

# High-precision simulation of polarization effects for processes on modern colliders using the Monte Carlo event generator **ReneSANCe**

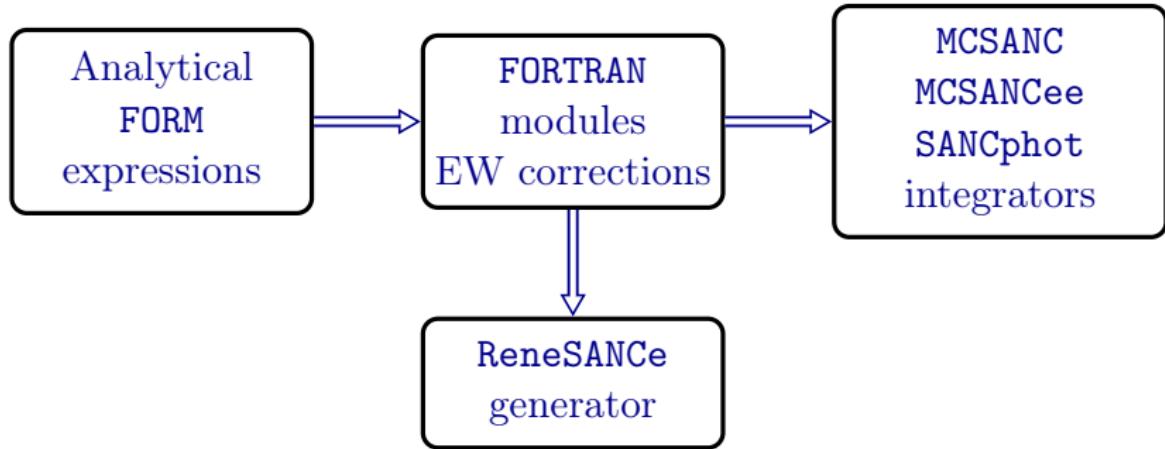
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on behalf of SANC team

JINR; INP BSU

The XV-th International School-Conference  
"The Actual Problems of Microworld Physics"  
01 September 2023



# The SANC framework and products family



## Publications:

SANC – CPC 174 481-517

MCSANC – CPC 184 2343-2350; JETP Letters 103, 131-136

SANCPHOT – arXiv:2201.04350

ReneSANCe – CPC 256 107445; CPC 285 108646

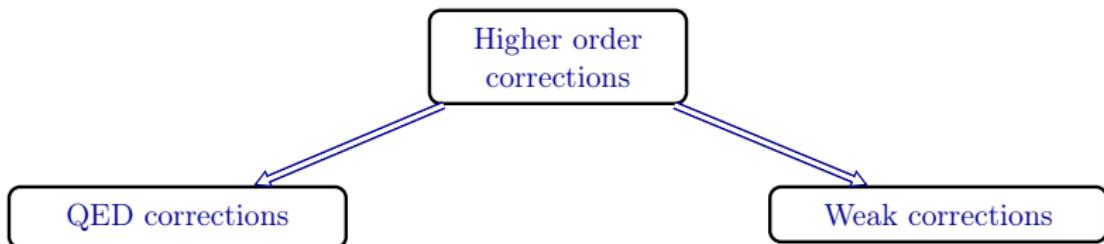
SANC products are available at <http://sanc.jinr.ru/download.php>

ReneSANCe is also available at <http://renesance.hepforge.org>

## SANC advantages:

- full one-loop electroweak corrections
- higher order corrections
- massive case
- accounting for polarization effects
- full phase space operation
- results of ReneSANCe event generator and SANC integrators are thoroughly cross checked

# Higher order improvements



- Leading logarithmic (LL) approximation.
- Corrections to  $\Delta\alpha$ .
- Shower with matching.
- Corrections to  $\Delta\rho$ .
- Leading Sudakov logarithms.

# Higher order improvements, QED

Basic formula:

$$\sigma^{\text{LLA}} = \int_0^1 dx_1 \int_0^1 dx_2 \mathcal{D}_{ee}(x_1) \mathcal{D}_{ee}(x_2) \sigma_0(x_1, x_2, s) \Theta(\text{cuts}),$$

where  $\sigma_0(x_1, x_2, s)$  – is the Born level cross section of the annihilation process with changed momenta of initial particles.

$\mathcal{D}_{ee}(x)$  describes the probability density of finding an electron with an energy fraction  $x$  in the initial electron beam.

[Kuraev, E.A.; Fadin, V.S. Sov. J. Nucl. Phys. 1985, 41, 466–472]

# Higher order improvements, QED

The leading log is  $L = \ln \frac{s}{m_l^2}$ .

LO	1			
NLO	$\alpha L$	$\alpha$		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
$N^3 LO$	$\frac{1}{6}\alpha^3 L^3$	$\frac{1}{6}\alpha^3 L^2$	...	

In the LL approximation we can separate pure photonic (marked “ $\gamma$ ”) and the rest corrections which include pure pair and mixed photon-pair effects (marked as “pair”).

$$e^+ e^- \rightarrow t\bar{t}, \sqrt{s} = 350 \text{ and } 500 \text{ GeV}$$

Multiple photon ISR relative corrections  $\delta$  (%) in the LLA approximation.

$\sqrt{s}$ , GeV	350	500
$\mathcal{O}(\alpha L), \gamma$	-42.546(1)	-3.927(1)
$\mathcal{O}(\alpha^2 L^2), \gamma$	+8.397(1)	-0.429(1)
$\mathcal{O}(\alpha^2 L^2), e^+ e^-$	-0.460(1)	-0.030(1)
$\mathcal{O}(\alpha^2 L^2), \mu^+ \mu^-$	-0.277(1)	-0.018(1)
$\mathcal{O}(\alpha^3 L^3), \gamma$	-0.984(1)	+0.021(1)
$\mathcal{O}(\alpha^3 L^3), e^+ e^-$	+0.182(1)	-0.012(1)
$\mathcal{O}(\alpha^3 L^3), \mu^+ \mu^-$	+0.110(1)	-0.008(1)
$\mathcal{O}(\alpha^4 L^4), \gamma$	+0.070(1)	+0.002(1)

## DGLAP

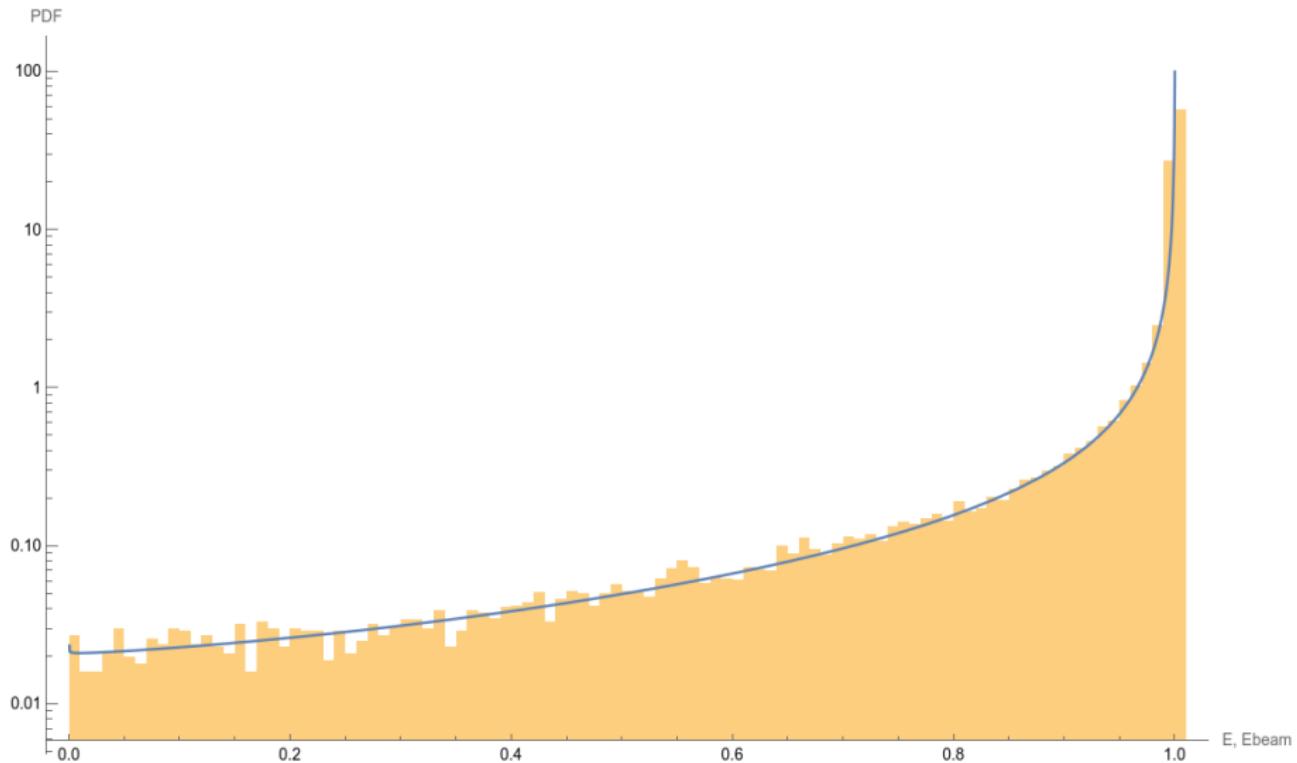
Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation:

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(y) D\left(\frac{x}{y}, Q^2\right)$$

Sudakov form factor:

$$\Delta(s_1, s_2) = \exp\left[-\frac{\alpha}{2\pi} \int_{s_2}^{s_1} \frac{dQ^2}{Q^2} \int_0^{1-\epsilon} dy P(y)\right]$$

# Energy distribution of electrons according shower algorithm



# Higher order improvements, weak

Higher order improvements added to NLO cross section through  $\Delta\rho$   
 parameter:  $s_W^2 \rightarrow \bar{s}_W^2 \equiv s_W^2 + \Delta\rho c_W^2$ .

- $\mathcal{O}(\alpha)$  A. Sirlin, PRD22, (1980) 971; W.J. Marciano, A. Sirlin, PRD22 (1980) 2695; G. Degrassi, A. Sirlin, NPB352 (1991) 352, P. Gambino and A. Sirlin, PRD49 (1994) 1160
- $\mathcal{O}(\alpha\alpha_s)$  A. Djouadi, C. Verzegnassi, PLB195 (1987) 265; B. Kiehl, NPB353 (1991) 567; B. Kniehl, A. Sirlin, NPB371 (1992) 141, PRD47 (1993) 883; A. Djouadi, P. Gambino, PRD49 (1994) 3499
- $\mathcal{O}(\alpha\alpha_s^2)$  L. Avdeev et al., PLB336 (1994) 560; K.G. Chetyrkin, J.H. Kuhn, M. Steinhauser, PLB351 (1995) 331; PRL75 (1995) 3394; NPB482 (1996)
- $\mathcal{O}(\alpha\alpha_s^3)$  Y. Schroder, M. Steinhauser, PLB622 (2005) 124; K.G. Chetyrkin et al., hep-ph/0605201; R. Boughezal, M. Czakon, hep-ph/0606232
- $\mathcal{O}(\alpha^2)$  G. Degrassi, P. Gambino, A. Sirlin, PLB394 (1997) 188; M. Awramik, M. Czakon, A. Freitas, JHEP0611 (2006) 048

$e^+e^- \rightarrow t\bar{t}$ ,  $\sqrt{s} = 350$  and 500 GeV

Integrated Born and weak contributions to the cross section and higher-order leading corrections in two EW schemes:  $\alpha(0)$  and  $G_\mu$ .

$\sqrt{s}$ , GeV	350	500
$\sigma_{\alpha(0)}^{\text{Born}}, \text{ pb}$	0.22431(1)	0.45030(1)
$\sigma_{G_\mu}^{\text{Born}}, \text{ pb}$	0.24108(1)	0.48398(1)
$\delta_{G_\mu/\alpha(0)}^{\text{Born}}, \%$	7.48(1)	7.48(1)
$\sigma_{\alpha(0)}^{\text{weak}}, \text{ pb}$	0.25564(1)	0.47705(1)
$\sigma_{G_\mu}^{\text{weak}}, \text{ pb}$	0.26055(1)	0.48420(1)
$\delta_{G_\mu/\alpha(0)}^{\text{weak}}, \%$	1.92(1)	1.50(1)
$\sigma_{\alpha(0)}^{\text{weak+ho}}, \text{ pb}$	0.25900(1)	0.48483(1)
$\sigma_{G_\mu}^{\text{weak+ho}}, \text{ pb}$	0.25986(1)	0.48289(1)
$\delta_{G_\mu/\alpha(0)}^{\text{weak+ho}}, \%$	0.33(1)	-0.40(1)

SANC

support different initial beams

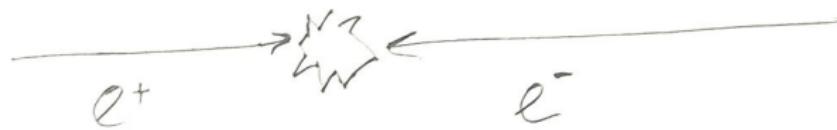
$ll$   
mode

$hh$   
mode

$\gamma\gamma$   
mode

$eh$   
mode

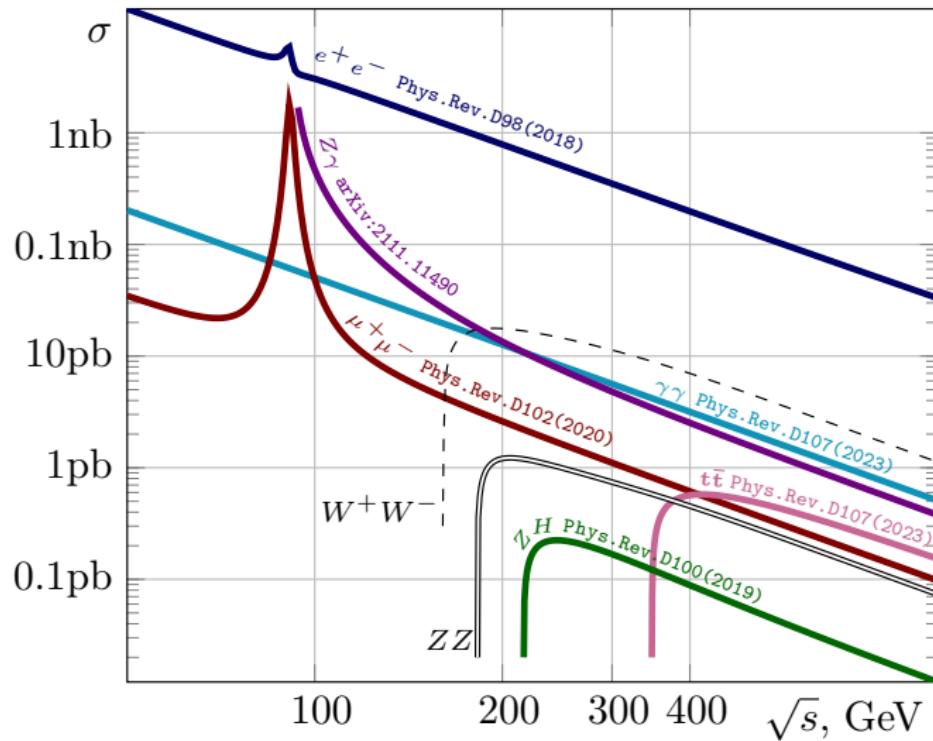
# ll mode



# Processes of interest

- Bhabha ( $e^+e^- \rightarrow e^-e^+$ ), Phys. Rev. D 98, 013001.
- ZH ( $e^+e^- \rightarrow ZH$ ), Phys. Rev. D 100, 073002.
- s-channel ( $e^+e^- \rightarrow \mu^-\mu^+$ ,  $e^+e^- \rightarrow \tau^-\tau^+$ ), Phys. Rev. D 102, 033004.
- Photon-pair ( $e^+e^- \rightarrow \gamma\gamma$ ), Phys. Rev. D 107, 073003.
- s-channel ( $e^+e^- \rightarrow t\bar{t}$ ), Phys. Rev. D 107, 113006.
- Muon-electron scattering ( $\mu^+e^- \rightarrow \mu^+e^-$ ), Phys. Rev. D 105, 033009.
- Møller ( $\mu^+\mu^+ \rightarrow \mu^+\mu^+$ ), JETP Lett. 115, 9.
- $Z\gamma$  ( $e^+e^- \rightarrow Z\gamma$ ).
- $ZZ$  ( $e^+e^- \rightarrow ZZ$ ).
- publication, available in release of the generator
- publication, in preparation for next release of the generator
- in preparation

# Basic processes of SM for $e^+e^-$ annihilation



The cross sections are given for polar angles between  $10^\circ < \theta < 170^\circ$  in the final state.

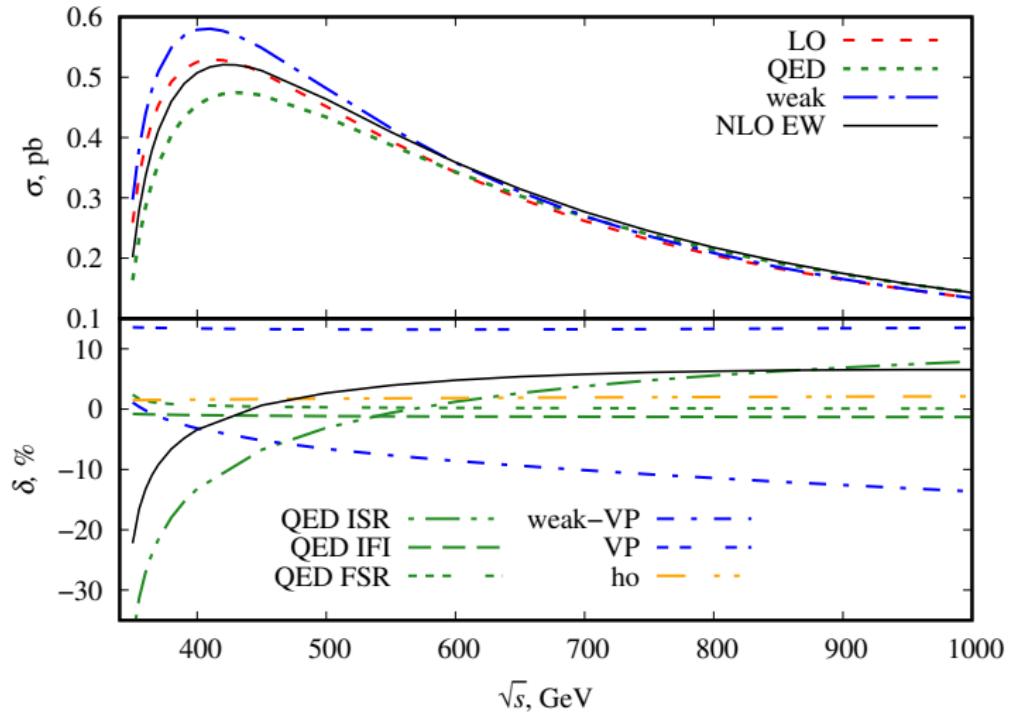
# Distributions

For each process we provide all important distributions:

- Cross section over energy and angle distribution
- Forward-Backward Asymmetry
- Left-Right Asymmetry
- Final-State Fermion Polarization

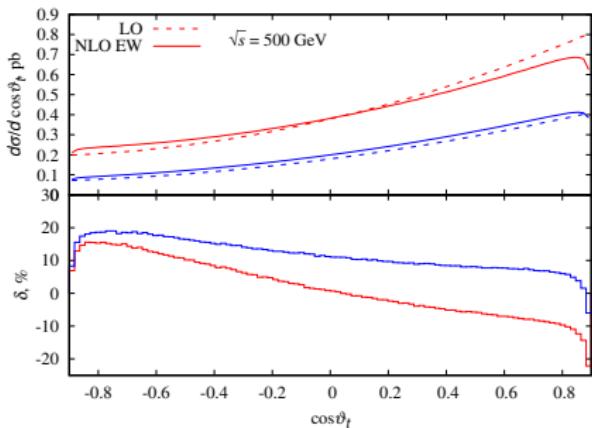
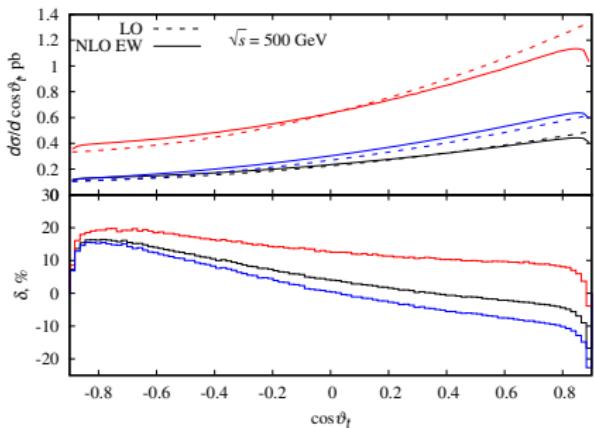
$e^+e^- \rightarrow t\bar{t}$ , energy dependence

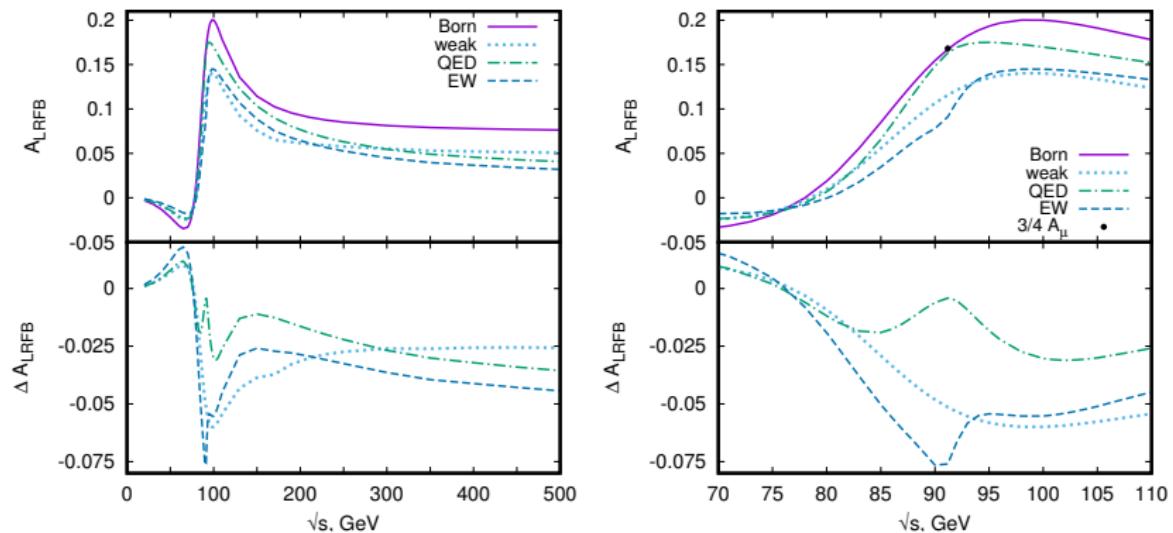
The LO and NLO EW corrected unpolarized cross sections and the relative corrections in parts as a function of the c.m.s. energy.



$e^+e^- \rightarrow t\bar{t}$ , angle dependence

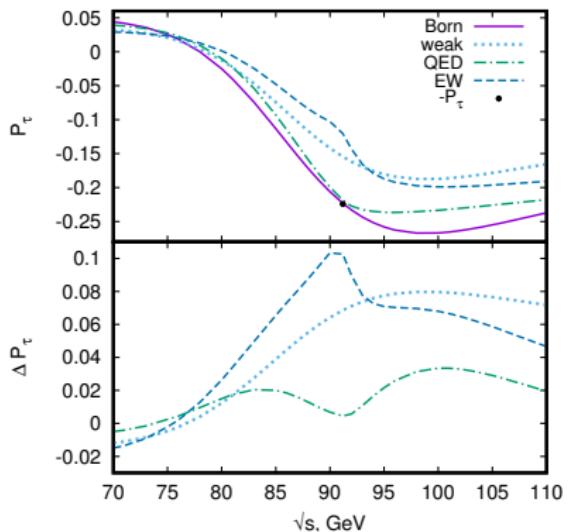
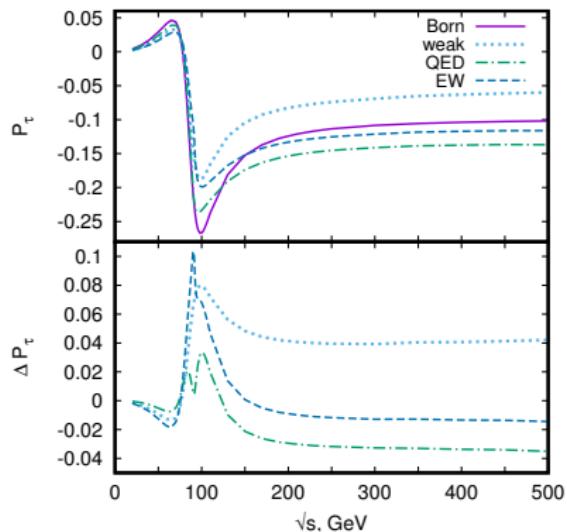
The left part corresponds to the unpolarized (black), and fully polarized, with  $(P_{e^+}, P_{e^-} = +1, -1)$  (red) and  $(-1, +1)$  (blue), initial beams, while the right one shows the partially polarized initial beams with  $(P_{e^+}, P_{e^-} = (+0.3, -0.8)$  (red) and  $(-0.3, +0.8)$  (blue) for the energy  $\sqrt{s} = 350$  GeV.



$e^+e^- \rightarrow \mu^+\mu^-$ , Left–Right Forward–Backward Asymmetry


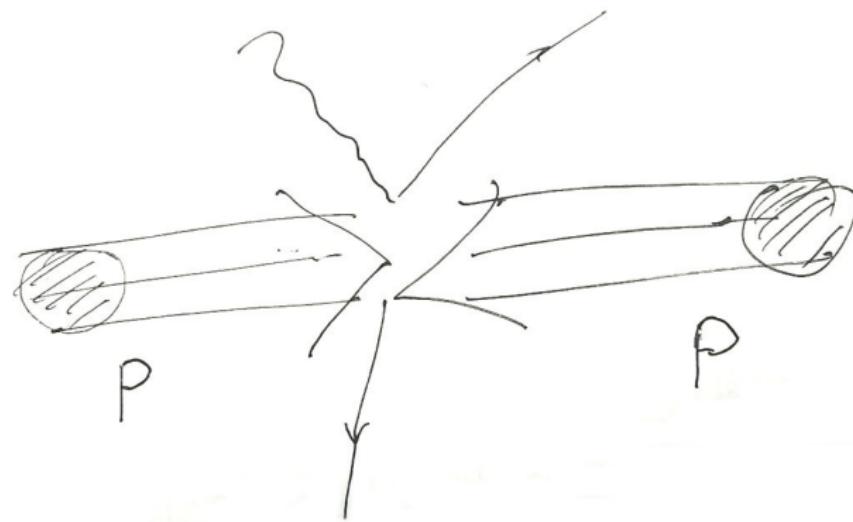
**(Left)** The  $A_{LRFB}$  asymmetry in the Born and 1-loop (weak, QED, EW) approximations and  $\Delta A_{LRFB}$  for c.m.s. energy range; **(Right)** the same for the  $Z$  peak region.

$$A_{LRFB} = \frac{(\sigma_{L_e} - \sigma_{R_e})_F - (\sigma_{L_e} - \sigma_{R_e})_B}{(\sigma_{L_e} + \sigma_{R_e})_F + (\sigma_{L_e} + \sigma_{R_e})_B},$$

$e^+e^- \rightarrow \tau^+\tau^-$ , Final-State Fermion Polarization


**(Left)** The  $P_\tau$  polarization in the Born and 1-loop (weak, pure QED, and EW) approximations and  $\Delta P_\tau$  vs. c.m.s. energy in a wide range; **(Right)** the same for the  $Z$  peak region. The black dot indicates the value  $P_\tau$  at the  $Z$  resonance.

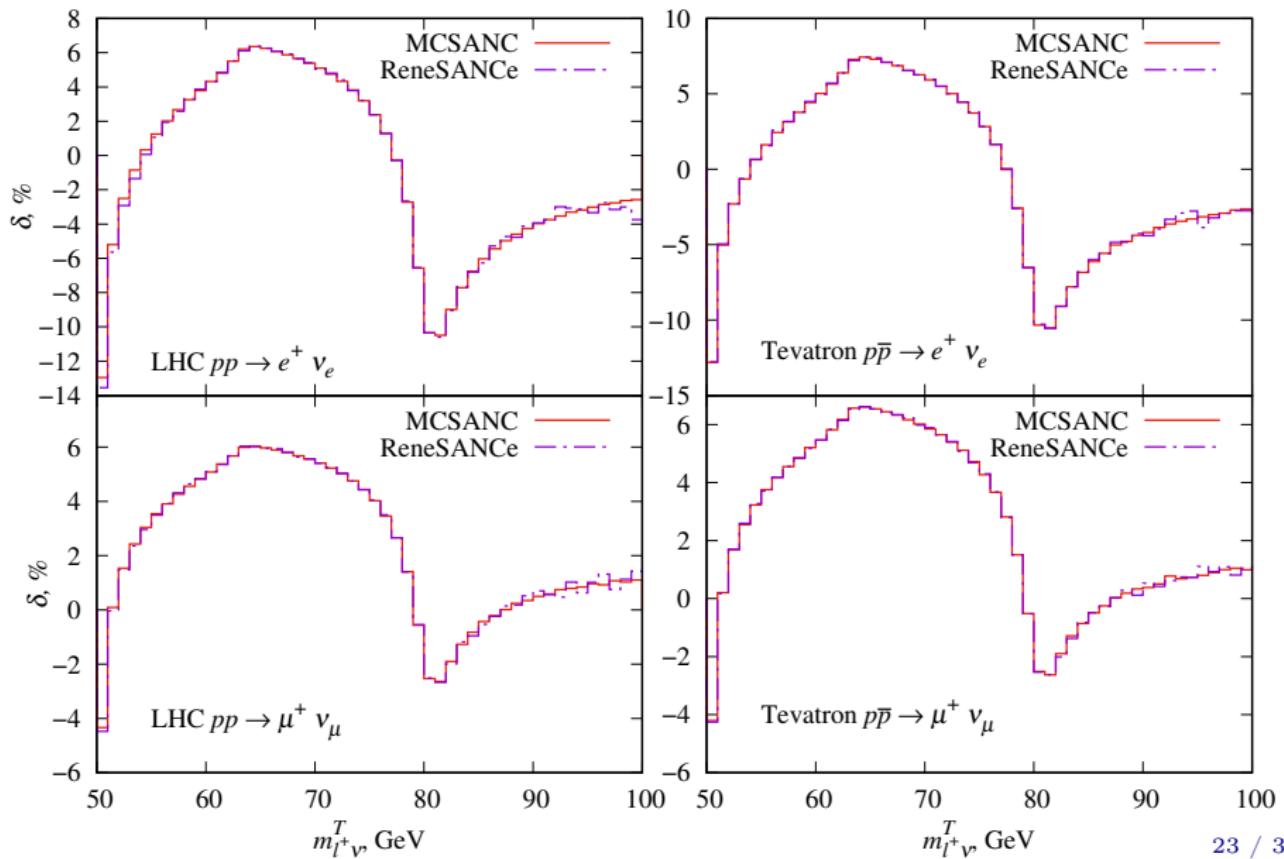
# hh mode



# hh mode

- The following processes are fully implemented in  $pp[p\bar{p}]$  mode:
  - $pp[p\bar{p}] \rightarrow Z, \gamma \rightarrow \ell^+ \ell^-$
  - $pp[p\bar{p}] \rightarrow W^- \rightarrow \ell^- \bar{\nu}_\ell$
  - $pp[p\bar{p}] \rightarrow W^+ \rightarrow \ell^+ \nu_\ell$
- Photon and gluon induced channels can be generated together
- Based on the **SANC** modules
- Complete one-loop and some higher-order electroweak radiative corrections
- Unweighted events in **ROOT** and **LHE** format
- Thoroughly cross checked against **MCSANC** integrator

# NLO EW: ReneSANCe vs MCSANC for CC DY



# Tuned comparisons of MCSANC with other codes

C. Buttar et al., ‘‘Les houches physics at TeV colliders 2005, standard model and Higgs working group: Summary report’’, in Physics at TeV colliders. Proceedings, Workshop, Les Houches, France, May 2-20, 2005, <http://www.arXiv.org/abs/hep-ph/0604120>.

C. E. Gerber et al., ‘‘Electroweak working group, tev4lhc-top’’, in Tevatron-for-LHC Report: Top and Electroweak Physics, 2007, <http://www.arXiv.org/abs/0705.3251>

S. Alioli et al., Eur. Phys. J. C77 (2017), no. 5 280,  
<http://www.arXiv.org/abs/1606.02330>.

## Use in data analysis

ReneSANCe was used in preparation of these notes:

- A precise measurement of the  $Z$ -boson double-differential transverse momentum and rapidity distributions in the full phase space of the decay leptons with the ATLAS experiment at  $\sqrt{s} = 8$  TeV [ATLAS-CONF-2023-013]
- A precise determination of the strong-coupling constant from the recoil of  $Z$  bosons with the ATLAS experiment at  $\sqrt{s} = 8$  TeV [ATLAS-CONF-2023-015]

# Polarized PDFs

<b>QUARKS</b>	<i>unpolarized</i>	<i>chiral</i>	<i>transverse</i>	<b>GLUONS</b>	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	$f_1$		$h_1^\perp$	U	$f_1^g$		$h_1^{\perp g}$
L		$g_{1L}$	$h_{1L}^\perp$	L		$g_{1L}^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^\perp$	$g_{1T}$	$h_{1T}, h_{1T}^\perp$	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{1T}^g, h_{1T}^{\perp g}$

- Collinear PDFs:  $f_1(x, Q^2)$  (Density),  $g_1 \equiv g_{1L}(x, Q^2)$  (Helicity),  $h_1(x, Q^2) \equiv h_{1T}(x, Q^2)$  (Transversity)
- TMD PDFs:  $f_{1T}^\perp(x, Q^2, k_T)$  (Sivers),  $g_{1T}^\perp(x, Q^2, k_T)$  (Worm-gear-T),  $h_{1L}^\perp(x, Q^2, k_T)$  (Worm-gear-L),  $h_1^\perp(x, Q^2, k_T)$  (Boer-Mulders),  $h_{1T}^\perp(x, Q^2, k_T)$  (Pretzelosity)

## Observables

We introduce the following combinations of fully polarized components of the hadron-hadron cross section  $\sigma^{++}, \sigma^{+-}, \sigma^{-+}, \sigma^{--}$ :

$$\sigma = \frac{1}{4} (\sigma^{++} + \sigma^{+-} + \sigma^{-+} + \sigma^{--}) = \sigma^{00},$$

$$\Delta\sigma_L = \frac{1}{4} (\sigma^{++} + \sigma^{+-} - \sigma^{-+} - \sigma^{--}) = \frac{1}{2} (\sigma^{+0} - \sigma^{-0}),$$

$$\Delta\sigma_{LL} = \frac{1}{4} (\sigma^{++} - \sigma^{+-} - \sigma^{-+} + \sigma^{--}).$$

*Single-spin* asymmetry is defined by

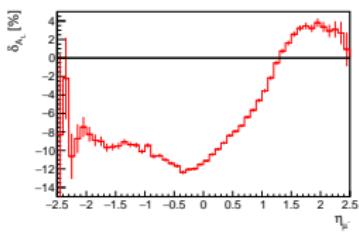
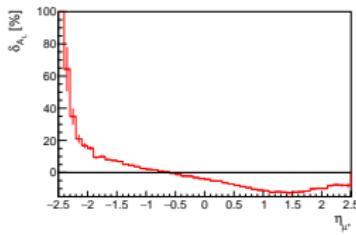
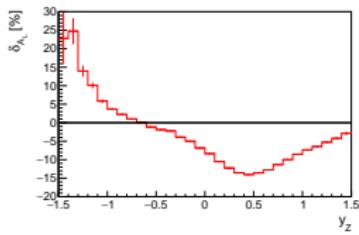
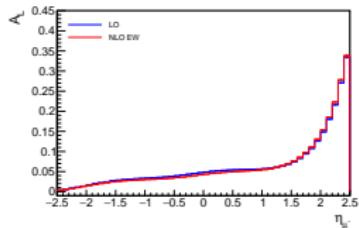
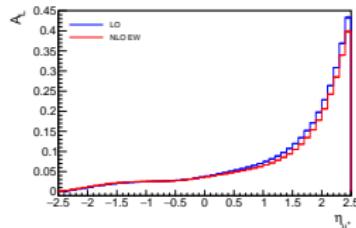
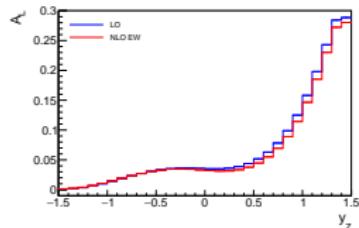
$$A_L(\mathcal{O}) = \frac{d\Delta\sigma_L/d\mathcal{O}}{d\sigma/d\mathcal{O}},$$

and *double-spin* asymmetry is defined by:

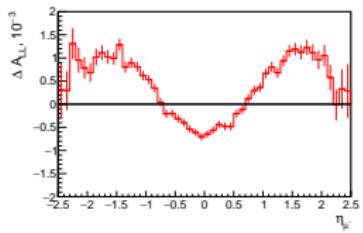
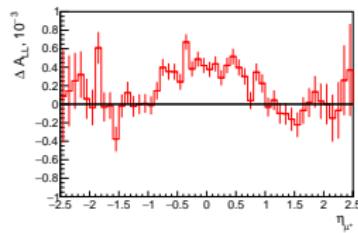
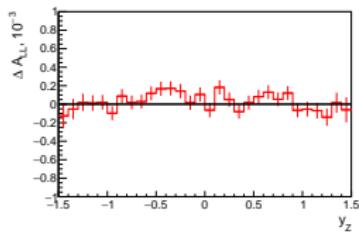
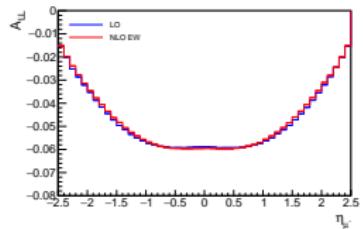
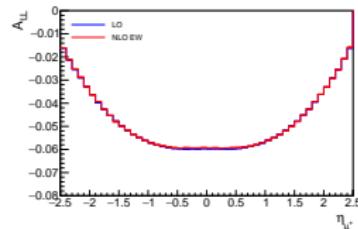
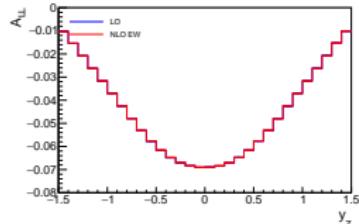
$$A_{LL}(\mathcal{O}) = \frac{d\Delta\sigma_{LL}/d\mathcal{O}}{d\sigma/d\mathcal{O}}$$

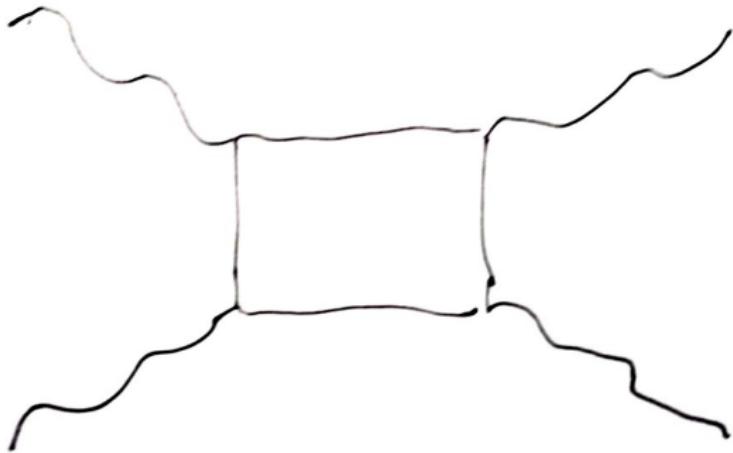
for observable  $\mathcal{O}$  (we show results for  $\mathcal{O} = y_{\mu^+\mu^-}, \eta_{\mu^+}, \eta_{\mu^-}$ ).

# Numerical results: $A_L$



# Numerical results: $A_{LL}$



$\gamma\gamma$  mode

# Processes of interest

- $\gamma\gamma \rightarrow \gamma\gamma$
- $\gamma\gamma \rightarrow Z\gamma$
- $\gamma\gamma \rightarrow ZZ$
- $\gamma\gamma \rightarrow ZH$
- $\gamma\gamma \rightarrow \nu\bar{\nu}$
- $\gamma\gamma \rightarrow l^-l^+$
- $\gamma\gamma \rightarrow W^-W^+$

First step to transversal polarization.

The SANC group is not going to support the codes Hector, polHector.  
In low priority some processes will be implemented in ReneSANCe.

# RESUME: SANC

- Monte Carlo tools of SANC provide:
  - Complete one-loop EW corrections
  - Initial & final state polarization support
  - Easy to investigate various asymmetries
  - LL-accuracy improvements to cross section
  - Higher order improvements throw  $\Delta\rho$
- ReneSANCe provide:
  - Events with unit weights
  - Output in Standard Les Houches Format
  - Simple installation & usage
- The research is supported by grant of the Russian Science Foundation (project No. 22-12-00021)

# Square of matrix element

$$|\mathcal{M}|^2 = L_{e-}^{\parallel} R_{e+}^{\parallel} |\mathcal{H}_{-+}|^2 + R_{e-}^{\parallel} L_{e+}^{\parallel} |\mathcal{H}_{+-}|^2 + L_{e-}^{\parallel} L_{e+}^{\parallel} |\mathcal{H}_{--}|^2 + R_{e-}^{\parallel} R_{e+}^{\parallel} |\mathcal{H}_{++}|^2$$

$$- \frac{1}{2} P_{e-}^{\perp} P_{e+}^{\perp} \operatorname{Re} \left[ e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}_{--}^* + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}_{-+}^* \right]$$

$$+ P_{e-}^{\perp} \operatorname{Re} \left[ e^{i\Phi_-} \left( L_{e+}^{\parallel} \mathcal{H}_{+-} \mathcal{H}_{--}^* + R_{e+}^{\parallel} \mathcal{H}_{++} \mathcal{H}_{-+}^* \right) \right]$$

$$- P_{e+}^{\perp} \operatorname{Re} \left[ e^{i\Phi_+} \left( L_{e-}^{\parallel} \mathcal{H}_{-+} \mathcal{H}_{--}^* + R_{e-}^{\parallel} \mathcal{H}_{++} \mathcal{H}_{+-}^* \right) \right],$$

где

$$L_{e\pm}^{\parallel} = \frac{1}{2}(1 - P_{e\pm}^{\parallel}), \quad R_{e\pm}^{\parallel} = \frac{1}{2}(1 + P_{e\pm}^{\parallel}), \quad \Phi_{\pm} = \phi_{\pm} - \phi,$$

$\mathcal{H}_{--}$ ,  $\mathcal{H}_{++}$ ,  $\mathcal{H}_{-+}$ ,  $\mathcal{H}_{+-}$  — helicity amplitudes.

Moortgat-Pick, G. et al. Phys.Rept. 460 (2008) 131-243