The XV–th International School–Conference “The Actual Problems of Microworld Physics” (27.08 – 3.09 2023)

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# Fractal effects of the space distribution of stars in the solar neighborhood

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# Plan of the talk

- Introduction. The study of Fractal structures in the stellar medium of the Galaxy

- Concept of "fractal" and "fractal dimension”
- Goal of this work
- Main stages in the development of ideas about the fractal structure of the stellar medium of galaxies. Law of Carpenter-Vaucouleurs
- Mandelbrot interpretation and introduction of "fractal dimension"
- Method “mass-radius” for calculation of mean stellar density
- The results of numerical calculations of mean stellar density and fractal properties of 200,000 stars in the solar neighborhood

- Conclusions

# Introduction

**The study of Fractal structures in the stellar medium of the Galaxy**

→ constructing internally consistent and more appropriate adequate to observational data kinetic theory of the stellar medium
(the kinetic parameters of fractal stellar media differ significantly from the corresponding parameters for quasi-homogeneous media with limited density fluctuations)

→ refining basic equations of stellar dynamics for fractal stellar media

The fractality of the structure of stellar medium at distances from 1 pc to 200 pc - from observations of **:**

- young population of galaxy (*Efremov and Elmegrin, 1998; Elias et al., 2009; Elmegrin et al., 2014*)
- F, G – type dwarf stars in the solar neighborhood from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015)*
- interstellar gas and dust clouds *(Larson, 1981; de Vega et.al, 1998)*

# Concept of "fractal"

"Fractal" means fractional, broken, - Mandelbrot called so self-similar geometric figures, each fragment of which is repeated when the scale is reduced, (*Mandelbrot*, *1988)*

→ The figure has the property of *scale invariance.*
This is the first basic property of fractal objects

Benua Mandelbrot – the founder of fractal analysis who introduced the term "fractal"

Benua Mandelbrot
1924 - 2010

# Concept of “fractal dimension“

Fractal dimension – is a measure, how the object fills the space of embedding

“ Fractal dimension“ :

$$D = \lim_{a \to 0} \frac{\log N(a)}{\log(1/a)}$$

***a*** **–** scale unit; ***N*** **–** number of scale units, covering the object;
***D*** **–** fractal dimension.

If such a limit exists and the *value is a fraction*, then this object is a fractal.
This is the second basic property of fractal objects

# Goal of this work

**Goal of this work**– to study fractal properties of the spatial distribution of 200,000 stars of all spectral types in the solar neighborhood at a distance of 1 pc to 100 pc from the Sun from observational data of the “GAIA” telescope (DR2, 2018)

Telescope “GAIA” has a goal – studying coordinates, direction of motion, spectral types of a billion stars in the **Milky Way,** search for exoplanets, asteroids and comets in the Solar System

Launched in 2013 and located at a distance 1, 5 million km from the Earth

DR2 – Data Release 2 (2018)

# The main stages in development of ideas about fractal structures in stellar medium of galaxies

Carpenter (1938) : number density of galaxies in cluster decreases with the growth of characteristic cluster sizes according to the fractional-power law

$$n \sim N/r^3 \sim r^{-1,5}$$

$N$ – number of galaxies in cluster; $r$ – size of cluster; $n$ – density of galaxies in cluster

Vaucouleurs (1970) (on base of *new* galaxy clusters data):

$$n(r) \sim r^{-1,7}$$

- galactic medium is arranged hierarchically
- any observer, included in the hierarchy, will find that the mean density around him decreases with distance
- any large identical volumes have the same mean density, regardless of the position of their centers relative to each other. This density can be called the invariant conditional density

**Vaucouleurs has extended power law to entire stellar medium**

# The interpretation of Mandelbrot and fractal dimension

**Mandelbrot (1988)** interpreted results by Carpenter and Vaucouleurs as a special case of stochastic self-similarity of three-dimensional random fractal sets for which the relation holds:

$$n(r) \sim r^{-\alpha}$$

$r$ – characteristic size of the increasing volume around observer included in hierarchy; $n(r)$ – invariant conditional density ; $\alpha$ – exponent

$$D = 3 - \alpha$$

$D$ – **is fractal dimension**

Mandelbrot showed that, depending on the characteristics of the medium in various gravitating media, for fractal structures :

$$0 \leq D \leq 3$$

$\rightarrow$ $\alpha$ – exponent can vary from 0 to 3

# Calculation of mean stellar density by the "mass - radius" method

**This method** consists of determination of number $N\,(r; R_i)$ of stars in spheres with increasing radius "*r*" with center in the ***i***-th star located at a radial distance $R_i$ from the observer.

The mean number of stars in spheres of radius "*r*":

$$N(r) = \frac{1}{m(r)} \sum_{i=1}^{m(r)} N(r; R_i)$$

$N\,(r)$ – mean number of stars in spheres with radius "*r*";

$N\,(r; R_i)$ – number of stars in sphere with radius "*r*" with center in the ***i***-th star;

$R_i$ – radial distance of ***i***-th star from the observer;

$m\,(r)$ – number of spheres of radius "*r*" is equel to number of studied stars;

**Mean stellar density n (r) in spheres of radius "*r*" around the stars:**

$$n(r) = N(r)/V(r)$$

$V\,(r)$ – volume of sphere of radius "*r*"

# The results of our calculations of spatial distribution and fractal properties of 200 000 stars in solar neighborhood from data of “GAIA” (DR2,2018)

200,000 stars of all spectral types at distances from 1 to 100 parsecs from Sun

**Mean star density n (r) in spheres of radius *r* around the stars is approximated by power laws of the form:**

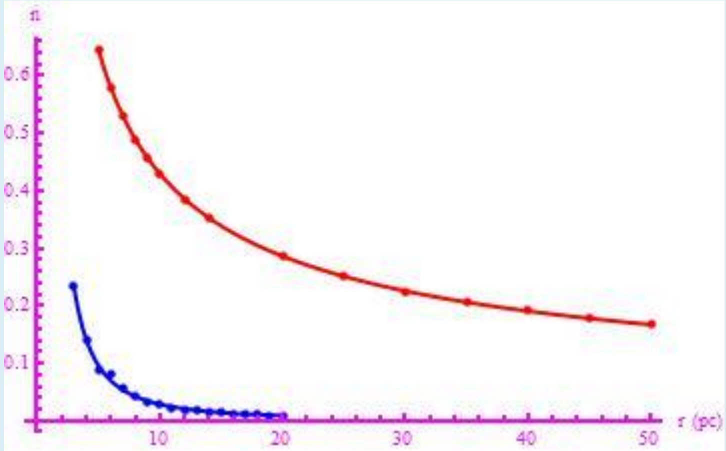
$$n(r) = hr^{-\alpha}$$

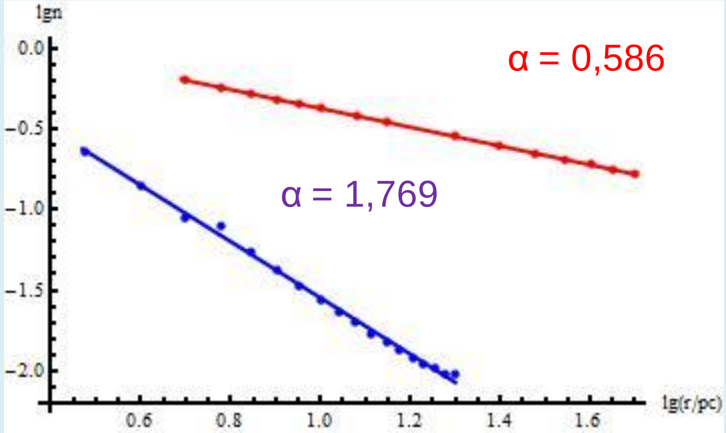
**where h = 1,654; α = 0,586; D = 3 - α ≈ 2,41**

$$n(r) = 1,654\, r^{-0,586}$$

This law confirms the conclusions by Vaucouleurs and Mandelbrot for fractal structures in gravitating media

## Mean stellar density vs. radius of the sphere around the stars




X-axis: **r** in parsecs (pc),
Y-axis: stellar density **n** in pc$^{-3}$.

Red line – **200,000 stars** of all spectral types at distances from **1 to 100 parsecs** from Sun from observational data of telescope "GAIA" (DR2, 2018)
h = 1,654; α = 0,586; D ≈ 2,41

Blue line – **13,000** F, G – type **dwarf stars** at distances from **1 to 20 parsecs** from Sun from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015),* h = 1,644; α = 1,769; D ≈ 1,23

# Conclusions

- Study of stellar medium in the solar neighborhood from observational data of telescope "GAIA" (DR2, 2018) showed the presence of fractal structures with fractal dimension D ≈ 2,41
- The result obtained is consistent with result for F, G – type **dwarf stars** from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015)* (fractal dimension D ≈ 1,23);
- Our fractal dimension D ≈ 2,41 is explained by: the greater completeness of observational data of the "GAIA" telescope:

| Our stars | F, G – type dwarf stars |
|---|---|
| 200 000 stars | 13 000 stars |
| 1 – 100 parsecs from Sun | 1 – 20 parsecs from Sun |
| all spectral classes | F, G – type **dwarf stars** (40% of star density in solar neighborhood) |

- stars more fill the space of embedding,
- more structure in stellar system

# Conclusions

- Stars in solar neighborhood from observational data of telescope "GAIA" form gravitationally bound structured formations, such as clusters, parts of spiral arms, clumps, which have fractal properties and obey the laws of fractal kinetics

M 31 Galaxy Nebula of Andromeda

# Estimation of the effective interparticle spacing for the fractal stellar medium in the solar neighborhood based on GAIA DR2 data

The data obtained were used to estimate one of the important parameters in stellar dynamics - the effective interparticle spacing for the fractal stellar medium in the solar neighborhood.

In *(Chumak, Rastorguev, 2015)* a formula was derived for the effective interparticle spacing $r_m$ for fractal medium from the distribution law of the distance to the nearest neighbor:

$$w(r)dr = 4\pi h \, exp\left(-\frac{4\pi h}{3} r^D\right) r^{D-1} dr$$

$w(r)$ – the distance distribution caused by the nearest neighbor in a spherical layer of radius $r$ and thickness $dr$;

coefficient **h = 1,654;** fractal dimension **D ≈ 2,41** were taken from our calculations for 200 000 stars .

A-priory:

$$r_m = \int_0^\infty r\, w(r)\, dr$$

where $w\ (r)$ – the distance distribution, which is presented above;

$w\ (r)dr$ it is better to rewrite in the form:

$$w(x)dx = \frac{3}{D} e^{-x} dx$$

where

$$x = \frac{4\pi h}{D} r^D$$

coefficient $h$ and fractal dimension $D$ were taken from our calculations.

Then, the effective interparticle spacing $r_m$ :

$$r_m = \frac{3}{D}\left(\frac{4\pi h}{D}\right)^{-1/D} \int_0^\infty e^{-x} x^{1/D} = \frac{3}{D}\left(\frac{D}{4\pi h}\right)^{1/D} \Gamma\left(\frac{D+1}{D}\right)$$

In the limit fractal dimension D≈3 and mean stellar density h≈n we obtain the well-known estimation from Chandrasekhar (1943) for the mean interparticle spacing for quasi-homogeneous media:

$$r_m = \left(\frac{3}{4\pi n}\right)^{1/3} \Gamma\left(\frac{4}{3}\right) \approx 0,554 n^{-1/3}$$

- Study of stellar medium in the solar neighborhood from observational data of telescope “GAIA” (DR2, 2018) showed the presence of fractal structures with fractal dimension D ≈ 2,41 ; h = 1,654 ; $r_m$ = 0,49 parsec
- The result obtained is consistent with result for F, G – type **dwarf stars** from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015)* (fractal dimension D ≈ 1,23; h =1,644 ; $r_m$ = 0,48 parsec );
- Traditional estimations of the effective interparticle spacing for stellar media - $r_m$ = 1,16 parsec ; $r_m$ = 1,06 parsec
-
-

# Thank you for your attention

NGC 3521 Galaxy

# Приветствие из «Золотых песков»

Ковер янтарный устилает
Дубраву, и встает туман.
Сквозь блики дня почти что тает
Небесный солнца океан.

Упало за горизонт светило…
Поникла арками трава…
И гладь зеркальная подстыла…
Направив взгляд свой в облака…

*М. Эмеральд*

Трещат стволы…пригнулись травы…
Шёпот Природы средь долин
Осенней утренней дубравы
Едва, как призрак, уловим.

*М. Эмеральд*

«Золотые пески»
(из сборника стихотворений
«Дыхание осени»)

# Backup slides

• Hertzsprung Russell diagram

•- shows the relationship between

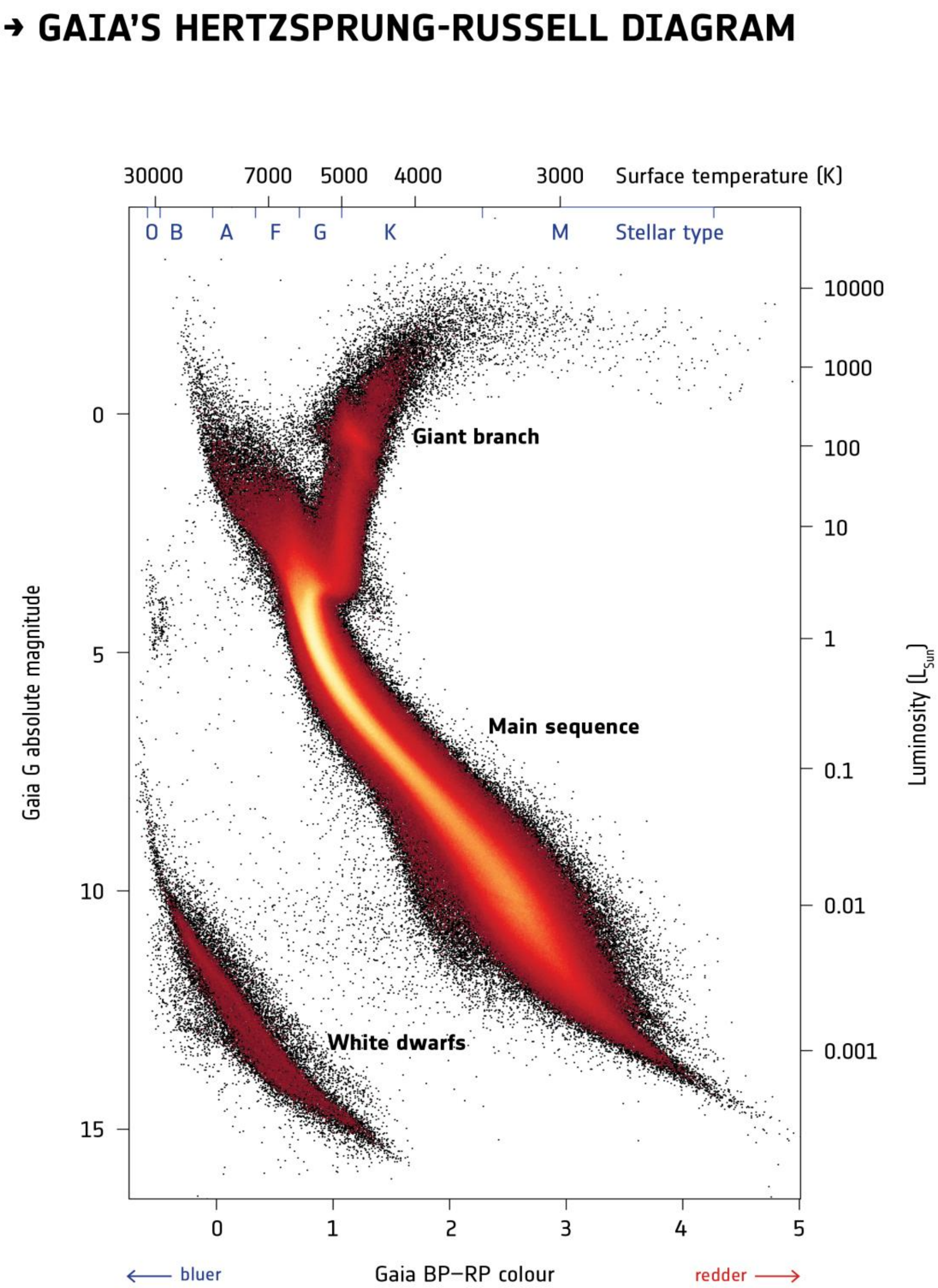
•the *spectral class* and

•the *luminosity* of stars

Main sequence-

the main grouping of stars on the diagram *"spectral class – luminosity"*

It contains most of the stars, since the main sequence corresponds to the *longest stage in the evolution of stars*, at which thermonuclear reactions involving hydrogen take place in the core of the star.



# Spectral classes of stars

- Spectral classes of stars - groups of stars distinguished by the nature of their spectra.
- Closely related to the temperature of stellar atmospheres.
- The hottest stars are blue stars of spectral classes O and B (50,000 K)
- The coldest stars are stars of spectral classes M and L (2000 K).
- Sun - spectral class G dwarf
- Dwarf - small main sequence star (up to one solar radius and not exceeding the luminosity of the Sun)
- Luminosity - the radiation power of a celestial body. That is the amount of energy emitted by it per unit of time
- For most main sequence stars, the relation $M^4 \sim R^5 \sim L$
- M – mass of star, R – radius of star, L –luminosity of star

# Distances in astronomy

- 1 parsec ~ 30 000 000 000 000 km.
- 1 light year ~ 0,3 parsec.
- 1 astronomical unit ~ 150 000 000 km

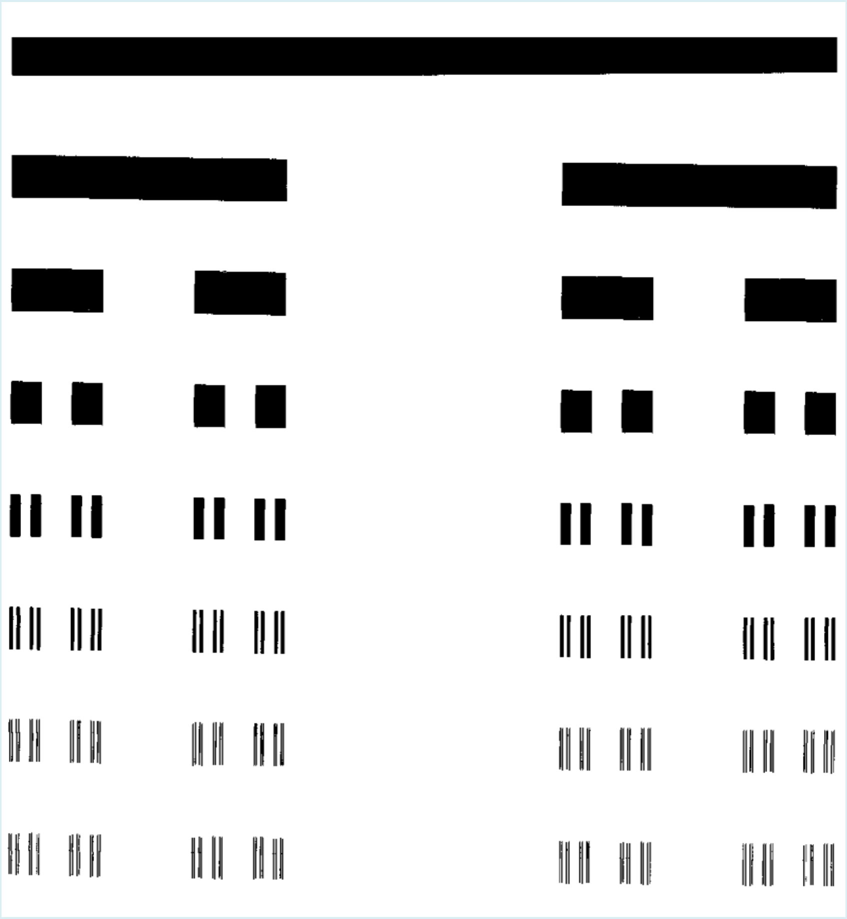
# Закон Карпентера – Вокулера и стохастические иерархии в гравитирующих средах - 1

Галактическая среда устроена иерархически, и потому любой наблюдатель, если он связан с объектом, включенным в иерархию, обнаружит, что средняя плотность вокруг него убывает с расстоянием. При этом в галактической среде нет выделенного положения, то есть любые достаточно большие, но *одинаковые* объемы, имеют одинаковую среднюю плотность независимо от положения их центров друг относительно друга. Эту плотность можно назвать инвариантной *условной плотностью (УП)*. Однако, если мы будем синхронно изменять размеры этих (одинаковых) объемов, то УП будет изменяться с характерным размером объема **r** по закону:

$$n(r) \sim r^{-\alpha}$$

где $\alpha = 1,7$ . Это и есть закон Карпентера – Вокулера для галактической среды

# Пыль Кантора



$$2^3 = 8 \qquad n = 2^N$$

$$1/3 \;\; 1/9 \;\; 1/27$$

$$a = (1/3)^N \quad 2^N(1/3)^N = (2/3)^N$$

$$na = 1, n = 1/a$$

$$na^2 = 1, n = 1/a^2$$

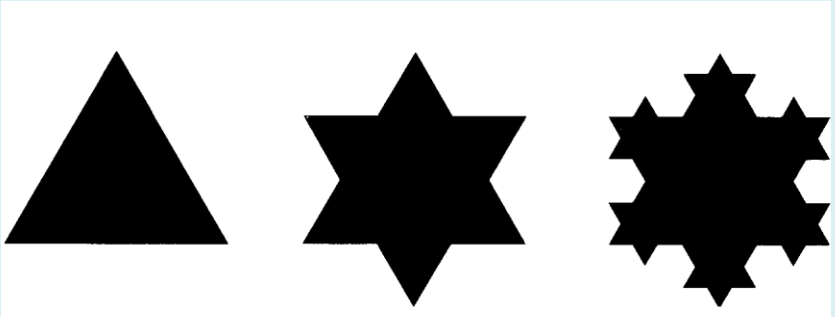
$$n = 1/a^3 \quad n = 1/a^d$$

$d$ – размерность объекта

$$d = \frac{\log n}{\log\left(\frac{1}{a}\right)}$$

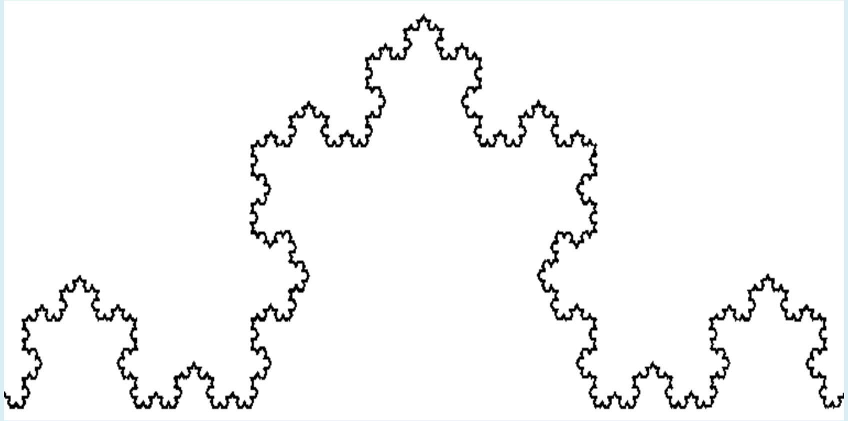
$$d = \log(2)/\log(3) = 0,63 \ldots$$

# Снежинка Коха



$n = 4 \qquad a = 1/3$

$n = 4^2 \qquad a = (1/3)^2$



$d = \log(4)/\log(3) = 1,26 \ldots$

# Соотношение «масса – радиус»

Далеко не всегда при определении фрактальной размерности природных объектов удобно применять бокс алгоритм. Для таких объектов удобнее рассматривать связь между массой и радиусом объекта, заключенного под эти радиусом:

$$\boldsymbol{M \sim R^{d}}$$

где $\boldsymbol{d}$ - фрактальная (Хауздорфова) размерность.

# Соотношение «периметр – площадь»

Рассмотрим соотношение «периметр – площадь» и его связь с фрактальной размерностью. У плоских эвклидовых фигур – окружностей, квадратов, многоугольников – отношение периметра ***L*** к корню квадратному из площади ***S*** есть величина постоянная:

$$L/(S)^{1/2} = const$$

Если периметр представляет собой фрактальный объект, и длина профиля объекта зависит от масштаба ***a*** , с помощью которого эта длина измеряется, то это соотношение может быть записано в виде:

$$L^{1/d}/(S)^{1/2} = const$$

где ***d*** **- фрактальная (Хауздорфова) размерность.**

# Соотношение «периметр – площадь» (продолжение)

Таким образом,

$$S \sim L^{2/d}$$

Этим соотношением можно воспользоваться для определения фрактальных размерностей различных природных объектов. Таким способом была определена Хауздорфова размерность теней солнечных пятен. Она оказалась равной

$$d_0 = 1{,}35 \pm 0{,}03$$

Оказалось также, что эта размерность равна фрактальной размерности границ облаков в земной атмосфере.

# Закон распределения ближайшего соседа (задача Герца) - 1

Найдем связь распределения Хольцмарка с распределением ближайшего соседа, $\boldsymbol{n}$ – однородная плотность

$$\boldsymbol{w}(\boldsymbol{r})\boldsymbol{dr} = \boldsymbol{P}\left\{\text{ближайший сосед} \in \underbrace{(\boldsymbol{r}, \boldsymbol{r} + \boldsymbol{dr})}_{\substack{\text{сферический слой} \\ \text{радиуса } \boldsymbol{r} \text{ и толщиной } \boldsymbol{dr}}}\right\} =$$

$$= \boldsymbol{P}\{\text{нет звезд ближе } \boldsymbol{r}\}$$

$$* \boldsymbol{P}\{\text{в слое } \boldsymbol{r} + \boldsymbol{dr} \text{ ровно } \mathbf{1} \text{ звезда}\}$$

Математическое ожидание числа звезд в слое $(\boldsymbol{r}, \boldsymbol{r} + \boldsymbol{dr})$

$$\boldsymbol{\lambda} = \mathbf{4}\boldsymbol{\pi}\boldsymbol{r}^{\mathbf{2}} \underbrace{\boldsymbol{n}}_{\text{мала}} \underbrace{\boldsymbol{dr}}_{\text{мала}} \ll \mathbf{1}$$

# Закон распределения ближайшего соседа (задача Герца) - 2

Распределение Пуассона

$$P_{\text{п}}(k) = \lambda^k \frac{e^{-\lambda}}{k!}$$

В нашем случае $k = 1 \Rightarrow P_{\text{п}}(1) \approx \lambda \left( e^{-\lambda} \text{ близко к } 1, \text{т. к. } \lambda \text{ мало} \right) \Rightarrow$

$$\Rightarrow P\{\text{в слое } r + dr \text{ ровно } 1 \text{ звезда}\} = 4\pi r^2 n dr$$

$$P\{\text{в пределах } r \text{ нет ни одной звезды}\} = 1 - \underbrace{\int_0^r w(r)dr}_{\substack{\text{вероятность того,что} \\ \text{есть хотя бы } 1 \text{ звезда}}}$$

$$w(r)dr = \left[1 - \int_0^r w(r)dr\right] 4\pi r^2 n dr$$

$$w(r)dr = \left[1 - \int_0^r w(r)dr\right] 4\pi r^2 n dr$$

$$\frac{w(r)}{4\pi r^2 n} = 1 - \int_0^r w(r)dr$$

$$\int_0^r w(r)dr = 1 - \frac{w(r)}{4\pi r^2 n}$$

$$\frac{d}{dr}\left[\int_0^r w(r)dr\right] = \frac{d}{dr}\left[1 - \frac{w(r)}{4\pi r^2 n}\right]$$

$$w(r) = -\frac{d}{dr}\left[\frac{w(r)}{4\pi r^2 n}\right]$$

$$\frac{d}{dr}\left[\frac{w(r)}{4\pi r^2 n}\right] = -w(r)$$

$$\frac{d}{dr}\underbrace{\left[\frac{w(r)}{4\pi r^2 n}\right]}_{y(r)} = \underbrace{\left[\frac{w(r)}{4\pi r^2 n}\right]}_{y(r)} * \left(-4\pi r^2 n\right)$$

$$\frac{dy(r)}{dr} = y(r)\left(-4\pi r^2 n\right)$$

$$\frac{1}{y}\frac{dy}{dr} = -4\pi r^2 n; \quad \frac{d}{dr}\ln y = -4\pi r^2 n$$

$$\frac{d}{dr}\int \ln y\, dr = -\int 4\pi r^2 n dr$$

$$\ln y = -\frac{4\pi r^3 n}{3} \qquad \underbrace{y(r)}_{\frac{w(r)}{4\pi r^2 n}} = C exp\left\{-\frac{4}{3}\pi R^3 n\right\}$$

При $\quad r \to 0 : w(r) \underset{r\to 0}{\longrightarrow} 4\pi r^2 n \Rightarrow C = 1$

# Закон распределения ближайшего соседа (задача Герца) - 4

Закон для распределения расстояний до ближайшего соседа

$$w(r) = C4\pi r^2 n exp\left\{-\underbrace{\frac{4}{3}\pi R^3}_{V} n\right\},$$

$V * n -$ математическое ожидание числа звезд

Вероятность малых сил $\to 0, w(r) \xrightarrow[r\to\infty]{} 0.$

Точная формула для среднего расстояния между звездами

$$\overline{d} = \int_0^\infty rw(r)dr = \int_0^\infty e^{-4\pi r^3 \frac{n}{3}} * 4\pi r^3 n dr = \frac{1}{\left(\frac{4\pi n}{3}\right)^{\frac{1}{3}}} \int_0^\infty e^{-x} x^{\frac{1}{3}} dx = \frac{\Gamma\left(\frac{4}{3}\right)}{\left(\frac{4\pi n}{3}\right)^{\frac{1}{3}}} \approx$$

$$\approx 0.554 n^{-\frac{1}{3}} \approx \frac{n^{-\frac{1}{3}}}{2} \qquad \boxed{\overline{d} \approx \frac{1}{2n^{\frac{1}{3}}}}$$

# Estimation of the effective interparticle spacing for the fractal stellar medium in the solar neighborhood based on GAIA DR2 data

The data obtained were used to estimate one of the important parameters in stellar dynamics - the effective interparticle spacing for the fractal stellar medium in the solar neighborhood.

In *(Chumak, Rastorguev, 2015)* a formula was derived for the effective interparticle distance $r_m$ for fractal medium from the distribution law of the distance to the nearest neighbor:

$$w(r)dr = 4\pi h\, exp\left(-\frac{4\pi h}{3} r^D\right) r^{D-1} dr$$

$w(r)$ – the distance distribution caused by the nearest neighbor in a spherical layer of radius $r$ and thickness $dr$;

coefficient **h = 1,654;** fractal dimension **D ≈ 2,41** were taken from our calculations for 200 000 stars .

the dependence of average density on distance (2) for a fractal medium, and D is the fractal dimension from formula (3):

# Distances in astronomy

- Study of stellar medium in the solar neighborhood from observational data of telescope "GAIA" (DR2, 2018) showed the presence of fractal structures with fractal dimension D ≈ 2,41
- The result obtained is consistent with result for F, G – type **dwarf stars** from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015)* (fractal dimension D ≈ 1,23);
- Our fractal dimension D ≈ 2,41 is explained by: the greater completeness of observational data of the "GAIA" telescope:

| Our stars | F, G – type dwarf stars |
|---|---|
| 200 000 stars | 13 000 stars |
| 1 – 100 parsecs from Sun | 1 – 20 parsecs from Sun |
| all spectral classes | F, G – type **dwarf stars** (40% of star density in solar neighborhood) |

- stars more fill the space of embedding,
- more structure in stellar system

# The results of calculations of spatial distribution and fractal properties of 200 000 stars in solar neighborhood from data of "GAIA" (DR2,2018)

200,000 stars of all spectral types at distances from 1 to 100 parsecs from Sun

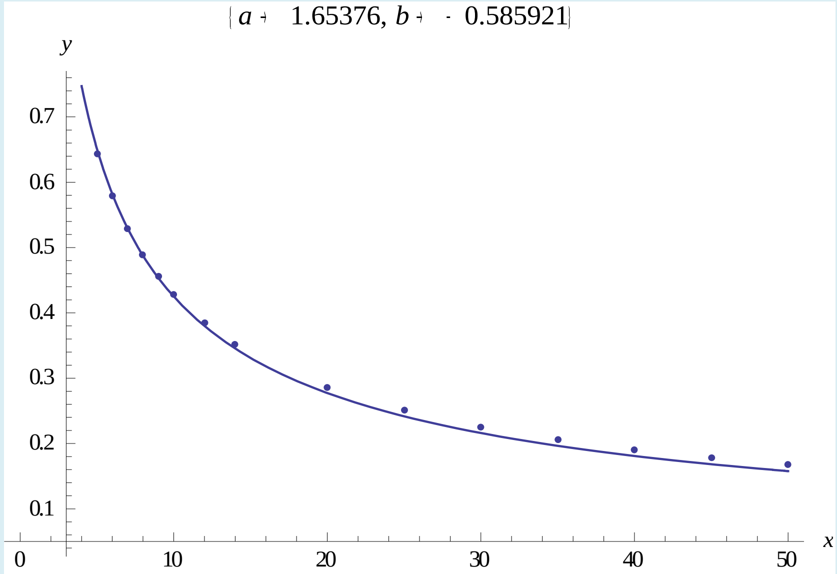
→ local star density "n (r)" in spheres with increasing radius "*r*" with center in the i-th star is approximated by power laws of the form:

$$n(r) = hr^{-\alpha}$$

where n (r) - local star density , h and α are coefficients, D – fractal dimension

→This confirms the conclusions of Vaucouleur - Mandelbrot for fractal structures in gravitating media. → In this case, h = 1,654, α = 0,586, D = 2,41

The dependence of the conditional stellar density on the radius of the volume under consideration is shown in Fig. (1). Here, along the x-axis, the values of r are expressed in parsecs (pc), along the y-axis, the values of the stellar density n are in pc-3.



# The main stages in development of ideas about fractal structures in stellar medium of galaxies

Carpenter (1938) : number density of galaxies decreases with the growth of their characteristic sizes according to the fractional-power law

$$\boldsymbol{n} \sim N/\boldsymbol{r}^3 \sim \boldsymbol{r}^{-1{,}5}$$

***N*** – number of galaxies in cluster; ***r*** – size of cluster; ***n*** – density of galaxies in cluster

Vaucouleurs (1970) (on base of *new* galaxy clusters data):

$$\boldsymbol{n}(\boldsymbol{r}) \sim \boldsymbol{r}^{-1{,}7}$$

- galactic medium is arranged hierarchically
- any observer, included in the hierarchy, will find that the average density around him decreases with distance
- any large identical volumes have the same average density, regardless of the position of their centers relative to each other. This density can be called the invariant conditional density

**Vaucouleurs has extended power law to entire stellar medium**

# The results of our calculations of spatial distribution and fractal properties of 200 000 stars in solar neighborhood from data of "GAIA" (DR2,2018)

200,000 stars of all spectral types at distances from 1 to 100 parsecs from Sun

**Mean star density n (r) in spheres of radius *r* around the stars is approximated by power laws of the form:**

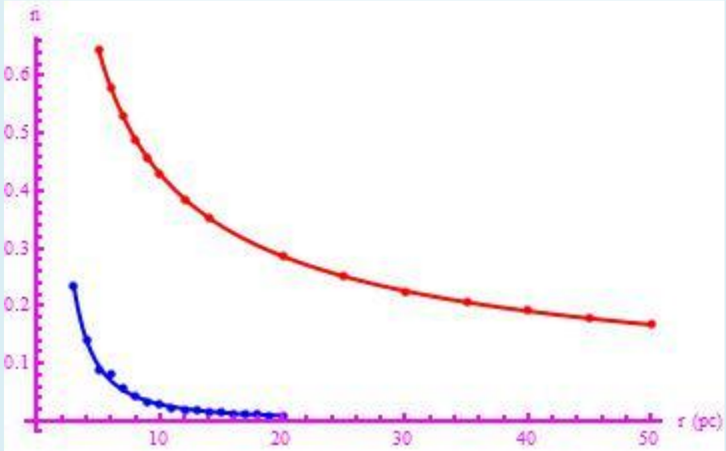
$$n(r) = hr^{-\alpha}$$

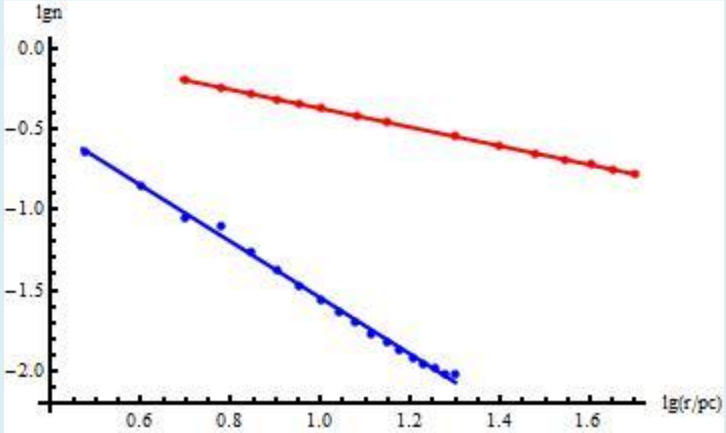
where h = 1,654, α = 0,586, D = 3 - α ≈ 2,41

$$n(r) = 1,654\, r^{-0,586}$$

This law confirms the conclusions by Vaucouleur and Mandelbrot for fractal structures in gravitating media

## Mean stellar density vs. radius of the sphere around the stars




Graphics of Mean stellar density vs. radius of the sphere around the stars

X-axis - the values of **r** in parsecs (pc),
Y-axis, the values of the stellar density **n** are in pc-3.

Red line – 200,000 stars of all spectral types at distances from **1 to 100 parsecs** from Sun from observational data of telescope “GAIA” (DR2, 2018)

Blue line – 13,000 F, G – type dwarf stars at distances from **1 to 20 parsecs** from Sun from observational data of the Geneva–Copenhagen Survey (*Chumak and Rastorguev, 2015),* h = 1.644, α = 1.769, D ≈ 1.23.