# Heavy-flavor-conserving nonleptonic weak decays of heavy baryons 

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## Charmed baryons

- SU(3) (u,d,s): M. Gell-Mann, G.Zweig (1964)
- SU(4) (u,d,s,c): J.D. Bjorken, S.L. Glashow (1964)
- GIM mechanism: S.L. Glashow, J. Iliopoulos, L. Maiani (1970)
- Problem: flavor changing neutral current at tree level

$$
\begin{gathered}
Q_{L}^{u}=\left(\begin{array}{c}
\boldsymbol{d} \cos \theta_{C}+\boldsymbol{s} \sin \theta_{C}
\end{array}\right) \\
\bar{Q}_{L}^{u} \gamma^{\mu} \frac{\tau^{3}}{2} \boldsymbol{Q}_{L}^{u}= \\
\\
\quad \frac{1}{2}\left(\bar{u}_{L} \gamma^{\mu} u_{L}-\cos ^{2} \theta_{C} \bar{d}_{L} \gamma^{\mu} \boldsymbol{d}_{L}-\sin ^{2} \theta_{C} \bar{s}_{L} \gamma^{\mu} s_{L}\right. \\
\\
\left.-\sin \theta_{C} \cos \theta_{C}\left[\bar{d}_{L} \gamma^{\mu} s_{L}+\bar{s}_{L} \gamma^{\mu} d_{L}\right]\right)
\end{gathered}
$$

- Solution: extra duplet with charm quark

$$
\begin{gathered}
Q_{L}^{c}=\binom{c}{-\boldsymbol{d} \sin \theta_{C}+\boldsymbol{s} \cos \theta_{C}} \\
\bar{Q}_{L}^{c} \gamma^{\mu} \frac{\tau^{3}}{2} Q_{L}^{c}= \\
\frac{1}{2}\left(\bar{c}_{L} \gamma^{\mu} c_{L}-\sin ^{2} \theta_{C} \bar{d}_{L} \gamma^{\mu} d_{L}-\cos ^{2} \theta_{C} \bar{s}_{L} \gamma^{\mu} s_{L}\right. \\
\left.+\sin \theta_{C} \cos \theta_{C}\left[\bar{d}_{L} \gamma^{\mu} s_{L}+\bar{s}_{L} \gamma^{\mu} d_{L}\right]\right) \\
\bar{Q}_{L}^{\mu} \gamma^{\mu} \frac{\tau^{3}}{2} Q_{L}^{u}+Q_{L}^{c} \gamma^{\mu} \frac{\tau^{3}}{2} Q_{L}^{c}=\frac{1}{2}\left(\bar{u}_{L} \gamma^{\mu} u_{L}+\bar{c}_{L} \gamma^{\mu} c_{L}-\bar{d}_{L} \gamma^{\mu} d_{L}-\bar{s}_{L} \gamma^{\mu} s_{L}\right)
\end{gathered}
$$

## Charmed baryons in one gluon exchange model

A. De Rújula, H. Georgi, S.L. Glashow, 1975.

- The model involves four types of fractionally charged quarks, each in three colors, coupling to massless gauge gluons.
- The short-range quark-quark interaction is taken to be Coulomb-like for the calculation of hadron masses due to the asymptotic freedom.
- The masses of charmed mesons and baryons have been predicted.

Numerous states with charm $\mathrm{C}=0$ and $\mathrm{C}=1$ were discovered.

There are precise results on the decays of single-charmed baryons

$$
\Lambda_{c}^{+} \rightarrow \boldsymbol{p} \phi, \quad \wedge \pi^{+}, \quad \Sigma^{+} \pi^{0} ; \quad \Xi_{c}^{+} \rightarrow p \bar{K}^{* 0}(892)
$$

Three weakly decaying baryons with $\mathrm{C}=2$ expected:

$$
\begin{aligned}
\Xi_{c c}^{++} & =c c u \text { and } \Xi_{c c}^{+}=c c d & & \text { isospin doublet } \\
\Omega_{c c}^{+} & =c c s & &
\end{aligned} \text { isospin singlet }
$$

The study of the heavy-flavor-conserving nonleptonic weak decays of heavy baryons has received a lot of attention due to their observation and measurement of branching fractions by the LHCb and Belle Collaborations.

The decay $\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}+\pi^{-}$was first observed at LHCb experiment and the branching fraction was measured to be $\mathcal{B}=(0.55 \pm 0.02 \pm 0.18) \%$

Recent experimental data obtained by the Belle collaboration gave the value of $\mathcal{B}=(0.54 \pm 0.05 \pm 0.12) \%$ which is in perfect agreement with the LHCb result.

Baryons containing both an $s$ quark and a heavy $c$ or $b$ quark, usually decay via the disintegration of the heavy quark, i.e. via the decay $s \rightarrow u(\bar{u} d)$. There is, however, the possibility of the weak scattering cs $\rightarrow \boldsymbol{d c}$.


Singly charmed $1 / 2^{+}$baryon states. Notation $[a, b]$ and $\{a, b\}$ for antisymmetric and symmetric flavor index combinations.

| Title | Content | $S U(3)$ | $\left(I, I_{3}\right)$ | Mass $(\mathrm{MeV})$ |
| :--- | :--- | :--- | :--- | :--- |
| $\Lambda_{c}^{+}$ | $c[u d]$ | $\overline{3}$ | $(0,0)$ | $2286.46 \pm 0.14$ |
| $\Xi_{c}^{+}$ | $c[u s]$ | $\overline{3}$ | $(1 / 2,1 / 2)$ | $2467.71 \pm 0.23$ |
| $\Xi_{c}^{0}$ | $c[d s]$ | $\overline{3}$ | $(1 / 2,-1 / 2)$ | $2470.44 \pm 0.28$ |
| $\Sigma_{c}^{++}$ | $c u u$ | 6 | $(1,1)$ | $2453.97 \pm 0.14$ |
| $\Sigma_{c}^{+}$ | $c\{u d\}$ | 6 | $(1,0)$ | $2452.65 \pm 0.22$ |
| $\Sigma_{c}^{0}$ | $c d d$ | 6 | $(1,-1)$ | $2453.75 \pm 0.14$ |
| $\Xi_{c}^{\prime+}$ | $c\{u s\}$ | 6 | $(1 / 2,1 / 2)$ | $2578.2 \pm 0.5$ |
| $\Xi_{c}^{\prime 0}$ | $c\{d s\}$ | 6 | $(1 / 2,-1 / 2)$ | $2578.7 \pm 0.5$ |
| $\Omega_{c}^{0}$ | $c s s$ | 6 | $(0,0)$ | $2695.2 \pm 1.7$ |

- Ground states of baryons with $J^{P}=1 / 2^{+}$can decay only weakly via the internal and external $W$-exchange.
- Two-body decays of baryons have five different quark topologies:


Ia


Ib

Tree diagrams


IIa


IIb


III

W-exchange diagrams

The quark diagrams that contribute to the Cabibbo-favored decay $\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}+\pi^{-}$are shown below.


Ia


IIb
$\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+} \pi^{-}$via $s \rightarrow u(d \bar{u})$ transitions.


IIa


III
$\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+} \pi^{-}$via $s c \rightarrow d c$ transitions.

After hadronizarion, the diagram la factorizes out into two parts: the weak transition $\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}$via the $W$-emission and the matrix element describing the pion leptonic decay.

The $W$-exchange diagrams IIa, IIb and III contribute into both the pure quark diagrams called the short distance (SD) contributions and effectively into the pole diagrams shown shown below. They describe the so-called long distance (LD) contributions. For instance, the diagrams Ila and III effectively generate the $\Sigma_{c}^{0}$-resonance diagram whereas the diagram IIb effectively generates the $\bar{\Xi}_{c}^{+}$and $\bar{\Xi}_{c}^{\prime+}$-resonance diagrams.


The effective Hamiltonian needed for calculations is written as

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}}^{\Delta \mathrm{S}=1}=\frac{G_{F}}{\sqrt{2}} & {\left[V_{u s}^{*} V_{u d}\left(C_{1}^{(u)}\left(\mu_{u}\right) Q_{1}^{(u)}+C_{2}^{(u)}\left(\mu_{u}\right) Q_{2}^{(u)}\right)\right.} \\
& \left.+V_{c s}^{*} V_{c d}\left(C_{1}^{(c)}\left(\mu_{c}\right) Q_{1}^{(c)}+C_{2}^{(c)}\left(\mu_{c}\right) Q_{2}^{(c)}\right)+\text { H.c. }\right]
\end{aligned}
$$

Here $Q_{1}$ and $Q_{2}$ is the set of effective four-quark operators given by

$$
\begin{array}{lll}
Q_{1}^{(u)}=\left(\bar{s}_{a} O_{L}^{\mu} u_{b}\right)\left(\bar{u}_{b} O_{\mu L} d_{a}\right), & & Q_{2}^{(u)}=\left(\bar{s}_{a} O_{L}^{\mu} u_{a}\right)\left(\bar{u}_{b} O_{\mu L} d_{b}\right), \\
Q_{1}^{(c)} & =\left(\bar{s}_{a} O_{L}^{\mu} c_{b}\right)\left(\bar{c}_{b} O_{\mu L} d_{a}\right), & Q_{2}^{(c)}=\left(\bar{s}_{a} O_{L}^{\mu} c_{a}\right)\left(\bar{c}_{b} O_{\mu L} d_{b}\right) .
\end{array}
$$

Here $O_{L}^{\mu}=\gamma^{\mu}\left(1-\gamma_{5}\right)$ is the left-handed chiral weak matrix. We adopt the numeration of the operators from the paper:
G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125-1144 (1996) where the $C_{2} Q_{2}$ means the leading order whereas the $C_{1} Q_{1}$ is for sub-leading order. The numerical values of the Wilson coefficients $C_{1}$ and $C_{2}$ are being equal to

$$
\begin{array}{ll}
C_{1}^{(u)}\left(\mu_{u}\right)=-0.625, & C_{2}^{(u)}\left(\mu_{u}\right)=1.361,
\end{array} \quad\left(\mu_{u}=O(1 \mathrm{GeV})\right), ~ 子-0.621, \quad \boldsymbol{C}_{2}^{(c)}\left(\mu_{c}\right)=1.336, \quad\left(\mu_{c}=\boldsymbol{O}\left(\boldsymbol{m}_{c}\right)\right) .
$$

We do not include penguin operators because their Wilson coefficients are small compare with those from current-current operators.

In the SM the relation $V_{c s}^{*} V_{c d}=-V_{u s}^{*} V_{u d}$ holds to an excellent approximation. For instance, in the Wolfenstein parametrization of the CKM-matrix, one has

$$
\begin{aligned}
& V_{u s}^{*} V_{u d}=+\lambda\left(1-\lambda^{2}\right)+O\left(\lambda^{4}\right) \\
& V_{c s}^{*} V_{c d}=-\lambda\left(1-\lambda^{2}\right)+O\left(\lambda^{4}\right)
\end{aligned}
$$

The global fit in the SM for the Wolfenstein parameter gives $\boldsymbol{\lambda}=0.22500 \pm 0.00067$. In what follows, we introduce the short notations

$$
V_{\mathrm{CKM}}^{(u)}=\left|V_{u s}^{*} V_{u d}\right|, \quad \text { and } \quad V_{\mathrm{CKM}}^{(c)}=-\left|V_{c s}^{*} V_{c d}\right|
$$

The numerical values of the CKM matrix elements are taken from PDG:

$$
\begin{array}{ll}
\left|\boldsymbol{V}_{u d}\right|=0.97373 \pm 0.00031, & \left|\boldsymbol{V}_{u s}\right|=0.2243 \pm 0.0008 \\
\left|\boldsymbol{V}_{c d}\right|=0.221 \pm 0.004, & \left|\boldsymbol{V}_{c s}\right|=0.975 \pm 0.006
\end{array}
$$

that approximately give $V_{\mathrm{CKM}}^{(u)} \approx 0.218$ and $V_{\mathrm{CKM}}^{(c)} \approx-0.215$.

## Covariant Constituent Quark Model

The CCQM is based on a phenomenological relativistic Lagrangian describing the coupling of a hadron $H$ to its constituents:

$$
\mathcal{L}_{\text {int }}=g_{H} \bar{H}(x) J_{H}(x)+\text { H.c. }
$$

The coupling constant $g_{H}$ is determined from the so-called compositeness condition, which was proposed by Salam and Weinberg.

The quark currents $J_{H}(x)$ have the nonlocal shapes as

$$
\begin{array}{rlrl}
J_{M}(x) & =\int \boldsymbol{d} x_{1} \int \boldsymbol{d} x_{2} F_{M}\left(x ; x_{1}, x_{2}\right) \cdot \overline{\boldsymbol{q}}_{f_{1}}^{a}\left(x_{1}\right) \Gamma_{M} \boldsymbol{q}_{f_{2}}^{a}\left(x_{2}\right) & \text { Meson } \\
J_{B}(x) & =\int \boldsymbol{d} x_{1} \int \boldsymbol{d} x_{2} \int \boldsymbol{d} x_{3} F_{B}\left(x ; x_{1}, x_{2}, x_{3}\right) & & \text { Baryon } \\
& \times \Gamma_{1} \boldsymbol{q}_{f_{1}}^{a_{1}}\left(x_{1}\right)\left[\varepsilon^{a_{1} a_{2} a_{3}} \boldsymbol{q}_{f_{2}}^{T a_{2}}\left(x_{2}\right) C \Gamma_{2} \boldsymbol{q}_{f_{3}}^{a_{3}}\left(x_{3}\right)\right] & \\
J_{T}(x) & =\int \boldsymbol{d} x_{1} \ldots \int \boldsymbol{d} x_{4} F_{T}\left(x ; x_{1}, \ldots, x_{4}\right) & \text { Tetraquark } \\
& \times\left[\varepsilon^{a_{1} a_{2} c} \boldsymbol{q}_{f_{1}}^{T a_{1}}\left(x_{1}\right) C \Gamma_{1} \boldsymbol{q}_{f_{2}}^{a_{2}}\left(x_{2}\right)\right] \cdot\left[\varepsilon^{a_{3} a_{4} c} \overline{\boldsymbol{q}}_{f_{3}}^{T a_{3}}\left(x_{3}\right) \Gamma_{2} C \overline{\boldsymbol{q}}_{f_{4}}^{a_{4}}\left(x_{4}\right)\right]
\end{array}
$$

## Vertex functions

Translational invariance of the vertex functions:

$$
F_{H}\left(x+a, x_{1}+a, \ldots, x_{n}+a\right)=F_{H}\left(x, x_{1}, \ldots, x_{n}\right), \quad \forall a .
$$

Our choice:

$$
F_{H}\left(x, x_{1}, \ldots, x_{n}\right)=\delta^{(4)}\left(x-\sum_{i=1}^{n} w_{i} x_{i}\right) \Phi_{H}\left(\sum_{i<j}\left(x_{i}-x_{j}\right)^{2}\right)
$$

where $w_{i}=m_{i} /\left(\sum_{j=1}^{n} m_{j}\right)$ and $m_{i}$ is the mass of the quark at point $x_{i}$.

The vertex function $\Phi_{H}$ is written as

$$
\begin{aligned}
& \Phi_{H}\left(\sum_{i<j}\left(x_{i}-x_{j}\right)^{2}\right)=\prod_{i=1}^{n-1}\left[\int \frac{d q_{i}}{(2 \pi)^{4}}\right] e^{-i \sum_{j=1}^{n-1} q_{j}\left(x_{j}-x_{n}\right)} \widetilde{\Phi}_{H}\left(-\vec{\Omega}_{q}^{2}\right), \\
& \widetilde{\Phi}_{H}\left(-\vec{\Omega}_{q}^{2}\right)=\exp \left(\vec{\Omega}_{q}^{2} / \Lambda_{B}^{2}\right), \quad \vec{\Omega}_{q}^{2}=\frac{1}{2} \sum_{i \leq j} q_{i} \boldsymbol{q}_{j}
\end{aligned}
$$

The matrix elements contributing to the baryon transitions $\bar{\Xi}_{c}^{0} \rightarrow \Lambda_{c}^{+} \pi^{-}$ are represented by a set of the quark diagrams shown in figure below.


They describe the so-called short distance contributions.

The diagrams describing the the building blocks of the long distance contributions look as following


Weak $B_{1}-B_{2}$ transition


Strong $B_{1} B_{2} M$ coupling

The quark propagators are chosen as ordinary Dirac propagators

$$
S_{q}\left(x_{1}-x_{2}\right)=\int \frac{d^{4} k}{(2 \pi)^{4} \boldsymbol{i}} \frac{e^{-i k\left(x_{1}-x_{2}\right)}}{m_{\boldsymbol{q}}-\not k}
$$

We use the Fock-Schwinger representation for them in evaluation of the Feynman integrals.

$$
\frac{\boldsymbol{m}+k \boldsymbol{k}}{\boldsymbol{m}^{2}-\boldsymbol{k}^{2}}=(\boldsymbol{m}+k) \int_{0}^{\infty} d \alpha \exp \left[-\alpha\left(\boldsymbol{m}^{2}-\boldsymbol{k}^{2}\right)\right]
$$

## Evaluation of the diagrams

Then one can arrive at the following representation for the Feynman diagram with n-propagators.

$$
\Pi=\int_{0}^{\infty} d^{n} \alpha F\left(\alpha_{1}, \ldots, \alpha_{n}\right)
$$

where $F$ stands for the whole structure of a given diagram. The set of Schwinger parameters $\alpha_{i}$ can be turned into a simplex by introducing an additional $t$-integration via the identity

$$
1=\int_{0}^{\infty} d t \delta\left(t-\sum_{i=1}^{n} \alpha_{i}\right)
$$

leading to

$$
\Pi=\int_{0}^{\infty} d t t^{n-1} \int_{0}^{1} d^{n} \alpha \delta\left(1-\sum_{i=1}^{n} \alpha_{i}\right) F\left(t \alpha_{1}, \ldots, t \alpha_{n}\right)
$$

## Infrared confinement

- Cut off the upper integration at $1 / \lambda^{2}$

$$
\Pi^{c}=\int_{0}^{1 / \lambda^{2}} d t t^{n-1} \int_{0}^{1} d^{n} \alpha \delta\left(1-\sum_{i=1}^{n} \alpha_{i}\right) F\left(t \alpha_{1}, \ldots, t \alpha_{n}\right)
$$

- The infrared cut-off has removed all possible thresholds in the quark loop diagram.
- We take the cut-off parameter $\lambda$ to be the same in all physical processes.


## Matrix elements of the SD-contributions

$$
\begin{aligned}
& M_{\mathrm{SD}}=\frac{\boldsymbol{G}_{\mathrm{F}}}{\sqrt{2}}\left\{\boldsymbol{V}_{\mathrm{CKM}}^{(u)} \overline{\boldsymbol{u}}\left(\boldsymbol{p}_{2}\right)\left[\left(\boldsymbol{C}_{2}^{(u)}+\boldsymbol{\xi} \boldsymbol{C}_{1}^{(u)}\right) D_{\mathrm{Ia}}+\left(\boldsymbol{C}_{2}^{(u)}-\boldsymbol{C}_{1}^{(u)}\right) D_{\mathrm{IIb}}\right] \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)\right. \\
&\left.+\boldsymbol{V}_{\mathrm{CKM}}^{(c)}\left(\boldsymbol{C}_{2}^{(c)}-\boldsymbol{C}_{1}^{(c)}\right) \overline{\boldsymbol{u}}\left(\boldsymbol{p}_{2}\right)\left[D_{\mathrm{IIa}}+D_{\mathrm{III}}\right] \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)\right\} .
\end{aligned}
$$

The tree diagram factorizes into two pieces according to

$$
\begin{aligned}
D_{\mathrm{Ia}}= & \boldsymbol{N}_{c} g_{M} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}}{(2 \boldsymbol{\pi})^{4} \boldsymbol{i}} \widetilde{\Phi}_{M}\left(-\boldsymbol{k}^{2}\right) \operatorname{tr}\left[\boldsymbol{O}_{L}^{\mu} S_{u}\left(\boldsymbol{k}-\boldsymbol{w}_{u} \boldsymbol{q}\right) \gamma_{5} \boldsymbol{S}_{\boldsymbol{d}}\left(\boldsymbol{k}+\boldsymbol{w}_{\boldsymbol{d}} \boldsymbol{q}\right)\right] \\
\times & 6 \boldsymbol{g}_{B_{1}} g_{B_{2}} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}_{1}}{(2 \pi)^{4} \boldsymbol{i}} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}_{2}}{(2 \pi)^{4} \boldsymbol{i}} \widetilde{\Phi}_{B_{1}}\left(-\vec{\Omega}_{\boldsymbol{q}}^{2}\right) \widetilde{\Phi}_{B_{2}}\left(-\vec{\Omega}_{r}^{2}\right) \\
& \times S_{c}\left(\boldsymbol{k}_{2}\right) \operatorname{tr}\left[\boldsymbol{S}_{u}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{2}\right) \boldsymbol{O}_{\mu L} S_{s}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{1}\right) \gamma_{5} \boldsymbol{S}_{\boldsymbol{d}}\left(\boldsymbol{k}_{1}+\boldsymbol{k}_{2}\right) \gamma_{5}\right] \\
= & -6 \boldsymbol{f}_{M} \boldsymbol{q}^{\mu} \boldsymbol{g}_{B_{1}} g_{B_{2}} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}_{1}}{(2 \boldsymbol{\pi})^{4} \boldsymbol{i}} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}_{2}}{(2 \boldsymbol{\pi})^{4} \boldsymbol{i}} \widetilde{\Phi}_{B_{1}}\left(-\vec{\Omega}_{\boldsymbol{q}}^{2}\right) \widetilde{\Phi}_{B_{2}}\left(-\vec{\Omega}_{r}^{2}\right) \\
& \times S_{c}\left(\boldsymbol{k}_{2}\right) \operatorname{tr}\left[\boldsymbol{S}_{u}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{2}\right) \boldsymbol{O}_{\mu L} S_{s}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{1}\right) \gamma_{5} \boldsymbol{S}_{\boldsymbol{d}}\left(\boldsymbol{k}_{1}+\boldsymbol{k}_{2}\right) \gamma_{5}\right]
\end{aligned}
$$

## Matrix elements of the SD-contributions, cont.

The calculation of the three-loop $W$-exchange diagrams is much more involved because the matrix element does not factorize. One has

$$
\begin{aligned}
D_{\mathrm{IIb}} & =12 \boldsymbol{g}_{B_{1}} \boldsymbol{g}_{B_{2}} \boldsymbol{g}_{M}\left[\prod_{\boldsymbol{i}=1}^{3} \int \frac{\boldsymbol{d}^{4} \boldsymbol{k}_{\boldsymbol{i}}}{(2 \boldsymbol{\pi})^{4} \boldsymbol{i}}\right] \widetilde{\Phi}_{B_{1}}\left(-\vec{\Omega}_{\boldsymbol{q}}^{2}\right) \widetilde{\Phi}_{B_{2}}\left(-\vec{\Omega}_{r}^{2}\right) \widetilde{\Phi}_{M}\left(-\boldsymbol{P}^{2}\right) \\
& \times S_{\boldsymbol{c}}\left(\boldsymbol{k}_{3}\right) \operatorname{tr}\left[\gamma_{5} \boldsymbol{S}_{\boldsymbol{d}}\left(\boldsymbol{k}_{2}+\boldsymbol{p}_{2}\right)\left(1+\gamma_{5}\right) \boldsymbol{S}_{u}\left(\boldsymbol{k}_{2}+\boldsymbol{k}_{3}\right)\right] \\
& \times \operatorname{tr}\left[\boldsymbol{S}_{u}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{2}\right) \gamma_{5} \boldsymbol{S}_{\boldsymbol{d}}\left(\boldsymbol{k}_{1}+\boldsymbol{p}_{1}\right) \gamma_{5} \boldsymbol{S}_{\boldsymbol{s}}\left(\boldsymbol{k}_{1}+\boldsymbol{k}_{3}\right)\left(1-\gamma_{5}\right)\right]
\end{aligned}
$$

Finally, the matrix element describing the SD-contributions are written as

$$
M_{\mathrm{SD}}\left(\bar{\Xi}_{c}^{0} \rightarrow \Lambda_{c}^{+} \pi^{-}\right)=\frac{\boldsymbol{G}_{F}}{\sqrt{2}} \overline{\boldsymbol{u}}\left(\boldsymbol{p}_{2}\right)\left(\boldsymbol{A}_{\mathrm{SD}}+\gamma_{5} \boldsymbol{B}_{\mathrm{SD}}\right) \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)
$$

## Matrix elements of the LD-contributions

$$
M_{\Sigma_{c}^{0}}=\frac{G_{F}}{\sqrt{2}} V_{\mathrm{CKM}}^{(c)}\left(\boldsymbol{C}_{1}^{(c)}-C_{2}^{(c)}\right) \overline{\boldsymbol{u}}\left(\boldsymbol{p}_{2}\right) D_{\Sigma_{c}^{0} \wedge_{c}^{+} \pi^{-}}\left(\boldsymbol{p}_{1}, \boldsymbol{p}_{2}\right) S_{\Sigma_{c}^{0}}\left(\boldsymbol{p}_{1}\right) D_{\bar{\Xi}_{c}^{0} \Sigma_{c}^{0}}\left(\boldsymbol{p}_{1}\right) \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)
$$

where $S_{\Sigma_{c}^{0}}\left(\boldsymbol{p}_{1}\right)=1 /\left(\boldsymbol{m}_{\Sigma_{c}^{0}}-\boldsymbol{p}_{1}\right)$.

$$
\begin{aligned}
M_{\bar{\Xi}_{c}^{\prime}+} & =\frac{G_{F}}{\sqrt{2}} \bar{u}\left(\boldsymbol{p}_{2}\right)\left\{\left[\boldsymbol{V}_{\mathrm{CKM}}^{(u)}\left(C_{1}^{(u)}-C_{2}^{(u)}\right) D_{\bar{E}_{c}^{\prime}+\Lambda_{c}^{+}}^{(u)}\left(\boldsymbol{p}_{2}\right)\right.\right. \\
& \left.+\boldsymbol{V}_{\mathrm{CKM}}^{(c)}\left(\boldsymbol{C}_{1}^{(c)}-C_{2}^{(c)}\right) D_{\Xi_{c}^{(c)} \Lambda_{c}^{+}}^{(c)}\left(\boldsymbol{p}_{2}\right)\right] \times S_{\left.\Xi_{c}^{\prime}+\left(\boldsymbol{p}_{2}\right) D_{\Xi_{c}^{0} \Xi_{c}^{\prime+} \pi^{-}}\left(\boldsymbol{p}_{1}, \boldsymbol{p}_{2}\right)\right\} \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)}
\end{aligned}
$$

where $S_{\bar{E}_{c}^{\prime}+}\left(\boldsymbol{p}_{1}\right)=1 /\left(\boldsymbol{m}_{\bar{\Xi}_{c}^{\prime}}+\boldsymbol{p}_{1}\right)$.

## Matrix elements of the LD-contributions

It appears that the strong transition $\Xi_{c}^{0} \rightarrow \Xi_{c}^{+}+\pi^{-}$is identically equal to zero due to the chosen form of the interpolating quark current $\epsilon^{a b c} c^{a}\left(u^{b} C \gamma_{5} s^{c}\right)$.
As a result, this transition is described by the diagram which contains the trace of a string with three quark propagators and three $\gamma_{5}$-matrices that gives zero contribution. Explicitly we one has

$$
\begin{aligned}
D_{\Xi_{c}^{0} \Xi_{c}^{+} \pi^{-}} & =6 g_{\Xi_{c}^{0}} g_{\Xi_{c}^{+}} g_{\pi^{-}}\left[\prod_{i=1}^{2} \int \frac{d^{4} k_{i}}{(2 \pi)^{4} i}\right] \widetilde{\Phi}_{\Xi_{c}^{0}}\left(-\vec{\Omega}_{q}^{2}\right) \widetilde{\Phi}_{\Xi_{c}^{+}}\left(-\vec{\Omega}_{r}^{2}\right) \widetilde{\Phi}_{\pi^{-}}\left(-P^{2}\right) \\
& \times S_{c}\left(k_{2}\right) \operatorname{tr}\left[S_{u}\left(k_{1}+p_{2}\right) \gamma_{5} S_{d}\left(k_{1}+p_{1}\right) \gamma_{5} S_{s}\left(k_{1}+k_{2}\right) \gamma_{5}\right] \equiv 0 .
\end{aligned}
$$

There are two kinds of the interpolating currents for the $\Lambda$-type baryons $\left(\Lambda_{Q}, \Xi_{Q}\right)$ where $Q=b, c$. They are written as

$$
\begin{array}{ll}
J_{S}=\epsilon^{a b c} Q^{a}\left(u^{b} C \gamma_{5} s^{c}\right), & \text { scalar diquark } \\
J_{V}=\epsilon^{a b c} \gamma_{\alpha} Q^{a}\left(u^{b} C \gamma^{\alpha} \gamma_{5} s^{c}\right) & \text { vector diquark. }
\end{array}
$$

The interpolating current with the vector diquark could provide the nonvanishing transition $\bar{\Xi}_{c}^{0} \rightarrow \bar{E}_{c}^{+}+\pi^{-}$.

## Full amplitude

It is widely accepted that S-wave amplitude is saturated by the $1 / 2^{-}$ resonances. Ordinarily, their contributions are calculated by using the well-known soft-pion theorem in the current-algebra approach. It allows one to express the parity-violating S -wave amplitude in terms of parity-conserving matrix elements. In our case, one has

$$
\boldsymbol{A}_{1 / 2-}\left(\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}+\boldsymbol{\pi}^{-}\right)=\frac{1}{\boldsymbol{f}_{\pi}} \boldsymbol{A}_{\Xi_{c}^{+} \Lambda_{c}^{+}},
$$

where

$$
\boldsymbol{A}_{\Xi_{c}^{+} \wedge_{c}^{+}}=V_{\mathrm{CKM}}^{(u)}\left(\boldsymbol{C}_{2}^{(u)}-C_{1}^{(u)}\right) \boldsymbol{a}_{\Xi_{c}^{+} \Lambda_{c}^{+}}^{(u)}+V_{\mathrm{CKM}}^{(c)}\left(C_{2}^{(c)}-C_{1}^{(c)}\right) \boldsymbol{a}_{\Xi_{c}^{+} \Lambda_{c}^{+}}^{(c)}
$$

Finally, the transition $\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}+\pi^{-}$amplitude is written in terms of invariant amplitudes as

$$
\left\langle\Lambda_{c}^{+} \pi^{-}\right| \mathcal{H}_{\mathrm{eff}}\left|\overline{\bar{E}}_{c}^{0}\right\rangle=\frac{\boldsymbol{G}_{F}}{\sqrt{2}} \bar{u}\left(\boldsymbol{p}_{2}\right)\left(\boldsymbol{A}+\gamma_{5} \boldsymbol{B}\right) \boldsymbol{u}\left(\boldsymbol{p}_{1}\right)
$$

where $A$ and $B$ are given by

$$
\begin{array}{ll}
A=A_{\mathrm{SD}}+A_{\mathrm{LD}}, & A_{\mathrm{LD}}=A_{\Sigma_{c}^{0}}+A_{\Xi_{c}^{\prime}+}+A_{1 / 2-}, \\
B=B_{\mathrm{SD}}+B_{\mathrm{LD}}, & B_{\mathrm{LD}}=B_{\Sigma_{c}^{0}}+B_{\Xi_{c}^{\prime}+}
\end{array}
$$

## Decay width

It is more convenient to use helicity amplitudes $H_{\lambda_{1} \lambda_{M}}$ instead of invariant amplitudes $A$ and $B$

$$
H_{\frac{1}{2} t}^{v}=\sqrt{Q_{+}} \boldsymbol{A}, \quad \boldsymbol{H}_{\frac{1}{2} t}^{A}=\sqrt{Q_{-}} \boldsymbol{B}
$$

where $\boldsymbol{m}_{ \pm}=\boldsymbol{m}_{1} \pm \boldsymbol{m}_{2}, Q_{ \pm}=\boldsymbol{m}_{ \pm}^{2}-\boldsymbol{q}^{2}$.

The two-body decay width reads

$$
\Gamma\left(\boldsymbol{B}_{1} \rightarrow \boldsymbol{B}_{2}+\boldsymbol{M}\right)=\frac{\boldsymbol{G}_{F}^{2}}{32 \pi} \frac{\left|\boldsymbol{p}_{2}\right|}{\boldsymbol{m}_{1}^{2}} \mathcal{H}_{s}, \quad \mathcal{H}_{s}=2\left(\left|\boldsymbol{H}_{\frac{1}{2} t}^{v}\right|^{2}+\left|\boldsymbol{H}_{\frac{1}{2} t}^{A}\right|^{2}\right)
$$

where $\left|p_{2}\right|=\lambda^{1 / 2}\left(m_{1}^{2}, m_{2}^{2}, q^{2}\right) /\left(2 m_{1}\right)$.
Also it would be instructive to evaluate the asymmetry parameter defined by

$$
\alpha=\frac{\left|\boldsymbol{H}_{1 / 2 \boldsymbol{t}}\right|^{2}-\left|\boldsymbol{H}_{-1 / 2 \boldsymbol{t}}\right|^{2}}{\left|\boldsymbol{H}_{1 / 2 \boldsymbol{t}}\right|^{2}+\left|\boldsymbol{H}_{-1 / 2 \boldsymbol{t}}\right|^{2}}=-\frac{2 \kappa \boldsymbol{A} B}{\boldsymbol{A}^{2}+\kappa^{2} B^{2}}
$$

where $\boldsymbol{\kappa}=\left|\boldsymbol{p}_{2}\right| /\left(E_{2}+\boldsymbol{m}_{2}\right)$ and $E_{2}=\left(\boldsymbol{m}_{1}^{2}+\boldsymbol{m}_{2}^{2}-\boldsymbol{q}^{2}\right) /\left(2 \boldsymbol{m}_{1}\right)$.

## Numerical results

Model parameters have been determined by a global fit to a multitude of decay processes.

| $\boldsymbol{m}_{u / \boldsymbol{d}}$ | $\boldsymbol{m}_{\boldsymbol{s}}$ | $\boldsymbol{m}_{\boldsymbol{c}}$ | $\boldsymbol{\lambda}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 0.241 | 0.428 | 2.16 | 0.181 | GeV |

The size parameters of light meson were fixed by fitting the data on the leptonic decay constant.

| Meson | $\Lambda_{M}(\mathrm{GeV})$ | $f_{M}(\mathrm{MeV})$ | $\boldsymbol{f}_{M}^{\text {expt }}(\mathrm{MeV})$ |
| ---: | ---: | ---: | :---: |
| Pion | 0.871 | 130.3 | $130.41 \pm 0.20$ |

Since the experimental data of the single charm baryon decays become to appear recently, we will assume for the time being that the size parameters of all single charm baryons are the same.

## Numerical results

Dependence of the branching fractions on the size parameter of a single charm baryon.


One can see that the measured branching fraction can be accommodated in the framework of this work by having $\Lambda_{c} \approx 0.61 \mathbf{G e V}$.

## Numerical results

In order to estimate the uncertainty caused by the choice of the size parameter we allow the size parameter to vary from $\Lambda_{\min }=0.54$ to $\Lambda_{\text {max }}=0.66 \mathrm{GeV}$ that correspond to the intersections of the theoretical curve for branching fraction with the experimantal lower and upper error bars. We evaluate the mean and the mean square deviation

$$
\bar{\Gamma}=\sum_{i=1}^{N} \Gamma_{i} / N \quad \sigma^{2}=\sum_{i=1}^{N}\left(\Gamma_{i}-\bar{\Gamma}\right)^{2} / N
$$

Finally, our result for the branching fraction reads as

$$
\mathcal{B}\left(\bar{\Xi}_{c}^{0} \rightarrow \Lambda_{c}^{+}+\pi^{-}\right)=(0.54 \pm 0.11) \%
$$

which should be compared with the data from LHCb and Belle:

$$
\mathcal{B}_{\mathrm{LHCb}}=(0.55 \pm 0.02 \pm 0.18) \% \quad \mathcal{B}_{\text {Belle }}=(0.54 \pm 0.05 \pm 0.12) \%
$$

| Amplitudes | SD | LD | SD+LD |
| :---: | :---: | :---: | :---: |
| A-ampl. $\left(\mathrm{GeV}^{2}\right)$ | 0.0156 | -0.0751 | -0.0595 |
| B-ampl. $\left(\mathrm{GeV}^{2}\right)$ | 0.166 | -5.378 | -5.212 |

## Asymmetry parameter

It is instructive to evaluate the asymmetry parameter defined by

$$
\alpha=\frac{\left|\boldsymbol{H}_{1 / 2 \boldsymbol{t}}\right|^{2}-\left|\boldsymbol{H}_{-1 / 2 \boldsymbol{t}}\right|^{2}}{\left|\boldsymbol{H}_{1 / 2 \boldsymbol{t}}\right|^{2}+\left|\boldsymbol{H}_{-1 / 2 \boldsymbol{t}}\right|^{2}}=-\frac{2 \kappa A B}{\boldsymbol{A}^{2}+\kappa^{2} B^{2}}
$$

Here $\boldsymbol{\kappa}=\left|\boldsymbol{p}_{2}\right| /\left(E_{2}+\boldsymbol{m}_{2}\right)$ and $E_{2}=\left(\boldsymbol{m}_{1}^{2}+\boldsymbol{m}_{2}^{2}-\boldsymbol{q}^{2}\right) /\left(2 \boldsymbol{m}_{1}\right)$.

The numerical value of the asymmetry parameter is found to be equal to

$$
\alpha=-0.75 .
$$

## Numerical results

## Comparison with other approaches.

| Approach | $\mathcal{B}\left(\bar{\Xi}_{c}^{0} \rightarrow \Lambda_{c} \pi^{-}\right) \%$ | Asymmetry |
| :---: | :---: | :---: |
| LHCb [1] | $0.55 \pm 0.02 \pm 0.1$ | - |
| Belle [2] | $0.54 \pm 0.05 \pm 0.12$ | - |
| Voloshin [3] | $>0.025 \pm 0.015$ | - |
| Gronau (constr) [4] | $0.194 \pm 0.070$ | - |
| Gronau (destr) [4] | $<0.01$ | - |
| Faller [5] | $<0.39$ | - |
| Cheng [6] | $0.72 \pm 0.07$ | $0.46 \pm 0.05$ |
| Niu [7] | $0.58 \pm 0.21$ | -0.16 |
| Our model | $0.54 \pm 0.11$ | -0.75 |

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## Summary

- We have studied two-body nonleptonic $\Delta C=0$ decay $\Xi_{c}^{0} \rightarrow \Lambda_{c}^{+}+\pi^{-}$ in the framework of the covariant confined quark model with account for both short and long distance effects.
- The short distance effects are induced by four topologies of external and internal weak $W$-interactions, while long distance effects are saturated by an inclusion of the so-called pole diagrams.
- Pole diagrams are generated by resonance contributions of the low-lying spin $1 / 2^{+}\left(\Sigma_{c}^{0}\right.$ and $\Xi_{c}^{\prime+}$ ) and spin $1 / 2^{-}$baryons. The last contributions are calculated by using the well-known soft-pion theorem.
- It is found that the contribution of the SD diagrams is significantly suppressed, by more than one order of magnitude in comparison with data. The most significant contributions are coming from the intermediate $1 / 2^{+}$and $1 / 2^{-}$resonances.
- We can get consistency with the experimental data for the value of size parameter being equal to $\Lambda \approx 0.60 \mathrm{GeV}$.

