Редкие распады мезонов

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THE XV-th INTERNATIONAL SCHOOL-CONFERENCE

"THE ACTUAL PROBLEMS OF MICROWORLD PHYSICS"

Minsk, Belarus, 27 August – 3 September, 2023

- ***Flavour-changing neutral currents:*** $K \to \pi\nu\bar{\nu}; K_L \to \pi^0 l^+ l^-; K^+ \to \pi^+ l^+ l^-; K_L \to \mu^+\mu^-$

- ***Lepton flavour universality tests:*** $\frac{Br(K^+ \to \pi^+\mu^+\mu^-)}{Br(K^+ \to \pi^+ e^+ e^-)}; \frac{Br(K \to \pi\mu\nu)}{Br((K \to \pi e\nu))}$

- ***Lepton flavour and number violating decays***

$$K^+ \to \pi^-(\pi^0) l_1^+ l_2^-, K^+ \to \pi^+\mu^\pm e^\mp, K^+ \to l_1^+\nu l_2^-\nu, K_L \to (\pi^0(\pi^0)\mu^\pm e^\mp)$$

- ***Tests of low-energy QCD***

$$K \to \pi\pi, K \to \pi\pi\pi, K \to \pi\gamma\gamma, K \to l\nu\gamma, K \to \pi\pi\gamma, K \to \pi\pi\gamma, K \to \pi\pi\pi\gamma, K \to \pi e^+ e^-, K \to \pi\pi l\nu$$

- ***CKM first-row unitarity tests*** $K^+ \to (\pi^0) l^+\nu,, K_L \to \pi^\pm l^\mp \nu$

- ***Production of feebly-interacting particles in kaon decays*** $K^+ \to l^+ N, K^+ \to \pi^+ S$

The HIKE Collaboration Letter of Intent
arXiv:2211.16586v1 [hep-ex] 29 Nov 2022

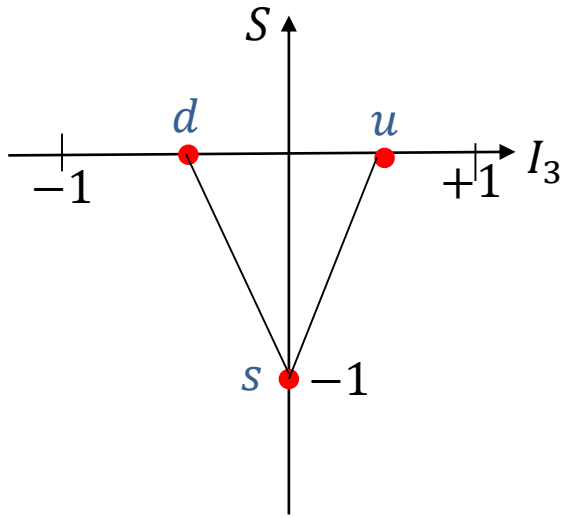
Слабые взаимодействия:

Лептонов:

$$\begin{array}{cccccc} e^- & \mu^- & \tau^- & e^+ & \mu^+ & \tau^+ \\ \downarrow & \downarrow & \downarrow W^- & \downarrow & \downarrow & \downarrow W^+ \\ \nu_e & \nu_\mu & \nu_\tau & \bar{\nu}_e & \bar{\nu}_\mu & \bar{\nu}_\tau \end{array} \qquad \Delta Q = 1$$

Кварков:

$$\begin{array}{ccc} \mathrm{u} & \mathrm{c} & \mathrm{t} \\ \uparrow & \uparrow & \uparrow \\ \mathrm{d} & \mathrm{s} & \mathrm{b} \end{array} \qquad Q = \frac{2}{3} \qquad Q = -\frac{1}{3}$$

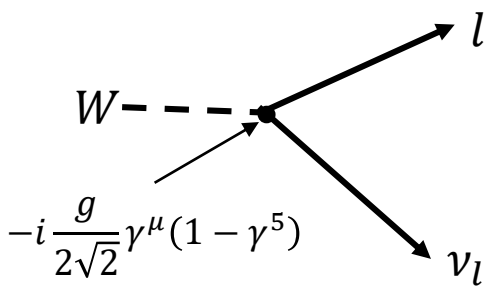


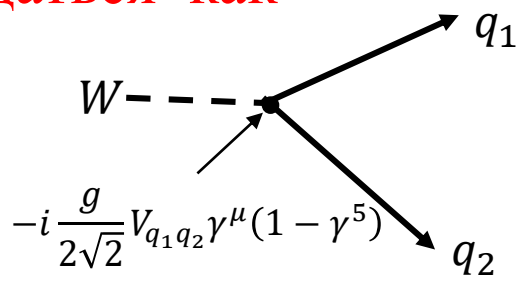
Проекция изоспина I_3 меняется в случае

$s \to u$ на $\frac{1}{2}$

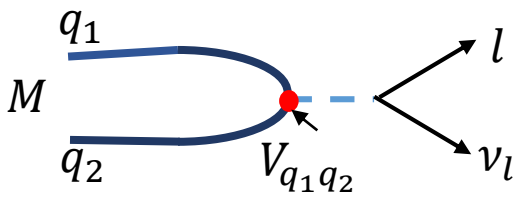
$\mathrm{d} \to u$ на 1

$W^{\pm}$ может распадаться как

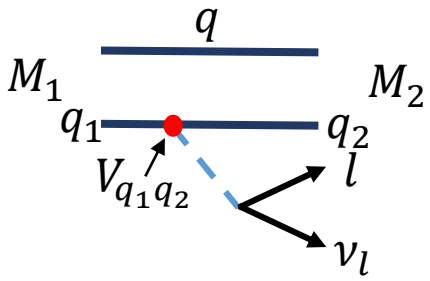




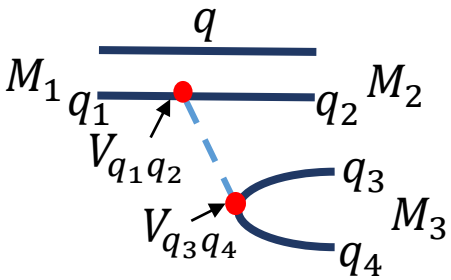
$V_{q_1q_2}$-матричный элемент матрицы смешивания Кабиббо-Кобаяши –Маскава V_{CKM}



Лептонные распады



Полулептонные распады



Нелептонные распады

Матричный элемент распада

$$M(M_1 \to M_2 l \nu_l) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} H^\mu L_\mu$$

Где лептонный и адронный токи имеют вид:

$$L_\mu = \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l$$

$$H^\mu = \langle M_2 | V^\mu - A^\mu | M_1 \rangle$$

Где $V^\mu = \bar{q}_1 \gamma^\mu q_2$ и $A^\mu = \bar{q}_1 \gamma^\mu \gamma_5 q_2$

- *Covariant Constintuent Quark Model (CCQM).*

T. Branz, A. Faessler, T. Gutsche, M.A. Ivanov, J.G. Körner, V. E. Lyubovitskij

Phys. Rev. D81, 034010 (2010)

$$L_{int}^{st}(x) = g_M M(x) \int dx_1 \int dx_2\, F_M(x, x_1, x_2)\bar{q}_1(x_1)\lambda_M \Gamma_M q_2(x_2)$$

$F_M(x, x_1, x_2)$-вершинная функция, характеризующая конечный размер адрона

Для обеспечения трансляционной инвариантности $F_M(x, x_1, x_2)$ должна удовлетворять требованию

$$F_M(x + a, x_1 + a, x_2 + a) = F_M(x, x_1, x_2)$$ для любого вектора a

$$F_M(x, x_1, x_2) = \delta^4\left(x - \sum_{i=1}^{2} w_i x_i\right)\Phi_M\left((x_1 - x_2)^2\right)$$

$$w_i = \frac{m_i}{m_1 + m_2}$$ m_1, m_2 – массы констэнтьюэнтных кварков.

Простейшая форма формфактора : $\Phi_M(-l^2) = \exp\left(-\frac{l^2}{\Lambda_M^2}\right)$

Λ_M^2-параметр модели, характеризующий размер мезона

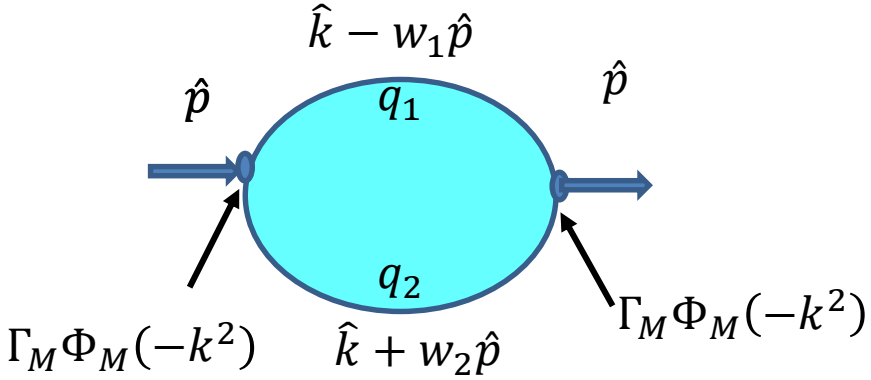
Константа связи g_M определяется из условия связности:

$$Z_M = 1 + \frac{3g_M^2}{4\pi^2}\widetilde{\Pi}'_M(m_M) = 0$$

При вычислении матричных элементов удобно использовать:

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\widetilde{\Pi}'_M(m_M)}$$

$\widetilde{\Pi}_M$-*массовый оператор в случае псевдоскалярных и поперечная часть поляризационного оператора в случае векторных и аксиально- векторных мезонов.*



$$w_{1,2} = \frac{m_{q_{1,2}}}{m_{q_1} + m_{q_2}}$$

$$\Pi_M(p^2) = 3g_M^2 \int \frac{d^4k}{(2\pi)^4 i} \Phi_M^2(-k^2) Tr\{\Gamma_M S_{q_1}(\hat{k} - w_1\hat{p})\Gamma_M S_{q_2}(\hat{k} + w_2\hat{p})\}$$

$$S_q(\hat{k}) = \frac{1}{m_q - \hat{k} - i\epsilon}$$ *-пропагатор свободного констинтьюэнтного кварка*

В представлении Фока-Швингера

$$S_q(\hat{k} + \hat{p}) = \frac{1}{m_q - \hat{k} - \hat{p}} = \frac{m_q + \hat{k} + \hat{p}}{m_q^2 - (k+p)^2} = (m_q + \hat{k} + \hat{p}) \int_0^\infty d\alpha\, e^{-\alpha(m_q^2 - (k+p)^2)}$$

Любая диаграмма может быть представлена в виде

$$G = N_c \frac{g_1 g_2 \cdots g_n}{4\pi^2} \int_0^\infty d^n\alpha\, F(\alpha_1, \alpha_2, \cdots, \alpha_n)$$

После использования тождества $$1 = \int_0^\infty dt\, \delta\left(t - \sum_{i=1}^n \alpha_i\right)$$

$$G = N_c \frac{g_1 g_2 \cdots g_n}{4\pi^2} \int_0^\infty dt\, t^{n-1} \int_0^1 d^n\alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$

$$\int_0^\infty dt \to \int_0^{\frac{1}{\lambda^2}} dt$$

$$G^c = N_c \frac{g_1 g_2 \cdots g_n}{4\pi^2} \int_0^{\frac{1}{\lambda^2}} dt\, t^{n-1} \int_0^1 d^n\alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$

Параметры модели

Параметры характеризующие размеры мезонов (ГэВ):

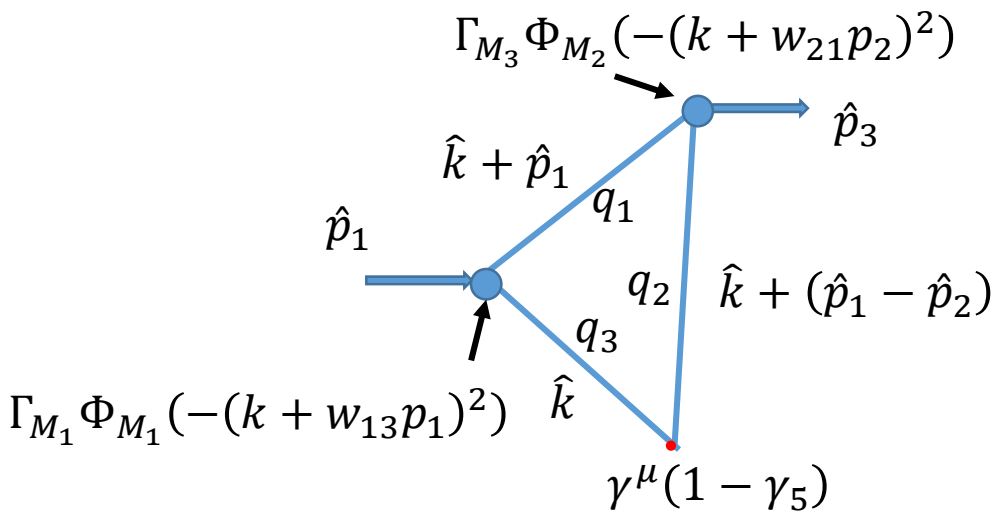
Λ_π	Λ_K	Λ_{K^*}	Λ_D	Λ_{D_s}	Λ_η^{qq}	Λ_η^{ss}	$\Lambda_{\eta'}^{qq}$	$\Lambda_{\eta'}^{ss}$
$0,711$	$1,014$	$0,805$	$1,600$	$1,750$	$0,881$	$1,014$	$0,257$	$2,707$

Массы конститнтьюэнтных кварков(ГэВ):

$m_{u/d}$	m_s	m_c	m_b
$0,241$	$0,428$	$1,672$	$5,05$

Параметр обрезания

$\lambda = 0,181\ GeV$



$$\boldsymbol{I^{\mu}_{M_1M_2}(\hat{p}_{M_1},\hat{p}_{M_2}) = \int \frac{d^4k}{(2\pi)^4 i}\Phi_{M_1}(-(k+w_{13}p_1)^2)\Phi_{M_2}(-(k+w_{32}p_3)^2)\,Tr\{\Gamma_{M_1}S_{q_1}(\hat{k}+\hat{p}_1)\Gamma_{M_2}S_{q_2}(\hat{k}+\hat{p}_2)\gamma^\mu(1-\gamma_5)S_{q_3}(\hat{k})\}}$$

Матричный элемент распада :

Псевдоскалярный мезон в конечном состоянии:

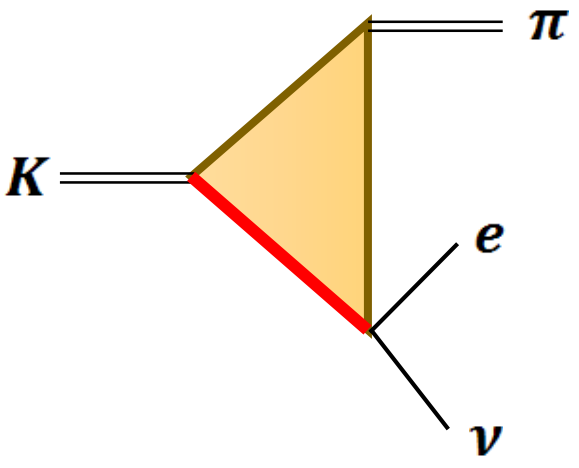
$$M^\mu(p_1,p_2) = F_+(t)P^\mu + F_-(t)Q^\mu \qquad P^\mu = (p_1+p_2)^\mu \quad Q^\mu = (p_1-p_2)^\mu \quad t = (p_1-p_2)^2$$

Векторный мезон в конечном состоянии

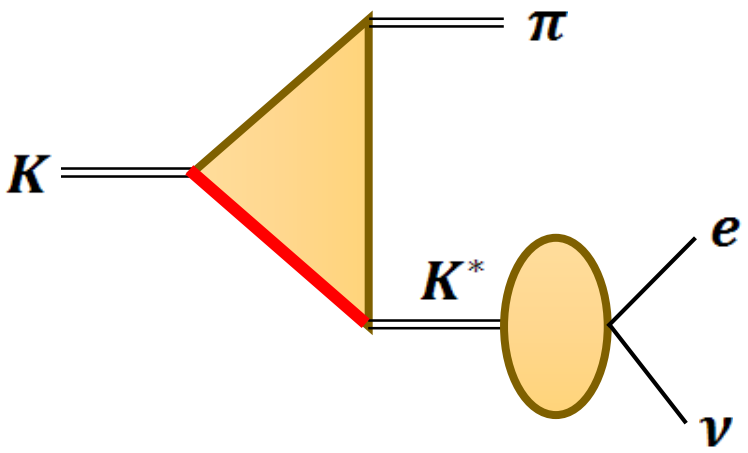
$$M^{\mu\nu}(p_1,p_2) = \frac{\epsilon_{V\alpha}}{M_1+M_2}\left[-g^{\mu\alpha}P\cdot QA_0(t) + P^\mu P^\alpha A_+(t) + Q^\mu P^\alpha A_-(t) + i\varepsilon^{\mu\alpha\nu\beta}P_\nu Q_\beta V(t)\right]$$

Распад $K \to \pi e \nu_e$

$$M^{\mu}(p_1, p_2) = F_+(t)(p_1 + p_2)^{\mu} + F_-(t)(p_1 - p_2)^{\mu}$$



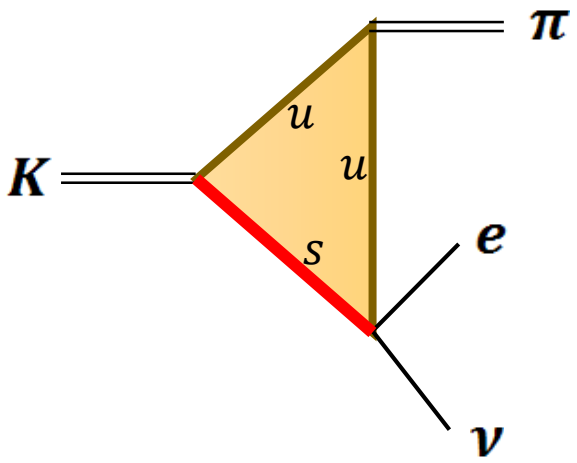
a



b

$$F_+(t) = F_+^a(t) + F_+^b(t) \qquad F_-(t) = F_-^a(t) + F_-^b(t)$$

$$t = (p_1 - p_2)^2$$



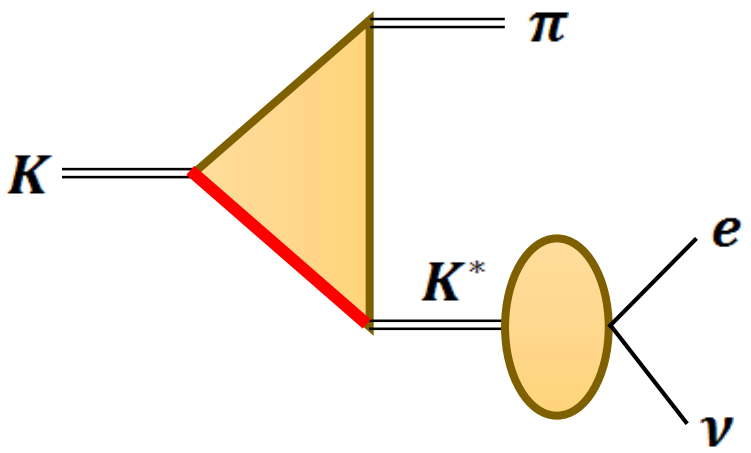
$$w_{us} = \frac{m_u}{m_u + m_s}$$

$$M_a^{\mu}(p_1,p_2) = 3g_k g_\pi \int \frac{d^4k}{(2\pi)^4 i} \Phi_K(-(k+w_{us}p_1)^2)\Phi_\pi\left(-\left(k+\frac{1}{2}p_2\right)^2\right)\frac{Tr\{i\gamma_5(m_u+\hat{k}+\hat{p}_1)i\gamma_5(m_u+\hat{k}+\hat{p}_2)\gamma^\mu(m_s+\hat{k})\}}{(m_u^2-(k+p_1)^2)(m_u^2-(k+p_2)^2)(m_s^2-k^2)}$$

$$= \frac{3g_k g_\pi}{4\pi^2}\int \frac{d^4k}{(2\pi)^2 i} Tr\{i\gamma_5(m_u+\hat{k}+\hat{p}_1)i\gamma_5(m_u+\hat{k}+\hat{p}_2)\gamma^\mu(m_s+\hat{k})\}\int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3$$

$$exp\left(-\alpha_1(m_u^2-(k+p_1)^2)-\alpha_2(m_u^2-(k+p_2)^2)-\alpha_3(m_s^2-k^2)+\frac{(k+w_{us}p_1)^2}{\Lambda_K^2}+\frac{\left(k+\frac{1}{2}p_2\right)^2}{\Lambda_\pi^2}\right)=$$

$$= \sqrt{h_K h_\pi}\int_0^{\frac{1}{\lambda^2}} du\, u^2 \int_0^1 d^3\alpha\,\delta\left(1-\sum_{i=1}^{3}\alpha_i\right)[F_1(u\alpha_1,u\alpha_2,u\alpha_3,t)(p_1+p_2)^\mu + F_2(u\alpha_1,u\alpha_2,u\alpha_3,t)(p_1-p_2)^\mu]$$



$$M_b^{\mu}(p_1,p_2) = g_K g_\pi T^{\nu}_{K\pi K^*}(t) g_{K^*} G^{\nu\rho}_{K^*}(t)\, g_{K^*} D^{\rho\mu}_{K^*}(t)$$

$T^{\nu}_{K\pi K^*}(t)$-форм фактор распада $K \to \pi K^*$(K^*-виртуальный)

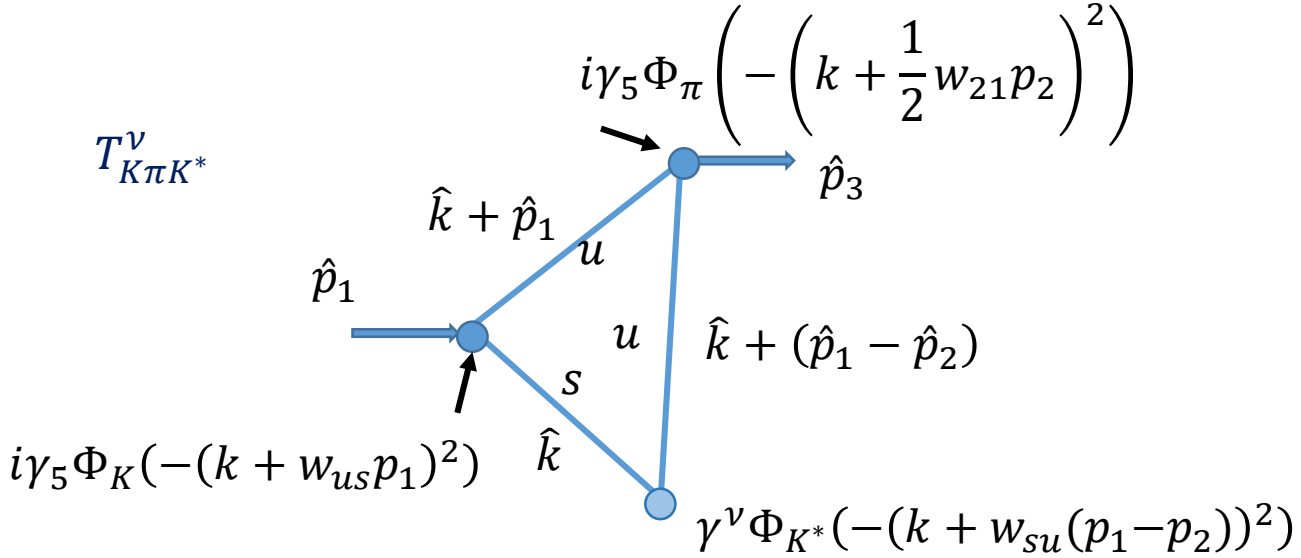
$D^{\rho\mu}_{K^*}(t)$-форм фактор перехода $K^* \to e\nu_e$

$G^{\nu\rho}_{K^*}(t)$ – пропагатор виртуального K^* мезона, в цепочном приближении определенный как:

$$h_V G_V^{\nu\rho}(p^2) = \frac{1}{\Pi_1(p^2)-\Pi_1(m_V^2)}\left\{-g^{\nu\rho} + \frac{p^\nu p^\rho \Pi_2(p^2)}{\Pi_1(p^2)-\Pi_1(m_V^2)+p^2\Pi_2(p^2)}\right\}$$

Где $\Pi_1(p^2)$ и $\Pi_2(p^2)$ - поперечная и продольная части поляризационного оператора

$T^{\nu}_{K\pi K^*}$



$$T^{\nu}_{K\pi K^*}(p_1,p_2)=\int\frac{d^4k}{(2\pi)^4 i}\Phi_K(-(k+w_{us}p_1)^2)\Phi_\pi\left(-\left(k+\frac{1}{2}p_2\right)^2\right)\Phi_{K^*}(-(k+w_{su}(p_1-p_2))^2)$$

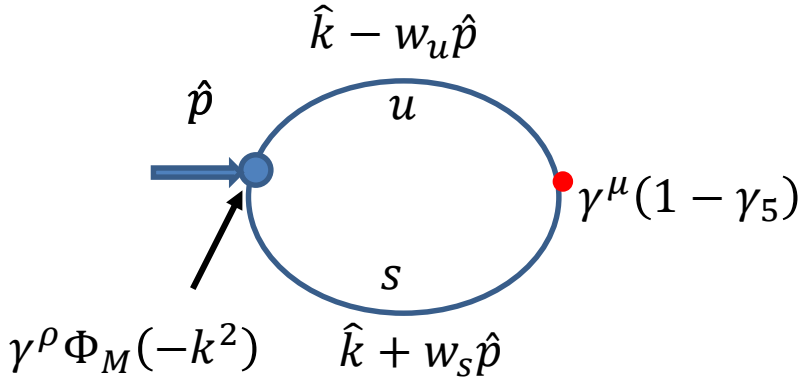
$$\frac{Tr\{i\gamma_5(m_u+\hat{k}+\hat{p}_1)i\gamma_5(m_u+\hat{k}+\hat{p}_2)\gamma^\nu(m_s+\hat{k})\}}{(m_u^2-(k+p_1)^2)(m_u^2-(k+p_2)^2)(m_s^2-k^2)}=$$

$$=\int\frac{d^4k}{(2\pi)^2 i}Tr\{i\gamma_5(m_u+\hat{k}+\hat{p}_1)i\gamma_5(m_u+\hat{k}+\hat{p}_2)\gamma^\nu(m_s+\hat{k})\}\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2\int_0^\infty d\alpha_3$$

$$=exp\left(-\alpha_1(m_u^2-(k+p_1)^2)-\alpha_2(m_u^2-(k+p_2)^2)-\alpha_3(m_s^2-k^2)+\frac{(k+w_{us}p_1)^2}{\Lambda_K^2}+\frac{\left(k+\frac{1}{2}p_2\right)^2}{\Lambda_\pi^2}+\frac{(k+w_{su}(p_1-p_2))^2}{\Lambda_K^2}\right)=$$

$$=\int_0^{\frac{1}{\lambda^2}}\boldsymbol{du\,u^2}\int_0^1\boldsymbol{d^3\alpha\delta}\left(\boldsymbol{1}-\sum_{i=1}^{3}\boldsymbol{\alpha_i}\right)[\boldsymbol{F_{1V}(u\alpha_1,u\alpha_2,u\alpha_3,t)(p_1+p_2)^\mu+F_{2V}(u\alpha_1,u\alpha_2,u\alpha_3,t)(p_1-p_2)^\mu}]$$

$D_{K^*}^{\rho\mu}$



$$D_{K^*}^{\rho\mu}(p^2) = \int \frac{d^4k}{(2\pi)^2 i}\Phi_{K^*}(-k^2)\frac{Tr\{\gamma^\rho\,(m_u + \hat{k} - w_u\hat{p})\gamma^\mu(1-\gamma_5)\,(m_s + \hat{k} + w_s\hat{p})\}}{(m_u^2 - (k - w_u\hat{p})^2)(m_s^2 - (k + w_s\hat{p})^2)} =$$

$$= \int \frac{d^4k}{(2\pi)^2 i} Tr\{\gamma^\rho\,(m_u + \hat{k} - w_u\hat{p})\gamma^\mu(1-\gamma_5)\,(m_s + \hat{k} + w_s\hat{p}))\}\int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3$$

$$exp\left(-\alpha_1(m_u^2 - (k - w_u\hat{p})^2) - \alpha_2(m_s^2 - (k + w_s\hat{p})^2) + \frac{k^2}{\Lambda_{K^*}^2}\right) =$$

$$= \int_0^{\frac{1}{\lambda^2}} du\, u \int_0^1 d^2\alpha\,\delta\left(1 - \sum_{i=1}^{2}\alpha_i\right)\left[D_1(u\alpha_1, u\alpha_2, p^2)g^{\rho\mu}p^2 + D_2(u\alpha_1, u\alpha_2, p^2)p^\rho p^\mu\right]$$

Формфакторы распада могут быть параметризованы как:

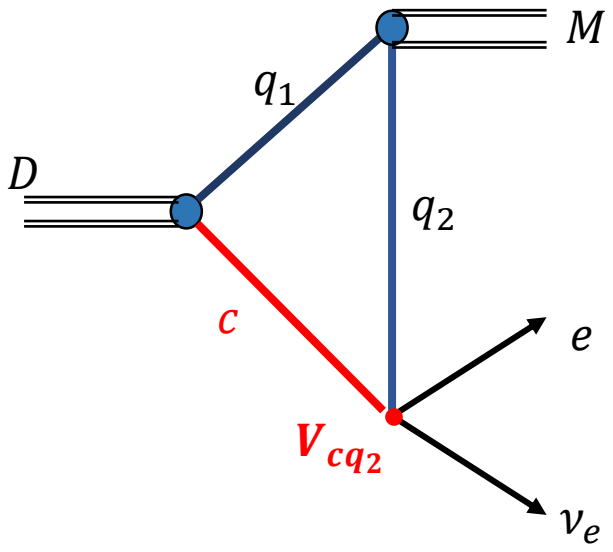
$$F_{\pm}(t) = F_{\pm}(0)\left[1 + \lambda_{\pm}\frac{t}{m_{\pi}^2}\right]$$

$$\lambda_{\pm} = m_{\pi}^2 \frac{F'_{\pm}(0)}{F_{\pm}(0)} \qquad \xi(0) = \frac{F_{-}(0)}{F_{+}(0)} \qquad \lambda_0 = \lambda_{+} + \frac{m_{\pi}^2}{m_K^2 - m_{\pi}^2}\xi(0)$$

Параметр	*Полученное значение*	*Эксперимент*
λ_+	$0{,}034 \pm 0{,}004$	$0{,}0298 \pm 0{,}0005$
λ_-	$0{,}028 \pm 0{,}0036$	0
$\xi(0)$	$-0{,}38 \pm 0{,}0047$	$-0{,}35 \pm 0{,}14$

$$F_{+}(m_K^2) + F_{-}(m_K^2) = 0{,}9\frac{f_K}{f_{\pi}}$$

Полулептонные распады D мезонов



$$M(D \to M_2 l \nu_l) = \frac{G_F}{\sqrt{2}} V_{cq_2} H^{\mu} L_{\mu}$$

$$|V_{cd}| = 0{,}2140 \pm 0.0029 \pm 0.0093$$

$$|V_{cs}| = 0{,}967 \pm 0.025$$

- BaBar Coliaboration*Phys.Rev.D* 91 (2015) 5, 052022Measurement of the$D^0 \to \pi^+ e^- \nu_e$ differential decay branching fraction as a function of q^2 and study of form factor parameterizations
- BESSIII Coliaboration *Phys.Rev.D* 92 (2015) 072012 Study of Dynamics of $D^0 \to K^- e^+ \nu_e$and $D^0 \to \pi^- e^+ \nu_e$ Decays
- CLEO Coliaboration *Phys.Rev.D* 80 (2009) 032005 Improved measurements of D meson semileptonic decays to π and K mesons

Формфакторы $F_+(0)$ для $D \to \pi, K$ переходов

	CCQM	LSCR	LFQM	CQM	LQCD	Эксперимент
$D \to \pi$	0.63 ± 0.09	$0.635^{+0.06}_{-0.057}$	0.66	0.69	$0.612 - 0.66$	0.637 ± 0.009
$D \to K$	0.77 ± 0.11	$0.661^{+0.067}_{-0.066}$	0.79	0.78	$0.73 - 0.765$	0.737 ± 0.004

LSCR-light-cone sum rules

- Y. L. Wu, M. Zhong, and Y. B. Zuo, Int. J. Mod. Phys. A 21, 6125 (2006)

LFQM-covariant light-front quark model

- R. C. Verma, J. Phys. G 39, 025005 (2012) [arXiv:1103.2973]

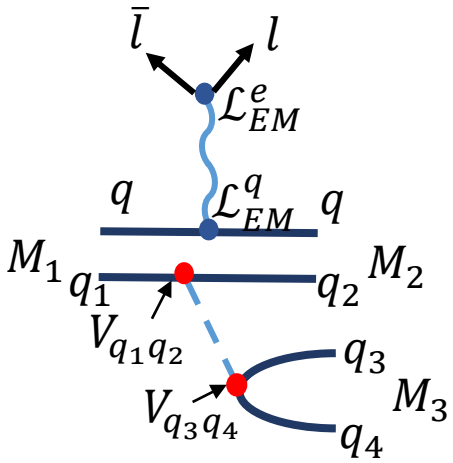
CQM –constituent quark model

- D. Melikhov and B. Stech, Phys. Rev. D 62, 014006 (2000) [hep-ph/0001113]

LQCD-lattice calculations

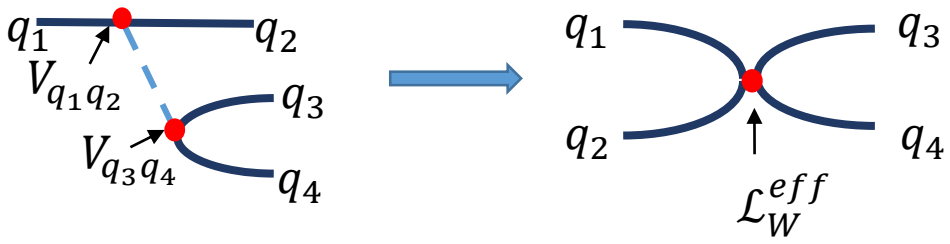
- V. Lubicz et al. (ETM Collaboration), Phys. Rev. D 96, 054514 (2017) [arXiv:1706.03017]
- H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage, and J. Shigemitsu (HPQCD Collaboration), Phys. Rev. D 84, 114505 (2011) [arXiv:1109.1501
- C. Aubin et al. (Fermilab Lattice and MILC and HPQCD Collaborations), Phys. Rev. Lett. 94, 011601 (2005) [hep-ph/0408306].

Распад $M_1 \to M_2 l^+ l^-$



$$\mathcal{L}_{EM}^{q} = \mathrm{e}A_\mu \bar{q}_i^a Q_{ij} \gamma^\mu q_j^a, Q = diag\left(\frac{2}{3}; -\frac{1}{3}; -\frac{1}{3}\right)$$

$$\mathcal{L}_{EM}^{e} = \mathrm{e}A_\mu \bar{l} Q_{ij} \gamma^\mu l$$



$$\mathcal{L}_{W}^{eff} = \frac{G_F}{2\sqrt{2}} V_{q_1 q_2} V_{q_3 q_4} \sum c_i(\mu) O_i$$

$c_i(\mu)$-Вильсоновские коэффициенты

O_i-четырехфермионные операторы

$$O_1 = (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_2 = (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) + 2(\bar{d}O_L^\mu s)(\bar{d}O_L^\mu d) + \\ +2(\bar{d}O_L^\mu s)(\bar{s}O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_3 = (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu s)(\bar{s}O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_4 = (\bar{d}O_L^\mu u)(\bar{u}O_L^\mu s) + (\bar{d}O_L^\mu s)(\bar{u}O_L^\mu u) - (\bar{d}O_L^\mu s)(\bar{d}O_L^\mu d) \qquad \Delta I = 3/2$$

$$O_5 = (\bar{d}O_L^\mu \lambda^a s) \sum_{q=u,d,s} (\bar{q}O_R^\mu \lambda^a q) \qquad \Delta I = 1/2$$

$$O_6 = (\bar{d}O_L^\mu s) \sum_{q=u,d,s} (\bar{q}O_R^\mu q) \qquad \Delta I = 1/2$$

$$O_L^\mu = \gamma^\mu(1-\gamma^5)\,; \quad O_R^\mu = \gamma^\mu(1+\gamma^5)$$

Распад $K^+ \to \pi^+ l^+ l^-$

$$M(K^+ \to \pi^+ l^+ l^-) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} e \frac{\sqrt{h_k h_\pi}}{8\sqrt{2\pi}} F(q^2, m_K^2, m_\pi^2) q^\mu \frac{-i g^{\mu\nu}}{q^2 + i\epsilon} (-ie) \bar{l}(k) \gamma^\nu l(k')$$

$$\Gamma(K^+ \to \pi^+ e^+ e^-) = \frac{{G_F}^2 \alpha^2}{128\pi^3 m_K^3} V_{ud}^2 V_{us}^2 \frac{9 h_K h_\pi}{8\pi} \times$$

$$\times \int_{4m_e^2}^{(m_K - m_\pi)^2} dq^2\, q^2 \left(1 + \frac{2m_e^2}{q^2}\right) \lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2}\right) \lambda^{1/2}\left(1, \frac{m_e^2}{q^2}, \frac{m_e^2}{q^2}\right) \left|F(q^2, m_K^2, m_\pi^2)\right|^2$$

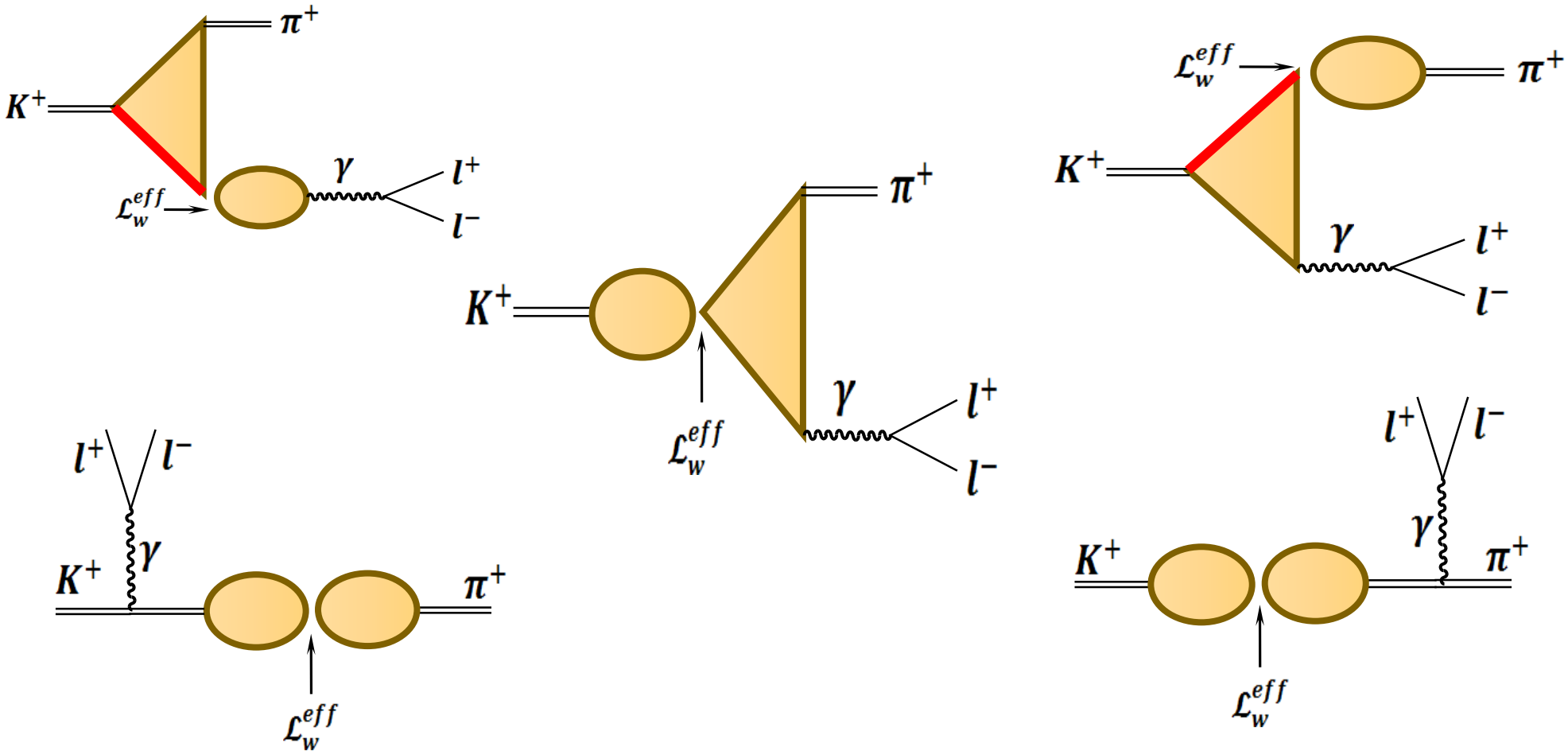
$$F(q^2, m_K^2, m_\pi^2)$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$$

$$F(q^2, m_K^2, m_\pi^2) = F_1(q^2, m_K^2, m_\pi^2) + F_2(q^2, m_K^2, m_\pi^2)$$

$$F_2(q^2, m_K^2, m_\pi^2) = F_A(q^2, m_K^2, m_\pi^2) + F_P(q^2, m_K^2, m_\pi^2)$$

Диаграммы, определяющие распад $K^+ \to \pi^+ l^+ l^-$

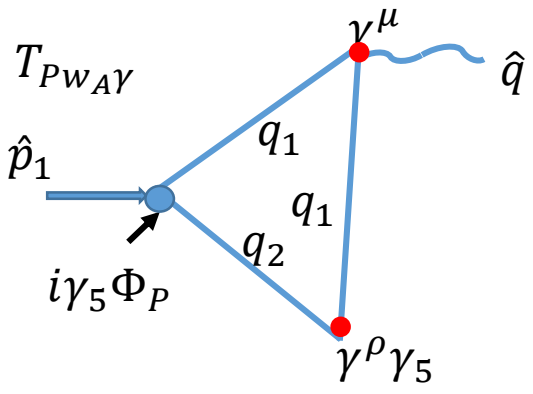


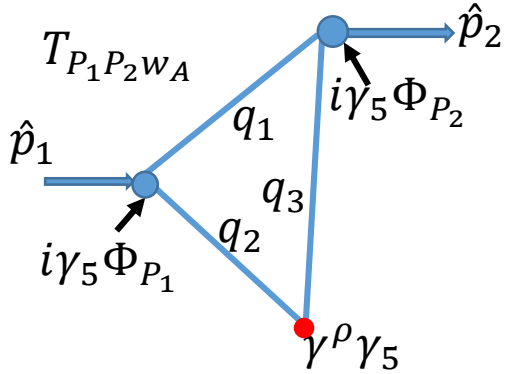
$$\boldsymbol{F_1(q^2, m_K^2, m_\pi^2) = \frac{2}{3}(-c_1 - 2c_2 - 2c_3 - 2c_4)G_1 + \frac{4}{9}(-c_1 - 2c_2 + 3c_3 + 3c_4)G_2 - \frac{2}{9}c_5 G_3}$$

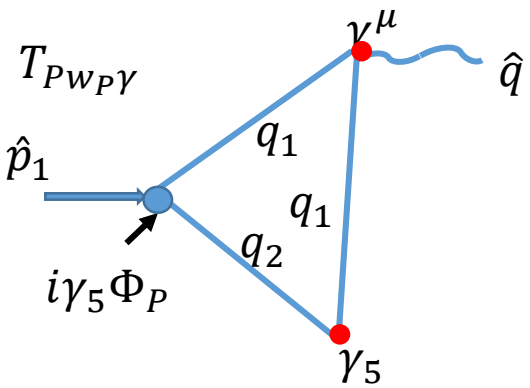
$$G_1 = D_{w_A K}(m_K^2) T_{\pi w_A \gamma}(m_K^2, m_\pi^2, q^2) + D_{w_A \pi}(m_\pi^2) T_{K w_A \gamma}(m_\pi^2, m_K^2, q^2)$$

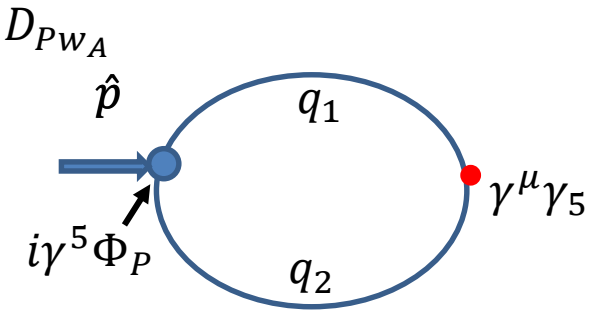
$$G_2 = q^2 D_{\gamma\gamma}(q^2) T_{K \pi w_A}(m_K^2, m_\pi^2, q^2)$$

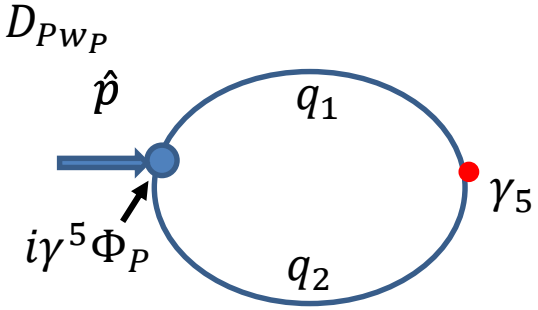
$$G_3 = D_{\pi w_P}(m_\pi^2) T_{K w_P \gamma}(m_K^2, m_\pi^2, q^2) + D_{K w_P}(m_K^2) T_{\pi w_P \gamma}(m_\pi^2, m_K^2, q^2)$$

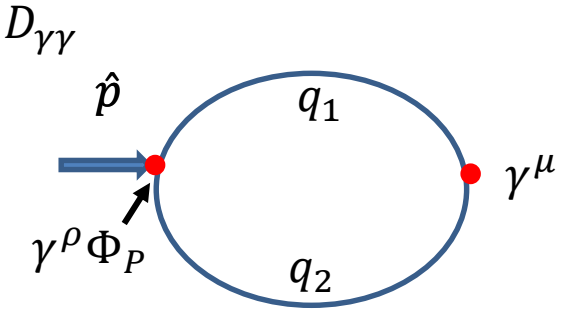




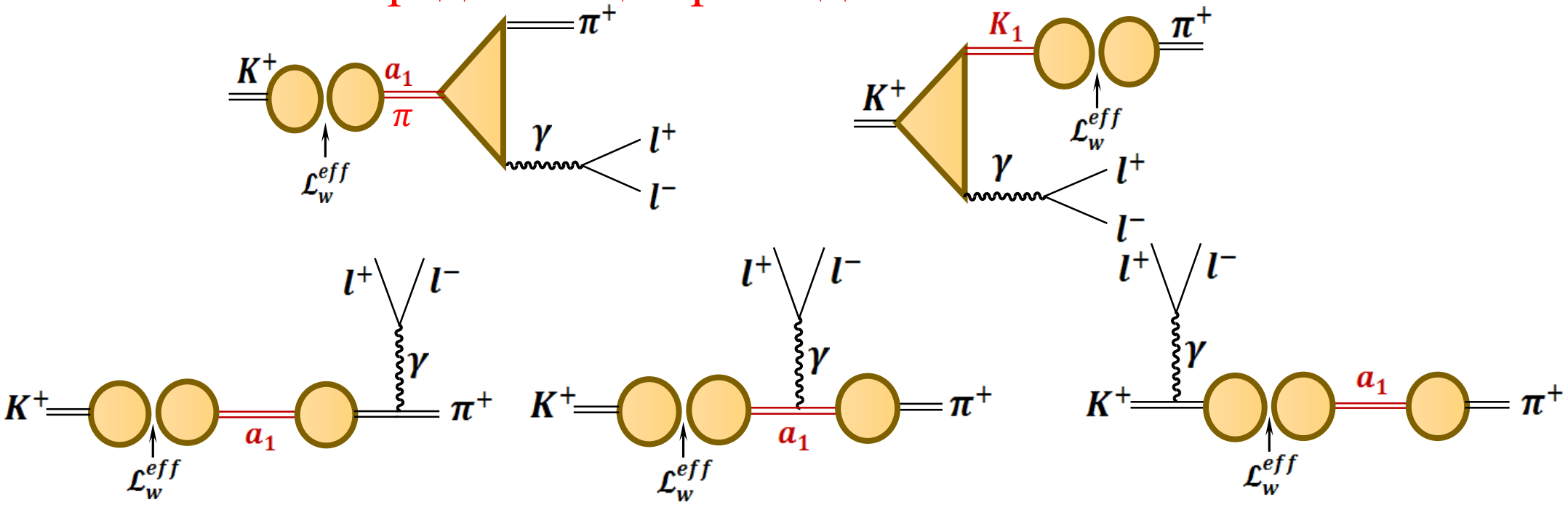






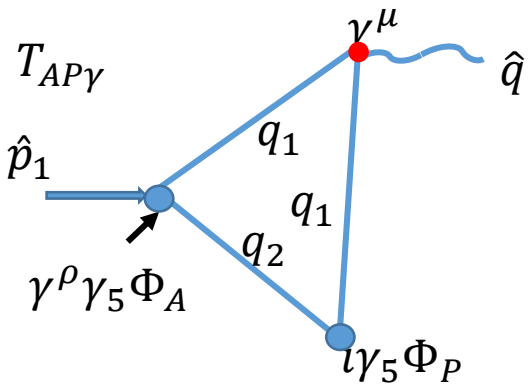


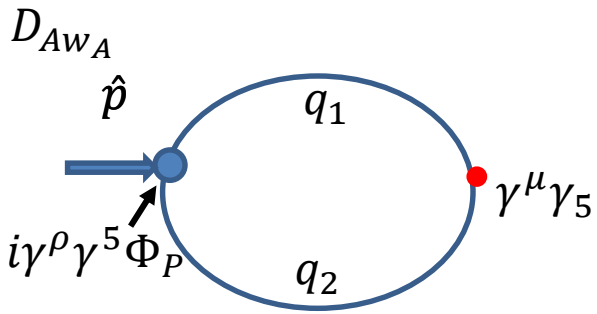
Диаграммы с промежуточными мезонами, определяющие распад $K^+ \to \pi^+ l^+ l^-$

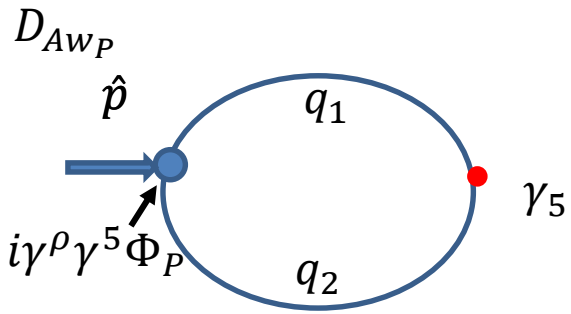


$$F_A(q^2, m_K^2, m_\pi^2) = \frac{2}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)G_{A1} + \frac{4}{9}(-c_1 - 2c_2 + 3c_3 + 3c_4)G_{A2} - \frac{2}{9}c_5 G_{A3}$$

$$F_P(q^2, m_K^2, m_\pi^2) = \frac{2}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)G_{P1} + \frac{2}{3}c_5 G_{P2}$$







$$G_{A1} = -D_{w_A K}(m_K^2) D_{Aw_A}(m_K^2) T_{AP\gamma}(m_K^2, m_\pi^2, q^2) \frac{\Pi_1(m_K^2) + m_K^2 \Pi_2(m_K^2)}{\Pi_1(m_K^2) - \Pi_1(m_A^2) + m_K^2 \Pi_2(m_K^2)} + K \leftrightarrow \pi$$

$$G_{A2} = q^2 D_{\gamma\gamma}(q^2) \left\{ \frac{D_{PwA}(m_\pi^2) T_{PP\gamma}(m_\pi^2, m_K^2, q^2)}{\Pi_1(m_\pi^2) - \Pi_1(m_A^2) + m_\pi^2 \Pi_2(m_\pi^2)} - K \leftrightarrow \pi \right\}$$

$$G_{A3} = \left\{ D_{Pw_P}(m_\pi^2) + D_{Pw_P}(m_K^2) \right\} \left\{ \frac{D_{AwA}(m_K^2) T_{AP\gamma}(m_K^2, m_\pi^2 q^2)}{\Pi_1(m_K^2) - \Pi_1(m_A^2) + m_K^2 \Pi_2(m_K^2)} - K \leftrightarrow \pi \right\}$$

$$G_{P1} = \left\{ m_K^2 D^2{}_{Pw_A}(m_K^2) - m_\pi^2 D^2{}_{Pw_A}(m_\pi^2) \right\} \frac{T_{PP\gamma}(m_K^2, m_\pi^2 q^2)}{2\left(D_{PP}(m_K^2) - D_{PP}(m_\pi^2) \right)}$$

$$G_{P2} = \left\{ D^2{}_{Pw_P}(m_K^2) - D^2{}_{Pw_P}(m_\pi^2) \right\} \frac{T_{PP\gamma}(m_K^2, m_\pi^2 q^2)}{2\left(D_{PP}(m_K^2) - D_{PP}(m_\pi^2) \right)}$$

Численные результаты

Распад	$\mathrm{Br}_{exp} \times 10^{-7}$	$\mathrm{Br}_1 \times 10^{-7}$	$\mathrm{Br}_2 \times 10^{-7}$
$K^+ \to \pi^+ e^+ e^-$	3.00 ± 0.09	5.58 ± 0.56	3.23 ± 0.56
$K^+ \to \pi^+ \mu^+ \mu^-$	0.917 ± 0.014	1.15 ± 0.012	0.73 ± 0.014

$$\boldsymbol{\alpha_s = 0,05 - 1,0\,; \quad \mu = 0,9 - 1,9\ GeV}$$

$$R\left(\frac{\mu^+\mu^-}{e^+e^-}\right) = \frac{Br(K^+ \to \pi^+\mu^+\mu^-)}{Br(K^+ \to \pi^+e^+e^-)}$$

$$R_{exp} = 0.309(43)$$

$$R_{ChT} \cong 0.23 \qquad R_{our} = 0.22 \pm 0.04$$

Перспективы:

- *Изучение остальных редких распадов K мезонов*
- *Изучение редких распадов D мезонов*
  $D^+ \to \pi^+ e^+ e^-, D^+ \to \pi^+ \mu^+ \mu^-, D_s^+ \to \pi^+ e^+ e^-, D_s^+ \to K^+ \mu^+ \mu^-$ и т.д
- *Изучение возможных взаимодействий D и η_c мезонов*
- *Изучение нелептонных распадов K и D мезонов*