Primary Black Holes in the Early Universe, Quantum-Gravitational Corrections and Inflationary Cosmology

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1 Introduction

It is clear that in cosmology, at least at very high energies, that is, in the Early Universe, quantum-gravitational effects must also be taken into account in any case, whether the inflationary scenario is realized or not. In this case, one can consider these small black holes as primordial black holes (**pbh**) , and investigate the question of how the latter can change the known cosmological parameters in the Early Universe.

2. Primordial Black Holes

The most common mechanism for the formation of **pbh** is the gravitational collapse of high-density matter, generated by cosmological perturbations arising, for example, in the process of inflation, (but not necessarily) in the Early Universe.

The mass **of pbh** $M(t_M)$, formed during the time t after the big bang

$$M(t_M) \approx \frac{c^3 t_M}{G} \approx 10^{15} \left(\frac{t}{10^{-23}} \sec\right) g. \qquad (1)$$

From (1) **pbh** have a wide range of masses, in particular for the Planck time $t_M \approx 10^{-43}$ s, **pbh** has a Planck mass $M(t_M) \approx 10^{-5} g$, quantum-gravitational effects will be significant.

It has been established that the study of the formation and evaporation **of pbh** is a powerful tool in the study of processes in the

Early Universe.

Quantum black holes **qbh** are currently understood as Schwarzschild black holes, $r = r_{qbh} \propto l_p$and mass $m = m_{qbh} \propto M_p$.

A natural question arises: how can **qbh be formed** ? The first and most obvious way for the formation of such objects is the evaporation of a large (classical) black hole due to the thermal radiation from this hole established by Hawking and the formation of a stable Planck remnant at the final stage of this evaporation.

However, due to formula (1), **qbh** can be formed in the Early Universe as **pbh** in the Planck time $\approx 10^{-43}$s. Regardless of the way **qbh is formed** , quantum gravitational effects are essential for them.

In all cases, one can find quantum-gravitational corrections for the main characteristics of black holes, which will be significant for **qbh .**

In particular, if the Generalized Uncertainty Principle (GUP) is valid in the transition to high (Planck) energies ***(Nouicer, PLB,2007)***:

$$(\boldsymbol{\delta X})(\boldsymbol{\delta P}) \geq \frac{\hbar}{2}\langle \mathbf{exp}\frac{\boldsymbol{\alpha}^2 \boldsymbol{l}_p^2}{\hbar^2}\boldsymbol{P}^2\rangle$$

In first order $(\boldsymbol{\delta X})(\boldsymbol{\delta P}) \geq \frac{\hbar}{2}\left(\mathbf{1} + \frac{\boldsymbol{\alpha}^2 \boldsymbol{l}_p^2}{\hbar^2}(\boldsymbol{\delta P})^2\right)$ (2*)

then there can exist a Planck black hole (which will be called "minimal" below) with a minimum mass $M_0$and radius of the event horizon r_{min}:

$$r_{min} = (\delta X)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}}m_p, \qquad (3)$$

where $\boldsymbol{\alpha}$ is the model-dependent order parameter 1, $\boldsymbol{e}$ is the base of natural logarithms and $\boldsymbol{r}_{min} \propto \boldsymbol{l}_p, \boldsymbol{M}_0 \propto \boldsymbol{m}_p$.

It is initially assumed that the original black hole is strictly larger than the "minimal" black hole (3)

$$r_M > r_{min}, M > M_0. \qquad (4)$$

Here $\hbar = c = k_{\mathrm{B}} = 1$, in which $\boldsymbol{l_p^2 = G, m_p{}^2 = 1/G}$

Semiclassical approximation , Hawking temperature, T_{H}

$$\boldsymbol{T_{\mathrm{H}} = \frac{1}{8\pi GM}} \qquad (5)$$

Within the framework of GUP, quantum-gravitational corrections, has the form:

$$T_{\mathrm{H},q} = \frac{1}{8\pi M l_p^2} \exp\left(-\frac{1}{2} W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right), \qquad (6)$$

$W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)\right)^2$ $--$value at the corresponding point of the Lambert function $W(u)$,

$$\boldsymbol{W(u)e^{W(u)} = u}. \qquad (7)$$

Lambert W-function briefly

$W(u)$ is the multifunction for complex variable $u = x + y\imath$. However for example for real $u = x, -\frac{1}{e} \leq u \leq 0,$, $W(u)$ is the single-valued continuous function with two branches $W_{0,}W_{-1}$ and this

is present picture .

It is clear that for a black hole of large size, that is, with large mass M and area of the event horizon A, the value of the deformation parameter $\frac{1}{e}\left(\frac{M_0}{M}\right)^2$ is vanishingly small and close to zero. In this case, the value $W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)$ is also close to $W(0)$. It is easy to see which $W(0) = 0$ is an explicit solution of Eq. (7). Then

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) \approx 1, \qquad (8)$$

However, for small black holes

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 1, \qquad (9)$$

The right side of (6) can be expanded into a series in terms of a small parameter $\frac{1}{e}(M_0/M)$ with the the leading first term:

$$\boldsymbol{T_{\mathrm{H},q} \simeq \frac{1}{8\pi M l_p^2}\left(1+\frac{1}{2e}\left(\frac{M_0}{M}\right)^2+\frac{5}{8e^2}\left(\frac{M_0}{M}\right)^4+\frac{49}{48e^3}\left(\frac{M_0}{M}\right)^6+\cdots\right)}, \quad (10)$$

Then, within the framework of GUP, $M \to M_q$:

$$\boldsymbol{T_{\mathrm{H},q} \simeq \frac{1}{8\pi M_q l_p^2}; M_q \doteq M\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)\right)^2\right)} \quad (11)$$

Since for Schwarzschild black holes,for $c = \hbar = 1$

$$r_M = 2MG, \qquad (12)$$

where $r = r_M$ is the radius of its event horizon, then formula (10) can be rewritten as

$$T_{\mathrm{H},q} \simeq \frac{1}{8\pi M l_p^2}\left(1+\frac{1}{2e}\left(\frac{r_{min}}{r_M}\right)^2+\frac{5}{8e^2}\left(\frac{r_{min}}{r_M}\right)^4+\frac{49}{48e^3}\left(\frac{r_{min}}{r_M}\right)^6+\cdots\right), \quad (13)$$

The the ”semiclassical” formula (12) has a ”quantum” analog:

$$r_{M_q} = 2M_q G. \qquad (14)$$

Obviously, formulas (11), (13) can be considered as a (quantum-gravitational) deformation with the deformation parameter of the $\left(\frac{M_0}{M}\right)^2$ of semiclassical temperature T_{H} and

$$\left(\frac{\boldsymbol{M_0}}{\boldsymbol{M}}\right)^{\mathbf{2}} = \boldsymbol{r}^{\mathbf{2}}_{\boldsymbol{min}}/\boldsymbol{R}^{\mathbf{2}}(\boldsymbol{A}) \doteq \boldsymbol{\alpha}_{\boldsymbol{R}(\boldsymbol{A})}, \qquad (15)$$

$$\boldsymbol{exp}\left(\pm\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{W}\left(-\frac{\mathbf{1}}{\boldsymbol{e}}\left(\frac{\boldsymbol{M_0}}{\boldsymbol{M}}\right)^{\mathbf{2}}\right)\right) = \boldsymbol{exp}\left(\pm\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{W}\left(-\frac{\mathbf{1}}{\boldsymbol{e}}\boldsymbol{\alpha}_{\boldsymbol{R}(\boldsymbol{A})}.\right)\right) \quad (16)$$

The notation $r_M = R(A), r_{M_q} = R(A_q)$, where $R(A), R(A_q)$are the radii of the event horizon of a black hole with masses $M$and, $M_q$respectively, and $A_q$is the area of the event horizon with the radius $R(A_q)$.

3 Primary Black Holes and "Quantum Shifts" (QS) for Cosmological Parameters in Inflation Models

The metric of a Schwarzschild black hole

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (18)$$

where M is the mass of this hole.

When studying the Early Universe for **PBHs** , (18) is replaced by the Schwarzschild-de Sitter (SdS) metric,

$$\boldsymbol{ds^2 = -f(\tilde{r})dt^2 + \frac{d\tilde{r}^2}{f(\tilde{r})} + \tilde{r}^2 d\Omega^2} \qquad (19)$$

where $\boldsymbol{f(\tilde{r}) = 1 - \frac{2GM}{\tilde{r}} - \frac{\Lambda\tilde{r}^2}{3} = f(\tilde{r}) = 1 - \frac{2GM}{\tilde{r}} - \frac{\tilde{r}^2}{L^2}}$,

$\boldsymbol{L = \sqrt{3/\Lambda}}$**,** $\boldsymbol{M}$ is the black hole mass , $\tilde{r}$ is a small quantity, and Λ is the cosmological term.

Two horizons corresponding to two different zeros $f(\tilde{r})$: the black hole's event horizon and the cosmological horizon. In particular, this will be the case when the value $\boldsymbol{M}$ is small In this case, the radius of the bh event horizon with metric (19) will have the form

$$r_{\mathrm{H}} \simeq 2GM\left[1+\left(\frac{r_M}{L}\right)^2\right], \qquad (20)$$

where $\boldsymbol{r_M}$ from (12) and $\boldsymbol{L} \gg \boldsymbol{r_M}$, the condition is satisfied with high accuracy $\boldsymbol{r_{\mathrm{H}}} = \boldsymbol{r_M}$, for usial Schwarzschild BH.

In the general case, in particular in inflationary cosmology, metric (19) is written in terms of conformal time η:

$$\boldsymbol{ds^2 = a^2(\eta)\{-d\eta^2 + \left(1+\frac{\mu^3\eta^3}{r^3}\right)^{4/3}\left[\left(\frac{1-\mu^3\eta^3/r^3}{1+\mu^3\eta^3/r^3}\right)^2 dr^2 + r^2 d\Omega^2\right]\}}, \quad (21)$$

where $\boldsymbol{\mu} = (\boldsymbol{GMH_0/2})^{\boldsymbol{1/3}}$, where $\boldsymbol{H_0}$ is the deSitter Hubble parameter

$$\boldsymbol{a = a(\eta) = -1/(H_0\eta), \eta < 0}. \qquad (22)$$

Here $\boldsymbol{r}$ it satisfies the condition $\boldsymbol{r_0} < \boldsymbol{r} < \infty$ and the value $\boldsymbol{r_0} = -\boldsymbol{\mu\eta}$ in (21) corresp. to the black hole singularity. Due to (12),

$$\boldsymbol{\mu} = (\boldsymbol{r_M H_0}/\mathbf{4})^{\mathbf{1/3}}, \quad (23)$$

with high accuracy.

<u>Two different Pictures.</u>

I .In this picture, $\boldsymbol{\mu} = \boldsymbol{const}$,(***Prokopec, Reska,JCAP 2011***). Then if in formula (23) ”shifts” r_M: $r_M \mapsto \widetilde{r_M}$, then ”shifts” accordingly, and H_0: $H_0 \mapsto \widetilde{H}_0$ so that

$$\mu = (r_M H_0/4)^{1/3} = (\widetilde{r_M}\widetilde{H}_0/4)^{1/3}, \widetilde{H}_0 = \frac{r_M}{\widetilde{r_M}} H_0. \ (24)$$

1.1 . Initially, a primordial black hole is considered in the absence of absorption and emission processes. Since $\mu = const$, the replacement $r_M \mapsto r_{M_q}$ leads to $H_0 \to H_{0,q}$ that satisfies

$$\mu = (r_M H_0/4)^{\frac{1}{3}} = (r_{M_q} H_{0,q}/4)^{\frac{1}{3}} \ , H_{0,q} = H_0 \exp\left(-\frac{1}{2} W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right) (25)$$

Since the potential energy of inflation $V(\phi_0)$: $\boldsymbol{H_0^2 = V(\phi_0)/(3M_p^2) = \Lambda/3}$, then we obtain a shift for $V(\phi_0)$, generated by quantum gravitational corrections for for a primary Schwarzschild hole with mass M

$$\boldsymbol{V(\phi_0) \to V(\phi_0)_q = \Lambda_q M_p^2 = 3M_p^2 H_{0,q}^2 =}$$

$$\boldsymbol{= 3\exp\left(-W\left(-\frac{1}{e}\alpha_{R(A)}\right)\right) M_p^2 H_0^2} \qquad (27)$$

where Λ is the effective cosmological constant and Λ_q the same constant, taking into account the above **qgc** .

Then **qgc** to all other inflationary parameters $a(\eta)$can be found

$$\boldsymbol{a(\eta) \to a(\eta)_q \doteq -1/(H_{0,q}\eta), \eta < 0,} \qquad (28)$$

to the Hubble parameter $H = a'(\eta)/a^2(\eta) \mapsto H_q(\eta) = a'(\eta)_q/a^2(\eta)_q$, as well as to the parameters in the slow roll-off mode, for example, to ϵ:

$$(\boldsymbol{\epsilon} = -\frac{\dot{\boldsymbol{H}}}{\boldsymbol{H}^2}) \mapsto (\boldsymbol{\epsilon_q} = -\frac{\dot{\boldsymbol{H}}_{\boldsymbol{0,q}}}{\boldsymbol{H}_{\boldsymbol{0,q}}^2}), \qquad (29)$$

The condition $\epsilon \ll 1$ for slow rolling in the inflationary scenario transformed into the condition $\epsilon_q \ll 1$, which should be additionally investigated to estimate the boundary $r_{M_q} = R(A,q)$, that is, in fact, the boundary of the deformation parameter $\alpha_{R(A,q)}$ from (15).

1.2 *Case of "minimal" absorption of particles by a black hole .*

Let M the initial mass of the black hole with the area of the event horizon be A. **Bekenstein 1973**: the minimum increment of the area of the black hole event horizon absorbing the particle of energy and size R was estimated E : $(\Delta A)_0 \simeq 4l_p^2(\ln 2)ER$. In quantum consideration $R \sim 2\delta X$ and $E \sim \delta P$. Semiclassical Picture.

low energies $\boldsymbol{E} \ll \boldsymbol{E_p}$, that is, **HUP,** $(\delta X)(\delta P) \geq \frac{\hbar}{2}$, $(\Delta A)_0 \simeq 4l_p^2(\ln 2)$.

For all energies: $\boldsymbol{E \leq E_p}$**, GUP,** $\left(\Delta\hat{A}\right)_{0,q} \approx 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{\hat{A}}\right)\right)$

qgc was obtained , which can be denoted by $(\Delta A)_{0,q}$, to the area of the event horizon of any Schwarzschild black hole, if GUP (2) is valid

$$\left(\Delta\hat{A}\right)_{0,q} \approx 4l_p^2 \ln 2 \exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{\boldsymbol{A_q}}\right)\right), \qquad (32)$$

where $\boldsymbol{A_q}$ the area of the event horizon of the given Schwarzschild black hole and $\boldsymbol{A_0 = 4\pi(\delta X)_0^2}$ is the area of the event horizon of the "minimal" **qbh** . In this case, the "quantum" shift for H_0, i.e., the analogue of formula (26) will have the form

$$\boldsymbol{\widehat{H}_{0,q} = H_0 \frac{R(\widetilde{A})}{R(\widehat{A_q})}}, \qquad (33)$$

where $\boldsymbol{R(\widetilde{A}), R(\widehat{A_q})}$ radii after minimal absorbing in semiclassical and quantum gravity pictures. *From last formula* :

$\hat{\boldsymbol{a}}(\boldsymbol{\eta})_{\boldsymbol{q}} \doteq -\mathbf{1}/(\hat{\boldsymbol{H}}_{\mathbf{0},\boldsymbol{q}}\boldsymbol{\eta}), \boldsymbol{\eta} < \mathbf{0},$ and other cosmological parameters.

1.3 *Black Hole Evaporation and QGC*

Similarly can be obtained analog (33) for ***Black Hole Evaporation***

$$\boldsymbol{H}_{\mathbf{0}} \mapsto \boldsymbol{\mathcal{H}}_{\mathbf{0},\boldsymbol{q}} = \frac{\boldsymbol{R}(\boldsymbol{M}_{\boldsymbol{Evap}})}{\boldsymbol{R}(\boldsymbol{M}_{\boldsymbol{q},\boldsymbol{Evap}})}\boldsymbol{H}_{\mathbf{0}},$$

where $\boldsymbol{R}(\boldsymbol{M}_{\boldsymbol{Evap}})$ – radius **qbh** after evaporation in semiclassical approximation and $\boldsymbol{R}(\boldsymbol{M}_{\boldsymbol{q},\boldsymbol{Evap}})$ – **in quant-grav. picture (qgc)** **for** $\boldsymbol{t}_{\boldsymbol{Evap}} = \boldsymbol{t}_{\boldsymbol{Infl}} - \boldsymbol{t}_{\boldsymbol{M}}$.

4 Black Holes Formation Probability Corrections

It is assumed that non-relativistic particles with a mass $m < m_p$ dominate in the pre-inflationary period and, for convenience, denote the Schwarzschild radius as R_S. $\boldsymbol{N}(\boldsymbol{R},\boldsymbol{t})$ the number of particles in ball with physical radius $R = R(t)$ and volume $V_R(t)$.

Due to (11), it is necessary to replace the Schwarzschild radius $R_S$with

$$R_{S,q} = R_S \exp\left(\frac{1}{2} W\left(-\frac{1}{e}\alpha_{R_S}\right)\right).$$

Then from the general formula $N(R_S,t) = \langle N(R_S,t)\rangle + \delta N(R_S,t)$.

Formation of a Schwarzschild black hole with a radius, $\boldsymbol{R_S}$ it is necessary that, due to statistical fluctuations, the number of $N(R_S,t)$ mass particles m in the volume of this hole $V_{R_S} = \frac{4}{3}\pi R_S^3$ satisfies the condition

$$\boldsymbol{N(R_S,t) \geq \frac{R_S m_p^2}{2m} = \frac{2GM}{2Gm}, \mathrm{i.e.\,m} N(R_S,t) \geq M} \quad (59)$$

which, by virtue of **qgs**

$$\boldsymbol{N(R_{S,q},t) \geq R_{S,q} m_p^2/(2m) = \exp\left(\frac{1}{2} W\left(-\frac{1}{e}\alpha_{R_S}\right)\right) R_S m_p^2/(2m).} \quad (60)$$

qgc is taken into account for the formation **of pbh** in the pre-inflationary period, the number of particles can be less than without this account, i.e. increases the probability of its formation. Applied in the framework of the replacement $R_S \mapsto R_{S,q}$we obtain an analogue of general formula

$$\delta N_q \geq \delta N_{cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S, t)_q \rangle = \frac{m_p^2 R_{S,q}}{2m} [1 - (HR_S)^2] =$$
$$= \frac{m_p^2 R_S}{2m} [1 - (HR_S)^2] \exp\left(\frac{1}{2} W\left(-\frac{1}{e} \alpha_{R_S})\right)\right) = \delta N_{cr} \exp\left(\frac{1}{2} W\left(-\frac{1}{e} \alpha_{R_S})\right)\right)$$

In the last formula, in square brackets, instead of $(HR_S)^2$should be $(H_q R_{S,q})^2$, however, since the case is considered $\mu = const$, these values coincide.

Note that here we use the standart condition

$$\boldsymbol{HR_S < 1}, \qquad (57)$$

i.e. what Schwarzschild radius $\boldsymbol{R_S}$ less than Hubble radius,

$$\boldsymbol{R_S < R_H = 1/H}.$$

Since the $\mathbf{exp}\left(\frac{1}{2}\boldsymbol{W}\left(-\frac{1}{e}\boldsymbol{\alpha_{R_S}}\right)\right) < \mathbf{1}$ value

$$\boldsymbol{\delta N_{\mathrm{cr},q} < \delta N_{\mathrm{cr}}} \qquad (58)$$

This conclusion can also be reached directly by comparing this probability in the semiclassical picture:

$$P(\delta N(R_S, t) > \delta N_{cr}(R_S, t)) = \int_{\delta N_{cr}}^{\infty} d(\delta N) P(\delta N) \qquad (61)$$

and taking into account **qgc**

$$P(\delta N(R_{S,q}, t) > \delta N_{\mathrm{cr}}(R_{S,q}, t)) = \int_{\delta N_{\mathrm{cr},q}}^{\infty} d(\delta N) P(\delta N) \qquad (62)$$

$$\int_{\delta N_{cr,q}}^{\infty} d(\delta N) P(\delta N) =$$

$$= \int_{\boldsymbol{\delta N_{cr,q}}}^{\boldsymbol{\delta N_{cr}}} \boldsymbol{d(\delta N) P(\delta N)} + \int_{\delta N_{cr}}^{\infty} d(\delta N) P(\delta N) > \int_{\delta N_{cr}}^{\infty} d(\delta N) P(\delta N)$$

pbh when **qgc is taken into account** increases.

It is possible to analyze what will change in the estimate of the probability of occurrence of **pbh** if, $\mu \neq const$ in calculating **qgc** for **pbh** , only the parameters change, while the cosmological parameters remain the same. Then in formula (21) there is a replacement

$$(\mu = (GMH_0/2)^{1/3}) \mapsto (\mu_q = (GM_qH_0/2)^{1/3}), \quad (64)$$

and M_q is the mass **pbh** , taking into account **qgc** from formula (11) and in the formula (58) the replacement

$$\delta N \geq \delta N_{cr,q} \doteq \frac{m_p^2 R_{S,q}}{2m} - \langle N(R_S,t)_q \rangle = \frac{m_p^2 R_{S,q}}{2m}[1-(HR_{S,q})^2] =$$

$$= \frac{m_p^2 R_S \exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)}{2m}[1 - H^2 R_S^2 \exp\left(W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)] \quad (65)$$

In order to understand how the probability of occurrence of **pbh** **changes** in comparison with the case when **qgc** are not taken into account in this consideration, it is necessary to compare the last expression with the corresponding value $\delta N_{cr} = \frac{m_p^2 R_S}{2m}[1-(HR_S)^2]$.

In this scenario formation probability **pbh** increases if condition (58) is satisfied $\delta N_{cr,q} < \delta N_{cr}$, if

$$H^2 R_S^2 < \frac{1-\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)}{1-\exp\left(\frac{3}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)} = \frac{1}{1+\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)+\exp\left(W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)}, i.e.$$

$$HR_S < \frac{1}{\sqrt{1+\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)+\exp\left(W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)}}, \quad (68)$$

Note that since it is R_S small, it is $\exp\left(\frac{1}{2}W\left(-\frac{1}{e}\alpha_{R_S}\right)\right), \exp\left(W\left(-\frac{1}{e}\alpha_{R_S}\right)\right)$ also small and the value on the right side of (68) is close to 1,i.e., the smaller the Schwarzschild radius **pbh** , the more, taking into account **qgc** , the probability of their occurrence increases.

5 Final Comments. Further Research

5.1. Einstein Equiv. Principle (*EEP*) in black hole field (for large (classical) BH) is break for semiclassical approximation (***D. Singleton and S. Wilburn,Hawking Radiation, Unruh Radiation, and the Equivalence Principle,Phys. Rev. Lett., 107, 081102 (2011); Phys. Rev. Lett., 108, 049002 (2012).)***

The remote detector in the black hole field detects two temperatures: the black hole temperature T_{BH} and the Unruh temperature T_{UN}.

If EEP remains true in this case of semiclassical approximations Then always $T_{BH} = T_{UN}$. However always $T_{BH} > T_{UN}$.

It was shown in ***A.E. Shalyt-Margolin, IJTP.2021,60,1858***, that this result remains true for the **qbh** if $\boldsymbol{T_{BH}} \to \boldsymbol{T_{BH,q}}$; $\boldsymbol{T_{UN}} \to \boldsymbol{T_{UN,q}}$ and $\boldsymbol{T_{BH,q}} > \boldsymbol{T_{UN,q}}$

where $\boldsymbol{T_{BH,q}}, \boldsymbol{T_{UN,q}}$ **–** corresp. temperatures with **qgc for this qbh**

But for well-known QFT *EEP* is true.

Therefore that if **quantum pbh** is the basis of space-time foam at Planck scale, then local well-known QFT has an upper limit of applicability $\boldsymbol{\tilde{E}}$ that is much lower than the Planck energy $\boldsymbol{\tilde{E}} \ll \boldsymbol{E_p}$.

Therefore question on black holes formation probability and its qgc is very actual.

5.2. It is shown how **"Quantum Shifts" (QS)** to the main cosmological parameters in the inflationary scenario generated by quantum **pbh** can be explicitly calculated. It directly follows that for small-radius **pbh**, these **QS** are especially large and can introduce significant corrections into the main parameters of inflation.

Question: *in what way those* **QS** *connected with the general theory of* ***qgc*** *in cosmology and in particular of* ***qgc*** *for cosmological perturbations* ***(Claus Kiefer and other).***

5.3. ***qgc*** to the scale invariant power spectrum of a scalar _field on de Sitter space from small black holes (***qbh***) that formed during a pre-inlationary era.