinematically mixes with SM $U(1)_Y$ hypercharge gauge field (kinematic mixing portal)

Such mixing can be generated by loops of massive particles charged under both $U(1)_Y$ and secluded U(1)' groups



Gauge Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\bar{F}'_{\mu\nu}\bar{F}'^{\mu\nu} - \frac{\varepsilon}{2c_W}\bar{F}'_{\mu\nu}B^{\mu\nu}$$

$B_{\mu\nu}, \bar{F}'_{\mu\nu}$ are field strength tensors of $U(1)_Y, U(1)'$

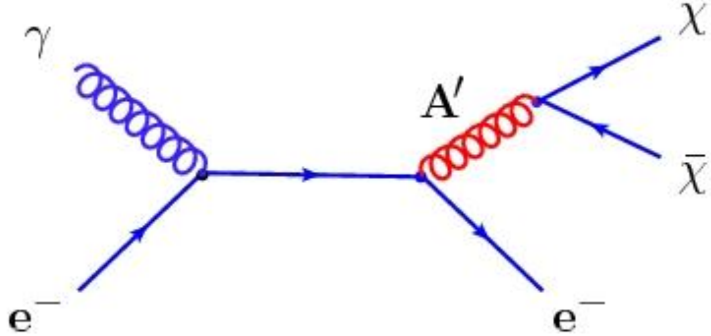
ε – kinetic mixing parameter

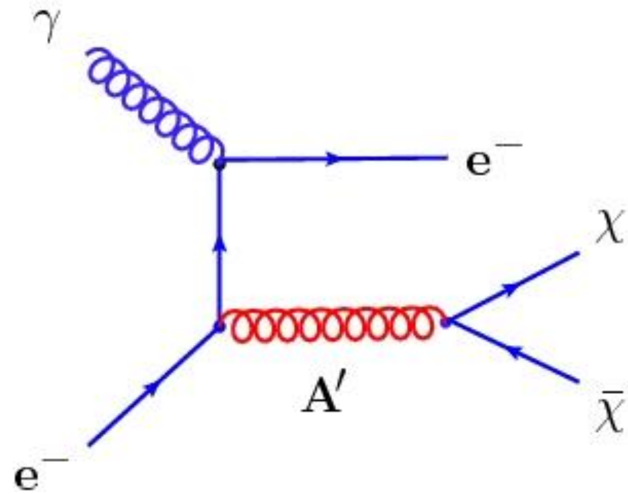
Interaction Lagrangian

(*after diagonalization of gauge and DP fields*)

$$\mathcal{L}_{\text{int}} = eJ_\mu A^\mu - \varepsilon e J_\mu A'^\mu + \varepsilon e' t_W J'_\mu Z_\mu + e' J'_\mu A'^\mu + \mathcal{L}_{A'\chi}$$

Production of DP in γe^- collision with subsequent invisible decay

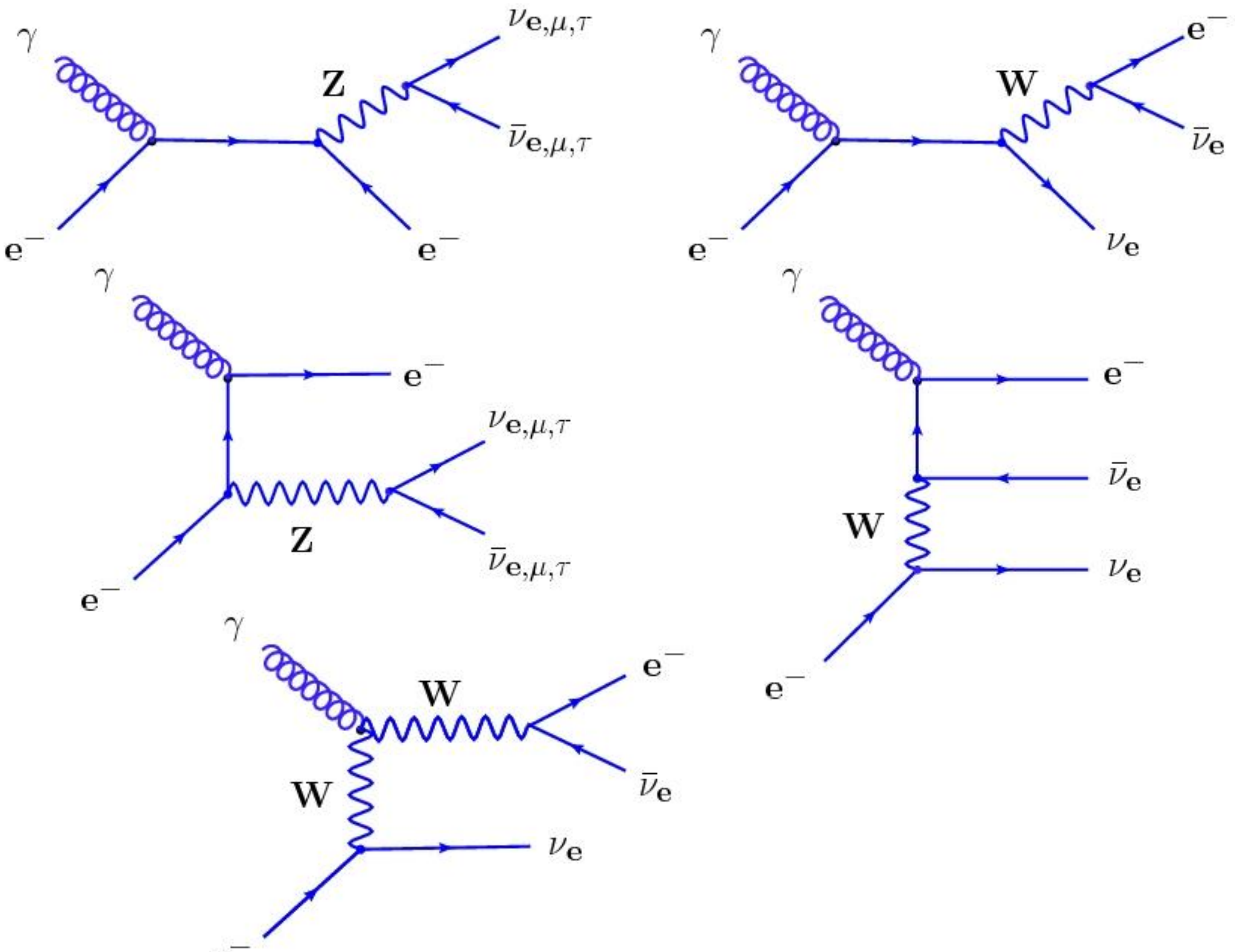




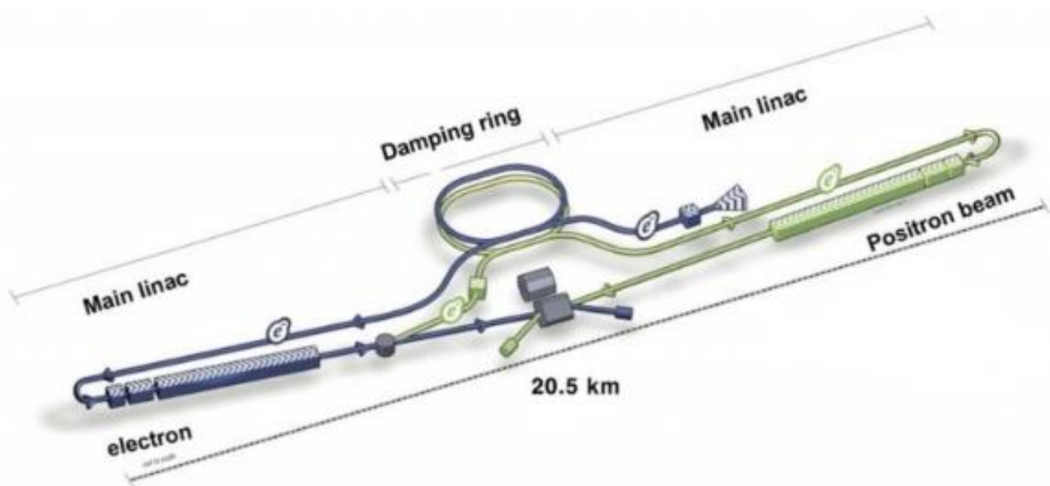
χ – DM particle

We assume that $\mathcal{B}$(A' → χ bar(χ)) = 1

SM background: $\gamma e^- \rightarrow e^- + \nu + \mathrm{bar}(\nu)$



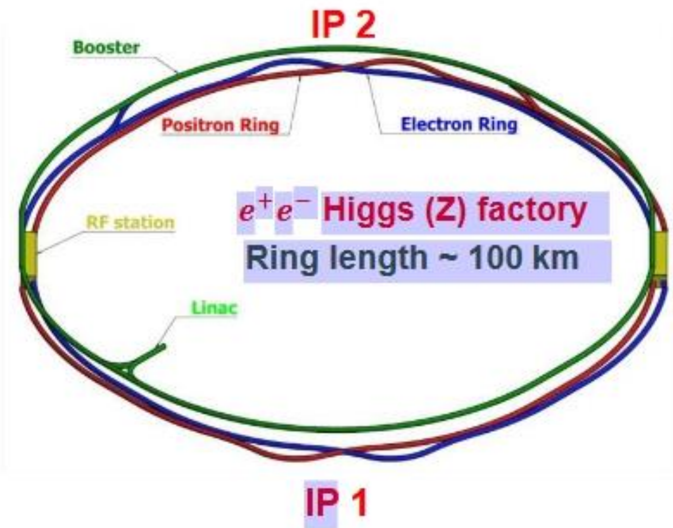
International Linear Collider (ILC)

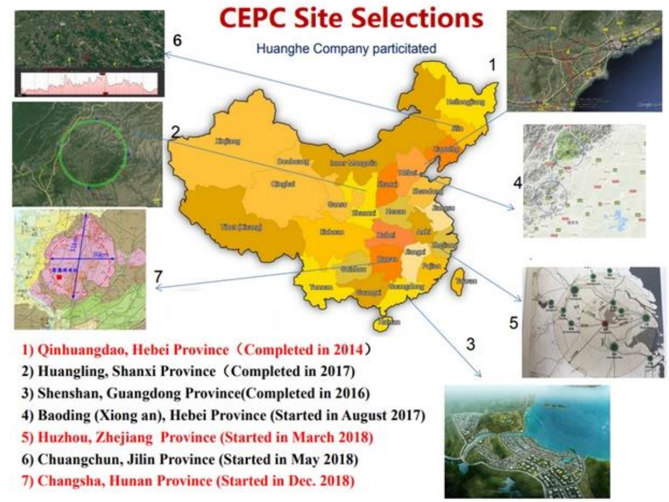


	Beam energy	Integrated luminosity (unpolarized beams)
1st stage	250 GeV	2.0 ab^{-1}
2nd stage	500 GeV	4.0 ab^{-1}
3rd stage	1000 GeV	5.4 ab^{-1}

(ILC Technical Design Report, arXiv:1306.6327, 1903.01629)

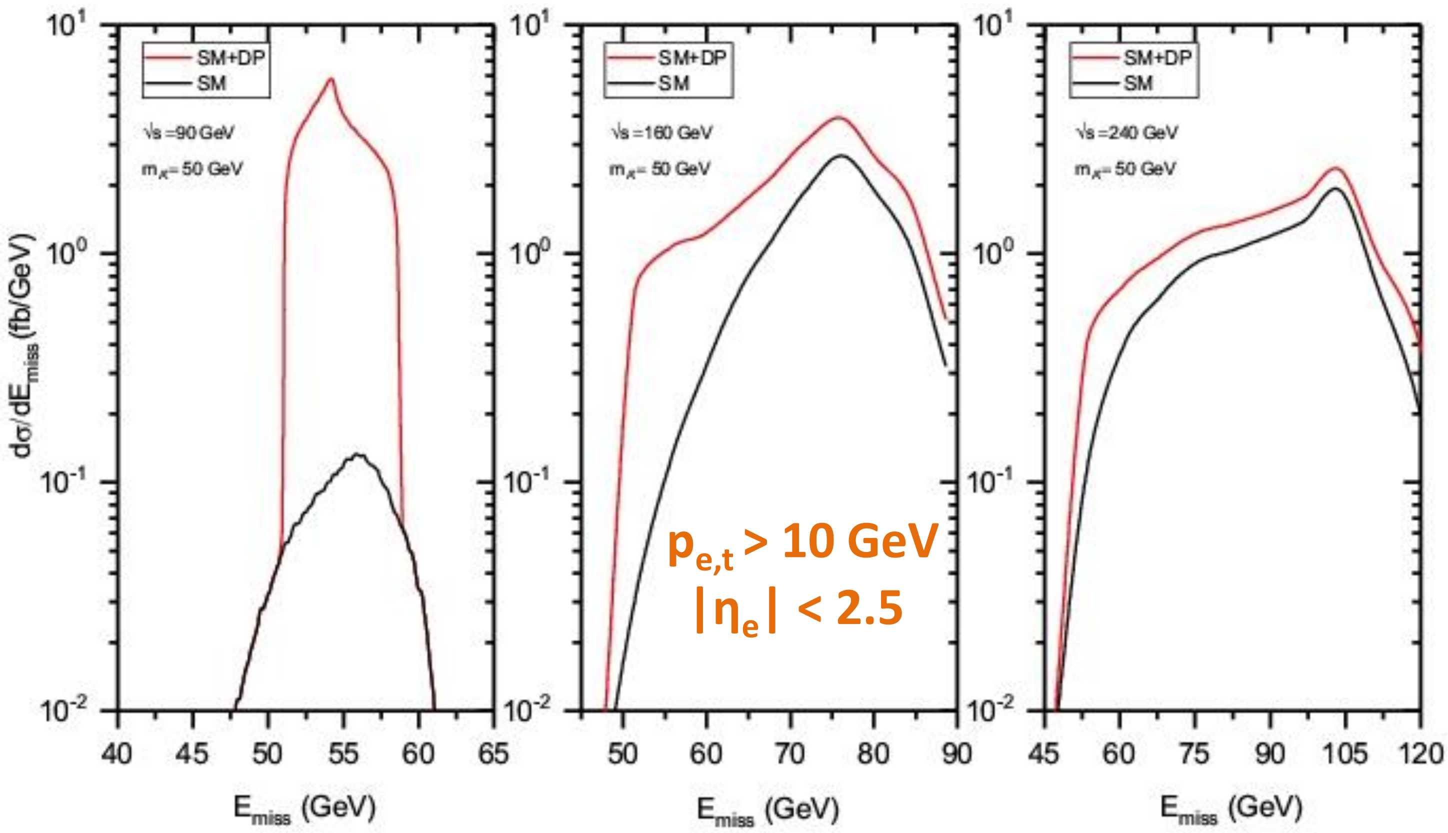
Circular Electron Positron Collider (CEPC)



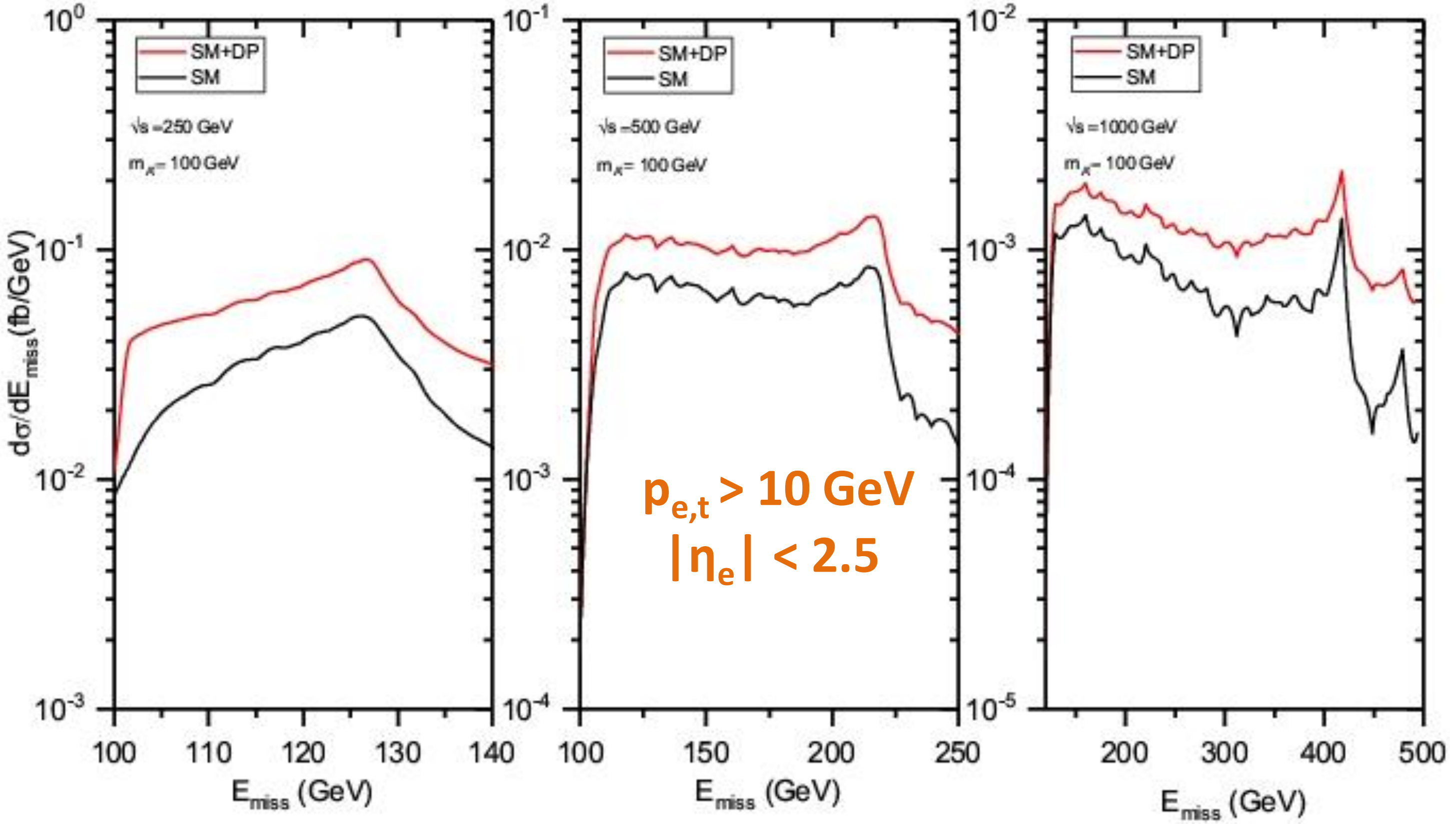


	Beam energy	Integrated luminosity (unpolarized beams)
1st stage	90 GeV	16 ab^{-1}
2nd stage	180 GeV	2.6 ab^{-1}
3rd stage	240 GeV	5.6 ab^{-1}

(CEPC Study Group, arXiv:1809.00285, 1811.10545)

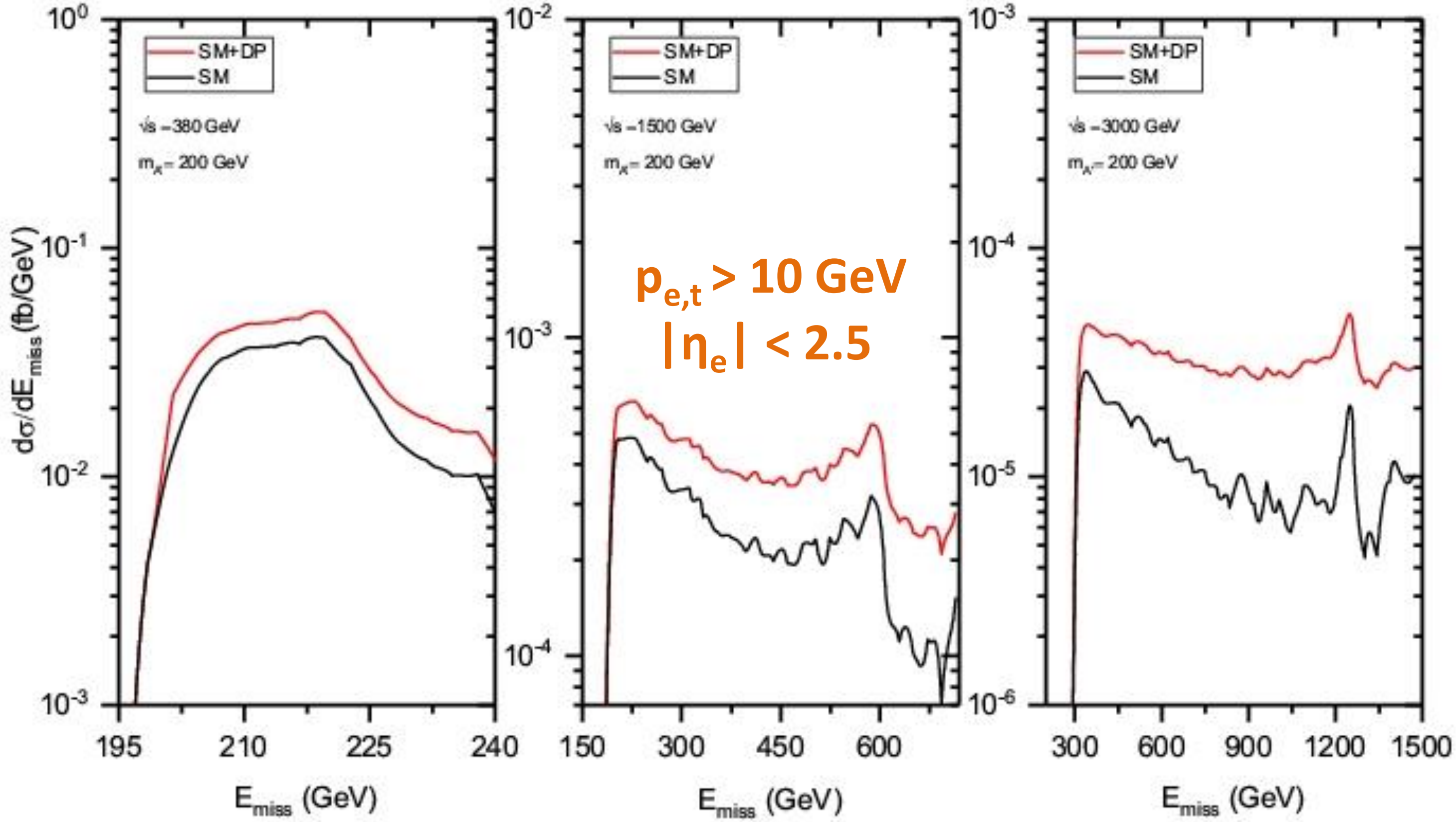


Differential cross sections for $\gamma e^- \rightarrow e^- + E_{miss}$ scattering at the collider CEPC. DP mass is $m_{A'}$ = 50 GeV, mixing parameter is ε = 0.1. Black curves are SM predictions.



$p_{e,t}$ > 10 GeV
$|\eta_e|$ < 2.5

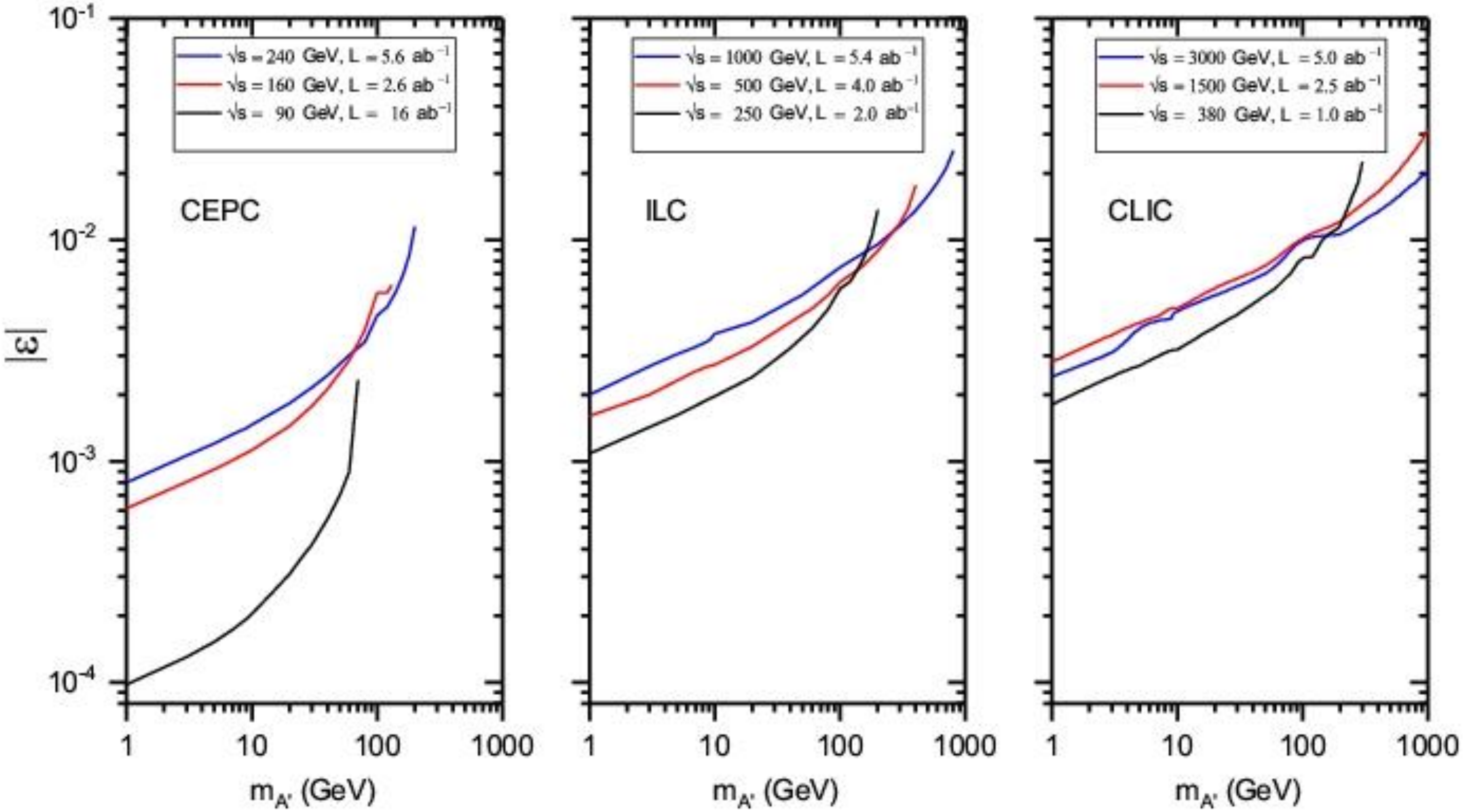
Differential cross sections for $\gamma e^- \rightarrow e^- + E_{miss}$ scattering at the collider ILC. DP mass is $m_{A'}$ = 100 GeV, mixing parameter is ε = 0.1. Black curves are SM predictions.



$p_{e,t}$ > 10 GeV
$|\eta_e|$ < 2.5

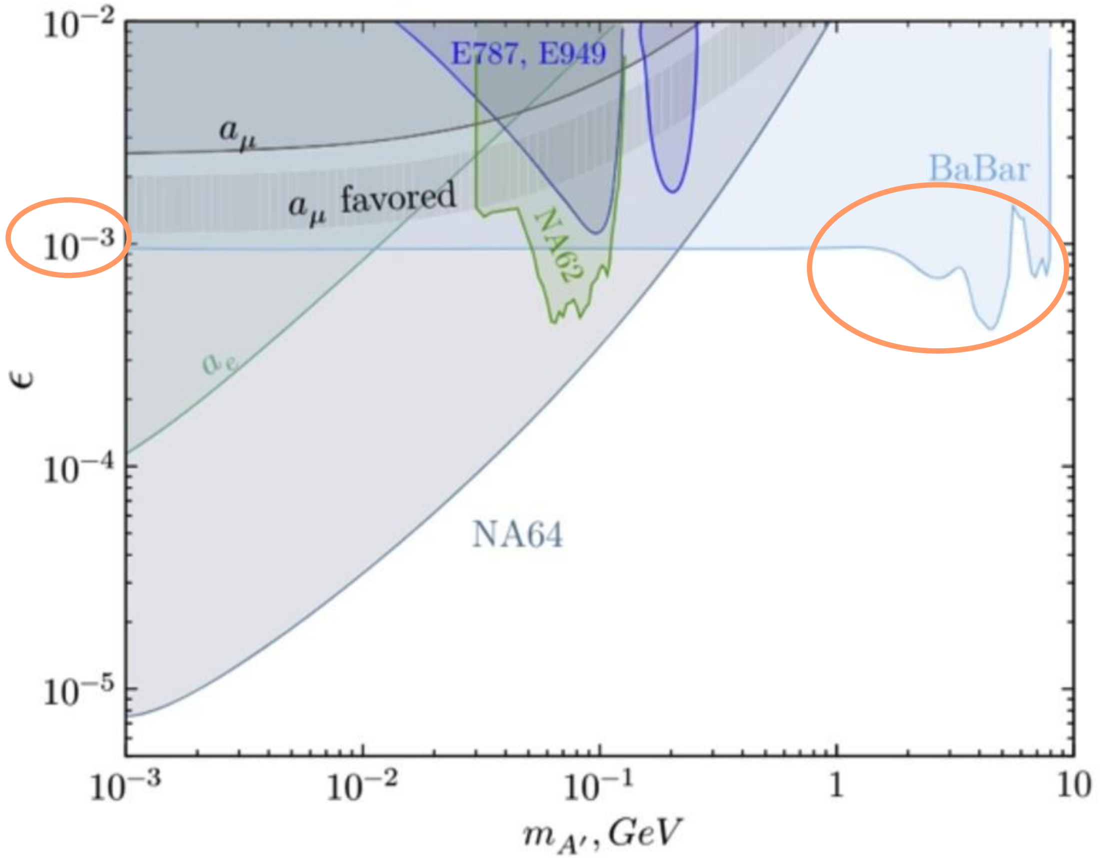
Differential cross sections for $\gamma e^- \rightarrow e^- + E_{miss}$ scattering at the collider CLIC. DP mass is $m_{A'}$ = 200 GeV, mixing parameter is ε = 0.1. Black curves are SM predictions.

(S.Inan & A.K., EPJC, 82, 592, 2022)



Excluded bounds at 95% C.L. on DP mass $m_{A'}$ and mixing parameter ε for invisible DP production in $\gamma e^- \rightarrow A'e^-$ *unpolarized* scattering at future lepton colliders

$p_{e,t} > 10$ GeV, $|\eta_e| < 2.5$, $|m_{A'} - m_{miss}| < 10$ GeV



Exclusion limits on mixing parameter ε for massive DP going into invisible states

(A.Filippi & M.De Napoli, Rev. Phys. 5, 100042, 2020)

Axion-like particles

Strong CP problem and QCD axion

U(1) problem
→ axial anomaly → θ-vacuum
→ strong CP problem → QCD axion

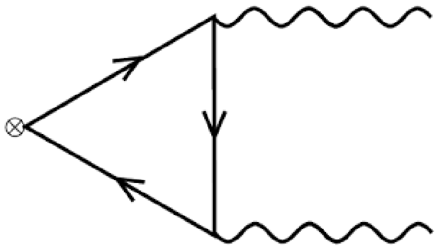
QCD Lagrangian: global symmetry
$U(3)_V \bullet U(3)_A = SU(2)_V \bullet SU(2)_A \bullet U(1)_V \bullet U(1)_A$

Quark condensates → spontaneously broken $U(1)_A$
→ NG massless boson should appear

Absence of NG boson is known as U(1) problem

One possible solution – ABJ chiral anomaly

$$\partial^\mu J^5_\mu = \frac{g^2 N_f}{16\pi^2} G_{a\mu\nu}\tilde{G}^{\mu\nu}_a$$



This term is a total divergency

$$G_{a\mu\nu}\tilde{G}^{\mu\nu}_a = \partial^\mu K_\mu$$

Chiral anomaly introduces a pure surface integral to QCD action

$$\Delta S_{\rm QCD} = \frac{g^2 N_f}{16\pi^2} \int ds_\mu K^\mu$$

As a result, QCD action acquires θ-term

$$S_{\rm eff} = S_{\rm QCD} + \theta \frac{g^2}{32\pi^2} \int dx \, G_{a\mu\nu} \tilde{G}_a^{\mu\nu}$$

Exp. limit on neutron electric dipole moments,
$|d_n| < 1.8 \bullet 10^{-13}$ e fm, requires $\theta_{phys} < 10^{-9}$

Smallness of θ_{phys} is known as strong CP problem

Elegant solution: Peccei-Quinn (PQ) mechanism with a new, spontaneously broken, global $U(1)_{PQ}$ symmetry

(R.Peccei & H.Quinn, 1977)

$U(1)_{PQ}$ invariant total Lagrangian acquires the term

$$\mathcal{L}_a = \xi \frac{g^2}{32\pi^2} \frac{a}{f_a} G_{a\mu\nu} \tilde{G}_a^{\mu\nu}$$

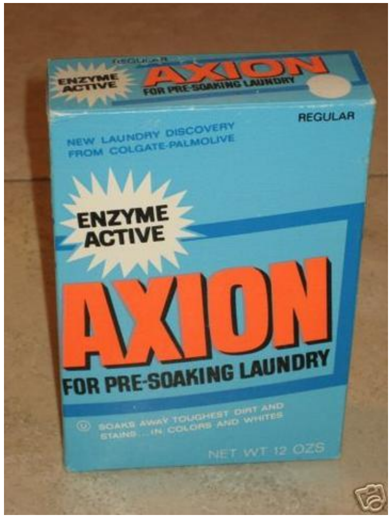
Lagrangian written in terms of $a = a_{phys} + \langle a \rangle$, where $\langle a \rangle = -f_a \theta_{phys}/\xi$, no longer has CP violating term

New symmetry effectively replaces static CP-violating angle θ with dynamically CP-conserving field, axion *a*

(S.Weinberg, F.Wilczek, 1978)

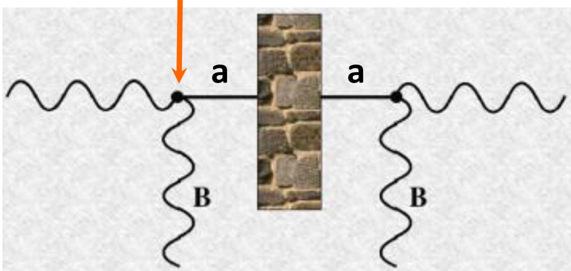
... axions (I named them after a laundry detergent, since they clean up a problem with an axial current)

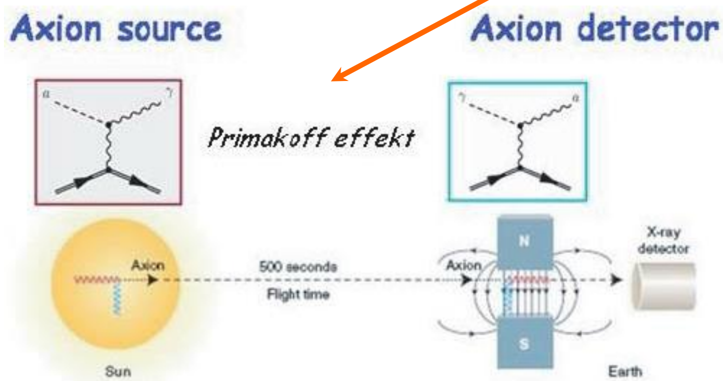
F. A. Wilczek, Nobel lecture (December 8, 2004)



Axion is a leading DM candidates. Axion phenomenology:

stellar evolution, axion mediated forces, DM detection, axion decays, axion-photon conversion, light shining through the wall, solar axions





Axion-like particles (ALPs):

no coupling to gluons, but nonzero coupling to photons
→ may be detected at colliders in light-by-light scattering

$$-\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \vec{E} \vec{B}$$

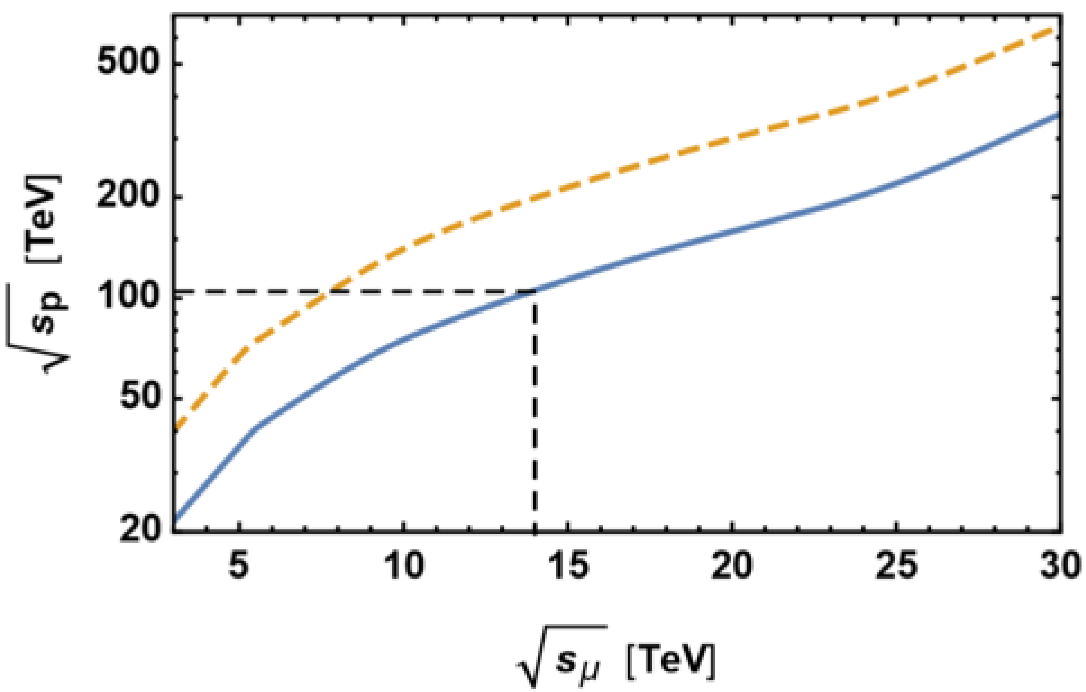
ALP mass are treated independently of its coupling

Why muons?

F.Tikhonin, *On the effects of the clashing mu-meson beams* (in Russian), JINR Report P2-4120, Dubna,1968
G. Budker, *Accelerators and colliding beams*, HEACC, 1969, Yerevan, USSR

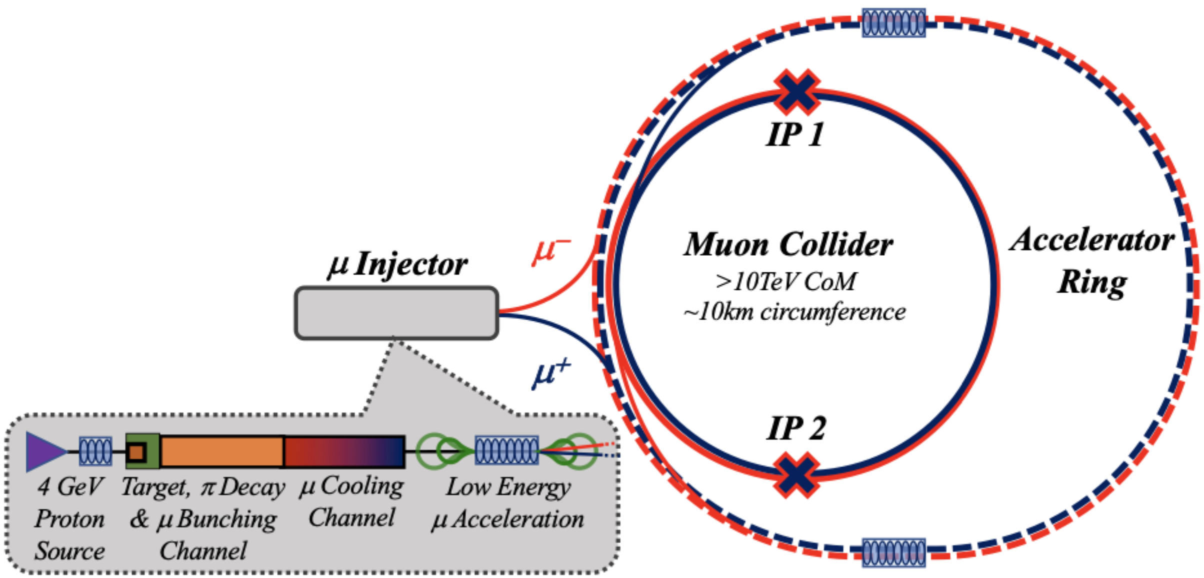
- *Muons can be made to collider at energy of 10 TeV or more in a compact ring without limitations from synchrotron radiation*
- *Being point-like particles, their centre-of-mass collision energy is entirely available to produce high-energy reactions*
- *Muon collider with a given energy and luminosity is more effective than a proton collider with comparable energy and luminosity (see next slide)*

The plot compares pair production cross-sections for heavy particles with mass equal approximately to half the muon collider energy



Yellow line assumes comparable processes for muon and proton production. Blue line accounts for possible QCD enhancement on proton production

Future muon collider (conceptual scheme)

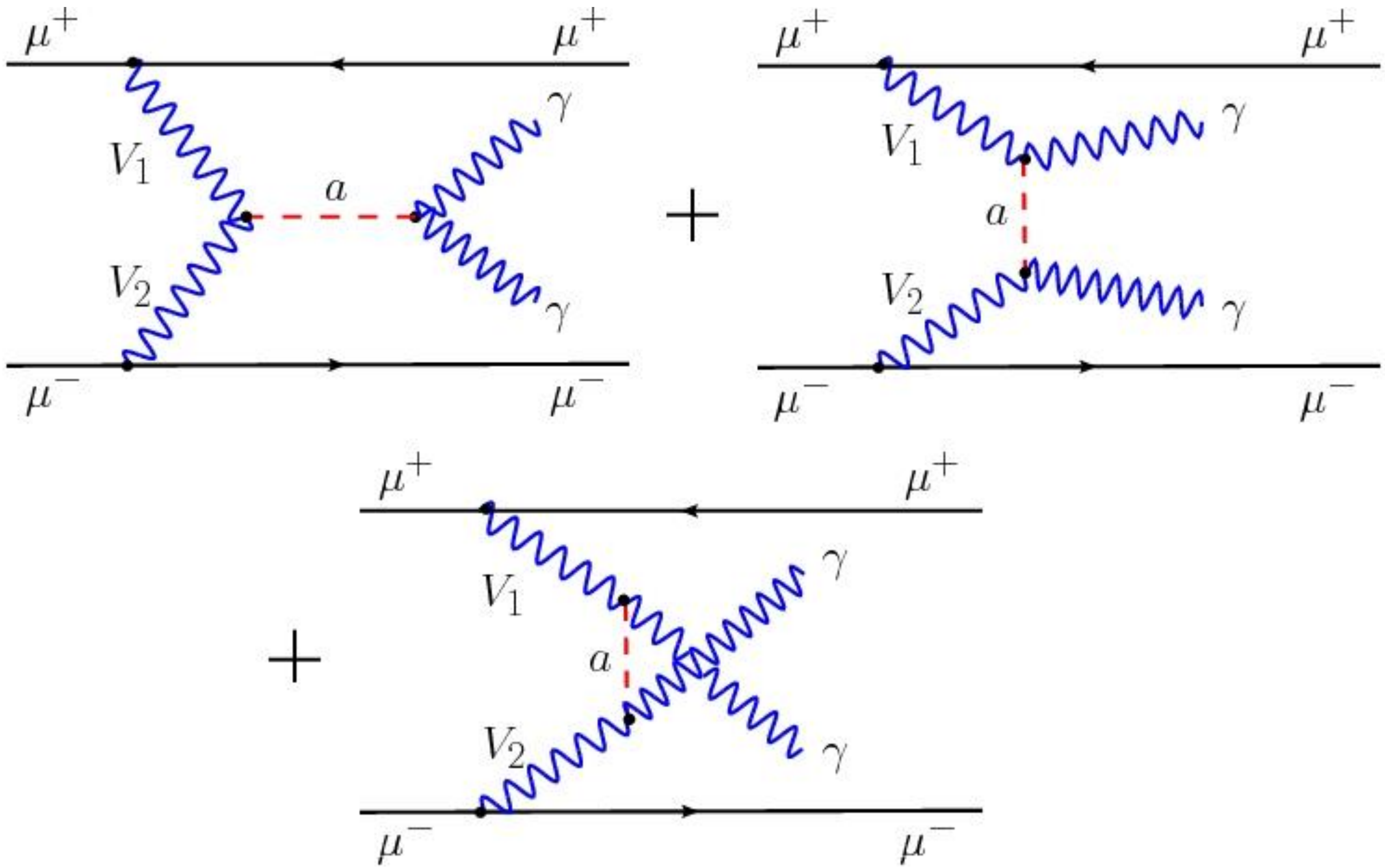


(Muon Collider Working Group, arXiv: 1901.06150)

Centre-of-mass energy	E_{cm}	TeV	3	10	14
Luminosity	$\mathfrak{L}$	$10^{34}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$	2	20	40
Collider circumference	C_{coll}	km	4.5	10	14

30 TeV and 100 TeV colliders are also under consideration

Production of ALPs in vector boson scattering at future muon collider



ALP - gauge boson Lagrangian

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a\,\partial^\mu a - \frac{1}{2}m_a^2 a^2 + g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{a\gamma Z} a F_{\mu\nu}\tilde{Z}^{\mu\nu} + g_{aZZ} a Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

ALP mass

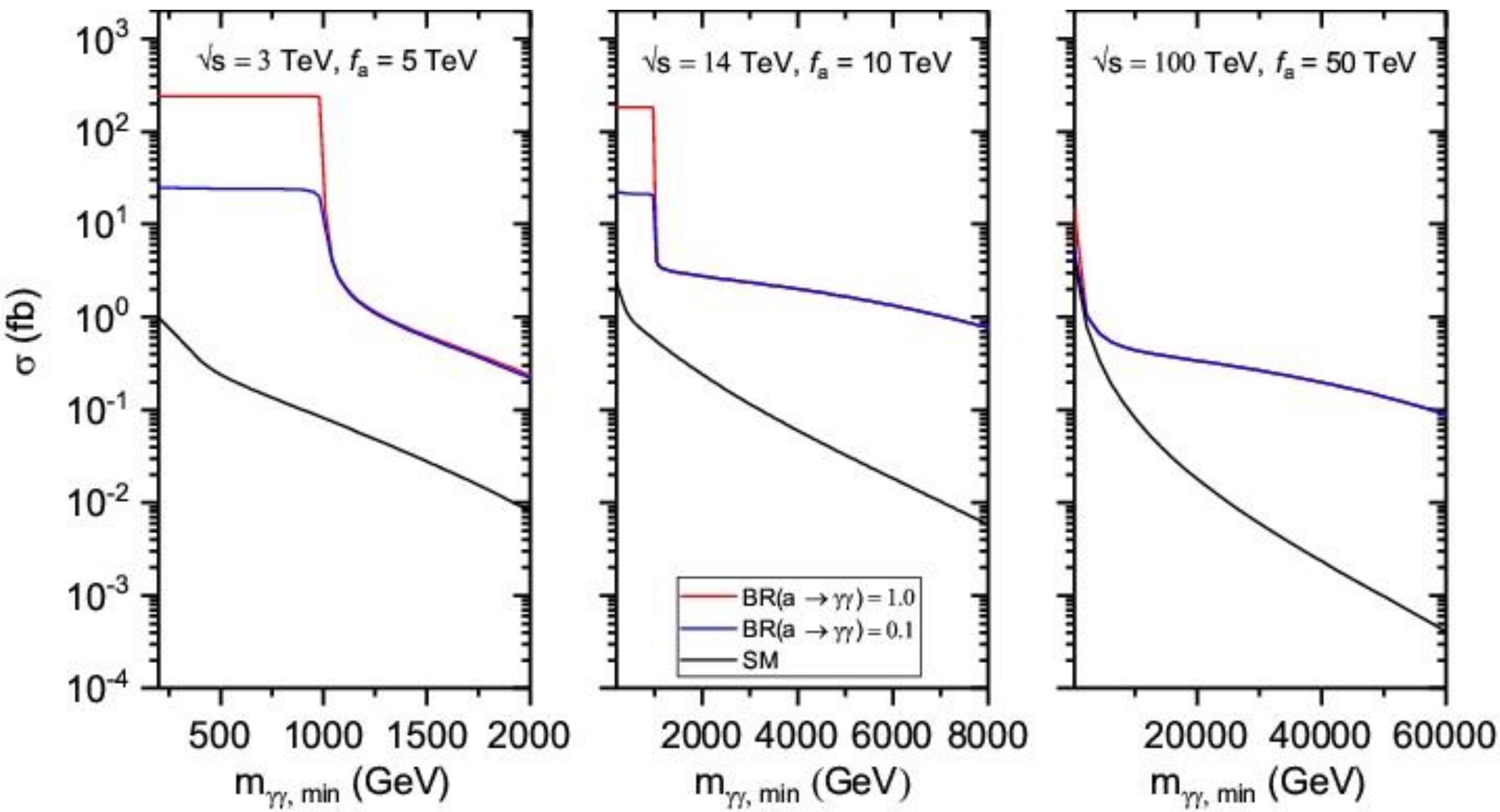
If ALP couples to hypercharge U $(1)_Y$, but not to $SU(2)_L$:

$$g_{a\gamma\gamma} = \frac{1}{f_a}\,, \quad g_{a\gamma Z} = -\frac{2s_w}{c_w}\frac{1}{f_a}\,, \quad g_{aZZ} = \frac{s_w^2}{c_w^2}\frac{1}{f_a}$$

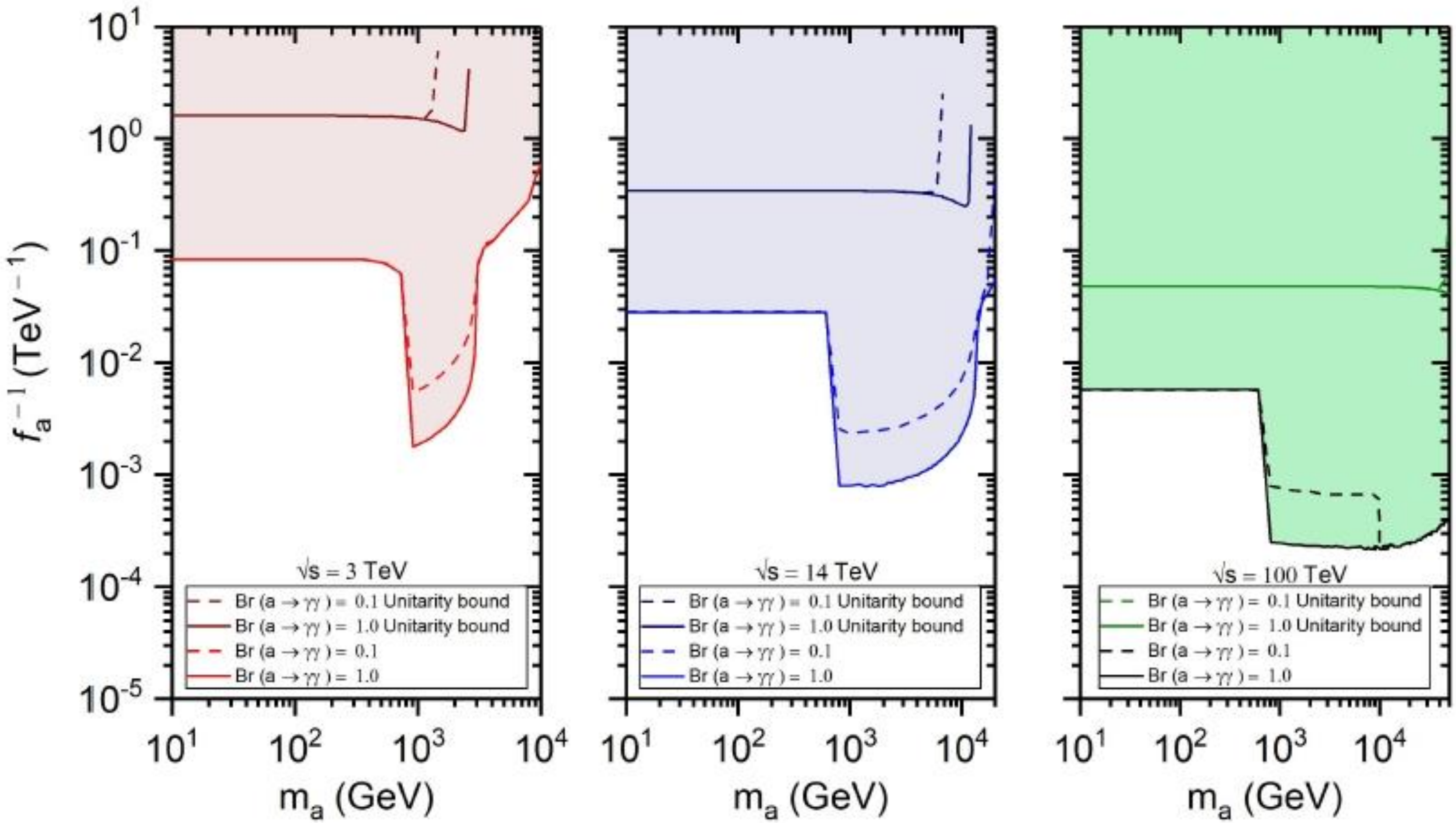
ALP coupling

ALP decay width

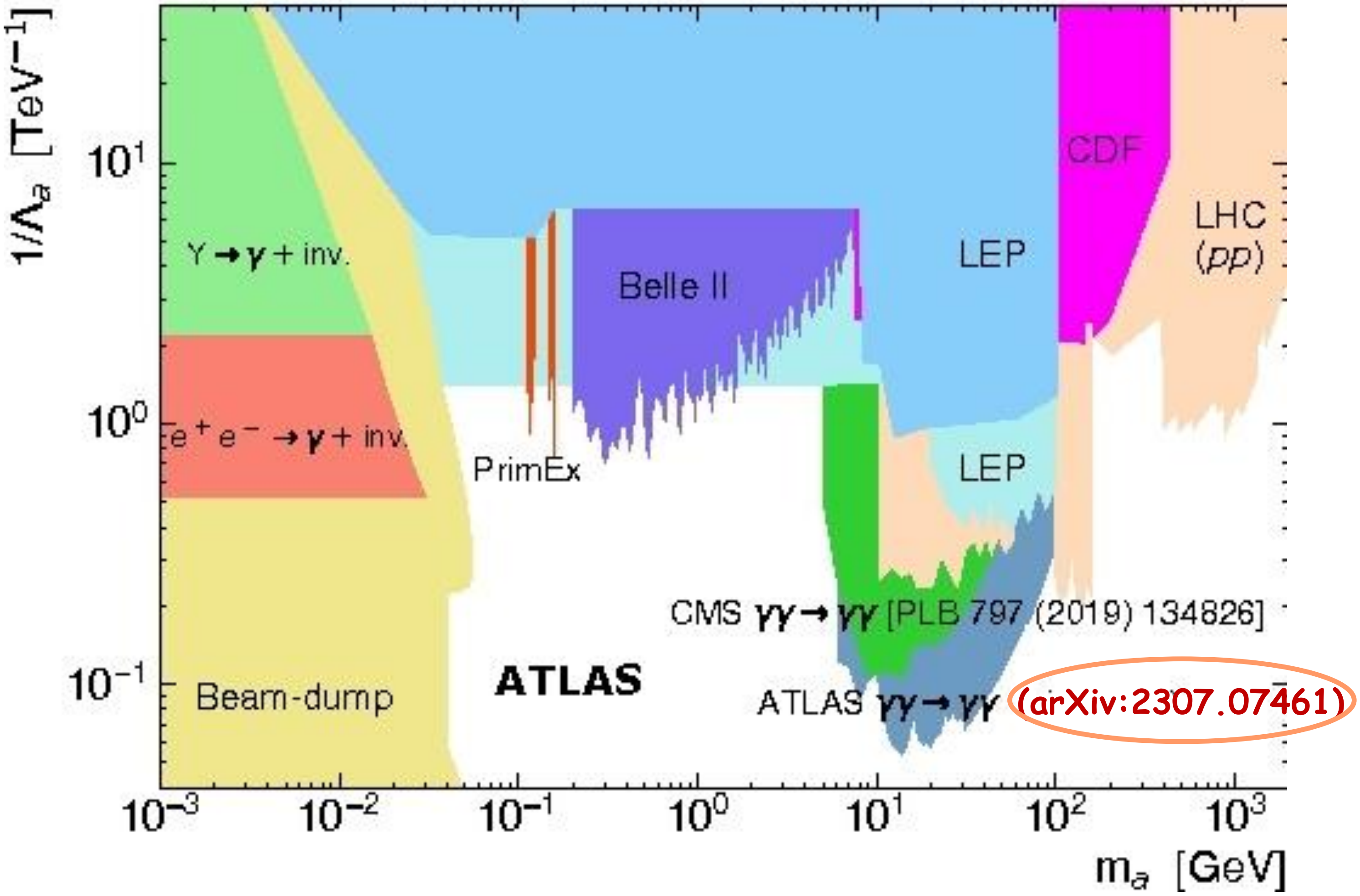
$$\Gamma(a \to \gamma\gamma) = \frac{m_a^3}{4\pi f_a^2}$$



Total cross section for scattering μ+μ- → μ+ γγ μ- at future muon collider versus minimal value of diphoton invariant mass $m_{\gamma\gamma, min}$



95% C.L. exclusion regions for ALP-gauge boson coupling f_a^{-1} and ALP mass m_a coming from collision $\mu^+\mu^- \rightarrow \mu^+ \gamma\gamma\, \mu$ *(S.Inan & A.K., J. Phys. G, 2023)*



Current 95% C.L. exclusion regions for ALP parameters from collider experiments

Conclusions

- We have studied anomalous quartic $\gamma\gamma\gamma\gamma$ couplings in unpolarized and polarized LBL scattering of CB photons at 3 TeV CLIC.
- Exclusion regions for couplings are obtained. Our best constraints are $\varsigma_1 = 6.9 \bullet 10^{-4}$ TeV^{-4} , $\varsigma_2 = 1.4 \bullet 10^{-3}$ TeV^{-4}, one order of magnitude stronger than HL-LHC bounds derived for integrated luminosity of 3 ab^{-1}.
- Sensitivity to anomalous $\gamma\gamma\gamma Z$ couplings in $\gamma\gamma \to \gamma Z$ scattering at 3 TeV CLIC is also obtained. The best limit is $5 \bullet 10^{-3}$ TeV^{-4}, while HL-LHC bound is $(1-0.7) \bullet 10^{-1}$ TeV^{-4}.

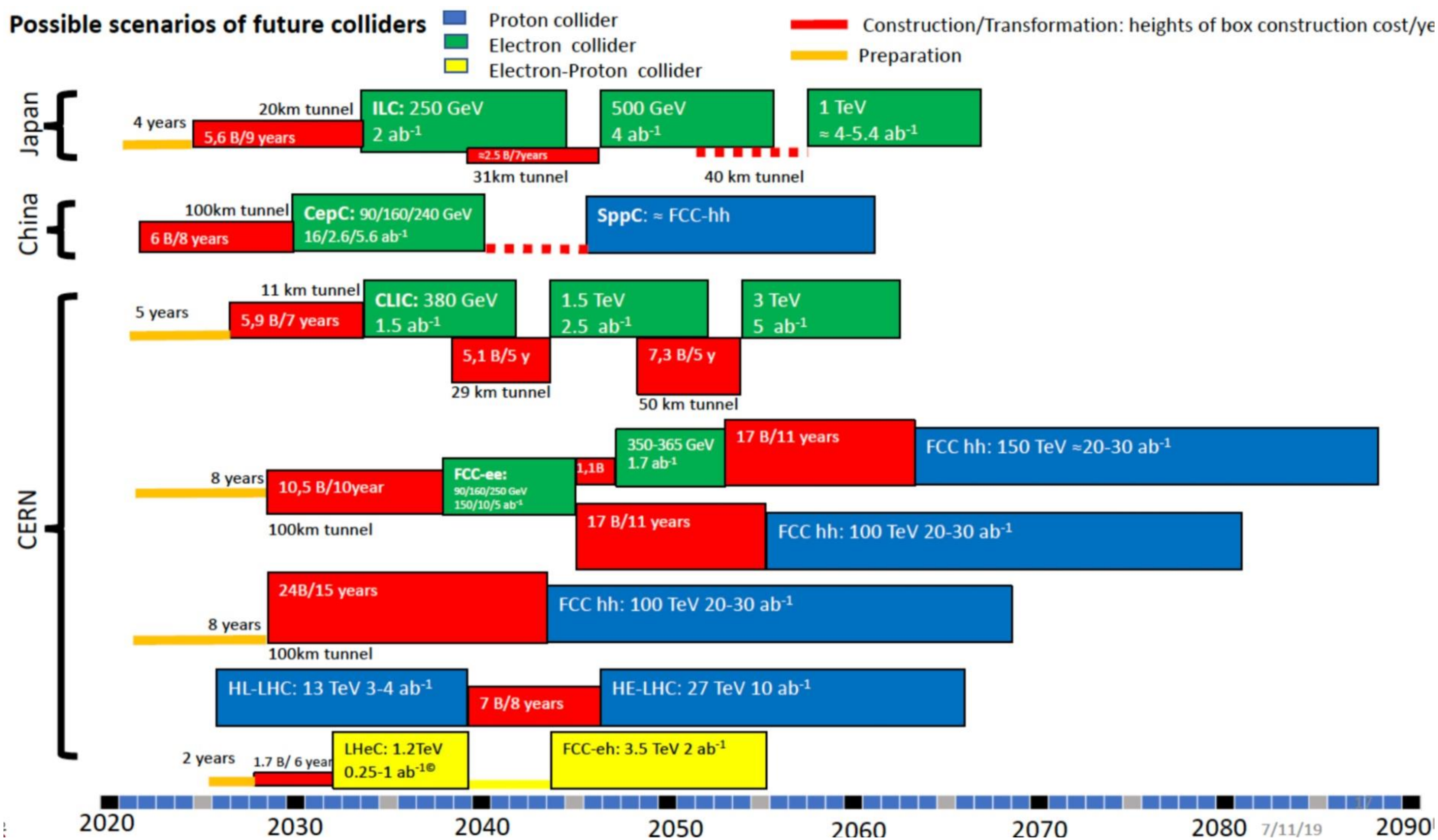
- Production of massive DP in γe scattering at future lepton colliders ILC, CLIC and CEPC is studied for the fist time.
- Invisible decay mode of DP is addressed. Wide range of DP mass $m_{A'}$ (1 GeV - 1 TeV) is considered. Exclusion regions in the plane $(m_{A'}, \varepsilon)$, where ε is kinetic mixing parameter, are derived.
- In low mass region, 1-10 GeV, our bounds on ε for the 90 CEPC are several times stronger than BaBar limit (~ 10^{-3}). The CEPC bounds for 160 and 240 GeV, as well as bounds for the 250 ILC, are close to BaBar constraints.

- Possibility of heavy ALP production in process $\mu+\mu- \rightarrow \mu+ \gamma\gamma \mu-$ at future nuon collider is examined. Collision energies of 3, 14 and 100 TeV with integrated luminosities 1,20 and 1000 ab^{-1} are considered.
- The 95% C.L.exclusion regions for ALP parameters are obtained. Results are presented as curves in the plane (m_a, f_a), where m_a is ALP mass and f_a is its coupling to gauge bosons.

Thank you for your attention

Back-up slides
(electron-positron colliders)

Possible scenario of future colliders



Accelerator-based projects proposed by the community in recent years

Collider (type)	$\sqrt{s}$ (GeV) [$\mathcal{L}_{int}$ (ab^{-1}), duration (years)]
HE-LHC (circular, *pp*)	27×10^3 [15, 20]
ILC (linear, e^+e^-)	91 [0.1, 1.5]; 250 [2, 11]; 350 [0.2, 0.75]; 500 [4, 9]
CLIC (linear, e^+e^-)	380 [1.0, 8], 1.5×10^3 [2.5, 7], 3×10^3 [5, 8]
FCC-ee (circular, e^+e^-)	88–94 [150, 4]; s-channel h [20, 3]; 157–163 [10, 2]; 240 [5, 2]; 340–365 [1.7, 5]
FCC-hh (circular, *pp*)	100×10^3 [20–30, 25]
FCC-eh (circular plus ERL, *ep*)	3.5×10^3 [3, 25]
MuC (circular, $\mu^+\mu^-$)	3 TeV [1, 5]; 10 TeV [10, 5]; 10 TeV [20, 5]
CepC (circular, e^+e^-)	91 [16, 2]; 160 [2.6, 1]; 240 [5.6, 7]; 360 [-, -]

Anomalous $\gamma\gamma\gamma\gamma$ vertex

Operators with gauge boson field strength tensor only

$$
\begin{aligned}
O_{T,0} &= Tr[W_{\mu\nu}W^{\mu\nu}] \times Tr[W_{\alpha\beta}W^{\alpha\beta}], \\
O_{T,1} &= Tr[W_{\alpha\nu}W^{\mu\beta}] \times Tr[W_{\mu\beta}W^{\alpha\nu}], \\
O_{T,2} &= Tr[W_{\alpha\mu}W^{\mu\beta}] \times Tr[W_{\beta\nu}W^{\nu\alpha}], \\
O_{T,5} &= Tr[W_{\mu\nu}W^{\mu\nu}] \times B_{\alpha\beta}B^{\alpha\beta}, \\
O_{T,6} &= Tr[W_{\alpha\nu}W^{\mu\beta}] \times B_{\mu\beta}B^{\alpha\nu}, \\
O_{T,7} &= Tr[W_{\alpha\mu}W^{\mu\beta}] \times B_{\beta\nu}B^{\nu\alpha}, \\
O_{T,8} &= B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}, \\
O_{T,9} &= B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha}.
\end{aligned}
$$

Operator with covariant derivatives only

$$O_{S,0} = [(D_\mu\Phi)^\dagger(D_\nu\Phi)] \times [(D^\mu\Phi)^\dagger(D^\nu\Phi)],$$

$$O_{S,1} = [(D_\mu\Phi)^\dagger(D^\mu\Phi)] \times [(D_\nu\Phi)^\dagger(D^\nu\Phi)],$$

$$O_{S,2} = [(D_\mu\Phi)^\dagger(D_\nu\Phi)] \times [(D^\nu\Phi)^\dagger(D^\mu\Phi)].$$

Operator with covariant derivatives and field strength tensors

$$O_{M,0} = Tr[W_{\mu\nu}W^{\mu\nu}] \times [(D_\beta\Phi)^\dagger(D^\beta\Phi)],$$

$$O_{M,1} = Tr[W_{\mu\nu}W^{\nu\beta}] \times [(D_\beta\Phi)^\dagger(D^\mu\Phi)],$$

$$O_{M,2} = [B_{\mu\nu}B^{\mu\nu}] \times [(D_\beta\Phi)^\dagger(D^\beta\Phi)],$$

$$O_{M,3} = [B_{\mu\nu}B^{\nu\beta}] \times [(D_\beta\Phi)^\dagger(D^\mu\Phi)],$$

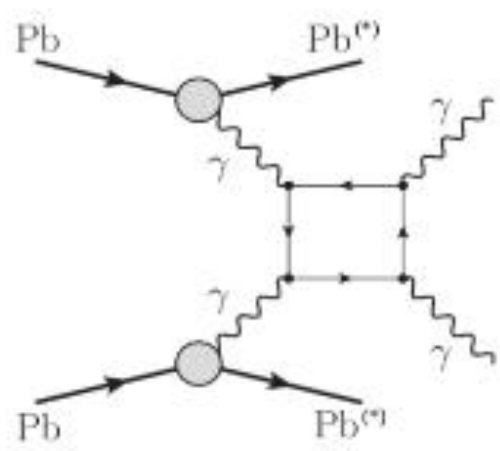
$$O_{M,4} = [(D_\mu\Phi)^\dagger W_{\beta\nu}(D^\mu\Phi)] \times B^{\beta\nu},$$

$$O_{M,5} = [(D_\mu\Phi)^\dagger W_{\beta\nu}(D^\nu\Phi)] \times B^{\beta\mu},$$

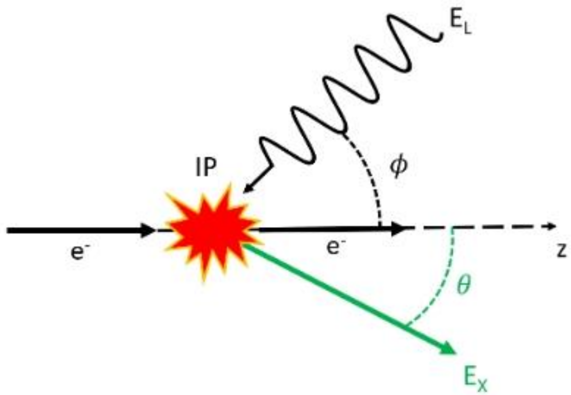
$$O_{M,7} = [(D_\mu\Phi)^\dagger W_{\beta\nu}W^{\beta\mu}(D^\nu\Phi)].$$

The first evidence of γγ → γγ process was observed by ATLAS & CMS in Pb-Pb collisions

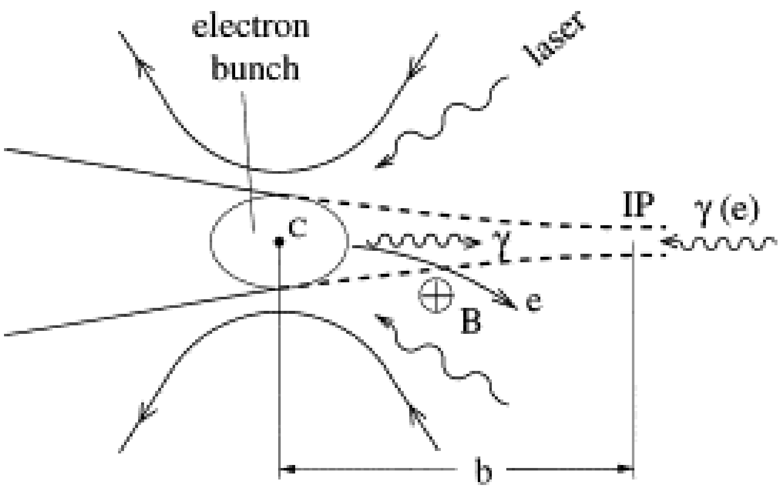
(ATLAS Collab., Nat. Phys., 13, 852, 2017)
(CMS Collab., Phys. Rev. Lett. 123, 052001, 2019)



Inverse laser-Compton scattering process



Scheme of γγ (γe) collider



Helicity of CB photon

$$\xi(E_\gamma, \lambda_0) = \frac{\lambda_0(1-2r)[1-x+1/(1-x)] + \lambda_e r\zeta[1+(1-x)(1-2r)^2]}{1-x+1/(1-x) - 4r(1-r) - \lambda_e\lambda_0 r\zeta(2r-1)(2-x)},$$

where $x = E_\gamma/E_e$, $r = x/\zeta(1-x)$, $\zeta = 4E_eE_0/m_e^2$, m_e being the electron mass.

Spectrum of CB photons

$$f_{\gamma/e}(x) = \frac{1}{g(\zeta)}\Big[1 - x + \frac{1}{1-x} - \frac{4x}{\zeta(1-x)} + \frac{4x^2}{\zeta^2(1-x)^2}$$
$$+ \lambda_0\lambda_e r\zeta(1-2r)(2-x)\Big],$$

where

$$g(\zeta) = g_1(\zeta) + \lambda_0\lambda_e\, g_2(\zeta)\ ,$$
$$g_1(\zeta) = \left(1 - \frac{4}{\zeta} - \frac{8}{\zeta^2}\right)\ln(\zeta+1) + \frac{1}{2} + \frac{8}{\zeta} - \frac{1}{2(\zeta+1)^2}\ ,$$
$$g_2(\zeta) = \left(1 + \frac{2}{\zeta}\right)\ln(\zeta+1) - \frac{5}{2} + \frac{1}{\zeta+1} - \frac{1}{2(\zeta+1)^2}\ .$$

Helicity amplitudes

$$M_{++++}(s,t,u) = \frac{(4g_1+3g_2)}{2}\, s^2 \; ,$$

$$M_{++--}(s,t,u) = \frac{(4g_1+g_2)}{2}\,(s^2+t^2+u^2) \; ,$$

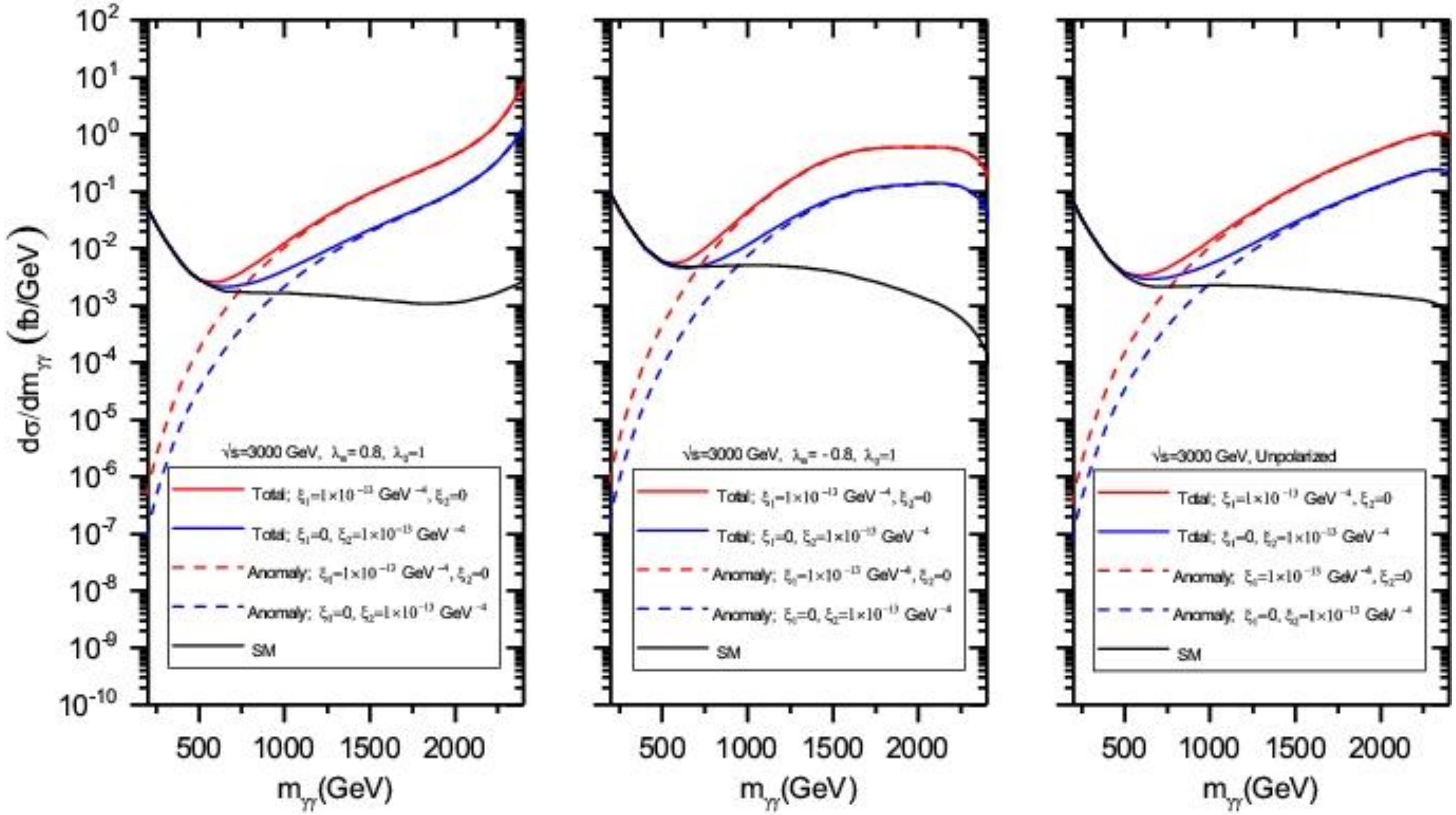
$$M_{+-+-}(s,t,u) = \frac{(4g_1+3g_2)}{2}\, u^2 \; ,$$

$$M_{+++-} = M_{++-+} = M_{+-++} = 0 \; .$$

$$M_{+--+}(s,t,u) = M_{+-+-}(s,u,t) = \frac{(4g_1+3g_2)}{2}\, t^2 \; ,$$

$$M_{+---}(s,t,u) = M_{+-++}(s,u,t) = 0 \; .$$

$$M_{-\lambda_2\lambda_3\lambda_4}(s,t,u) = M_{+-\lambda_2-\lambda_3-\lambda_4}(s,t,u) \; .$$



Differential cross sections for the process γγ → γγ versus invariant mass of outgoing photons at √s = 3 TeV. The left, middle and right panels correspond to λ_e= 0.8, -0.8, 0.

Solid curves (from top downwards):

(ς_1 = 10^{-13} GeV^{-4}, ς_2 = 0), (ς_1 = 0, ς_2 = 10^{-13} GeV^{-4}), SM.

Discovery significance (δ = percentage systematic error)

$$S_{\text{dis}} = \sqrt{2}\left[(s+b)\ln\left(\frac{(s+b)(1+\delta^2 b)}{b+\delta^2 b(s+b)}\right) - \frac{1}{\delta^2}\ln\left(1+\frac{\delta^2 s}{1+\delta^2 b}\right)\right]^{1/2}$$

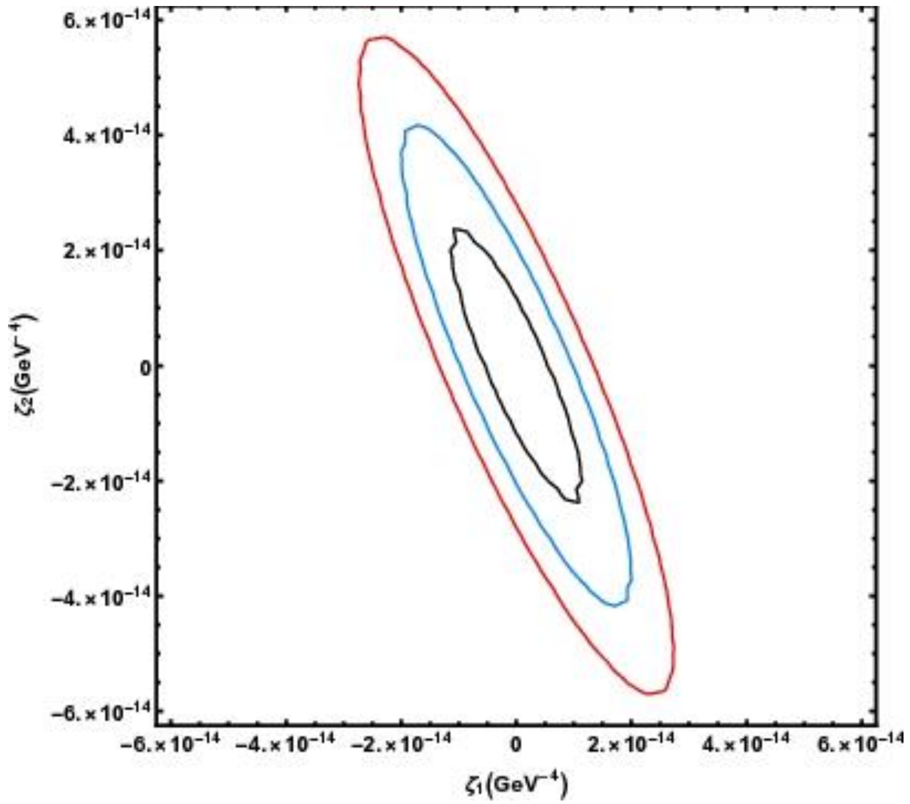
In the limit δ = 0

$$S_{\text{dis}} = \sqrt{2\left[(s+b)\ln\left(1+\frac{s}{b}\right) - s\right]}$$

s << b

$$S_{\text{dis}} = \frac{s}{\sqrt{b}}$$

$S_{dis} \geq 5$ as discovery region



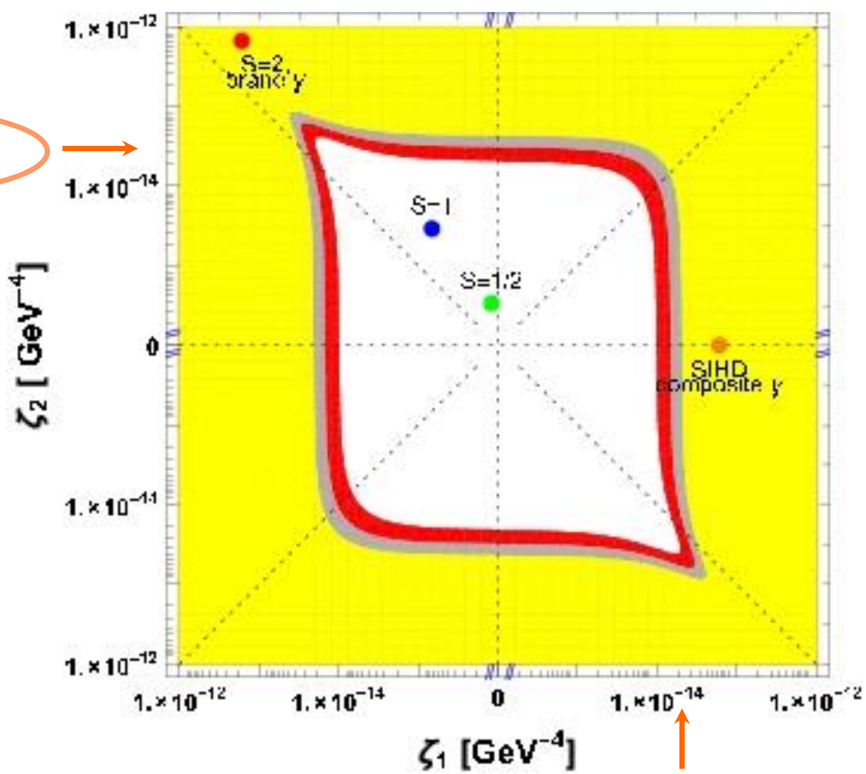
$|\eta_{\gamma\gamma}| < 2.5$
$m_{\gamma\gamma} > 1$ TeV

95% C.L. exclusion region for anomalous couplings ς_1, ς_2 for unpolarized LBL scattering at the CLIC. Systematic errors are 0%, 5%, and 10%. The collision energy is 1.5 TeV, the integrated luminosity is 2.5 ab^{-1}. Couplings ς_1, ς_2 are in GeV^{-4}.

(S.Inan & A.K., EPJC, 81, 664, 2021)

Bounds on anomalous couplings for LHC

(S.Fichet et al., JHEP 02, 165, 2015)



3 $*10^{-14}$

2 $*10^{-14}$

Anomalous $\gamma\gamma\gamma Z$ vertex

Another effective Lagrangian for anomalous γγγZ couplings

$$L_{\gamma\gamma\gamma Z} = \tilde{g}_1 (F_{\mu\nu} F^{\mu\nu})(F_{\rho\sigma} Z^{\rho\sigma}) + \tilde{g}_2 (F_{\mu\nu} \tilde{F}^{\mu\nu} F_{\rho\sigma} \tilde{Z}^{\rho\sigma})$$

(C. Baldenegro et al., JHEP 06, 142, 2017)

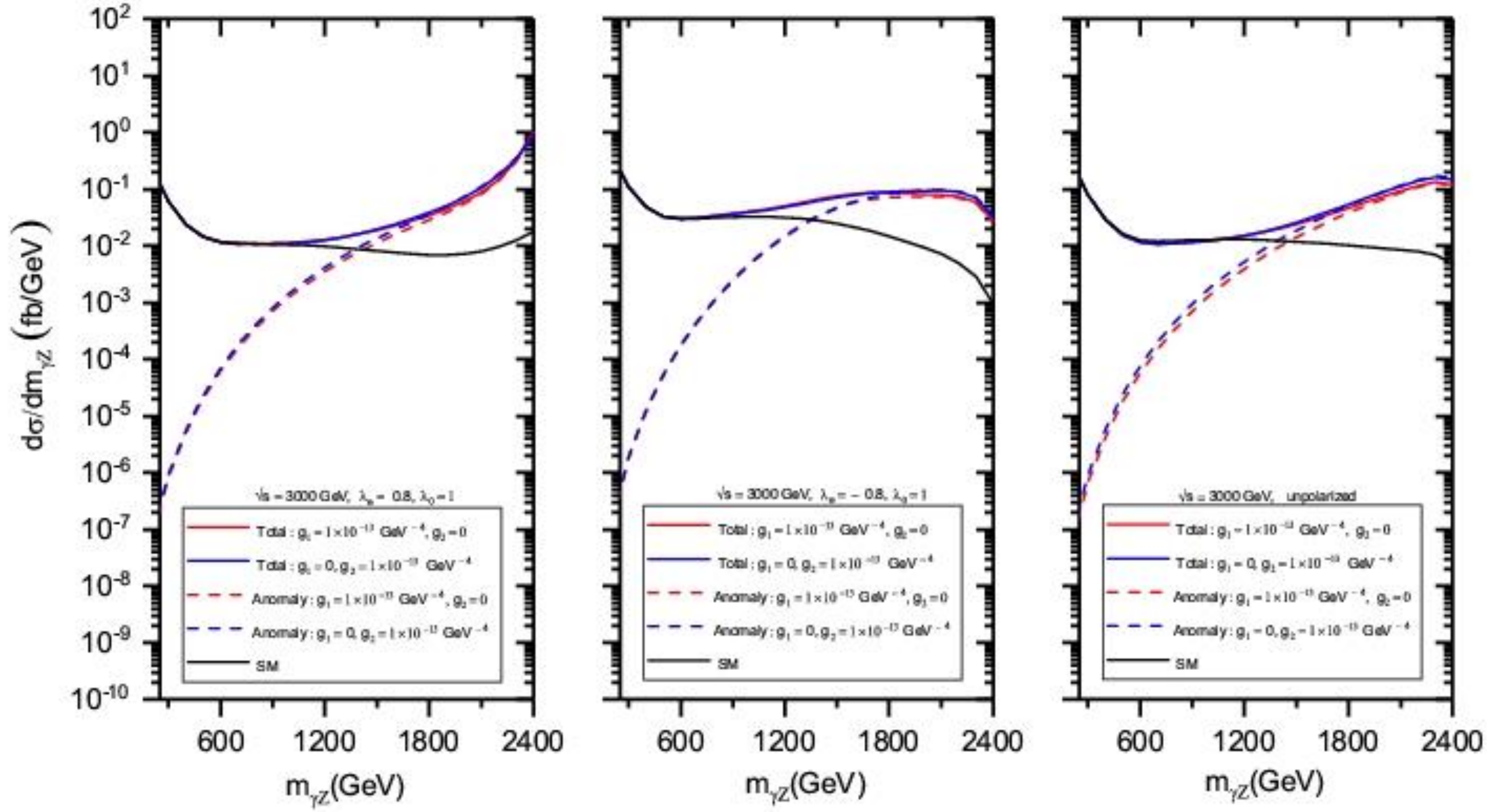
→ **Relations between two sets of coupling constants**

$$g_1 = 8(\tilde{g}_2 - \tilde{g}_1), \quad g_2 = 8\tilde{g}_2$$

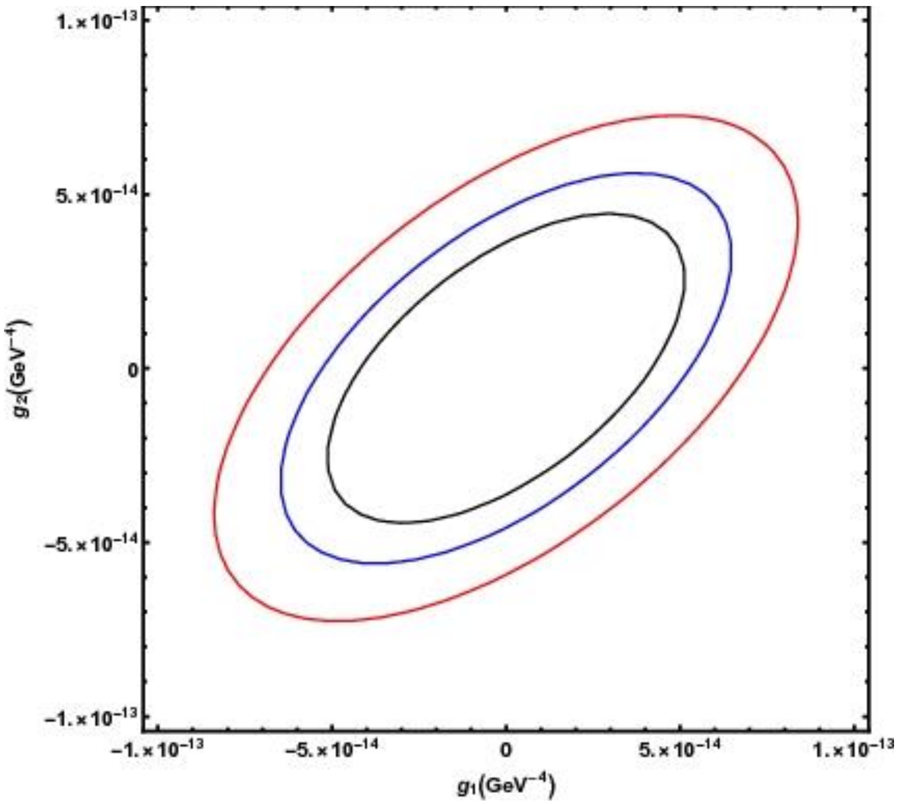
Unpolarized case

$$\sum_{\lambda_1 \ldots \lambda_4} |M_{\lambda_1\lambda_2\lambda_3\lambda_4}|^2 = \frac{1}{4}[g_1^2(3A + 2B) - 4g_1g_2(A + B) + 4g_2^2(A + B)]$$

$$A = s^2t^2 + t^2u^2 + u^2s^2\,, \quad B = stu\, m_Z^2$$



Differential cross sections for the process $\gamma\gamma \rightarrow \gamma Z$ versus invariant mass of outgoing photons at $\sqrt{s}$ = 3 TeV. The left, middle and right panels correspond to λ_e= 0.8, -0.8, 0.
Solid curves (from top downwards): (g_1 = 10^{-13} GeV^{-4}, g_2 = 0), SM.



$|\eta| < 2.5$
$m_{\gamma Z} > 1$ TeV

95% C.L. exclusion region for anomalous couplings g_1, g_2 for unpolarized $\gamma\gamma \to \gamma Z$ scattering at the CLIC. Systematic errors are 0%, 5%, and 10%. The collision energy is 1.5 TeV, the integrated luminosity is 2.5 ab^{-1}. Couplings g_1, g_2 are in GeV^{-4}.

(S.Inan & A.K., JHEP, 10, 121, 2021)

Partial-wave expansion of helicity amplitude

(M.Jacob & G. Wick, Ann. Phys. 7, 404, 1959; ibid 281, 774 2000)

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\theta,\varphi) = 16\pi \sum_J (2J+1)\sqrt{(1+\delta_{\lambda_1\lambda_2})(1+\delta_{\lambda_3\lambda_4})} \times e^{i(\lambda-\mu)\phi}\, d^J_{\lambda\mu}(\theta)\, T^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s)$$

$d^J_{\lambda\mu}$ = Wigner's (small) d-function

$$T^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{32\pi}\frac{1}{\sqrt{(1+\delta_{\lambda_1\lambda_2})(1+\delta_{\lambda_3\lambda_4})}}\int_{-1}^{1} M_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,z)\, d^J_{\lambda\mu}(z)\, dz$$

Partial-wave unitary bound

$$\left|T^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s)\right| \le 1$$

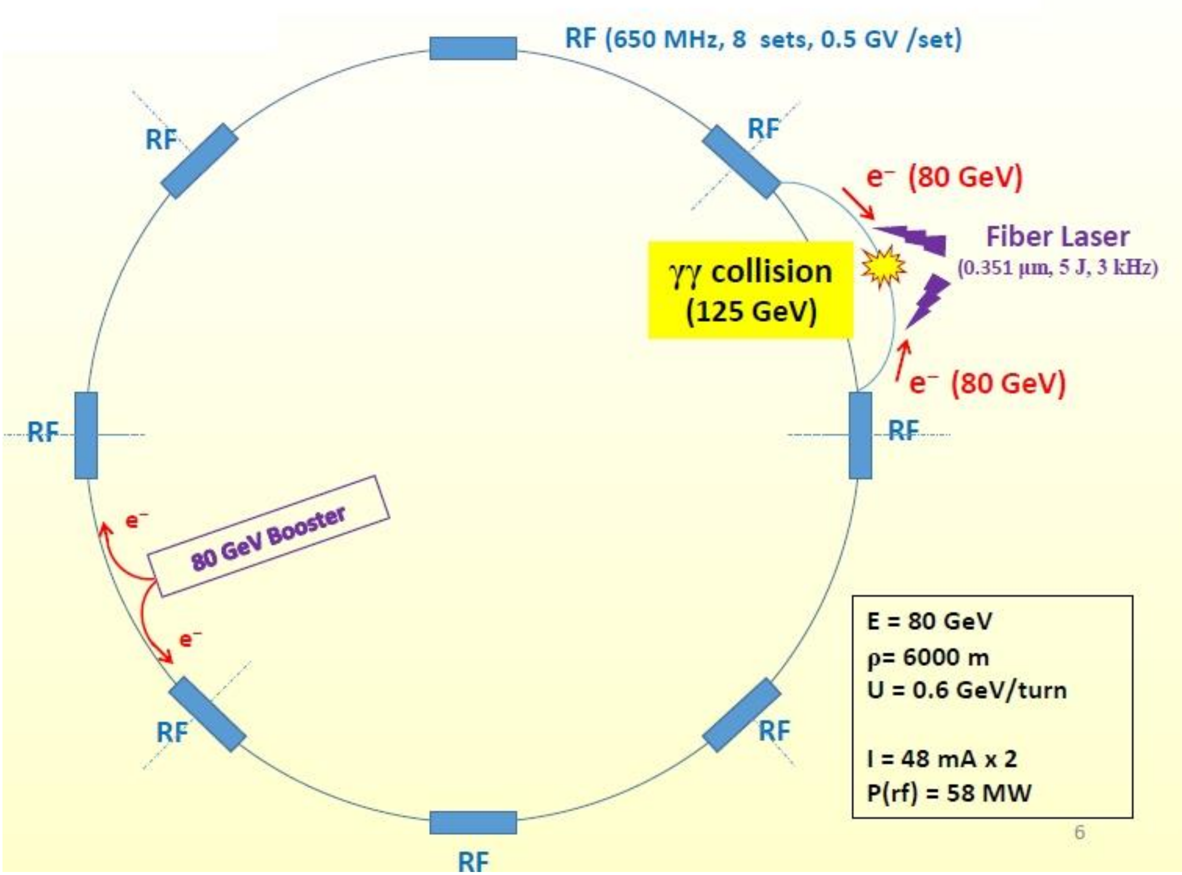
Invisible dark photons

Diagonalization of gauge fields W^3, B and DP field A’

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \\ \bar{A}'_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W & -s_W\varepsilon \\ -s_W & c_W & -c_W\varepsilon \\ t_W\varepsilon & 0 & 1 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \\ A'_\mu \end{pmatrix}$$

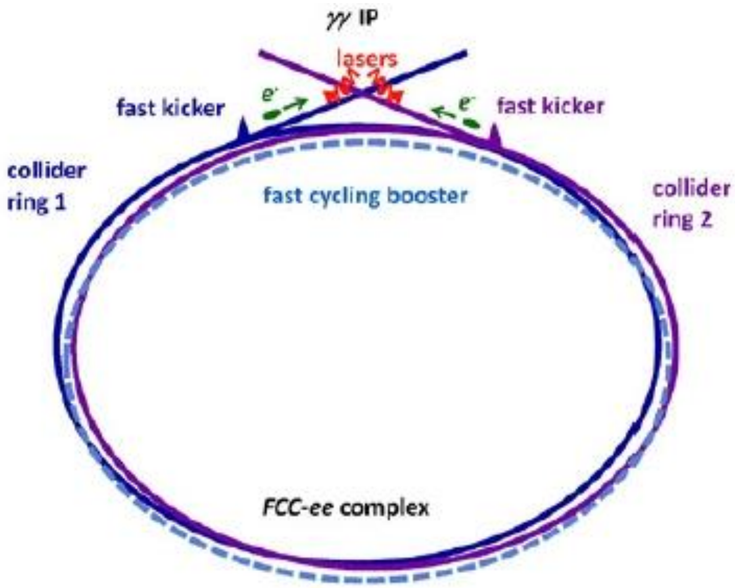
CEPC as a γγ factory

(Ch. Zhang & W. Chou, Photon Beam Workshop, Padova, Italy, 2017)



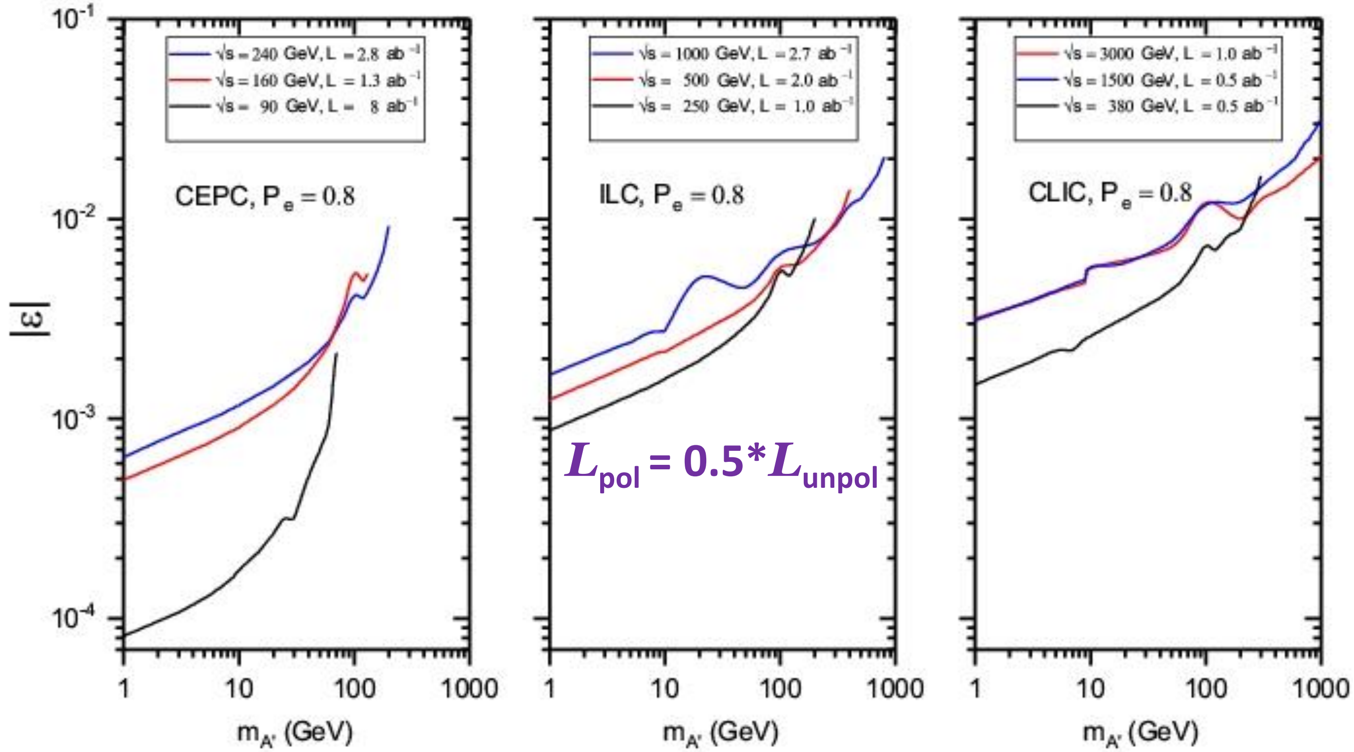
FCC-ee as a γγ factory

(R. Alesan et al., IPAC 2015, Richmond, VA, USA, 2015)



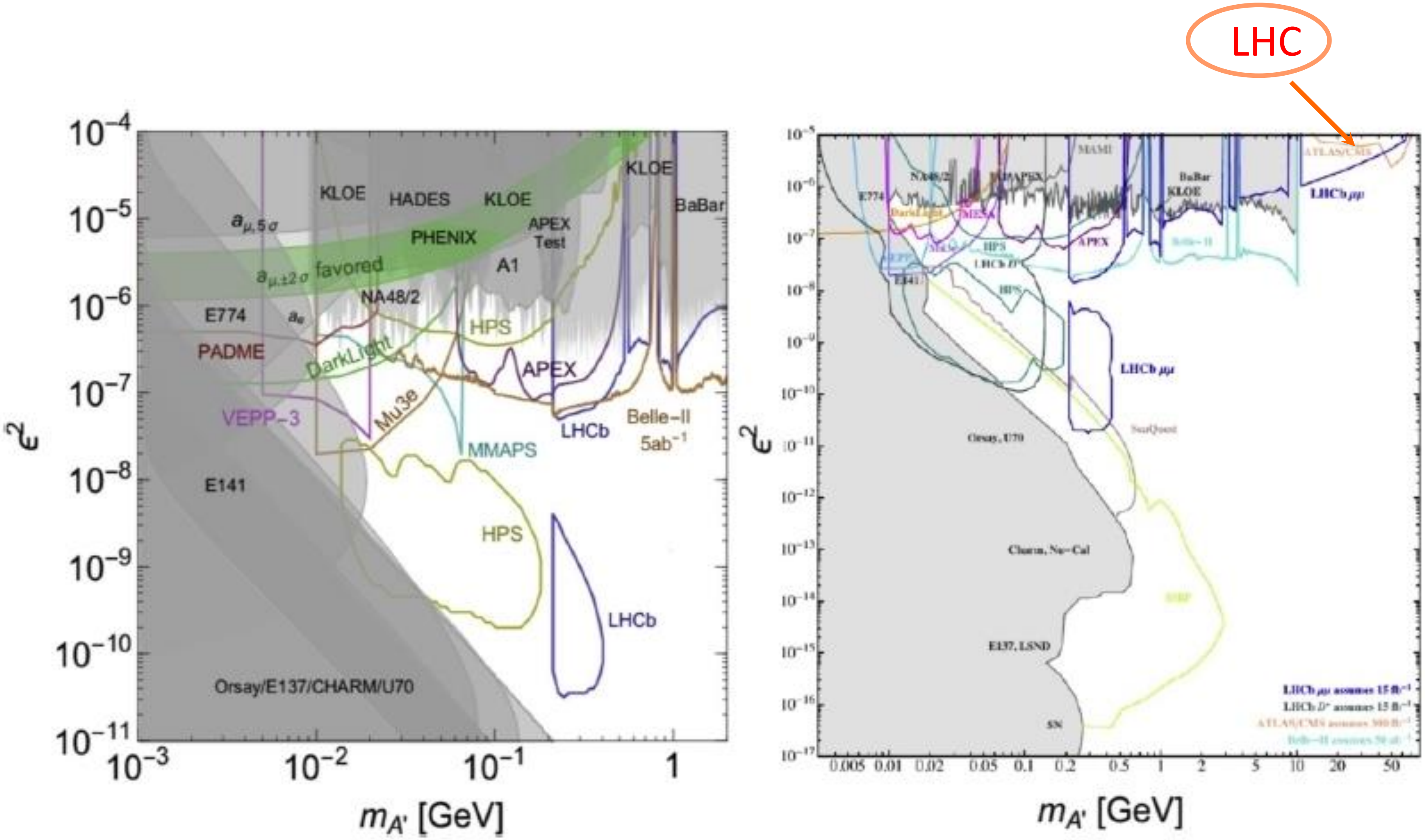
γγ Higgs factory can be realized by back-scattering two counter-propagating electron bunches off laser pulses

(S.Inan & A.K., EPJC, 82, 592, 2022)



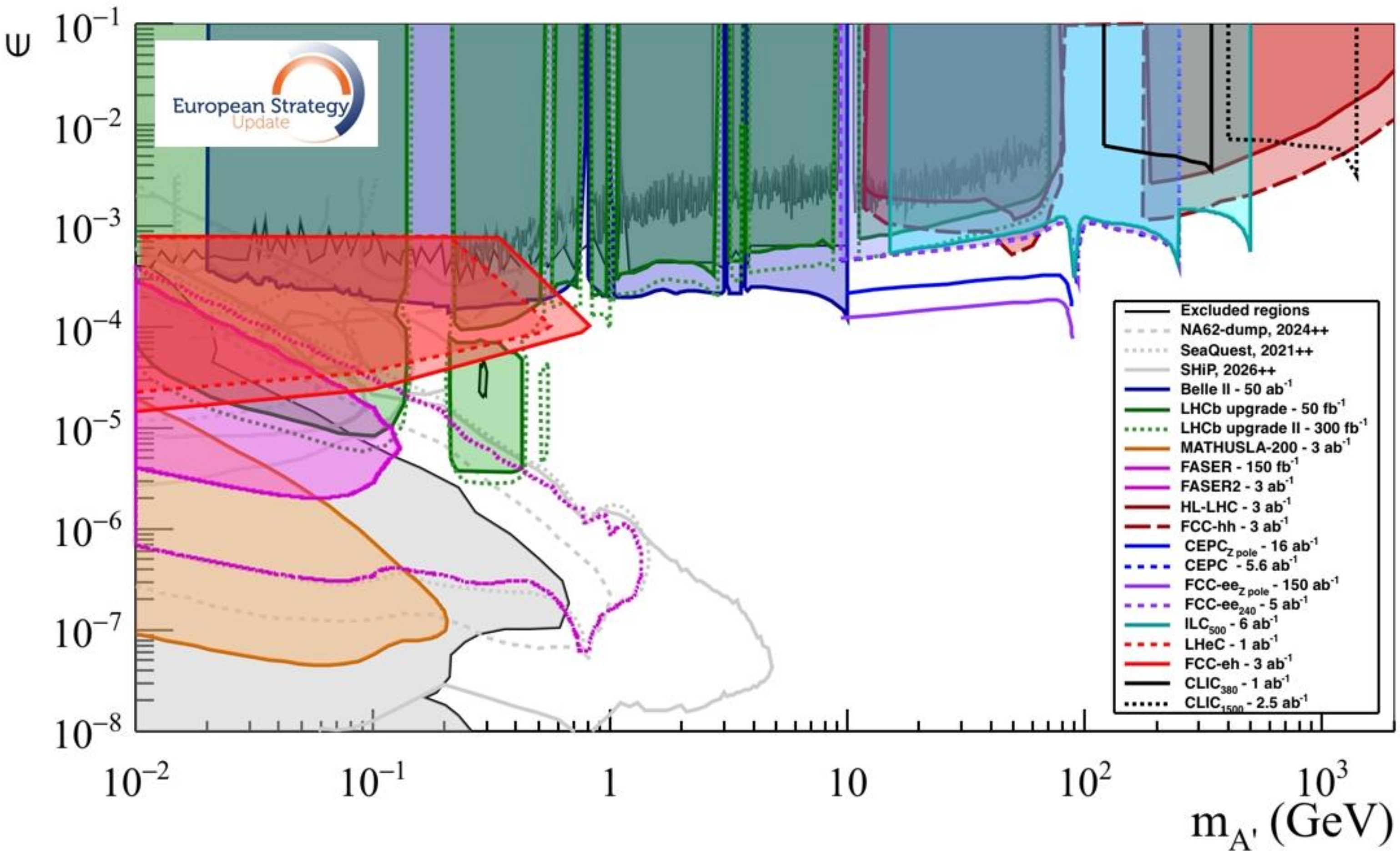
Excluded bounds at 95% C.L. on DP mass $m_{A'}$ and mixing parameter ε for invisible DP production in collision of *unpolarized* photon and electron whose *polarization is* 80%

$p_{e,t} > 10$ GeV, $|\eta_e| < 2.5$, $|m_{A'} - m_{inv}| < 10$ GeV



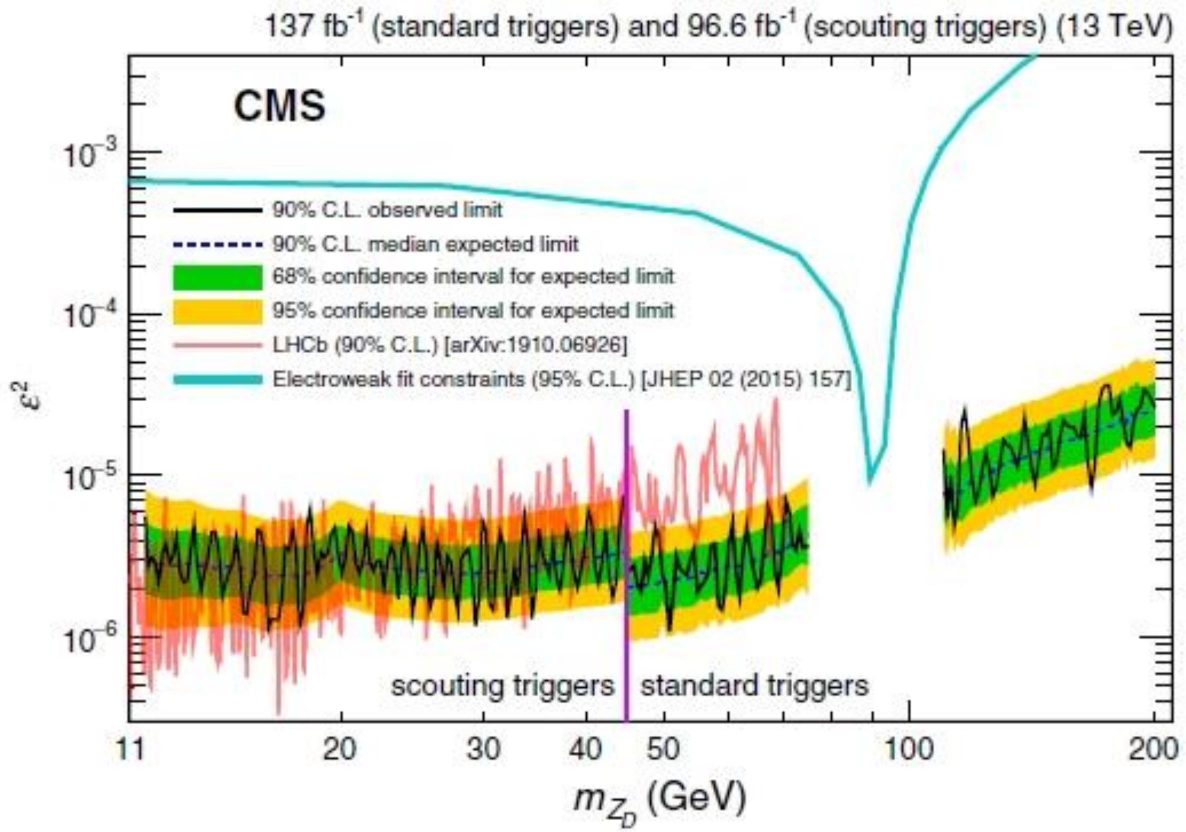
Existing bounds (shaded regions) for DP in visible decays

(A. Filippi & M. De Napoli, Rev. Phys. 5, 100042, 2020)



Sensitivity for DPs in the plane mixing parameter ε versus DP mass. HL-LHC, CEPC, FCC-ee & FCC-hh curves correspond to 95% C.L. exclusion limits, all other curves to 90% C.L. exclusion limits

(R.Ellis at al., arXiv:1910.11775)

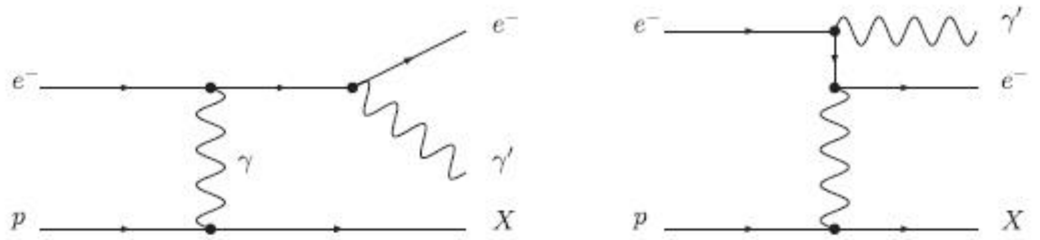


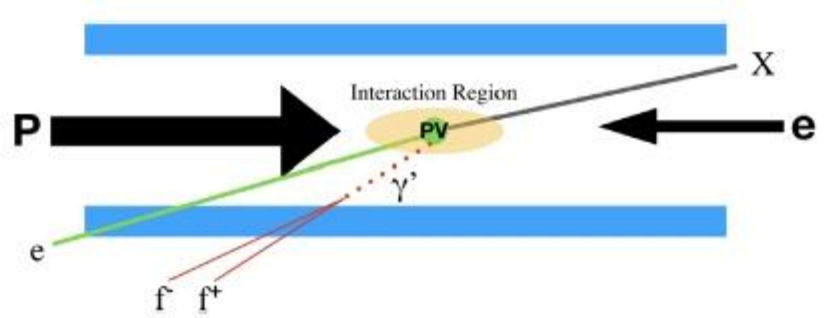
CMS limit on mixing parameter for DP decaying into muon pair

(CMS Collab., Phys. Rev. Lett. 124, 131802, 2020)

DP at future ep colliders LHeC and FCC-he

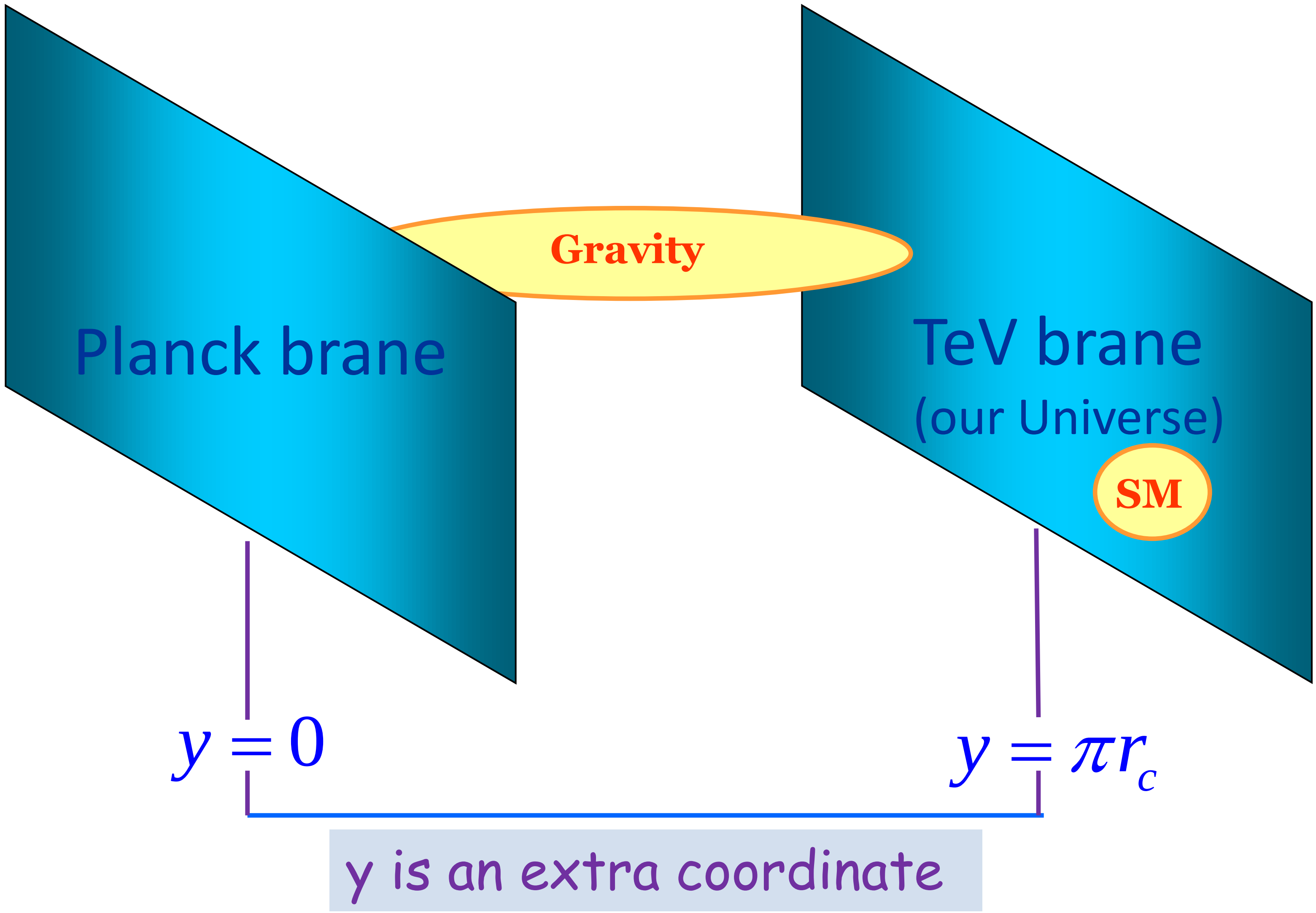
(CMS Collab., Phys. Rev. D 101, 015020, 2020)





Back-up slides
(KK gravitons at muon collider)

AdS_5 space-time with two branes



Background metric

$$ds^2 = e^{-2\sigma(y)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dy^2$$

Five-dimensional action

$$S = \int d^4x \int_{-\pi r_c}^{\pi r_c} dy\,\sqrt{G}\,(2\bar{M}_5^3\mathcal{R} - \Lambda) + \int d^4x \sqrt{|g^{(1)}|}\,(\mathcal{L}_1 - \Lambda_1) + \int d^4x \sqrt{|g^{(2)}|}\,(\mathcal{L}_2 - \Lambda_2)$$

General solution of Einstein-Hilbert's equations

$$\sigma(y) = \frac{kr_c}{2}\left[\left|\mathrm{Arccos}\left(\cos\frac{y}{r_c}\right)\right| - \left|\pi - \mathrm{Arccos}\left(\cos\frac{y}{r_c}\right)\right|\right] + \frac{\pi\,|k|r_c}{2} - C$$

(A.K. Nucl. Phys. B 909, 218 2016)

Hierarchy relation

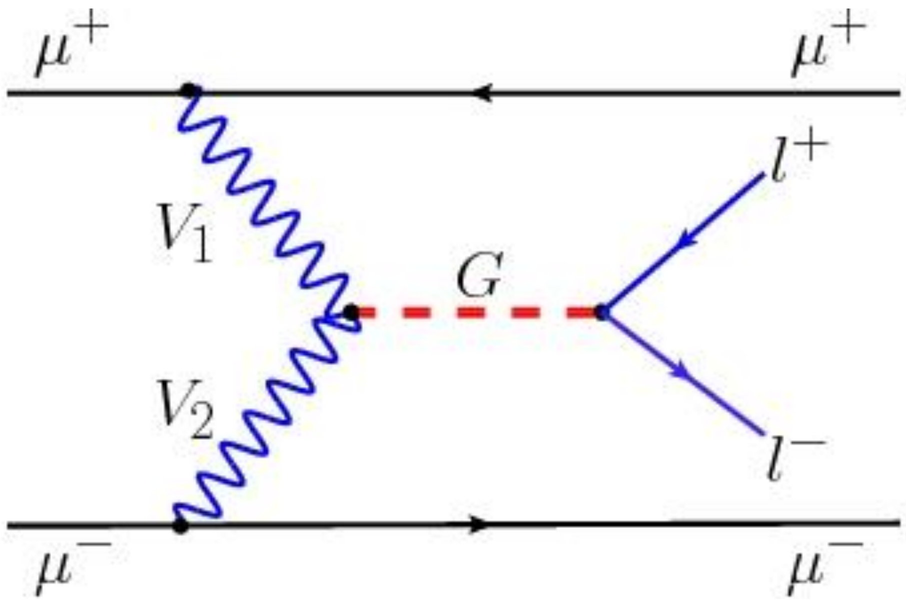
$$\bar{M}_{\rm Pl}^2 = \frac{\bar{M}_5^3}{k}\left[e^{2\pi k r_c} - 1\right]$$

Graviton Lagrangian

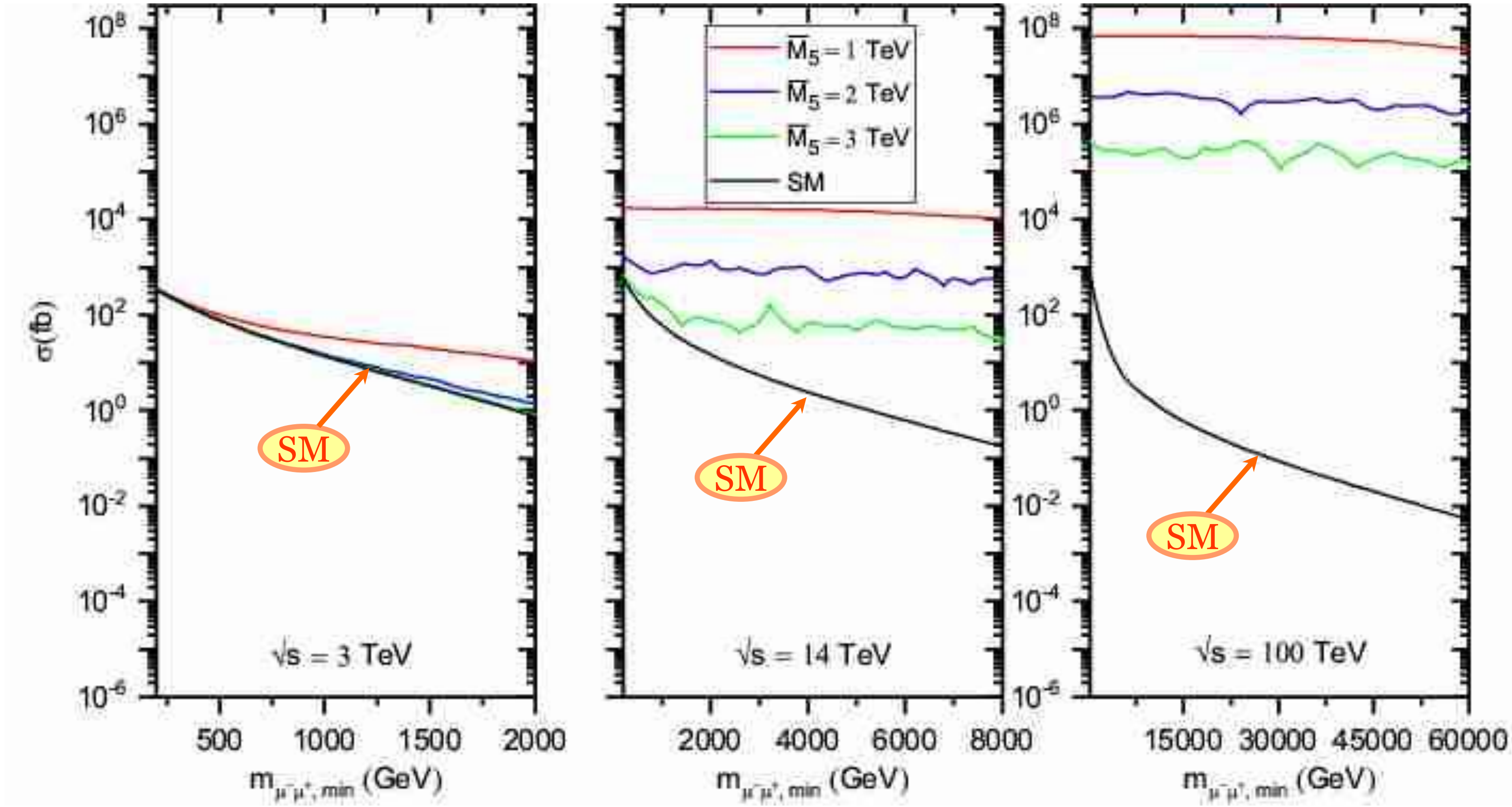
$$\mathcal{L}_{\rm int} = -\frac{1}{\bar{M}_{\rm Pl}}\, h_{\mu\nu}^{(0)}(x)\, T_{\alpha\beta}(x)\, \eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{\Lambda_\pi}\sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x)\, T_{\alpha\beta}(x)\, \eta^{\mu\alpha}\eta^{\nu\beta}$$

Masses of KK gravitons

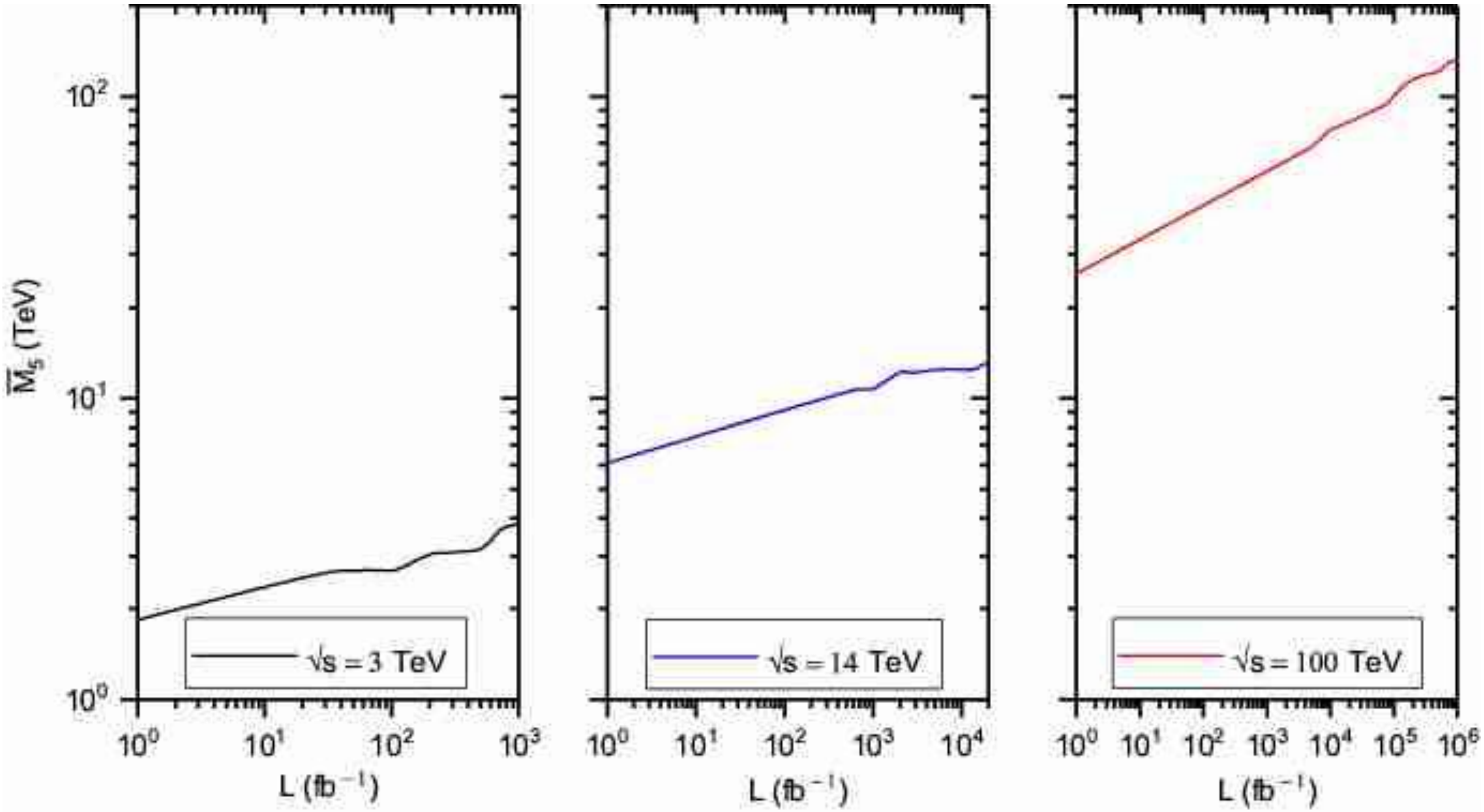
$$m_n = x_n k\,, \quad n = 1, 2, \ldots$$



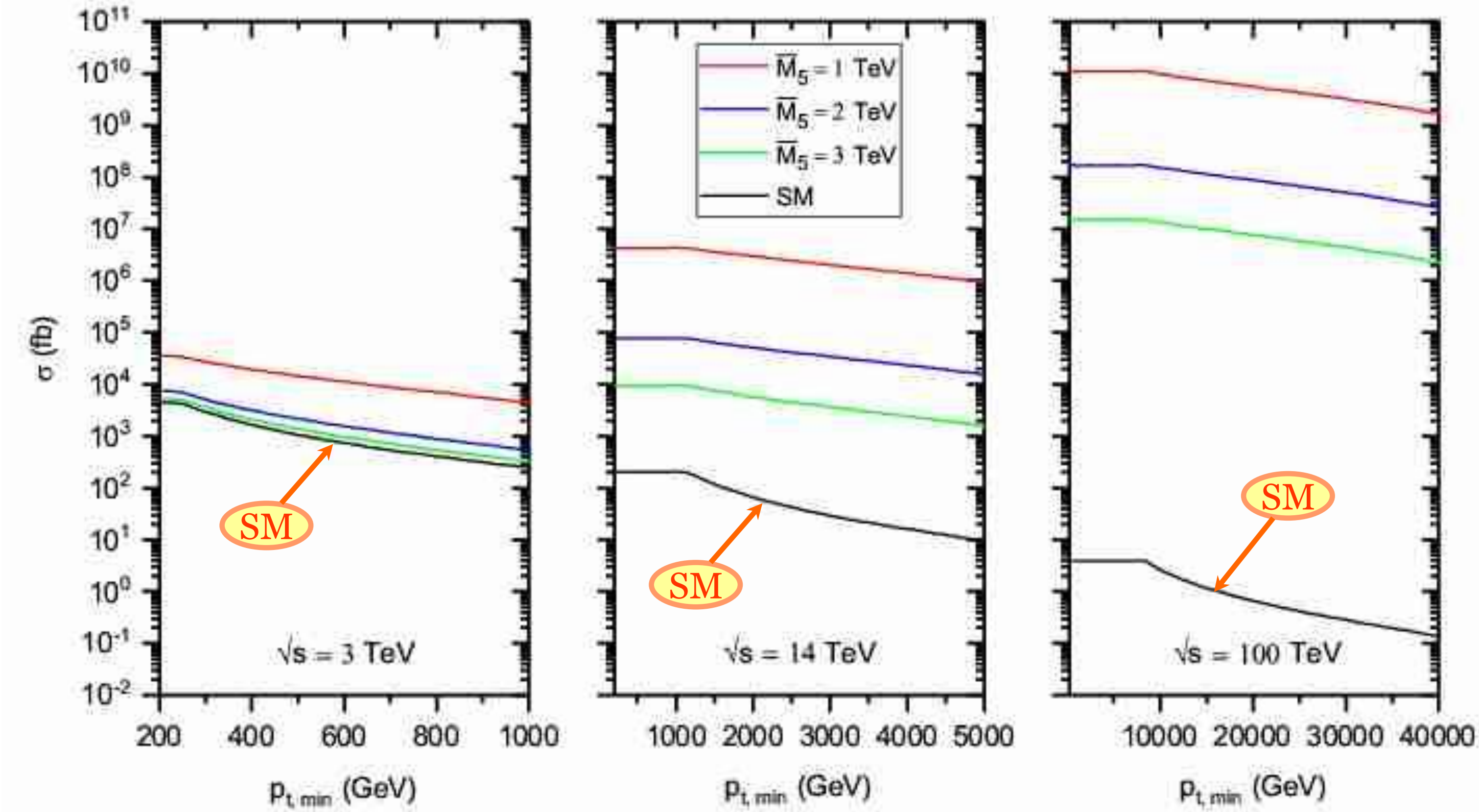
Contribution of the KK graviton G to collision of two vector bosons V_1, V_2 = γ or Z, with two outgoing charged leptons at muon collider



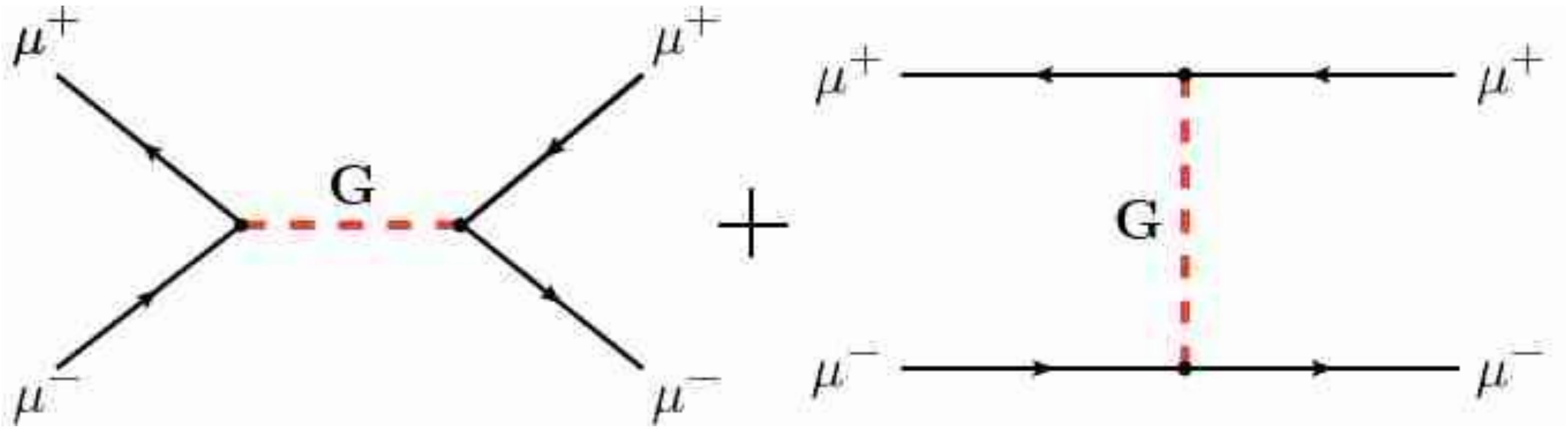
Total cross sections for the process μ+μ- → 2 (μ+μ-) via minimal invariant mass of *two* detected muons $m_{\mu+\mu-,\,min}$ at muon collider for different values of 5-dimensional Planck scale M_5



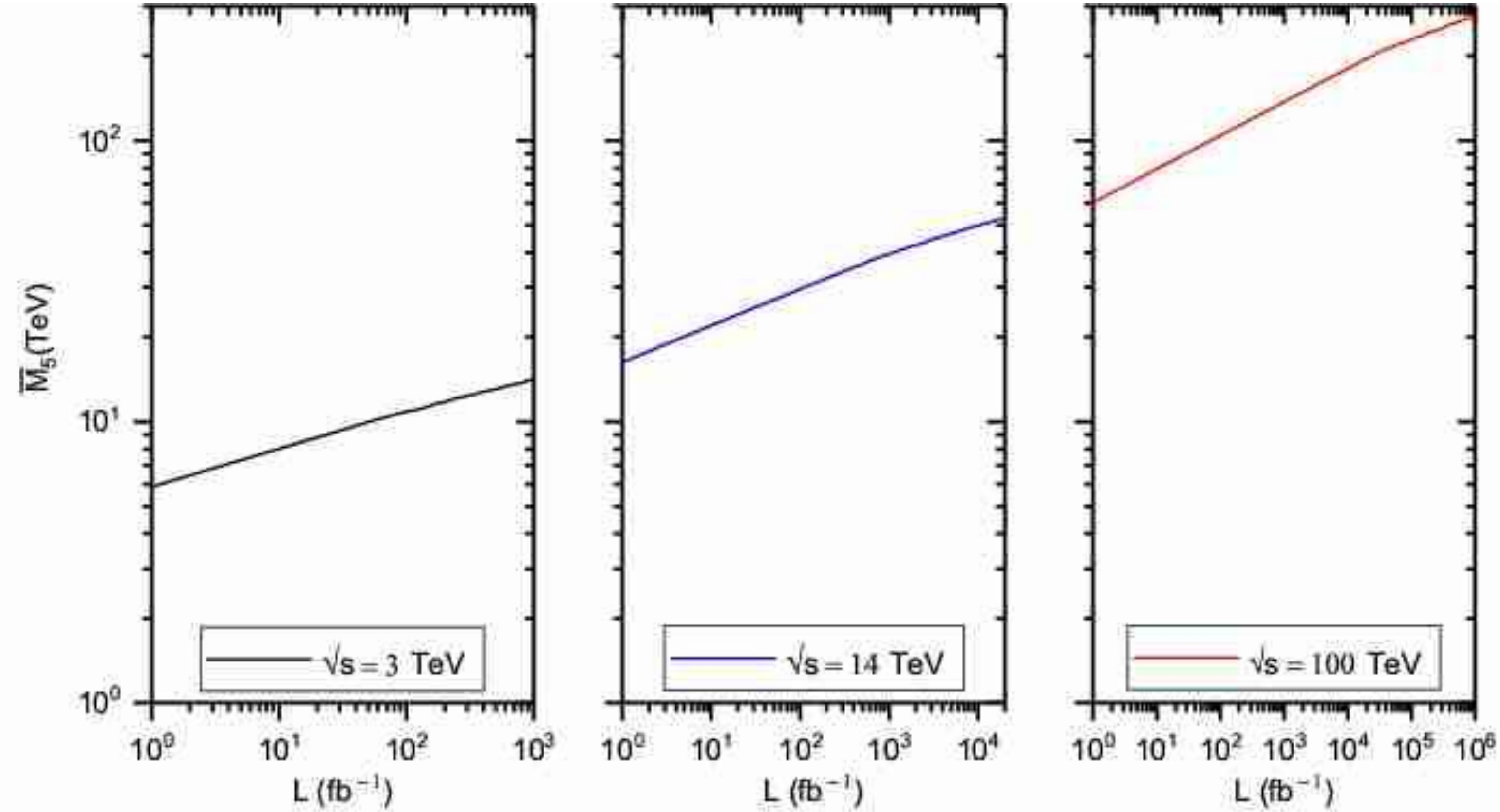
Excluded bounds on 5-dimensional Planck scale M_5 via integrated luminosity of the muon collider L for the process $\mu^+\mu^- \to 2\,(\mu^+\mu^-)$



Total cross sections for the process μ+μ- → 2 (μ+μ-) via transverse momentum of detected muons at muon collider for different values of 5-dimensional Planck scale M_5



Contribution of KK graviton G to the process
μ+μ- → μ+μ- at muon collider



Excluded bounds on 5-dimensional Planck scale M_5 via integrated luminosity of the muon collider L for the process $\mu^+\mu^- \rightarrow \mu^+\mu^-$

Back-up slides
(anomalous couplings for $\gamma\gamma\gamma Z$ vertex)

Feynman rule for anomaly vertex γγγZ

$$
\begin{aligned}
P^{\mu\nu\rho\alpha} = \mathcal{P}\{ & g_1[(p_1\cdot p_2)(p_2\cdot p_3)g^{\mu\nu}g^{\rho\alpha} - (p_1\cdot p_3)p_2^\mu p_1^\nu g^{\rho\alpha} \\
& - (p_1\cdot p_3)p_1^\nu p_2^\alpha g^{\mu\rho} + p_2^\mu p_1^\nu p_1^\rho p_3^\alpha] \\
& + g_2[-(p_1\cdot p_2)(p_1\cdot p_3)g^{\mu\alpha}g^{\nu\rho} + (p_2\cdot p_3)p_1^\nu p_1^\alpha g^{\mu\rho} \\
& - (p_2\cdot p_3)p_1^\nu p_1^\rho g^{\mu\alpha} + (p_2\cdot p_3)p_1^\nu p_2^\alpha g^{\mu\rho} + 2(p_2\cdot p_3)p_2^\mu p_1^\rho g^{\nu\alpha} \\
& - (p_1\cdot p_3)p_2^\rho p_1^\alpha g^{\mu\nu} + p_3^\mu p_1^\nu p_2^\rho p_1^\alpha]\}
\end{aligned}
$$

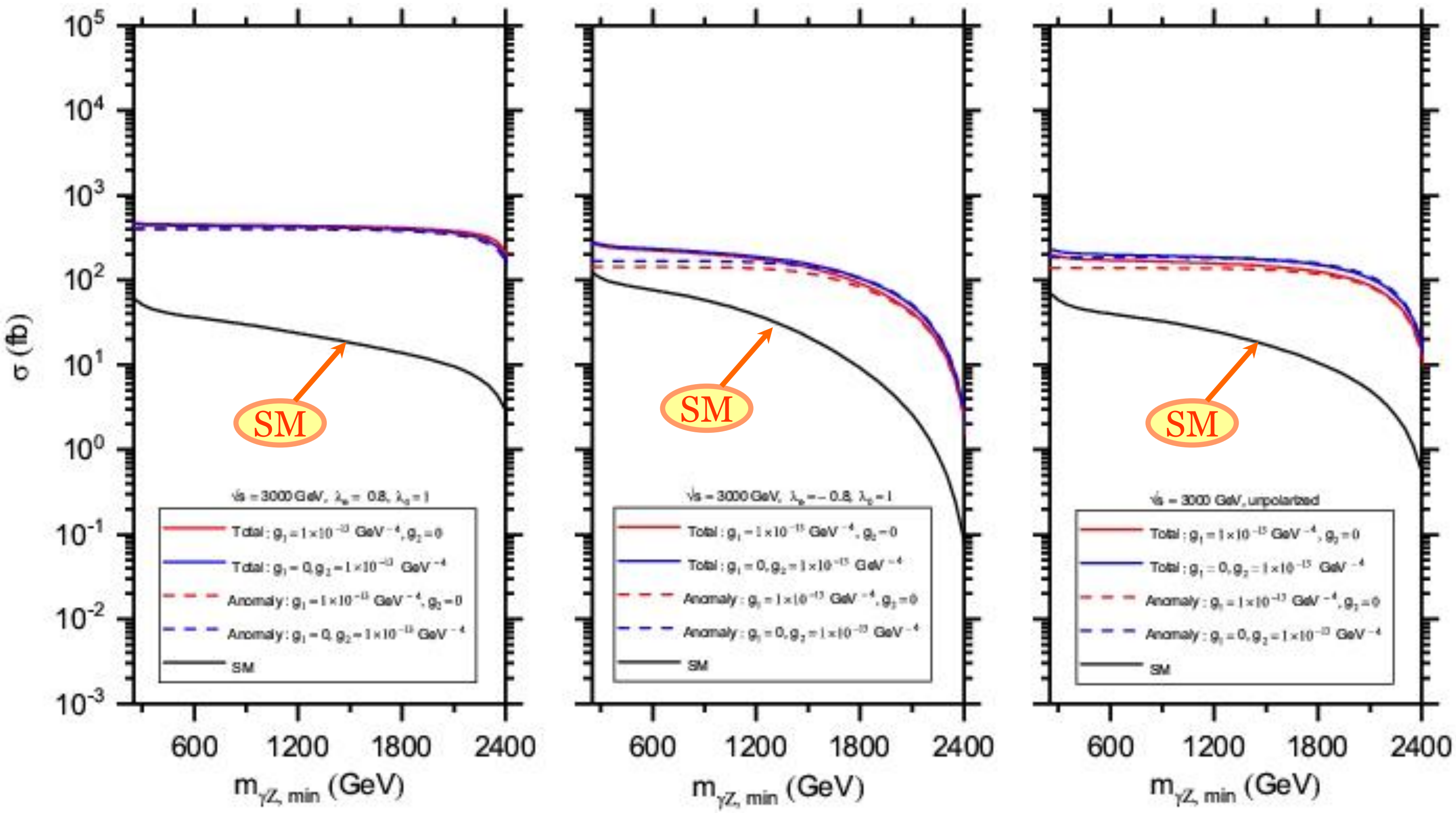
$\mathcal{P}$ – permutations (symmetrization with respect to photon's momenta and indices)

$$
M_{\lambda_1\lambda_2\lambda_3\lambda_4}(p_1,p_2,p_3) = P_{\mu\nu\rho\alpha}(p_1,p_2,p_3)\,\varepsilon_\mu^{\lambda_1}(p_1)\varepsilon_\nu^{\lambda_2}(p_2)\varepsilon_\rho^{*\lambda_3}(p_3)\varepsilon_\alpha^{*\lambda_4}(p_4)
$$

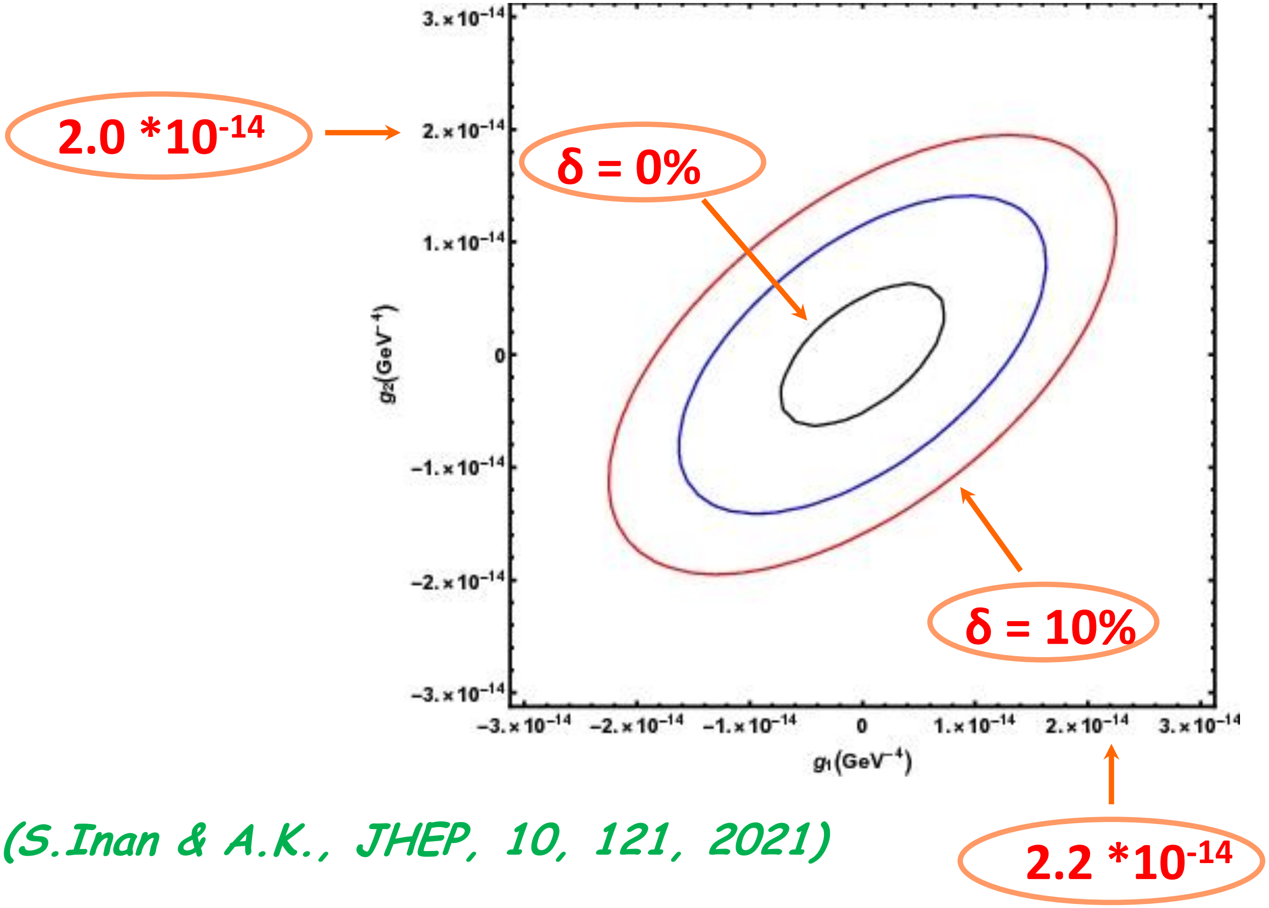
→ 24 helicity amplitudes proportional to g_1
24 helicity amplitudes proportional to g_2

(S.Inan & A.K., JHEP 10, 121, 2021)

SM amplitudes are taken from *G.Gounaris et al., EPJC 10, 499, 1999*



Total cross sections for γγ → γZ scattering versus minimal invariant mass of outgoing photons for √s = 3 TeV. The left, middle and right panels correspond to λ_e= 0.8, -0.8, 0. Solid curves correspond to (g_1 = 10^{-13} GeV^{-4}, g_2 = 0), SM.



$|\eta| < 2.5$

$m_{\gamma Z} > 1$ TeV

(S.Inan & A.K., JHEP, 10, 121, 2021)

95% C.L. exclusion region for anomalous couplings g_1, g_2 for unpolarized $\gamma\gamma \rightarrow \gamma Z$ scattering at the CLIC. Systematic errors are 0%, 5%, and 10%. The collision energy is 3 TeV, the integrated luminosity is 5 ab^{-1}. Couplings g_1, g_2 are in GeV^{-4}.

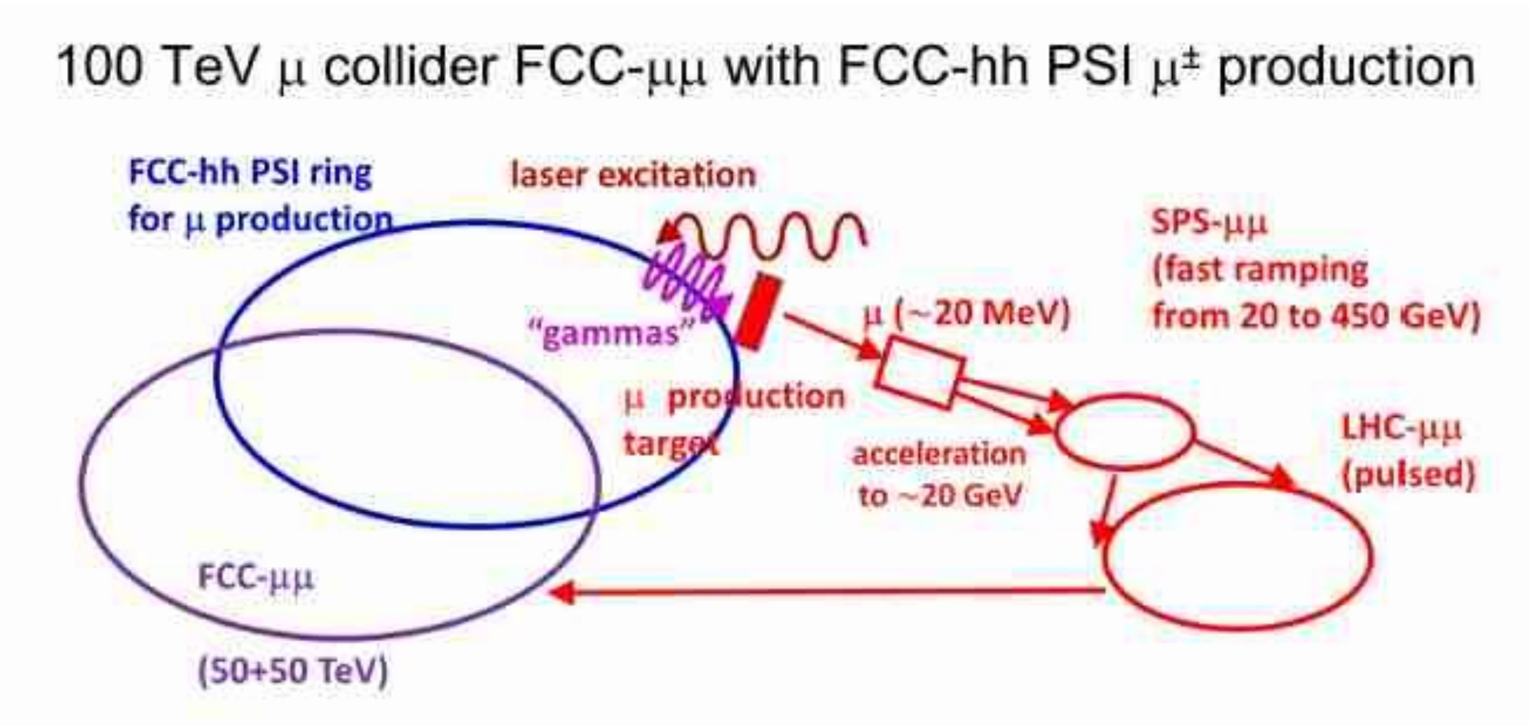
Exclusion limits on anomalous couplings at the CLIC for energy 3.0 TeV. Polarized case

λ_e		0	-0.8	0.8
L, fb^{-1}		5000	4000	1000
$\lvert g_1 \rvert$, GeV^{-4} ($g_2 = 0$)	$\delta = 0\%$	5.98×10^{-15}	7.14×10^{-15}	$\boxed{5.13 \times 10^{-15}}$
	$\delta = 5\%$	1.33×10^{-14}	1.73×10^{-14}	7.79×10^{-15}
	$\delta = 10\%$	1.85×10^{-14}	2.39×10^{-14}	1.04×10^{-14}
$\lvert g_2 \rvert$, GeV^{-4} ($g_1 = 0$)	$\delta = 0\%$	5.18×10^{-15}	6.62×10^{-15}	$\boxed{5.19 \times 10^{-15}}$
	$\delta = 5\%$	1.16×10^{-14}	1.60×10^{-14}	7.87×10^{-15}
	$\delta = 10\%$	1.62×10^{-14}	2.21×10^{-14}	1.05×10^{-14}

(S.Inan & A.K., JHEP, 10, 121, 2021)

Back-up slides
(muon colliders)

100 TeV muon collider based on FCC complex



Gluon field is a pure gauge at spatial infinity

$$A_\mu\big|_{r\to\infty} \to -\frac{i}{g}\partial_\mu \omega \omega^{-1} \qquad \omega_n \to e^{i2\pi n} \text{ as } r \to \infty$$

Winding number

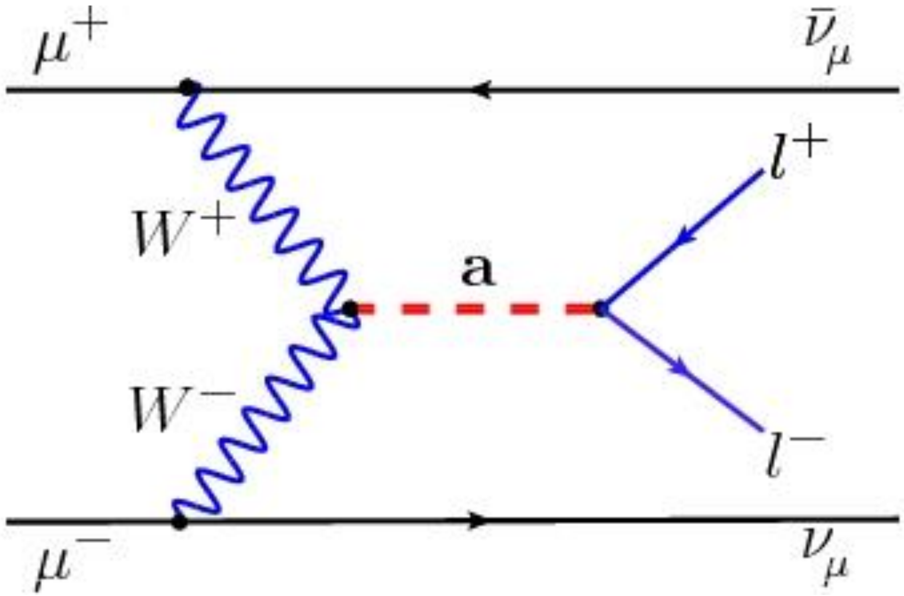
$$n = \frac{g^2}{32\pi^2}\int ds_\mu K^\mu$$

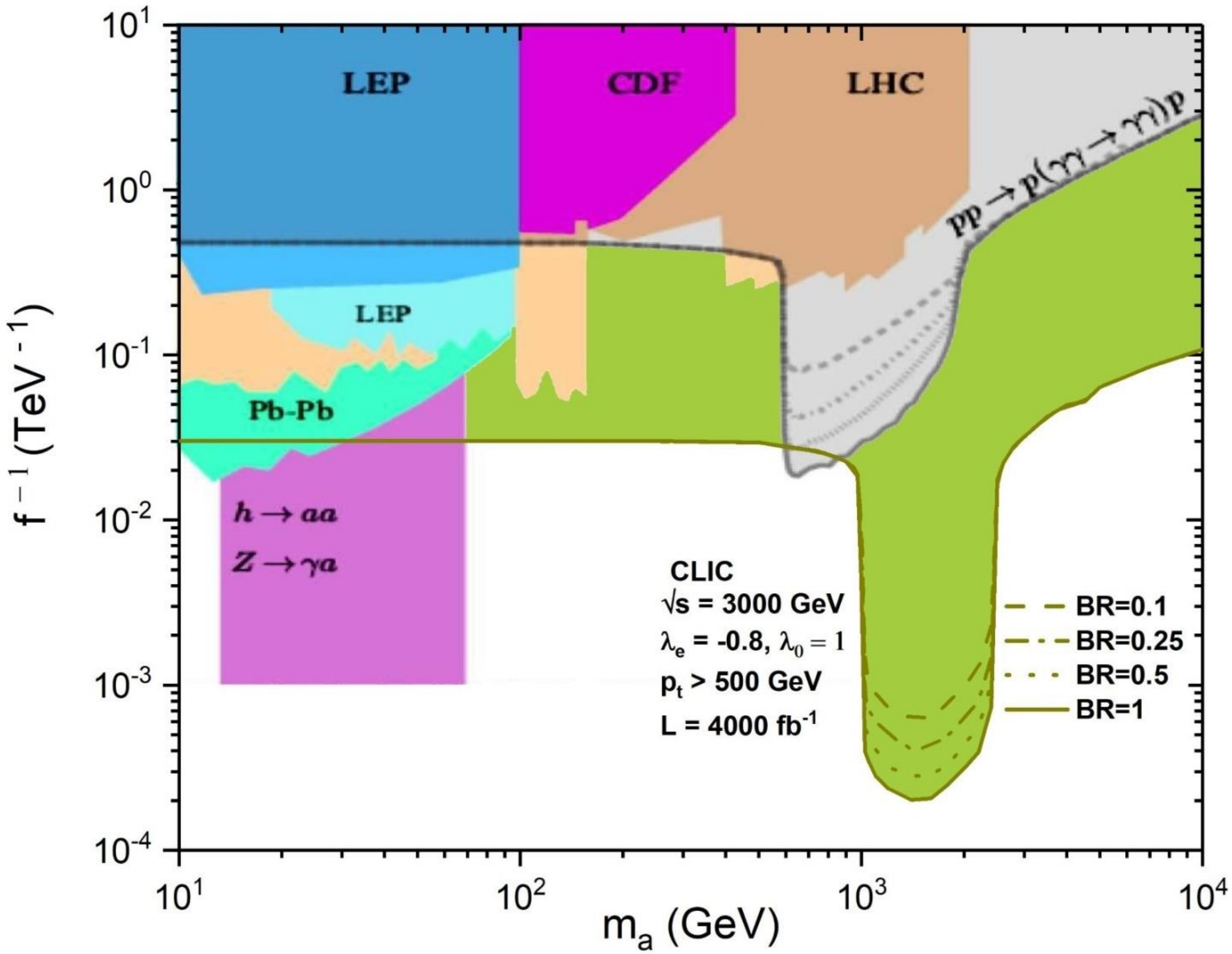
SU(2): map of 3-dimensional sphere S^3 on sphere S^3 of SU(2) group

True or θ-vacuum

$$|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$$

Production of ALPs via W boson fusion at future muon collider

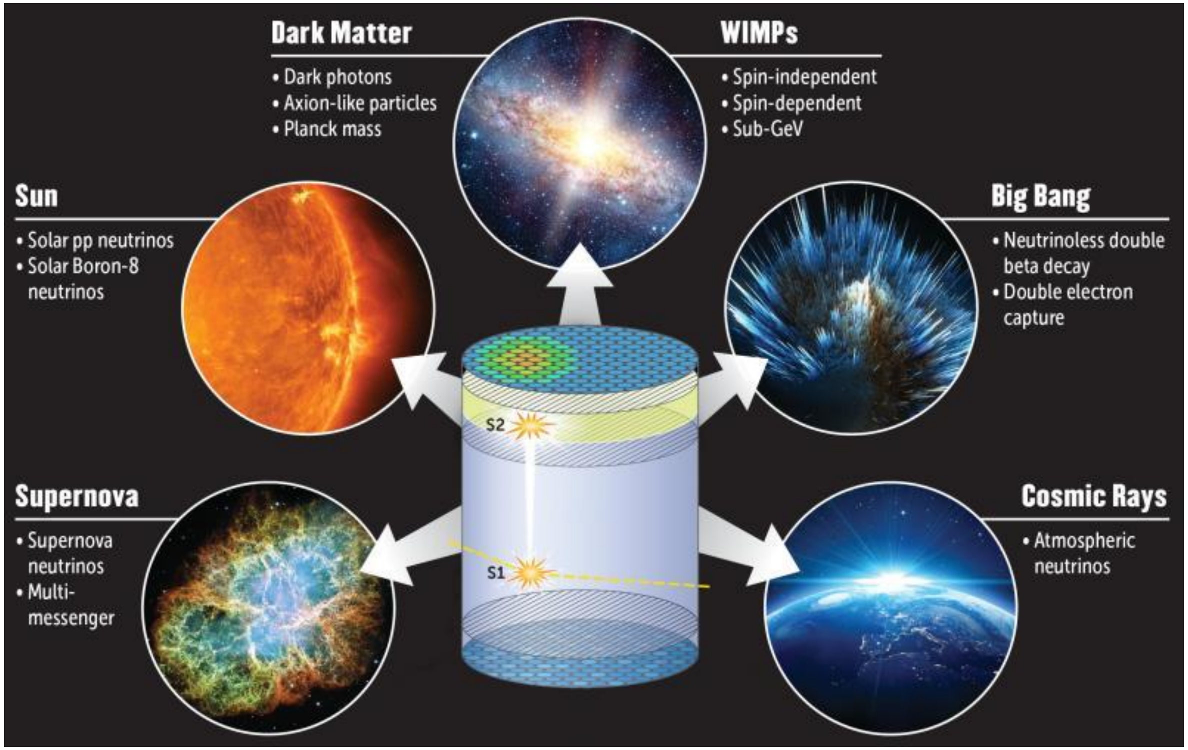


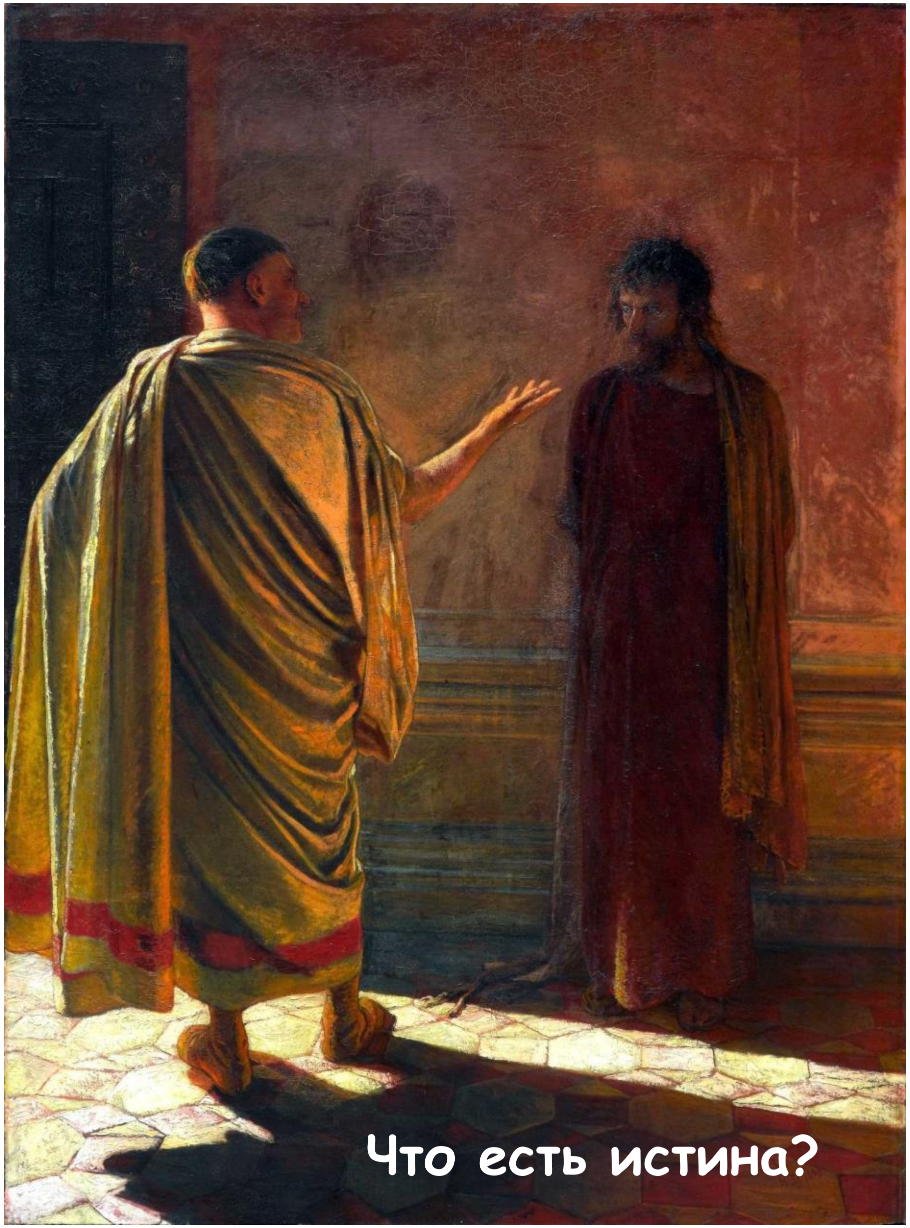


Current 95% C.L. exclusion regions for ALP parameters from collider experiments

Detector XENONnT

(aiming to detect DM & WIMPs)




Что есть истина?