Double spin asymmetries in the elastic $e\vec{p} \to e\vec{p}$, $e\vec{p} \to \vec{e}p$ and $\vec{e}\vec{p} \to ep$ processes in the case of parallel spins

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Rosenbluth Method or Rosenbluth Technique (RT)

The Rosenbluth method for measuring of the electric (G_E) and magnetic (G_M) proton form factors (so-called Sachs form factors) ratio is based on the measurements of the unpolarized cross section of the elastic process of the electron scattering on the proton

$$e(p_1) + p\,(q_1) \to e(p_2) + p\,(q_2)$$

in the laboratory reference frame ($q_1 = (M, \vec{0})$) and $m_e = 0$ in the one-photon exchange approximation [1]:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau_p} \left(G_E^2 + \frac{\tau_p}{\varepsilon} G_M^2 \right) . \quad (1)$$

$$G_E = F_1 - \tau_p F_2 \,,\; G_M = F_1 + F_2 \,, \quad (2)$$

$$\varepsilon = (1 + 2(1+\tau_p)\tan^2(\theta_e/2))^{-1}. \quad (3)$$

Here $\tau_p = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1E_2\sin^2(\theta_e/2)$, $q = q_2 - q_1$, $\alpha = 1/137$ – fine structure constant, **ε is the degree of the linear polarization of the virtual photon**, $0 \leqslant \varepsilon \leqslant 1$

[1]. M. Rosenbluth, Phys. Rev. **79**, 615 (1950)

My question to the audience

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau_p} \left(G_E^2 + \frac{\tau_p}{\varepsilon} G_M^2 \right) .$$

$$G_E = F_1 - \tau_p F_2 \, , \; G_M = F_1 + F_2 \, .$$

My question: What is the physical meaning of the decomposition of the Rosenbluth formula into two terms containing only the squares of the Sachs form factors ?

It is usually stated in the modern literature, in particular, in textbooks on the physics of elementary particles [1], that the Sachs form factors are simply convenient because they allow the representation of the Rosenbluth formula in the simple and compact form of the sum of two terms containing only G_E^2 and G_M^2. These formal reasons for advantages of the Sachs form factors are included, in particular, in known monographs [2,3], are not criticized, and are reproduced until now, e.g., in dissertation [4].

[1] F. Halzen and A. Martin, *Quarks and Leptons*, 1984.
[2] A. I. Akhiezer and V. B. Berestetskii, *QED*, Nauka, Moscow, 1969.
[3] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Course of Theoretical Physics, Vol. 4: QED,* Nauka, Moscow, 1989.
[4] A. J. R. Puckett, Thesis, arXiv: nucl-ex/1508.01456v1.

Erroneous terminology (red color)

In [1]: ε is the virtual photon longitudinal polarization parameter...

In [5]: Let us introduce another set of kinematical variables: Q^2, and the degree of the linear polarization of the virtual photon, ε .

In [7]: The ε is often erroneously called in literature as a degree of longitudinal polarization. In fact, it is a degree of linear polarization.

In [8]: If the scattering is described by the one-photon exchange approximation, then for unpolarized electrons the virtual photons are linearly polarized, whereas for polarized electrons the photons are elliptically polarized.

[1]. I. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015).
[2]. N. Dombey, Rev. Mod. Phys. **41**, 236 (1969).
[3]. A. Akhiezer, M. Rekalo, Fiz. Elem. Chast.Atom.Yadra **4**, 662 (1973).
[4]. M. V. Galynskii and M. I. Levchuk, Phys. At. Nucl. **60**, 1855 (1997).
[5]. G.I. Gakh, E. Tomasi-Gustafsson, Nuclear Physics A **799**, 127 (2008).
[6]. F. Gil-Dominguez, J.Alarcon, C. Weiss, arXiv: 2306.01037 [hep-ph].
[7]. N.Korchagin, and A.Radzhabov, arXiv: 2106.06883v1 [nucl-th].
[8]. M.J. Alguard *et al.* PRL **37**, 1261 (1976).

Akhiezer–Rekalo method of the Polarization transfer (PT)

A.I. Akhiezer and M.P. Rekalo proposed a method for measuring of the Sachs form factors ratio in the process $\vec{e}p \to e\vec{p}$ [2]:

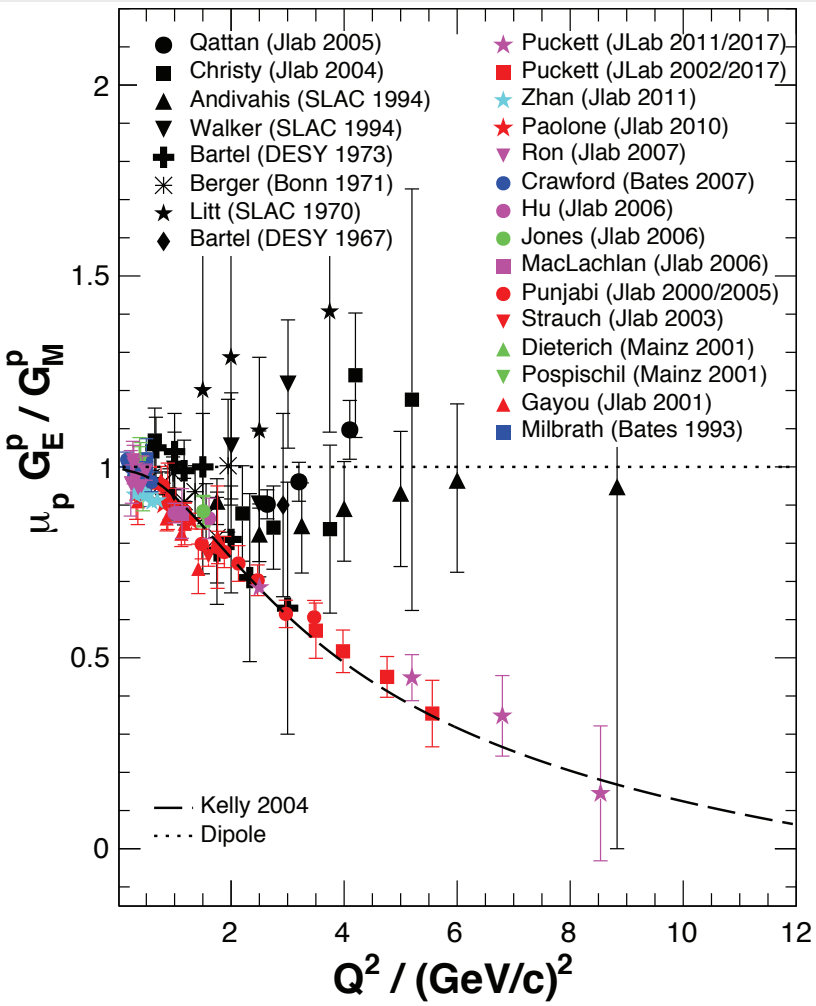
$$e\left(p_1, s_{e_1}\right) + p\left(q_1\right) \to e\left(p_2\right) + p\left(q_2, s_{p_2}\right) \tag{4}$$

Their method relies on the phenomenon of polarization transfer from the longitudinally polarized initial electron to the final proton and requires measurement of the spin-dependent cross section. This method is called by the polarization transfer (PT) method. In paper [2] was shown that the ratio of the degrees of longitudinal (P_l) and transverse (P_t) polarizations of the scattered proton has the form

$$\frac{P_l}{P_t} = -\frac{G_M}{G_E}\frac{E_{e1} + E_{e2}}{2M}\tan\frac{\theta_e}{2}\,. \tag{5}$$

[2] A. Akhiezer, M. Rekalo, Fiz.Elem.Chast.Atom.Yadra **4**, 662 (1973).

Discrepancy between RT and PT JLab-experiments



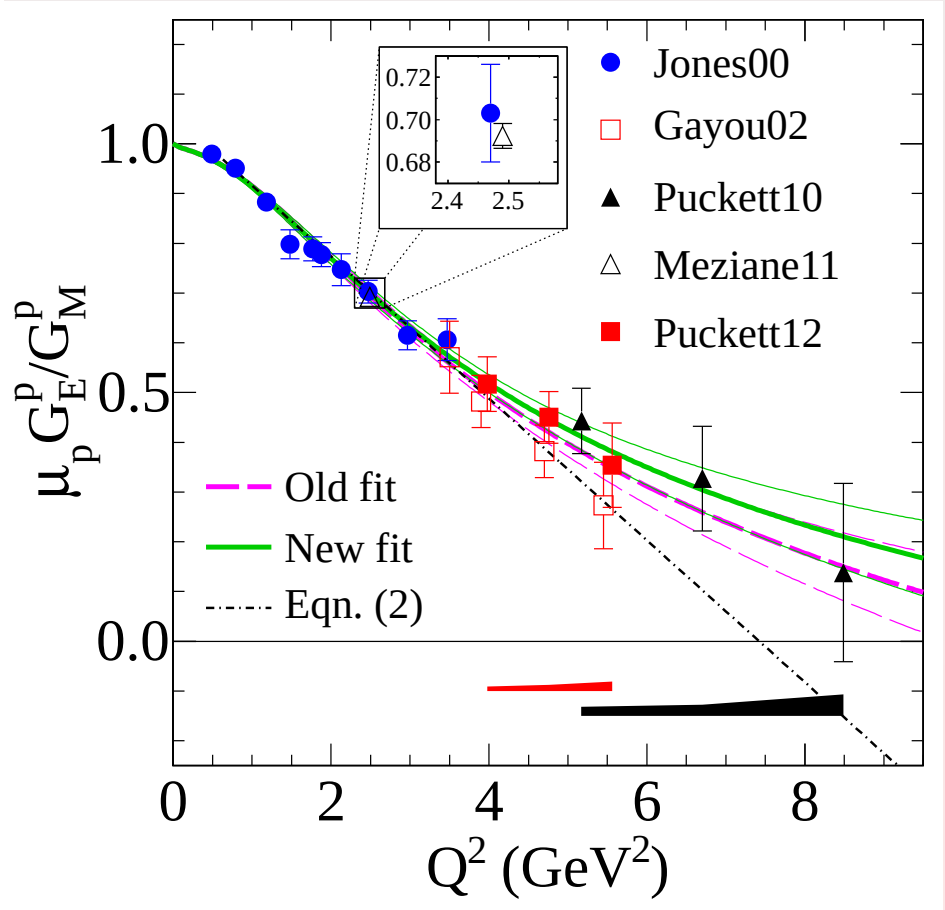


Рис. : **1.** The world data on the proton form factor ratio, $\mu_p G_E/G_M$. left panel: black symbols – TR (without TPE correction), color symbols – method AR.

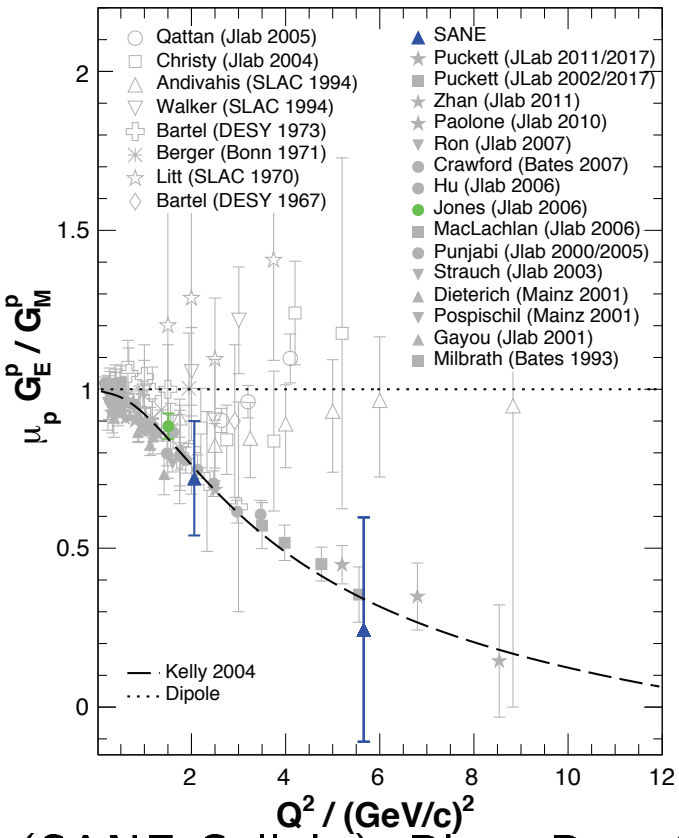
$$R \equiv \mu_p G_E/G_M \approx 1 - 0.13\,(Q^2 - 0.04) \approx 1 - \frac{1}{8}\,Q^2\ . \qquad (6)$$

$\mu_p G_E/G_M$ from double spin asymmetry (3th Method)

In the experiment [2], the ratio of R were extracting by the method [3] from the results of measurements of double spin asymmetry in the process

$$e\left(p_1, s_{e_1}^{\parallel}\right) + p\left(q_1, s_{p_1}^{\perp}\right) \to e\left(p_2\right) + p\left(q_2\right)$$

E_1=4.725 (5.895) GeV, Q^2=2.06 (5.66) GeV^2, $P_t = (70 \pm 5)$%, $P_e = (73 \pm 1.5)$ %.



[2] A. Liyanage et. al. (SANE Collab.), Phys. Rev. C **101**, 035206 (2020).
[3] T.W. Donnelly and A.S. Raskin, Annals of Physics **169**, 247 (1986).

Feynman diagrams for the $ep \to ep$ process

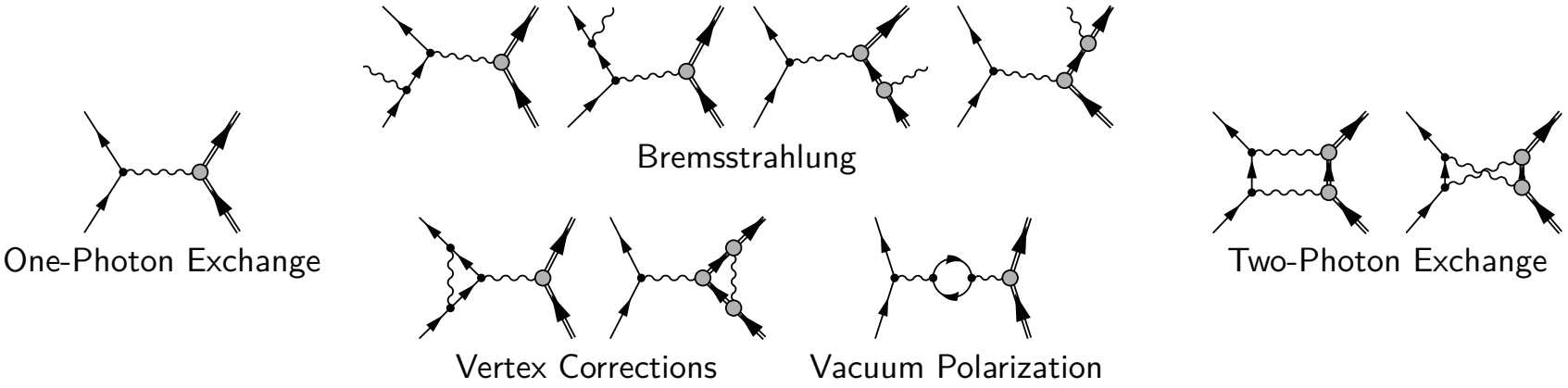


Рис.: **1.** Feynman diagrams for the $ep \to ep$ process:
(a) corresponds to the one-photon exchange or first Born approximation.
(b)–(e) show the first-order bremsstrahlung process $\ell^{\pm} p \to \ell^{\pm} p \gamma$ in the cases when the photon is emitted by the initial-state lepton (b),
final-state lepton (c), initial-state proton (d), or final-state proton (e).
(f)–(j) represent the processes contributing to the virtual-photon corrections: the vacuum polarization correction (f), the lepton (g) and proton (h) vertex corrections, and the TPE corrections (i), (j).

Present status of the question

In order to resolve this contradiction, it was assumed that the discrepancy in question may be caused by disregarding, in the respective analysis, the contribution of two-photon exchange [1,2]:

[1] P. Guichon, M. Vanderhaeghen, PPL. **91**, 142303 (2003),
[2] P. Blunden, W. Melnitchouk, J. A. Tjon, PRL. **91**, 142304 (2003).

At the present time, three experiments aimed at studying the contribution of TPE are known:
1) experiment at the VEPP-3 storage ring in Novosibirsk,
2) the EG5 CLAS experiment at JLab,
3) the OLYMPUS experiment at the DORIS accelerator at DESY.

[exp1] I. A. Rachek, *et al.*, Phys. Rev. Lett. 114 (2015) 062005.
[exp2] D. Adikaram *et al.*, Phys. Rev. Lett., 114 (2015) 062003.
[exp3] B.S. Henderson *et al.* Phys. Rev. Lett. 118 (2017) 092501.

Ranalysis of three experiments taking into account TPE [1]

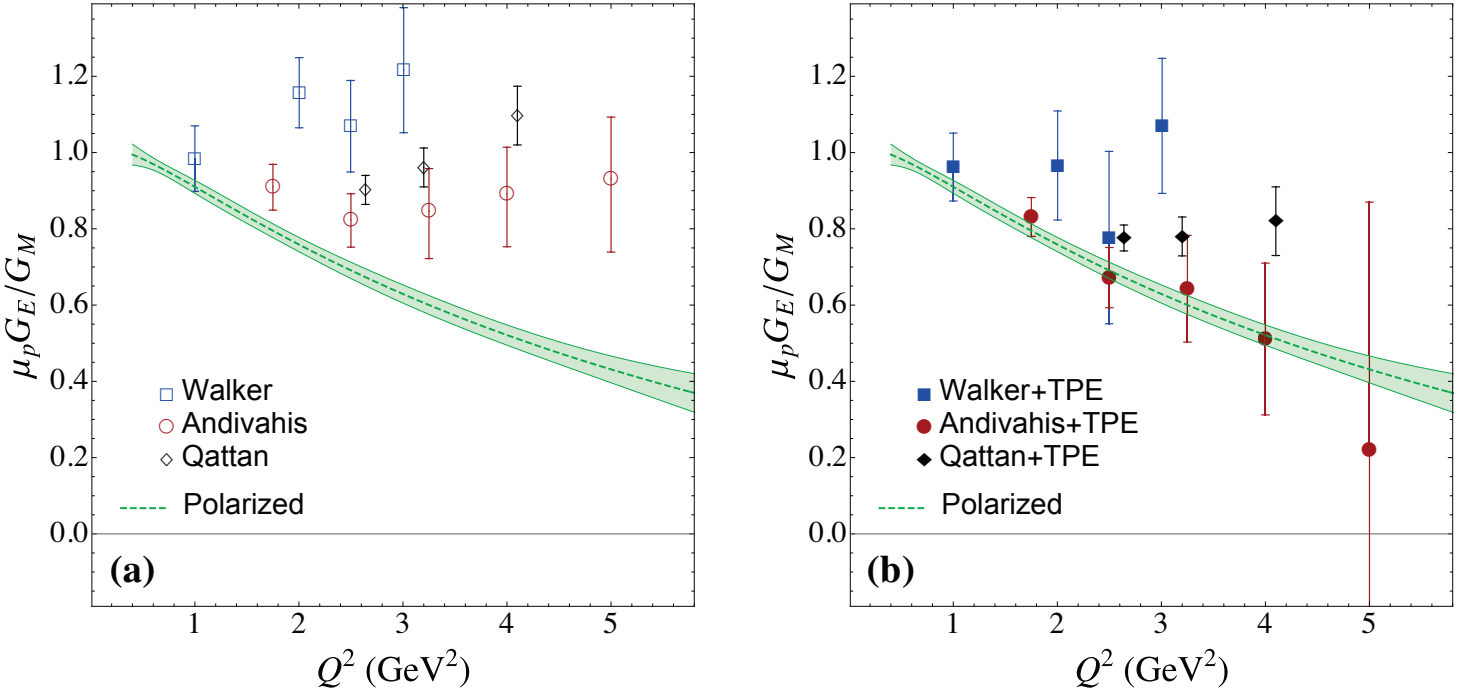


Рис.: **3. (a)** $\mu_p\, G_E/G_M$, versus Q^2, extracted using RT and PT separation in experiments Walker1994, Andivahis1994, Qattan2005. **(b)** The ratio $\mu_p\, G_E/G_M$ extracted from a reanalysis of the RT data using improved standard RCs together with the TPE effects from [1].

[1] J. Ahmed, P. Blunden, W. Melnitchouk, PRC **102**, 045205 (2020).
[2] M. Christy et al. PRL **128**, 102002 (2022) ($Q^2 \approx 15, 76$ GeV^2).
[3] M. V. Galynskii, Phys. Part. Nucl. Lett. **19**, 26 (2022).

New Rosenbluth separations [1]

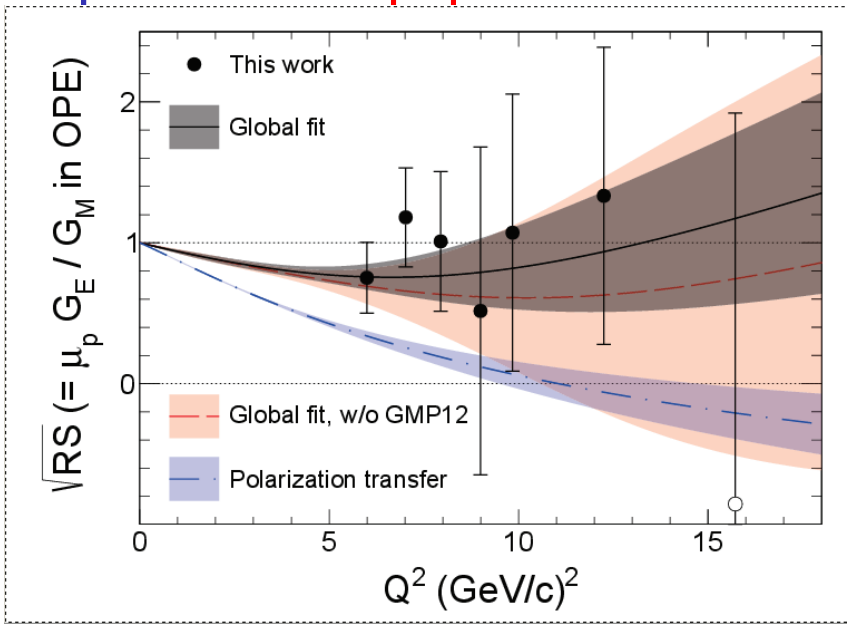


Рис.: Direct Rosenbluth separation in a new Q^2 regime with energy beam from 2.2 to 11 GeV and Q^2 from 1.577 to 15.76 GeV^2.

Таблица: Values Δ_σ at $R = R_d$ for E_1 (GeV) and Q^2 (GeV^2) in [1].
$\Delta_\sigma = \sigma^{\uparrow\uparrow}/(\sigma^{\uparrow\uparrow} + \sigma^{\downarrow\uparrow})$, $\Delta_\sigma > \Delta_0$ not gold; LHRS(1.6%); RHRS (*2.0 %).

$E_1 \backslash Q^2$	1.577	1.858	4.543	5.947	6.993	7.992	9.002	9.053	9.807	11.19	12.07	12.57	15.76
10.587													**0.009**
10.587	0.221	0.193	0.086	0.064	0.053	0.045	0.038	0.038	0.033	0.026	0.022	0.020	
8.518							0.031		0.026	***0.018**	***0.013**	***0.011**	
6.427			0.076	0.051	0.037	0.026		***0.016**					
2.222	0.168	*0.130											

[1] M. Christy et al. PRL **128**, 102002 (2022).

Double spin asymmetries in the elastic processes $e\vec{p} \to e\vec{p}$, $e\vec{p} \to \vec{e}p$, $\vec{e}\vec{p} \to ep$ (a case of parallel spins [1])

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2) \quad [2-7] \tag{7}$$

$$e(p_1) + p(q_1, s_{p_1}) \to e(p_2, s_{e_2}) + p(q_2) \tag{8}$$

$$e(p_1, s_{e_1}) + p(q_1, s_{p_1}) \to e(p_2) + p(q_2). \tag{9}$$

$$\boldsymbol{a} = \boldsymbol{q}_2/q_{20} - \boldsymbol{q}_1/q_{10} \quad [1]. \tag{10}$$

In the frame, where $q_1 = (m, \mathbf{0})$, $q_2 = (q_{20}, \boldsymbol{q}_2)$

$$\boldsymbol{c} = \boldsymbol{c}_{p_1} = \boldsymbol{c}_{p_2} = \boldsymbol{n}_{p_2} = \boldsymbol{q_2}/|\boldsymbol{q}_2|\,. \tag{11}$$

[1] F.I. Fedorov, Theor. Math. Phys. **2**, 248 (1970).
[2] M.Galynskii, E.Kuraev, Yu.Bystritskiy, JETP Lett. **88**, 481 (2008).
[3] M.V. Galynskii, JETP Lett. **109**, 1 (2019).
[4] M.V. Galynskii and R.E. Gerasimov, JETP Lett. **110**, 646 (2019).
[5] M.V. Galynskii, JETP Lett. **113**, 555 (2021)
[6] M.V. Galynskii, Phys. Part. Nucl. Lett. **19**, 26 (2022).
[7] M.V. Galynskii, JETP Lett. **116**, 420 (2022).

Helicity spin bases

The spin 4-vector $s = (s_0, \boldsymbol{s})$ of the fermion with 4-momentum p ($p^2 = m^2$) satisfying the conditions $sp = 0$ and $s^2 = -1$, is given by

$$s = (s_0, \boldsymbol{s}), \quad s_0 = \frac{\boldsymbol{c}\,\boldsymbol{p}}{m}, \quad \boldsymbol{s} = \boldsymbol{c} + \frac{(\boldsymbol{c}\,\boldsymbol{p})\,\boldsymbol{p}}{m(p_0 + m)}\ , \tag{12}$$

where $\boldsymbol{c}$ ($\boldsymbol{c}^2 = 1$) is the axis of spin quantization. If the 4-vector s is known, then the spin quantization axis $\boldsymbol{c}$ is given by

$$\boldsymbol{c} = \boldsymbol{s} - \frac{s_0}{p_0 + m}\,\boldsymbol{p}, \tag{13}$$

At present, the most popular in high-energy physics is the helicity basis [1], in which the spin quantization axis is directed along the momentum of the particle ($\boldsymbol{c} = \boldsymbol{n} = \boldsymbol{p}/|\boldsymbol{p}|$), while the spin 4-vector (12) defined as

$$s = (s_0, \boldsymbol{s}) = (|\boldsymbol{v}|, v_0\,\boldsymbol{n}), \tag{14}$$

where $v = (v_0, \boldsymbol{v})$, $v = p/m$, $v^2 = 1$.

[1] M. Jacob and G. Wick, Ann. Phys. **7**, 404 (1959).

Diagonal spin bases (for the $e\vec{p} \to e\vec{p}$ process)

For the process under consideration

$$e(p_1) + p(q_1, s_{p_1}) \to e(p_2) + p(q_2, s_{p_2}), \tag{15}$$

where p_1, q_1 (p_2, q_2) – the 4-momenta of the initial (final) electrons and protons with masses m_0 and m, it is possible to project the spins of the initial proton and the final electron in one common direction given by [1]

$$\boldsymbol{a} = \boldsymbol{q}_2/q_{20} - \boldsymbol{q}_1/q_{10}. \tag{16}$$

The geometric image of (16) is the diagonal of the parallelogram, it is natural to call it the diagonal spin basis (DSB). In DSB s_{p_1} and s_{p_2} are

$$s_{p_1} = \frac{m^2 q_2 - (q_1 q_2)\, q_1}{m\sqrt{(q_1 q_2)^2 - m^4}}\,, \quad s_{p_2} = \frac{(q_1 q_2)\, q_2 - m^2 q_1}{m\sqrt{(q_1 q_2)^2 - m^4}}\,. \tag{17}$$

In the frame, where $q_1 = (m, \mathbf{0})$, $q_2 = (q_{20}, \boldsymbol{q}_2)$ the spin 4-vectors (17) reduces to

$$s_{p_1} = (0, \boldsymbol{n}_2)\,,\ s_{p_2} = (|\boldsymbol{v}_2|, v_{20}\, \boldsymbol{n_2})\,,\ \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|\,, \tag{18}$$

where $v_2 = (v_{20}, \boldsymbol{v_2}) = q_2/m$.

$$\boldsymbol{c} = \boldsymbol{c}_{p_1} = \boldsymbol{c}_{p_2} = \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|. \tag{19}$$

[1] F.I. Fedorov, Theor. Math. Phys. **2**, 248 (1970).

Diagonal spin bases (for the $e\vec{p} \to \vec{e}p$ process)

For the process under consideration

$$e(p_1) + p\,(q_1, s_{p_1}) \to e(p_2, s_{e_2}) + p\,(q_2), \tag{20}$$

where p_1, q_1 (p_2, q_2) – the 4-momenta of the initial (final) electrons and protons with masses m_0 and m, it is possible to project the spins of the initial proton and the final electron in one common direction given by [1]

$$\boldsymbol{a} = \boldsymbol{p}_2/p_{20} - \boldsymbol{q}_1/q_{10}. \tag{21}$$

The geometric image of (21) is the diagonal of the parallelogram, it is natural to call it the diagonal spin basis (DSB). In DSB s_{p_1} and s_{e_2} are

$$s_{p_1} = \frac{m^2 p_2 - (q_1 p_2) q_1}{m\sqrt{(q_1 p_2)^2 - m^2 m_0^2}},\ s_{e_2} = \frac{(q_1 p_2) p_2 - m_0^2 q_1}{m_0\sqrt{(q_1 p_2)^2 - m^2 m_0^2}}. \tag{22}$$

In the frame, where $q_1 = (m, \mathbf{0})$, the spin 4-vectors (22) reduces to

$$s_{p_1} = (0, \boldsymbol{n_2})\,,\ s_{e_2} = (|\boldsymbol{v_2}|, v_{20}\,\boldsymbol{n_2})\,, \tag{23}$$

where $\boldsymbol{n}_2 = \boldsymbol{p_2}/|\boldsymbol{p}_2|$, $v_2 = (v_{20}, \boldsymbol{v_2}) = p_2/m_0$.

$$\boldsymbol{a} = \boldsymbol{c} = \boldsymbol{c}_{p_1} = \boldsymbol{c}_{e_2} = \boldsymbol{n}_2 = \boldsymbol{p_2}/|\boldsymbol{p}_2|\,. \tag{24}$$

[1] F.I. Fedorov, Theor. Math. Phys. **2**, 248 (1970).

Diagonal spin bases (for the $\vec{e}\vec{p} \to ep$ process)

For the process under consideration

$$e(p_1, s_{e_1}) + p(q_1, s_{p_1}) \to e(p_2) + p(q_2) \tag{25}$$

the common axis of spin quantization given by [1]

$$\boldsymbol{a} = \boldsymbol{p}_1/p_{10} - \boldsymbol{q}_1/q_{10}. \tag{26}$$

The geometric image of (26) is the diagonal of the parallelogram, it is natural to call it the diagonal spin basis (DSB). In DSB s_{p_1} and s_{e_1} are

$$s_{p_1} = \frac{m^2 p_1 - (q_1 p_1)\, q_1}{m\sqrt{(q_1 p_1)^2 - m^2 m_0^2}}\,, \quad s_{e_1} = \frac{(q_1 p_1)\, p_1 - m_0^2\, q_1}{m_0\sqrt{(q_1 p_1)^2 - m^2 m_0^2}}\,. \tag{27}$$

In the frame, where $q_1 = (m, \mathbf{0})$, the spin 4-vectors (27) reduces to

$$s_{p_1} = (0, \boldsymbol{n}_{e_1})\,,\ s_{e_1} = (|\boldsymbol{v}_{\boldsymbol{e_1}}|, v_{e_{10}}\, \boldsymbol{n}_{\boldsymbol{e_1}})\,,\ \boldsymbol{n}_{e_1} = \boldsymbol{p_1}/|\boldsymbol{p}_1|\,. \tag{28}$$

$$\boldsymbol{c} = \boldsymbol{c}_{p_1} = \boldsymbol{c}_{e_1} = \boldsymbol{n}_{e_1} = \boldsymbol{p_1}/|\boldsymbol{p}_1|\,. \tag{29}$$

[1] F.I. Fedorov, Theor. Math. Phys. **2**, 248 (1970).

Ultrarelativistic limit

In the ultrarelativistic limit, when the electron mass can be neglected (i.e. at $p_{10}, p_{20} \gg m_0$), the spin 4-vectors (22), (27) reduces to

$$s_{p_1} = \frac{m^2 p_2 - (q_1 p_2)\, q_1}{m\,(q_1 p_2)}, \qquad s_{e_2} = \frac{p_2}{m_0}\,. \tag{30}$$

$$s_{p_1} = \frac{m^2 p_1 - (q_1 p_1)\, q_1}{m\,(q_1 p_2)}, \quad s_{e_1} = \frac{p_1}{m_0}\,. \tag{31}$$

Q^2-dependence θ_e and θ_p by energies in experiment [1]

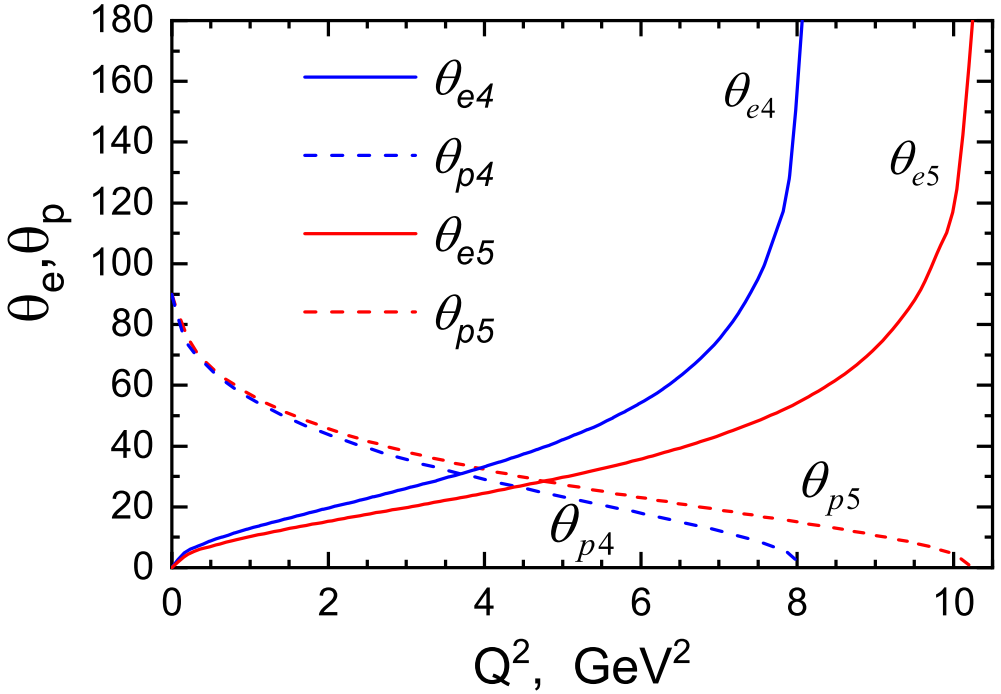


Рис.: **4.** Electron scattering angle θ_e and proton scattering angle θ_p versus Q^2 at the electron beam energies used in the experiment [1]. The θ_{e4}, θ_{p4}, and θ_{e5}, θ_{p5} lines are plotted for $E_1 = 4.725$ and 5.895 GeV.

Таблица: **1.** Electron scattering angle θ_e and proton scattering angle θ_p

E_1 (GeV)	Q^2 (GeV^2)	$\theta_e\,(rad)$	$\theta_p\,(rad)$	Q^2_{max} (GeV)^2
5.895	2.06	0.27	0.79	10.247
5.895	5.66	0.59	0.43	10.247
4.725	2.06	0.35	0.76	8.066
4.725	5.66	0.86	0.35	8.066

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).

Differential cross section of the $e\vec{p} \to e\vec{p}$ process

$$e(p_1) + p(q_1, s_{p_1}) \to e(p_2) + p(q_2, s_{p_2}),$$

$$\frac{d\sigma_{\delta_1,\delta_2}}{dt} = \frac{\pi\alpha^2}{\lambda_s(1+\tau_p)} \left(\omega_+ G_E^2 Y_1 + \omega_- \tau_p G_M^2 Y_2\right) \frac{1}{q^4}. \tag{32}$$

$$\begin{aligned} Y_1 &= (p_+ q_+)^2 + q_+^2 q_-^2 \,, && (33)\\ Y_2 &= (p_+ q_+)^2 - q_+^2 (q_-^2 + 4m_0^2) \,, && (34)\\ \lambda_s &= 4((p_1 q_1)^2 - m_0^2\, m^2) = \lambda(s, m_0^2, m^2), \\ \omega_+ &= (1 + \lambda_{p_1}\lambda_{p_2})/2,\ \omega_- = (1 - \lambda_{p_1}\lambda_{p_2})/2, \end{aligned}$$

λ_s is the Källén function, $p_+ = p_1 + p_2$, $q_\pm = q_2 \pm q_1$, $q_- = q, t = q^2$.
Longitudinal polarization degree of the final proton

$$P_r = P_t \frac{G_E^2 Y_1 - \tau_p G_M^2 Y_2}{G_E^2 Y_1 + \tau_p G_M^2 Y_2} = P_t \frac{R^2 - \tau_p \mu_p^2 (Y_2/Y_1)}{R^2 + \tau_p \mu_p^2 (Y_2/Y_1)}. \tag{35}$$

$$P_r = P_t \frac{(R_\sigma - 1)}{(R_\sigma + 1)}, R_\sigma = \frac{\sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow}} = \frac{Y_1 G_E^2}{\tau_p Y_2 G_M^2} = \frac{Y_1}{Y_2} \frac{R^2}{\tau_p\, \mu_p^2}. \tag{36}$$

$$R^2 = \mu_p^2\, \tau_p \frac{Y_2}{Y_1} \frac{1 + R_p}{1 - R_p},\ R_p = \frac{P_r}{P_t}\,, P_t = \lambda_{p_1},\ P_r = \lambda_{p_2}^f \tag{37}$$

Angular dependence of the polarization transfer

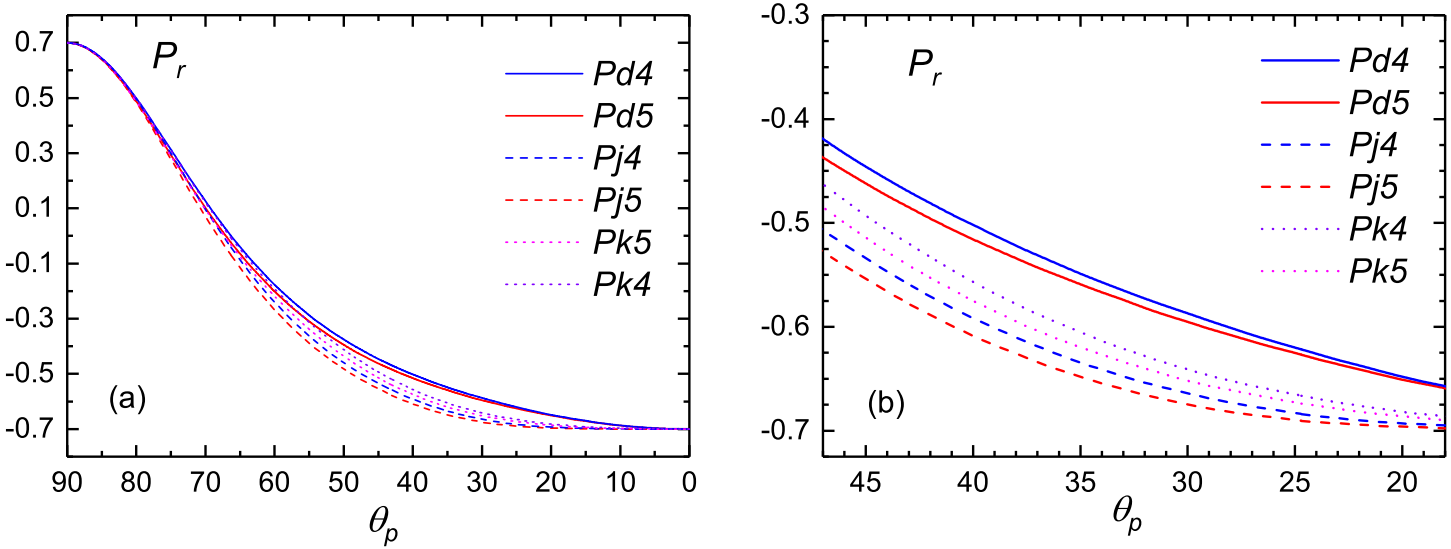


Рис.: (a) Dependence of P_r (36) on the proton scattering angle θ_p for the E_1 and P_t values used in the experiment [1] in the entire range of variation in the angle $\theta_p \in (90^\circ, 0^\circ)$. (b) The same dependence in the range $\theta_p \in (47^\circ, 18^\circ)$, in which $2.06\,\mathrm{GeV}^2 \leqslant \mathrm{Q}^2 \leqslant 5.66$ GeV^2. The Pd, Pj, and Pk lines correspond to the dipole dependence (38), the Qattan parameterization (58) from [2], and the Kelly parameterization from [3].

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I. A. Qattan [et al.], Phys. Rev. C **91**, 065203 (2015).
[3] J. J. Kelly, Phys. Rev. C **70**, 068202 (2004).

Q^2-dependence of the polarization transfer

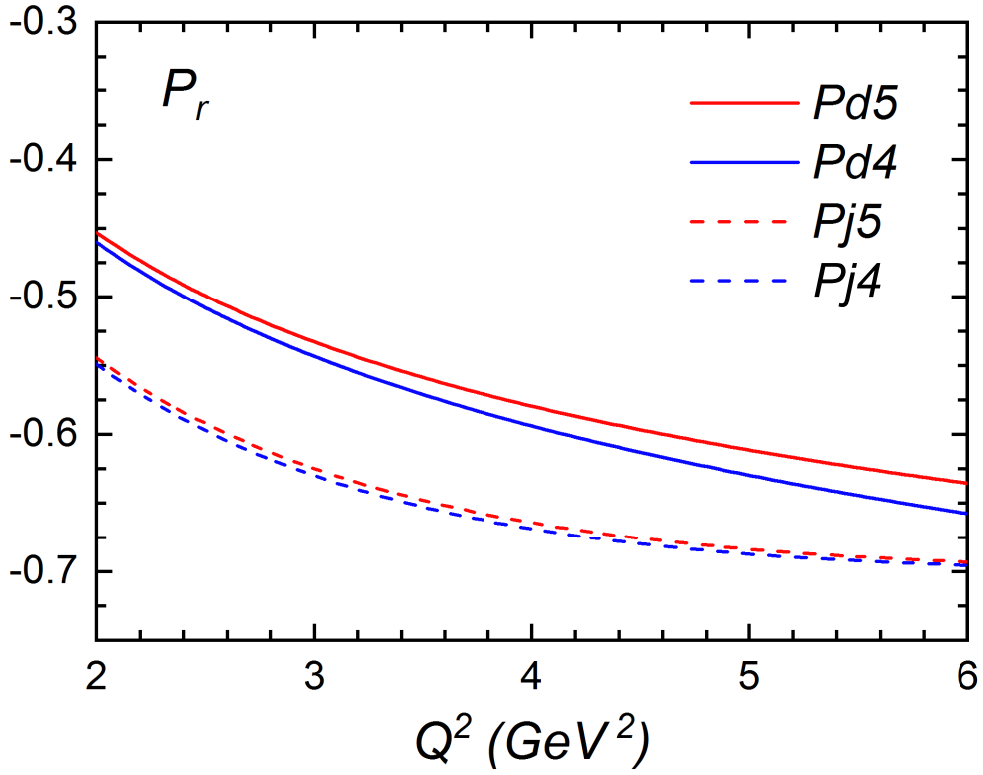


Рис.: Q^2-dependence of the polarization transfer to the recoil proton in the case $R = R_d$ and $R = R_j$ from [2] for $E_1 = 4.725$ (5.895) GeV and $P_t = 0.70$.

$$R_d = 1, \tag{38}$$

$$R_j = (1 + 0.1430\, Q^2 - 0.0086\, Q^4 + 0.0072\, Q^6)^{-1}, \tag{39}$$

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I. A. Qattan [et al.], Phys. Rev. C **91**, 065203 (2015)
[3] J. J. Kelly, Phys. Rev. C **70**, 068202 (2004).

Relative differences of the polarization transfer

Table 4 contains the polarizations $Pj5$, $Pd5$, $Pj4$, $Pd4$, $Pk5$, and $Pk4$, and their relative differences (in percent) Δ_{dj} and Δ_{dk}

$$\Delta_{dj} = \left|\frac{\mathrm{Pd} - \mathrm{Pj}}{\mathrm{Pd}}\right|, \ \Delta_{jk} = \left|\frac{\mathrm{Pj} - \mathrm{Pk}}{\mathrm{Pj}}\right|$$

Таблица: Degree of longitudinal polarization of the recoil proton P_r (36) at electron beam energies $E_1 = 4.725$ (5.895) GeV and values $Q^2 = 2.06$ (5.66) GeV^2. The values in the columns for Pd, Pj, and Pk correspond to the dipole dependence (38) $R = R_d$, the Qattan parameterization R_j (58) and the Kelly parameterization $R = R_k$.

E_1 (GeV)	Q^2 (GeV^2)	θ_e (deg)	θ_p (deg)	Pd	Pj	Pk	Δ_{dj}, %	Δ_{dk}, %
5.895	2.06	15.51	45.23	–0.460	–0.552	–0.511	16.6	9.98
5.895	5.66	33.57	24.48	–0.628	–0.691	–0.675	9.1	6.96
4.725	2.06	19.97	43.27	–0.467	–0.556	–0.517	16.1	9.67
4.725	5.66	49.50	19.77	–0.649	–0.693	–0.682	6.4	4.84

It follows from Table 4 that, at $Q^2 = 2.06$ GeV^2, the relative difference between $Pj5$ and $Pd5$ is 16.6 %; the difference between $Pj4$ and $Pd4$ is approximately the same: 16.1 %. At $Q^2 = 5.66$ GeV^2 this difference decreases to 9.1 and 6.4 %, respectively.

Differential cross section of the $e\vec{p} \to \vec{e}p$ process

$$\frac{d\sigma_{e\vec{p}\to\vec{e}p}}{dt} = \frac{\pi\alpha^2}{2\lambda_s(1+\tau_p)}\frac{|T|^2}{t^2}, \tag{40}$$

$$|T|^2 = I_0 + \lambda_{p_1}\lambda_{e_2} I_1, \tag{41}$$

$$I_0 = G_E^2 Y_1 + \tau_p G_M^2 Y_2, \tag{42}$$

$$I_1 = \tau_p(G_E G_M Y_3 + G_M^2 Y_4), \tag{43}$$

where $t = q^2$, $\lambda_s = 4((p_1 q_1)^2 - m_0^2 m^2)$, λ_{p_1} (λ_{e_2}) – the degree of polarization of the initial proton (of the final electron).
Here the functions Y_i ($i = 1, \ldots 4$) defined as

$$Y_1 = (p_+ q_+)^2 + q_+^2 q_-^2, \tag{44}$$

$$Y_2 = (p_+ q_+)^2 - q_+^2(q_-^2 + 4m_0^2), \tag{45}$$

$$-Y_3 = 2\,\kappa\, m^2\,((p_+ q_+)^2 + q_+^2(q_-^2 - 4\,m_0^2))\,z^2, \tag{46}$$

$$Y_4 = 2\,(m^2 p_+ q_+ - \kappa q_+^2)(\kappa p_+ q_+ - m_0^2\, q_+^2)z^2, \tag{47}$$

$$z = (\kappa^2 - m^2 m_0^2)^{-1/2}\,,\ \kappa = q_1 p_2.$$

Polarization of the final electron in the $e\vec{p} \to \vec{e}p$ process

Expression (41) for $|T|^2$ can be written as

$$|T|^2 = I_0 + \lambda_{p_1}\lambda_{e_2} I_1 = I_0\,(1 + \lambda_{e_2}\lambda^f_{e_2}). \tag{48}$$

$\lambda^f_{e_2}$ is the longitudinal polarization degree of the final electron.

$$\lambda^f_{e_2} = \lambda_{p_1}\frac{I_1}{I_0} = \lambda_{p_1}\frac{\tau_p(G_E G_M Y_3 + G_M^2 Y_4)}{G_E^2 Y_1 + \tau_p G_M^2 Y_2}. \tag{49}$$

Dividing the numerator and denominator in the last expression by $Y_1 G_M^2$

$$\lambda^f_{e_2} = \lambda_{p_1}\,\frac{\mu_p\tau_p\,((Y_3/Y_1)R + \mu_p(Y_4/Y_1))}{R^2 + \mu_p^2\,\tau_p\,(Y_2/Y_1)}. \tag{50}$$

Inverting relation (50), we obtain a quadratic equation with respect to R:

$$\alpha_0 R^2 - \alpha_1 R + \alpha_0\alpha_3 - \alpha_2 = 0 \tag{51}$$

$$\begin{aligned} &\alpha_0 = \lambda^f_{e_2}/\lambda_{p_1},\ \alpha_1 = \tau_p\,\mu_p\,Y_3/Y_1, \\ &\alpha_2 = \tau_p\,\mu_p^2\,Y_4/Y_1,\ \alpha_3 = \tau_p\,\mu_p^2\,Y_2/Y_1. \end{aligned} \tag{52}$$

Solutions to equation (51) have the form:

$$R = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_0(\alpha_0\alpha_3 - \alpha_2)}}{2\alpha_0}. \tag{53}$$

Utrarelativistic limit in laboratory frame

In the ultrarelativistic limit, when the electron mass can be neglected, expressions (44)–(47) for Y_i $(i = 1, \ldots 4)$ in LF are given by

$$Y_1 = 8m^2(2E_1E_2 - mE_-), \tag{54}$$

$$Y_2 = 8m^2(E_1^2 + E_2^2 + mE_-), \tag{55}$$

$$Y_3 = -(2m/E_2)\,Y_1, \tag{56}$$

$$Y_4 = 8m^2E_+E_-(m - E_2)/E_2, \tag{57}$$

where $E_\pm = E_1 \pm E_2$. Below we use the notation $R \equiv \mu_p\,G_E/G_M$. The formulas (54)–(57) were used to numerically calculate the Q^2-dependence of the longitudinal polarization degree of the scattered electron $\lambda_{e_2}^f$ (50) at electron beam energies ($E_1 = 4.725$ and 5.895 GeV) and the polarization degree of the proton target ($P_t = \lambda_{p_1} = 0.70$) in experiment [1] as while conserving the scaling of the SFF in the case of a dipole dependence ($R = R_d = 1$), and in case of its violation. In the latter case, the parametrization $R = R_j$ from the paper [2] was used

$$R_j = (1 + 0.1430\,Q^2 - 0.0086\,Q^4 + 0.0072\,Q^6)^{-1}, \tag{58}$$

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I.A. Qattan, J. Arrington, A. Alsaad, Phys. Rev. C **91**, 065203 (2015).

Longitudinal polarization degree of the scattered electron

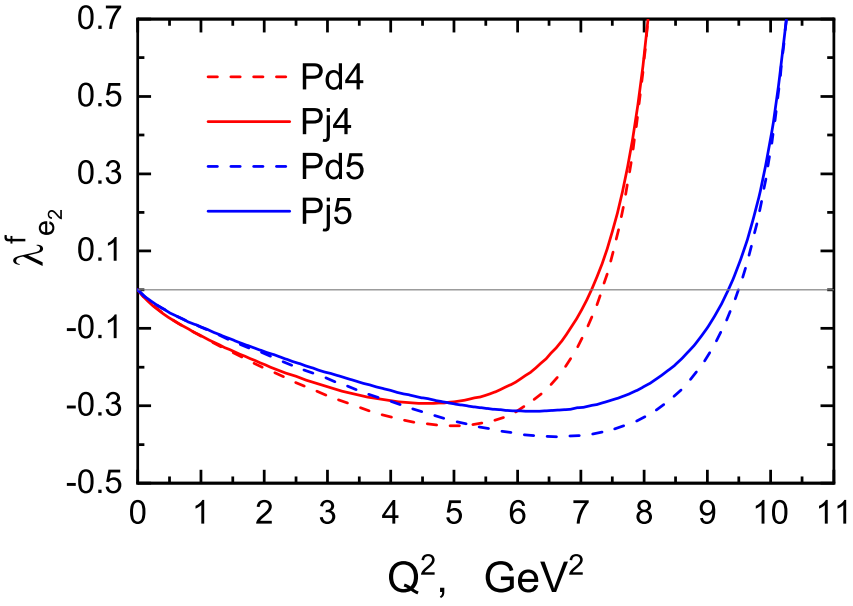


Рис.: Q^2-dependence of the longitudinal polarization degree of the scattered electron $\lambda^f_{e_2}$ (50) at electron beam energies in the experiment [1]. The lines $Pd4$, $Pd5$ (dashed) and $Pj4$, $Pj5$ (solid) correspond to the ratio $R = R_d$ in the case of dipole dependence and parametrization $R = R_j$ (58) from the paper [2]. The lines $Pd4$, $Pj4$ ($Pd5$, $Pj5$) correspond to the energies $E_1 = 4.725$ (5.895) GeV.

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I.A. Qattan, J. Arrington, A. Alsaad, Phys. Rev. C **91**, 065203 (2015).

Relative difference Δ_{dj} of the polarization effects at different parametrizations of the ratio $R \equiv \mu_p G_E/G_M$

The parametrizations of Qattan [1] (58) and Kelly [2] allow us to calculate the relative difference Δ_{dj} between the polarization effects

$$\Delta_{dj} = \left|\frac{\mathrm{Pd} - \mathrm{Pj}}{\mathrm{Pd}}\right|, \ \Delta_{jk} = \left|\frac{\mathrm{Pj} - \mathrm{Pk}}{\mathrm{Pj}}\right|, \tag{59}$$

where P_d, P_j and P_k are the polarizations calculated by formula (50) for $\lambda^f_{e_2}$ when using the corresponding parametrizations R_d, R_j and R_k.

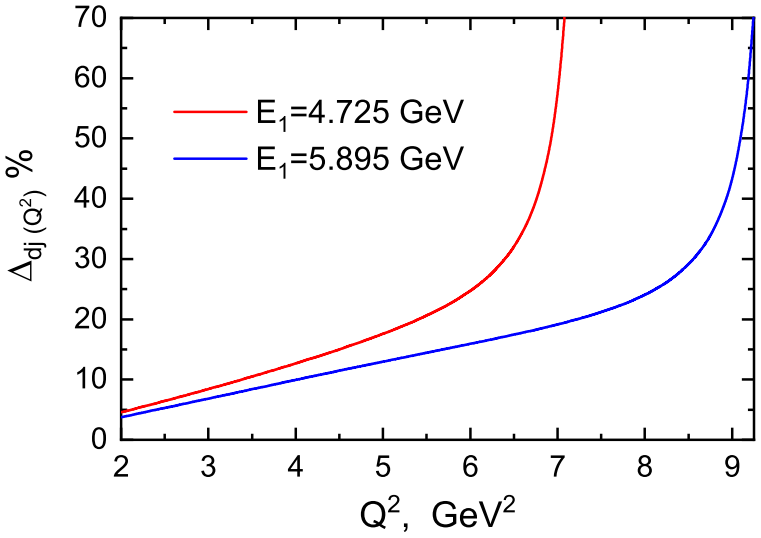


Рис.: Q^2-dependence of the relative difference Δ_{dj} (59) at electron beam energies $E_1 = 4.725$ GeV (red line) and $E_1 = 5.895$ GeV (blue line).

[1] I.A. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015).

[2] J.J. Kelly, Phys. Rev. C **70**, 068202 (2004).

Comparison of the transferred to electron polarization at different parametrizations of the ratio $R \equiv \mu_p G_E/G_M$

Таблица: The degree of longitudinal polarization of the scattered electron $\lambda^f_{e_2}$ (50) at electron beam energies $E_1 = 4.725$ and 5.895 GeV and two values $Q^2 = 2.06$ and 5.66 GeV^2 in the experiment [1]. The values in the columns for P_d, P_j, P_k correspond to the polarization transferred to the electron $\lambda^f_{e_2}$ (50) with dipole dependence, the parametrization of Qattan [2] (58) and Kelly [3].

E_1, GeV	Q^2, GeV^2	θ_e (°)	θ_p (°)	P_d	P_j	P_k	Δ_{dj}, %	Δ_{jk}, %
5.895	2.06	15.51	45.23	–0.170	–0.163	–0.163	4.1	0.0
5.895	5.66	33.57	24.48	–0.363	–0.309	–0.308	14.9	0.3
4.725	2.06	19.97	43.27	–0.207	–0.197	–0.197	4.8	0,0
4.725	5.66	49.50	19.77	–0.336	–0.263	–0.262	21.7	0.6

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I.A. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015).
[3] J.J. Kelly, Phys. Rev. C **70**, 068202 (2004).

Differential cross section of the $\vec{e}\vec{p} \to ep$ process

$$\frac{d\sigma_{\vec{e}\vec{p}\to ep}}{dt} = \frac{\pi\alpha^2}{\lambda_s(1+\tau_p)}\,\frac{|T|^2}{t^2}, \tag{60}$$

$$|T|^2 = I_0 + \lambda_{e_1}\lambda_{p_1} I_1, \tag{61}$$

$$I_0 = G_E^2 Y_1 + \tau_p\, G_M^2 Y_2, \tag{62}$$

$$I_1 = \tau_p(G_E G_M Y_3 + G_M^2 Y_4), \tag{63}$$

$$Y_1 = (p_+q_+)^2 + q_+^2 q_-^2, \tag{64}$$

$$Y_2 = (p_+q_+)^2 - q_+^2(q_-^2 + 4m_0^2), \tag{65}$$

$$-Y_3 = 2\kappa_2\, m^2\,((p_+q_+)^2 + q_+^2(q_-^2 - 4\,m_0^2\,))\,z_2^2, \tag{66}$$

$$Y_4 = 2(m^2\,p_+q_+ - \kappa_2\,q_+^2)(\kappa_2\,p_+q_+ - m_0^2\,q_+^2)\,z_2^2, \tag{67}$$

$$z_2 = (\kappa_2^2 - m^2 m_0^2)^{-1/2}\,,\ \kappa_2 = q_1 p_1.$$

where $t = q^2$, $\lambda_s = 4((p_1q_1)^2 - m_0^2\,m^2)$, λ_{p_1} (λ_{e_1}) – the degree of polarization of the initial proton (of the initial electron).

Double spin asymmetry in the $\vec{e}\vec{p} \to ep$ process

$$A = \frac{|T|^2(\lambda_{e_1} = -1) - |T|^2(\lambda_{e_1} = +1)}{|T|^2(\lambda_{e_1} = -1) + |T|^2(\lambda_{e_1} = +1)}. \tag{68}$$

$$A = -\lambda_{p_1} \frac{\tau_p \left(G_E G_M Y_3 + G_M^2 Y_4\right)}{G_E^2 Y_1 + \tau_p G_M^2 Y_2}. \tag{69}$$

Dividing the numerator and denominator in (69) by $Y_1 G_M^2$ and introducing the experimentally measured ratio $R \equiv \mu_p G_E / G_M$, we get

$$A = -\lambda_{p_1} \frac{\mu_p \tau_p \left((Y_3/Y_1)R + \mu_p (Y_4/Y_1)\right)}{R^2 + \mu_p^2 \tau_p \left(Y_2/Y_1\right)}. \tag{70}$$

In the ultrarelativistic limit, when the electron mass can be neglected, in laboratory frame expressions for Y_i $(i = 1, \ldots 4)$ are given by

$$Y_1 = 8m^2(2E_1E_2 - mE_-), \tag{71}$$

$$Y_2 = 8m^2(E_1^2 + E_2^2 + mE_-), \tag{72}$$

$$-Y_3 = (2m/E_1)\, Y_1, \tag{73}$$

$$-Y_4 = 8m^2 E_+ E_- (m + E_1)/E_1. \tag{74}$$

Q^2-dependence of the polarization asymmetry A

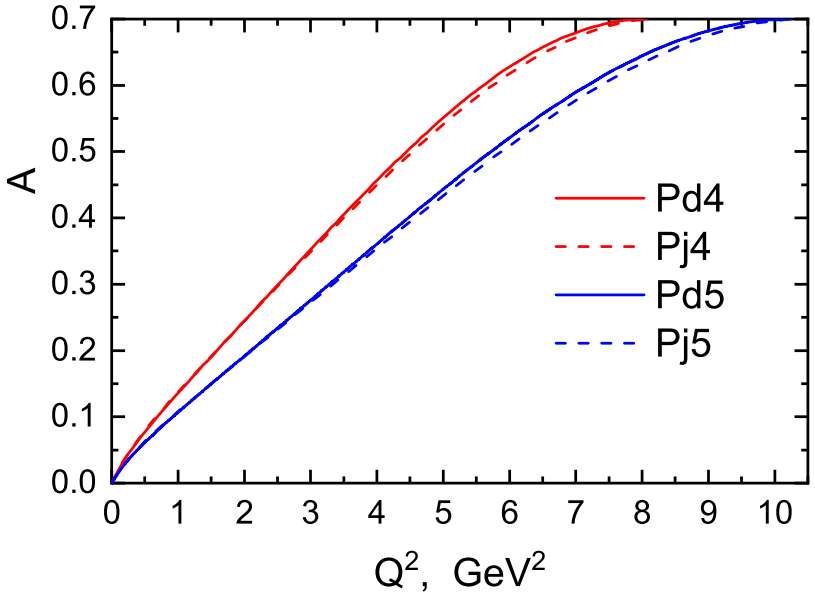


Рис.: Q^2-dependence of the polarization asymmetry A (70) in the $\vec{e}\vec{p} \to ep$ process at electron beam energies in the experiment [1]. The lines $Pd4$, $Pd5$ (dashed) and $Pj4$, $Pj5$ (solid) correspond to the ratio $R = R_d$ in the case of dipole dependence and parametrization $R = R_j$ (58) from [2]. The lines $Pd4$, $Pj4$ ($Pd5$, $Pj5$) correspond to the energies $E_1 = 4.725$ (5.895) GeV.

In [3] the double spin asymmetry in the process $\vec{e}\vec{p} \to ep$ was measured for the first time at $E_1 = 6.47$ GeV, $\lambda_{e_1} = 0.51$, $\lambda_{p_1} = 0.34$, $\theta_e = 8.0°$, $Q^2 = 0.76$ GeV^2, $A(0.76) = 0.11$. By our calculations, $A(0.76) = 0.038$.

[1] A. Liyanage *et al.* (SANE Collaboration), PRC **101**, 035206 (2020).
[2] I.A. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015).
[3] M.J. Alguard *et al.* PRL **37**, 1261 (1976).

Conclusion (a)

We have considered a possible method for measuring the ratio $R \equiv \mu_p G_E/G_M$ based on the transfer of polarization from the initial proton to the final electron in the $e\vec{p} \to \vec{e}p$ process, in the case when their spins are parallel, i.e. when an electron is scattered in the direction of the spin quantization axis of the resting proton target. For this purpose, in the kinematics of the SANE collaboration experiment [1], using the parametrizations of Qattan [2] and Kelly [3], a numerical analysis was carried out of the dependence of the degree of polarization of the scattered electron on the square of the momentum transferred to the proton, as well as from the scattering angles of the electron and proton. As it turned out, the parametrizations of Qattan [2] and Kelly [3] give almost identical results in calculations. It is established that the difference in the degree of longitudinal polarization of the final electron in the case of conservation and violation of the SFF scaling can reach 70 %, which can be used to conduct a new type of polarization experiment to measure the ratio R.

[1] A. Liyanage *et al.* (SANE Collaboration), PRC. **101**, 035206 (2020).
[2] I.A. Qattan, J. Arrington, A. Alsaad, PRC **91**, 065203 (2015).
[3] J.J. Kelly, Phys. Rev. C **70**, 068202 (2004).

Conclusion (b)

Proceeding from the results of JLab's polarization experiments on measuring the ratio of the Sachs form factors in the $\vec{e}p \to e\vec{p}$ process, using the Kelly (2004) and Qattan (2015) parametrizations for this ratio, in the kinematics of SANE's experiment (2020) on measuring the double spin asymmetry in the $\vec{e}\vec{p} \to ep$ process, a numerical analysis of the dependence of the longitudinal polarization transfer to the proton in the $e\vec{p} \to e\vec{p}$ process on the square of the momentum transferred to the proton, as well as from the scattering angles of the electron and proton was performed for the case where an recoil proton is scattered in the direction of the spin quantization axis of the resting proton target. It has been found that the polarization transfer to the proton is fairly sensitive to the parametrization of the ratio of the Sachs form factors, which opens possibilities for a new measurement of this ratio in the $e\vec{p} \to e\vec{p}$ process. It follows from the calculations that the violation of the scaling of the Sachs form factors leads to a significant increase in the magnitude of the polarization transfer to the proton, $|P_r|$, as compared to the case of the dipole dependence; the $|P_r|$ value, obtained with the Kelly parametrization is between the results

THANK FOR YOUR ATTENTION

The matrix elements of the proton current in the reaction $ep \to ep$

The matrix elements of the proton current corresponding to the proton transitions without and with spin-flip calculated in the DSB have the form

$$(J_p^{\delta,\delta})_\mu = 2M\, G_E (b_0)_\mu \,, \tag{75}$$

$$(J_p^{-\delta,\delta})_\mu = -2M\, \delta\sqrt{\tau} G_M (b_\delta)_\mu \,, \tag{76}$$

In expressions (75), (76) we used an orthonormalized basis (tetrad) of four-vectors b_A $(A = 0, 1, 2, 3)$; that is,

$$b_0 = q_+/\sqrt{q_+^2}\,,\ b_3 = q_-/\sqrt{-q_-^2}\ \ ,$$
$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma,\ (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} q_1^\nu q_2^\kappa p_1^\sigma/\rho\,. \tag{77}$$

Here, $q_+ = q_2 + q_1$, $q_- = q = q_2 - q_1$, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), p_1 is the four-momentum of the initial electron, and ρ is determined from the normalization conditions $b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1$, where $b_{\pm\delta} = b_1 \pm i\delta b_2$, $b_\delta^* = b_{-\delta}$, and $b_\delta b_\delta^* = -2$, $\delta = \pm 1$.