Rare Semileptonic $B^+ \to \pi^+\tau^+\tau^-$ Decay in the Standard Model

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Outline

1. Brief overview of rare semileptonic B-decays
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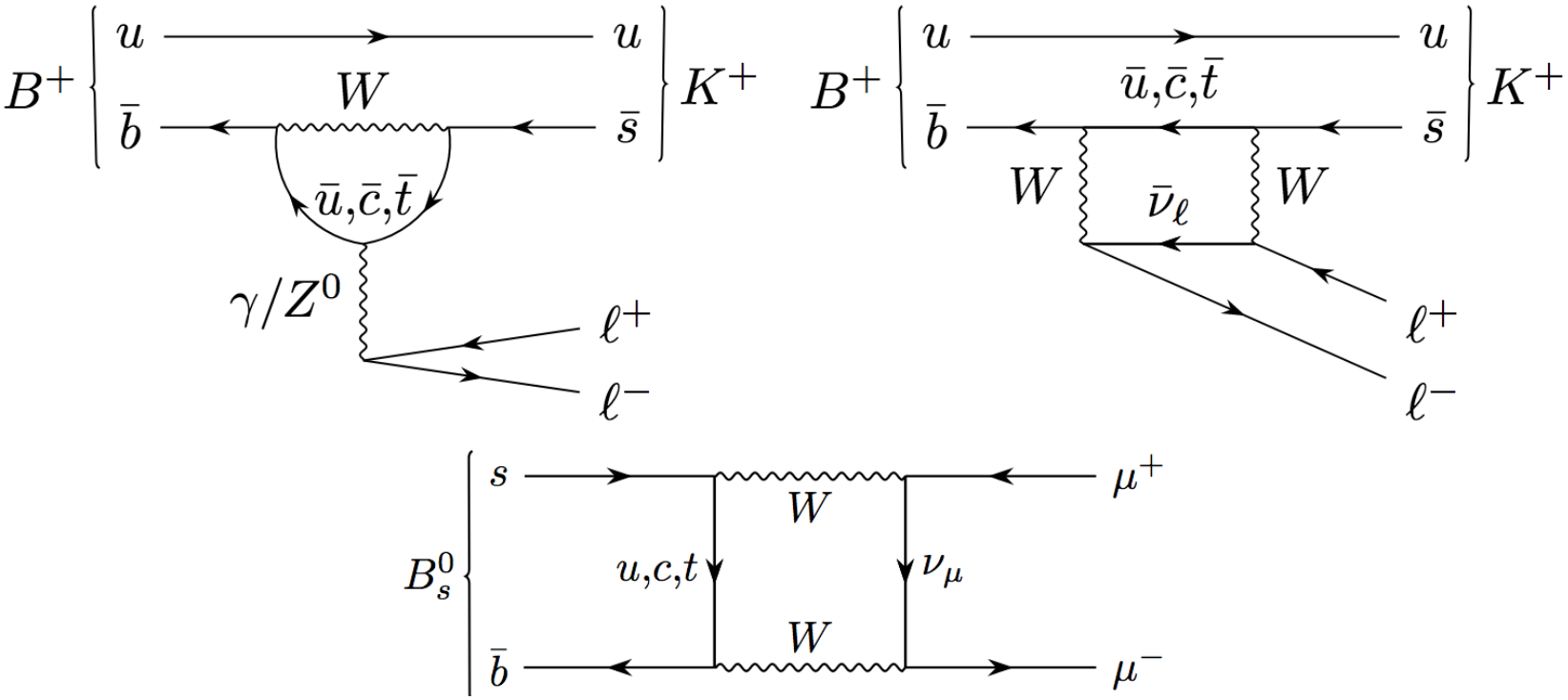
3. Branching fraction of $B \to P\ell^+\ell^-$ decay
4. Parameterizations of $B \to P$ transition form factors
5. Long-distance contributions in $B \to P\ell^+\ell^-$ decay
6. Numerical analysis of $B^+ \to \pi^+\tau^+\tau^-$ decay
7. Summary and outlook

Brief overview of semileptonic B-decays

- Rare semileptonic decays of B-mesons and Λ_b-baryons due to $b \to s$ and $b \to d$ transitions, where b, s, and d are down quarks, are sensitive to "New Physics"
- Branching fractions of semileptonic B-meson decays due to $b \to s$ transition, such as $B^\pm \to K^{(*)\pm}\mu^+\mu^-$, $B^0 \to K^{(*)0}\mu^+\mu^-$, $B_s^0 \to \phi\mu^+\mu^-$, dimuon invariant mass distributions, and coefficients in angular distributions are experimentally measured quite precisely
- Experimental information on the $b \to d$ semileptonic decays is rather sparse at present. The first exclusive decay induced by the $b \to d$ FCNC current, $B^\pm \to \pi^\pm\mu^+\mu^-$, was observed by the LHCb Collab. in 2012 and analyzed in detail in 2015
- Information on the FCNC $b \to s(d)\tau^+\tau^-$ transitions is highly unknown, and current interest involving $\tau^\pm$-lepton is concentrated on $B \to D^{(*)}\tau^\pm\nu_\tau$ due to the charged-current transition
- During the last several years, this field was driven by the anomalies indicating Lepton-Flavor-Univrsality (LFU) violation in the FCNC and CC decays. However, these anomalies are receding due to improved experimental precision

GIM Mechanism in FCNC B-Meson Decays

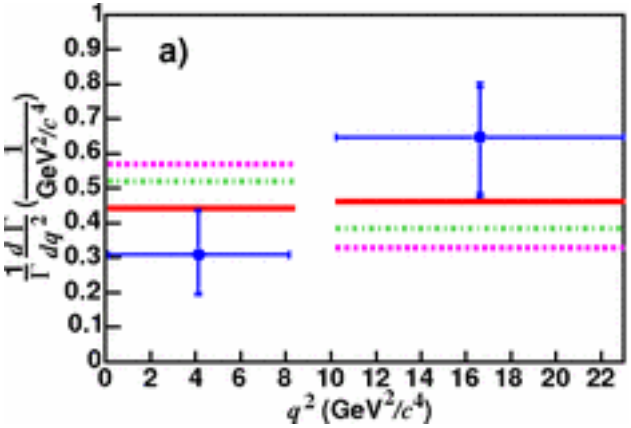
Lowest-order amplitudes for $B^+ \to K^+\ell^+\ell^-$ and $B_s^0 \to \mu^+\mu^-$ in the Standard Model



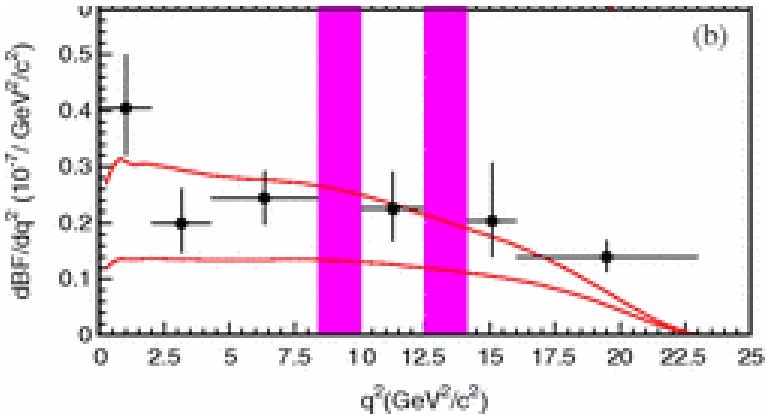
"New Physics" can generate additional amplitudes

FCNC semileptonic B-meson decays at e^+e^--colliders

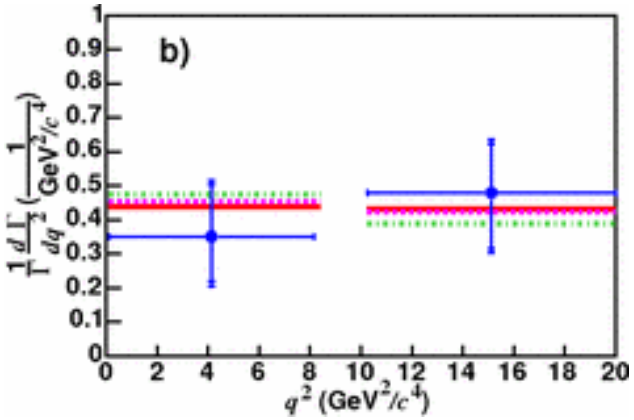
$B \to K\mu^+\mu^-$ [BaBar, PRD 73 (2006) 092001]



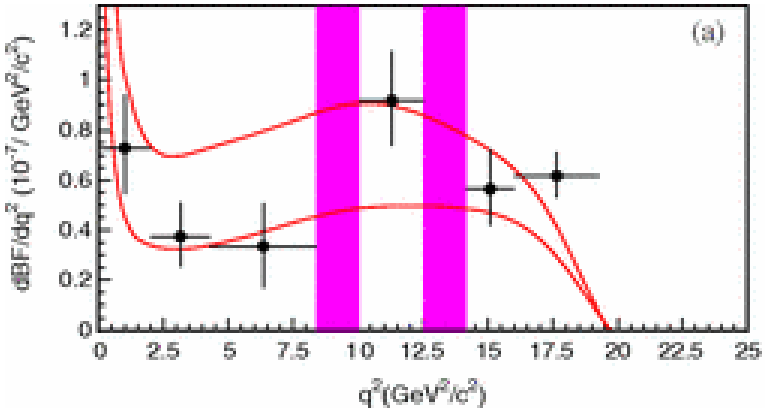
[Belle, PRL 103 (2009) 171801]



$B \to K^*\mu^+\mu^-$ [BaBar, PRD 73 (2006) 092001]

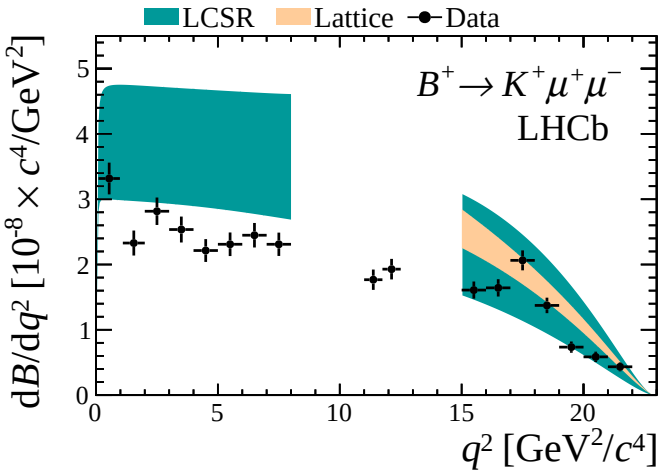


[Belle, PRL 103 (2009) 171801]

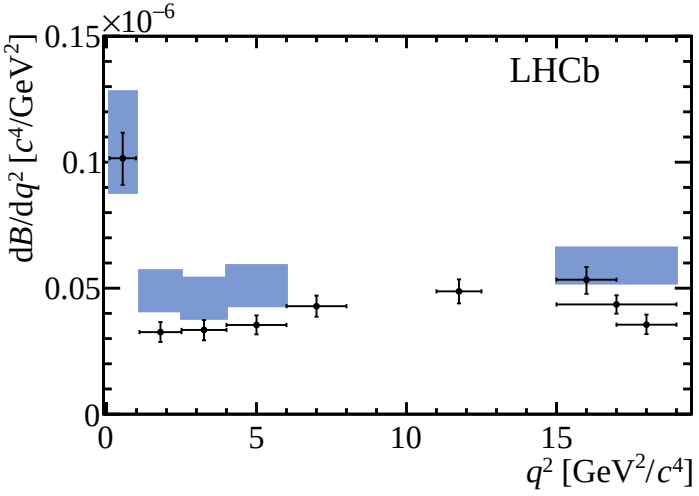


FCNC semileptonic B-meson decays from LHCb

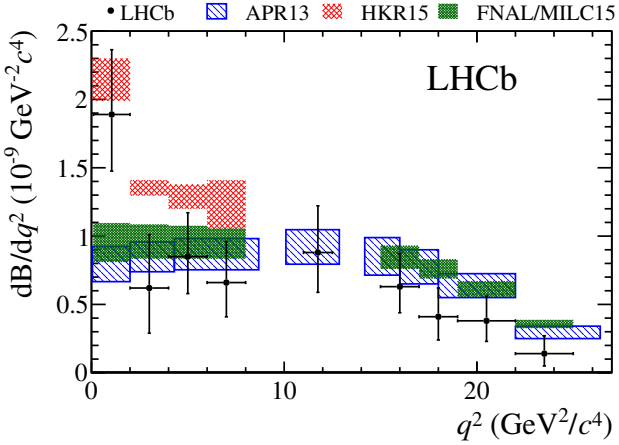
$B^+ \to K^+\mu^+\mu^-$ [JHEP 06 (2014) 133]



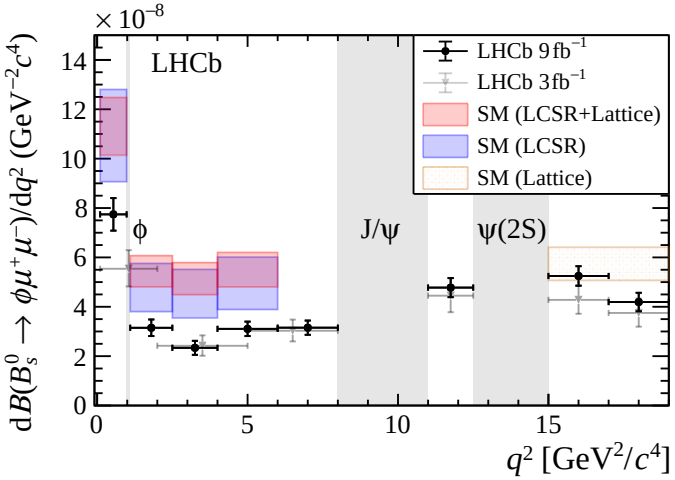
$B^0 \to K^{*0}\mu^+\mu^-$ [JHEP 11 (2016) 047]



$B^+ \to \pi^+\mu^+\mu^-$ [JHEP 10 (2015) 034]

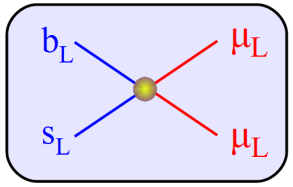


$B_s^0 \to \phi\mu^+\mu^-$ [PRL 127 (2021) 151801]

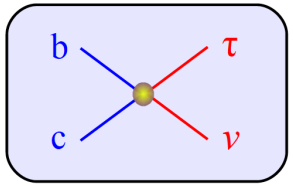


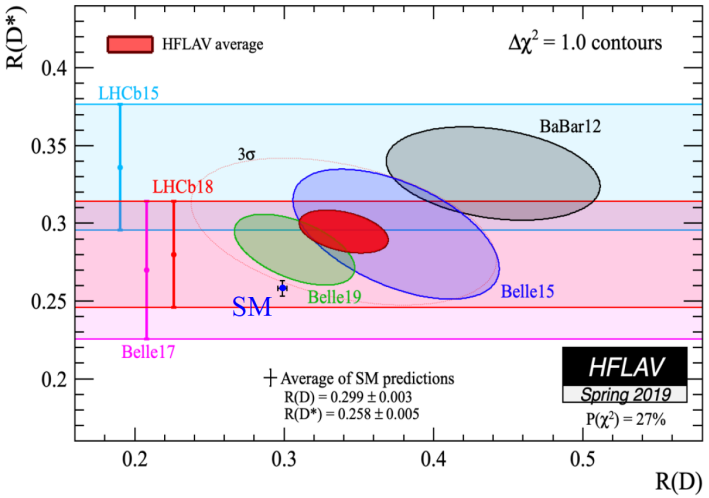
The anomalies: a brief recap

- $b \to s\, l^+ l^-$ (neutral currents): μ vs. e



- $b \to c\, l\nu$ (charged currents): τ vs. light leptons (μ, e)





$$R(X) = \frac{\Gamma(B \to X\, \tau\nu)}{\Gamma(B \to X\, l\nu)} \quad X = D \text{ or } D^*$$

- Clean SM predictions (*uncertainties cancel in the ratios*)
- Consistent results by 3 different exp.ts: 3.1σ excess over SM
- Slower progress

- $R_{D^{*-}} = 0.247 \pm 0.015 \pm 0.015$ [LHCb '22]
- $R_D = 0.307 \pm 0.037 \pm 0.016$ [Belle '20]
- Both are in agreement with their SM expectations [HFLAV '22]

Test of LFU using $B^\pm \to K^\pm \ell^+ \ell^-$ decays

- LHCb measurement of the $B^\pm \to K^\pm \mu^+ \mu^-$ to $B^\pm \to K^\pm e^+ e^-$ BF ratio [R. Aaij *et al.* (LHCb), PRL 113 (2014) 151601]

$$R_K \equiv \frac{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} d\Gamma/dq^2[B^\pm \to K^\pm \mu^+ \mu^-]dq^2}{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} d\Gamma/dq^2[B^\pm \to K^\pm e^+ e^-]dq^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

- Evidence of LFU in this ratio [R. Aaij *et al.* (LHCb), Nature Phys. 18 (2022) 277]

$$R_K = 0.846^{+0.042\ +0.013}_{-0.039\ -0.012}$$

- SM prediction [L. S. Geng *et al.*, Phys. Rev. D 96 (2017) 0930096]

$$R_K^{\mathrm{th}} = 1.0004 \pm 0.0008 \implies 3.1\sigma \text{ deviation}$$

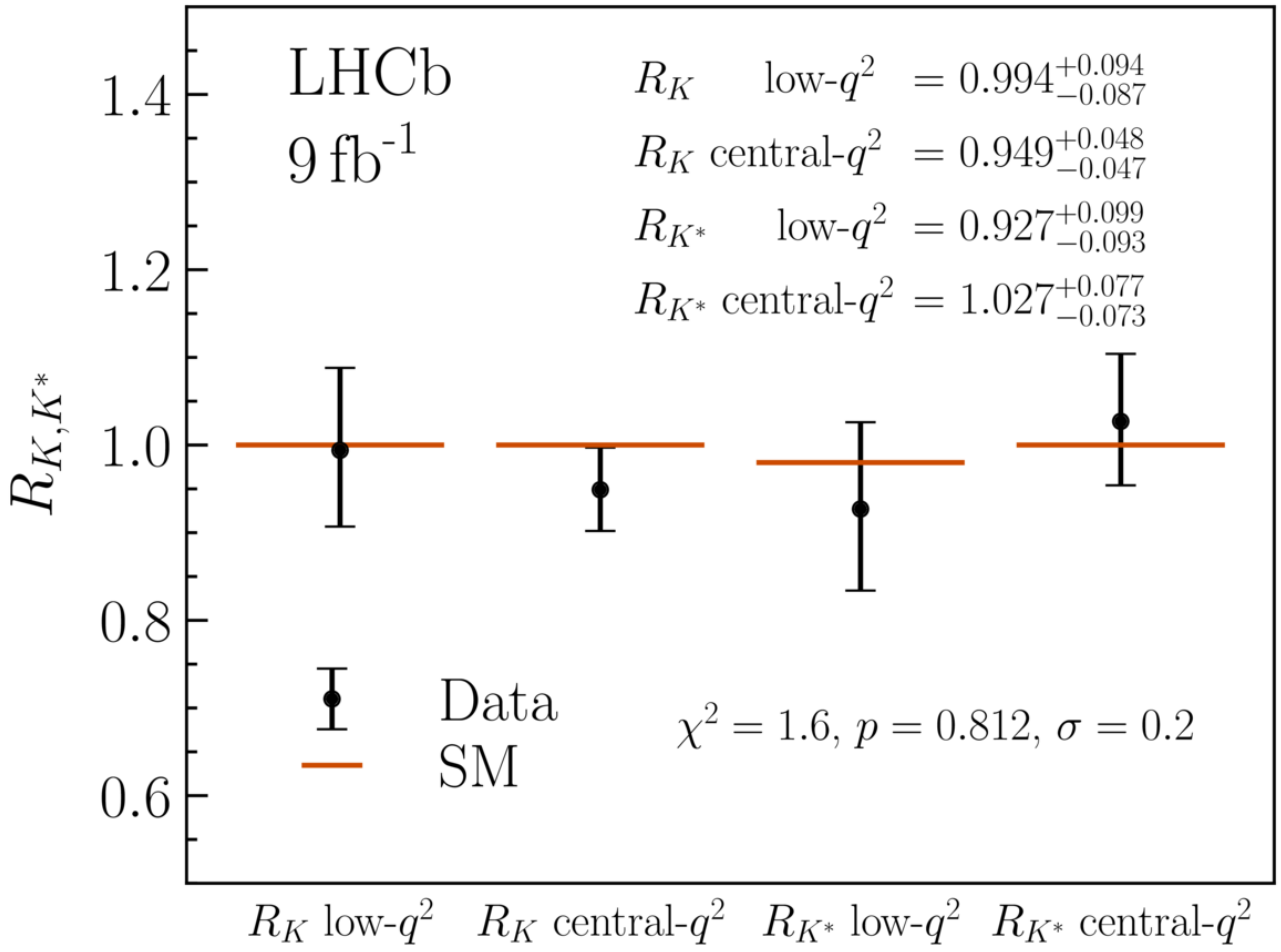
- Led to theoretical model building, such as $\mathcal{O}$ (TeV) leptoquarks
- Belle result [S. Choudhury *et al.* (Belle), JHEP 03 (2021) 105]

$$R_K = 1.03^{+0.28}_{-0.24} \pm 0.01$$

- However, this apparent LFU-violation is now resolved through improved electron identification

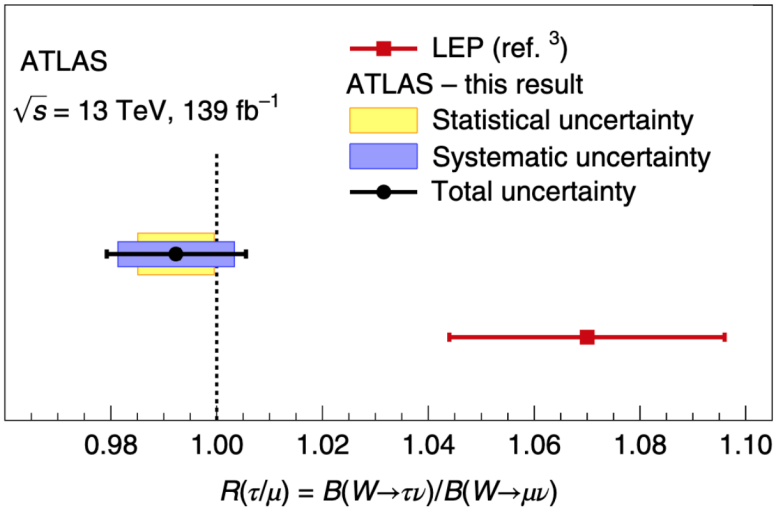
2022 LHCb Update of R_K

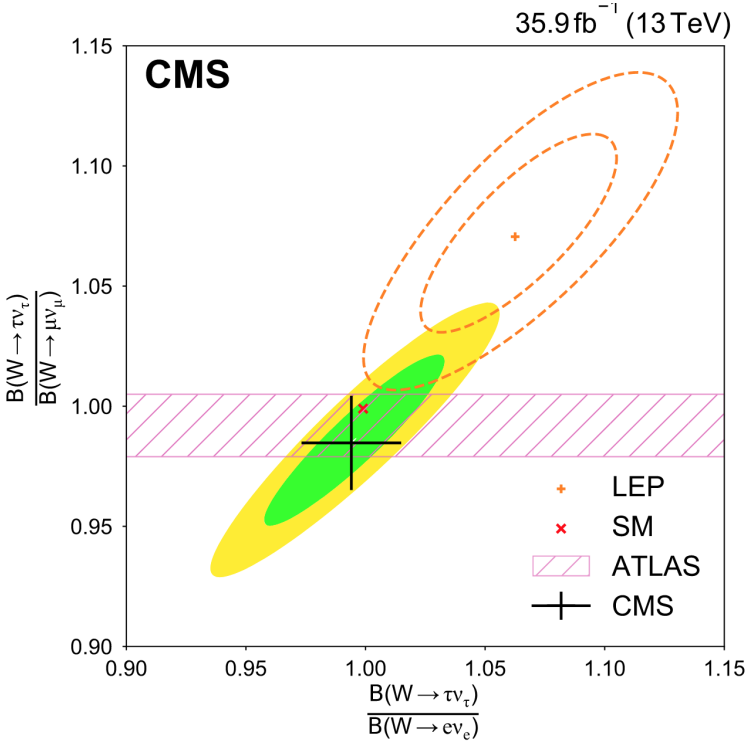
R. Aaji *et al.* (LHCb), Phys. Rev. Lett. 131 (2023) 051803
R. Aaji *et al.* (LHCb), Phys. Rev. D 108 (2023) 032002



Tests of Lepton Universality in $W^{\pm}$-Decays at the LHC

- LEP avarege: $R(\tau/\mu) = 1.066 \pm 0.025$
- $R(\tau/\mu) = 0.992 \pm 0.013$ [ATLAS: Nature Phys. 17 (2021) 813]
- $R(\tau/\mu) = 0.985 \pm 0.020$ [CMS: Phys. Rev. D 105 (2022) 072008]





SM vs. experimental data for $B \to K^{(*)}\tau^+\tau^-$ (current status)

- SM theoretical estimate of the total branching fraction
  [D. Du *et al.*, Phys. Rev. D 93 (2016) 034005]

$$\mathcal{B}_{\rm SM}(B^\pm \to K^\pm\tau^+\tau^-) = (16.036 \pm 0.873 \pm 0.787 \pm 0.546) \times 10^{-8}$$

$$\mathcal{B}_{\rm SM}(B^0 \to K^0\tau^+\tau^-) = (14.745 \pm 0.803 \pm 0.722 \pm 0.502) \times 10^{-8}$$

- Search at the BaBar experiment
  [J. P. Lees *et al.* (BaBar), Phys. Rev. Lett. 118 (2017) 031802]

$$\mathcal{B}_{\rm BaBar}(B^\pm \to K^\pm\tau^+\tau^-) < 2.25 \times 10^{-3} \quad (@\ 90\%\ \text{CL})$$

- Search at the Belle experiment
  [T. V. Dong *et al.* (Belle), Phys. Rev. D 108 (2023) L011102]

$$\mathcal{B}_{\rm Belle}(B^0 \to K^{*0}\tau^+\tau^-) < 2.0 \times 10^{-3} \quad (@\ 90\%\ \text{CL})$$

Effective Electroweak Lagrangian for $b \to s\,(d)$ FCNCs

- Theoretical calculations are convenient to perform within the Effective Electroweak Hamiltonian framework
- Lagrangian density includes quark flavors $q = u,\, d,\, s,\, c,\, b$

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}_{\rm QED}(x) + \mathcal{L}_{\rm QCD}(x) - \mathcal{H}^{b\to d}_{\rm weak}(x) - \mathcal{H}^{b\to s}_{\rm weak}(x)$$

- Flavor-changing neutral current (FCNC) term $\mathcal{H}^{b\to d}_{\rm weak}$ describes $b \to d$ transition

$$\mathcal{H}^{b\to d}_{\rm weak} = \frac{4G_F}{\sqrt{2}} \Bigg\{ V_{ud} V^*_{ub} \left[C_1(\mu)\, \mathcal{P}^{(u)}_1(\mu) + C_2(\mu)\, \mathcal{P}^{(u)}_2(\mu) \right] + V_{cd} V^*_{cb} \left[C_1(\mu)\, \mathcal{P}^{(c)}_1(\mu) + C_2(\mu)\, \mathcal{P}^{(c)}_2(\mu) \right] - V_{td} V^*_{tb} \sum_{j=3}^{10} C_j(\mu)\, \mathcal{P}_j(\mu) \Bigg\} + \text{h. c.}$$

- G_F is the Fermi constant; $V_{q_1 q_2}$ is the CKM matrix element; $C_j(\mu)$ are Wilson coefficients; $\mathcal{P}_j(\mu)$ are local $b \to d$ transition operators

The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

- Charged weak currents involving quarks

$$J^\mu W^+_\mu = \bar{U}_L \gamma^\mu W^+_\mu V_{\rm CKM} D_L$$

- Quark mixing matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- In terms of the Wolfenstein parameters (λ, A, ρ, η)

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2(1+i\lambda^2\eta) & 1 \end{pmatrix}$$

- Values are determined phenomenologically
  [R. L. Workman *et al.* (PDG), Prog. Theor. Exp. Phys. 2022 (2022) 083C01]

$$\lambda = 0.22500 \pm 0.00067 \quad \bar{\rho} = \rho(1-\lambda^2/2) = 0.159 \pm 0.010$$
$$A = 0.826^{+0.018}_{-0.015} \quad \bar{\eta} = \eta(1-\lambda^2/2) = 0.348 \pm 0.010$$

- $b \to d$ transitions involve the highly suppressed CKM matrix element V_{td} which, however, has a large weak phase

Local Operator Basis

- $\mathcal{P}_j(\mu)$ basis consists of dimension-6 operators
- Tree operators $(p = u,\, c)$
  $\mathcal{P}_1^{(p)} = (\bar{d}\gamma_\mu L T^a p)(\bar{p}\gamma^\mu L T^a b) \quad \mathcal{P}_2^{(p)} = (\bar{d}\gamma_\mu L p)(\bar{p}\gamma^\mu L b)$
- Penguin operators $(q = u,\, d,\, s,\, c,\, b)$
  $\mathcal{P}_3 = (\bar{d}\gamma_\mu L b)\sum_q(\bar{q}\gamma^\mu q) \quad \mathcal{P}_4 = (\bar{d}\gamma_\mu L T^a b)\sum_q(\bar{q}\gamma^\mu T^a q)$
  $\mathcal{P}_5 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L b)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho q)$
  $\mathcal{P}_6 = (\bar{d}\gamma_\mu\gamma_\nu\gamma_\rho L T^a b)\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T^a q)$
- Electromagnetic and chromomagnetic dipole operators
  $\mathcal{P}_{7\gamma} = \frac{e}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L) b\right] F_{\mu\nu}$
  $\mathcal{P}_{8g} = \frac{g_{st}}{16\pi^2}\left[\bar{d}\sigma^{\mu\nu}(m_b R + m_d L) T^a b\right] G^a_{\mu\nu}$
- Semileptonic operators
  $\mathcal{P}_{9\ell} = \frac{\alpha}{2\pi}(\bar{d}\gamma_\mu L b)\sum_\ell(\bar{\ell}\gamma^\mu \ell)$
  $\mathcal{P}_{10\ell} = \frac{\alpha}{2\pi}(\bar{d}\gamma_\mu L b)\sum_\ell(\bar{\ell}\gamma^\mu\gamma_5 \ell)$
- d-quark mass is neglected $(m_d = 0)$

$B \to P$ Transition Matrix Elements

$$\langle P(k)|\bar{p}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + k^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

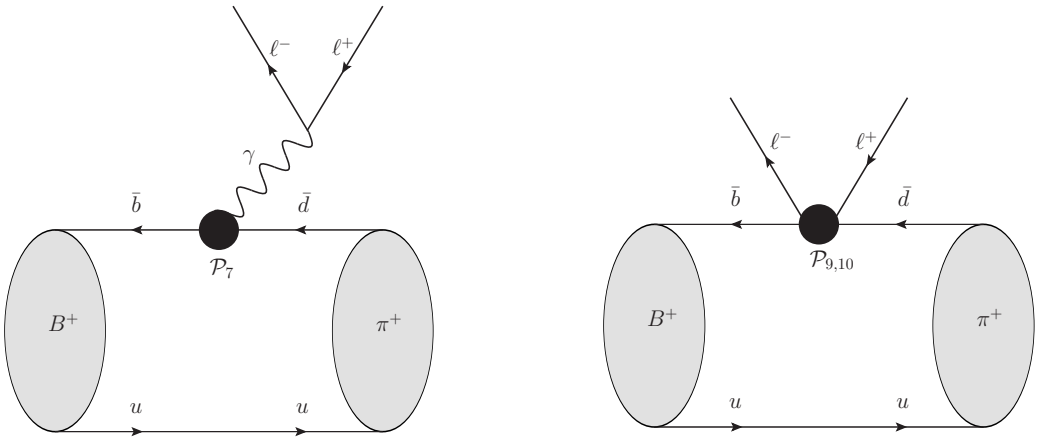
$$\langle P(k)|\bar{p}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 0$$

$$\langle P(k)|\bar{p}\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = i\left[(p_B^\mu + k^\mu)q^2 - q^\mu\left(m_B^2 - m_P^2\right)\right]\frac{f_T(q^2)}{m_B + m_P}$$

$$\langle P(k)|\bar{p}\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 0$$

- $q^\mu = p_B^\mu - k^\mu$ is the momentum transferred to leptons
- $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$ are the transition form factors

$B^+ \to \pi^+ \ell^+ \ell^-$ decay



Operators $\mathcal{P}_7$, $\mathcal{P}_9$, and $\mathcal{P}_{10}$ give the dominant contributions to the decay amplitude ($N = \frac{G_F}{\sqrt{2}}\, V_{tb} V_{td}^* \, \frac{e^2}{g_{st}^2}$)

$$\mathcal{M}_9 = N\, C_9 \left\langle \pi(p_\pi) | \bar{d}_L \gamma^\mu b_L | B(p_B) \right\rangle \left[\bar{u}(q_1) \gamma_\mu u(-q_2) \right]$$

$$\mathcal{M}_{10} = N\, C_{10} \left\langle \pi(p_\pi) | \bar{d}_L \gamma^\mu b_L | B(p_B) \right\rangle \left[\bar{u}(q_1) \gamma_\mu \gamma_5 u(-q_2) \right]$$

$$\mathcal{M}_7 = -i\, N \frac{2 m_b}{q^2}\, C_7 \left\langle \pi(p_\pi) | \bar{d}_L \sigma^{\mu\nu} q_\nu b_R | B(p_B) \right\rangle \left[\bar{u}(q_1) \gamma_\mu u(-q_2) \right]$$

$B \to P\ell^+\ell^-$ differential branching fraction

$$\frac{d\mathrm{Br}\,(B \to P\ell^+\ell^-)}{dq^2} = S_P \frac{2G_F^2\alpha^2\tau_B}{3(4\pi)^5 m_B^3} |V_{tb}V_{tp}^*|^2 \beta_\ell \lambda^{3/2}(q^2) F^{BP}(q^2)$$

Dynamical function

$$F^{BP}(q^2) = F_{97}^{BP}(q^2) + F_{10}^{BP}(q^2)$$

$$F_{97}^{BP}(q^2) = \left(1 + \frac{2m_\ell^2}{q^2}\right) \Big| C_9^{\mathrm{eff}}(q^2)\, f_+^{BP}(q^2)$$

$$+ \frac{2m_b}{m_B + m_P}\, C_7^{\mathrm{eff}}(q^2)\, f_T^{BP}(q^2) + L_A^{BP}(q^2) + \Delta C_V^{BP}(q^2) \Big|^2$$

$$F_{10}^{BP}(q^2) = \left(1 - \frac{4m_\ell^2}{q^2}\right) \left| C_{10}^{\mathrm{eff}}\, f_+^{BP}(q^2) \right|^2 + \frac{6m_\ell^2}{q^2} \frac{\left(m_B^2 - m_P^2\right)^2}{\lambda(q^2)} \left| C_{10}^{\mathrm{eff}}\, f_0^{BP}(q^2) \right|^2$$

S_P is the final-meson flavor factor ($S_{\pi^\pm} = 1$ and $S_{\pi^0} = 1/2$)

$p = s$, d denotes light quark in the $b \to p$ transition

$\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$, $\quad \lambda(q^2) = (m_B^2 + m_P^2 - q^2)^2 - 4m_B^2 m_P^2$

Effective Wilson Coefficients

- Branching fractions of B-meson decays induced by $b \to s\,(d)\,\ell^-\ell^+$ transitions are expressed through $C_7^{\rm eff}$, $C_9^{\rm eff}$, and $C_{10}^{\rm eff}$
- In NNLO, effective Wilson coefficients are given as [Asatrian et al. PRD 69 (2004) 074007]

$$
\begin{aligned}
C_7^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_7(\hat{s})\right] A_7 - \frac{\alpha_s}{4\pi}\left[C_1^{(0)}\,F_{1,c}^{(7)} + C_2^{(0)}\,F_{2,c}^{(7)} + \sum_{k=3}^{6} C_k^{(0)}\,F_k^{(7)} + A_8^{(0)} F_8^{(7)}\right] \\
&\quad - \frac{\alpha_s}{4\pi}\,\xi^{(q)}\left\{C_1^{(0)}\left[F_{1,c}^{(7)} - F_{1,u}^{(7)}\right] + C_2^{(0)}\left[F_{2,c}^{(7)} - F_{2,u}^{(7)}\right]\right\} \\
C_9^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_9(\hat{s})\right]\left\{A_9 + T_9\,h(\hat{m}_c^2,\hat{s}) + U_9\,h(1,\hat{s}) + W_9\,h(0,\hat{s}) + \xi^{(s)}\,T_{9a}\right. \\
&\quad \left.\times\,\left[h(\hat{m}_c^2,\hat{s}) - h(0,\hat{s})\right]\right\} - \frac{\alpha_s}{4\pi}\left[C_1^{(0)}\,F_{1,c}^{(9)} + C_2^{(0)}\,F_{2,c}^{(9)} + \sum_{k=3}^{6} C_k^{(0)}\,F_k^{(9)} + A_8^{(0)} F_8^{(9)}\right] \\
&\quad - \frac{\alpha_s}{4\pi}\,\xi^{(q)}\left\{C_1^{(0)}\left[F_{1,c}^{(9)} - F_{1,u}^{(9)}\right] + C_2^{(0)}\left[F_{2,c}^{(9)} - F_{2,u}^{(9)}\right]\right\} \\
C_{10}^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_{10}(\hat{s})\right] A_{10}
\end{aligned}
$$

- $\hat{m}_c = m_c/m_b; \quad \xi^{(q)} = V_{ub}V_{uq}^*/(V_{tb}V_{tq}^*) \quad (q = d,\, s)$
- $\hat{s} = q^2/m_b^2$ is scaled lepton-pair momentum squared

Form Factor Parameterizations

- Boyd-Grinstein-Lebed (BGL) parameterization ($i = +,\ 0,\ T$)

$$f_i(q^2) = \frac{1}{P_i(q^2)\,\phi_i(q^2, q_0^2)} \sum_{k=0}^{N} a_k^{(i)}\, z^k(q^2, q_0^2)$$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}$$

$$m_+ = m_B + m_\pi, \quad q_0^2 = 0.65\,(m_B - m_\pi)^2$$

- Blaschke factor: $P_{i=+,T}(q^2) = z(q^2, m_{B^*}^2)$ and $P_0(q^2) = 1$
- $\phi_i(q^2, q_0^2)$ is an outer function depending on isospin factor and three parameters K_i, α_i, and β_i
- FFs $f_{+,T}(q^2)$ have poles at the mass squared of vector B^*-meson while $f_0(q^2)$ is free from poles
- Values of $a_k^{(+)}$, $a_k^{(0)}$ & $a_k^{(T)}$ are borroed from [A. Ali, AP & A. Rusov, Phys. Rev.D 89 (2014) 094021]

Form Factor parametrizations

- Bourrely-Caprini-Lellouch (BCL) parameterization ($i = +,\ T$)

$$f_i(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{N-1} b_k^{(i)} \left[z^k(q^2, q_0^2) - (-1)^{k-N} \frac{k}{N} z^N(q^2, q_0^2) \right]$$

$$f_0(q^2) = \sum_{k=0}^{N-1} b_k^{(0)} z^k(q^2, q_0^2)$$

$$m_+ = m_B + m_\pi, \quad q_0^2 = m_+ \left(\sqrt{m_B} - \sqrt{m_\pi}\right)^2$$

- Form factors are considered as truncated series at $N = 4$
- Values of $b_k^{(+)}$, $b_k^{(T)}$ & $b_k^{(0)}$ are borroed from [J. A. Bailey *et al.* [Fermilab Lattice and MILC], Phys. Rev. Lett. 115 (2015) 152002; Phys. Rev. D 92 (2015) 014024]

Form Factor parametrizations

- Modified Bourrely-Caprini-Lellouch (mBCL) parameterization ($i = +,\ T$)

$$f_i(q^2) = \frac{f_i(q^2=0)}{1 - q^2/m_{B^*}^2}\left\{1 + \sum_{k=1}^{N-1} b_k^{(i)}\left[\bar{z}_k(q^2, q_0^2) - (-1)^{k-N}\,\frac{k}{N}\,\bar{z}_N(q^2, q_0^2)\right]\right\}$$

$$f_0(q^2) = \frac{f_+(q^2=0)}{1 - q^2/m_{B_0}^2}\left[1 + \sum_{k=1}^{N} b_k^{(0)}\,\bar{z}_k(q^2, q_0^2)\right]$$

- $\bar{z}_n(q^2, q_0^2) = z^n(q^2, q_0^2) - z^n(0, q_0^2)$
- q_0^2 is chosen as the optimal value
- $f_0(q^2)$ has also pole but at scalar B_0-meson mass squared
- Values of $b_k^{(+)}$, $b_k^{(T)}$ & $b_k^{(0)}$ are borroed from [D. Leljak, B. Melić & D. van Dyk, JHEP 07 (2021) 036]

SM vs. experimental data (current status)

- SM theoretical estimate of the total branching fraction [A. Ali, AP & A. Rusov, Phys. Rev.D 89 (2014) 094021]

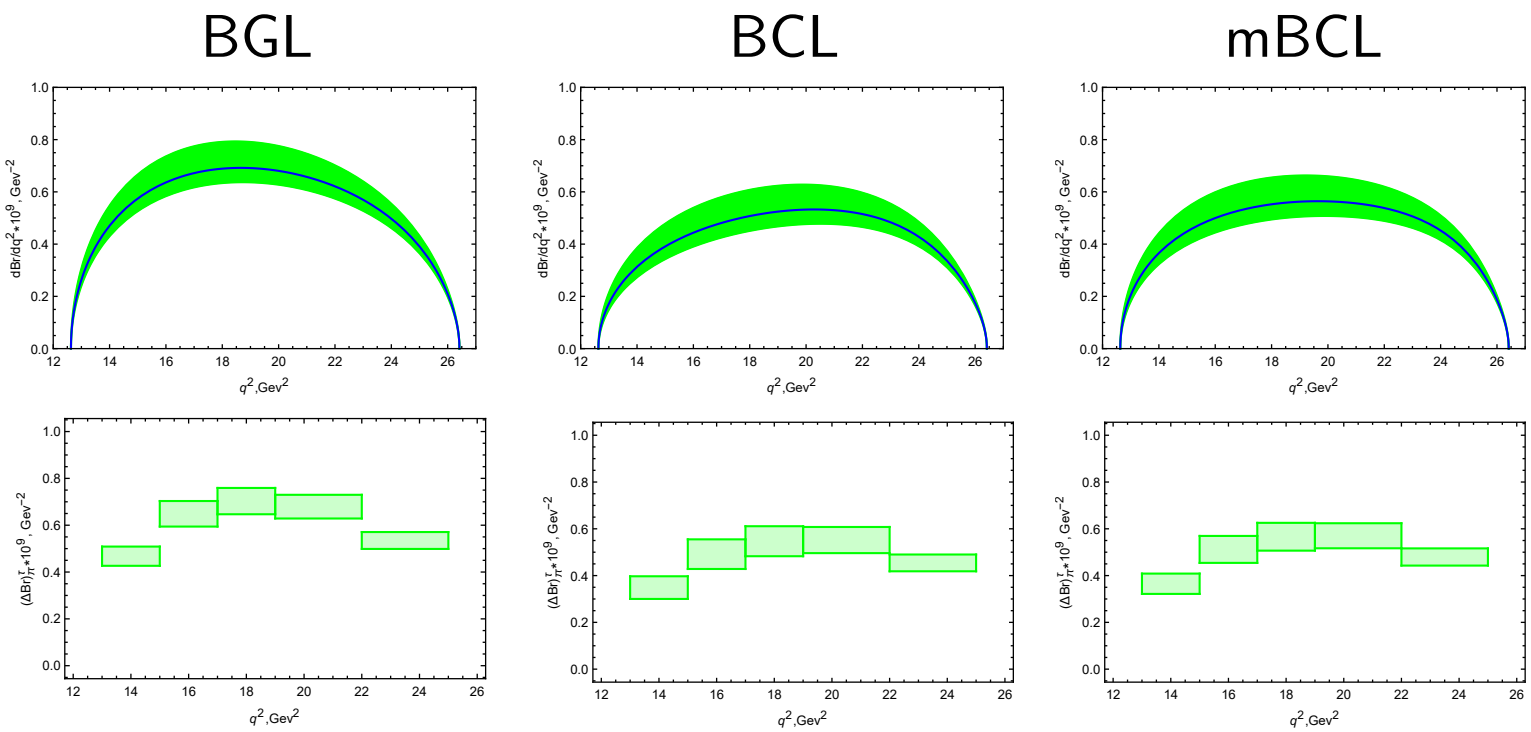
$$\mathrm{BR}_{\mathrm{SM}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- LHCb has measured the BR and dimuon invariant mass distribution in $B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}$ based on 3 fb^{-1} integrated luminosity [R. Aaij *et al.* (LHCb): JHEP 10 (2015) 034, arXiv:1509.00414]:

$$\mathrm{BR}_{\mathrm{LHCb}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = (1.83 \pm 0.24(\mathrm{stat}) \pm 0.05(\mathrm{syst})) \times 10^{-8}$$

- Allow to determine V_{td} but not precise: $|V_{td}| = (7.2^{+0.8}_{-0.9}) \times 10^{-3}$ vs. $|V_{td}| = (8.0 \pm 0.3) \times 10^{-3}$ from ΔM_{B_d}
- Precise measurements at the LHCb and Belle II will search for BSM physics in Boxes and Penguins in the b-quark sector involving $b \to d$ transitions

Dilepton invariant-mass distribution in $B^+ \to \pi^+\tau^+\tau^-$



Total branching fraction for $B^+ \to \pi^+\tau^+\tau^-$ decay

	BGL	BCL	mBCL	Faustov, Galkin EPJC 74 (2014) 2911	Wang, Xiao PRD 86 (2012) 114025
$\mathrm{Br_{th}} \times 10^9$	$7.56^{+0.74}_{-0.43}$	$6.00^{+0.81}_{-0.49}$	$6.28^{+0.76}_{-0.46}$	7.00 ± 0.70	$6.00^{+2.60}_{-2.10}$

Long Distance Contributions (LDCs) in $B^+ \to K^+\mu^+\mu^-$ decay

R. Aaij et al. [LHCb Collab.], EPJC 77 (2017) 161

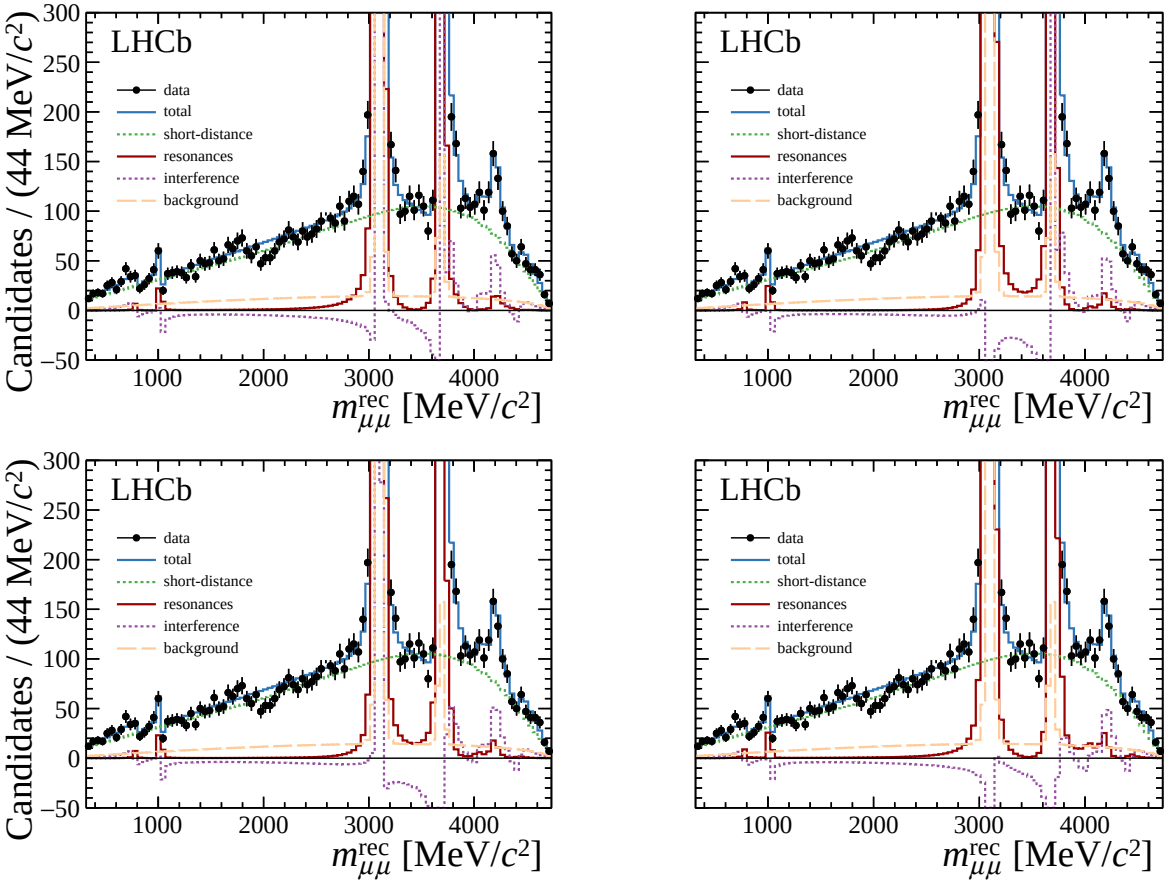
- Measurement of phase differences between the Short- and Long-Distance contributions in $B^+ \to K^+\mu^+\mu^-$ decay is performed by analysing the dimuon mass distribution
- Analysis is based on data corresponding to 3 fb^{-1} integrated luminosity collected by LHCb experiment in 2011 and 2012
- Resonances are entering C_9^{eff} Wilson coefficient

$$C_9^{\text{eff}} = C_9 + \sum_V \frac{\eta_V e^{i\delta_V} m_V \Gamma_V^{(0)}}{(m_V^2 - q^2) - i m_V \Gamma_V(q^2)}, \quad \Gamma_V(q^2) = \frac{p}{p_{0V}} \frac{m_V}{\sqrt{q^2}} \Gamma_V^{(0)}$$

- Phases δ_V of relativistic Breit–Wigner amplitudes representing different vector mesons $V = \rho^0, \omega, J/\psi, \psi(2S)$, $\psi(3S)$, $\psi(4040)$, $\psi(4160)$, $\psi(4230)$, $\psi(4415)$ decaying to muon pair in $B \to V(\to \mu^+\mu^-) + K$ are defined
- In dependence on signs of J/ψ- and $\psi(2S)$-meson amplitude phases, four solutions are found

LDCs in $B^+ \to K^+\mu^+\mu^-$ decay

R. Aaij et al. [LHCb Collab.], EPJC 77 (2017) 161



Long-Distance Contributions (LDCs)

- LDCs are from $B \to V(\to \ell^+\ell^-) + \pi$ decays, where $V = \rho^0, \omega, \phi, J/\psi, \psi(2S), \ldots$ are neutral vector mesons
- LDC representation [C. Hambrock *et al.*, PRD 92 (2015) 074020]

$$\Delta C_V^{B\pi}(q^2) = -16\pi^2 \frac{V_{ub}V_{ud}^* H^{(u)}(q^2) + V_{cb}V_{cd}^* H^{(c)}(q^2)}{V_{tb}V_{td}^*}$$

$$H^{(p)}(q^2) = (q^2 - q_0^2) \sum_V \frac{k_V f_V A_{BV\pi}^p}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})}$$

- k_V is valence quark content factor, m_V, f_V and $\Gamma_V^{\rm tot}$ are the mass, decay constant and total width of vector meson, $A_{BV\pi}^p$ $(p = u,\, c)$ are transition amplitudes, free parameter $q_0^2 = -1.0$ GeV^2
- Spectrum of $B^+ \to \pi^+\tau^+\tau^-$ is inside $q^2 \in [4m_\tau^2, (m_B - m_\pi)^2]$
- $\psi(2S)$ and higher charmonia contribute

Data on vector charmonia used: $\psi(2S)$ and higher

[Workman et al. [PDG], PTEP 2022 (2022) 083C01]

- Decay through intermediate $\psi(2S)$-meson

$$\mathrm{Br}(B^+ \to \pi^+\psi(2S) \to \pi^+\tau^+\tau^-) = (7.6 \pm 1.3) \times 10^{-8}$$

- Decay through $\psi(3S) = \psi(3770)$ (used $SU(3)_F$-symmetry)

$$\mathrm{Br}(B^+ \to \pi^+\psi(3S) \to \pi^+\tau^+\tau^-) = (5.4 \pm 1.9) \times 10^{-11}$$

- Electronic decays of $\psi(3770)$ and higher charmonia are compatable

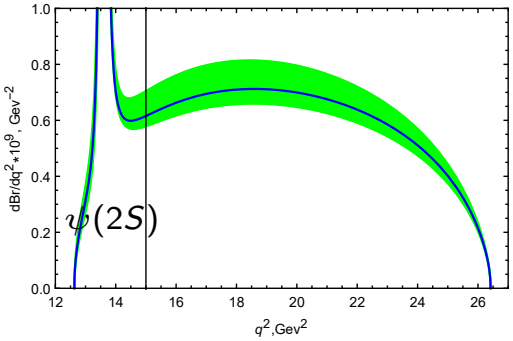
$$\mathrm{Br}(\psi(3770) \to e^+e^-) = (9.6 \pm 0.7) \times 10^{-6}$$
$$\mathrm{Br}(\psi(4040) \to e^+e^-) = (10.7 \pm 1.6) \times 10^{-6}$$
$$\mathrm{Br}(\psi(4160) \to e^+e^-) = (6.9 \pm 3.3) \times 10^{-6}$$
$$\mathrm{Br}(\psi(4230) \to e^+e^-) = (31 \pm 28) \times 10^{-6}$$
$$\mathrm{Br}(\psi(4360) \to e^+e^-) = (0.10 \pm 0.05) \times 10^{-6}$$
$$\mathrm{Br}(\psi(4415) \to e^+e^-) = (9.4 \pm 3.2) \times 10^{-6}$$

- $\mathrm{Br}(V \to \tau^+\tau^-)$ follows after lepton-pair phase space is corrected
- $\mathrm{Br}(B^+ \to \pi^+ V \to \pi^+\tau^+\tau^-)$ are also comparable

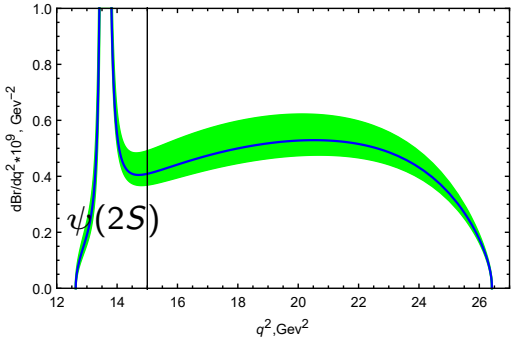
$B^+ \to \pi^+\tau^+\tau^-$ dilepton invariant-mass distribution with LDCs

- q^2-threshold in $B^+ \to \pi^+\tau^+\tau^-$ is $4m_\tau^2 = 12.6$ GeV^2
- Tauonic invariant-mass distribution includes perturbative, $\psi(2S)$-meson and higher charmonia contributions
- LDCs from $\psi(3S)$ and higher charmonia have three orders of magnitude suppression in comparison with $\psi(2S)$ LDC; comparable with the perturbative uncertainty and, hence, neglected
- Take into account $B \to \psi(2S)(\to \tau^+\tau^-) + \pi$ contribution only

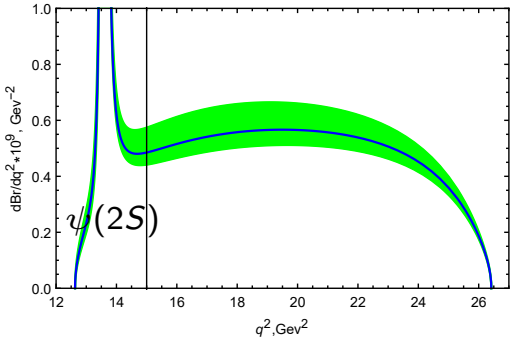
BGL



BCL



mBCL



- Phase Variation in $\psi(2S)$-amplitude gives $\lesssim 10\%$ effect on total BF
- For $\delta^{(u)}_{\psi(2S)} = 0$ & $\delta^{(c)}_{\psi(2S)} = 3\pi/4$, $\mathcal{B}_{\rm th} = (6.05^{+0.80}_{-0.47}) \times 10^{-9}$ (BCL FFs.)

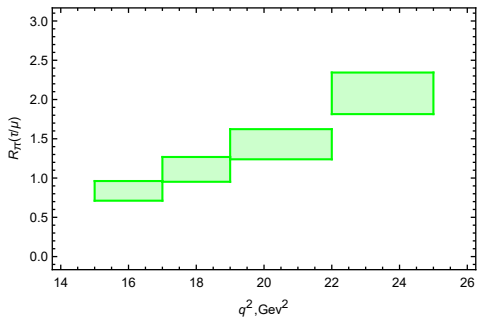
Theoretical predictions for the ratio $R_\pi^{(\tau/\mu)}$

$$R_\pi^{(\tau/\mu)}(q_1^2, q_2^2) = \frac{(\Delta\mathrm{Br})_\pi^\tau(q_1^2, q_2^2)}{(\Delta\mathrm{Br})_\pi^\mu(q_1^2, q_2^2)}, \quad (\Delta\mathrm{Br})_\pi^\ell(q_1^2, q_2^2) \equiv \int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\mathrm{Br}(B^+ \to \pi^+\ell^+\ell^-)}{dq^2}$$

Theoretical predictions for partial ratios $R_\pi^{(\tau/\mu)}(q_1^2, q_2^2)$

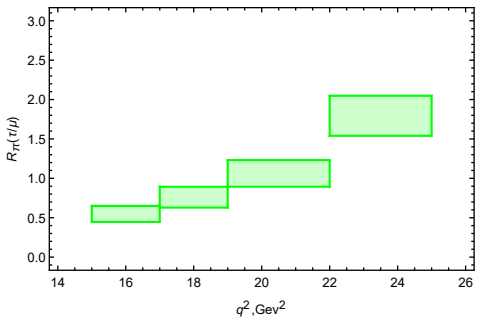
	$R_\pi^{(\tau/\mu)}(q^2_{\mathrm{min}}, q^2_{\mathrm{max}})$		
$[q^2_{\mathrm{min}}, q^2_{\mathrm{max}}]$ (GeV^2)	BGL	BCL	mBCL
$[15.0, 17.0]$	0.84 ± 0.13	0.55 ± 0.10	0.64 ± 0.12
$[17.0, 19.0]$	1.11 ± 0.16	0.76 ± 0.13	0.86 ± 0.15
$[19.0, 22.0]$	1.43 ± 0.19	1.06 ± 0.17	1.17 ± 0.18
$[22.0, 25.0]$	2.08 ± 0.27	1.79 ± 0.25	1.86 ± 0.26

BGL



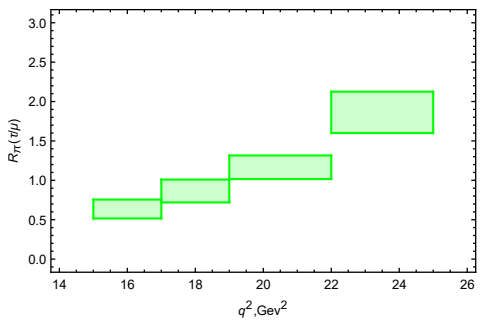
$R_\pi^{\mathrm{BGL}}(\tau/\mu) = 0.44 \pm 0.16$

BCL



$R_\pi^{\mathrm{BCL}}(\tau/\mu) = 0.31 \pm 0.12$

mBCL



$R_\pi^{\mathrm{mBCL}}(\tau/\mu) = 0.37 \pm 0.15$

Summary and outlook

- Searches of Lepton-Flavor-Violation in FCNC B-decays is still a work in progress. The decays $b \to s(d)\tau^+\tau^-$ remain essentially unexplored - a target for Belle II
- The exclusive decay rate for $B^\pm \to \pi^\pm\mu^+\mu^-$ and dimuon invariant mass spectrum measured by LHCb are in agreement with the SM. $\mathcal{B}(B \to \pi e^+e^-) = \mathcal{B}(B \to \pi\mu^+\mu^-)$ in the SM. Theoretical predictions for $\mathcal{B}(B^\pm \to \pi^\pm\tau^+\tau^-)$ and ditauon mass spectrum are obtained for three types of $B \to \pi$ form factors
- Total branching fraction depends somewhat on the FFs. For the BCL parameterization $\mathcal{B}_{\rm th}(B^\pm \to \pi^\pm\tau^+\tau^-) = (6.05^{+0.80}_{-0.47}) \times 10^{-9}$, typically a factor of 3 suppression in comparison with the already observed decay $B^\pm \to \pi^\pm\mu^+\mu^-$
- Impact of Long-Distance contributions from $\psi(2S)$ and higher charmonium resonance is analyzed, dominated by $\psi(2S) \to \tau^+\tau^-$
- A large number of exclusive decays induced by $b \to d\ell^+\ell^-$ remains to be measured. Due to the large weak phases in V_{td} (in penguins) and V_{ub} (in annihilation), a large CP violation is expected

Backup Slides

Wilson Coefficients

- At the matching scale μ_W, Wilson coefficients can be calculated as a perturbative expansion

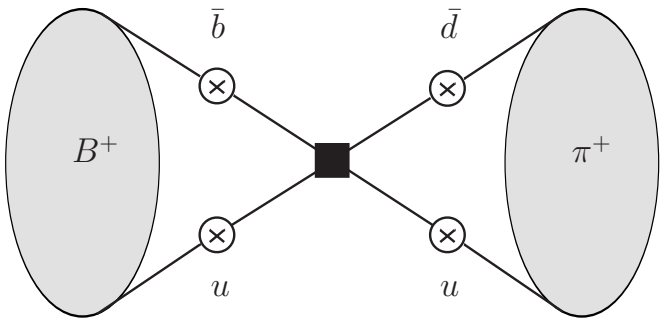
$$C_j(\mu_W) = \sum_{k=0}^{\infty} \left[\frac{\alpha_s(\mu_W)}{4\pi}\right]^k C_j^{(k)}(\mu_W)$$

- Mixed under the operator renormalization
- Evolved to the b-quark scale by RGE
- Hierarchy in the Wilson coefficients exists; in the naive dimensional regularization scheme at the next-to-leading logarithmic (NLL) order

$C_1(m_b)$	-0.146	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	1.056	$C_4(m_b)$	-0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	-0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	-0.039	$C_{10}(m_b)$	1.9×10^{-3}

Weak annihilation contribution in $B^+ \to \pi^+ \ell^+ \ell^-$ decay

Can be calculated within the LEET [Beneke et al. Eur.Phys.J.C41 173-188 (2005)]



$$L_A^{B\pi(t)}(q^2) = Q_q \, \frac{\pi^2}{3} \frac{4 f_B f_\pi}{m_b} \, \lambda_{B,-}^{-1}(q^2) \, C_{34}$$

$$L_A^{B\pi(u)}(q^2) = -Q_q \, \frac{\pi^2}{3} \frac{4 f_B f_\pi}{m_b} \, \lambda_{B,-}^{-1}(q^2) \, C_{12}$$

- Q_q is relative charge of spectator quark
- f_B and f_π are B- and π-meson decay constants
- $C_{34} = C_3 + \frac{4}{3}(C_4 + 12C_5 + 16C_6)$; $C_{12} = 3C_2$ are combinations of Wilson coefficients
- First inverse moment of B-meson LCDA enters these contributions (Grozin-Neubert model is used)

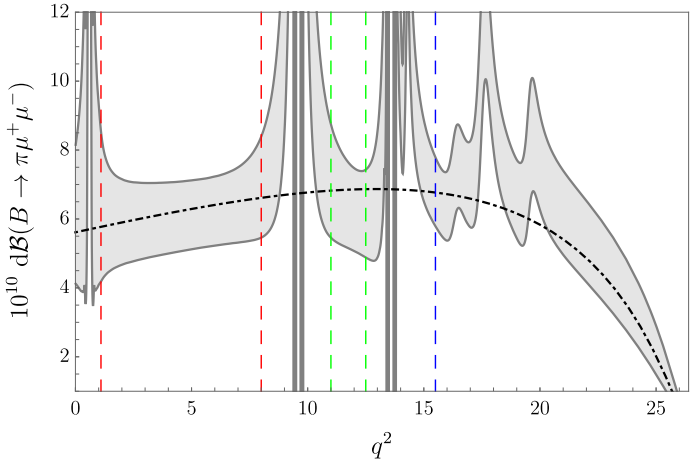
$$\lambda_{B,-}^{-1}(q^2) = \frac{e^{-q^2/(m_B \omega_0)}}{\omega_0} \left[i\pi - Ei(q^2/(m_B \omega_0)) \right]$$

- Here, $Ei(z) = \int\limits_z^{-\infty} dt \, e^t/t$ is the Exponential integral

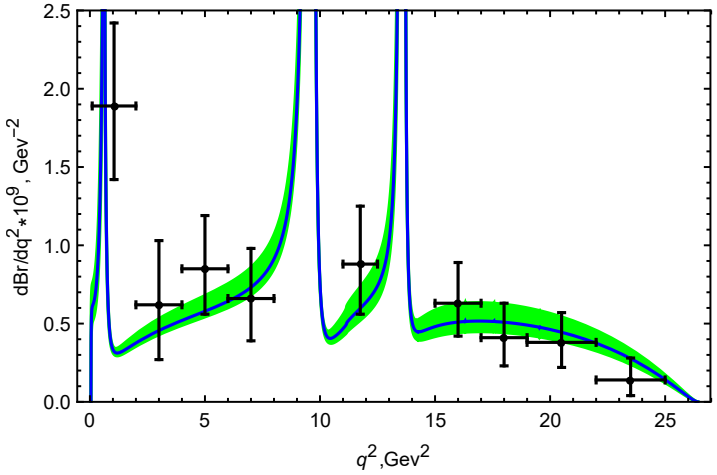
LDCs in $B^+ \to \pi^+\mu^+\mu^-$ decay

- Results of the LHCb analysis were used to calculate Long Distance contributions to $B^+ \to \pi^+\mu^+\mu^-$ decay

[M. Bordone et al., EPJC 81 (2021) 850]



Our BCL-based predictions



- Our theoretical estimates show that charmonia above the open charm threshold give contributions that do not exceed uncertainty of perturbative contribution